Studies on p-t-diagnosable system

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Abstract: The paper presents an algorithm for testing the probabilistically t-diagnosability of a system represented by a weighted digraph. In the second part of the paper, the design of a probabilistically t-diagnosable system (when the probability of failure of each subsystem is known) is described.

1 Introduction

In recent years, the problem of diagnosis of faults in a system (represented by a collection of interconnected components of the system) has achieved considerable importance. Some works available in the literature [1–5] concern themselves mainly with systems whose components have equal probability of failure. In general, all the components in a large system are not equally reliable. A more general model of diagnosability takes into account the fact that each of the components is associated with a definite probability of failure. Some necessary and sufficient conditions for diagnosability of such systems were given by Maheshwari *et al.* [6] and Fujiwara *et al.* [7]. In this paper we present an algorithm to test the diagnosability of such systems, where the probability of failure of each individual component is known beforehand. We also describe an algorithm for designing the interconnection between the components of a system such that it may be diagnosable.

2 P-t-diagnosable system in graph-theoretical model

A system is composed of several units or components and these units are able to test other units singly or in combination. The testing unit evaluates the tested unit as either fault-free or faulty. The test outcome is reliable only if the testing unit itself is fault-free, otherwise the test outcome is unreliable. As proposed by Preparata [1] a system S is represented by a digraph $G = (V, E)$ where each unit corresponds to a node in V and there is a directed edge from the ith node to the jth node in G if the ith unit u_i tests the jth unit u_j. The result of testing is represented by labelling the corresponding edge. Let a_{ij} be the label associated with the edge from node u_i to node u_j. The value of a_{ij} will be as follows:

a_{ij} = 1 if u_i is fault-free and u_j is faulty
= 0 if both u_i and u_j are fault-free
= '–' if u_i is faulty and irrespective of the status of u_j, since the testing unit u_i is itself faulty, the test outcome is unreliable and hence can have value either 0 or 1 without conveying any information about the tested unit.

For any subset $Q \subset V$, if k is the total number of outgoing edges from the nodes in Q, then there are a maximum of 2^k possible ways of labelling the edges of G, when all nodes in Q are the only faulty nodes. Let us denote the set of these test results as $R(Q)$. Two sets Q_1 and Q_2 of faulty units are distinguishable if and only if $R(Q_1) \cap R(Q_2) = \phi$, i.e. if the test result uniquely identifies the set of faulty units, then Q_1 and Q_2 are distinguishable. When the different units of a system have different probabilities of failure, its representation is given by a weighted digraph, each node representing each unit of the system, the edges representing the testing capabilities as discussed above and some weight is assigned to each node, which gives the probability of failure of that unit.

Let p_i be the weight of the ith node, i.e. p_i is the probability of failure of the ith unit. A set Q of nodes of G can fail simultaneously, if the joint probability of failure of all the nodes in Q is greater than or equal to some preset value t. Thus, Q will be a possible set of faulty nodes, if

$$\prod_{\substack{i\\u_i \in Q}} p_i \prod_{\substack{j\\u_j \notin Q}} (1 - p_i) \geqslant t$$

i.e.

$$\prod_{\substack{i\\u_j \in V}} (1 - p_j) \prod_{\substack{i\\u_i \in Q}} \frac{p_i}{(1 - p_i)} \geqslant t$$

denoting $\log\, p_i/(1 - p_i) = W_i$ and $\sum_i \log(1 - p_i) = K$, a constant for all Q, we find Q will be a possible set of faulty nodes if $\sum_{\substack{i\\u_i \in Q}} W_i \geqslant \log t - K$

This can be rewritten as

$$\sum_{\substack{i\\u_i \in Q}} W_i \geqslant f(t) \quad \text{where} \quad f(t) = \log t - K$$

A system will be said to be p-t-diagnosable (probabilistically t-diagnosable) if for every pair $Q_1, Q_2 \subset V$, and $W(Q_1) \geqslant f(t)$ and $W(Q_2) \geqslant f(t)$ {where $W(Q) = \sum_{\substack{i\\u_i \in Q}} W_i$, Q_1 and Q_2 are distinguishable}.

The different aspects of p-t-diagnosability or simply t-diagnosability (in the case when p_i is a constant for all i) have been discussed by several authors. In this paper we present an algorithm to test whether a system given as a weighted digraph is p-t-diagnosable. The design of a p-t-diagnosable system has also been discussed, i.e. given the set of nodes and the weight of each node, the problem is to find the edge connections so that the system is p-t-diagnosable for some given t. In general for practical situations, $p_i < 0.5$ and hence throughout our discussion we shall assume W_i to be negative for all i.

3 Definitions and notations

We shall denote the set of nodes testing the ith node u_i of G by $C(u_i)$. Any subset Q of V will be denoted by a Boolean team $T(Q)$. If $u_i \in Q$, u_i will appear as uncomplemented in the term, otherwise complemented. Conversely, any Boolean term containing the u_is as literals (primed or unprimed) represents a set of subsets of V. The set will contain a single

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subset of V if all the u_is appear as literals in the term. If u_i appears in the term in the primed form, then the set gives all the subsets of V in which u_i does not appear and similarly if u_j appears in the unprimed form, the set gives all the subsets of V in which u_j does appear. If T is a Boolean term involving the nodes of G, then $W(T)$ will denote the value of $W(Q)$ where Q is the smallest subset in the set of subsets represented by T. If T_1 and T_2 are two Boolean terms involving the nodes of G, then $W(T_1 T_2)$ will denote the value of $W(Q_1 \cup Q_2)$ where Q_1, Q_2 are, respectively, the smallest subsets in the sets of subsets represented by T_1 and T_2.

A Boolean term T involving the nodes of G will be called an admissible term if $W(T) \geqslant f(t)$. Two admissible terms T_1, T_2, involving the nodes of G, will be called pairwise admissible if $W(T_1 T_2) \geqslant 2f(t)$. If $Q_1, Q_2 \subset V$, then $Q_1 \oplus Q_2$ will denote the set of nodes belonging to Q_1 or Q_2 but not both.

4 Testing for *p-t*-diagnosability

With the above definitions and notations, we now proceed to formulate a few necessary conditions for *p-t*-diagnosability and then our method for testing *p-t*-diagnosability.

Theorem 1 [6]:
G may be *p-t*-diagnosable only if

$$W\{u_i \cup C(u_i)\} < f(t) \quad \text{for all } i$$

Theorem 2:
If Q_0 is a subset of V with $W(Q_0) \geqslant f(t)$ and there is no $Q \subset V$ with $W(Q_0) > W(Q) \geqslant f(t)$ then G may be *p-t*-diagnosable only if $W(V) < 2W(Q_0)$.

Proof:
If $W(V) \geqslant 2W(Q_0)$, form $Q_1 = Q_0$ and $Q_2 = V - Q_1$, then $W(Q_2) \geqslant W(Q_0) \geqslant f(t)$. Since Q_1 and Q_2 give two block partition on V and both of them are valid sets of faulty nodes, G is not *p-t*-diagnosable.

Theorem 3:
Two sets of faulty nodes Q_1, Q_2 are distinguishable if and only if there is an edge from at least one node in $V - (Q_1 \cup Q_2)$ to at least one node in $Q_1 \oplus Q_2$.

Proof:
Let us define, $V_1 = (V - Q_1) \cap (V - Q_2)$, $V_2 = Q_1 \cap (V - Q_2)$, $V_3 = (V - Q_1) \cap Q_2$, $V_4 = Q_1 \cap Q_2$. Then if the condition does not hold, there is no edge from any node in V_1 to any node in $V_2 \cup V_3$. Then label the edges of G as follows. Any edge from a node in $V_1 \cup V_2$ (or $V_1 \cup V_3$) to a node in $V_1 \cup V_2$ (or $V_1 \cup V_3$) is labelled as 0 and that to a node in $V_3 \cup V_4$ (or $V_2 \cup V_4$) as 1. Assign any labelling to the edges coming from any node in V_4. It can be seen that, for this labelling, Q_1, Q_2 are indistinguishable.

If the condition holds, there is an edge from a node in V_1 to a node in $V_2 \cup V_3$. If the nodes in Q_1 are faulty, the labelling of this edge is 0 (or 1) if the tested node is in V_3 (or V_2). If the nodes in Q_2 are faulty, the labelling of this edge is 0 (or 1) if the tested node is in V_2 (or V_3). Thus, $R(Q_1) \cap R(Q_2) = \phi$, i.e. Q_1, Q_2 are distinguishable.

We shall be using theorem 3 for testing the *p-t*-diagnosability of G after the necessary conditions given by theorems 1 and 2 are found to hold good. If there exist two faulty sets of nodes Q_1 and Q_2 which are not distinguishable, then there must exist at least one node u_i which belongs to Q_1 but not to Q_2. Let us find the pair of faulty sets of nodes, one containing u_i and other not. Thus they will belong to the sets given by T_1 and T_2, respectively, where $T_1 = u_i$ and $T_2 = \bar{u}_i$. If more literals are added to these two terms T_1 and T_2, the sets given by them will be refined and to have this refinement, we shall be using theorem 3.

In order that the sets of subsets of V given by T_1 and T_2 may contain pairwise-indistinguishable sets of faulty nodes, addition of literals to T_1 and T_2 must be done obeying the following conditions:

(*a*) All the nodes testing u_i should be added to T_1 and T_2: but no node should appear in the primed form in both T_1 and T_2 – this follows directly from theorem 3 (otherwise all the subsets given by modified T_1 will be distinguishable from those given by modified T_2).

(*b*) All the nodes testing u_i must not be added to T_1 in the unprimed form; because then the modified T_1 will no longer remain an admissible term as we have already assumed theorem 1 to hold good.

(*c*) Nodes should be added to T_1 and T_2 in a way to keep both the modified terms admissible, otherwise it cannot represent a faulty set of nodes as an element of the represented set.

(*d*) Nodes should be added to T_1 and T_2 in a way to have the modified terms as an admissible pair; the justification of this requirement is given by the following theorem.

Theorem 4:
If Q_1, Q_2 are two faulty sets of nodes, then $W(Q_1 \cup Q_2) \geqslant 2f(t)$.

Proof:
With $V_1, V_2, \ldots$ etc. as defined in the proof of theorem 3, $Q_1 = V_2 \cup V_4$ and $Q_2 = V_3 \cup V_4$. Thus $W(Q_1 \cup Q_2) = W(V_2) + W(V_3) + W(V_4)$. Hence, $W(Q_1 \cup Q_2) = W(Q_1) + W(Q_2) - W(V_4)$. As $W(Q_1) \geqslant f(t)$ and $W(Q_2) \geqslant f(t)$, the result follows as all the Ws are negative.

Taking consideration of the above conditions, literals should be added to the terms T_1 and T_2 so as to refine the sets of subsets of V which are pairwise indistinguishable. Suppose u_i is tested by the nodes $u_{i1}, u_{i2}, \ldots, u_{ik}$. Combining the conditions (*a*) and (*b*) we find that among these k nodes, there will be at least one node, say u_{i1}, which is to be added to T_1 as primed and to T_2 as unprimed. In fact, there will be, in general, a number of ways of adding these k nodes to T_1 and T_2 so as to satisfy the above conditions. For example, suppose u_1 is being tested by u_2 and u_3 and their Ws are such that sum of the Ws of any two of them is greater than or equal to $f(t)$. Then, if we choose $T_1 = u_1$ and $T_2 = \bar{u}_1$, all the possible refined forms of T_1 and T_2 (called T_{11} and T_{21}, respectively), after the nodes testing u_1 have been added to T_1 and T_2 as literals, are as follows:

$$\begin{array}{lll} T_{11} = u_1 u_2 \bar{u}_3 & T_{11} = u_1 \bar{u}_2 u_3 & T_{11} = u_1 \bar{u}_2 \bar{u}_3 \\ T_{21} = \bar{u}_1 u_2 u_3 & T_{21} = \bar{u}_1 u_2 u_3 & T_{21} = \bar{u}_1 u_2 u_3 \\ T_{11} = u_1 u_2 \bar{u}_3 & T_{11} = u_1 \bar{u}_2 u_3 & \\ T_{21} = \bar{u}_1 \bar{u}_2 u_3 & T_{21} = \bar{u}_1 u_2 \bar{u}_3 & \end{array}$$

Since u_{i1} appears as primed in one refined form and unprimed in another, each of these pairs can be further refined taking into consideration the nodes which are testing u_{i1} and excluding those which have already appeared in the refined forms. As in theorem 3, if any node appears in T_{11} and T_{21}, both in unprimed form, no refinement using this node is possible. As we have found from T_1, T_2, a number of refined pairs of terms T_{11} and T_{21} has been formed, repeating the same process on every pair T_{11} and T_{21}, we will get a number of T_{12} and T_{22}, the second refinement on T_1 and T_2 and so on. The process will terminate either when refining T_{1i} and T_{2i} to $T_{1(i+1)}$ and $T_{2(i+1)}$ no new nodes can be included, which shows that there exists at least one pair of indistinguishable sets of faulty nodes; hence G is not diagnosable, or, when any of the above four conditions no longer holds, indicating indistinguishable pairs of sets of faulty nodes (one containing u_i and another not) G does not exist.

For example if, in the above example, u_3 is tested by u_2 and u_4, then after the second refinement of the first T_{11} and T_{21} we will have

$$T_{12} = u_1 u_2 \bar{u}_3 u_4$$

$$T_{22} = \bar{u}_1 u_2 u_3 \bar{u}_4$$

This is only the second refined pair of the first T_{11} and T_{21} to satisfy the conditions (a) and (b) above. Now, it is to be checked whether this T_{12} and T_{22} satisfy conditions (c) and (d). If they do, in the third refinement we will have to consider the nodes testing u_4, as u_4 appears as primed in T_{22} and unprimed in T_{12} and so on.

As discussed earlier, the process will terminate under two conditions. If the first condition holds, G is not p-t-diagnosable and if the second condition holds, all pairs of faulty sets, one containing u_i and other not, are distinguishable. In the second case, select another node with which we have not as yet started and form the T_1 and T_2. Then repeat the same process as above to have the jth refinement, $j = 1, 2, \ldots$. If for some j, one pair of T_{1j} and T_{2j} contain u_i as primed in one of T_{1j} and T_{2j} and as unprimed in the other, no further refinement of that pair is obviously needed. When all the nodes of G have been used as the starting node and, in each case, the process of refinement terminated according to the second condition, G is p-t-diagnosable; on the other hand, if the process terminated according to the first condition in any of the cases, G is not diagnosable.

Example 1:
Let us consider a system consisting of 5 units. $V = \{u_1, u_2, u_3, u_4, u_5\}$. Let the probability of failure of the units be as follows:

$p(u_1) = \frac{1}{4}$, $p(u_2) = \frac{1}{3}$, $p(u_3) = \frac{1}{5}$, $p(u_4) = \frac{1}{4}$, $p(u_5) = \frac{1}{3}$ and $t = \frac{1}{50}$. Then we have $W(u_1) = -\log 3$, $W(u_2) = -\log 2$, $W(u_3) = -\log 4$, $W(u_4) = -\log 3$, $W(u_5) = -\log 2$ and $f(t)$

$$= \log t - \quad \log \{1 - p(u_i)\}$$

$$= -\log 50 + \text{lof}\ 5$$

$$= -\log 10$$

Let the digraph G corresponding to the system S be represented by the following adjacency matrix $B = (b_{ij})$

$$B = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$b_{ij} = 1 \quad \text{if } u_i \text{ tests } u_j$$
$$0 \quad \text{otherwise}$$

Now we have a check if there is any pair of consistent fault sets which are also indistinguishable.

Let us start with $T_1 = u_3$ and $T_3 = \bar{u}_3$. Following the method described above, we see that T_1 and T_2 can be refined as follows:

$$T_1 = u_3, T_2 = \bar{u}_3$$
$$T_{11} = u_3 \bar{u}_1,\ T_{21} = \bar{u}_3 u_1$$
$$T_{12} = u_3 \bar{u}_1 \bar{u}_4 u_2, T_{22} = \bar{u}_3 u_1 u_4 \bar{u}_2$$

Then we see that T_{12} and T_{22} cannot be refined any more so that both of them remain consistent as well as indistinguishable. Thus we see that any two fault sets Q_1 and Q_2, one of them containing u_3 and the other not containing it, are always distinguishable.

If we start with u_4, we have

$$T_1 = u_4, T_2 = \bar{u}_4$$
$$T_{11} = u_4 u_5, T_{21} = \bar{u}_4 u_5$$

and we see that T_{21} cannot be refined any further. Hence the sets $Q_1 = \{u_4, u_5\}$ and $Q_2 = \{u_5\}$ are both indistinguishable and consistent since

$$W(Q_1) = -\log 6 \geqslant f(t)$$
$$W(Q_2) = -\log 2 \geqslant f(t)$$

Hence the system S given above is not p-t-diagnosable.

5 Design of p-t-diagnosable system

In this Section, we shall discuss about the design of a p-t-diagnosable system. The design problem can be specified as follows. Given a set of nodes V together with the probability of failure of each node, the set of edges E is to be determined in order that the graph $G = (V, E)$ is p-t-diagnosable for some preset t. It will be shown that whenever the condition of theorem 2 holds, it is always possible to find the set E.

The design is given by the following steps and we shall prove that the following algorithm gives a p-t-diagnosable system.

Step 1:
Compute, from each probability, the W-value of each node. Sort them according to ascending values of Ws. Call the nodes $u_1, u_2, \ldots, u_n$ (where there are n nodes) so that u_1 has the lowest W and u_n the highest.

Step 2:
Select any u_i. If $i < n$ and

$$\sum_{j=i}^{n} W_j < f(t),$$

find the lowest k for which

$$\sum_{j=i}^{k} W_j < f(t),$$

else go to step 3. Have u_i tested by all of $u_{i+1}, u_{i+2}, \ldots, u_k$. Go to step 4.

Step 3:
Find the lowest k such that

$$\sum_{j=i}^{n} W_j + \sum_{j=i}^{k} W_j < f(t)$$

Have u_i tested by $u_{i+1}, \ldots, u_n, u_1, \ldots, u_k$.

Step 4:
Repeat steps 2 and 3 for all i.

Now we shall prove that the graph, obtained by the edges given by the above steps, is p-t-diagnosable. We shall always assume that the condition given by the theorem 2 holds for the set of nodes V.

Theorem 5:
With the above design, any pair of sets of faulty nodes, where u_1 appears in one set of the pair and not in the other set, is distinguishable.

Proof:
To prove the theorem, we shall show that if we apply our diagnosability-testing algorithm starting with u_1 on the digraph obtained by the above algorithm, no indistinguishable set pair can be obtained. Thus, to start with, $T_1 = u_1$ and $T_2 = \bar{u}_1$. Then, in T_{11} and T_{21} all the nodes $u_2, u_3, \ldots, u_k$ testing u_1 will appear and, as discussed earlier, at least one node will appear as complemented in T_{11} and uncomplemented in T_{21}. Let l be the highest number, $l \leqslant k$, such that u_l appears so. Then in T_{12} and T_{22} all the nodes testing u_l must appear excluding those which have appeared in T_{11} and T_{21}. Let $u_{l+1}, u_{l+2}, \ldots, u_m$ be the nodes testing u_l. If $m \leqslant l+1$, i.e. if $\sum_{j=1}^{n} W_j < f(t)$, then all the n nodes will appear in T_{12} and T_{22} in complemented or uncomplemented form as we started with u_1; but as the condition of theorem 2 holds, T_{12} and T_{22} are not admissible pairs. So we have proved the theorem. If $m \geqslant l+1$, then we will show that $m > k$. As per design, $\sum_{j=1}^{k} W_j < f(t)$ and k is the lowest integer to fulfil this condition. Similarly, $\sum_{j=1}^{m} W_j < f(t)$ and m is the lowest integer to fulfil the condition. Since $W_1 \leqslant W_2 \leqslant \ldots \leqslant W_n$, $\sum_{j=l}^{m} W_j < f(t)$ implies $\sum_{j=1}^{m-l+1} W_j < f(t)$. Thus $k \leqslant m - l + 1$ and since $l > 1$, $m > k$. Hence we find that in T_{11} and T_{21} all the nodes $u_1, u_2, \ldots, u_k$ appeared and in the next refinement, T_{12} and T_{22}, more nodes are included. Proceeding in this way, the refinement will stop when all the nodes are included, but as the condition of theorem 2 holds, the final refined terms are not admissible terms. Thus, we have proved the theorem.

Now, we will generalise theorem 5 for any arbitrary u_i instead of u_1. Consider any u_i as the starting node and, thus, $T_1 = u_i$ and $T_2 = \bar{u}_i$. Then T_{11} and T_{21} will be formed as discussed earlier. In T_{1i} and T_{2i} for any i, if u_l is the node with highest l which appears complemented in one of T_{1i} and T_{2i} and uncomplemented in the other so long as $\sum_{j=1}^{n} W_j < f(t)$. Using the same argument as in theorem 5, we find that the nodes included in $T_{1(i+1)}$ and $T_{2(i+1)}$ are always more than the nodes included in T_{1i} and T_{2i}. If, however, for some i, $\sum_{j=1}^{n} W_j \geqslant f(t)$ where l is as before, the number of nodes included in $T_{1(i+1)}$ and $T_{2(i+1)}$ may be equal or greater than that in T_{1i} and T_{2i}. If it is greater, then using the same argument as in theorem 5, we can prove distinguishability. If it is equal and u_l is tested by $u_{l+1}, \ldots, u_n, u_1, \ldots, u_k$, then from the description of the design

$$\sum_{j=1}^{n} W_j + \sum_{j=1}^{k} W_j < f(t)$$

and since u_l is the node with highest $l < n$, no node among $u_{l+1}, \ldots, u_n$ appears as primed in one of T_{1i} and T_{2i} or unprimed in the other, then, as per condition (b) discussed in Section 2, at least one of the nodes among $u_1, u_2, \ldots, u_k$ must appear as primed in one of $T_{1(i+1)}$ and $T_{2(i+1)}$ and unprimed in the other. Hence, the nodes included in $T_{1(i+2)}$ and $T_{2(i+2)}$ must be more than the nodes included in $T_{1(i+1)}$ and $T_{2(i+1)}$ and thus extending the argument of theorem 5 henceforth, we find that the distinguishability will hold whenever theorem 2 holds.

Thus we have proved that the system designed by the four steps above will give a p-t-diagnosable system.

Example 2:
Let us consider a system S with 5 units as given in example 1. We have to give the test pattern so that S will be p-t-diagnosable. It can easily be checked that the necessary conditions given in theorem 2 are satisfied. Nodes $u_1, \ldots, u_5$ are sorted in ascending order of W values. Call the sorted nodes $v_1, v_2, \ldots, v_5$ so that $W(v_1) = -\log 4$, $W(v_2) = -\log 3$, $W(v_3) = -\log 3$, $W(v_4) = -\log 2$, $W(v_5) = -\log 2$.

Then for each v_i, if $\sum_{j=i}^{n} W_j < f(t)$, find the smallest k such that $\sum_{j=i}^{i+k} W_j < f(t)$, otherwise find the smallest k such that $\sum_{j=i}^{n} W_j + \sum_{j=1}^{k} W_j < f(t)$. In the first case, have v_i tested by $v_{i+1}, v_{i+2}, \ldots, v_{i+k}$ and in the second case, have v_i tested by $v_{i+1}, v_{i+2}, \ldots, v_n, v_1, v_2, \ldots, v_k$. Thus we see that v_1 is tested by v_2 only, since

$$W_1 + W_2 = -\log 12 < -\log 10 = f(t)$$

Following this procedure the digraph G is obtained as in

Fig. 1. Using the procedure given in Section 4 it can be checked that system S given by Fig. 1 is *p-t*-diagnosable.

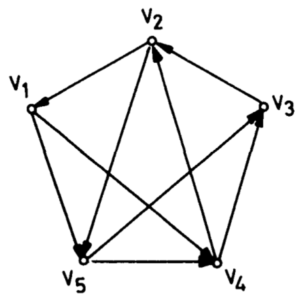

Fig. 1 *P-t-diagnosable system*

6 Remarks

A computer network can be represented as a digraph as in the graph-thoeretical model described above, where each individual computer in the network is represented by a node in the graph. Each computer in the graph can be tested by a number of other computers so that, if the tested computer appears to be faulty, its job load will be shared by the testing computers. These testing links are represented by corresponding directed edges in the graph. Now each individual computer is tested in offline and its probability of failure is calculated, which is associated with the corresponding node as its weight. Thus a computer network can be represented by a weighted digraph and it can be tested in the method described in this paper in order to locate any faulty unit(s).

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