INDIAN STATISTICAL INSTITUTE
Mid-semester Examination: 2012-2013 (First Semester)

M. Tech (QR & OR) II Year

Applied Stochastic Processes

Date: September 17, 2012 Maximum Marks: 100 Duration: 3 hours.

Note: Answer all the questions

1. Let $\{N(t), t \geq 0\}$ be a counting process with rate λ, which satisfies the following postulates.
(i) $N(0) = 0$.
(ii) The process has independent increments.
(iii) $P\{N(t, t+h] = 1\} = \lambda h + o(h)$.
(iv) $P\{N(t, t+h] \geq 2\} = o(h)$.

   (a) Is the process stationary? Justify your answer.

   (b) Define$P_n(t) = P\{N(t) = n\}$. Show that $P_0(t+h) = P_0(t)P_0(h)$.

   (c) Let $P(s, t)$ be the probability generating function of $N(t)$. Derive a differential equation for $P(s, t)$ and hence find $P_n(t)$ by solving the differential equation [You can use the result in (b)].

   (d) Determine the covariance between $N(t)$ and $N(t+s)$ for $t > 0,\ s > 0$.

$[2 + 4 + 12 + 4 = 22]$

2. (a) Suppose that customers arrive at a service station in accordance with a Poisson process having rate λ. Given that two customers arrived during the first hour. What is the probability that both arrived during the first 30 minutes? What is the probability that at least one arrived during the first 30 minutes?

   (b) The number of accidents in a town follows a Poisson process with a mean of 2 per day and the number X_i of people involved in the ith accident has the distribution

$$P\{X_i = k\} = \frac{1}{2^k}, \quad k \geq 1.$$

   Find the mean and the variance of the number of people involved in accidents per week.

$[6 + 10 = 16]$

P.T.O.

3. (a) Consider an NHPP $\{N(t), t \geq 0\}$ with intensity function $\lambda(t)$, where $N(t)$ follows Poisson with mean $m(t) = \int_0^t \lambda(x)dx$. Then show that for $t_2 > t_1$, $N(t_2) - N(t_1)$ follows Poisson with mean $\int_{t_1}^{t_2} \lambda(x)dx$.

   (b) Consider an NHPP with intensity function $\lambda(t) = 0.2t^{-1/2}$. Then derive the pdf of T_1.

[10 + 5 = 15]

4. Consider a a repairable system, which is observed until the time of the eleventh failure. The failure times are

$$9, 20, 65, 88, 104, 107, 138, 143, 149, 186, 208, 227.$$

Assume that the failure process can be modelled by a homogeneous Poisson process with parameter λ.

   (a) Find the maximum likelihood estimate of λ.

   (b) Derive 95% confidence interval for λ.

[4 + 8 = 12]

5. (a) Let $\{N(t), t \geq 0\}$ be a renewal process having mean interarrival time μ. Then *show that with probability* 1

$$\lim_{t \to \infty} \frac{N(t)}{t} = \frac{1}{\mu}.$$

   (b) A man works on a temporary basis. The mean length of each job he gets is three months. If the amount of time he spends between jobs exponentially distributed with mean 2(months), then at what rates does the man gets new jobs?

[7 + 5 = 12]

6. Consider a delayed renewal process $\{N_D(t), t \geq 0\}$ whose first interarrival has distribution G and the others have distribution F. Let $M_D(t) = E[N_D(t)]$. Then show that

$$M_D(t) = G(t) + \int_0^t M(t-x)dG(x),$$

where $M(t) = \sum_{n=1}^{\infty} F^{(n)}(t)$ and $F^{(n)}$ is the n-fold convolution of F with itself.

[10]

7. Derive the distribution of residual life at time t and also find its limiting distribution.

[13]

Indian Statistical Institute

Mid-Semester Examination: 2012-2013

Course Name: M. Tech. (QR & OR) 2^{nd} YEAR

Subject Name: Advanced Statistical Methods

Date: 18.09.2012 Maximum Marks: 50 Duration: 2 hours

Note:
1. This paper carries 60 marks.
2. Answer all four questions but the maximum you can score is 50.
3. All notations have their usual meanings

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1. Let Y be $N_3(\mu, \Sigma)$ where

$$\underline{\mu} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}, \quad and \quad \Sigma = \begin{bmatrix} 9 & 0 & 7 \\ 0 & 12 & 0 \\ 7 & 0 & 10 \end{bmatrix}$$

a) Which of the following random variables are independent? State the necessary results.

i) y_1 and y_2 ii) y_1 and y_3 iii) y_2 and y_3

iv) (y_1,y_2) and y_3 v) (y_1,y_3) and y_2

b) Explain how will you obtain a random variable Z from Y such that Z follows $N_3(\underline{0}, \underline{I})$. State the necessary results that you use.

[5 + 4= 9]

2. Consider an iid sample of size n = 5 from a bivariate normal distribution $X \sim N_2\left(\mu, \begin{bmatrix} 3 & \rho \\ \rho & 1 \end{bmatrix}\right)$

where ρ is a known parameter.
Suppose $\bar{x}' = (1, 0)$. For what values of ρ would the hypothesis $H_0 : \bar{\mu}' = (0, 0)$ be rejected in favour of $H_1 : \mu' \neq (0, 0)$ (at the 5% level)?

[10]

3. (a) Write down the multiple linear regression model in matrix form

(b) State the assumptions

(c) Write down the normal equations

(a) Show that $\hat{\beta}$ is unbiased for β

(d) Derive the expression for $V(\hat{\beta})$

The symbols used above have their usual meaning

[2+2+2+5+3 = 14]

4. Observations on two responses were collected for three treatments. The observation vectors $(y_1, y_2)'$ are as follows:

Treatment 1: $\begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

Treatment 2: $\begin{bmatrix} 3 \\ 5 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \end{bmatrix}$

Treatment 3: $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

a) Identify the underlying model; write down the same and the associated assumptions.

b) Test for the equality of treatment effects.

c) If the null hypothesis is rejected test for each variable separately

d) Draw conclusions

e) What are the alternative test procedures ?

[5+10+6+2+4 = 27]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination: (2012 - 2013)

Course Name: M. Tech. (QROR) **Year: 2nd year**

Subject Name: Database Management Systems

Date: September 19, 2012 **Maximum Marks: 50** **Duration: 2 hrs**

Answer to all the questions.

1. A big insurance company wants to design a database for its emploee and their dependents. The following information are to be kept for the employee and their dependents.

   i. Employee(emp_no, address, date_of_birth, PAN, basic_salary, house_rent_allowance, total_salay, department_name);

   ii. Dependent(emp_no, dependent_name, relationship).

   The attribute names are self explanatory. The company owns a big residentdential campus, where some of the employees and their dependents are accommodated. The campus includes some flats of different areas but with a unique flat number. If an employee is accommodated in the residential campus, he/she will no get the house rent allowance.

   For the benefit of the employees, the company runs a dispensary where doctors of different specializations are employed by the company, and the free medical services are offerred to the employees and their dependents. Note that all the doctors have to stay in the residential campus of the company.

   The company runs a school for the dependents of its employees who stay in the residential campus. The teachers of the school are also employees of the company. They have different specializations, and each of them can teach at most two subjects.

   The insurance company sells poilicies to its employee as well as to the dependents of the employees. Each policy is characterized by a policy number, sum assured, date of acceptance, date of maturity, amount of premium, maturity amount; policy number being unique.

The company wants to maintain the following information.

i. Patients of the dispensary;

ii. Students of the school;

iii. Information related to the policy holders who are also the employees or their dependents.

Draw an appropriate ER diagram to show the relationship among the entity sets so that all the above information can be obtained whenever necessary. You may need to consider some more entity sets besides those given above. 30

2. Explain the following terms with appropriate examples. 3+3+10+3+3 = 22

   i. DDL;

   ii. DML;

   iii. Mapping cardinalities;

   iv. Atomicity property;

   v. Aggregation.

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2012-13

M. Tech (QR& OR) II year

Subject: Reliability II

Maximum Marks: 100 Duration: 3 hours Date:21.9.2012

Notations used in this paper are usual notations used in the class.

Answer all questions. Marks alloted for each question are given in [ ].

1. (a) Define likelihood ratio ordering of probability distributions.

(b) If $\pi'(h_1, t_1) \overset{LR}{>} \pi(h_2, t_2)$ then show that $g(\pi'(h_1, t_1)) \geq g(\pi(h_2, t_2))$, where $g_k(\pi(h,t)) = max\{0, -c + \alpha(\pi(h,t))[1 + g_{k-1}(\pi(hs, t+1))] + [1 - \alpha(\pi(h,t))]\, g_{k-1}(\pi(hf, t+1))\}$ and $lim_{k \to \infty} g_k(\pi(h,t)) = g(\pi(h,t))$.

[Note you have to state and prove all associated results needed for the above proof]

[5 + 40 = 45]

2. (a) Consider the case of a reliability allocation model using effort minimization algorithm. Define $k_0$as the maximum value of j such that $R_j < \left[\frac{R^*}{\prod_{i=j+1}^{n+1} R_i}\right]^{1/j}$, $R_{n+1} = 1$ and the final optimum allocation is $R_i^* = R_0^*$ if $i \leq k_0$
R_i if $i > k_0$

Formulate the following problem:

We are given R^*, the target reliability of the system consisting of five components having reliabilities as R_1,R_2,R_3,$R_4$and R_5 in increasing order. We need to find out R_i^*, $i = 1, 2, ...5$, the allocated values of the reliabilities of the components. We have to find out R_i, $i = 1, 2, ...5$ for $R^* = 0.80$, so that $k_0 = 3$ if we use effort minimization algorithm.

(b) Give an example with R_i, $i = 1, 2, ...5$ values and $R^* = 0.80$ such that $k_0 = 3$.

(c) Use effort minimization algorithm to find out R_i^*, $i = 1, 2, ...5$ for the example in **(b)** above.

[15+5+10=30]

3. **(a)** Assume that both stress and strength of a component follow log-normal distribution with different parameters. Derive an expression of reliability of the component in terms of the parameters.

(b) Let the strength(X) of a component be log-normal with $E(X) = 150,000KPa$ and σ_X as its variance. Similarly the stress (Y) follows log-normal with $E(Y) = 100,000KPa$ and $\sigma_Y = 15,000KPa$. Find out the maximum allowable σ_X of strength so that the reliability does not fall below 0.990.

[13 + 12=25]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2012-13

Course name	:	M. Tech. (QR & OR)-II
Subject Name	:	Industrial Experimentation
Date: 24/09/2012	Maximum Marks: 100	Duration 3 hours

NOTE: (i) This paper carries 106 marks. Answer as much as you can but the maximum you can score is 100. The marks are indicated in [ ] on the right margin.

(ii) The symbols and notations have the usual meaning as introduced in your class.

1. Define the following five terms with suitable example wherever feasible: Experiment, Factor & levels, Main effect, Random factor, Sum of squares of contrast.

(3×4+4) = [16]

2. Four treatments are being compared using a completely randomized design. Write down an appropriate fixed effect model for the design when each treatment is replicated n times. In order to detect existing differences between treatments, we consider the following three sets of contrasts C_1 to C_3. Are contrasts in each set orthogonal? What questions can be answered by testing each set of contrasts? Outline the Scheffe's method for comparing all contrasts.

Treat-	Contrast set 1		
ment	C_1	C_2	C_3
T_1	3	0	0
T_2	-1	2	0
T_3	-1	-1	1
T_4	-1	-1	-1

Treat-	Contrast set 2		
ment	C_1	C_2	C_3
T_1	0	1	2
T_2	0	1	0
T_3	1	-1	-1
T_4	-1	-1	-1

Treat-	Contrast set 3		
ment	C_1	C_2	C_3
T_1	1	1	-1
T_2	1	-1	1
T_3	-1	1	1
T_4	-1	-1	-1

(3+2×3+2×3+6) = [21]

3. A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to use a randomized block design, with the bolts of cloth consider as blocks. She selects five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths follow.

Chemical	Bolt				
	1	2	3	4	5
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

Perform an analysis of variance of the data and draw appropriate conclusions. Which chemical(s) will you recommend for use and why? Suppose the observation from Chemical 2 on Bolt 4 is missing. Derive an expression to estimate this observation and estimate the missing value.

($F_{0.05}(3,12)=3.49$, $F_{0.05}(4,12)=3.26$, $F_{0.01}(3,12)=5.95$, $F_{0.01}(4,12)=5.41$)

(7+5+6+2) = [20]

4. How do you implement the principle of randomization for a Latin Square Design?

[5]

5. Define a balance incomplete block design, BIBD(a, b, r, k, λ). Show that $b \geq a + r - k$, where notations have their usual meaning as discussed in the class. Write the least squares normal equations for an appropriate BIBD model. Obtain the least squares intrablock estimators of the treatment effects. Prove that $k\sum_{i=1}^{a} Q_i^2/(\lambda a)$ is the adjusted sum of squares for treatments in a BIBD. Show that the variance of the intrablock estimators $\{\hat{\tau}_i\}$ is $k(a-1)\sigma^2/(\lambda a^2)$. Examine whether a BIBD with parameters $a = b = 22$, $r = k = 7$, $\lambda = 2$ exists!

(3+7+6+6+8+6+2) = [38]

6. Identify the following design involving seven treatments applied to seven blocks:

Block	Treatment		
1	1	2	4
2	2	3	5
3	3	4	6
4	4	5	7
5	5	6	1
6	6	7	2
7	7	1	3

[6]

INDIAN STATISTICAL INSTITUTE
M. Tech. (QR & OR) 2nd YEAR
Year: 2012
MID SEMESTER EXAMINATION

Subject: Operations Research-II

Date of Exam: 25.09.2012 **Max. Marks: 100**

Time: 3 hours

Answer any five.

1. State the geometrical aspects for finding the solution of LP and NLP problems. Suppose A is an $m \times n$ matrix and c is an n vector. Then, exactly one of the following two systems has a solution:

$$\text{System 1} \quad Ax < 0 \quad \text{for some } x \in R^n$$

$$\text{System 2} \quad A^t y = 0 \text{ and } y \geq 0 \quad \text{for some } y \in R^m.$$

Define convex function, pseudo-convex function and quasi-convex function.

[6 + 8 + 6 = 20]

2. Define epigraph and sub-gradient of a function.
Let S be a nonempty convex set in R^n and let $f : S \to R$. Then show f is convex if and only if $epi\ f$ is a convex set.
Let $f(x_1, x_2) = 2x_1 + 2x_1^2 - 2x_1x_2 + x_2^2$. Find the Hessian matrix $H(x)$ and show that $H(x) \in$ PD.

[6 + 8 + 6 = 20]

3. State the duality theorem. Explain complementary slackness condition.
Let S be a nonempty closed convex set in R^n and $y \notin S$. Then there exist a nonzero vector p and a scalar α such that $p^t y > \alpha$ and $p^t x \leq \alpha$ for each $x \in S$.

[7+7 + 6 = 20]

4. Suppose that $f: R^n \to R$ is differentiable at $\bar{x}$. Prove that if there is a vector d such that $\nabla f(\bar{x})^t d < 0$, then there exists a $\delta > 0$ such that $f(\bar{x} + \lambda d) < f(\bar{x})$ for each $\lambda \in (0, \delta)$.

   Suppose that $f: R^n \to R$ is differentiable at $\bar{x}$. Prove that $\nabla f(\bar{x}) = 0$ and $H(\bar{x})$ is positive semi-definite if $\bar{x}$ is a local minimum.

   State the diagonalization algorithm for checking PD/PSD matrix.

[5 + 8+7 = 20]

6. State an additive type model for dynamic programming problem using Bellman's principle of optimality.

   The sales manager for a publisher of college text books has four sale persons to assign to three different regions of the country. Determine how many sales persons should be assigned to the respective regions in order to maximize sale using dynamic programming problem. The following table gives the estimated increase in sales in each region:

Salesman	Estimated increase in sales (in appropriate unit) for three regions		
	1	2	3
1	35	21	28
2	48	42	41
3	70	56	63
4	89	70	75

[8+12=20]

7. Describe Johnson's rule for an *n* jobs, 2 machines sequencing problem. Under which conditions an *n* jobs, 4 machines problem is solvable? Solve the following 2 jobs, 4 machines problem graphically.

	Machines			
Jobs	A	B	C	D
1	2	4	5	1
2	2	5	3	6

Technological ordering of Job-1 is A-B-C-D and of Job-2 is D-B-A-C.

[8+4+8=20]

INDIAN STATISTICAL INSTITUTE
Semester Examination: 2012-2013 (First Semester)

M. Tech (QR & OR) II Year

Applied Stochastic Processes

Date: November 26, 2012 Maximum Marks: 100 Duration: 3 hours.

Note: This paper carries 114 marks. Answer as many questions as you can.

1. Consider a renewal process $\{N(t), t \geq 0\}$ having interarrival time X_n, $n \geq 1$ and $\mu = E[X_1] < \infty$. Define $S_{N(t)} = \sum_{i=1}^{N(t)} X_i$.

   (a) Show that $E[S_{N(t)+1}] = \mu(M(t)+1)$, where $M(t) = E[N(t)]$.

   (b) Using (a), prove that $\dfrac{M(t)}{t} > \dfrac{1}{\mu} - \dfrac{1}{t}$.

   (c) Define residual life and spent life.

[10+3+2=15]

2. Consider a renewal process with interarrival time distribution F. If F is the uniform [0, 1] distribution function, prove that

$$M(t) = e^t - 1, \quad 0 \leq t \leq 1.$$

[10]

3. Let $\{N(t), t \geq 0\}$ be a renewal process having interarrival time X_n, $n \geq 1$ and suppose that each time a renewal occurs we receive a reward. Let R_n be the reward earned at the time of the nth renewal. Assume that the R_n, $n \geq 1$, are independent and identically distributed. Let $R(t)$ represents the total reward earned by time t. Then show that if $E[R_1] < \infty$ and $E[X_1] < \infty$,

$$\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E[R_1]}{E[X_1]}, \quad \text{with probability 1,}$$

[12]

4. The lifetime of a car (in years) is uniformly distributed over [0, 10]. A person buys a new car as soon as his old car either breaks down or reaches the age of T_0 years. A new car costs C_1 units and an additional cost of C_2 units is incurred whenever a car breaks down. Assume that a T_0-year old car in working order has resale value $R(T_0)$. There is no resale value of a failed car. Suppose that a cycle begins

each time a new car is purchased. Derive the expected length of the renewal cycle. Calculate the long-run average cost per unit time.

[6+6=12]

5. Consider a pure birth process $X(t)$ with rate λ_n (for $n \geq 1$). Define $P_n(t) = P[X(t) = n]$.
   (a) Derive the differential equations for $P_n(t)$'s and obtain a recursive solution for $P_n(t)$'s.
   (b) Define T_k as the time between kth and $(k+1)$th birth. Then find the distribution of T_0.

[$(4+6)+4=15$]

6. Consider a machine that works for an exponentially distributted amount of time having mean $1/\lambda$ before breaking down; and suppose that it takes an exponentiallly distributed amount of time having mean $1/\mu$ to repair the machine. Let $X(t)$ denotes the state of the machine at time t.
   (a) Write this as a general continuous time Markov process by specifying the matrix R of infinitesimal transition probabilities.
   (b) If the machine is in working condition at time 0, then derive the probability that it will be under repair at time $t = 10$.

[3+7=10]

7. Consider a linear birth and death process with immigration, $X(t)$, with rates $\lambda_n = n\lambda + \alpha$, $n \geq 0$ and $\mu_n = n\mu$, $n \geq 1$.
   (a) Show that $E[X(t)]$ satisfies $\frac{d}{dt}E[X(t)] = (\lambda - \mu)E[X(t)] + \alpha$. Hence find $E[X(t)]$ when $X(0) = N$.
   (b) Find the equilibrium distribution. Derive the condition for the existence of the same.

[$(10+4)+6=20$]

8. Describe the $M|M|s|k$ queueing system (s= number of servers and k=system capacity) as birth and death process. Write down the differential equations and obtain the steady-state probabilities.

[3+2+5=10]

9. Consider a discrete time branching process in which in each generation an individual either dies or is replaced by two offspring, the probability of two events being p_0 and p_2 respectively. Let X_n be the population size of the nth generation. Assuming $X_0 = 1$, derive the recursive relationship for the probability generating function of X_n. Hence find $P[X_2 = 2]$.

[7+3=10]

Indian Statistical Institute

3rd semester Examination: 2012-2013

M. Tech (QR & OR) II Year

Subject: Reliability II

Maximum Marks: 100 **Duration: 3 hours** **Date:** 27.11.12

Notations used in this paper are usual notations used in the class.

Answer all questions. Marks alloted for each question are given in [ ].

1. (i) Describe the Cox's proportional hazard model explaining the parameters involved.

(ii) Why is it called proportional hazards model?

(iii) Why is it called a semi-parametric model?

(iv) A group of 6 lives was observed over a period of time as part of a mortality investigation. Each of the lives are under observation from the age of 55 until they died or were censored. The table below shows sex, age at exit and reason for exit from the investigation. Write the partial likelihood of these observations.

Life	Sex	Age at exit	Reason for exit
1	M	56	death
2	F	62	censored
3	F	63	death
4	M	66	death
5	M	67	censored
6	M	67	censored

(v) Explain Breslow's approximation to partial likelihood method of estimation with reference to the above model.

(vi) An investigation was carried out into the survival times (measured in months of patients in hospital) following liver transplants. The covariates are

$z_{1i} = 0$ for Placebo
1 for treatment X
and z_{2i} =weight of patient (measured in kg).
The observed life times with weights in brackets are as follows:-

Placebo	Treatment X
4(86)	7*(59)
10(70)	12(76)
15(78)	15(69)
17(88)	15*(51)

Observations with * represents censored.
Using Breslow's approximation, find out what contribution to the partial likelihood is made by the deaths at time 15.

[3+1+1+4+2+4=15]

2.(i) Explain what is meant by the following:

(a) Non- Renewing Three stage warranty policy
(b) Renewing Pro-rata warranty policy
(c) Basic Rebate warranty policy
(d) Money Back Guarantee policy
(e) Two Dimensional Non-Renewing Free Replacement warranty policy
(f) Cumulative Free Replacement warranty policy

(ii) Describe different classes of warranty.

(iii) Mention the respective classes for each of the policies given in (i) above.

[(2+2+2+2+2+2)+4+3=19]

3. Prove that a coherent system of independent IFRA components has an IFRA life distribution.

[20]

4. (a) Discuss the steps involved in accelerated life testing experiments.

(b) A manufacturer of Bourdon tubes (used as a part of pressure sensors in avionics) wishes to determine its MTTF. The manufacturer defines the failure as a leak in the tube. The tubes are manufactured from 18 Ni (250) maraging steel and operate with dry 99.9% nitrogen or hydraulic fluid as the internal working agent. Tubes fail as a result of hydrogen embrittlement arising from the piting corrosion attack. Because of the critically of these tubes, the manufacturer decides to conduct ALT by subjecting them to different levels of pressures and determining the time for a leak to occur. The units are continuously examined using an ultrasound method for detecting leaks, indicating failure of the tube. Assume that lifetimes at each stress level follows Weibull distribution. Units are subjected to 3 stress levels of gas pressures(in psi) and the times(in hours) for the tubes to show leaks are given in the following table:

100 psi	120 psi	140 psi
4331	2055	426
5759	2127	435
6529	2801	451
6930	3377	528
7277	3433	613
7668	3947	670
7885	4333	710
8468	4932	836
9652	5264	894
10471	5570	959
11728	5829	1067
12256	6200	1139
13429	6952	1198
14160	7343	1376
17606	9183	1780

Determine the mean lifes for design pressures of 70 and 80 psi.

[3+12=15]

5. (i) Derive the expression for reliability of a component with gamma distributed stress and strength.

(ii) The strength of a component has a gamma distribution with parameters $\lambda = 1$ and $m = 4$. The failure inducing stress is also gamma distributed with $\mu = 1$ and $n = 2$. Compute the reliability of the component.

(iii) The strength of a component is normally distributed with mean 100Mpa and standard deviation 10 Mpa. The stresses acting on the component follow exponential distribution with a mean value of 50 Mpa. Compute the reliability of the component.

[9+4+8=21]

6. Fill in the following blanks from the given choices :

(a) The largest class of life distributions among the following is – – – – – –– *(DFR, DFRA, NWU, NWUE)*.

(b) If $\pi'(h_1, t_1) \overset{LR}{>} \pi(h_2, t_2)$ then the ratio $\frac{\pi'_n(h_1,t_1)}{\pi_n(h_2,t_2)}$ is ------ *(increasing, decreasing)* in n.

(c) *DFR* property is ------ (preserved, not preserved) in mixture of distributions.

(d) If $\pi'(h_1, t_1) \overset{LR}{>} \pi(h_2, t_2)$ then $\alpha(\pi'(h_1, t_1))$ ------$(\geq, \leq)\alpha(\pi(h_2, t_2))$.

(e) *IFR* property is ------ (preserved, not preserved) in formation of coherent systems.

[2×5=10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: 2012-13

Course Name: M. Tech. (QR OR); II Year

Subject Name: Advanced Statistical Methods

Date: 30 .11.2012 Maximum Marks: 100 Duration: 3½ hours

Note: This paper carries 123 marks. You can answer any part of any question, but maximum you can score is 100.

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1) Write Agree or Disagree and briefly Justify

a) One may carry out several ANOVAs for each variable instead of a MANOVA.

b) A multiple linear regression model should not contain terms like X_1^2, X_2X_3, $X_3^2X_4$ etc.

c) A model developed by multiple linear regression method represents the underlying causal model.

d) Single linkage method & complete linkage method would lead to the same cluster.

[3 x 4 =12]

2) An engineer approaches you and gives the following information:

- He has carried out a multiple linear regression analysis with 4 independent variables X_1, X_2, X_3 and X_4 and obtained multiple correlation coefficient $R_1^2 = 0.79$.
- Later on he added two more variables X_5 and X_6 and obtained multiple correlation coefficient $R^2 = 0.84$.
- The total number of observations were 50

He wanted to know if X_5 and X_6 add significantly to the information.

a) State the statistical hypothesis he wanted to test?

b) Outline the method of testing the hypothesis stating clearly all the required assumptions.

c) Show that the test statistic to test the significance of the engineer's hypothesis can be written as

$$F_0 = \frac{(R^2 - R_1^2)/r}{(1-R^2)/(n-p-1)}$$

Where,

r = number of variables whose effect has been hypothesized to be insignificant

n = total number of observations

p = number of independent variables.

d) Test the hypothesis and interpret the results.

[2 + 7 + 6 + 3 = 18]

3) An experiment was conducted to study the relationship of abrasion index (y) for a tire tread compound in terms of three factors:

x_1 : hydrated silica Level, **x_2** : Silane Coupling agent level and **x_3** : Sulfur level

Following table gives the results. Abrasion index (y) is coded

x_1	x_2	x_3	y
-1	-1	1	2
1	-1	-1	20
-1	1	-1	17
1	1	1	98
-1	-1	-1	3
1	-1	1	32
-1	1	1	32
1	1	-1	39
0	0	0	33
0	0	0	33
0	0	0	40
0	0	0	42
0	0	0	45
0	0	0	42

a) Fit a multiple linear regression model relating y to x_1, x_2 and x_3 and test for lack of fit.
b) Is it possible to get the contribution of each of the regressors unconditionally on the others? Explain your answers?

[10 + 5 = 15]

4) a) What is multicollinearity?

b) What is variance inflation factor? How does it help to detect the multicollinearity?

[2 + 4 = 6]

5) a) Samples of size three are drawn from two bivariate normal populations with a common dispersion matrix and possibly different mean vectors μ_1 and μ_2

$$X_1 = \begin{bmatrix} 3 & 2 & 4 \\ 7 & 4 & 7 \end{bmatrix} \quad and \quad X_2 = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 7 & 9 \end{bmatrix}$$

i) Test for the difference between the population mean vectors using an appropriate test procedure.

ii) Construct sample linear discriminant function to separate the groups.

iii) Using the linear discriminant function obtained in (ii) can you say which variable is responsible for the difference?

iv) Classify the observation $x_o' = [2, 7]$ as coming from population-1 or population-2 with equal prior probability and equal misclassification costs.

b) What is optimum error rate of a classification rule? Derive the expression for optimum error rate for a classification rule for two normal populations with equal covariance matrices.

c) Write down the expression for expected cost of misclassification for classification to several populations.

[(8 + 5 + 4 + 2) + (2 + 8) + 5 = 34]

6) (a) What is the difference between principal component analysis and factor analysis?

(b) Explain the terms communality and specific variance in a factor model.

(c) Show that the assumptions of the factor models and communality remain unchanged under orthogonal transformation.

(d) In a consumer-preference study with five variables, the factor loadings were estimated by principal component method using correlation matrix. The estimated factor loadings are given on the following page.

Variable	Estimated factor loadings	
	F_1	F_2
X_1	0.56	0.82
X_2	0.78	-0.53
X_3	0.65	0.75
X_4	0.94	-0.11
X_5	0.80	-0.54

Calculate communalities and specific variance for each variable. Calculate the proportion of total sample variance due to the first factor.

e) What is the purpose of rotation in factor analysis? How is the varimax rotation achieved?

[3 +4+ 6 +4 +5 = 22]

7) (a) What is the difference between classification analysis and cluster analysis?

(b) The distance matrix between pairs of five items is given below.

$$\begin{bmatrix} 0 & 4 & 6 & 1 & 6 \\ 4 & 0 & 9 & 7 & 3 \\ 6 & 9 & 0 & 10 & 5 \\ 1 & 7 & 10 & 0 & 8 \\ 6 & 3 & 5 & 8 & 0 \end{bmatrix}$$

Cluster the five items using the single-linkage method. Draw the dendogram and interpret.

[4+8 + 4 = 14]

INDIAN STATISTICAL INSTITUTE

First Semester Examination : 2012-13

Course Name : M.Tech (QR & OR) 2nd YEAR

Subject Name : Operations Research-II

Date: 04-12-2012 Maximum Marks: 100 Duration: 10:30-13:30

Note if any: Answer any seven from (1) to (8).

1. a) Let $f: R^n \to R$, $g_i: R^n \to R$ for $i=1, \ldots m$. Consider the problem to minimize $f(x)$ subject to $g_i(x) < 0$ for $i = 1, \ldots, m$. Let $\bar{x}$ be a feasible solution and suppose that f and g_i are differentiable at $\bar{x}$. If $\bar{x}$ locally solves the problem, then prove that there exist scalars u_i, $i = 1, 2, \ldots, m$ such that

$$\nabla f(\bar{x}) + \sum_{i=1}^{m} u_i \nabla g_i(\bar{x}) = 0$$

$$u_i g_i(\bar{x}) = 0 \quad \text{for } i = 1, \ldots, m$$

$$0 \neq u \geq 0$$

b) State the dual feasibility of the problem in (a).

[6+4=10]

2. a) Formulate quadratic programming problem as a linear complementarity problem LCP (q, M). State the same formulation in case of linear programming problem.

b) State the Lemke's algorithm mentioning the termination criteria for the LCP formulation in (a).

[4+6=10]

3. a) Can you suggest a method to solve a linear fractional programming problem as linear programming problem?

b) Suggest a method by which a separable nonlinear programming problem can be formulated as linear programming problem.

[5+5=10]

4. a) Define primal feasibility, dual feasibility and complementary slackness conditions of a nonlinear programming problem.

b) State the KKT sufficient conditions of optimality.

[5+5=10]

5. a) Define co-positive plus and co-positive star matrices with examples.
   b) Solve the following linear complementarity problem, LCP (q, M) by using complementary pivoting algorithm.

$$M = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -2 \\ 1 & -1 & 2 & -2 \\ 1 & 2 & -2 & 4 \end{bmatrix} \qquad q = \begin{bmatrix} 2 \\ 2 \\ -2 \\ -6 \end{bmatrix}$$

[4+6=10]

6. Define unimodular and totally unimodular matrix. What is the necessary condition for a matrix to be totally unimodular? Show that, if a matrix A is totally unimodular, then every basic solution of the LP: $Ax = b, x \geq 0$, where A and b are assumed to be integer, is integer. Formulate the situation in which K out of N possible constraints hold.

[4+4+2=10]

7. Consider the following matrix.

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

   a) Is B totally unimodular?

   b) Describe Gomory's cutting plane algorithm.

[4+6=10]

8. Consider the following project for which three time estimates are given.

Activity	t_o	t_m	t_p
1-2	1	2	3
2-3	3	4	5
2-4	2	3	10
3-5	2	2	2
4-5	3	4	11
5-6	1	3	11

a) Draw the network.
b) Estimate the most expected project completion time and its variance from the given three time estimates, after finding the critical path.
c) Compute the probability that the project will be completed within 12 days.

[4+4+2=10]

9. Assignment

[30]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2012 - 2013)

Course Name: M. Tech. (QROR) **Year: 2nd year**

Subject Name: Database Management Systems

Date: December 05, 2012 **Maximum Marks: 100** **Duration: 3 hrs**

Answer all the questions.

1. Describe the advantages of a database management system over a file processing system. [20]

2. a) Define functional dependency with two real life examples.
   b) State the union, decomposition and pseudotransitivity rules of functional dependency, and prove them from the definition of functional dependencies. [3 + (3+4+4+6) = 20]

3. Consider the following relation schemas.

   Student (RollNo, Name, Degree, Year, Gender, DeptNo, ClassTeacher);
   Department (DeptID, DeptName, DeptHead, Phone);
   Professor (EmpID, Name, Gender, StartYear, DeptNo, Phone);
   Course (CourseID, CourseName, Credit, DeptNo);
   Enrolment (RollNo, CourseID, Semester, Year, Grade);
   Teaching (EmpID, CourseID, Semester, Year, ClassRoomNo);
   PreRequisite (PreRequisiteCourse, CourseID).

   a) Write a relational algebra expression for determining the roll numbers and names of the students who are enrolled for every course taught by Professor A. B. and the course does not require any prerequisite course.
   b) Write the equivalent code in SQL for the above query.
   c) Convert the above relational algebra expression into a safe tuple relational calculus expression.
   d) Convert the above safe tuple relational calculus expression into a safe domain relational calculus expression. [7 + 5 + 5 + 3 = 20]

(Please turn over)

4. Consider the following relational schema of a patient database, Patient.

Patient (WardNo, WardName, Location, ChargeNurseName, ChargeNurseID, Telephone, PatientID, PatientName, DateonWaitingList, ExpectedStay, DateAdmitted, DateLeave, DateActualLeave, BedNumber)

The following functional dependencies need to hold on this relation schema:

PatientID DateAdmitted → DateonWaitingList ExpectedStay DateLeave DateActualLeave BedNumber,
PatientID → PatientName,
BedNumber → WardNo,
WardNo → WardName Location ChargeNurseID ChargeNurseName Telephone,
ChargeNurseID → ChargeNurseName,
ExpectedStay DateAdmitted → DateLeave.

a) Determine, with reasons, the normal form which the above relation schema belongs to.
b) Decompose the above relation schema into the schemas that are in 3NF, if the original one is not in 3NF. Give reasons for each step of decomposition. State with reasons whether the functional dependencies remain preserved on this decomposition.
c) Decompose the given relation schema into the schemas that are in BCNF, if the given relation schema is not in BCNF. Give reasons for this decomposition. State with reasons whether this decomposition is dependency preserving.

[2 + (10+2) + (8+2) = 24]

5. Write short notes on any one of the following topics. [20]
a) Transaction management, Concurrency control and Recovery system;
b) Storage & File structure, Indexing & Hashing, and Query Processing;
c) Hierarchical model, Network model and Object based model