INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: (2012-2013)
MS (Q.E.) II Year
International Economics I

Date: 03.09.12   Maximum Marks 40   Duration 3 hours

Group A

Answer all questions

1. Examine the validity of any two of the following statements in the context of 2x2 HOS model.
   a) Rybczynski theorem is valid in the presence of factor intensity reversal.
   b) Price output response is positive even if the production functions in the two sectors are of fixed coefficient type.
   c) Trade is always balanced.

   (10)

2. Examine the validity of any two of the following statements in the 3x2 specific factor model.
   a) One cannot determine the level of outputs and factor prices if production functions in both the sectors are of fixed coefficient type.
   b) If the price of a commodity rises, given the price of the other commodity and factor endowments, price of the mobile factor of production remains unchanged.
   c) If the capital stock specific to one sector is increased given other factor endowments and price of all the commodities, level of output of the other sector remains unchanged.

   (10)

Group B

Answer all questions

1. Show that in a two agent setting, with the equilibrium being Walras' stable, the recipient of a transfer is always a gainer.

   (10)

2. In the Brecher-Alejandro set up, show that foreign capital inflow is necessarily welfare immiserizing.

In this set up, could you work out what happens to imports consequent to capital inflow?

(7+3)

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2012-13

Course name: MSQE II
Subject name: Game Theory II
Date: 05.09.12
Maximum marks: 40
Duration: 2 hours
Please answer parts A and B on separate answer books. Each part is worth 20 marks.

Part A

Q.1. (a) Clearly define the core and the dominance core. [2 + 2]
(b) Establish the relation between them (you must state any result you use for proving (b)). [4]

Q.2. Show that if two games are strategically equivalent and one of them is essential, then the other is essential as well. [5]

Q.3. A set of coalitions is called minimally balanced if it does not have a proper subset which is balanced. Give an example of a minimally balanced collection for the game with the player set $N = \{1, 2, 3\}$. [7]

Part B

Please give rigorous answers to all questions.

Consider the sealed-bid, independent private value auction model done in class. There are N bidders. Values are drawn from the set $[0, \omega]$ according to the continuously differentiable, increasing distribution function F.

Suppose the seller wishes to sell using a first-price auction.

(I) Assuming bidders with 0 value submit a 0 bid, is there a (perfect Bayesian) symmetric Nash equilibrium bidding strategy in this game? [6]

(II) If so, what does the bid submitted by a bidder with value z converge to as the number of competitors becomes unboundedly large? [4]

Suppose additionally $\omega = 1$, and $F(x) = x^k$, $k > 0$ over [0,1]. Assuming an equilibrium exists:

(III) For a given value $x \in (0, 1)$, how does the equilibrium bid change as k changes? Is the equilibrium bid a concave or convex function of k? [1 + 2]

(IV) What is the expected revenue to the seller? What is her optimal reserve price, should she decide to set one? [4 + 3]

MSQE II - Theory of Finance I
Midsem. Exam. / Semester I 2012-13
Time - 2 hours/ Maximum Score - 30

10 · 08 · 2012

1. Let $X_n = X_0 + R_1 + \cdots + R_n$ and $Y_n = Y_0 + S_1 + \cdots + S_n$, for all $n \geq 1$, for all $n \geq 1$, where (R_k, S_k) are i.i.d. random variables with Bivariate Normal distribution with mean $(0,0)$ and covariance matrix $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ with $-1 < \rho < 1$.

   (a) (4 marks) Assuming $X_0 \equiv 0 \equiv Y_0$, give brief reason that X_n and Y_n are martingale with respect to appropriate filtering $\{\mathcal{F}_n\}$ after describing this $\{\mathcal{F}_n\}$.

   (b) (6 marks) Find the compensator for $\{X_n^2\}$ and for $\{X_n Y_n\}$.

2. (a) (4 marks) Let S_t be the price of an equity that satisfies the Black-Scholes model $dS_t = \mu S_t dt + \sigma S_t dB_t$, where $\{B_t \ : \ t \geq 0\}$ is the standard Brownian motion. Use Itô's lemma (after stating it carefully) to show, $S_t = S_0 \exp\{(\mu - \sigma^2/2)t + \sigma B_t\}$.

   (b) (6 marks) Assume that $\{\log S_t\}$ is a martingale. A trader gains 100 Rupees if $\log S_t$ hits an upper level 'b' before a lower level 'a' (where $a < \log S_0 < b$). Otherwise, the trader loses.

      i. Find the condition for which $\{\log S_t\}$, given above, is a martingale with respect to an appropriate filtration $\{\mathcal{F}_t\}$.

      ii. Find the probability that the trader gains 100 Rupees.

      iii. Find the expected time for his (or her) gain or loss.
      [If you use any theorem that was done in the class you need to state it carefully.]

3. A trader buys an European Call option on a stock (S) for $Rs.C$ with a maturity in T months. The initial stock price was $Rs.S_0$ and the strike price was $Rs.K$. Assume the price of the option follows risk-neutral valuation and the stock follows the Black-Scholes model with the risk-free interest rate r and volatility σ.

   (a) (6 marks) Find the probability that the option will be exercised. Would the probability increase if (i) σ increases? (ii) T increases? Justify your answers.

   (b) (3 marks) Calculate the expected value of S_T when the option is exercised.

   (c) (5 marks) Can you find the Call price in terms of **this expected value** and the **probability of exercise the option**, found above? Should the Call price increase if (i) S_0 increases? (ii) r increases? Justify your answer.

All the best.

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2012-2013
M.S. (Q.E.), 2nd Year
Econometric Applications I

Date: 11 September 2012   Maximum marks: 100   Duration: 3 hours

[Answer question no. 1 and any **three** from the rest of the questions]

1. Suppose the share of bottom 10% and the median daily income of people in a community are 0.03 and Rs. 148 respectively. Assuming that the distribution of income is lognormal, (i) find the parameters μ and σ^2 of the distribution, and hence or otherwise find (ii) the mean income of bottom 10% and (iii) the mean income of top 10% people in the community. [10+5+10=25]

2. State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? State and prove the properties of Pareto distribution. [2+6+5+12=25]

3. Describe Positive and Normative Measures of Inequalities. Write down the desirable properties of a measure of inequality. Examine whether Relative Mean Deviation satisfy Scale Invariance Property and Pigou-Dalton Property of Transfer. [12+5+8=25]

4. Suppose X is a size distribution having the distribution function F(.).

   (i) Define Lorenz Ratio (LR) of X. Show that it can be expressed as

   $$LR = 1 - 2\int_0^1 F_1 \, dF,$$

   where the symbols have their usual meaning.

   (ii) Prove that LR can also be found from $E|Y_1 - Y_2|/(2M)$, where Y_1 and Y_2 are i.i.d. having the same distribution function F(.) and M is their common mean.

   (iii) Show that if $X \sim LN(\mu, \sigma^2)$ then $LR = 2\Phi(\sigma/\sqrt{2}) - 1$. [2+12+11=25]

5. (a) Examine the incomes of the following three groups of persons and arrange the groups in increasing order of their inequalities, if possible, giving sufficient reasons.

   Group – I: 10 20 30 40 50 60 70 80 90 100
   Group – II: 20 37 57 76 95 114 133 152 172 189
   Group – III: 21 21 84 84 105 126 147 147 210 210

   (b) Calculate the Lorenz Ratio of the incomes of persons in Group – I only.
   (c) Derive Lorenz Curve and Lorenz Ratio of Pareto distribution [5+5+(8+7)=25]

6. Write short notes on any two of the following:
   (a) Estimation of parameters of lognormal distribution from grouped data.
   (b) The problems with the data that are faced in econometric analysis.
   (c) Graphical Test of lognormal distribution. [12½+12½=25]

Find the expression for β_{iM}. What is meant by the *security market line*? Draw it in a diagram and interpret it.

(b) One way of estimating β_{iM}'s is to fit regression equations to the past data on rates of return on individual securities as well as on the market portfolio (say, M). For this, one may hypothesize the following relation for each risky asset i,

$$r_i = r_f + \beta_{iM}\{r_M - r_f\} + \varepsilon_i.$$

where ε_i's are random errors , say identically and independently distributed with mean zero and standard deviation, σ_ε. Further, suppose, each ε_i is also distributed independently of r_M. Consider now any other portfolio P with weights attached to the risky asset i being w_{P_i} (and $\sum w_{P_i} = 1$). Show that

$$\sigma^2(r_P) = \beta_{PM}^2 \, \sigma^2(r_M) + \sum_{i=1}^{N} w_{P_i}^2 \sigma_\varepsilon^2.$$

The first term on the RHS of this relation is called the *systematic risk* and the second term, the *diversifiable risk*. Explain these concepts.

$$[6 + 4] = [10]$$

3. (a) Consider two risky assets a and b, with rates of return r_a and r_b. The means and variances of these returns are μ_a, μ_b and σ_a^2, σ_b^2, respectively. Suppose both variances are equal: $\sigma_a = \sigma_b = \sigma$ (say). Consider now a portfolio P which has x proportion of the investors' funds put in the asset a and the remaining, i.e. $(1-x)$, proportion in the asset b. Let σ_P^2 be the variance of this portfolio return. Obviously, σ_P changes with x. On an (x, σ_P) diagram show that if ρ (the correlation coefficient between r_a and r_b) is less than 1, then σ_P falls as x rises for all $x < \frac{1}{2}$. How does σ_P change with x, if $\rho = 1$?

(b) Construct a *two period* consumption choice problem in which saving in the current period may be invested in a portfolio consisting of one risk-free and N risky assets and the second period consumption ($\tilde{C}$) as well as the gross rate of return from the i-th risky asset (R_i) are random (for all i). Find the first-order conditions for the optimal *choice* between investment in the *risk-free asset* and

each risky asset. Find the relation between $E(R)$ and the risk-free rate (r) and also the expression of **risk-premium** in this model. *Interpret* all the results.

[5 + 5] [10]

Group B

Answer all

1) a) Show that the initial period consumption in the Ramsey model is a function of the initial asset and the present discounted value of lifetime wage earnings.

b) Using that expression, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital market.
In this context, discuss the problems associated with either a very low or a very high rate of interest. [3 + 7] [10]

2) a) Show that all paths other that the convergent one in the Ramsey model would either violate the transversality condition or one of the other necessary conditions for an optimum.
Show that in such an infinite horizon model, financing any given profile of government expenditure either by lump sum taxes or by government borrowing amounts to the same thing.
b) Assuming a Cobb-Douglas production function, work out the transitional dynamics of the savings rate in the Ramsey model, when the initial capital stock is less than the steady state capital stock. [5 + 5] [10]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2012-2013)
MS (Q.E.) II Year
International Economics I

Date: 19.11.12   Maximum Marks 60   Duration 3 hours

Group A
Note: Answer any two

1. Using a specific factor model of an open economy analyse the effect of international trade and technological progress on the skilled unskilled wage inequality. (15)

2. Analyse how the world economy grows along a balanced growth path in the North- South model of Findlay (AER, 1980). (15)

3. Examine the validity of the following statements.

   a) An improvement in the terms of trade always raises the rate of growth of industrial sector in the Matsuyama (JET, 1992) model.
   b) Production possibility curve always slopes negatively in the 2x 3 specific factor model of Jones. (10+5)

Group B
Note: Answer all questions

1. Show that in a three-agent setting, a transfer paradox might occur even when the equilibrium is Walras stable. In this context discuss the role of substitution effects in ensuring normal results. (15)

2. Consider a 2 country, 2 commodity trading world, with perfectly competitive markets and show that imposing an ad- valorem export tax is the same as imposing an ad-valorem

import tariff when the government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax.

[Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.]

(15)

INDIAN STATISTICAL INSTITUTE
End-Semestral Examination: 2012-13

Course name: MSQE II
Subject name: Game Theory II
Date: 21·11·12
Maximum marks: 100
Duration: 3 hours
Note: Please answer parts A and B on separate answer books. Each part is worth 50 marks.

Part A

1. Define a convex game. Develop an equivalent condition for convexity of a game. [1+7]

2. Show that the Shapley value of a player in a game is the average of his marginal contributions that are paid out over all the possible orders. [10]

3. Establish the relationship between the kernel and the bargaining set of a game. [10]

4. State the Bondareva- Shapley Theorem. Construct a game to show how this theorem can be used to show non-emptiness of its core. [3+3]

5. Demonstrate rigorously that the reasonable set can be treated as a core catcher. Give an example to show that a payoff vector in a reasonable set need not be in the core. [6+2]

6. Consider a partitioning of a player set into two groups. Each member of one of the groups has one right shoe and each member of the other has one left shoe. The shoes are identical in all respects except for left-right difference. A single shoe has no value, whereas a left-right pair is worth 1 unit. Model this situation as a cooperative game by constructing the characteristic function. [8]

Part B

1. Consider the common value model. Show that there is a unique symmetric equilibrium in the second-price, winner-pay auction. [15]

2. Consider the independent private value model. Show that there is a unique symmetric equilibrium in the first-price, all-pay auction. [15]

3. Consider the common value model. Assuming the seller wishes to sell using a winner-pay auction, does she prefer selling through a first-price or a second-price auction? Answer rigorously. [10]

4. Suppose there is an object for sale and there are three bidders with a common value V for the object that is uniformly distributed over [0, 1]. Given V = v, bidders' signals X_i are uniformly and independently distributed over [0, v]. Assume sale takes place through a second-price, winner-pay auction.

(a) What is the symmetric equilibrium bid function? [5]

(b) What is the expected revenue of the seller? [5]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108

FIRST SEMESTRAL EXAMINATION 2012 - 13
M. Stat. II year and M.S. (Q.E.) II year
Econometric Methods / Econometric Methods II

Date: 24 November 2012   Maximum Marks: 100   Time: 3 hours

Answer any *five* questions. Marks allotted to each question are given within parentheses.

1. (a) For the multiple linear regression model $Y = X\beta + \varepsilon$ with $D(\varepsilon) = \sigma^2 I_n$ and $p\lim_{n\to\infty}(X'\varepsilon/n) = 0$, where notations have their usual meanings, derive the IV estimator of β. Also obtain the standard error of the IV estimator.

   (b) Consider the following model for stationary $\{y_t\}$:

$$y_t = \Phi\, y_{t-1} + u_t, \quad t = 1,2,\ldots,n$$
$$u_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \text{ where } \varepsilon_t \sim WN\,(0,\sigma_\varepsilon^2).$$

   Show that the OLS is not an appropriate method of estimation of the parameters of this model.
   Could y_{t-2} and y_{t-3} be used as instruments? Justify your answer with derivations wherever necessary.

$$[8+\overline{6+6} = 20]$$

2. (a) Let $y_1, y_2, \ldots, y_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. Consider the problem of testing the null hypothesis $H_0 : \mu = \mu_0$ against the alternative $H_1 : \mu \neq \mu_0$ by the LR and RS tests. Then show that $LR \geq RS$, where the abbreviations stand for the respective test statistics.

   (b) Describe White's IM test and comment on its power.

   (c) Discuss briefly how encompassing principle may be used to test between two nonnested hypotheses.

$$[8+6+6 = 20]$$

3. (a) State, along with assumptions, the random effects model, and then describe how you would estimate the parameters of this model.

(b) Describe how you can test between the fixed effects and random effects models for panel data.

(c) Consider the panel data case when
$$v_{it} = \mu_i + \lambda_t + \epsilon_{it}$$
where $i = 1, \ldots, N$, $t = 1, \ldots, T$, and $\mu_i, \lambda_t,$ and ϵ_{it} are random variables having zero means, independence among themselves as also with each other, with variances σ_μ^2, σ_λ^2 and σ_ϵ^2, respectively. Show that
$$V = E(vv') = \sigma^2 [\rho A + wB + (1 - \rho - w)I_{NT}]$$
where $A = I_N \otimes J_T$,
$$B = J_N \otimes I_T,$$
$$\sigma^2 = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_\epsilon^2 , \quad \rho = \sigma_\mu^2 / \sigma^2, \quad w = \sigma_\lambda^2 / \sigma^2$$
and J_T is a $T \times T$ matrix of unity.

[6+6+8 = 20]

4. (a) Consider estimating the standard probit model where $prob(y_i = 1)$ $= \Phi(\beta'x_i / \sigma)$, and y is a binary variable that takes on the value 0 or 1. How would the estimated coefficients compare if you run a probit model on the same data, except that y has been recoded on a value of 0 or 10? Give justifications for your answer.

(b) Describe the ML method of estimation of logit model when only one observation of the dependent variable is available for each 'setting' of the independent variables, and also sketch a proof of consistency of this estimator.

(c) Distinguish between censored and truncated samples. Also argue why the OLS estimator of the slope parameter of a truncated regression model is inconsistent.

[6+6+8 = 20]

5. (a) Explain what you mean by cointegration. In case of relationship involving two time series variables, is cointegration unique? Justify.

(b) Discuss how you would test for the presence of cointegration in a system of equations with K (>2) variables.

(c) What is Granger causality? Discuss the relationship between Granger causality and cointegration.

[6+6+8 = 20]

6. (a) Show that an ARCH (1) process entails a symmetric distribution with tails heavier than the normal distribution.

(b) Show that a GARCH (p,q) process for a time series $\{X_t\}$ can be represented as an ARMA process of suitable orders for $\{X_t^2\}$

(c) Describe a test for testing 'no ARCH' *versus* 'ARCH' in a dynamic linear regression model. Also discuss, with justifications, if the test statistic would change if 'ARCH' is replaced by 'GARCH' model.

$$[6+6+8 = 20]$$

*[Note: Answer question no. 1 and any **three** from the rest of the questions. Marks allotted to each question are given at the end of the question]*

1. The following data show the percentage distribution of different races in Malaysia in a certain year among all households and non-poor households.

Races	Percentage distribution among	
	All households	Non-poor households
Malay	60	40
Chinese	30	40
Other	10	20
Total	100.0	100.0

   If the overall Head Count Ratio of Malaysia is 30%, then compute the Head Count ratio of each of the three groups in Malaysia. If the overall HCR happens to be 60% instead of 30%, then what should be the percentage change in the HCR in each racial group?   [13+12=25]

2. (a) Is it true that the demand for a commodity will always increase if the mean income increases and the inequality of income decreases for a given group of people? Explain your answer assuming a suitable form of the Engel curve and a specific income distribution.
   (b) Describe how you will estimate Engel elasticity using Specific Concentration Curve. (*You should show the derivations of the associated results.*)   [12+13=25]

3. (a) Describe how you will fit the following form of Engel curve

   $$y = \alpha(x+\beta)/(x+\gamma), \qquad \alpha>0, (x+\beta)>0 \ \& \ (x+\gamma)>0,$$

   given the observations on y and x values. Hence explain how you will classify a commodity as necessary, luxury or inferior. Also draw a rough sketch of the curve in each case.

   (b) Describe Prais–Houthakker (PH) Model to see the effect of household size on the consumption pattern. What is the difficulty in estimating the Model? How did PH tackle it?   [10+15=25]

4. Discuss the properties including cost minimization and profit maximization conditions of Cobb-Douglas Production function. What are the statistical problems in the estimation of this function?   [13+12=25]

5. Write down the criteria for a good measure of concentration in business and industry. Show how these criteria are satisfied by (i) Herfindahl – Hirschman index and (ii) Hall and Tideman index. Also discuss why Lorenz Ratio cannot be taken as a measure of concentration in business and industry.   [5+(8+8)+4=25]

6. Write short notes on <u>any two</u> of the following:

   (a) Linear Expenditure System.
   (b) Normative measures of inequality
   (c) Measures of poverty.   [12½+12½ =25]

MSQE II - Theory of Finance I
Final Exam. / Semester I 2012-13
Time - 3 hours/ Maximum Score - 50

Date : 27.11-2012

NOTE : SHOW ALL YOUR WORK. RESULTS USED MUST BE CLEARLY STATED.

1. Let $X_n = X_0 + R_1 + \cdots + R_n$ and $Y_n = Y_0 + S_1 + \cdots + S_n$, for all $n \geq 1$, where (R_k, S_k) are i.i.d. random variables with $P(R_k = 1, S_k = 1) = 1/3 = P(R_k = -1, S_k = -1)$ and $P(R_k = -1, S_k = 1) = 1/6 = P(R_k = 1, S_k = -1)$.

   (a) (3 marks) Assuming $X_0 \equiv 5 \equiv Y_0$, give brief reason that X_n and Y_n are martingale with respect to appropriate filtering $\{\mathcal{F}_n\}$ after describing this $\{\mathcal{F}_n\}$.

   (b) (5 marks) Find the compensators for $\{X_n^2\}$ and $\{X_n Y_n\}$.

   (c) (5 marks) If the price of a foreign asset on the nth day is $X_n Y_n$ Rupees, then find the expected time the price will take to hit either 10 or 50 Rupees.

2. Suppose there are 3 large stocks A, B and C and the covariance matrix of their returns is

$$\begin{pmatrix} 6 & 4 & -4 \\ 4 & 6 & -4 \\ -4 & -4 & 14 \end{pmatrix}$$

and their mean return is $(1, 1, 2)$.

   (a) (8 marks) Suppose you have 1 crore of rupees to invest in these stocks. Calculate the best portfolio (in the sense of minimum variance), by giving the portfolio weights, so that expected return would be 37/27.

   (b) (5 marks) Can you find out the probable loss from this portfolio that may happen with 5% probability? If so, find the loss. [You may assume joint normality of returns.]

3. In 1985, Bankers Trust developed the index currency option notes (ICONs). For Yen-US dollar exchange rate, let S_t be the yen per dollar at time t. The amount (in US dollar) it pays to the holder of the notes at the time of maturity T is

$$1000 - \max(0, 1000(\frac{169}{S_T} - 1))$$

and it pays nothing if S_T goes below 84.5 yen per dollar.

Assume that under risk-neutral measure, S_t satisfies,

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where $\{W_t\}$ is a standard Brownian motion, $\mu = r_j - r_a$, where r_j and r_a are the risk-free interest rates per year in Japan and US, respectively, and $\sigma > 0$ is assumed to be the constant volatility during the whole period up to maturity time T.

(a) (4 marks) Let $X_t = \frac{c}{S_t}$. Use Itô's formula and derive the SDE for X_t.

(b) (2 marks) Sketch the payoff function of the ICONs, clearly indicating the cutoff points.

(c) (5 marks) Calculate the arbitrage-free price of the ICON when $S_0 = 120$, $r_j = 0.25\%$, $r_a = 2\%$, $T = 6$ months and $\sigma = 0.3$.

(d) (5 marks) Calculate the price using a suitable Binomial model for $\Delta T = 2$ months, and then compare it with that obtained in part (3c).

4. (6 marks) Suppose that there is a riskless asset in strictly positive supply, and investors prefer to hold efficient frontier portfolios. Assume that borrowing at the riskless rate r_f is prohibited. Show that for any feasible portfolio q,

$$E[r_q] = E[r_{zc(m)}] + \beta_{mq}(E[r_m] - E[r_{zc(m)}]), \quad E[r_m] > E[r_{zc(m)}] \quad \text{and} \quad E[r_{zc(m)}] \geq r_f.$$

5. ($4 \times 3 = 12$ marks) Write TRUE or FALSE and justify your answer (the justification should be Mathematical as much as possible).

(a) In the Black-Scholes-Merton model, the call option price is a decreasing function of volatility.

(b) If the market is incomplete, Put-Call parity may not hold.

(c) The explicit difference method is always more accurate than the implicit difference method.

(d) ARCH(2) is a better model for the returns than GARCH(1,1) when it comes to capturing the higher kurtosis of the returns.

All the best.

Date: 3ᴇ 11·12—   Maximum Marks 60   Duration 3 hours

Group A

Answer any two

1. Robinson Crusoe lives forever in his island economy which experiences productivity shock, In z_t (i.i.d. with zero mean). In period t he gives labour l_t (which yields disutility), produces output y_t, consumes c_t and maximizes expected present value of lifetime utility:

$$E_0\left[\sum_{t=0}^{\prime}\beta^t u(c_t . l_t)\right] = E_0\left[\sum_{t=0}^{x}\beta^t \{\ln c_t - l_t^\sigma\}\right] \quad (\sigma>1; 0<\beta<1), \quad \text{subject to}$$

(i) $c_t + k_{t+1} = y_t = z_t k_t^\alpha l_t^{1-\alpha}$, $(0<\alpha<1$ and capital, k, depreciates fully after one period).

Let $v(k_t . z_t)$ denote the value function which satisfies the following *Bellman* equation

(ii) $\quad v(k_t . z_t) = \max_{c_t, k_{t+1}, l_t} \{\ln c_t - l_t^\sigma\} + \beta E_t\{v(k_{t+1} . z_{t+1})\}]$

An educated guess about the solution of (ii) is that v (.) takes the following form:

(iii) $v(k_t . z_t) = \theta_0 + \theta_1 \ln k_t + \theta_2 \ln z_t$ $\{\theta_0, \theta_1, \theta_2$ are constants to be determined.$\}$

(a) Using (i) and (iii), maximize (ii) with respect k_{t+1} (or c_t) and l_t.

Find θ_1 and θ_2 as well as optimal values of k_{t+1} and l_t in terms of α, β, σ and y_t.

Does the optimal value of k_{t+1}/y_t or that of l_t depend on k_t and z_t? Explain.

(b) Suppose there was no shock up to period 0 and the economy was in equilibrium

at an output level y^*. Suppose further that at period zero there is a *one-time* productivity

shock, i.e. $\ln z_0 = \varepsilon > 0$ and $\ln z_t = 0$ for all $t \neq 0$.

Find $\ln y^*$ and trace the time path of y_t for all $t \geq 0$.

$$[9 + 6] = [15]$$

2. How is a real business cycle model tested empirically? Describe briefly the methodology used.

$$[15]$$

3. Describe in detail a model of CAPM with the absolute minimum of assumptions, namely that there be no opportunities for pure arbitrage. What is the expression of risk premium of an asset? Derive it and explain.

$$[15]$$

Group B
Answer all

1. In the Blanchard-Yaari model with cohort dependent wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation? Explain.
   Consider the same model, but now with zero population growth, cohort independent wages and open to international asset market with a constant rate of interest. Can you show that the aggregate savings in this model is negatively related to the level of assets? [Savings = Total income – Total Consumption, where Total income = wage income + interest income on assets. Also assume that the steady state exists in the model.]

   (10+5)

2. Show that with investments having convex adjustment costs, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest.
   Find out the conditions on the production function and on the investment adjustment cost function under which Tobin's marginal q would be equal to the average q.

   (10+5)

Group A

Answer all

1. Consider a world of two countries, home and foreign, and two produced goods, consumption good (C) and capital good (K). In addition there is a non-produced factor called labour (L) which grows at the rate g in each country. Each good is produced with K and L using well behaved Neo-Classical technologies. Finally assume all wage income is consumed and a fixed fraction of profit income is saved. The saving ratios at home and foreign are s and s^* respectively.

   i)   Construct the equilibrium under autarky in each country.

   ii)   Show that in trade equilibrium the country with higher saving ratio exports the capital intensive good.   (5+5=10)

2. Construct a general dynamic activity analysis model of production and show that the set-up described in question 1 is a special case of the general activity analysis model. Prove gains from international trade for the general activity analysis model.

   (3+2+5=10)

Group B

Answer all

1. Show how a tariff can depress domestic price in presence of transport cost in trade and with technology exhibiting increasing returns to scale. Explain the mechanics behind the result.   (10)

P.T.O

2. Show that the condition ensuring positive price-output response is also sufficient to validate the Stolper- Samuelson theorem in its magnification version. when one of the sectors is subject to increasing returns to scale.

(10)

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2012-2013)
MS (Q.E.) II Year
Incentives and Organisations

Maximum Marks: 30
Duration: 2 hours

DATE - 2/02

1. Consider the following hidden information model. A monopolist can produce a good in different qualities. The cost of producing a unit of quality s is s^2. The consumer can buy 0 or 1 unit. If she buys 0, her utility is 0, while if she buys 1 unit, her utility is θs.

It is common knowledge that θ is either θ_L, with probability α, or θ_H. The monopolist knows only this distribution, while the consumer knows her θ. Assume $0 < \alpha < 1$, $\theta_H > \theta_L > 0$.

The monopolist decides on qualities and prices and offers a contract which the consumer can accept or reject. Production takes place if the consumer accepts.

(a) What is the optimal contract, assuming that the consumer is served, irrespective of type?

(b) When is it optimal for the principal to offer a contract designed to serve only the high value consumer?

$$[10 + 5] = [15]$$

2. Consider the following costly state verification model. A monopoly financier can fund an entrepreneur in exchange for half the return from the project. The project return can be 0 or 1 equiprobably. The entrepreneur observes the outcome and if it is 1, can give the financier the appropriate share, or misreport and give 0. The financier normally does not observe the outcome, but can do so by auditing at cost c after the entrepreneur has made a payment. The entrepreneur cannot make a payment exceeding the project return.

If on receiving 0, the intermediary audits and finds the outcome has been 1, he claims the full return from the project. But he cannot commit to an audit strategy prior to the project getting initiated. Assuming $0 < c < 0.5$, what is the financier's net expected payoff from the interaction?

$$[15]$$

MSQE II - Theory of Finance II
Midterm / Semester II 2012-13
Time - 2 hours/ Maximum Score - 30

NOTE : SHOW ALL YOUR WORK. RESULTS USED MUST BE CLEARLY STATED.

1. (6+6=12 marks)

   Let f be the price of a derivative based on two risky assets whose prices are given by S_1 and S_2 where the market has risk-free interest rate r.

   (a) If the asset prices follow the binomial model find the Δs and the risk-neutral probabilities, if possible. Is the market complete? Is the market arbitrage-free?

   (b) Answer the same question when the asset prices follow the trinomial model after finding the Δs and the risk-neutal probabilities, if possible.

2. (5+3=8 marks)

   Consider the Black-Scholes model for a risky asset S with a continuous dividend (with a constant rate q). Prove the European put-call parity relation,

   $$P_q(t) + S_t e^{-q(T-t)} = C_q(t) + K e^{-r(T-t)}$$

   where $P_q(t)$, $C_q(t)$ and S_t are the prices of put, call and asset prices, respectively. Here r is the risk-free interest, and K and T are the strike price and maturity time, respectively, for both.

   Is the market complete and arbitrage-free? Justify.

3. (5+5=10 marks)

   (a) Let r_i be risk-free interest in the in the In the continuous model regime, give an example of a market, with two risky assets, which is complete and arbitrage free.

   What is the market price of risk in this case?

   (b) In the same continuous model regime, give an example of a market, with two risky assets, which is incomplete.

   What is the market price of risk in this case?

4. (3+3=6 marks) Write TRUE or FALSE and justify your answer (the justification should
   Mathematical as much as possible).

   (a) Short rate $r(t)$ is the instantaneous rate with which money account grows and can
       found as a limit of the term structure interest rate.

   (b) For $0 \le t \le T$, let $p(t, T)$ be the price of the zero coupon bond at time t, that matures
       time T. Then
       $$p(t, T) = exp\{-\int_t^T r(u)du\}$$
       where r is the short rate.

All the best.

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: (2012-2013)

MS(QE) II

Social Choice and Political Economy

Date: 28.02.2013   Maximum Marks: 40   Duration: 2 hours.

Note: Answer Group A and Group B in separate answer scripts.

Group-A

Note: Answer all questions.

(1) (a) Develop a necessary and sufficient condition for the Blackorby-Donaldson-Kolm inequality index to be an absolute index. Clearly state the preliminaries you require here. (8)

 (b) Provide (with appropriate explanation) a policy interpretation of the index and give its graphical representation. (3)

(2) (a) Show that the strong Pareto principle implies the weak Pareto principle using a welfare function defined on a given domain. (7)

 (b) Define a rank preserving progressive transfer in a non-decreasingly ordered income distribution from person $(i + 4)$ to person i, $1 \leq i < i + 4 \leq n$. (2)

Group-B

Note: Answer all questions.

(1) Consider the social aggregation problem with a finite number of agents and two alternatives. Show that the majority voting social welfare function is symmetric among agents, neutral between alternatives and positive responsive. Find a Paretian social welfare function that satisfies neutrality between alternatives, that fails to satisfy symmetry among agents and that also fails to satisfy positive responsiveness. (6+4=10)

(2) State and prove the group contraction lemma giving all the relevant definitions and stating all the relevant results. (10)

Date: Maximum Marks: **50**   Duration: 2 hrs.

(Answer any two questions)

1. (a) Define Lorenz Curve (LC) that incorporates both discrete and continuous type size distributions of income, given the distribution function
$$F : [0,\ T] \to [0,\ 1]$$

(b) Find the LC for log-logistic distribution using the above definition.

(c) Suppose you are given a data set of K income classes (x_{i-1} to x_i) with the corresponding frequency of population in each income class (n_i, $i=1,2,....,K$).

    (i)   Describe a method to fit the log-logistic distribution to the above data set.

    (ii)   How do you judge the goodness-of-fit?

                                        [4+4+(8+9)=25]

2. (a) Consider two income profiles $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$ with $x_1 \le x_2 \le \cdots \le x_n$ and $y_1 \le y_2 \le \cdots \le y_n$.

    (i)   Define Lorenz dominance of x with respect to y. State the assumptions clearly.

    (ii)   Why is the ordering of income profiles, generated by the LC comparison, a quasi-ordering? Illustrate with an example.

    (iii)   What is a generalized Lornez curve (GLC)? What is the advantage of GLC over LC?

(b) Define 'proportional failure rate' (PFR) in the context of income distribution. Show that the Pareto distribution has constant PFR and the Lognormal distribution has increasing PFR throughout the domain.

(c) Suppose the income variate x follows a Gamma distribution with density function
$$f(x) = \frac{1}{\Gamma\alpha}\ e^{-x}\ x^{\alpha-1}, \quad x \ge 0$$
Find the Lorenz curve and Lorenz Ratio.

                                         [9+8+8= 25]

P.T.O.

3. (a) What do you mean by a "Bistochastic" matrix? Explain its relevance in the context of welfare comparison.

(b) In the context of measurement of poverty, state the axioms (i) Replication invariance, (ii) Scale invariance and (iii) Decomposability.

What would be the problem if a poverty index does not satisfy these axioms?

(c) Consider the income profile $x_1 \leq x_2 \ldots \leq x_n$ and the poverty measure

$$P = \left[\frac{1}{q} \sum_{i=1}^{q} (z - x_i)^\alpha \right]^{1/\alpha}, \text{ where } q: \text{ number of poor and } z: \text{ poverty line.}$$

Examine the index in light of the above axioms.

(d) What are "ethical" poverty measures?

[8+9+ 6 +2 = 25]

Answer any five questions. Marks allotted to each question are given within parenthesis.

1. (a) Suppose a paper mill located by the side of a river disposes off wastewater, generated during production, in the river. A fishery firm operates its fishing activities in the down stream of the river. Assuming zero transaction cost of bargaining, examine if it is possible to regulate externalities through bargaining if property rights are clearly defined between polluter and victim. Explain your answer.

    (b). The cost function of producing the quantity of paper P is given as $C_p = P^2 + 8$ and that of producing fish product F is given as $C_F (F, P) = F^2 + FP + 4$. The per unit prices of fish and paper are Rs. 10.0 and Rs. 11.0, respectively. Calculate the production and profit of each firm if (i) two firms are independent, (ii) two firms are merged, (iii) paper mill has the right to pollute and (iv) fishery has the right to get clean water.

    [12+8=20]

2. Consider a non-renewable resource X, say oil, stock of which at the initial period is X_0. Let p_t and q_t be respectively the price and quantity of extraction in period t. Assume cost of extraction is affected only by quantity of extraction and the rate of interest is fixed at r for all periods.

    (a) State and derive the optimal extraction policy that would be followed by a competitive mine owner.

    (b) Assume price $p_t = p$, which is constant and cost function is $C(q_t) = a + bq_t^2$. Starting extraction at 0th period, what will be the optimal extraction path and optimal time (T) when the firm will stop extraction?

    [10+10=20]

3. In a single species competitive fishery the initial stock of fish is X_0 and production function is $q = \theta XE$, where q is the harvest rate, θ is a positive constant and E is effort involved in catching fish. The simple cost function is $c(X,q) = c(X)q$, where $c(X)$ is average cost depending on stock of fish and w is the cost per unit of effort. The biological growth function is given by $g(X) = \gamma X(k - X)$, where k is carrying capacity level and γ is a positive constant. The firm's objective is to maximize present value of net benefit over an infinite time horizon, given the discount rate r, which is same as the market rate of interest. Competitive price is p, which is constant.

(a) Determine the 'Fundamental Rule' for capital accumulation for renewable resources.
(b) How does a change in the rate of interest change the optimal decision for capital accumulation?

[12+8=20]

4. What do you mean by total economic value of environmental goods and services? Explain travel cost method (TCM) and problems in using this method for empirical estimation of environmental demand function for a natural site.

[5+15=20]

5. (a). As a forest supplies both timber and amenity value, the optimal forest management criterion should be extended to include the amenity value along with timber supply from the forest. In this situation, how does a private forest land owner decide the rotation age in a rotational forestry, when optimal decision is affected by market determined factors like stumpage price, forest establishment cost and rate of interest? Explain your answer analytically.
   (b). Explain how timber taxation policies may affect the timing of harvest (assuming that only timber value is considered for estimating rotation age)?

[12+8=20]

6. (a). Explain the management methods used to prevent the over-exploitation of an open access fishery.
   (b) Explain the non-declining natural capital stock rule for sustainable development.

[10+10=20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2012-2013)
MS(QE) II
Social Choice and Political Economy

Date: 26.04.2013   Maximum Marks: 100   Duration: 3 hrs.

Note: Answer Group A and Group B in separate answer scripts.

Group A

1) (a) Define Harsanyi's aggregation equation by giving necessary preliminaries. (3)

(b) Clearly state the necessary and sufficient condition for the Harsanyi aggregation equation to be unique. (2)

(c) Consider a two-person society in which

$$v_1(p) = 25p_0 + 10p_1 + 20p_2,$$

$$v_2(p) = 5p_0 + 15p_1 + 25p_2, \text{ and}$$

$$v(p) = 10p_0 + 10p_1 + 15p_2.$$

Establish the Harsanyi aggregation equation in this case. Is it unique? (8)

(2) (a) Define the lexicographically equitable transfer principle. (2)

(b) Consider the social welfare function $\sum_{i=1}^{n} \left(H\left(\frac{i}{n}\right) - H\left(\frac{i-1}{n}\right) \right) x_i$, where x_i's are non-decreasingly ordered incomes. Show that the corresponding Atkinson-Kolm-Sen inequality index satisfies the lexicographically equitable transfer principle if and only if $\frac{H\left(\frac{i}{n}\right)}{\frac{i}{n}}$ is decreasing in $\frac{i}{n}$. (8)

(c) Identify the necessary and sufficient condition for the generalized Gini welfare function $\sum_{i=1}^{n} a_i x_i$ to satisfy the Pigou-Dalton transfer principle, where $a_i > 0$ for all i and the incomes x_i's are ordered non-decreasingly. (8)

(3) Characterize the utilitarian rule using anonymity as one of the axioms. (6)

(4) Consider two income distributions whose total incomes as well as population sizes are different. Develop a quasi-ordering of the two distributions using a strictly S-concave social welfare function. (7)

(5) Make a systematic comparison between cardinally measurable non-comparable utilities with ordinally measurable non-comparable utilities by giving examples. (6)

Group B

(1) Consider a social welfare function F defined over a finite set of alternatives A with $|A| \geq 3$ for a finite set of agents N. Show that if F satisfies unrestricted domain, independence of irrelevant alternatives and non-imposition, then one of the following statements must be true.

(a) F is null.

(b) There exist $x, y \in A$ such that for all profiles with the property that $x P_i y$ for all $i \in N$, we have $x P y$.

(c) There exist $x, y \in A$ such that for all profiles with the property that $x P_i y$ for all $i \in N$, we have $y P x$. **(12)**

(2) Consider the preference relation $\mathcal{R}$ defined on the finite set of alternatives A with $|A| \geq 3$. Answer the following questions.

(a) Define reflexivity, completeness, quasi-transitivity and acyclicity of $\mathcal{R}$ on A.

(b) Show that $\mathcal{R}$ on A is reflexive, complete and quasi-transitive if and only if for all $x, y, z \in A$ such that for $x P y$ and $y R z$ we get $x R z$.

(c) Demonstrate via an example that acyclicity over triples is not sufficient for maximal elements to exist. **(8+10+5=23)**

(3) Let $\mathcal{R}_{\geq}$ be the set of all possible single-peaked preferences defined on the real line. Let n be the odd number of agents in a society with single-peaked preferences. Define group strategyproofness of a social choice function. Consider a social choice function that selects the median ranked peak for each profile, that is, given a profile $x = (x_1, \ldots, x_n)$ of announced peaks of the n agents, the social choice function picks the $\left(\frac{n+1}{2}\right)^{\text{th}}$ ranked peak. Show that this social choice function is group strategyproof. **(15)**

Indian Statistical Institute
Second Semestral Examination: 2012 –2013
M.S.(Q.E) – II Year
Econometric Applications II

Date: 30.4.2013   Maximum Marks: **100**   Duration: 3 hrs.

(Answer any <u>four</u> questions)

1. (a) What are 'relative' and 'absolute' measures of income inequality?
Define 'normative' and 'positive' measure of inequality. Give examples.

(b) State and explain the desirable properties that an inequality index should satisfy.

(c) Examine the following inequality index in the light of the above properties.

$$I(x) = \frac{\sqrt{\dfrac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}}{\mu}$$, where μ is the mean income.   [5+8+12= 25]

2. (a) Consider the following logarithmic form of the cost function $C\,(u,\,p)$:

$$\log C\,(u,\,p) = a_0 + \sum_j a_j \log p_j + \frac{1}{2}\sum_j \sum_k c^*_{jk} \log p_j \log p_k + u b_0 \Pi_j\, p_j^{b_j}\,,$$

where notations have their usual meanings.

(i) Under what restrictions on the parameters is $C(u,\,p)$ linear homogeneous in prices? Explain.

(ii) Derive the demand system in budget share equation form from the above cost function. What is this system called?

(iii) Derive the conditions under which the demand system satisfies adding–up, homogeneity and symmetry properties..

(b) What are 'exact' and 'consistent' aggregations in the context of consumer demand analysis? What type of aggregation does the above system conform to?

[(3+5+12)+5 = 25]

3. (a) What is the difference between a 'censored' and a 'truncated' distribution?

(b) Describe the 'Tobit model' in the context of modelling zero expenditure.

(c) Describe the 'Double hurdle model' (with discrete random preference regimes) for modelling zero expenditure.

[3+10+12=25]

4. (a) What is 'sample selection bias'? Give an example of a bivariate sample selection model with a participation and an outcome equation. How would you estimate the parameters using Heckman's procedure?

(b) Discuss the idea of Propensity Score Matching (PSM) and its applicability. Describe the different matching procedures.

(c) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?

[12+10+3=25]

5. (a) Suppose a production function $f(x)$ is strictly monotonic, quasiconcave and positively linearly homogeneous. Then show that $f(x)$ is concave.

(b) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans. "D-F technical efficiency is necessary, but not sufficient for Koopmans technical efficiency" – explain this statement diagrammatically for both input and output oriented measures.

(c) Show graphically, using a single input-single output framework, that the input and output oriented measures of technical efficiency are identical when the technology exhibits constant returns to scale, but it need not be true for variable returns to scale. Show how these definitions relate to the D-F measures.

(d) Describe the Modified Least Squares (MOLS) method of estimating technical efficiency from cross section data using a deterministic production frontier and a Half Normal distribution of efficiency (u).

$$[u \sim Half\ Normal(0,\sigma_u^2) \Rightarrow f(u) = \frac{2}{\sigma_u \sqrt{2\pi}} exp(-u^2 / 2\sigma_u^2)]$$

[6+8+6+5=25]

6. (a) State the assumptions under which it is possible to construct a production possibility set from the observed data without any explicit specification of a production function.

(b) Describe the input and output oriented model due to Charnes, Cooper and Rhodes in the context of Data Envelopment Analysis (DEA).

(c) Assume that we have the following data:

- Unit 1 produces 100 pieces of items per day and the inputs are 10 Rupees of materials and 2 labour-hours;

- Unit 2 produces 80 pieces of items per day and the inputs are 8 Rupees of materials and 4 labour-hours;

- Unit 3 produces 120 pieces of items per day and the inputs are 12 Rupees of materials and 1.5 labour-hours.

Define the objective function and the constraints to calculate the efficiency of unit 1. Reformulate the problem in a linear programing framework.

[7+10+8=25]

Indian Statistical Institute
Semestral Examination: 2012-2013
MS(QE) II: 2012-2013
Industrial Organization

Date: --09/05/2013   Maximum Marks: 40   Duration: 3 Hours

Answer Each Group on a Separate Answer Script

Group A: Answer any Two questions. Your score cannot exceed 20 marks in this Group.

1. Consider Cournot oligopoly with n number of identical firms; $n > 2$. Each firm has constant marginal cost, $b > 0$. The market demand is linear and in inverted form given by, $p = a - \sum_i x_i$; $0 < b < a$. (i) Show that a two-firm (horizontal) merger is never privately profitable but a k-firm merger is profitable if and only if $k/n \geq .0.8$ and $n \geq 5$. [Use $\pi(n) = [(a-b)/(n+1)]^2$]

[4+8=12]

2. A multinational firm is to decide whether it will export to a host country or open a 100% owned subsidiary in the host country. In case of export it faces a tariff $t > 0$ per unit for its exports, and in case of subsidiary formation it has to incur a set up cost, $F > 0$. In either case the MNC will have to compete with a host firm. The marginal costs of production of the MNC and the host firm are respectively, c_m and c_h; $c_m < c_h$. The market demand function is downward sloping. How do you decide the optimal entry strategy of the MNC?

[12]

3. Assume that two firms compete in prices, given the market demand function $D(p)$ with $D'(p) < 0, D''(p) \leq 0 \ \forall p > 0$ and $D(0) < \infty$. The firms have already come up with quantities $\overline{Q}_1$ and $\overline{Q}_2$; $0 < \overline{Q}_1, \overline{Q}_2 < D(0)$, and now they are to decide prices simultaneously and non-cooperatively. Show that in pure strategy equilibrium both firms will charge the same price $p(\overline{Q}_1 + \overline{Q}_2)$ where $p(.)$ is the inverted demand.

[12]

4. Consider n firm Cournot oligopoly where each firm holds a share θ of each of the other firms; $0 < \theta < 1$ and accordingly it has claim on other firm's profit. However, each firm production decision is taken by the respective firm independently. The market demand for the product is $P = A - bQ$ and marginal cost of production for each firm is constant and same, say, $c < A$. Show that a suitable choice of θ will lead to monopoly outcome.

[12]

Group B: Answer any Two questions. Your score cannot exceed 20 marks in this Group

1. Consider Nunn's (2007) model of entrepreneurship under colonization. An economy has a continuum of agents who are presented by [0,1]. Agents play the game G, where they simultaneously choose one of the two actions: productive (P) or unproductive (U). Let τ ($0 < \tau < 1$) be the rate of extraction by the colonizer and q ($0 < q < 1$) be the parameter standing for security of property rights.

A "P agent" (agent who chooses P) produces a good that has value 1. An "U agent" steals from the P agent. Let x be the fraction of U agents. Then

$s(x) = \Pr(\text{a P agent good is stolen}) = \min\{x/(1-x), 1\}$

$t(x) = \Pr(\text{an U agent steals successfully}) = \min\{(1-x)/x, 1\}$

Regardless of whether there is theft or not, a P agent loses fraction τ of his good to the colonizer. In the event of theft he further loses fraction q. An U agent gains fraction q of the good in the event of successful theft and his payoff is not subject to colonizer's extraction.

(a) Let $\tau > 1-q$. After writing the payoffs of P agents and U agents for the game G, determine all of its Nash Equilibrium (NE).

(b) Consider a specific example where $\tau = 3/4$ and q $= 1/2$ (note that $\tau > 1-q$, so you can use your answer in (a)). Does this game have a Correlated Equilibrium which is not an NE?

[7+5=12]

2. Consider the model of Aghion and Bolton (1987) where there is one buyer (B), one incumbent firm (I) and one entrant (E). B buys at most one unit of the good and has a valuation of 1 for the good. Both I and E have constant unit costs. I's unit cost k ($0<k<1$) is commonly known. However, the unit cost c of E is not known. It is only known that c is uniformly distributed in [0,1]. Note that E will make entry only if it can make positive profit.

I and B can sign a prior contract (P, P_0), which is implemented as follows: (i) if B buys from I, then I must charge price P to B and (ii) if B does not buy from I, B must pay a penalty P_0 to I.

    (a) Showing all steps of your work, determine optimal (P, P_0) for I.

    (b) Does optimal (P, P_0) always block entry from E?

<div align="right">[10+2=12]</div>

3. Consider the big push theory of industrialization of Murphy et al. (1989) where there are n sectors, each sector producing a distinct good. Labor is the only input in this economy. Each sector has a competitive fringe that can produce 1 unit of good from 1 unit of labor at wage 1. Each sector also has an entrant which can produce $\alpha > 1$ unit of good from 1 unit of labor at wage 1+v, where v>0. Any entrant also has to make an initial investment of F units of labor to make entry.

For i =1,..,n, denote the entrant of sector i by firm i. The n entrants 1,..,n simultaneously decide whether to make entry ("e") or stay out ("o"). This can be considered as an n-person game G where each player has two strategies. A firm that stays out obtains 0. Let $\pi(k)$ be the payoff of a firm that has made entry when there are k entrants. Suppose the initial conditions of the economy is such that (i) $\pi(k)$ is increasing in k, (ii) $\pi(1) <0$ and (iii) $\pi(n) >0$.

    (a) Determine all values of k that can be sustained as Nash Equilibrium.(NE) of G.

    (b) Determine if G has Correlated Equilibria (CRE) which are not NE. Are there CRE that result in "partial industrialization" (i.e. k>0 but k<n)?

<div align="right">[5+7=12]</div>

4. Consider the O-ring theory of productivity of Kremer (1993) where the production of a good requires completion of n tasks. A worker of skill q can successfully complete a task with probability q. The skill of workers of the economy is drawn uniformly from the interval $[0,1]$. If a firm hires a worker of skill q_i for task $i = 1..,n$, the probability that it will produce a good is $q_1.q_2..q_n$. Assume that any firm is a price-taker, which takes the price of the good as given. Let this price be 1. Let $w(q)$ be the wage for skill q.

(a) After writing the profit function of a firm, clearly state and prove the skill-clustering theorem.

(b) Assuming that firms operate in a perfectly competitive industry, determine $w(q)$.
  · Show that it is strictly increasing and convex in q.

$$[9+3=12]$$

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2012-13

Course: Masters in Quantitative Economics Year II

Subject: Political Economy and Comparative Economic Systems

Date: 6th May 2013 **Maximum Marks:** 100 **Duration:** 3 hours

ANSWER ANY FOUR QUESTIONS (All questions carry equal marks (25)).

1. Consider a society with two income levels: $I_H > I_L > 0$. Average income in this society is $\bar{I}$, total population is n and income class $i \in \{H, L\}$ has n_i members, with $n_L \equiv \rho n$, $\rho \in (0,1)$. Individuals have identical preferences, represented by the utility function $lnx + lny$, where x is the amount of a private good consumed by the individual and y is the total amount of a public good generated by voluntary contributions. Define $I_H \equiv \bar{I}(1 + \theta), \theta > 0$.

(i) Find the range of values of θ for which individual consumption bundles in the Cournot-Nash equilibrium are independent of the income distribution.

(ii) Show how this range changes with a change in: (a) the total population (n), and (b) the population distribution (ρ). Explain your results.

2. Using the model of communal conflict developed by Dasgupta and Kanbur, *derive* the range of income distributions for which the Cournot-Nash equilibrium involves *separation* between

communities. Explain the implications of your findings for policy interventions intended to reduce tensions between communities.

3. Using the model of ethnic conflict developed by Esteban and Ray, identify the key factors that determine the activism response function of a group. Explain how greater aggression in supplying activists, on the part of one group, impacts on the equilibrium activism level for *both* groups.

4. Set up the model of economic interaction between a majority and a minority discussed in class. Clearly identifying all relevant assumptions, **prove** that social segmentation is a necessary, but not sufficient, condition for cooperation to be observed.

5. Suppose an agent can either (a) produce for the market, or (b) engage in rent-seeking activities. Income from market production is c, some of which may be appropriated by rent-seekers. The maximum amount that can be appropriated by rent-seekers from any producer is $(c - s), (c - s) > 0$. The return to rent-seeking is r. $r \in [0, R]$. Thus, each rent-seeker can receive, at most, R; $s < R < c$. Let the ratio of people engaged in rent-seeking and production be n. Returns to productive activity are given by: $C = \max[c - nr, s]$. Let $n' = \frac{(c-s)}{R}$. Returns to rent-seeking are given by: $r = R$ if $n \le n'$, $r = \frac{(c-s)}{n} < R$ otherwise. Identify the equilibria in this economy and explain their stability characteristics. Use your results to illustrate how a growing economy with low rent-seeking can move permanently to one with low growth and high social strife in response to an external shock.

SECOND SEMESTRAL EXAMINATION (2012-2013)

M.S.(QE) II

Advanced Topics in International Economics

Date: *10·05·13*   Maximum Marks: 60   Duration 3 hours.

Group A

Answer any two questions. All questions carry equal marks.

1. Consider a two country two good scenario where one of the goods exhibit production uncertainty and where the countries are identical *ex ante*. Making suitable assumptions about technology and demand argue that free trade between the countries can be Pareto inferior.

2. Set up a model of a small open economy with moral hazard and private insurance. Argue heuristically that when insurance companies are competitive and risk neutral, free trade is the (constrained) Pareto optimum.

3. In a two sector short run Keynesian model where investment is autonomously given, show that free trade unambiguously benefits the country exporting investment goods and unambiguously hurts the country exporting consumption goods.

Group B

Answer all

1. Refer to the Krugman (1991) model of Economic Geography. Begin with a situation where all footloose factors of production are located in one single location. Now, how would changes in transport cost destabilize such an allocation of factors? Assume that the elasticity of substitution is reasonably high and share of expenditure on manufacturing low.   (15)

2. Consider an economy producing a final good Y with the following production function:

$Y = X^\alpha L_Y^{1-\alpha}$, where $X = \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}}$ and $0 < \alpha, \rho < 1$. L_1 is the amount of labour and x is

the amount of the ith intermediate input (they come in n varieties) used in the production of Y. To produce these intermediate goods one has to spend one unit of capital to begin with (this constitutes the fixed cost), and thereafter, each successive units of the good is produced by employing one unit of labour alone. The total amount of labour and capital available to the economy is $\bar{L}$ and $\bar{K}$ respectively. The producers of the intermediate inputs are monopolistically competitive in the usual Dixit-Stiglitz sense. Labour is perfectly mobile between the Y sector and the intermediate goods sector and full employment of both the factors, labour and capital, prevails. Furthermore the Y producers can only earn zero profits in equilibrium (i.e. price is equal to average cost in Y sector). This constitutes the description of the economy.

If this economy was allowed to trade (where trade is at the level of both the final good Y and the intermediate goods x_i's) with an economy which is in all respect similar to this country, except that they may differ in their total labour and capital endowments.

a) Show that the wage rate would be equalized in both the countries.

b) Show that the rental rate would be equalized in both the countries.

(7.5 + 7.5 15)

MSQE II - Theory of Finance II
Final Exam. / Semester II 2012-13
Time - 3 hours/ Maximum Score - 40
Date: 11th May, 2012

1. (6+2+4=12 marks)

   Assume the following short rate model for r,

   $$dr = a(b - r)dt + \sigma\sqrt{r}dW_t$$

   with $a(> 0)$, b, σ, $r(0)$ are constants and W is a standard Brownian motion.

   (a) Show that, if $r(0) > 0$ then $r(t)$ remain positive. Find the distribution of $r(t)$ as $t \to \infty$.

   (b) Can this $r(\cdot)$ be judged as a reasonable model for interest rate? Justify your answer.

   (c) Does this model for interest rate posses Affine Term Structure? Explain briefly with reasons.

2. (4+6=10 marks)

   (a) In the continuous model regime, give an example of a market, at least with two risky assets, which is complete and arbitrage free.

   (b) Let the prices of two stocks, at time $t \geq 0$, be denoted by S_{1t} and S_{2t}, such that $dS_{it} = \mu_i S_{it}dt + \sigma_i S_{it}dW_t$, for $i = 1, 2$. Assume that a derivative pays $\max((S_{1T} + S_{2T}) - K)^+$, where T is the maturity time and K is a positive constant (that may be called the strike price).

   (i) Find the risk-neutral price of the derivative at time $t \geq 0$.

   (ii) Find the corresponding risk-neutral measure.

3. (5+5=10 marks)

   (a) Consider a coupon bond, starting at T_0, that pays the owner an amount c_i at intermediate dates T_i, $i = 1, \ldots, n$ ($T_i < T_{i+1}$, for all i). At time T_n owner gets the face value K. For $t < T_1$, find the price of this bond, say $p_f(t)$, in terms of the price of the zero coupon bond.

   (b) Assume $c_i = r(T_i - T_{i-1})L(T_{i-1}, T_i)K$, where $L(.,.)$ denote the LIBOR rate. Determine r and show that the price p_f at time T_0 is K.

4. (4+4+4+4=16 marks)

Justify if the following holds or not.

(a) If the investors and the firms gets a different rate of interest for borrowing then from Modigliani-Miller's theorem, we get $\bar{X}_{jk}/V_{jk} = \rho_k$ for all firm j, irrespective of the firms' financial structure (or capital structure), where V_{jk} is the value and $\bar{X}_{jk}$ is the total expected return of the firm j in the kth class.

(b) In an incomplete market if a portfolio h_1 is replicable with another one, say h_2, then both the portfolios have the same price with respect to all Martingale measures, if the market is arbitrage free.

(c) In an arbitrage free market there exist a unique market price of risk process.

(d) Any price process with constant rate of growth can be made a martingale with an appropriate measure by a change of numeraire.

All the best.

Course name: MSQE II

Subject name: Incentives and Organisations

Date: / 3 ᴜ ᶜ / ᴣ

Maximum marks: 50

Duration: 3 hours

1. Consider an economy with a continuum of agents. Agent i produces output q_i by supplying input a with the individual production function $q_i = \theta_i a$, where θ_i is an idiosyncratic productivity parameter. For any agent, the productivity parameter is a random draw from support $[\theta_L, \theta_H]$ according to density $f(\theta)$ where $f(\theta) > 0$ for all θ. All agents have the same utility function $u(c) - a$ with $u' > 0$ and $u'' < 0$, where c is consumption.

a. What is the distribution of output and consumption in the economy when each agent lives and works in autarchy? [5]

Suppose now that all agents in this economy can write an insurance contract on a competitive insurance market before they know their productivity type.

b. What is the optimal insurance contract when θ and a are observable ex post? [5]
c. What is the optimal insurance contract when only θ is observable? [5]
d. What is the optimal contract when neither θ nor a are observable ex post and $f(\theta)/(1 - F(\theta))$ is monotonically increasing? [5]

2. Consider a principal-agent problem with three exogenous states of nature, $\theta_1, \theta_2, \theta_3$; two effort levels, a_L and a_H; and two output levels, distributed as follows as function of the state of nature and effort level:

State	θ_1	θ_1	θ_1
Probability	0.25	0.5	0.25
Output under a_H	18	18	1
Output under a_L	18	1	1

The agent implements effort before he learns the state of nature. The principal is risk-neutral, while the agent receives utility $w^{0.5} - c(a)$ when he receives monetary compensation w and puts in effort a, $c(a)$, the cost of effort, is 0 for a_L and 0.1 for a_H. The agent's reservation utility is 0.1.

a. Derive the optimal contract when effort is observable. [10]
b. Derive the optimal contract when only output is observable. [10]
c. Assume that principal can buy for a price p an information system that allows the parties to verify whether state of nature θ_3 happened or not. What maximum price will he pay for this system? [10]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2012-2013)
MS(QE) II
Back Paper
Social Choice and Political Economy

Date: **22.07.13**   Maximum Marks: 100   Duration: 3 hrs.

Note: Answer Group A and Group B in separate answer scripts.

Group A

(1) Define the generalized utilitarian rule. State and prove a theorem on the generalized utilitarian rule. **(2+2+8=12)**

(2) Define independent prospects. Show that independence of prospects holds if and only if utilities are affinely independent. **(2+8=10)**

(3) Axiomatically characterize the social welfare function $\lambda(1-cI)$ where $\lambda > 0$ is the mean income , $0 < c < 1$ is a constant and I is a relative inequality index bounded between zero and one. **(14)**

(4) Each of the following statements is either true or false. If the statement is true, prove it. Give a counterexample if it is false.

    (a) Given ordinally measurable, non-comparable utilities, continuity and the strong Pareto principle cannot be satisfied simultaneously.

    (b) For ordinally measurable, non-comparable utilities, the weak Pareto principle and anonymity are incompatible. **(7+7=14)**

Group B

(1) Define quasi-transitivity and acyclicity. Consider a reflexive and complete binary relation R defined on the set of alternatives A which is finite but has more than two elements. Using this information prove the following:

    (a) The binary relation R is quasi-transitive if and only if for all $x, y, z \in A$, xPy and yRz implies xRz.

    (b) The binary relation R is acyclic if and only if the maximal set $M(S, R)$ is non-empty for all non-empty $S \subseteq A$.

    (4+8+8=20)

(2) State and prove the Median Voter Theorem, giving all the relevant definitions. **(10+5=15)**

(3) Define strategyproofness, unanimity and the full range property of a social choice function when agents have strict preferences and the set of alternatives is A which is finite and has more than two elements. Show that if the social choice function satisfies strategyproofness and the full range property then it also satisfies unanimity. (Note that the set of agents may be more than two) $(4+2+2+7=15)$