BACK PAPER

INDIAN STATISTICAL INSTITUTE

SECOND SEMESTRAL EXAMINATION (2009-2010)
M.S.(QE) II
Advanced Topics in International Economics

Date: 16·08·10   Maximum Marks: 100   Duration 3 hours.

Answer all

1) Show that optimal tariff can be positive even for a small open economy when there is increasing returns to scale. (25)

2) Show that the growth rate that would be arrived at with international spill over of knowledge but without commodity trade might be the same as that which would obtain with both international spill over of knowledge and commodity trade. (25)

3) Show that a temporary monopoly, in spite of the static distortionary loss that it entails, can be welfare enhancing in a dynamic sense. (25)

4) Show that when knowledge spill over is localized and countries engage in commodity trade, the larger country captures the entire share of the global market in the long run. What can you say about the growth rates of the two countries in the steady state? Comment on the growth rate of the larger country when in transition. (25)

Date: 30.08.10 Maximum Marks 40 Duration 3 hours

Use separate booklets for group A & B

Group A

Answer all

1. If an asset i is bought currently at a price p_i and *sold later* at a *price* q_i, the rate of return (r_i) is given by: $r_i = (q_i - p_i)/p_i$. This rate equals the rate of return on a risk-free asset (r_f) in a world of certainty in which case the asset's price is given by:

(A) $$p_i = \frac{q_i}{1+r_f}$$

In an uncertain world, however, q_i is random and the CAPM says that, if $E(q_i)$ is the mean value of the asset's later price and $E(r_M)$ is the mean (rate of) return on an (efficient) market portfolio M with variance σ_M^2, then

(B) $$\frac{E(q_i) - p_i}{p_i} = r_f + \beta_{iM}[E(r_M) - r_f]$$

(a) Derive the expression for β_{iM} from a mean-variance (optimisation) exercise in world with a risk-free asset and interpret the relation (B).

(b) How does the relation (A) then get modified in an uncertain world?

[**Hint:** Note that p_i is known so that $E(r_i) = \{E(q_i) - p_i\}/p_i$. Using this result to express $Cov(r_i, r_M)$ in terms of $Cov(q_i, r_M)$ and then using the relation (B), the new expression for p_i is to be obtained in which the term, $Cov(q_i, r_M)/\sigma_M^2$, will appear.

[7 + 3] = [**10**]

2. (a) Two assets a and b have random rates of return, r_a and r_b, which have the same mean (μ) and the same variance (σ^2) and the correlation coefficient between the two is ρ which is *less than unity*. Show that an equally weighted portfolio achieves the *minimum* variance (among all possible portfolios), *independently* of the value of ρ.

(b) In a *two-period* model a consumer, in the current period, consumes a part of his initial wealth W and invests the rest in N risky assets and one risk-free asset. State the consumer's utility maximisation problem algebraically and find the first- order conditions for utility maximisation. Interpret these conditions.

$$[4+6] = [10]$$

Group B
Answer all

1) a) Show that the initial period consumption in the Ramsey model is a function of the initial asset and the present discounted value of lifetime wage earnings.

b) Using that expression, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital market. In this context discuss the problems associated with either a very low or a very high rate of interest. $[3+7] = [10]$

2) a) Show that all paths other that the convergent one in the Ramsey model would either violate the transversality condition or one of the other necessary conditions for an optimum.
Show that the No Ponzi condition together with optimality delivers what is required by the transversality condition

b) Assuming a Cobb-Douglas production function, work out the transitional dynamics of the savings rate in the Ramsey model. $[5+5] = [10]$

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2010-11

M.S. (Q.E.) II Year
Modern Growth Theory

Date: 06.09.10   Maximum Marks: 40   Duration: 2 Hours

(All questions carry equal marks).

1. State the assumptions of the neoclassical one section growth model. How is the model developed by Mankind Romen and Weil an improvement over this one section model ?

    [7+3]

2. Can you prove the existence of a steady-state growth equilibrium in the neoclassical one section growth model with Hicks-neutral technical progress ? Explain your answer.

    [10]

3. Show that the steady-state growth equilibrium is unique and stable in the model developed by Mankind, Romen and Weil

    [10]

4. Analyse the properties of optimal investment policy in a one section growth model when the objective of the planner is to attain a tangled pen capita capital stock in minimum time. Can you always find a solution to this time minimization problem ? Explain your answer.

    [7+3]

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Mid-Semestral Examination: (2010 –2011)

MSQE II Year

Econometric Methods II

Date:10. 09.2010   Duration: 2 Hours

This question paper carries a total of 35 marks. You can answer any part of any question, but the maximum that you can score is 30.

(All symbols in this paper are self-explanatory)

1. Consider the simple linear regression equation: $y_i = \alpha + \beta x_i + \varepsilon_i$, with usual conditions on the regressor, x, ε_i's are i.i.d. It is given that the true distribution of ε is symmetric and is such that $E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma^2 I_n$. However, you assumed (wrongly) normality for ε.

   (a) Find the QMLE for α and β.

   (b) Show that QMLE estimates of α and β are consistent.

   (c) Show that information matrix equality does not hold.

   (d) Suppose you want to test $H_0:\beta=1$ against $H_1: \beta>1$. Show that the Wald test is not invariant to the algebraically equivalent null hypothesis, $H_{01}: \beta^k = 1$, where k is appositive integer. It is known that β is positive.

   (e) Let $g = \frac{1}{N}\sum x_i \varepsilon_i$. Define $J = \min_\beta ng' Wg$, W is a positive definite matrix. Show that this minimizer is a weighted least square estimate. Show that J follows asymptotically χ^2 distribution for a particular choice of W.

   [2+6+7+10+ (5+5)=35]
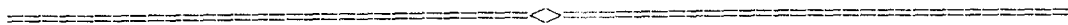
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Mid-semester examination: (2010-2011)

MS(QE) II

Game Theory II

Date: 13.09.2010   Maximum Marks: 60   Duration: 3 hrs.

Note: Answer Group A and Group B in separate answerscripts.

GROUP A

(1) When do you say that a collection of coalitions in a coalitional form game is balanced? State and prove a sufficient condition for the core of such a game to be non-empty using balancedness. (2+2+13=17)

(2) Define the Nash bargaining solution. Show that the solution exists and is unique. (2+5=7)

(3) Define a stable set in an n-person game with a characteristic function V. Can one stable set be subset of another? (4+2=6)

GROUP B

(1) Show that for the non-excludible public goods problem an efficient mechanism satisfies dominant strategy incentive compatibility if and only if it is a VCG mechanism. (20)

(2) Show that in the model of price discrimination where the buyers privately known taste parameter $\theta \in [\underline{\theta}, \bar{\theta}]$, the type-contingent mechanism $\langle q(\theta), T(\theta) \rangle$ is incentive compatible only if

   (i) $q(\theta)$ is non-decreasing in θ, and

   (ii) $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V(q(x)) dx$. (10)

Maximum Marks: 40

Time: 2 hours

Date of Examination: 17.9.10

Answer Question 1 and either Question 2 or Question 3

1. Consider an informal insurance arrangement between two individuals A and B. Each individual has an uncertain income stream over an infinite horizon. In particular, for each person, in each period, income can take a value $y > 0$ with probability p ($0 < p < 1$) and a value 0 with probability $(1-p)$. Every period, each person has a strictly increasing and concave utility function $U(y)$ with $U(0) = 0$. Also the individuals have a common discount rate $r > 0$.

   (a) Define the first best insurance contract and show that it is implementable if the rate of discount is sufficiently small.

   (b) Show that no insurance contract is implementable if the agents are risk neutral having a common utility function $U(y) = y$. Find the intuition behind the result.

   [10 + 10]

2. Using a suitable model, show that a less developed economy might get trapped in a low-level equilibrium and a big push can take this economy out of that equilibrium. [20]

3. Show that if capital markets are imperfect, and individuals accumulate human capital through costly education, initial distribution of wealth matters in determining the long run distribution of wealth. What, if any, is the efficiency implication of this result? [20]

INDIAN STATISTICAL INSTITUTE
Supplementary Mid-Semester Examination: 2010-11

M.S. (Q.E.) II Year
Modern Growth Theory

Date: 04.10.10   Maximum Marks: 40   Duration: 2 Hours

(Answer all question).

1. (a) Condition a Neo-classical one sector growth model with the following production function.

$$Y = K^{\alpha} \, L^{1-\alpha} - \beta K - (1-\beta) L$$

with $0 \le \alpha, \beta \le 1$.

Explain the problem of existence, uniqueness and stability of the steady-state equilibrium in this case when other assumptions of that model remain unchanged.

(b) Do you find any additional problem regarding the existence of steady-state equilibrium in this modified model if the consumption function is $C = Y^{\theta}$ with $0 \le \theta \le 1$. Explain your answer.

[12+8]

2. Consider the Mankiw-Romen-Weil model with an additional assumption that physical capital depreciates at a constant rate. Then show that the steady state equilibrium in this model is unique and stable. Also analyse the effect of an increase in the depreciation rate on the steady-state equilibrium level of per capita consumption.

[20]

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INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2010-2011)

MS(QE) II
Game Theory II

Date: 22.11.2010   Maximum Marks: 100   Duration: $3\frac{1}{2}$ hrs.

Note: Answer Group A and Group B in separate answerscripts.

Note: Answer all questions.

Group A

(1) Show that the nucleolus of a coalitional form game contains exactly one point. (You may assume non-emptiness of the nucleolus.) **(6)**

(2) Define a market game and show that a market game has a non-empty core. **(3+8=11)**

(3) When do you say that a game is strictly competitive? Prove the interchangeability property of such a game. **(2+5=7)**

(4) Demonstrate rigorously the relationship between the bargaining set and the kernel of a coalitional form game. **(10)**

(5) Consider the weighted majority game $(N, v; w, q)$ with the player set $N = \{1, 2, 3, 4\}$, $\underline{w} = (2, 1, 1, 1)$, $q = 3$ and $v : 2^N \to \Re$ is defined by

$$v(S) = \begin{cases} 1 & \text{if } \sum w_i \geq 3 \\ 0 & \text{otherwise.} \end{cases}$$

Identify the bargaining set of this game for the coalition structure $\{\{1, 2\}, \{3\}, \{4\}\}$. **(10)**

(6) Do you agree or disagree with the statement that for a superadditive game the core and dominant core coincide? In either case justify your answer. **(6)**

Group B

(1) Consider the model of discriminating monopolist, where type (taste parameter) θ of a buyer lies in the interval $[\underline{\theta}, \overline{\theta}]$. $\underline{\theta} > 0$. Show that the mechanism $(q(\theta), T(\theta))$ is incentive compatible *only if*

(i) $q(\theta)$ is non-decreasing in θ, and

(ii) $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V(q(x)) dx$.

(20)

1

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(2) Consider the bilateral trading model under incomplete information. Assume that the seller's valuation is distributed with positive probability density over the interval $[a_1, b_1]$, the buyer's valuation is distributed with positive probability density over the interval $[a_2, b_2]$ and that the interiors of these intervals have a non-empty intersection. Then show that no incentive compatible and individually rational trading mechanism can be ex-post efficient. State the relevant definitions and results. (30)

MSQE II

Economic Development I

Semestral Examination

Maximum Marks 60

Time 3 hours

Date 24.11.10

Answer question 1 and any two from the remaining.

1. Consider an agricultural market consisting of two periods: period 1 (the busy season) and period 2 (the lean season). In period t, ($t = 1, 2$) market demand is given by the inverse demand function

$$p_t = 100 - Q_t$$

Where p_t is market price at period t and Q_t is <u>total</u> quantity sold at period t. There are two types of sellers in the market. There are cash constrained small sellers who are compelled to sell their stocks in period 1. Together they possess 50 units of stocks. In addition, there is a large seller who is large enough to affect the market price. The large seller has a total of 40 units of stocks which he allocates for sales between the two periods to maximize his profits. There is no storage cost or discounting.

(a) Find equilibrium price, total market sales and sales by the large seller for each period.

(b) Suppose instead of 50 units, total amount of stocks of small sellers is 90 units while the initial stocks of the large seller remains at 40 units. Find the large seller's optimal policy and the equilibrium market price and quantity in each period.

(c) Assume that initial stocks are as in (b) above. Find equilibrium price, sales by the large seller and sales by the small (price taking) sellers in each period if the small sellers are not cash constrained and are free to allocate their sales between the two periods to maximize their profits. Comment on the equilibrium outcomes of (b) and (c).

(d) Go back to the initial specification of total stocks, i.e., 50 for the small sellers and 40 for the large seller. Suppose there is a government who announces a procurement price of 46 in period 1 at which it is willing to buy any amount. Find market sales and sales to the government by the large seller and the small sellers in the two periods.

[5+5+5+5]

2. Using a model of increasing returns, show how the long run equilibrium may depend upon the interplay of initial conditions and future expectations. Indicate, in particular, the possible paths to the long run equilibrium.

[20]

3. Characterize an interlinked contract between a farmer (agent) and a trader-cum-lender (principal) where the agent takes a production loan from the principal and is compelled to sell his produce to the principal at the contracted interest and price. Show that the contract is efficient.

[20]

4. Using a model of majority voting, show that an increase in inequality reduces the rate of growth of an economy by increasing the divergence between the actual tax rate and the growth maximizing tax rate. Can a perfectly egalitarian society achieve the growth maximizing tax rate?

[20]

Note: Answer Question 1 and any **three** from the rest of the questions

1. Using the following data on the Per-capita Expenditure (PCE) compute (i) the Head Count Ratio (H), (ii) the Income Gap Ratio (I) and (iii) the Sen's Index of Poverty (P). Assume that the poverty line was Rs. 20 per 30 days in the base period and the current CPI for agricultural labourers with that base is Rs. 400.

Table 1: The size distribution of population by PCE

PCE (Rs./30 days)	Percentage of population	Average PCE (Rs./30 days)
0 – 30	0.91	24.80
30 – 40	2.48	35.79
40 – 50	5.10	45.42
50 – 60	7.98	55.23
60 – 70	9.75	65.15
70 – 85	15.35	77.35
...	...	...

[25]

2. What do you mean by concentration of firms in an industry? Write down the axioms proposed by Hall and Tideman in this context. Define Absolute Concentration Ratio (ACR), Herfindahl-Hirschman index (HHI) and Hall and Tideman index (HTI). Prove that HHI and HTI satisfy all these axioms. Is there any other index which satisfies all these axioms? [25]

3. (a) Suppose income of each person is increased by a fixed proportion. Assuming a constant elasticity form of an Engel curve describe the changes in demand of a specific commodity.

(b) Is it true that the demand for a commodity will always increase if the mean income increases and the inequality of income decreases for a given group of people? Explain your answer assuming a suitable form of the Engel curve and a specific income distribution.[7+18]

4. Define Cobb-Douglas (CD) production function. State and prove its properties. Describe the cost minimization and profit maximization procedures of obtaining the optimum quantities of inputs and outputs under CD production function set up. [25]

5. Write short notes on any two of the following:

   a. Prais Houthakker Formulation of an Engel Curve.
   b. Pareto law of income distribution.
   c. Popular forms of Engel Curves.
   d. Properties of Lorenz curve of two-parameter lognormal distribution.   [12½+12½]

Date: 30.11.10   Maximum Marks 60   Duration 3 hours

Group A

Answer any two

1. With only risky assets, the covariance between returns on any *two frontier* portfolios P and Q (with respective weight vectors w_P, w_Q and return vectors r_P, r_Q,) is given by

$w'_P V w_Q$ = $(C/D)\{ E(r_P) - A/C \}\{ E(r_Q) - A/C \} + 1/C$, [A, C, D are positive constants involving asset returns' variance- covariance matrix V and other parameters].

 (a) Given the above result show graphically, for any efficient frontier portfolio P, its zero covariance frontier portfolio, $zc(p)$ and find its mean return, $E(r_{zc(P)})$.

[*Hint* Using the expression for $\sigma^2_{r_P}$ find that $E(r_P) - \dfrac{\partial E(r_P)}{\partial \sigma(r_P)} \cdot \sigma(r_P)$ equals $E(r_{zc(P)})$.]

(b) Add to the above set of risky assets, a risk-free asset with the rate of return, r_f. Define the **Sharpe Ratio** of a portfolio consisting of only risky assets. Argue how one can find the efficient portfolio frontier with the help of this Ratio in this case. How does this frontier look like?

$$[9 + 6] = [\ 15\ ]$$

2. Write a note on the development of macroeconomic theory and policy prescription from the so-called Keynesian revolution up to the New Classical School.

$$[\ 15\ ]$$

3. What does *inter-temporal substitution of* leisure mean in the context of a RBC model? Develop a suitable two-period utility maximization model of a household (with perfect foresight) to show how such a substitution may arise. If wage rates in both periods were

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to rise in the same proportion, would the household increase its labour in the current period/ in the future period/ in both periods/ in neither period? Give reasons.

[15]

Group B
Answer all

1. a) Show that an increased contribution to a "Pay as you go" pension fund will lower the steady state capital labour ratio.
   In this context what can you say about the new steady state welfare of the economy compared to the earlier steady state.

   b) Show that a fully funded pension system is neutral in terms of its effect on capital accumulation.

   $[12+3]=[15]$

2. a) In the Blanchard Yaari model, assume that wages are age independent (i.e. $\alpha = 0$, in the usual notation) and population growth rate (n) is zero. Now, show that the steady state capital accumulation does not exhibit dynamic inefficiency.

   b) Assume that the representative household's lifetime utility function is given by $V = V(C_1,\ C_2)$, where C_i is consumption in period i, $V_i \equiv \frac{\partial V}{\partial C_i} > 0$ and $V_{ii} \equiv \frac{\partial^2 V}{\partial C_i^2} < 0$. No restriction is put on V_{12} .
   Households' intertemporal budget constraint is given by
   $C_1 + \frac{C_2}{1+r_1} = (1+r_0)A_0 + \left[Y_1 + \frac{Y_2}{1+r_1}\right] \equiv \Omega$, where Y_i is the exogenous income in period 'i', A_0 is the initial financial wealth and r_i is the interest rate in period 'i'.

   What can you say about the changes in period consumption C_i's due to a ceteris paribus change in wealth Ω?

   $[10+5]=[15]$

INDIAN STATISTICAL INSTITUTE
First Semester Examination : 2010-11
M. S. (QE) II Year
Modern Growth Theory

Date : 3.12.2010   Maximum Marks: 60   Time: 3 Hrs.

Answer any three questions.

1.(a) How does the model of Benhabib and Farmer (1994) differ from the Ramsey-Solow model ?

(b) Show that the steady state equilibrium is unique in the model of Benhabib and Farmer.

(c) Is the transitional growth path also unique in this model? Explain your answer.

(4+8+8)

2.(a) State the assumptions of the FMS (1993) model of endogenous growth.

(b) Derive the optimum income tax rate in this model when the government maximizes the long run growth rate.

(c) Show that the steady-state equilibrium is unique and is a saddle point in this model.

(6+6+4+4)

3.(a) In the context of Lucas (1988) model of endogenous growth, analyse the validity of the following statements:

(i) In the absence of external effect of human capital on production, competitive equilibrium growth rate is equal to the socially efficient growth rate.

(ii) Rate of growth varies positively with the consumption rate of discount.

(iii) If the human capital accumulation sector is unproductive, the steady-state equilibrium is a 'no growth' equilibrium.

(b) Derive the long-run growth rate in the model of Romer (1990).

(3 × 3 + 11)

4.(a) Show that the steady-state equilibrium in the Solow (1956) model is unique and stable.

(b) Is the existence of steady-state equilibrium consistent with capital augmenting technical progress? Explain your answer in the context of the Solow model.

(14+6)

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Date: 6.12.10   Maximum Marks: 100   Duration: 3 hours

This question paper carries a total of **120** marks. You can answer any part of any question; but the maximum that you can score is **100**. Marks allotted to each question are given within parentheses. Answer **PART–I and PART–II on separate answerscripts**.

PART – I

1. (a) Explain the nature of dependence implied by an ARCH (q) process.

 (b) Show that a GARCH (p,q) process for $\{x_t\}$ can be represented as an ARMA process of suitable orders for $\{x_t^2\}$.

 (c) Find the unconditional second and fourth order central moments of a GARCH $(1,1)$ process, and hence obtain its kurtosis coefficient as

$$\frac{3\left[1-(\alpha_1+\beta_1)^2\right]}{1-(\alpha_1+\beta_1)^2-2\alpha_1^2}.$$

 (*Notations have their usual meaning.*)

[3+5+12=20]

2. (a) Show that the information matrix is block diagonal between the mean and the variance parameters of an ARCH (p) model in regression framework. Do you think that the same holds for an ARCH-M model? Justify your answer.

 (b) Explain what is meant by 'leverage effect' in financial variables. Discuss how the EGARCH model captures this effect as well as volatility contained in financial time series.

 (c) Consider a GARCH model in the regression set-up in which the dependent variable is transformed by the Box-Cox transformation. Do you think that this extension is a better formulation than the usual GARCH model? Justify your answer.

[10+6+4= 20]

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3. (a) Describe how error correction model (ECM) is related to cointegration.

(b) Discuss the main shortcomings of the standard residual-based test of cointegration in single-equation models. Also describe the ECM test and indicate why this test can be considered to be better than the residual-based test.

(c) Discuss briefly the identification problem in the context of cointegrated system of equations.

[5+10+5= 20]

PART – II

1. Consider the fixed effects panel data model with CLRM assumptions as:
$$y_{it} = \alpha_i + x_{it}{}' \beta + \epsilon_{it}$$
    (a) Show that $\hat{\beta}_{FE}$ is consistent.
    (b) Show that $\hat{\beta}_B$ is consistent.
    (c) Show that $\hat{\beta}_{OLS}$ for the (wrong) Model
    $$y_{it} = \alpha_i + x_{it}{}' \beta + \epsilon_{it} \text{, is inconsistent.}$$

[7+7+6=20]

2. Consider the random effects panel data model with usual CLRM assumptions as
$$y_{it} = \alpha_i + x_{it}{}' \beta + \epsilon_{it}$$

    (a) State all the assumptions clearly.
    (b) Show that the $\hat{\beta}_{RE}$ is consistent.
    (c) Show that the $\hat{\beta}_{FE}$ is also consistent.
    (d) How will you test for random effects model vs fixed effects model? Derive the test.

[3+4+6+7= 20]

3. Consider the multiple linear regression model with CLRM assumptions as
$y = X\beta + \epsilon$, ϵ follows normal distribution.

Let $\theta = \left(\sigma^2, \beta' \right)'$.

    (a) Find the information matrix for θ.
    (b) Derive LM, Wald & LR test for $\beta = \beta_0$.

[5+ (5+5+5)=20]

Date 12.01.2011   Maximum Marks: 100   Duration: 3 hours

Group A

Answer any two

1. (a) In a CAPM with only risky assets, two **frontier** portfolios P and Q will satisfy

$$Cov\,(r_P, r_Q) = \frac{C}{D}\left[\left\{E(r_P) - \frac{A}{C}\right\}\left\{E(r_Q) - \frac{A}{C}\right\}\right] + \frac{1}{C},$$

where $A = e'V^{-1}1$, $B = e'V^{-1}1$, $C = 1'V^{-1}1$, $D = BC - A$, e is the vector of assets' *mean* returns, 1 is the *sum vector* and V is the *variance-covariance matrix* of the assets' returns. **Given this result**, show that for any portfolio Q (*not necessarily* a frontier portfolio), $E(r_Q)$ can be expressed as a linear combination of $E(r_P)$ and $E(r_{zc(P)})$ where $ZC(P)$ is the zero covariance portfolio of the portfolio P.

(b) Rates of return of two risky assets a and b have identical means (μ) and identical variances (σ^2) and the correlation coefficient between the two (ρ) is 0.5. The variance of a portfolio of these two assets (denoted by σ_P^2) depends on ρ, σ^2 and the proportion (x) of the total fund invested in the asset a. Draw σ_P against x in a (σ_P, x) diagram.

[16 + 9] = [25]

2. Suppose, the central planner seeks to maximize intertemporal utility of an (infinitely lived) representative household in an economy subject to $C_t + K_{t+1} = Y_t$ and the production function for Y_t (specified below). Suppose, the *optimal* path is given by

$K_t = a Y_{t-1}$ (a depends on the parameters of the model and is a positive constant);

$C_t = (1-a)Y_t$ and $Y_t = L^{1-\alpha} K_t^{\alpha} A_t$ (optimal labour is a constant L every period);

where C_t, Y_t and A_t are respectively consumption, output and productivity in period t and K_t is the capital stock at (the *beginning* of) period t.

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Suppose, z_t ($\equiv \ln A_t$) is given by the process: $z_t = z_{t-1} + d + \varepsilon_t$ (ε_t, a *white noise*)

(a) Interpret the process z_t.

(b) Find the effects of a one-time shock to ε on the paths of Y_t and K_t.

$$[10 + 15] = [25]$$

.Write a note on the macroeconomic policies pursued in India.

$$[25]$$

Group B
Answer all

1. In the Blanchard Yaari model with cohort dependent wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation? Explain.

[2

2. In an infinite horizon model, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital mark

In this context discuss the problems associated with either a very low or a very high rate of interest.

[2

INDIAN STATISTICAL INSTITUTE
First Semester Back Paper Examination: 2010-11

M.S. (Q.E.) II Year
Modern Growth Theory

Date: 21.01.11   Maximum Marks: 100   Duration: 3 Hours

(Answer all questions).

1. (a) Consider an otherwise identical solow model with the following modified consumption function :

$$C = a + (1 - s)\ Y$$

where $a > 0$ is a constant and $0 < s < 1$ is the constant MPS.

Can you prove the existence of a steady state equilibrium in this modified model ? Explain your answer.

(b) Show that the steady-state equilibrium in the Mankiw-Romen-Weil (1992) model is unique and stable.

[13+20]

2. (a) Derive the long run rate of growth in Barro (1990) model of endogenous growth.

(b) Explain why transitional dynamic properties do not arise in this model.

[25+9]

3. Human capital accumulation is necessary but not sufficient to explain endogenous growth in Lucas (1988) model – Discuss.

[33]

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MSQE II

Economic Development II

Back Paper Examination

24·01·11

Maximum Marks 100

Time 3 Hours

Answer all questions.

1. Consider an informal insurance arrangement between two individuals A and B. Each individual has an uncertain income stream over an infinite horizon. In particular, for each person, in each period, income can take a value y > 0 with probability p (0 < p < 1) and a value 0 with probability (1-p). Every period, each person has a strictly increasing and concave utility function U(y) with U(0) = 0. Also the individuals have a common discount rate r > 0.

   (a) Define the first best insurance contract and show that it is implementable if the rate of discount is sufficiently small.

   (b) Show that no insurance contract is implementable if the agents are risk neutral having a common utility function U(y) = y. State the intuition behind the result.

   [15+15]

2. Discuss how a sharecropping arrangement can (a) reduce risks when both the landlord and the tenant are risk averse; (b) solve the incentive problem when a risk neutral landlord is unable to observe the work-effort put in by a risk averse tenant.

   [20+20]

Suppose, $z_t \, (\equiv \ln A_t)$ is given by the process: $z_t = z_{t-1} + d + \varepsilon_t$ (ε_t, a *white noise*)

(a) Interpret the process z_t.

(b) Find the effects of a one-time shock to ε on the paths of Y_t and K_t.

$$[10 + 15] = [25]$$

3. Write a note on the macroeconomic policies pursued in India.

$$[25]$$

Group B
Answer all

1. In the Blanchard Yaari model with cohort dependent wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation? Explain.

[2$

2. In an infinite horizon model, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital marke

In this context discuss the problems associated with either a very low or a very high rate of interest.

[2

INDIAN STATISTICAL INSTITUTE
First Semester Back Paper Examination: 2010-11

M.S. (Q.E.) II Year
Modern Growth Theory

Date: 21.01.11   Maximum Marks: 100   Duration: 3 Hours

(Answer all questions).

1. (a) Consider an otherwise identical solow model with the following modified consumption function :

$$C = a + (1 - s)\ Y$$

where $a > 0$ is a constant and $0 < s < 1$ is the constant MPS.

Can you prove the existence of a steady state equilibrium in this modified model ? Explain your answer.

(b) Show that the steady-state equilibrium in the Mankiw-Romen-Weil (1992) model is unique and stable.

[13+20]

2. (a) Derive the long run rate of growth in Barno (1990) model of endogenous growth.

(b) Explain why transitional dynamic properties do not arise in this model.

[25+9]

3. Human capital accumulation is necessary but not sufficient to explain endogenous growth in Lucas (1988) model – Discuss.

[33]

*****

MSQE II

Economic Development II

Back Paper Examination

24·01·11

Maximum Marks 100

Time 3 Hours

Answer all questions.

1. Consider an informal insurance arrangement between two individuals A and B. Each individual has an uncertain income stream over an infinite horizon. In particular, for each person, in each period, income can take a value $y > 0$ with probability p $(0 < p < 1)$ and a value 0 with probability $(1-p)$. Every period, each person has a strictly increasing and concave utility function $U(y)$ with $U(0) = 0$. Also the individuals have a common discount rate $r > 0$.

   (a) Define the first best insurance contract and show that it is implementable if the rate of discount is sufficiently small.

   (b) Show that no insurance contract is implementable if the agents are risk neutral having a common utility function $U(y) = y$. State the intuition behind the result.

   [15+15]

2. Discuss how a sharecropping arrangement can (a) reduce risks when both the landlord and the tenant are risk averse; (b) solve the incentive problem when a risk neutral landlord is unable to observe the work-effort put in by a risk averse tenant.

   [20+20]

3. Using a suitable model of inter-temporal stock holding sales, demonstrate (a) large sellers stay in the market for a longer period than small sellers; (b) social welfare (sum of consumers' and producers' surplus) is maximized when a sufficiently large number of sellers are price-takers.

[15+15]

Date: 23-02-2011   Maximum Marks 40   Duration 3 hours

Use separate booklets for group A & B

Group A

Answer all

1. Consider a two-country-three-good Ricardian model of international trade. Good 1 is produced by the home country alone and good 3 is produced only by the foreign country. Good 2 is produced by both home and foreign. Analyze the effects of a technical progress (i.e. a fall in the labour coefficient) in (a) good 1, (b) good 2 and (c) good 3 on home and foreign real wages. How would your answer change if home produces goods 1 and 2 while foreign produces only good 3? [15]

2. In a two-country-two-good pure exchange model of international trade, the home country is endowed with 100 units of good 1 and 100 units of good 2 while the foreign country is endowed with 150 units of good 1 and 50 units of good 2. Each country has the same utility function $U = C_1^\alpha C_2^{1-\alpha}$, $0 < \alpha < 1$, where C_1, C_2 are consumption of the two goods.
If $\alpha = 1/4$, find the free trade equilibrium relative price, export and import of each country.
[5]

Group B

Answer all

1. Show that in a two agent setting, Walras stability guarantees that the recipient of a transfer necessarily gains. [10]

P.T.O

2. Consider a two commodity trading world in which a country is small in the sense that it faces <u>constant</u> international prices of commodities (p^* in our class room notation).

What can you say about the change in

a) Imports

b) welfare

of the country if it were to impose a tariff on its imports (beginning from a position of free trade)?

[7 · 3=10]

Date: 25.02.11   Maximum Marks: 100   Time: 3 hours

Answer question no.1 and any three from the rest.

Questions are of values as indicated in the margin.

1. Are the following statements 'TRUE' or 'FALSE'? Give reasons for your answer:

(a) Technical efficiency of farmers can best be judged by the index of 'scale efficiency'.

(b) When uncertainty and risk are taken into account the adoption of sharecropping becomes more plausible.

(c) Social efficiency prices should be closer to the prices confronted by small farmers than the large ones.

(d) 'Cheap labour hypothesis' refers to the intense use of hired labour by large farms.

(4 × 10 = 40)

2. Define allocative efficiency. Formalize the neo-classical concept of allocative efficiency for two sets of inputs for producing two types of crops, say rice and wheat, respectively.

(6+14= 20)

3. Distinguish between frontier and non-frontier production functions. Examine the problem of technical efficiency of farms using non-frontier profit function technique.

(6 + 14=20)

4. Distinguish between 'fixed tenancy' and 'share tenancy' in agriculture. Elucidate the basic micro-economic models of sharecropping in a competitive environment. Discuss theoretically the conditions under which share tenancy becomes efficient.

(4 +6 +10 = 20)

5. Write short notes on the following:

(a) 'Catching up' phenomenon previling during the post-green revolution period in Indian agriculture.

(b) Statistical validity of the inverse relationship between farm size and productivity.

(10 +10= 20)

INDIAN STATISTICAL INSTITUTE

Mid-semestral Examination: (2010-2011)

MS(QE) II

Social Choice and Political Economy

Date: .03.2011   Maximum Marks: 60   Duration: $2\frac{1}{2}$ hrs.

Note: Answer Group A and Group B in separate answerscripts.

Group A

(1) (a) Define second order stochastic dominance.

(b) Give an example of a utility function which is consistent with first and third order stochastic dominance criterion but not with second order stochastic dominance criterion. **(2+3=5)**

(2) (a) Define Harsanyi's aggregation equation. State and prove Harsanyi's aggregation theorem for Pareto Indifference. **(2+13=15)**

(b) The expected utility functions of two persons are

$$V_1(p) = 15p_0 + 10p_1 + 20p_2,$$
$$V_2(p) = 5p_0 + 15p_1 + 25p_2.$$

The society expected utility function is

$$V(p) = 10p_0 + 10p_1 + 20p_2.$$

Determine the vector $(\lambda_1, \lambda_2, \mu)$ in the Harsanyi aggregation equation. **(10)**

Group B

(1) For the social aggregation problem with two alternatives give an example of a social welfare function that satisfies Pareto and neutrality but fails to satisfy symmetry and positive responsiveness. Justify your answer. **(8)**

(2) State and prove Arrow's Impossibility Theorem by giving all the relevant definitions. **(5+15+2=22)**

Indian Statistical Institute
Mid Semestral Examination: (2010 – 2011)
M.S. (QE) – II year
Econometric Applications II

Date: **04·03·11**   *Maximum Marks – 50*   *Duration: 2 hours*

Answer any two questions

1. a) Consider the following income distribution $x = (6, 1, 8)$ with distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{1}{3} & 1 \le x < 6 \\ \dfrac{2}{3} & 6 \le x < 8 \\ 1 & 8 \le x \end{cases}$$

Find the Lorenz curve and Gini coefficient.

b) Suppose the Lorenz curve $L(p)$ $(0 \le p \le 1)$ of a continuous type income distribution is given by

$$L(p) = 1 - (1 - p)^{1 - \frac{1}{a}}$$

Find the underlying distribution.

c) Let the income variate x be continuous and

$$F(x) = \frac{1}{1 + \left(\dfrac{x}{x_0}\right)^{-a}}, \quad x \ge 0, \quad a > 1, \quad x_0 : \text{median}.$$

Find the Lorenz Curve and Lorenz ratio.

d) Show that the proportional failure rate (PFR) for Pareto distribution is constant.

[8+6+8+3=25]

2. a) What are 'relative' and 'absolute' measures of income inequality? Give examples.

b) State and explain the desirable properties that an inequality index should satisfy.

c) Examine the following inequality index in the light of the above properties.

$$I(x) = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}}{\mu}$$

[5 : 8 : 12 25]

3. Suppose you are given a data set of K income classes (x_{i-1} to x_i) with the corresponding frequency of population in each income class (n_i, $i = 1, 2, ..., K$).

a) Describe a method to fit the log-logistic distribution to the above data set.
b) How do you judge the goodness-of-fit?
c) Briefly describe the method of fitting a non parametric distribution to the same data set.

[8+9+8=25]

Answer Each Group on a Separate Answer Script. Each question carries 8 marks.

Group A: Answer any THREE questions.

1. Consider the following Stackelberg equilibrium with free entry. The market demand for the product is linear and all firms have the same cost function, $C(q) = cq$, $c > 0$ but there is a positive sunk entry fee. In the first stage, a firm, leader, enters the market by paying the entry fee, and then commits the level of its production. In the second stage, after knowing the decision of the leader, all the potential followers simultaneously decide "in" or "out"; the firm which chooses "in", pays the entry cost. In the third stage, all the followers who have entered choose their quantities simultaneously as Cournot players. Find the subgame perfect Nash equilibrium of this game.

2. Assume that the market demand function for a product is linear, and production involves fixed cost as well as constant marginal cost. Find the condition under which an unregulated market system will not lead to any production at all despite there being a positive production level that is socially desirable. Find the socially optimal output level given the demand and the cost functions. How do you implement, by means of a tax-subsidy policy, the socially optimal production level by the monopolist?

3. Assume an n (≥ 2)- firm Cournot equilibrium. Each firm has a linear cost function but they are subject to specific unit taxes. Now, if the tax distribution changes, how does it affect (i) the tax revenue of the government, and (ii) welfare of the country?

4. (a) Show that the free entry Cournot equilibrium leads to "excess" entry.

    (b) If two firms, each producing a substitute product, compete in prices and face similar cost and demand structures that give positive sloping reaction functions, then show that each firm must prefer being the follower to being the leader.

5. Consider two firms with unit costs c_1 and c_2 respectively. $0 < c_1 < c_2$. The firms compete in prices. Find the equilibrium in the following cases.

(a) The firms choose their prices sequentially.

(b) They choose their prices simultaneously but there is a predetermined capacity constraint for each firm.

• Group B: Answer any TWO questions

1. Suppose that consumers have the following preferences:

    $U = \theta V(q) - T$, if they pay T and consume q units. $V(q) = \left[1 - (1 - q)\right] q$

    $= 0$ if they do not consume

    There are two groups of consumers: those with taste parameter θ_1 in proportion λ and those with taste parameter θ_2 in proportion $1 - \lambda$. The monopolist produces the good at constant marginal cost c. $c < \theta_1 < \theta_2$.

    Show that for any linear tariff $T(q) = pq$ with $p > c$ there exists a two part tariff $\widetilde{T}(q) = \widetilde{A} + \widetilde{p}q$ such that if both type of consumers are offered the choice between T and $\widetilde{T}$ both types of customers and the firm are made better off.

2. Consider a monopsonist buying a product valued at $R = bq^{1}$ from a subcontractor whose unit cost of production c may be low or high with probabilities p and $(1 - p)$. The actual cost is known to the subcontractor while the monopsonist observes only the probability distribution.

Show that only the efficient (low cost) contractor sells the optimum (first–best) amount to the monopolist and obtains an information rent.

3. There are two goods. The price of the y-good is 1 and each consumer has an initial wealth of 15. A monopolist supplies the x-good. He has a constant marginal cost of 1.2 up to his capacity constraint of 10. He will offer at most two price-quantity packages, (r_1, x_1) and (r_2, x_2), where r_i is the total cost of purchasing x_i units. Two consumers have utility functions $u_1(x_1, y_1) = a_1 \log(1 + x_1) - r_1$ and $u_2(x_2, y_2) = a_2 \log(1 + x_2) - r_2$ where $2 < a_1 < a_2 < 7$. He cannot identify which consumer is which, but he can prevent resale.

(a) Write down the monopolist's profit maximization problem. You should have four constraints plus the capacity constraint.

(b) Which constraints will be binding at the optimal solution?

4. A Monopolist produces a durable good that does not depreciate in the first period but lasts for only two periods. The cost of production is zero and the monopolist and consumers have the same discount factor δ where $\delta = 1/(1 + r)$. The consumption demand for the good in each period is $D(p) = 1 - p$.

a) Show that if the monopolist sells the good in each period and a resale market exists in which goods sold in the first period can change hands, then the first period price always exceeds the second period price.

b) Would the monopolist gain by reducing the durability of the good to one period?

Date: 2/5/2011   Maximum Marks: 40   Duration: 3 Hours

Use separate answer script for each group.

Group A: Answer any <u>two</u> questions.

1. Two firms are engaged in Bertrand competition. The demand function is: $X(p) = 2 - p$, $p \in [0.2]$. Firms' constant unit costs are c_1 and c_2 where, $2 > c_2 > c_1 > 0$. Suppose unit costs are originally $c_2 = 1.5$ and $c_1 = 1.0$. Then an outside innovator discovers a new technology that leads to a lower unit cost $c^* = 0.75$. The outside innovator sells the exclusive right to use the new technology to the highest bidder. (Assume that there is an arbitrarily small monetary unit.)

   (a) How does the equilibrium solution change if the innovation is acquired by firm 2 or by firm 1?

   (b) Who will get the technology under the bidding game and at what price?

   (c) Suppose the innovation was discovered by firm 2, rather than an outside innovator. Firm 2 has option to sell the innovation to firm 1. What is the equilibrium outcome?

   $$[4+2+4=10]$$

2. (a) A monopolist (M) faces demand curve $p = 1 - q$ for a product in each of the two periods, 1 and 2. Its unit cost of production is $0 < c < 1$ in period 1 and $c - \lambda q_1^M$ in period 2, where $\lambda > 0$ and q_1^M is the first period output (the firm learns by doing). Assume the discounting rate to be zero. Derive q_1^M in equilibrium.

   (b) Suppose now that M faces an entrant (E) in the second period; E has unit cost c and an entry cost $F > 0$. If entry occurs, the firms play Cournot game. Find M's first period output.

P.T.c

(c) If M follows entry deterrence strategy (and accordingly decides q_1^V), what will be the corresponding first period output of M? Also derive its total profits under entry deterrence.

[3+3+4=10]

3. Consider a Hotelling duopoly, with firms located at points 0 and 1, and consumers uniformly spread over the interval $[-a, 1+a]$ where $a > 0$. Firms have constant and equal marginal costs $c > 0$. Each consumer wants just one unit of the good, and is willing to pay a lot for it if necessary but naturally wants to pay as little as possible. Consumers bear a 'transportation cost' td, where d is the distance to the firm patronized and t is a constant.

(i) Calculate the equilibrium prices charged by the firms. Explain your results compared to the case when $a = 0$.

(ii) How does the transport cost paid by all consumers together depend on a?

[6+4=10]

4. Consider a Cournot oligopoly with linear demand $p(Q) = \max\{0, A - Q\}$ and with N firms, and suppose that each firm i has constant unit costs $c_i > 0$, but the values of c_i may differ across firms. Firms know each other's costs and compete in Cournot fashion.

(a) Show that as long as each firm is "active" (producing a positive quantity), the price p depends *only* on N and on $\bar{c}$, the simple average value of c_i's.

(b) Continuing to assume that all firms are active, what is the industry average cost of production in the Cournot equilibrium? In particular, show that industry average cost of production falls as cost asymmetry goes up.

(c) In this context also derive the relation between the industry average cost and Herfindahl index of market concentration (H).

[3+4+3=10]

Group B: Answer any <u>two</u> questions

1. A natural monopolist has total costs $C(Q) = 300 + 15Q$ and faces market demand $Q = 200 - 2P$. Solve for the monopolist's profits, output, and consumer surplus when:

(a) Price is set equal to marginal cost.

(b) Price is set equal to average cost.

(c) There is two-part pricing, and the monopolist sets them to maximize profits.

(d) There is two-part pricing, and the regulator sets them to maximize consumer surplus, subject to the monopolist breaking even.

[For parts (c) and (d), you can assume that there are 10 identical consumers.]

[2+2+3+3=10]

2. An Entrepreneur (E) has a project which needs an initial investment of k. The project's random output, x, depends on E's choice of effort as follows: $x = ey$, where $e > 0$ is E's choice of effort and y is a random variable distributed uniformly on $[0, 2]$. E's private cost of effort is $g(e) = (1/2)e^2$, and this effort is unobservable, while the output is observable and verifiable. E makes a take-it-or-leave-it offer to an investor (I). Assume that E has no starting capital, so that I will have to pay the start-up cost k. The contract also specifies the sharing rule of output, so that when output is x, E leaves $w(x)$ to himself and the rest goes to I. E has limited liability which constrains $w(x) > 0$ for all x. Both parties are risk-neutral, and the market interest rate is normalized to zero.

(a) Suppose that E's effort is observable and verifiable. Solve for the first best level of effort. For what values of k would the project be worth undertaking if there were no moral hazard?

(b) From now on assume that the project is worth undertaking. Show that despite the unobservability of effort by E, an optimal contract implements the first-best level of effort. [Hint: Consider contracts of the form

$$w(x) = 0 \quad \text{if } x < a,$$
$$= w_o \quad \text{if } x > a. \ ]$$

[4+ 6=10]

3. A team of two workers produces and sells chairs for the principal. Each worker chooses high or low effort. An agent's utility is $U = w - 2v$ if his effort is high, and $U = w$ if it is low, with a reservation utility of $\overline{U} = 0$. Nature chooses business conditions to be excellent, good, or bad, with probabilities θ_1, θ_2 and θ_3. The principal observes output but not business conditions, as shown in the table.

P.T.O

	Excellent θ_1	Good θ_2	Bad θ_3
High, High	100	100	60
High, Low	100	50	20
Low, Low	50	20	0

(a) Suppose $\theta_1 = \theta_2 = \theta_3$. Why is $\{(w(100) = 30, w(\text{not } 100) = 0), (\text{High, High})\}$ not an equilibrium?

(b) Suppose $\theta_1 = \theta_2 = \theta_3$. Is it optimal to induce high effort? What is an optimal contract with nonnegative wages?

(c) Suppose $\theta_1 = 0.5$, $\theta_2 = 0.5$, $\theta_3 = 0$. Is it optimal to induce high effort? What is an optimal contract (possibly with negative wages)?

$$[3 + 3 + 4 = 10]$$

4. Assume that ex ante investment affects the quality of investment and thus the value to the buyer. The buyer's ex post value is

$$v(I) = 3I - \tfrac{1}{2}I^2.$$

The buyer's and the seller's surplus from the trade are respectively

$$v(I) - p \quad \text{and} \quad p - c - I \quad (\; 0 < c < \tfrac{1}{2} \text{ is the constant production cost}).$$

Suppose that I and hence v are observable by the buyer but not verifiable by the court so that it cannot be specified by a contract.

(i)   Determine the efficient amount of investment.

(ii)   Suppose that there is no contract and the two parties bargain ex post according to the Nash Bargaining solution. Is the investment optimal?

(iii)   Suppose that the parties sign a contract specifying that the buyer has the right to buy the good at a given price p. Is this contract efficient? What if the seller has the right to sell at a given price?

$$[3 + 3 + 4 = 10]$$

4

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2010-2011)
MS (Q.E.) II Year
International Economics I

Date: Maximum Marks 60   Duration 3 hours

Answer Group A and Group B in separate scripts

Group A
Answer both questions

1. (a) In a static trade model with many goods and a general structure of tariffs and subsidies, find a sufficient condition for gains from trade.

   (b) In a dynamic model with many goods and factors, show that free trade is better than autarky when one considers the steady state as well as the off steady state paths.

   [7 + 8]

2. Consider a two-good, three-factor small open economy with immobile capital in the two sectors and freely mobile labour across the two sectors. Find the effects of the following on the three factor returns:

   (a) an increase in the endowment of labour;

   (b) an increase in the endowment of capital specific to sector 1;

   (c) an increase in *all* factor endowments in the same proportion.

   [5 + 5+ 5]

Group B
Answer both questions

1. Show how pattern of trade (Inter/Intra-industry) is related to factor endowment ratio and economies of scale.

   (15)

2. a) Show that with the scale economies not too strong, Stolper- Samuelson theorem in its magnification version remains valid.

PTO

b) Recall the Ethier model.

In the $m - X_1$ plane, you have the usual PPF; downward sloping and concave to the origin (both m and X_1 are produced under constant returns to scale using labour and capital and both these factors are intersectorally mobile).

Formally put; $m = T(X_1)$, where $T' < 0$, $T'' < 0$.

i) Derive an expression for the curvature $\left(\frac{d^2 X_2}{dX_1^2}\right)$ of the PPF in $X_1 - X_2$ plane (where X_2 is subject to IRS, as in Ethier).

ii) Show that the condition $(1 - (\gamma - 1)A) > 0$ ensures that this PPF is usual looking, i.e. concave to the origin in $X_1 - X_2$ plane.

[Note: All notations bear my class lecture meanings.]

[Hints: 1. X_2 is related to m and m is related to X_1 through T.

2. Use the relation; $T' = \frac{dm}{dX_1} = -\frac{1}{p_m}$.]   [7+3+5]

INDIAN STATISTICAL INSTITUTE
End-Semestral Examination: (2010-2011)
MS (Q.E.) II Year
Incentives and Organisations

Answer all questions. Maximum Marks: 60. Duration: 3 hours

1. Consider the following principal agent model with hidden action, set in an insurance context. A risk-averse individual with initial wealth $W_0 > 0$ can lose an amount x of her wealth. The amount x is contractible. A risk-neutral insurance company can offer to insure the individual by setting a premium as well as a payment to the insured conditional on x. The distribution of x depends on accident-prevention effort a taken by the individual, which is unobserved. The cost of effort a is $((a^2)/2)$. Given a, assume that $f(x,a)=p(a)g(x)$, for $x>0$, where $\int g(x)dx=1$ and $f(0,a)=1-p(a)$, and $p''(a)>0>p'(a)$. Suppose the insurance company has full bargaining power. What is the full information optimal contract? How does it compare with the private information optimal contract?

    $[10 + 10] = [20]$

2. A risk-neutral principal has a project which requires one and exactly one worker to be initiated. There are two workers, and the principal can hire any one of them. They are both risk-neutral, and if hired, must be paid at least 0 irrespective of project outcome. If the project is initiated, the benefit to the principal is either 0 or $y > 0$. If a hired worker takes low effort, the probability of high output is x. If high effort is taken, the corresponding probability is y_1 for worker 1, and y_2 for worker 2. Assume $1 > y_1 > y_2 > x > 0$. The cost of high effort is c_1 and c_2 for workers 1 and 2 respectively, with $c_1 > c_2 > 0$. Whom does the principal hire? What is the optimal contract?

    $[10 + 10] = [20]$

3. Suppose firm1 is the monopoly producer of an intermediate good consumed by firm 2. The quality of the intermediate good is known only to firm 1 and is denoted by q. Firm 1 has a surplus of $\alpha_0 q$ if it does not sell the good to firm 2 and a surplus of p if it sells it at price p. Firm 2 has a surplus of $\alpha_1 q - p$ if it obtains the good at price p. Firm 2 knows that q is a priori uniformly distributed on [0, Q]. Is there a price p at which both firms always agree to trade? Explain your answer.

    $[20]$

INDIAN STATISTICAL INSTITUTE

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Second Semestral Examination 2010 – 11

M.S.(Q.E.) – 2nd year

Agricultural Economics

Date: 12·5·11   Maximum Marks: 100   Time: 3 hours

Answer any <u>five</u> questions.

Marks allotted to each question are given within parentheses.

1. Define translog production function. Explain the advantages of using this functional form to estimate allocative and technical efficiencies of farming households. Justify your answer with a suitable empirical illustration.

[6+14 = 20]

2. Define welfare improving growth rate as conceived by Kakwani. Do you believe that agricultural technology and liberalization policies in India have been effective for enhancing welfare oriented growth in Indian states? Give reasons for your answer.

[6+14 = 20]

3. What are the major deficiencies of the neo-classical approach in measuring sharecropping efficiency? Outline a theoretical model taking into account the size-class distinction of tenants in the context of measuring efficiency of share tenants.

[6+14 = 20]

4. What do you mean by labour use efficiency? Explain how the labour use efficiency could be tested statistically in a less developed economy. Justify your answer with an empirical illustration.

[6+14 = 20]

P.T.O

5. Define resource cost ratio (RCR). How could RCR be used to estimate the degree of comparative advantage of producing crops in different regions of India. Do you think that production of rice represents an efficient utilization of domestic resources compared to other crops in India? Give reasons for your answer.

[6+7+7 = 20]

6. Write notes on the following:

(a) Unmarketed productive inputs and different land tenurial contracts.

(b) Adoption behaviour of modern agricultural technology by Indian farmers.

[10+10 = 20]

Indian Statistical Institute
Second Semestral Examination: 2010 –2011
M.S.(Q.E) – II Year/ M.Stat. – II Year
Econometric Applications II

Date: 16.5.2011   Maximum Marks: **100**   Duration: 3 hrs.

NOTE: The question carries 102 marks. Question number 7 is compulsory.
Answer any *four* questions (choose from 1-6).

1. (a) (i) Define the Atkinson-Kolm-Sen index of income inequality. (ii) Show that it is invariant under a positive linear transformation of the underlying individual utility functions. (iii) Show that, if it is a relative measure, then the underlying social welfare function is homothetic.

(b) Let $\gamma(x, x+h) = \dfrac{\int_{x}^{x+h} uf(u)du}{\mu}$. If *income share elasticity* (η) is defined as

$\lim_{h \to 0} \dfrac{d \log \gamma}{d \log x}$, then derive the expression for η for a Lognormal distribution.

(c) What do you mean by a "Bistochastic" matrix?
Suppose the income distribution $y = (1,6)$ is obtained from a distribution $x = (2,5)$ through income transfer. Show that if we write $y = Ax$, then A can not be Bistochastic.

$$[(2+4+5)+8+4 = 23]$$

2 . (a) State and explain the properties of the Marshallian demand functions.

(b) Consider the Rotterdam model given by
$$w_i d \log q_i = b_i (d \log x - \sum_k w_k d \log p_k) + \sum_j c_{ij} d \log p_j \ , \ i=1,2,...n,$$

where $w_i = \dfrac{p_i q_i}{x}$ and $c_{ij} = \dfrac{p_i p_j S_{ij}}{x}$, S_{ij} being the compensated price effects and the symbols have their usual meanings.

What are the restrictions to be imposed on the parameters for the system to satisfy the properties of demand? Show how the restrictions are obtained.

1

(c) Show that for the Linear Expenditure System, all commodities are price inelastic, that is, $|\gamma_{ii}| < 1$, γ_{ii} being the non-compensated own price elasticity of item i. State the assumptions clearly.

[8+8+7=23]

3. (a) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?
   (b) How can one estimate the sampling error of Laspeyres' price index using a regression framework?
   (c) Show that Laspeyres' price index is always greater than the corresponding Paasches' index.
   (d) Describe the Ramsey-Samuelson-Diamond-Mirlees approach to the determination of optimal commodity taxation.

[3+5+5+10=23]

4. (a) State the difference between a 'censored distribution' and a 'truncated distribution'.
   (b) Explain the 'Double hurdle model' (with discrete random preference regimes) for modelling 'zero expenditure'.
   (c) Give an example of a bivariate sample selection model with a participation and an outcome equation. How would you estimate the parameters using Heckman's procedure?

[3+10+10=23]

5. (a) Describe the properties of a production function.
   (b) Examine the Leontief production function $f(x) = min\{\alpha_1 x_1, \alpha_2 x_2, ... \alpha_n x_n\}$, $\alpha_i > 0$ in the light of the properties mentioned above.
   (c) For both input and output oriented technical efficiencies, demonstrate (graphically) the relationship between Debreu-Farrell and Koopmans measures.

[6+9+8=23]

6. (a) Describe a parametric method of estimating technical efficiency.
   (b) Describe a non-parametric method of estimating technical efficiency.

[11+12=23]

7. Practical Exercise

[10]

27/6/11

Date: ~~00-00-2011~~   Maximum Marks: 100   Duration: 3 Hours

Use separate answer script for each group.

Group A: Answer any two questions

1. (a) Clearly explain the following concepts:

   (i) Entry effectively blockaded; (ii) Entry effectively impeded; (iii) Entry ineffectively impeded.

   (b) Consider sequential entry in a market. After entry decisions are settled, firms compete in a Cournot fashion. The market demand for the product (in inverted form) is given by $P(\sum_i x_i) = 20 - \sum_i x_i$. There is a large number of potential entrants and each of them has access to the same production technology, as given by the cost function, $C(x_i) = F + cx_i = 12 + 4x_i$ for $x_i > 0$; otherwise, $C(0) = 0$. Suppose that n firms have already entered the market, $n = 1, 2, 3 \ldots \ldots$; Suppose further that they together decide whether they will deter entry of the next firm (and hence all other firms) by spending an equal amount on deterrence. The entry deterrence technology is given by $D(nI) = 2(nI)/3$, which states that if there are n firms and each invests an amount I on entry deterrence, then the $(n+1)$-th entrant will face an entry cost $D(nI)$. [The payoff of a firm in an n-firm Cournot oligopoly, leaving out the entry cost, is given by: $\pi(n) = [(a-c)/(n+1)]^2 - F$.]

   a. Construct the game tree showing moves of the players and payoffs at each terminal node.

   b. Also find the SPNE of the game.

   [6+(15+4)=25]

2. Consider Bertrand duopoly with zero cost of production. The demand function, as faced by the two firms, are respectively,

$$x_1(p_1, p_2) = 1 + (p_2)^2 - 0.5p_1 \quad for \ 0 \le p_2 \le 1,$$
$$= 2p_2 - 0.5p_1 \quad for \ 1 \le p_2 \le 4,$$
$$= 4\sqrt{p_2} - 0.5p_1 \quad for \ 4 \le p_2.$$

and

$$x_2(p_1, p_2) = p_1 - 0.5a p_2 \quad for \ some \ a, 0 \le a \le 4.$$

Solve the model for equilibrium prices, given that $0 \le a \le 4$. Use diagram to show your results.

[25]

3. Consider the circular city model of Salop (1979) with a modification that transportation costs are quadratic (of the form, td^2, where t is the transportation cost per unit distance and d is the consumer's distance to the selected shop). Derive the price and the number of firms in the symmetric zero profit equilibrium. How do you compare this equilibrium with the social optimum?

[15+10=25]

Group B: Answer any <u>two</u> questions

1. A Monopolist produces a durable good that lasts for two periods at zero (marginal) cost. The inverse demand curve for the good in each period is

$$p = 2 - 0.02Q,$$

where Q is the total number of goods used in the period. The discount factor is $\delta = 0.8$.

(a) Find the per-period output, rental price and the aggregate profit of the monopolist, if he chooses to rent the good in every period.

(b) Suppose the monopolist can only sell the good and cannot commit in period 1 not to produce or sell additional units in period 2. Find the monopolist's output and sales in every period as well as his aggregate profit.

(c) Why are output, sales and profits different in the rental and the sales models?

[10+10+5=25]

2. Consider the **Baron-Myerson** model: a firm has low marginal cost c_L with probability φ and high marginal cost c_H with probability $1 - \varphi$: $c_H > c_L$. It has known fixed cost F. The consumer demand function is $Q(p)$ and consumer surplus is $v(p)$, where $v'(p) = -Q(p)$. The regulator places no weight on the firm's rent (i.e., $\alpha = 0$). Suppose the regulator wishes to **set** a single ("pooling") contract to the firm, regardless of the firm's marginal costs, and the contract takes the form of a price p and a transfer T.

   (a) Assuming that the regulator wishes to ensure that the high-cost firm is willing to produce, what is the optimal form of this single contract?

   (b) Consider exactly the same **problem** as in question 1 above, but find a condition on φ that makes it optimal for the regulator to have production from only the low-cost firm.

$$[15+10=25]$$

3. The production function for a process is

$$x = 16(e_1 + e_2)$$

   if e_1 and e_2 are both greater than zero. If either equals zero, then $x = 0$. The effort exerted by team member i is e_i and x is measured in Rupees. Team member i's utility is given by

$$u_i = m_i - 2e_i^2,$$

   where m_i is income. The sharing rule is that they split the yield of their joint effort equally, i.e. $m_i = (\frac{1}{2})x$.

   (a) Find the efficient effort and income levels.

   (b) What is the income of the partnership if each partner maximizes his individual utility?

   (c) What is the maximum amount the partners are willing to pay someone to monitor their effort levels if this monitoring can eliminate any shirking?

$$[8+7+10=25]$$

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2010-2011) (Back paper)
MS (Q.E.) II Year
International Economics I

Date ᒐ ४ ᑕᑭ] �1   Maximum Marks: 100   Duration: 3 hours

Answer Group A and Group B in separate scripts

Group A
Answer both questions

1. Using a theoretical model show that interests of immobile factors regarding the opening up of trade are conflicting in the short run but converge in the long run when these factors are free to move.

   [25]

2. In a static model of international trade, show that free trade is superior to autarky. How far can this result be extended to a dynamic context?

   [25]

Group B

Answer both questions

1. Explain the mechanism through which capital inflow in a small tariff ridden economy can reduce welfare.

   [25]

2. Show how distributional conflict is related to differences in factor endowment ratios and economies of scale.

   [25]

Syllabus: (International Economics I): Professor Abhirup Sarkar

(Group A)

1. Pure Exchange Model
2. Ricardian Model
3. Specific Factors Model
4. Heckscher-Ohlin Model
5. Trade, Development and Growth
6. Static and Dynamic Gains from Trade

Syllabus: (International Economics I): Dr. Brati Sankar Chakraborty

(Group B)

1. Comparative Statics in a Basic Trade Model: growth, terms of trade and welfare (the immiserization result).

2. Transfers

3. Tariffs, terms of trade, volume of trade, optimal tariff, domestic prices (Metzler Paradox).

4. Growth/capital inflow under tariff regime (Johnson, Brecher, Diaz-Alejandro), capital inflow under alternative protectionist regime (s) vis-à-vis tariff.

5. Increasing Returns and trade.

Back Paper

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2010-2011)

MS(QE) II

Social Choice and Political Economy

Date: ᐧᐧ.0᎐2011   Maximum Marks: 100   Duration: 3 hrs.

Note: Answer all questions. Answer Group A and Group B in separate answerscripts.

Group A

(1) Define first order stochastic dominance. State and prove its welfare significance. (1+9=10)

(2) State Harsanyi's aggregation rule and prove his first aggregation theorem using Pareto indifference. (2+14=16)

(3) Explain the equity-efficiency trade-off that arises in distributional ranking problem. Prove a theorem which shows preference for higher efficiency under a fixed relative equity level. (2+12=14)

(4) (a) Define positional dictatorship and illustrate the concept by an example.

    (b) Give a graphical exposition of 'ordinal measurability and non-comparability' information invariance condition.

    (5+5=10)

Group B

(1) State and prove the Group Contraction Lemma, giving all the relevant definitions. (15)

(2) State and prove the Gibbard-Satterthwaite Theorem when there are two agents and there are finite but more than two alternatives. (35)

Second Semestral Examination: 2010 –2011
M.S.(Q.E) – II Year/ M.Stat. – II Year
Econometric Applications II

Back paper

Date: $01|07|11$ Maximum Marks: 100   Duration: 3 hours

Answer **all** questions.

1. a) What are the different approaches to specifying a demand system?

   b) What is the Gorman-Polar form of cost function? What is the form of the Engel curve implied by this form of cost function?

   c) Show that for the Linear Expenditure System (LES) the non-compensated own price elasticities are approximately proportional to the corresponding expenditure elasticities.

   d) In the LES, an amount z of good 1 must be bought. Show that for $i = 2,3...,n$.

   $$p_i q_i = p_i c_i + \frac{b_i}{\sum\limits_{i=2}^{n} b_i}(x - p_1 z - \sum\limits_{k=2}^{n} p_k c_k)$$

   where the symbols have their usual meanings.

   $[6 + 4 + 7 + 8 = 25]$

2. a) Explain the difference between a 'censored distribution' and a 'truncated distribution?

   b) Write down the Tobit model to incorporate zero consumption with full specification of the distribution of the error term. Why are the assumptions underlying the standard linear regression model not tenable in such a case?

c) Given the dynamic model for 'clothing'

$$q(t) = \alpha + \beta s(t) + \gamma x(t)$$

where $q(t)$: rate of demand at time t

$x(t)$: income during the same time

$s(t)$: inventory of 'clothing' at time t.

and assuming that the stock is used up at a constant depreciation rate δ, find the short and long term derivatives of consumption with respect to income.

[3 + 12 + 10 = 25]

3. Write short notes on the following:

a) Application of complete demand models to demand projection.

b) The Almost Ideal Demand System.

[25]

4. a) Explain the 'deterministic' and 'stochastic' production frontiers assuming a log-linear relationship between inputs and output. State clearly the assumptions you make.

b) Describe the corrected least squares method of estimating the parameters of a deterministic production frontier.

c) Give a brief outline of the estimation of a random effects model using generalised least squares method. What type of data would you use for this estimation? How would you estimate 'inefficiency' assuming a half normal distribution of the inefficiency component?

[If $u \sim$ Half normal, $E(u) = \dfrac{2\sigma_u}{\sqrt{2\pi}}$ and $Var(u) = \dfrac{\pi - 2}{\pi} \sigma_u^2$]

[6 + 6 + 13 = 25]