A Model of HF Impulsive Atmospheric Noise

M. P. SHINDE AND S. N. GUPTA

Abstract—**Determination of optimal receiver or detector and suboptimal estimator in the presence of additive atmospheric noise depends on the application of a mathematically tractable model of noise. In the tropics the atmospheric radio noise occurring mostly in the burst form above a relatively small continuous background does not deliver energy at a constant rate. This type of noise is non-Gaussian and has a very large dynamic range. The noise bursts consist of a number of short impulses. They are modelled here as the product of a narrow-band Gaussian noise and the reciprocal of a non-Gaussian random process. This paper includes the derivation of statistical information for the above noise viz, the probability distribution of amplitudes, and of separation between pulses, required for determining the error probabilities using various digital methods. This information can be used in filter optimization in digital systems, where the error probability is to be minimized.**

INTRODUCTION

In the tropics, atmospheric radio noise is the principal source of interference to communications in the high-frequency band. The noise mostly arises in the form of distinct bursts above a relatively small continuous background [1], [2]. The continuous background arises as a result of the superposition of a large number of sources distant from the receiving location while the pulses arise from near or local sources. These long duration noise bursts rather than the continuous background noise embrace several hundreds or even thousands of symbols and cause most errors in digital data communications.

The burst form of noise does not deliver energy at a constant rate; clustering of the noise pulses is observed [3]–[5]. The noise is, therefore, non-Gaussian in character with a large dynamic range. There is a breakdown of statistical independence of the envelope of the received noise for receiver bandwidth of 10 KHz [6]. These are the some of the characteristics of noise investigated and reported by several investigators [2]–[6].

A model of this type noise has been derived by Hall [3], assuming the noise as the quotient of two independent Gaussian processes. This model called the T model explains some of the observed properties of noise. However, this model leads to an infinite amount of power. It, therefore, fails to represent a physical process. In view of this shortcoming of the T-model a new model has been derived taking into consideration the known properties of the burst form of noise. This model is compared with the available data.

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The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Delhi, India.

II. DISTRIBUTION OF FIRST-ORDER STATISTICS OF HF ATMOSPHERIC RADIO NOISE

In order to incorporate the non-Gaussian nature with a large dynamic range in the model it is proposed that the clustering of the HF atmospheric radio noise pulses be predicted by

$$Y(t) = Z(t)N(t) = \left[\frac{1}{B(t)}\right]\cdot N(t) \tag{1}$$

where

$$Z(t) = \frac{1}{B(t)}$$

and $N(t)$ and $B(t)$ are statistically independent random processes; $N(t)$ is a zero-mean Gaussian noise component having its distribution given by

$$p_N = N[0,\sigma]$$

so that

$$p_N(x) = \frac{1}{(2\pi\sigma_1^2)^{1/2}} \exp\left(-x^2/2\sigma_1^2\right), \qquad -\infty < x < \infty. \tag{2}$$

$B(t)$ is a non-Gaussian random process with strong correlation among its samples. The density function of distribution of $B(t)$ is assumed to be

$$p_B(b) = \begin{cases} \dfrac{2^{1/2}b^2}{\pi^{1/2}\sigma^{(3+(1/2n))}} \exp\left(-b^2/2\sigma^2\right), & \text{if } b \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

It follows that [7]

$$p_Y(y) = \int_0^{+\infty} x p_B(x) p_N(yx)\cdot dx, \qquad 0 < y < \infty. \tag{4}$$

Assuming $\sigma_1 = \sigma$ for simplicity and substituting values of $p_B(x)$ and $p_N(yx)$ from (A-2) and (A-3) in the appendix yields

$$p_Y(y) = \frac{1}{\pi\sigma^{(4+(1/2n))}} \int_0^{\infty} x^3 \cdot \exp\left[-\left(\frac{1+y^2}{2\sigma^2}\right)x^2\right]\cdot dx. \tag{5}$$

Evaluating (5) by the change of variables:

$$\exp\left[-\left(\frac{1+y^2}{2\sigma^2}\right)x^2\right] = w$$

yielding

$$p_Y(y) = \frac{1}{2\pi\sigma^{(4+(1/2n))}} \cdot \left(\frac{2\sigma^2}{1+y^2}\right)^2 \int_0^1 \log(1/w)\cdot dw.$$

Referring to standard mathematical tables by Hodgeman [8] item 436,

$$\int_0^1 \left(\log\frac{1}{w}\right)^n dw = n!.$$

Hence

$$p_Y(y) = \begin{cases} \dfrac{2}{\pi\sigma_{(1/2n)}(1+y^2)^2}, & 0 < y < \infty \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Equation (6) defines the distribution of the first-order statistics of the non-Gaussian component of the HF atmospheric radio noise.

The characteristic function is useful for obtaining the moments of a random variable. It is defined for a random variable x with density function $p(x)$ as

$$F(jv) = E\{\exp(jvx)\} \tag{7}$$

where $j = (-1)^{1/2}$. For the continuous random variable x, this becomes

$$F(jv) = \int_{-\infty}^{\infty} p_Y(x) \exp(jvx)\cdot dx. \tag{8}$$

Substituting value of $p_Y(x)$ from (6), we get

$$\begin{aligned} F(jv) &= \frac{2}{\pi\sigma^{1/2n}} \int_{-\infty}^{\infty} \frac{\exp(jvx)}{(x^2+1)^2}\cdot dx \\ &= \frac{2}{\pi\sigma^{1/2n}} \int_{-\infty}^{\infty} \frac{\exp(jvx)}{(x^2+1)^{(3+1)/2}}\cdot dx \\ &= \left[\frac{2}{\pi\sigma^{1/2n}}\right] \frac{2\pi^{1/2}}{\Gamma[(3+1)/2]} \left(\frac{|v|}{2}\right)^{3/2} K_{3/2}(|v|) \end{aligned} \tag{9}$$

where $K_{3/2}$ is a modified Bessel function of the second kind [9], [10] given by

$$K_{3/2}(|v|) = K_{1+1/2}(|v|) = (\pi/2|v|)^{1/2}\left(1+\frac{1}{|v|}\right) \cdot \exp[-|v|].$$

Thus

$$F(jv) = \frac{|v|}{\sigma^{1/2n}}\left(1+\frac{1}{|v|}\right)\cdot \exp[-|v|]. \tag{10}$$

III. FIRST-ORDER DISTRIBUTION OF ENVELOPE

The narrowband received noise is denoted by

$$Y(t) = v(t)\cdot\cos[w_0 t + \theta(t)]. \tag{11}$$

Let $\hat{y}$ be the quadrature component corresponding to $Y(t)$, $Z(f)$, and $N(f)$ be the Fourier transforms of $Z(t)$ and $N(t)$, respectively with negligible overlap between them, as seen by

$$v = (y^2 + \hat{y}^2)^{1/2}, \qquad \theta = \tan^{-1}(\hat{y}/y)$$

and

$$\hat{y} = z(t)\cdot\hat{n}(t) \tag{12}$$

where $n(t)$ and $\hat{n}(t)$ at the same instant t are independent and identically distributed random variables.

For $z = 1/B$ it follows that [11]

$$p_z(z) = \left.\frac{p_B(B)}{|dz/dB|}\right|_{B=1/z}$$

or

$$\begin{aligned} p_z(z) &= \frac{1}{z^2}p_B(1/z) = \frac{1}{z^2}\cdot\frac{2^{1/2}}{\pi^{1/2}\sigma^{(3+(1/2n))}} \cdot (1/z^2)\exp(-1/2\sigma^2x^2) \\ &= \frac{2^{1/2}}{\pi^{1/2}\sigma^{(3+(1/2n))}z^4}\cdot\exp(-1/2\sigma^2z^2). \end{aligned} \tag{13}$$

Now

$$\begin{aligned} p_{y,\hat{y}}(y,\hat{y}) &= p_{zn,z\hat{n}}(y,\hat{y}) \\ p_{y,\hat{y}}(y,\hat{y}) &= \int_{-\infty}^{\infty} (1/x^2)\cdot p_{z,n,\hat{n}}(x,y/x,\hat{y}/x)\cdot dx \\ &= \int_{-\infty}^{\infty} (1/x^2)p_z(x)\cdot p_n(y/x)\cdot p_{\hat{n}}(\hat{y}/x)\cdot dx. \end{aligned}$$

Substituting

$$p_n(y/x) = \frac{1}{(2\pi\sigma^2)^{1/2}}\cdot\exp(-y^2/2\sigma^2x^2)$$

$$p_{\hat{n}}(\hat{y}/x) = \frac{1}{(2\pi\sigma^2)^{1/2}}\exp(-\hat{y}^2/2\sigma^2x^2)$$

and $p_z(z)$ from (13) and $y^2 + \hat{y}^2 = v^2$, we get

$$p_{y,\hat{y}}(y,\hat{y}) = \frac{1}{2^{1/2}\pi^{3/2}\sigma^{(5+(1/2n))}}\cdot\int_0^{\infty}\frac{1}{x^6}\exp\left(-\frac{1+v^2}{2\sigma^2x^2}\right)\cdot dx. \tag{14}$$

Evaluating (14) by the change of variables

$$\exp\left(-\frac{1+v^2}{2\sigma^2x^2}\right) = w$$

we get

$$p_{y,\hat{y}}(y,\hat{y}) = \frac{2}{\pi^{3/2}\sigma^{1/2n}(1+v^2)^{5/2}}\cdot\int_0^1 (\log 1/w)^{3/2}\cdot dw.$$

Using Dwight [12] item 866.3

$$\int_0^1 (\log 1/w)^{3/2} = \Gamma(5/2).$$

Therefore,

$$p_{y,\hat{y}}(y,\hat{y}) = \frac{2\Gamma(5/2)}{\pi^{3/2}\sigma^{1/2n}(1+v^2)^{5/2}}. \tag{15}$$

The joint probability density of envelope and phase of the narrow-band received noise is given by

$$p_{1-v,\phi}(v,\phi) = v p_{y,\hat{y}}(y,\hat{y}) \tag{16}$$

and

$$p_{1-v,\phi}(v,\phi) = p_{1-v}(v)p_{\phi}(\phi)$$

and assuming uniform phase distribution, i.e.,

$$p_{\phi}(\phi) = 1/2\pi$$

we get

$$p_{1-v}(v) = \frac{3v}{\sigma^{1/2n}(v^2+1)^{5/2}} \tag{17}$$

Hence

$$p_v(v) = \frac{3Kv}{\sigma^{1/2n}(v^2+1)^{5/2}},$$

where K is a constant that must be determined from the condition

$$\int_{-\infty}^{\infty} p_v(v)\, dv = 1.$$

Substituting value of $p_v(v)$ in this condition we get

$$K = \sigma^{1/2n}.$$

Hence

$$p_v(v) = \frac{3v}{(v^2+1)^{5/2}}. \tag{18}$$

The expected value of v is

$$E[v] = \int_{-\infty}^{\infty} x \cdot p_v(x) \cdot dx = 3\int_0^{\infty} \frac{x^2\, dx}{(1+x^2)^{5/2}}$$

$$= \left[\left(\frac{1}{1+1/x^2}\right)^{3/2}\right]_0^{\infty}$$

$$= 1. \tag{19}$$

Also

$$E[v^2] = \int_0^{\infty} x^2 p_v(x) \cdot dx.$$

must be finite, and

$$E[v^2] = 3\int_0^{\infty} \frac{x^3\, dx}{(x^2+1)^{5/2}}$$

$$= 3\left[-\frac{1}{(x^2+1)^{1/2}} + \frac{1}{3(x^2+1)^{3/2}}\right]_0^{\infty}$$

$$= 2. \tag{20}$$

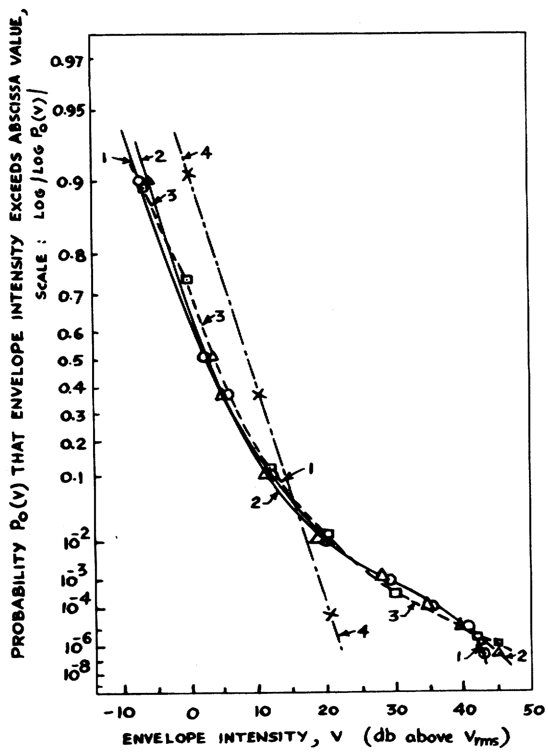

Fig. 1. Composite probability distribution function of the envelope of received HF noise; comparison of model results with data measured at Singapore and at Slough, Eng.
—○— Measured data, Singapore. Band Centre frequency, f_0 = 11 MHz. Receiver power bandwidth = 425 Hz. (Clarke, et. al, [10])
—△— Measured data, Slough, England. (Same conditions as above)
—□— New Model $v_{rms}^2 = E[v^2] \approx 10$ dB above v_{rms}^2
—×— Rayleigh-distributed envelope $E[v^2] = \hat{v}_{rms}^2$

As the value of $E[v^2]$ is finite, the model represents physical noise. Formula (18) is convenient in analysing noise immunities.

The probability $P_0(v)$ that the envelope intensity exceeds level V is given by

$$P_0(v) = 1 - P(v) = \int_v^{\infty} p_v(x)\, dx \cdot = 3\int_v^{\infty} \frac{x \cdot dx}{(x^2+1)^{5/2}}$$

$$= 3\left[-\frac{1}{3(x^2+1)^{3/2}}\right]_v^{\infty}$$

$$= \frac{1}{(v^2+1)^{3/2}}. \tag{21}$$

This model is compared with experimental results. The new model results are plotted in Fig. 1 based on the assumption that $E[v^2]$ is 10 dB above v_{rms}^2 of atmospheric radio noise at 11 MHz. Results obtained from measured data and published work [13]–[15] are also plotted in Fig. 1. This shows a close agreement between the observed values and values calculated from the above model.

IV. DISTRIBUTION OF ENVELOPE LEVEL CROSSINGS

It is assumed that the observed atmospheric noise is bandlimited by the receiver to an RF bandwidth $2B$. The envelope of the received noise is, therefore, bandlimited to a frequency band of width $2B$ centred about zero frequency. It can be shown [16] that this bandlimited envelope $v(t)$ is approximately described in the time interval $[0,T_0]$ by its 2B T_0 independent equidistant samples provided 2B $T_0 \gg 1$. If T is the separation time between a downcrossing and the next upcrossing of the level V_0 by the envelope of the received noise, the probability that T exceeds T_0 is given [2] by

$$P(T_0) = [1 - P_0(V_0)]^{(2BT_0-1)}. \tag{22}$$

Substituting the value of $1 - P_0(V_0)$ from (21) yields

$$P_0(T_0) = \left[1 - \frac{1}{(v^2+1)^{3/2}}\right]^{(2BT_0-1)}. \tag{23}$$

The probability density function of spacing between successive envelope crossings is given by the derivative of $P(T_0)$ as

$$p_0(T_0) = \frac{3(2BT_0-1)}{(v^2+1)^{5/2}}\left[1 - \frac{1}{(v^2+1)^{3/2}}\right]^{(2BT_0-2)}. \tag{24}$$

$[dp_0(T_0)/dT_0] = 0$ gives the location of $p_{0-\max}$ at

$$T_0 = (1/2B)\left\{1 - 1/\log\left[1 - \frac{1}{(v^2+1)^{3/2}}\right]\right\}.$$

Thus the occurrence of maximum value of $p_0(T_0)$ indicates that the noise pulses are dependent and they have tendency to cluster. This result is in agreement with the observed correlation between pulses [17].

For large values of v, $1/(v^2+1)^{3/2} \ll 1$, the terms containing second and all higher powers of $1/(v^2+1)^{3/2}$ in the series expansion of right-hand side of (18) may be neglected to give

$$p_0(T_0) = 1 - \frac{2BT_0}{(v^2+1)^{3/2}}. \tag{25}$$

Equation (25) indicates that $P_0(T_0)$ goes on decreasing with increasing values of bandwidth B.

Another formula for the distribution of time interval between successive bursts is shown to be approximately in the lognormal form by using log-normal distribution of the duration of burst q and pulse repetition period r given below [18], [19] and techniques of Fenton [20] seen as

$$p(\alpha) = \frac{1}{(2\pi)^{1/2}\sigma_\alpha \alpha}\exp\left\{-\left[\frac{\ln(\alpha/\alpha_m)}{(2)^{1/2}\sigma_\alpha}\right]^2\right\} \tag{26}$$

where α represents q or r, α_m is the median value of α, and σ_α is the standard deviation of $\ln \alpha$.

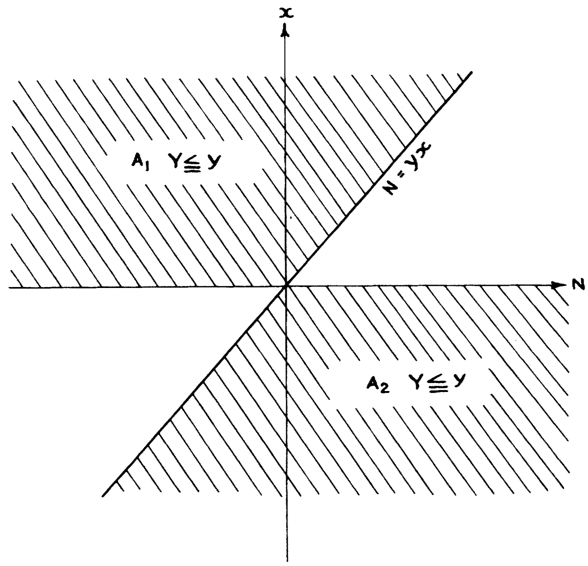

Fig. 2. N–x plot for determining limits of integration for ratio of random variables.

V. DISCUSSION AND CONCLUSIONS

It is seen from Fig. 1 that the calculated results derived from the new model of the HF atmospheric noise are in agreement with those obtained from measured data. As the first and second moment of the density function of the proposed distribution of noise are finite, representation of the noise by the model in this paper seems reasonable. The higher order statistics of the noise is given by (23) and (24) for $P_0(T_0)$ and $p(T_0)$ respectively, which could not be verified for want of measured data. This model can be used to find the probability of error and hence the performance of digital communication systems. Design of signals or techniques for overcoming the interfering effect of this type of noise in communication systems forms the subject matter of a subsequent paper to be communicated.

VI. APPENDIX

Let

$$Y(t) = \frac{N(t)}{B(t)} \leftrightarrow Y = N/x \tag{A1}$$

for a given value of $Y = y$, we must determine the region in $N - x$ plane for $Y \leq y$. Plotting the equation

$$N = xY$$

as in Fig. 2, we see that for x positive N is limited by $-\infty \leq N \leq yx$ for $x > 0$. For x negative we have $yx \leq N \leq +\infty$ for $x < 0$. The probability is that $Y \leq y$ can now be written as

$$P(Y \leq y) = \int_0^{+\infty}\int_{-\infty}^{yx} p(N,x)\, dN\, dx + \int_{-\infty}^{0}\int_{yx}^{+\infty} p(N,x)\, dN\, dx.$$

Differentiating with respect to y gives the resulting pdf

$$p_Y(y) = \int_0^{+\infty} x^p(N = yx,x)\, dx - \int_{-\infty}^{0} xp(N = yx,x)\, dx. \tag{A2}$$

If N and x are independent, we get

$$p_Y(y) = \int_0^{+\infty} xp_N(yx)\cdot p_B(x)\, dx - \int_{-\infty}^{0} xp_N(yx)\cdot p_B(x)\cdot dx. \tag{A3}$$

Since we have assumed that $p_B(x) = 0$ for $x < 0$, the second integral vanishes, and we get

$$p_Y(y) = \int_0^{+\infty} xp_B(x)\cdot p_N(yx)\cdot dx, \qquad 0 < y < \infty \tag{A4}$$

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