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INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108

MID-SEMESTRAL EXAMINATION, 2011-2012
M.S.(Q.E.) 1st Year

Statistics

Date: 02.09.11 Maximum Marks: 100 Time: 3 hours

Answer all questions. Marks allotted to each question are given within parentheses.

1. Examine if the following statements are *true* or *false* or *uncertain*. Give brief explanations in support of your answers.

   (i) If $\{A_n\}$ is a sequence of sets, then there always exists a sequence of disjoint set $\{D_n\}$ such that $\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} D_n$.

   (ii) For a sequence of random variables $\{X_n\}$,

   $$|X_m - k| > \varepsilon \Leftrightarrow \sup_{n \geq m} |X_n - k| > \varepsilon \text{ for any } \varepsilon > 0$$

   where k is a constant.

   (iii) Let X_1 and X_2 be two independent random variables each following a negative binomial distribution. Then a new random variable, y, defined as $y = \frac{1}{2}(X_1 + X_2)$ also follows a negative binomial distribution.

   (iv) Suppose a quadratic form $Q = y'By$ is negative semidefinite where B is a symmetric matrix of order m and y is a $(m \ x \ 1)$ vector. Then Q would necessarily remain negative semidefinite under any nonsingular transformation.

   $[4 \times 5 = 20]$

2. (i) Show that for any m events $A_1, A_2, \ldots A_m$,

   $$P\left(\bigcap_{i=1}^{m} A_i\right) \geq \sum_{i=1}^{m} P(A_i) - (m-1).$$

   (ii) Suppose an urn contains M balls of which X are black and $(M-X)$ are white. A sample of size n is drawn with replacement from the urn. Find the probability that the jth ball drawn is black given that the sample

contains exactly x black balls. Would this probability change if the balls are drawn without replacement? Give explanations/ intuitions to your answer.

[9 + 13 = 22]

3. (i) Suppose that for some random variable X, the second moment exists. Then show that $\mu_2^1 \geq \mu_2$ where μ_2^1 and μ_2 are the second raw and central moments of X, respectively.

   (ii) Prove that if μ_s^1 exists, then μ_r^1 necessarily exists for $r < s$, where μ_s^1 stands for the s^{th} raw moment of a continuous random variable.

[4+8 = 12]

4. (i) For a nonnegative continuous random variable X, show that $L = G/2\mu$ where μ is the expectation of X, L is the Lorenz ratio and G the Gini mean difference.

   (ii) Find the coefficients of skewness and kurtosis of a Gamma distribution, and then find under what condition(s) on the parameter(s) the distribution tends to be symmetrical and leptokurtic. Find also the moment generating function of a Gamma distribution.

[9+15 = 24]

5. (i) Suppose $X_1, X_2, \ldots, X_n$ are independently and identically distributed continuous random variables with common *p.d.f.* $f_x(\cdot)$ and *c.d.f.* $F_X(\cdot)$. Further, let $y_n = \max[X_1, X_2, \ldots, X_n]$. Then find the *p.d.f.* of y_n.

   (ii) Let X_1 and X_2 be independent binomially distributed random variables with parameters $(n_1, 1/2)$ and $(n_2, 1/2)$, respectively. Find the distribution of a new random variable y defined as $y = X_1 - X_2 + n_2$. Is the distribution a standard one? Justify.

[9+13 = 22]

Indian Statistical Institute

Mid-Semester Examination: 2011-2012

MS(QE) I/ M/Stat.II: 2011-2012

Game Theory I

Date: 05·09·11 Maximum Marks: 40 Duration: 3 Hours

Answer any **FOUR** Questions

1. (a) Consider the following payoff matrix of player 1 (row player) and player 2 (column player).

	X	Y
X	3, 2	1, 1
Y	4, 3	2, 4

Now suppose that the players choose their actions sequentially. Player 1 moves first, but her choice is not perfectly observed by player 2 when it is his turn to move. If player 1 chooses X, player 2 observes it to be X with probability $1-\varepsilon$ and Y with probability ε; $0<\varepsilon<1$. Similarly, if player 1 chooses Y, player 2 observes it to be Y with probability $1-\varepsilon$ and X with probability ε. (i) Represent this in an extensive form game. (ii) Given this extensive form game, derive the corresponding strategic form game. [2+4]

(b) Prove that the set of pure strategy Nash equilibria of a strategic game G is a subset of the mixed strategy Nash equilibria of the mixed extension game G'. [4]

2 Suppose each of two players announces a non-negative integer equal to at most 100. If $x_1+x_2 \leq 100$ where x_i is the number announced by player i, then each player receives payoff of x_i. If $x_1+x_2>100$ and $x_i<x_j$, then player i gets x_i and player j gets $100-x_i$. If $x_1+x_2>100$ and $x_i=x_j$, then each player receives 50. Construct the payoff matrix and show that the game can be solved by iterated elimination of the (weakly) dominated strategies of the players. [4+6]

3 There are two players, 1 and 2, competing in prices in a homogeneous good market; prices can be only integer quantities. The market demand for the product is $D(p)=\max[0,20-p]$, where p is the price of the product. Consumers always

contains exactly x black balls. Would this probability change if the balls are drawn without replacement? Give explanations/ intuitions to your answer.

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Mid-Semester Examination: 2011-2012
MS(QE) I/ M/Stat.II: 2011-2012
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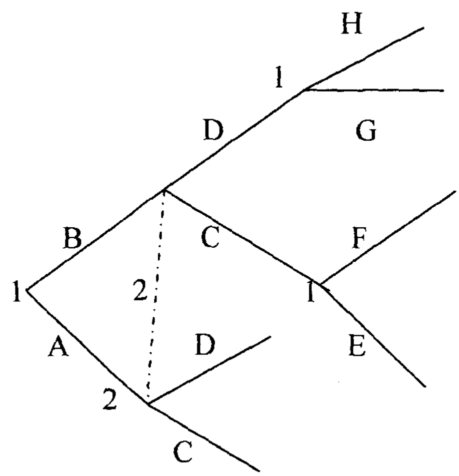
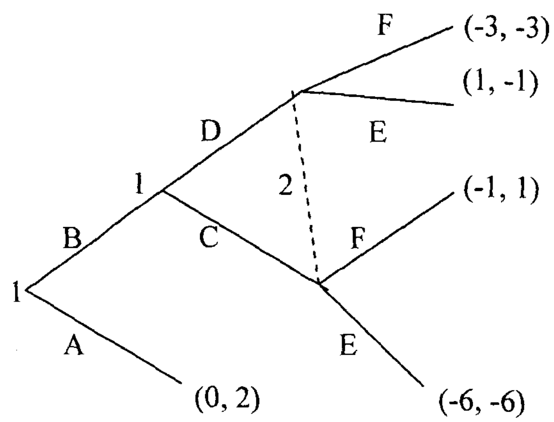
P.T.O

buy from the low-price seller. In case the firms charge the same price, each firm gets one-half of the customers. The unit costs of firm 1 and firm 2 are 3 and 5 respectively. Find the equilibrium prices to be charged by the firms. [10]

4 Consider the following extensive form game. (i) Find the strategy set of each player. (ii) Assume that mixed strategies of the players are given. Find the corresponding equivalent behavior strategies of the players such that the probability distributions over the terminal nodes are the same. (iii) Let $u_1(t)$ be the payoff of player 1 at the terminal node t. Find his expected payoff, given the mixed strategies of the players. [1+6+3]



5 Find all Nash and subgame perfect Nash equilibria (in pure strategy) of the following extensive form game. [4+6]



INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: (2011-2012)

MS(QE) I & MSTAT II

Microeconomic Theory I

Date: 07.09.2011 **Maximum Marks:** 100 **Duration:** $3\frac{1}{2}$ hrs.

Note: Answer all questions.

(1) Show that if R on X is rational, then we have the following:

(a) P is both irreflexive and transitive.

(b) I is reflexive, transitive and symmetric.

(c) If $xPyRz$, then xPz. Similarly, if $xRyPz$, then xPz.

(4+6+4=14)

(2) Consider a rational preference relation R on X. Show that if $u(x) = u(y)$ implies that xIy and $u(x) > u(y)$ implies that xPy, then $u(.)$ is a utility function representing R. **(6)**

(3) Suppose that X is finite and R is a rational preference defined on X. Consider the function $u^* : X \to \Re$ such that $\forall\, x \in X$, $u^*(x) = |X| - |\{z \in X \mid zPx\}|$. Is the function $u^*(.)$ a valid utility representation of the preference relation R on X? Justify your answer. **(10)**

(4) Show that a choice structure $(\mathcal{B}, C(.))$ for which a rationalizing preference relation exists, satisfies the path-invariance property: For every pair $B_1, B_2 \in \mathcal{B}$ such that $B_1 \cup B_2 \in \mathcal{B}$ and $C(B_1) \cup C(B_2) \in \mathcal{B}$, we have $C(B_1 \cup B_2) = C(C(B_1) \cup C(B_2))$. **(18)**

(5) Define the weak axiom of revealed preference for the market economy. Show that if the Walrasian demand function $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law, then the weak axiom of revealed preference holds if and only if it holds for all compensated price changes. **(2+16=18)**

(6) Show that if the Walrasian demand function $x(p, w)$ is generated by a rational preference relation, then it must satisfy the weak axiom of revealed preference. **(10)**

(7) If $u(.)$ is a continuous utility function representing R on X, then show that R must be continuous. **(9)**

(8) Let R be a preference relation defined on X and let $u(.)$ be a utility function representing it. Define convexity of R and quasi-concavity of $u(.)$. Show that R is convex if and only if the utility function $u(.)$ representing it is quasi-concave. **(1+2+12=15)**

Mid-semester Examination: (2011)
M. S. (Q. E.) I Year
Computer Prog. & Applications

Date:09.09.11 Duration: 2 hours 30 mins

Answer as many questions as you like. But you can at most score 60.

1. Perform the following decimal additions in binary number system and express the results in hexadecimal and octal number systems:

(i) 41.50 + 2.25
(ii) 11.75 + 9.25

5+5

2. Write brief notes on any three of the following:

(i) Software and hardware (ii) Arithmetic logic unit (iii) Memory address register (iv) positional number systems

12

3. Write a flow chart for executing the program:

$S=|A|+|B|$ (where, $|X|$ =>modulus(X))

6

4. State what will be the outputs of the following programs with proper justifications:

(i)
```
#include<stdio.h>
main()
{
  int i;
  printf("Life's greatest happiness is to be convinced we are loved by others.\n");
  for(i=1; i<=10; i++)
  main();
}
```

(ii)
```
#include<stdio.h>
main()
{
  int num=50, *temp, total=0;
  temp=&num;
  *temp=200;
  temp=&total;
  *temp=num;
```

.... page 2

```
printf("%d %d %d", num, *temp, total);
}
```

(iii)
```
# include <stdio.h>
main()
{
    int var1=2,var2=2,var3=2;
    var1=var2= =var3;
    printf("%d",var1);
}
```

Will the result differ for var2=22?

4+4+5

5. Can you write a C program to swap two variables, without using a third variable.

5

6. Is the following code error free? If no, identify the error/s? If yes, how many times can this code be executed?

```
#include<stdio.h>
main( )
{
        char another ;
        int num ;
        do
        {
                printf ( "Enter a number " ) ;
                scanf ( "%d", &num ) ;
                printf ( "cube of %d is %d", num, num * num * num ) ;
                printf ( "\nWant to enter another number y/n " ) ;
                scanf ( " %c", &another ) ;

        } while (another = = 'y' ) ;
}
```

6

7. Write a C program to obtain the factorial of any number. 8

8. Write a C program *without using any semicolon* whose output will be: We Make a Living by What We GET, We Make a Life by What We GIVE. 5

Mid-Semester Examination
Economic Development I
MSQE I & II

Date: 9.9.11
Maximum Marks: 40
Time : 2 hours

Answer question 1 and either 2 or 3.

1. Consider two infinitely-lived agents with identically and independently distributed income streams every period. For each agent, income can take two possible values, 100 and 0, with equal probability. In any period, the utility function of each agent is given by $U(c) = c^{1/2}$, where c is consumption in that period. The agents are in an informal insurance arrangement in which if in a period income realizations are different, the high income agent makes a transfer to the low income agent. By assumption, the agents are unable to save or store. Also assume that the rate of discount r = 25%.
   (a) Find the first best contract and check whether it is implementable.
   (b) Suppose the income is in terms of grains and the agents are able to store their grains only for the next period. After two periods, the grains are rotten. Can storage be a good alternative to informal insurance?
   [15+5]
2. A risk neutral landlord gives his land to a risk-averse farmer for cultivation but cannot observe his work effort. The output, which is observable, depends on the effort and the realization of a random term. Assuming that the output can be either high (H) or low (L) and the effort level can be either 0 or 1, with high effort increasing the probability of high output, show that share tenancy is the best incentive contract. [20]
3. Show, in terms of a suitable model, how a less developed economy can get stuck in a bad equilibrium due to coordination failure and how a big push by the government can take the economy to a good equilibrium. [20]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2011-12

M.S. (QE) I year

Mathematical Methods

12.09.11

Date : [redacted] Maximum Marks :80 Duration :3 hrs

Note: Answer as many as you can. The maximum you can score is 80.

1. (a) Find the infimum of all numbers of the form $\frac{1}{2^p} + \frac{1}{5^q}$, where p, q take on all positive integer values.

   (b) Find the supremum and infimum of

   $$S = \{x \in I\!R : (x-a)(x-b)9x-c)(x-d) < 0\}$$

   , where $a < b < c < d$.

   (c) Let A, B be two sets of positive real numbers bounded above and let $a = supA$ and $b = supB$. Let

   $$C = \{xy : x \in A, y \in B\}.$$

   Prove that $supC = ab$.

   Show that the result is not true if we drop the 'positivity' condition. [4+3+(5+3)]

2. Let $f : X \to Y$ be a function

   - For sets $A, B \subseteq X$, show that $f(A \bigcap B) \subseteq f(A) \bigcap f(B)$
     Give an example to show that equality may not hold.
   - For $C, D \subseteq Y$, prove that

   $$f^{-1}(C \bigcup D) = f^{-1}(C) \bigcup f^{-1}(D)$$

   $$f^{-1}(C \bigcap D) = f^{-1}(C) \bigcap f^{-1}(D)$$

   [5+6]

3. - Let Z^+ denote the set of all positive integers. Show that $Z^+ \times Z^+ \times Z^+$ is countable.

- Show that any collection of disjoint intervals of positive length is countable.
  (Hint: Assume that rationals are countable) [6+5]

4. Let $\mathbb{R}^n$ denote the n-dimensional Eulidean space with the usual norm.

   - Define an open set in $\mathbb{R}^n$. Show that the union of any collection of open sets is open.
   - Give examples of open sets $U_1, U_2, U_3, \ldots$ such that $\bigcap_{k=1}^{\infty} U_k$ is not open.

   [7+5]

5. Let $U \subseteq \mathbb{R}$ be a non-empty open set. For each $x \in U$, let I_x denote the largest open interval contained in U.

   Show that $\{I_x : x \in U\}$ is a countable collection of disjoint intervals. [8]

6. Show that if K is a compact subset of $\mathbb{R}^n$, then K is closed and bounded. Is the result true for any metric space? [8]

7. Let (X, d) be a metric space

   - Define a Cauchy sequence in X. Show that any convergent sequence in X is Cauchy. Give an example to show that the converse need not be true.
   - Let $\{x_n\}$ be a Cauchy sequence and let $T = \{x_1, x_2, x_3, \ldots\}$ be its range. Show that if p is an accumulation point of T then $x_n \to p$ as $n \to \infty$

     Use this fact to show that every compact metric space is complete. (You may use a standard result for compact metric space.) [8+9]

8. Let (X, d) be a compact metric space and $f : X \to \mathbb{R}$ be a continuous function. Show that $f(X)$ is compact.

   Hence, show that there exist $p, q \in X$ such that

$$f(p) = inff(X),$$
$$f(q) = supf(X).$$

[6+5]

Ram Bera

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2011-12

M.S. (QE) I year
Mathematical Methods

Date :14.11.11 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The maximum you can score is 100. Notation is as used in the class.

1. (a) Let $f : X \to Y$ be continuous function from a metric space X to a metric space Y. Show that if X is connected then $f(X)$ is a connected subset of Y

   (b) Use the above result to prove the following. Suppose f is a real-valued continuous function on X and X is connected. Assume that there exist $x, y \in X$ such that $f(x) = a$, $f(y) = b$ and $a < b$. Show that for every $c \in (a, b)$ there exists $z \in X$ such that $f(z) = c$. [7+7]

2. - Let $f : X \to Y$ be a uniformly continuous function from a metric space X to a metric space Y. Prove that if $\{x_n\}$ is a Cauchy sequence in X then $\{f(x_n)\}$ is a Cauchy sequence in Y
   - Let $f(x) = \frac{1}{2}(x + \frac{2}{x})$. Take $X = [1, \infty)$. Show that f is a contraction of X with contraction constant $\alpha = \frac{1}{2}$. What is its fixed point? [6+6]

3. - Find a basis of the following subspace of $I\!R^4$.

     $$S = \{(x_1, x_2, x_3, x_4) \in I\!R^4 : x_1 - 2x_2 + x_3 = 0\}.$$

     What is its dimension?
   - let $\mathcal{P}_n$ denote the space of all real polynomials in $t$of degree $< n$. Let $x_1, x_2, ..., x_n$ be distinct real numbers. For $1 \leq i \leq n$ define

     $$l_i(t) = \prod_{j \neq i}(t - x_j).$$

     Show that $l_1(t), .., l_n(t)$ form a basis of $\mathcal{P}_n$ [5+6]

p.T.o

4. Define the sum of two subspaces S, T. When is it a direct sum? Prove that if $V = S \oplus T$, where S, T are subspaces, then

$$dim(V) = dim(S) + dim(T).$$

[10]

5. Let V be an n-dimensional real vector space . Show that it is isomorphic to $I\!R^n$. [10]

6. In each of the following , find the matrix of the linear transformation $f : V_1 \to V_2$ with respect to the bases $\mathcal{X}$ and $\mathcal{Y}$.

- $V_1 = I\!R^2, V_2 = I\!R^3$, $f(x_1, x_2) = (2x_1 - 3x_2, x_2, x_2 + 5x_1)$. $\mathcal{X}$ and $\mathcal{Y}$ are the canonical bases.
- $V_1 = \mathcal{P}_4, V_2 = \mathcal{P}_3$, $f(p(t)) = p'(t)$, $\mathcal{X} = \{1, t, t^2, t^3\}$ and $\mathcal{Y} = \{1, t, t^2\}$. Here $\mathcal{P}_n$ is as in Qs 3.

[5+5]

7.
- Show that a square matrix of order n is non-singular iff its rank is n
- Find the inverse of the following matrix if it exists.

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

[8+6]

8. Show that for any two square matrices of order n

$$\mathcal{C}(AB) \subseteq \mathcal{C}(A).$$

Hence deduce that $\rho(AB) \leq min\{\rho(A), \rho(B)\}$. [8]

9. Consider the system of equations

$$A\mathbf{x} = \mathbf{b},$$

P.7

where A is an $m \times n$ matrix, $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}$ and $\mathbf{b}$ is an $m \times 1$ matrix.

Show that if $\rho(A) = \rho([A : \mathbf{b}])$ then the above system is consistent.

Find whether the system $A\mathbf{x} = \mathbf{b}$ is consistent and find its general solution, when

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -4 \\ 0 & 1 & -3 \\ -1 & 0 & -2 \end{pmatrix},$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

[6+7]

10.
- Define the characteristic polynomial $\chi_A(\lambda)$ of a square matrix A. Show that the sum of the roots of $\chi_A(\lambda)$ is the trace of A and the product is the determinant
- Show that the minimal polynomial divides the characteristic polynomial of a matrix A.
(You may assume the Caley-Hamilton Theorem) [10+8]

First Semestral Examination: (2011-12)
MS (QE) I Year
Computer Programming and Applications

Date: 16.11.11 Maximum Marks: 100 Duration: 3 hours

Answer as many as you wish. But you may at most score 100.

1. Write short notes on any 5 of the following: a) *extent of a variable* b) *scope of an identifier* c) *local and global variables* d) *accumulator* e) *program counter* f) *memory stack and heap* h) *low and high level languages*. 3x5=15

2. What are the logical operators in C language?
Suppose the Government introduces a new insurance scheme so that the employees of Indian Statistical Institute get insured provided any one of the following conditions is satisfied:
The employee is married.
The employee is unmarried, male, and more than 35 years old.
The employee is unmarried, female, and more than 30 years old.
Write a C program *without using* any logical operator to determine if an employee is insured. Then modify the code *using* logical operator/s. 3+6+6=15

3. When would you prefer to use a *break* statement for a loop? Demonstrate through a C code, how you can verify whether a number is prime or not, while demonstrating the use of the *break* statement. Then write a suitable C code to print out all prime numbers less than 100, where the code implements the following algorithm from *Eratosthenes' sieve* method:
Create a list of consecutive integers from 2 to n: 2, 3, 4,..., n (n=100 in the present case).
Initially, let p equal 2, the first prime number.
Strike out from the list all multiples of p greater than p.
Find the first number remaining on the list greater than p (this number is the next prime); let p equal this number.
Repeat steps 3 and 4 until p^2 *is greater than n.*
All the remaining numbers on the list are prime. 2+6+ 8=16

4. (i) Explain the concepts behind different types of *function call* through the example of swapping of the values of any two variables.
(ii) Make a comparison between an iterative function and a recursive function. Write a C program to find the factorial of a number in *non-recursive* method. Then use the *recursive* method and do the same. 6+(3+5+6)=20

5. How is a *structure* different from an *array*? How is a *union* different from a *structure*? Hence, explain the difference between a *structure of structures* and a *union of structures*?

P.T.O

3+3+3+3=12

6. Write C programs to (i) insert a *new* element into an array (ii) delete *duplicates* in an array. Make a comparison between linear and binary search techniques. Sort any given array using any technique you wish by writing a suitable program. Then show how you can programmatically implement the binary search algorithm in such a sorted array.

(4+4)+3+6+7=24

7. (i) Show using Matlab language how you can (a) vertically concatenate two matrices (b) replicate a matrix? Also show some applications of the colon (:) operator in Matlab?
(ii)What are strings? Write a C program that prints its input with one word per line.
(iii) Is the following program error free? If yes, predict the output; if no, correct it and write the output:

```
#include<stdio.h>
main ( )
        {
                char name[ ]="Siddhartha Sharma";
                char *ptr;
                ptr=name;
                 while(*ptr!='\0')
                {
                        printf("%c",*ptr);
                        ptr++;
                }
        }
```

(3+3+4)+(2+6)+3=18

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2011-2012)

MS(QE) I & MSTAT II

Microeconomic Theory I

Date: 18.11.2011 **Maximum Marks:** 100 **Duration:** 3 hrs.

Note: Answer all questions.

(1) Suppose that $u(.)$ is a continuous utility function representing a locally non-satiated preference relation R on $X = \Re^L_+$. Show that the indirect utility function $v(p, w)$ satisfies the following properties, where the symbols p and w have their usual meanings.

(a) $v(p, w)$ is homogeneous of degree zero in its arguments.

(b) $v(p, w)$ is strictly increasing in w and non-increasing in p_l for any $l \in \{1, \ldots, L\}$.

(c) $v(p, w)$ is quasi-convex, that is, the set $\{(p, w) : v(p, w) \leq \bar{v}\}$ is convex for any $\bar{v}$. **(5+10+9=24)**

(2) Suppose that $u(.)$ is a continuous utility representation of a locally non-satiated preference relation R defined on $X = \Re^L_+$ and that the price vector is $p >> 0$. Show that for $u > u(0)$, if x^* is the optimal in the expenditure minimization problem, then x^* is also optimal in the utility maximization problem at the wealth level $p.x^*$. Also show that the maximized utility level in this utility maximization problem is exactly u. **(12+4=16)**

(3) Denote by Y^+ the additive closure of Y, that is, the smallest production set that is additive and contains Y. Show that if Y is convex, then $Y^+ = \cup_{n=1}^{\infty} nY$, where for any positive integer n, $nY = \{ny \in \Re^L : y \in Y\}$. **(16)**

(4) Suppose that $c(w, q)$ is the cost function of a single-output technology Y with production function $f(z)$ and that $z(w, q)$ is the associated conditional factor demand correspondence, where the symbols z, w and q have their usual meanings. Prove the following statements.

(a) The cost function $c(w, q)$ is non-decreasing in q.

(b) If $f(z)$ is homogeneous of degree one, then $c(w, q)$ and $z(w, q)$ are homogeneous of degree one in q.

(c) If $f(z)$ is concave, then $c(w, q)$ is a convex function of q.

(4+12+8=24)

(5) Define Pareto efficiency and weak Pareto efficiency. Show that if the consumption set $X_i = \Re^L_+$ for all $i = 1, \ldots, I$, and all consumers' preferences are continuous and strongly monotonic, then Pareto efficiency and weak Pareto efficiency are equivalent. **(4+16=20)**

Indian Statistical Institute
First Semester Examination
MSQE I &II
Economic Development I

Date: 24.11.11
Maximum Marks: 60
Duration: 3 hours

Answer question number 1 and any two from the remaining three

1. Ramu is a cobbler. He collects hides from carcasses of dead animals, dries them up in the sun and makes shoes. On a sunny day he can dry up a skin and make one pair of shoes which earns him X. On a rainy day, skins cannot be dried and therefore Ramu earns nothing. A sunny day occurs with probability p and a rainy day occurs with probability $(1-p)$. In addition, Ramu has some money which he lends and earns R per day as interest. Ramu is risk neutral, has a daily discount factor $\beta < 1$ and an infinite life time.
   (a) Let V and W denote the discounted value of Ramu's future life time income on a sunny day and on a rainy day respectively. Express V and W in terms of X, β, p and R.

   Now suppose that Ramu can buy a drying machine with the money he has. This allows him to dry the skin he gathers on a rainy day as well as on a sunny day. Accordingly, if he buys the machine, he earns an income X every day irrespective of whether it is rainy or sunny. But, of course, he has to forego his daily interest income. Moreover, the machine has a probability $(1-q)$ of getting stolen on any particular day in future. If the machine is stolen, Ramu comes back to his old technology and his money is gone forever.

   (b) On which day is he likely to buy the machine, a sunny day or a rainy day? Give reasons for your answer.
   (c) Let U be his life time income when he buys the machine. Express U in terms of X, β, p and q.
   (d) Prove the following proposition: If $X(1-p\beta) > R > X(1-\beta)$, then there exists $\bar{q} \in (0,1)$ such that for all $q \leq \bar{q}$, Ramu has no incentive to buy the machine. What does this tell you about property rights?

   [9+3+9+9=30]

2. Demonstrate, in terms of a suitable model, that in a less developed region with a large informal sector, where property rights are imperfect and political patronage is essential for survival, democracy might reduce efficiency.

   [15]

P.T.O

3. Characterize the optimal contract between a farmer who takes a production loan and a trader-cum-money lender who gives the loan and markets the product, when the product market and the credit market are interlinked. Show that the contract is Pareto efficient.

[15]

4. In a model of increasing returns, show how history and expectations can interplay to determine the long run equilibrium as well as the dynamic path to the long run equilibrium.

[15]

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M.S.(Q.E.) 1st Year (2011 – 12)
SEMESTRAL- I EXAMINATION
Statistics

Date: 26·11·11 Maximum Marks: 100 Time: 3 hours

This question paper carries a total of *120* marks. You can answer any part of any question; but the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. (a) If A and B are two events in a sample space, then prove or disprove the following.
   (i) $P(A) \leq P(AUB)$;
   (ii) If $P(A \mid B) \geq P(A)$, then $P(B \mid A) \geq P(B)$.

   (b) If $\{A_n\}$ is a non-decreasing monotone sequence of events belonging to a σ-field of events, then prove that

   $$P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n).$$

   (c) The completion of a construction job may be delayed because of a labour strike. The probabilities are 0.60 that there will be a strike, 0.85 that the construction job will be completed on time if there is no strike, and 0.35 that the constructed job will be completed on time if there is a strike. What is the probability that the construction job will be completed on time?

[6+8+6= 20]

2. (a) Show that for a random variable, the mean deviation about the median is minimum.

   (b) Suppose X and Y are two continuous random variables, and $V(X), V(Y), E(Y \mid X)$ and $V(Y|X)$ exist. Then prove that

   $$V(Y) = E[V(Y \mid X)] + V[E(Y \mid X)].$$

   (c) Show that the *m.g.f.* of the sum of two independent random variables is the product of the *m.g.f. s* of the two random variables.

[8+8+4 = 20]

3. (a) Explain the concept of 'convergence in probability' of a sequence of random variables, and hence define 'consistency' of an estimator.
(b) What is 'law of large numbers'? Explain.
(c) State and prove Khinchine's theorem.

[8+4+8 = 20]

4. (a) Let X be a uniform random variable over the interval [-1, 3]. Find the *p.d.f.* of $Y = X^2$.
(b) State the Cramer-Rao inequality along with all the assumptions and then prove this inequality. Is it ever possible that the least attainable variance of an estimator is less than the Cramer-Rao lower bound? Give justifications in support of your answer.

[6+14 = 20]

5. Consider a classical k-variable linear regression model as given below.

$$Y = X\beta + \varepsilon, \quad E(\varepsilon) = 0, \; D(\varepsilon) = \sigma_\varepsilon^2 I_n$$

(Notations have their usual meanings.)

(a) Prove that the ordinary least squares (OLS) estimator of β, say $\hat{\beta}$, has the minimum variance in the class of linear unbiased estimators of β.
(b) Under the assumption of normality of the errors, find the distributions of $\hat{\beta}$ and $e'e$ where e is the OLS residual vector.

[10+10 = 20]

6. (a) Explain the concepts of type I error, type II error and power of a statistical test.

(b) A national equal employment opportunties committee is conducting an investigation to determine if women employees are as well paid as their male counterparts in comparable jobs. Random samples of 75 males and 65 females in junior academic positions are selected, and the following calculations are obtained from their salary data.

	Male	Female
Mean	Rs. 11530.00	Rs. 10620.00
Standard deviation	Rs. 780.00	Rs. 750.00

Do the data provide strong evidence that the mean salaries in junior academic positions are different between the males and the females? Also construct a 95% confidence interval for the difference between the mean salaries.

[9+11 = 20]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2011-12

M.S. (QE) I year

Mathematical Methods

Date :30.12.11 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The maximum you can score is 100. Notation is as used in the class.

1. Let $f : X \to \{0.1\}$ be a continuous function from a metric space X. Show that X is connected iff $f(X)$ is constant. [10]

2.
   - Show that the intersection of an arbitrary family of subspaces of a vector space V is a subspace of V.
   - Define a basis of a vector space.
   - Find a basis of the following subspace of $I\!R^4$.

     $$S = \{(x_1, x_2, x_3, x_4) \in I\!R^4 : x_1 - x_2 + 5x_3 = 0\}.$$

     What is its dimension?
   - let $\mathcal{P}_n$ denote the space of all real polynomials of degree $< n$. Find a basis of $\mathcal{P}_n$. [6+4+5]

3. Define the sum of two subspaces.
   Prove that for any subspace S of a finite-dimensional vector space V, there exists a subspace T such that $S \oplus T = V$. [10]

4. Let V be an n-dimensional complex vector space . Show that it is isomorphic to C^n. [10]

5. In the following , find the matrix of the linear transformation $f : V_1 \to V_2$ with respect to the bases $\mathcal{X}$ and $\mathcal{Y}$.
   - $V_1 = I\!R^3, V_2 = I\!R^1,\ f(x_1, x_2, x_3) = (2x_1 - 3x_2, x_2, x_2 + 5x_1)$. $\mathcal{X}$ is the canonical basis and $\mathcal{Y} = \{\in\}$..
   - $V_1 = \mathcal{P}_4, V_2 = \mathcal{P}_3, f(p)(t) = \int p(t)dt, \mathcal{X} = \{1, t, t^2, t^3\}$ and $\mathcal{Y} = \{1, t, t^2\}$.

7 · 7

6. • Define rank of a matrix. When is a matrix said to be non-singular? Show that a permutation matrix is non-singular.

• Find the inverse of the following matrix if it exits

$$\begin{pmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[10 · 6]

7. Deduce that $\rho(AB) \leq min\{\rho(A), \rho(B)\}$.

8. Consider the system of equations

$$A\mathbf{x} = \mathbf{b},$$

where A is an $m \times n$ matrix, $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{pmatrix}$ and $\mathbf{b}$ is an $m \times 1$ matrix.

Show that if the above system is consistent then $\rho(A) = \rho([A : \mathbf{b}])$.

Find its general solution, when

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 0 & 4 & 4 \\ 1 & -1 & 2 & 1 \\ -1 & -2 & 2 & 0 \end{pmatrix},$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

[6 · 8]

9. • Define the minimal polynomial $\chi_A(\lambda)$ of a square matrix A. Show that the product is the determinant 10.

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M.S.(Q.E.) 1st Year (2011 – 12)
SEMESTRAL- I BACK PAPER EXAMINATION
Statistics

Date: 30.12.11 Maximum Marks: 100 Time: 3 hours

Answer **all** questions. Marks allotted to each question are given within parentheses.

1. (a) Suppose the sample space of a statistical experiment consists of four elementary events i.e., $S = \{e_1, e_2, e_3, e_4\}$. Assuming that the elementary events are equally likely, check if three events A, B and C defined as

$$A = \{e_1, e_2\}, \quad B = \{e_2, e_3\} \quad \text{and} \quad C = \{e_3, e_4\}$$

are independent pairwise as well as when all the three are taken together.

(b) For any three events A, B and C in a sample space, show that

$$P(ABC) = P(A)P(B|A)\,P(C|AB).$$

(c) Show that for any n events $A_1, A_2, \ldots, A_n$ in a sample space,

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i).$$

[5+5+10 = 20]

2. (a) The probability of a high school student being male is 1/3 and that being female is 2/3. Further, the probability that a male student completes a course successfully is 7/10 and that a female student does it is 4/5. A student selected at random is found to have completed the course. What is the probability that the student is a male?

(b) Find the coefficients of skewness and kurtosis of an exponential distribution.

[10 + 10 = 20]

3. (a) Prove that an MVU estimator is unique in the sense that if there are two MVU estimators, T_0 and T_1, then $T_0 = T_1$ with probability one.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with p.m.f.

$$f(x) = \begin{cases} p^x(1-p)^{1-x}, \text{ if } x = 0,1 \quad , \quad 0 < p < 1 \\ 0 \qquad\qquad\qquad\quad , \text{ otherwise.} \end{cases}$$

Show that variance of the sample mean $\overline{X}$ attains the Cramer-Rao lower bound.

(c) Suppose $\{T_n\}$ is a sequence of estimators for $\gamma(\theta)$. State two sufficient conditions for consistency and then show that these two conditions together are indeed sufficient conditions for T_n to be a consistent estimator for $\gamma(\theta)$.

[7+6+7=20]

4.(a) (i) Discuss what you understand by sufficiency for the parameter θ of a family of distribution functions.

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with parameter λ. Show that sample mean is a sufficient statistic for λ.

(b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a $N(\mu, \sigma^2)$ population. Find the ML estimators of μ and σ^2. Also obtain the variance of the ML estimator of μ.

[5+5+10=20]

5. (a) Let $X_1, X_2, \ldots, X_n$, be independent Poisson variables with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$ and Y be defined as $Y = \sum_{i=1}^{n} X_i$. Then show that the conditional distribution of $X_1, X_2, \ldots, X_n$ given Y takes a specific value y is a multinomial distribution with parameters y and $\lambda_i \big/ \sum_{i=1}^{n} \lambda_i$, $i = 1, 2, \ldots, n$.

(b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a $N(\mu, \sigma^2)$ population. Discuss how you would test the null hypothesis $H_0 : \mu = \mu_0$ (given) against the alternative hypothesis $H_1 : \mu \neq \mu_0$ when (i) σ^2 is known, and (ii) σ^2 is unknown.

[10 +10 = 20]

INDIAN STATISTICAL INSTITUTE
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MID-SEMESTRAL EXAMINATION 2011 - 12
M.S.(Q.E.)
Social Accounting

Date: 20·02·12 Maximum Marks: 40 Time: $2\frac{1}{2}$ hours

1. Write notes on:

   (a) Disposable income
   (b) GDP deflator
   (c) Chian index
   (d) Net National Product

$2 \times 4 = 8$

2. (a) Distinguish between GDP and GNP.

   (b) Suppose GDP of a Country in 2010 is \$15000 million. Residents of the Country receive \$ 2500 million as factor payments and foreigners living in the Country receive \$1750 million as factor payment from the Country they are staying. What is the value of the GNP of the Country?

$4 + 4 = 8$

3. During the second week of January 2012, in a small town of West Bengal, the households bought rice worth Rs. 3500.00 from the rice mill, ata and flour worth Rs. 2500.00 from the flour mill, vegetables worth Rs. 500.00 from the vegetable producer, eggs worth Rs. 150.00 from the poultry, fish worth Rs. 400.00 from the fisherman, bread worth Rs. 1000.00 from the bakery. The bakery bought flour for Rs. 400.00 from the flour mill, the rice miller bought paddy worth Rs. 2000.00 from the farmers the flour miller purchased wheat worth Rs. 2000.00 from the farmer, the poultry purchased foodstuff for his hens worth Rs. 60.00. The fisherman and the vegetable producer did not investment any amount on intermediate inputs during the week. Compute the gross domestic product of the region for the week.

8

4. (a) One day you give a beggar Rs. 10.00. What was the impact of this payment on GDP.

Next month when the same beggar visited your home again, you asked him to help you in your garden and he helped you. You paid him Rs. 25.00. What was the impact of this payment on GDP? Compare the two cases.

(b) A household purchased a Godrej refrigerator made in India valued Rs. 15000.00. His next door neighbour purchased a refrigerator made in South Korea for Rs. 12000.00. What were the impacts of these two purchases on GDP of India.

(c) During January 2010 to December 2010 a family resided in a rented house in Baranagar paying Rs. 60,000.00 per annum. In January 2011 the same family purchased the same house for Rs. 11,00,000.00 and continued to reside in the same house. What was the impact of this rent on GDP in 2010? What was the impact of this purchase on GDP in 2011?

(d) A women used to provide tution to two children of a family during 2008 to 2010. During 2008 and 2009 she received Rs. 2000.00 per month and during 2010 she received Rs. 2250.00 per month. In 2011 the women was married to a family member of the same family. She continued to taught the same children, but she did not get any payment for that.

What was the impact of this payment on GDP in 2008, 2009, 2010?

What was the impact of this free teaching on GDP in 2011?

$4 \times 4 = 16$

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2011-12

M. S. (Q.E.) I Year
Modern Growth Theory

Date: 20.02.12 **Maximum Marks: 40** **Duration: 2 Hours**

<u>Answer any four</u>

1. State the assumptions of the Neo-Classical one sector growth model.

[10]

2. Can an otherwise identical Solow economy attain the steady state equilibrium when the production function satisfies DRS? Explain your answer.

[10]

3. Is the introduction of capital augmenting technical progress consistent with the existence of steady-state equilibrium in the Neo-classical one sector growth model? Explain your answer.

[10]

4. Consider a dynamic model given by

$$\dot{x} = xy - 9$$

$$\dot{y} = y - x$$

where x and y are two state variables taking non negative values. Find out the intertemporal equilibrium point drawing a phase diagram and analyse its nature of stability.

5. a) How does Mankiw-Romer-Weil (MRW) model differ from Solow model ? Derive equations of motion in MRW model.

b) Solve the following time minimization problem :

$$Min. \int_0^T dt \quad \text{S.T. } \dot{x} = x + u^2 ; \; x(0) = 1 ; \; x(T) = 16 ; \text{ and}$$
$$u \in [-3, 0]$$

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INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2011-2012)
MS (Q.E.) I Year
Macroeconomics I

Date: 23.02.12 Maximum Marks 40 Duration 3 hours

Group A **Answer any two**

1. Consider an open economy having *perfect capital mobility* and a *flexible exchange rate*. Let e be the nominal exchange rate of the domestic currency and suppose the price level relevant for the LM curve - call it p - is a weighted average of domestic price (P) and foreign price (P^*) :

$$\frac{M}{p} = L(Y, i) \quad [p = \alpha P + (1 - \alpha) e P^*; \quad \alpha > 0]$$

Now, if i^* be the foreign interest rate (given). Perfect capital mobility implies that $i = i^* + \eta^e$ in equilibrium where η^e is the expected rate of depreciation of the domestic currency . Assume that P, P^*, are all given. Use this relation to replace i and draw the IS- LM curves on an e - y diagram. Find the impact of an expansionary fiscal policy in this model on the e - y diagram. Contrast it with the corresponding result for a standard Mundell - Fleming model with **P replacing p** in the LM curve.

[3 + 7] = [10]

2. The country A (under a *fixed exchange rate* regime) has lost access to the international financial markets as a result of a bloody military coup and a subsequent violent civil war.
   (a) Using the IS-LM framework examine the effects of a domestic currency devaluation in this economy on equilibrium values of domestic output, trade balance and interest rate. [*Hint*: Assume *zero* capital mobility and *given* values of foreign variables.]
   (b) Suppose, after the disturbances die down, the country again opens up, allowing *perfect capital mobility*, but keeping the exchange rate *fixed*. Trace once again the effects of a domestic currency devaluation on equilibrium values of domestic output, trade balance and interest rate. Compare these results with the corresponding ones in (a).

[5 + 5] = [10]

3. An economy (with no public institution, i.e. government) has only **five** producing units – a Wheat Farm (WF), a Flour Mill (FM), a Coal Mine (CM), an Electricity Plant (EP) and a Textile Firm (TF). Figures (in Rs. crores) on their production activities during 2011 are given below. For instance, output of wheat produced by WF is Rs. 50 crores. The notations used are: C = consumption, GFI = gross fixed investment, ΔS = change in stocks, EX = exports.

	for *Intermediate Use* by				for *Final Use* of (purchased/ self-produced) items by			
Purchase from (item)	Flour Mill	Coal Mine	Electric Plant	Textile Firm	Households C	TF GFI	WF ΔS	CM EX
WF (wheat)	40						10	
FM(wheat flour)					60			
CM (coal)			25		35			60
EP (electricity)	5	20		40	50			
TF (cloth)					125			
Abroad (raw cotton)				40				
(powerloom)						100		

(a) Argue, algebraically or otherwise, that GDP can be measured by the value added method or alternatively by the final expenditure method.

(b) Compute the value of GDP of this economy for 2011 by *both* the **Value Added** method and the **Final Expenditure** method.

(c) Suppose, given the structure of this economy, GFI in powerloom by TF in this year were *higher* by Rs. 10 crores. Would the GDP have been higher by Rs. 10 crores also? Argue.

[3 + 5 + 2] = [10]

Group B

Answer all

1. Consider an economy producing a final good Y with the following production function: $Y = X^{\alpha} L_Y^{1-\alpha}$, where $X = \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}}$ and $0 < \alpha, \rho < 1$. L_Y is the amount of labour and x_i is the amount of the ith intermediate input (they come in n varieties) used in the production of Y. To produce these intermediate goods, one has to spend one unit of capital to begin with (this constitutes the fixed cost), and thereafter, each successive units of the good is produced by employing one unit of labour alone. The total amount of labour and capital available to the economy is $\bar{L}$ and $\bar{K}$ respectively. The producers of the intermediate inputs are monopolistically competitive in the usual Dixit-Stiglitz sense. Labour is perfectly mobile between the Y sector and the intermediate goods sector and full employment of both the factors, labour and capital, prevails. Furthermore, the Y producers can only earn zero profits in equilibrium (i.e. price is equal to average cost in Y sector). With this above given description of the economy:

   a) Solve for the equilibrium allocation of labour in Y and the intermediate goods sector in terms of the parameters of the model.
   b) Solve for the wage rate of labour. What would happen to the wage rate if total labour endowment, $\bar{L}$ were to increase.
   c) Solve for the rental rate of capital. What would,happen to the rental rate, if total labour endowment, $\bar{L}$ were to increase.

[Hint: First, arrive at an equation to solve for the price of good Y, by invoking zero profit condition in production of good Y. It would then be convenient if you pick the Y good to be the numeraire.] **(10)**

2. Show that the short run multiplier (with fixed number of firms) essentially works itself out through the channel of profits.

   In that same model, what would be the effect of balanced budget increase in government expenditure on the labour supply.

   [Labour Supply = Total available Labour time – Leisure] [5+5]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2011-2012)
MS (Q.E.) I Year
Econometric Methods I

Date: Maximum Marks 35   Duration 2.30 hours

All notations are self-explanatory. You can answer any part of any question.

1. Consider the linear regression model $y_n = \mu + \beta x_n + \varepsilon_n$, $n = 1,2,\dots\dots\dots N$. Assume that $\{\varepsilon_n\}$ is a sequence of independent random variables with the following distribution: $P\{\varepsilon_n = \pm 1\} = \frac{1}{2}(1 - 2^{-n})$, $P\{\varepsilon_n = \pm 2^{-n}\} = \frac{1}{2^{n+1}}$.
   a. Check if CLRM conditions hold for ε_n.
   b. Show that OLS estimator of β is consistent.
   c. Show that OLS estimator of β has asymptotic normal distribution. [6+3+10=19]
2. Consider the above regression model with $x_n = n\ \forall\, n = 1, \dots, N$.
   a. Check if CLRM assumptions on 'X' hold or not.
   b. Show that OLS estimator of β is consistent. [4+4=8]
3. Define R^2 and $\bar{R}^2$ and discuss their relative merits and demerits, if any. [2+2+1=5]
4. Consider the multiple linear regression model $Y = X\beta + \epsilon$. Assume that all CLRM assumptions hold. Show that the OLS estimator of β is consistent and BLUE. [4+4=8]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108

SECOND SEMESTRAL EXAMINATION 2011 - 12
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 07·05·12 Maximum Marks: 100 Time: 3 hours

Answer any **five** questions. Marks allotted to each question are given within parentheses.

1. (a) Prove that the conditions for stationarity of an AR(p) process are that all the roots of the underlying characteristic equation must lie outside the unit circle. Also explain what happens if one of the roots lies on the unit circle.

   (b) Find the coefficients Φ_j, $j = 1,2,\ldots$ in the time series representation

   $X_t = \sum_{j=0}^{\infty} \Phi_j a_{t-j}$ of the following ARMA (2, 1) process.

   $$(1 - 0.8B + 0.15B^2)\ x_t = (1 - 0.4B)\ a_t,\ a_t \sim WN(0,1).$$

   $[\overline{8+4}+8 = 20]$

2. (a) Find the 3-step ahead minimum MSE forecast at origin n of the following time series.

   $$(1 - 0.6B)(1 - B)\ x_t = (1 + 0.3B)\ a_t,\ a_t \sim WN(0,1).$$

   (b) Describe how out-of-sample forecasts are obtained efficiently in a given time series. Also suggest, with justifications, two criteria for evaluating the out-of-sample forecast performance of a given time series model.

   $[8+\overline{8+4} = 20]$

3. (a) Consider the following system of equations:

   $$x_{1t} = 0.9x_{1,t-1} - 0.1x_{2,t-1} + \varepsilon_{1t}$$
   $$x_{2t} = 0.3x_{1,t-1} + 0.5x_{2,t-1} + \varepsilon_{2t}$$

   where $E(\varepsilon_{1t}) = E(\varepsilon_{2t}) = 0$ for all t, $V(\varepsilon_{1t}) = 9$ and $V(\varepsilon_{2t}) = 25$ for all t, and $Cov\ (\varepsilon_{1t}, \varepsilon_{2s}) = 12$ for $t = s$,and 0 for $t \neq s$.

   Find the status of stationarity/nonstationarity of both the time series $\{x_{1t}\}$and $\{x_{2t}\}$.

(b) Assume that (i) $x_t = [2,3]'$ at $t = 0$, (ii) there is a two-standard-deviation innovation in the first equation and a one-standard-deviation innovation in the second equation at t = 1, and (iii) both the innovations are zero in all subsequent time periods.

Find the impulse responses on each of the two time series in periods 1, 2 and 3, and then comment on the nature of these impulse responses in the system of equations considered here.

[10 + 10 = 20]

4. (a) State the main limitations of the Chow test. Give justifications for your answer.
(b) Discuss what, in your opinion, is the most important problem if the ADF test is carried out without any consideration to structural breaks.
(c) Describe briefly the Bai-Perron procedure of testing for multiple structural breaks in a given time series.

[4+8+8 = 20]

5. (a) Discuss the nature of unit roots in a quarterly time series.
(b) Describe the HEGY test for detecting the presence of seasonal and non-seasonal unit roots in a quarterly time series, and then comment on its size and power.

[6+14 = 20]

6. (a) Suppose that a time series $\{X_t\}$ is represented by

$$\{X_t\} = \sqrt{2} \sum_{j=1}^{q} \sigma_j \cos(\lambda_j t - \gamma_j)$$

where $\gamma_1, \gamma_2, \ldots, \gamma_q$ are independent random variables, each being uniformly distributed in the interval $[0, 2\Pi]$, and σ_j and λ_j, $j = 1, 2, \ldots, q$, are constants. Show that $\{X_t\}$ is a weakly stationary process.

(b) State and prove the theorem on finding the spectral density function of a linear combination of stationary stochastic process, and hence find the spectral density function of an ARMA (1, 2) process.

[10 + 10 = 20]

INDIAN STATISTICAL INSTITUTE

Mid-semester examination: (2011-2012)

MS(QE) I & MSTAT II

Microeconomics II

Date: 02.03.2012 Maximum Marks: 40 Duration: 3 hrs.

Note: Answer Group A and Group B in separate answer scripts.

GROUP A

Note: Answer both questions.

(1) Consider an economy capable of producing two goods, x and y, with two factors of production-labour L and capital K. To produce one unit of x, 2 units of labour and 2 units of capital are required; to produce one unit of y, 1 unit of labour and 3 units of capital are required. The economy is endowed with 100 units of labour and 200 units of capital. Finally each agent has a utility function $U = \max(x, y)$.

(a) Derive the relative supply curve as a function of the relative price.

(b) Derive the relative demand curve as a function of the relative price.

(c) Find the equilibrium relative price and the equilibrium factor prices.

(4+4+2=10)

(2) State and prove the Second Fundamental Theorem of Welfare Economics. What does the theorem tell us about the desirability of a free and competitive capitalist economy? **(8+2=10)**

GROUP B

(1) On the space of all simple lotteries (where the underlying set of consequences is finite), define the continuity axiom and the independence axiom of individual preferences. **(1+1=2)**

(2) Assume that the set of consequences is finite. If the rational preference relation $\mathcal{R}$ defined on the space of all simple lotteries $\mathcal{L}$ satisfy the independence axiom, then show the following.

(a) For all $\alpha \in (0,1)$ and $L, L', L'' \in \mathcal{L}$,
$L \; \mathcal{P} \; L' \Leftrightarrow \alpha L + (1-\alpha)L'' \; \mathcal{P} \; \alpha L' + (1-\alpha)L''$ and
$L \; \mathcal{I} \; L' \Leftrightarrow \alpha L + (1-\alpha)L'' \; \mathcal{I} \; \alpha L' + (1-\alpha)L''$. **(3+3=6)**

(b) There exists $\underline{L}, \overline{L} \in \mathcal{L}$ such that $\overline{L} \; \mathcal{R} \; L \; \mathcal{R} \; \underline{L}$ for all $L \in \mathcal{L}$. Do you need the continuity axiom to prove this result? **(9+3=12)**

INDIAN STATISTICAL INSTITUTE
Second Semester Examination: 2011-12
M. S. (Q.E.) I Year and II Year
Modern Growth Theory

Date: 23.04.2012 **Maximum Marks: 60** **Duration: 3 Hours**

Answer any three questions.

1 a) Explain why steady-state equilibrium can not exist in a Neo-classical one section growth model with Hicks-neutral technical change.

b) Show that the steady –state equilibrium is unique and stable in the Mankiw-Romer-Weil model.

[10+10]

2. Derive the optimum tax-rate in the FMS model when the objective of the government is to maximize the balanced growth rate of the economy.

[20]

3. a) State the assumptions of the Lucas model of endogenous growth. How does the Lucas model differ from the Ramsey-Solow model ?

[6+4]

b) Examine the validity of the following statements in the context of Lucas model.

i) In the steady-state equilibrium, note of growth of physical capital varies positively with the learning ability of the individual.

ii) In the absence of external effect of human capital, rate of growth in the market economy exceeds that in the planned economy.

[5+5]

4. 'If there is positive external effect of labour on the productivity of the economy and if the consumer derives disability from labour, then one can explain indeterminacy of transitional growth path even in the absence of external effect of capital' – Analyse the validity of this statement in the context of Benhabib and Farmen model.

[20]

□□□□□

INDIAN STATISTICAL INSTITUTE
203 B.T. Road, Kolkata – 700108
M.S. (Q.E) I Year
Second Semester Examination 2011 - 12
Social Accounting

Full Marks – 50 Date: 23.04.12 **Time: two and half hours**

Answer any FOUR questions : $12\frac{1}{2} \times 4 = 50$

1. (a) What is the System of National Accounts (SNA) ?

(b) What is the Material Product System (MPS)?

(c) What are the major points of differences between the SNA and the MPS? $3+3+6\frac{1}{2} = 12\frac{1}{2}$

2. (a) What are the uses of National Accounting ?

(b) What are the main features of Indian National Accounting System ? $6+6\frac{1}{2} = 12\frac{1}{2}$

3. (a) Is per capita gross domestic product a suitable measure of national well-being? Give reasons in support of your answer.

(b) Give an account of the Index of Sustainable Economic Welfare .

(c) Write a note on the Physical Quality of Life Index. $4+4+4\frac{1}{2} = 12\frac{1}{2}$

4. Given the following data for 2011, formulate Human Development Index for the countries. Write a note on your findings. $12\frac{1}{2}$

Countries	PCRGDP	LIFE	ADLI	Mean years of schooling
	PPP $	Years	%	Years
1	2	3	4	5
Bangladesh	1416	68.9	55.9	4.8
China	6828	73.5	94.0	7.5
Hong Kong	43229	82.8	100.0	10.0
India	3296	65.4	62.8	4.4
Indonesia	4119	69.4	92.2	5.8
Japan	32418	83.4	100.0	11.6
Malaysia	14012	74.2	92.5	9.5
Nepal	1155	68.8	59.1	3.2
Pakistan	2609	65.4	55.5	4.9
Philippines	3542	68.7	95.4	8.9
Republic of Korea	27100	80.6	100.0	11.6
Singapore	50633	81.1	94.7	8.8
Sri Lanka	4772	74.9	90.6	8.2
Thailand	7995	74.1	93.6	6.6

5. (a) Discuss the assumptions of the inter-industry system of Leontief

(b) For an industrial complex we prepared the following input-output matrix for year 2010 :

Output of industry	Intermediate use in industry					Gross Output
	Coal	Electricity	Railway	Machinery	Steel	
1	2	3	4	5	6	7
Coal (thousand tonnes)	30	270	200	50	50	650
Electricity (thousand KW)	20	10	100	30	10	200
Railway (thousand tonnes)	100	20	20	30	50	350
Machinery (thousand HP)	50	50	50	150	100	500
Steel (thousand tonnes)	10	20	20	50	10	160

(i) Estimate the input-coefficients.

(ii) Estimate the final consumption

$$4+8\frac{1}{2}=12\frac{1}{2}$$

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2011-2012)
MS (Q.E.) I Year
Macroeconomics I

Date: 27.04.12 Maximum Marks 60 Duration 3 hours

Answer Group A and Group B in separate scripts

Group A: Answer any two

1. Consider the following version of a two-period staggered wage model in which the *supply* of the **total output** of the economy (y_t) in period t is given by

$$y_t = y^* + \frac{1}{2} \times [\{p_t - E_t(p_t)\} + \{p_t - E_{t-1}(p_t)\}] \quad (\alpha > 0)$$

All variables are in logarithms and symbols are self-explanatory. $E_t(x_t)$ is the mathematical expectation (and also the rational expectation) of x_t conditional on the set of **information available** upto the **beginning of period** t.

Take α = **1** and assume that the *aggregate demand* is given by

$y_t = x_t - p_t$. [where $x_t \equiv m_t + v_t$, m is nominal money and v is velocity]

Solving for the equilibrium price p_t and taking expectations, it can be shown that

(*) $p_t - E_t(p_t) = \frac{1}{2}\{x_t - E_t(x_t)\}$ and $E_t(p_t) - E_{t-1}(p_t) = \frac{2}{3}\{E_t(x_t) - E_{t-1}(x_t)\}$

(a) Using the result (*) show that the equilibrium price also satisfies

$$p_t - E_{t-1}(p_t) = \frac{1}{2}\{x_t - E_{t-1}(x_t)\} + \frac{1}{6}\{E_t(x_t) - E_t(x_t)\}.$$

Find also the expression for the equilibrium y_t in terms of x_t, $E_t(x_t)$, $E_{t-1}(x_t)$.

(b) Suppose velocity follows random walk: $v_t = v_{t-1} + \varepsilon_t$.

Consider two alternative cases:

Case I: money supply is kept *constant* every period.

Case II: money supply every period is based on the past shocks to velocity in the following way :

$$m_t = a\,\varepsilon_{t-1}$$

where the value of a is *determined* by the authority.

Can any one of these two policies reduce long-run fluctuations of output around the natural rate? Explain. (Note that the authority can choose the value of a).

[6 + 9 = 15]

2. Suppose the supply curve of aggregate output (y) is the expectations-augmented Phillips curve:

(AS) $\quad y = y^* + \alpha(p - p^e)$ $\quad$ [y^* = natural output; $\alpha > 0$.]

where variables are in logarithms; p and p^e are respectively the *actual* and the *expected* price level. Suppose, the aggregate demand curve is given by

(AD) $\quad y = m - p$ $\quad$ [m = nominal money supply, a policy variables]

Consider *two* alternative mechanisms of formation of expectations:
(i) *naïve* expectations: $p^e = p_{-1}$ and (ii) *rational* expectations: $p^e = E(p)$, mathematical expectation based on the uptodate information. Suppose, $p_0^e = 0$ and m in period 0 (m_0) is such that $p_0 = 0$ and $y_0 = y^*$. Suppose, money supply from period 1 onwards is kept *growing* at a rate g: $m_t - m_{t-1} = g > 0$ for all $t \geq 1$.

(a) Show the initial equilibrium point and draw the AS curve for period 0 in a ($p - y$) diagram. Consider *naïve* expectations and comment on the output- inflation trade-off, when money (m) grows at the rate g. Show it graphically on the same diagram.

(b) Consider rational expectations. What happens to the trade-off now, when money (m) grows at the rate g? Explain. Can you show it graphically?

[8+7 = 15]

3. A simplified version of a *credit rationing* model is described below. There is a continuum of entrepreneurs, each having a project and an initial fund, W. Project i yields, R_i^S (its *success* value) with probability p_i and, R^f (the common *failure* value) with probability $(1 - p_i)$. All projects yield the *same expected* return, R and the density function $g(p_i)$ gives the distribution of p_i across entrepreneurs. To take up a project an entrepreneur has to take a loan of B from banks at an interest rate r (to be decided by banks) and repay $(1+r)$ B. Further, p is the probability of success of the marginal project for which entrepreneurs take loans. It is *given* that

$$R_i^S > (1+r)\,B > R > R^f \qquad \text{for all } i.$$

(a) Find the expected return to the investor in project i, $E(\Pi_i)$. Show how p is determined and how it varies with r.

(b) Find the expected pay off to the bank giving loans, $E(\Pi_b)$. Argue how the expected return to an individual bank, $E(\Pi_b)/B$, could have an inverted U-shape with respect to r. Discuss the implication of this result.

[6+9] = [15]

Group B

Answer all

1. Show that in an OLG model, introducing a 'Pay as you go' pension scheme will reduce the steady state per capita capital stock.

What would be the effect of such a pension scheme on the steady state welfare?

[5+10]

2. a) In a flex price, monopolistically competitive equilibrium of the Blanchard-Kiyotaki kind; show that money is neutral.

Beginning from such an equilibrium, show that a coordinated, equiproportionate reduction in all prices and wages would increase output, employment and welfare.

b) Consider an economy with the representative agent having the utility function:

$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}$, $0 < \alpha, \gamma < 1$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i^{\rho}\right]^{1/\rho}$, $0 < \rho < 1$ and c_i is the consumption of the i^{th} variety.

L is the labour supply, P is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. M is the money balances (and suppose M_0 is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where w is the money wage rate and π is the economy wide profits and T is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \leq F$$

$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

Y_i is the output of i^{th} variety and L_i is the labour employed in the production of the i^{th} variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed n).

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where G takes the form:

$$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$$ and g_i is the government consumption of the i^{th} variety.

(ii) What would be the effect of such increase in government expenditure on P ?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ($M = M_0$).]

[10+5]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2011-2012
MS (Q.E.) I Year
Econometric Methods I

Date: 30.04.12 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 110 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. Consider the linear regression model

$$Y_t = \alpha + \beta X_t + e_t, \ \ t = 1, 2, \ldots, T,$$

where each of $\{X_t\}$ and $\{e_t\}$ is independently and identically distributed with mean zero and variances σ_x^2 and σ_e^2, respectively. Further, X_t and e_t are mutually independent random variables. Now suppose, for some reason one attempts to fit the following regression model

$$\Delta Y_t = \theta_0 + \theta_1 \Delta X_t + u_t$$

by using the ordinary least square (OLS) method.

(a) Show that $E(\hat{\theta}_0) = 0$ and $E(\hat{\theta}_1) = \beta$, where $\hat{\theta}_0$ and $\hat{\theta}_1$ are the OLS estimators of θ_0 and θ_1, respectively.

(b) Obtain the autocorrelation function of u_t.

(c) Propose another unbiased estimator for β, and compare between the variances of this estimator and the other estimator already obtained, *viz.*, $\hat{\theta}_1$. **[10+ 5 + (3+7)=25]**

2. Consider the linear regression model

$$Y_t = \alpha + \delta Y_{t-1} + \beta X_t + e_t, \ Y_0 = 0, \ \ t = 1, 2, \ldots, T; \text{ and}$$
$$e_t = \rho e_{t-1} + u_t, \ e_0 = 0, |\delta| < 1, |\rho| < 1,$$

where each of $\{X_t\}$ and $\{u_t\}$ is independently and identically distributed with mean zero and variances σ_x^2 and σ_u^2, respectively. Further, X_t and u_t are mutually independent random variables. Let $\theta = (\alpha,\ \delta,\ \beta)'$.

(a) Show that $\hat{\theta}_{ols}$ is inconsistent.

(b) Assume that $\rho \neq 0$. How would you test for the null hypothesis, $H_0 : \delta = 0$? Provide analytical derivations.

(c) Assume that $\delta \neq 0$. How would you test for the null hypothesis, $H_0 : \rho = 0$? **[5+15+ 5 =25]**

3. Consider the linear regression model

$$y_i = \alpha + \beta x_i + e_i, \quad i = 1, 2, \ldots, n,$$

where $X = (x_1, x_2, \ldots, x_n)'$ is non-stochastic with finite second moment and $\{e_i\}$ is independently distributed random variable with mean zero and variances σ_i^2. Consider the quantity $S^2 = (Y - \alpha - \beta X)' W (Y - \alpha - \beta X)$, where $Y = (y_1, y_2, \ldots, y_n)'$, and W is $n \times n$ known symmetric matrix. Let Σ be the variance covariance matrix of $e = (e_1, e_2, \ldots, e_n)'$. Let $\theta = (\alpha, \beta)'$.

(a) Let $W = XX'$. Minimize S^2 with respect to θ. Show that this minimizer is a consistent estimate of θ.

(b) Let $W = \Sigma^{-1}$. Minimize S^2 with respect to θ. Show that this minimizer is a consistent estimate of θ.

(c) Compare the relative efficiencies of the above two estimators, *viz.* the estimator from (a) and the estimator from (b), respectively.

(d) Suppose you want to test the null hypothesis, $H_0 : \beta = 0$. How would you test the hypothesis using the estimator you obtained in (a)? **[8+7+ 10+10 =35]**

4. Briefly discuss about the problem of multicollinearity in a multiple linear regression model. How would you detect this problem for a given data set. **[5+5 =10]**

5. Consider the linear regression model

$$Y = \beta X + e,$$

where X is $n \times k$ stochastic matrix and is known to be endogenous. Assume that all other CLRM assumptions hold. Let Z be a $n \times k$ matrix of stochastic instruments with rank k.

(a) Propose an appropriate estimator using Z.

(b) Prove / disprove that your estimator is biased. **[5+10 =15]**

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: (2011-2012)

MS(QE) I & MSTAT II

Microeconomics II

Date: 03.05.2012 Maximum Marks: 60 Duration: 3 hrs.

Note: Answer Group A and Group B in separate answerscripts.

Group A

Note: Answer questions 1 and 2 and either 3 or 4.

(1) (a) Suppose there are two securities whose returns are specified as follows:

$$r^1 = (4, 4, 4, 0)$$
$$r^2 = (0, 0, 0, 2)$$

The price of security 1 is 2.3 and the price of security 2 is 0.2. Also, one can borrow or lend any amount at a risk free interest rate of 20%. Are the asset prices arbitrage free? **(5)**

(b) Suppose there are two securities whose returns are specified as follows:

$$r^1 = (1, 2)$$
$$r^2 = (2, 1)$$

It is known that the price of security 1 is 2 and that of security 2 is 3. Find the state prices μ_1, μ_2. **(5)**

(2) Consider an Arrow-Debreu economy with two goods, two periods and two states. Realization of endowments and consumption take place in the second period. In the first period, contingent markets open. Let p_{ls} represent the Arrow-Debreu price of commodity l in state s. Suppose these prices are given by $p_{11} = 1$, $p_{21} = 2$, $p_{12} = 2$, $p_{22} = 3$. Consider a particular agent with an endowment vector $(w_{11}, w_{21}, w_{12}, w_{22}) = (1, 2, 2, 1)$ and a consumption vector $(x_{11}, x_{21}, x_{12}, x_{22}) = (1.5, 1.5, 1.5, 1.5)$.

(a) Show that the consumption vector is feasible.

(b) Suppose there are two securities which pay $(1, 2)$ and $(2, 1)$ units of good 1 in the two states. Find the portfolio which supports the consumption vector. [Fractional purchase of securities is allowed.] **(5+5=10)**

(3) Define arbitrage free security prices. Suppose security prices are arbitrage free and there are three securities with return vectors r^1,r^2,r^3 and prices q^1,q^2,q^3. Show that if $r^1 = \alpha r^2 + \beta r^3$ then $q^1 = \alpha q^2 + \beta q^3$. **(3+7=10)**

(4) Define upper semi continuity and lower semi continuity. Stating your assumptions prove that the budget correspondence is both upper semi continuous and lower semi continuous. **(1+1+4+4=10)**

Group B

Note: Answer all the questions.

(1) Show that in any sub-game perfect Nash equilibrium of the screening game with unknown worker types, the low ability worker accepts $(\theta_L, 0)$ and the high ability worker accepts $(\theta_H, t^{(1)})$, where $t^{(1)}$, the task level assigned to the high type, satisfies $\theta_H - c(t^{(1)}, \theta_L) = \theta_L - c(0, \theta_L)$. Here the marginal (average) productivity of a worker is $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H < \infty$, the probability that a worker is of high type is $\gamma \in (0, 1)$ and the opportunity cost of accepting employment to each type of worker is zero. **(20)**

(2) Set up the problem of regulating a monopolist and derive the mechanism (or contract) when the constant marginal cost and fixed cost of the monopolist are common knowledge. **(10)**

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MID-SEMESTRAL EXAMINATION:MS(QE) 2011 -12

Time Series Analysis and Forecasting

Date: 29·2·12 Maximum Marks: 100 Time: 3 hours

Answer **all** questions. Marks allotted to each question are given within parentheses.

1. Examine whether the following statements are *true* or *false* or *uncertain*. Give brief explanations in support of your answers.

   (i) Estimating trend by moving average procedure is always the 'best'.
   (ii) A white noise process may exhibit seasonality.
   (iii) Strong stationarity never implies weak stationarity.
   (iv) The ACF of an AR(2) process always decays.

[4 × 5 = 20]

2. (a) Check whether the following time series $\{X_t\}$ is stationary and invertible.

$$X_t - 0.6X_{t-1} = a_t + 0.2a_{t-1} + 0.7a_{t-2}, a_t \sim WN\ (0, \sigma^2).$$

   (b) Find the ACF of an ARMA (1, 1) process in terms of its parameters. Also explain what would be the nature of the PACF of this process.

[6+(10+4) = 20]

3. (a) Discuss what you understand by seasonality and noise in a time series.

   (b) Distinguish between deterministic trend and stochastic trend.

   (c) Find the range of values of c for which the AR (2) process $x_t = x_{t-1} + c\,x_{t-2} + a_t,\ a_t \sim WN(0, \sigma^2)$, is stationary. Further, is the AR (3) process given by $x_t = x_{t-1} + c\,x_{t-2} - c\,x_{t-3} + a_t$ stationary for all values of c? Justify your answer.

[6+4+ 10 = 20]

4. (a) What are SACF and SPACF? Discuss briefly how these are used

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Answer any **four** questions. Marks allotted to each question are given within parentheses.

1. (a) Let $Z_t, t = 0,1,2,\ldots$ be independent normal variables each with mean zero and variance σ^2. Check if the time series $\{X_t\}$ given by $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$ where c is a constant, is weakly stationary. Is it also strongly stationary? Give explanations for your answer.

   (b) Suppose that a seasonal (say, quarterly) time series has trend which is quadratic in nature. Find appropriate differencing operator(s) for removing the deterministic components i.e., trend and seasonality, from this series.

   (c) Let the time series $\{X_t\}$ be defined by

   $X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} = a_t$

   where a_t's are uncorrelated random variables with mean zero and variance σ_a^2. Further, the roots r_1 and r_2 (both real) of the quadratic equation $z^2 + \alpha_1 z + \alpha_2 = 0$ are less than one in absolute value. Show that the time series $\{Y_t\}$ defined as $Y_t = X_t - r_1 X_{t-1}$, follows an autoregressive process of order 1 with parameter r_2.

   [8+5+12 = 25]

2. (a) Show that the variance of forecast error increases with increase in the time horizon of the forecast.

   (b) Find the 3-step ahead minimum MSE forecast at origin n of the following time series $\{x_t\}$.

   $(1+0.7B)\ (1-B^2)x_t = (1-0.3B)\ a_t$

   where $\{\ a_t\ \}$ is white noise with zero mean and unit variance.

(c) Briefly describe the maximum likelihood method of estimation of an AR(1) process.

[6 + 8 + 11 = 25]

3. (a) Explain why the augmented Dickey-Fuller (ADF) test is very important in time series analysis.

(b) Starting with an AR (p) process, derive an appropriate expression for the ADF estimating equation. Also distinguish between the ADF and PP tests.

(c) Discuss briefly how the power of ADF test can be improved.

[6 + 12 + 7 = 25]

4. (a) Describe the Quandt-Andrews test for detecting the presence of a single structural break in a time series. Also discuss the limitations of this test, if any.

(b) Describe the HEGY test for detecting the presence of seasonal and non seasonal unit roots in a quarterly time series.

[13 + 12 = 25]

5. (a) State the standard spectral representation of a stationary time series along with all assumptions, and then show that this representation indeed satisfies all the conditions of a weakly stationary time series.

(b) Prove that the spectral density function, $f(\lambda)$, of a stationary time series is nonnegative for all $\lambda \in [-\Pi, \Pi]$. Also find the spectral density function of an AR(2) process.

[10+15 = 25]