INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2010-11

B.Stat. (Hons.) II Year
Analysis III

Date: 30.08.10 **Maximum Marks: 40** **Duration: 2 Hours**

(Note : The paper carries 50 marks. Maximum you can score 40)

1. (i) Let $A \subset \mathbb{R}^2$ be a non-empty subset.

<u>Definition</u> : Call A 'complete' if every Cauchy sequence in A converges to an element of A.

Show that a non-empty subset A of $\mathbb{R}^2$ is complete if and only if A is closed. Hence, or otherwise, deduce that every non-empty compact subset of $\mathbb{R}^2$ is complete.

(ii) Let $C \subset \mathbb{R}^2$ be a non-empty compact subset and $U \subset \mathbb{R}^2$ be an open set such that $C \subset U$. Show that there exist an open set V and a compact set D such that $C \subset V \subset D \subset U$.

(iii) Let A, B be two non-empty connected subsets of $\mathbb{R}^2$. Suppose $A \cap B \neq \phi$, then show that $A \cup B$ is connected.

[(3+3)+10+6=22]

2. (i) Let $f : \mathbb{R}^2 \to \mathbb{R}$ and a be a point of $\mathbb{R}^2$. Let u be a unitvector in $\mathbb{R}^2$. Define the directional derivative of f at a in the direction of u. Define the function $F : \mathbb{R} \to \mathbb{R}$ by $F(t) = f(a + t\,u)$. What is the relationship of the above directional derivative with the function F

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x_1\, x_2) = x_1^2\, x_2 + x_2^3\, x_1$. Compute the directional derivative of f at the point (1, 2) in the direction (3, 4).

(ii) Let $F : \mathbb{R}^2 \to \mathbb{R}$ be a real-valued function and $a = (a_1, a_2) \in \mathbb{R}^2$. State a sufficient condition for F to be differentiable at a.

Consider the function $f(x_1,x_2) = x_1^2\, x_2 + x_2^3\, x_1$. Using the above stated sufficient condition show that f is differentiable at every point. Find the gradient of f at $(2, 1)$.

(iii) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \frac{xy}{\sqrt{x^2 + y^2}}, \quad (x,y) \neq (0,0)$$

$$= \quad 0 \quad , (x,y) = (0,0)$$

Show that $D_1 f$ and $D_2 f$ exist at $(x,y) \in \mathbb{R}^2$. Prove that both $D_1 f$ and $D_2 f$ are not continuous at (0, 0). Is f differentiable at $(0, 0)$?

[(3+2+5) + (3+5+3) +7 = 28]

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Indian Statistical Institute

Periodical Examinations, 2010-2011, Semester I

B. Stat.(Hons.) 2nd year

C and Data Srtructures

Date : September 3, 2010 Maximum Marks : 60 Time : 2½ Hours

Attempt all questions. *The paper carries a total of 73 marks. Maximum you can score is 60. Figures in the right margin indicate the marks on the different parts of a question.*

1. Write C programs to implement an efficient Data structure to store a set of elements classified into two groups. Each group should be stored using the LIFO principle. [12]

2. Write a C program to delete all occurrences of a given value from an unsorted singly linked list. [10]

3. Write a **non-recursive** algorithm for Quicksort. Derive the time complexity of your algorithm. [16+8=24]

4. Derive an expression for the number of binary trees which can be constructed, which have a given sequence as its pre-order traversal? [12]

5. Write an algorithm to reconstruct a binary tree from its pre-order and in-order traversals? [15]

INDIAN STATISTICAL INSTITUTE

Mid Semester Examination, 1^{st} Semester, 2010-11

Statistical Methods III, B.Stat 2^{nd} Year

Date: September 6, 2010 Time: 2 hrs 15 mins

This paper carries 32 marks. Answer all questions.
The maximum you can score is 30.

1. (a) Consider the usual regression model:

$$y_i = \alpha + \beta x_i + e_i;\ i = 1, 2, \ldots, n$$

where, x_is are fixed and e_is are i.i.d. N(0,σ^2). Compute the Fisher's information matrix for the vector $(\alpha, \beta, \sigma^2)$.

(b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with probability mass function:

$$f(x) = \frac{c(x)\theta^x}{\sum_x c(x)\theta^x};\ x = 0, 1, 2\ldots$$

(This is known as a *power series* distribution).

Show that the method of moments estimator and the m.l.e. of θ are identical. [7+5]

2. Assume that the monthly incomes of the employees in a particular firm are distributed as *Pareto* with density:

$$f(x) = \frac{cA^c}{x^{c+1}};\ x \geq A \geq 0$$

Suppose $X_1, X_2, \ldots, X_n$ are the monthly incomes of a random sample of n employees in the firm.

(a) Obtain the maximum likelihood estimator of the proportion of employees in the firm with monthly income in the interval $[I_1, I_2]$.

(b) Consider estimators of the form $k\widehat{A}$ to estimate A where $\widehat{A}$ is the m.l.e. of A. For what value of k does the estimator have the minimum mean squared error? [7+5]

3. Suppose $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are observations from a bivariate normal distribution. If X_n is equal to the mean of $X_1, X_2, \ldots, X_{n-1}$; and Y_n is missing, show that an EM algorithm used to estimate the parameters of the bivariate distribution with initial values, the m.l.e.s of the parameters based on the first $(n-1)$ observations, converges in two steps. [8]

Indian Statistical Institute
B Stat Second Year 2010
Mid Semester Examination
Economics I
Microeconomics

08·09·10

Maximum Marks: 40 Time : 180 minutes

Answer the following questions

1. a) Consider the optimization problem, max $U(x_1, x_2, \ldots\ldots\ldots, x_n)$ $U_i > 0$ $x_i \geq 0$ s.t. $\sum r_i x_i = w$. Suppose the utility function is strictly quasiconcave. Derive the optimality condition for the following two cases: (i) When there is interior solution and (ii) When there is corner solution. Explain your answers with the help of diagrams. (notations have usual meanings.)

b) Suppose x and y are the only two commodities in the consumption basket of an individual consumer. It is given that the marginal utility of x denoted $mu_x = 40 - 5x$ and marginal utility of y denoted $mu_y = 30 - y$. Prices of x and y are Rs.5 and Re.1 respectively. The budget of the consumer is Rs.40. **(i)** Derive the optimum consumption bundle the consumer will choose. **(ii)** How will your answer to **(i)** change if the consumer's budget falls to Rs.10?

c) Write down the equation of the income expansion path in the above problem. Can two different income expansion paths corresponding to different price ratios intersect? Explain.

(10+8+4=22)

P.T.O

2. a) Consider the production function $q = f(x_1, x_2, \ldots\ldots, x_n)$ $f_i > 0$ which is twice continuously differentiable and strictly concave. Explain the properties of the relevant cost functions, marginal cost function and average cost function when not all the inputs are variable. Use diagrams to explain your answer.(Assume that the cost function is differentiable).

b) Suppose a firm has the following cost function $C \begin{array}{l} = q^2 + 1 \ \forall\, q > 0 \\ = 0 \ \text{ if } q = 0 \end{array}$. Derive the supply function of the firm.

c) How does the answer to **b)** change if $C = 1/3$, when $q = 0$ instead ?

(12+4+6=22)

Indian Statistical Institute

Mid-Semester Examination: 2010-2011

Course Name: B. Stat II

Subject Name: Biology I

Date: September 8, 2010 *Maximum Marks:* 40; *Duration: 2.5 hrs*

All questions carry equal marks, answer any five, answer should be brief

1. (a). Why must each of giant DNA molecules in eukaryotic chromosomes contain multiple origin of replication? [4]
   (b) Transcription and translation are coupled in prokaryotes. Why is this not the case in eukaryotes? [4]

2. Mention the similarities and dissimilarities in oxidative pathways of palmitic acid and palmitoleic acid to generate ATP. Which of these two acids will generate more ATP and why? (Palmitic acid : $CH_3(CH_2)_{14}COOH$; Palmitoleic acid : $CH_3(CH_2)_5\,CH{=}CH\,(CH_2)_7\,COOH$) [8]

3. Distinguish between the following:

   a. Aerobic and anaerobic metabolism of glucose [2]
   b. Metabolism of glucose and lactose in human [2]
   c. Human and bacterial cells [2]
   d. Human and bacterial DNA [2]

4. Explain the following: (a) Nucleosome (b) quaternary structure of proteins (c) Isoelectic pH of protein (d) Melting temperature, Tm, of DNA. [8]

5. Mention the functions of three different types of RNA that are involved in protein synthesis. What do you mean by the statement that "genetic code is degenerate and comma-less"? [6+2]

6. How are the amino-acids, alanine and aspartic acid, metabolized to generate ATP? How many ATPs will be generated in both cases? [4+4]

7. In a random copolymer of $(AC)_n$, what will be frequencies of different triplet codons and different amino acids after translation, if (a) A:C= 5:1 and (b) A:C=1:5 in the copolymers? [8]

INDIAN STATISTICAL INSTITUTE

Mid-semestral Examination 2010-11

Physics I

Maximum Marks: 60 Date:08/09/10 Duration: 2hrs 30 minutes

Note: Use different Answer Sheets for different groups

Group A

All questions carry equal marks ($30 = 5 \times 6$)

1. (a) Show that the vector $\mathbf{A} = yz\hat{\mathbf{x}} + zx\hat{\mathbf{y}} + xy\hat{\mathbf{z}}$ can be written both as the gradient of a scalar and as the curl of a vector.
   (b) Let F be a conservative field such that $\mathbf{F} = -\nabla\phi$. Suppose a particle of constant mass m is moving in this field. If P and Q are two points in space, prove that

$$\phi(P) + \frac{1}{2}mv_P^2 = \phi(Q) + \frac{1}{2}mv_Q^2$$

   where v_P and v_Q are the magnitudes of the velocities of the particle at the points P and Q respectively.

2. Compute the line integral of

$$\mathbf{A} = 6\hat{\mathbf{i}} + yz^2\hat{\mathbf{j}} + (3y + z)\hat{\mathbf{k}}$$

   along the triangular path shown in Fig. 1. Check your answer using Stoke's theorem.

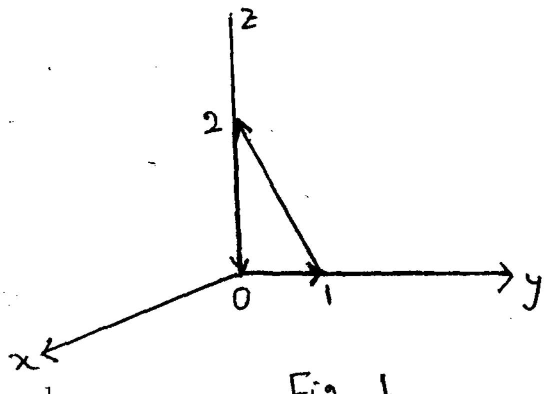


Fig. 1

or

Check Stokes' theorem using the function $\mathbf{v} = ay\hat{\mathbf{x}} + bx\hat{\mathbf{y}}$ (a and b are constants) and the circular path of radius R, centered at the origin in the xy plane.

3. A hollow spherical shell carries charge density $\rho = \frac{k}{r^2}$ in the region $a \leq r \leq b$ (See Fig. 2).
(a) Find the electric filed in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$. Plot $|\mathbf{E}|$as a function of r.
(b) Also, find the potential at the centre, using infinity as your reference point.

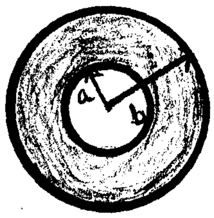
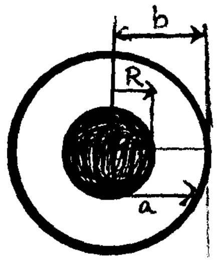


Fig. 2

4. (a) Find the energy stored in a uniformly charged solid sphere of radius R and charge q.
(b) A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as in the figure below). The shell carries no net chatge. Find the surface charge density σ at R, at a and at b.



5. A metal sphere of radius R_1 carries a charge q. It is surrounded, out to radius R_2, by a linear dielectric material of permittivity ϵ. Find the potential at the centre relative to infinity. Also compute the polarization and the bound charges explicitly.

Group B

Answer all questions

1(a). Show that the total kinetic energy T of a system of particles is given by

$$T = \frac{1}{2}MV^2 + \frac{1}{2}\sum_i m_i(\frac{dr_i'}{dt})^2$$

where M is the total mass of the system of particles. $\vec{V}$ is the velocity of the centre of mass relative to the origin O. and $\vec{r_i}'$ is the radius vector from the center of mass to the ith particle.

(b). Two masses m_1 and m_2 are interacting by a potential energy $V(\vec{r_1}, \vec{r_2})$. $\vec{r_1}$ and $\vec{r_2}$ being the position vectors of the masses m_1 and m_2 respectively relative to the origin O of some inertial reference frame. Calculate the total angular momentum in the centre of mass reference frame and give the physical significance.

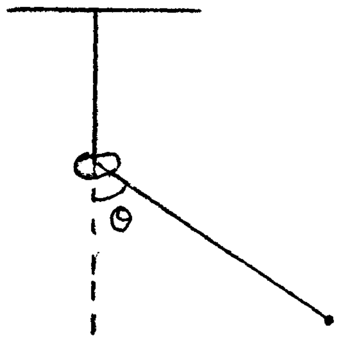
(c) Suppose a particle is subjected to a time dependent generalised force of the form $\vec{F} = f(t) grad W(\vec{r})$. What is the corresponding generalised potential U? What is the value of U when $\vec{F} = f(t)\vec{i}$? [5+(3+1)+(2+1)]

2. A particle P of unit mass moves on the positive x-axis under the force field $F = \frac{36}{x^3} - \frac{9}{x^2}$, $x > 0$. Show that each motion of P consists of either (i) a periodic oscillation between two extreme points or (ii) an unbounded motion with one extreme point, depending upon the value of the total energy. Give necessary diagram.
Initially P is projected from the point $x = 4$ with speed 0.5. Show that P oscillates between two extreme points and find the period of the motion. [You may make use of the formula $\int_a^b \frac{x\,dx}{\sqrt{(x-a)(b-x)}} = \frac{\pi(a+b)}{2}$]. [3+4]

3. A uniform circular pulley of mass $2m$ can rotate freely about its axis of symmetry. Two masses m and $3m$ are connected by a light inextensible string which passes over the pulley without slipping. The whole system undergoes planar motion with the masses moving vertically. Take the rotation angle of the pulley as generalized coordinate and obtain Lagrange's equation for the motion. Deduce the upward acceleration of the mass m. [4+1]

4. A particle is suspended from a support by a light inextensible string which passes through a small fixed ring vertically below the support. The particle moves in a vertical plane with the string taut. At the same time the support is made to move vertically having an upward displacement $Z(t)$ at time t. The effect is that the particle oscillates like a simple pendulum (as shown in the figure below) whose string length at time t is $a - Z(t)$, where a ia a positive constant.

Take θ-the angle between the string and the downward vertical as generalised coordinate and obtain Lagrange's equation. [6]



Indian Statistical Institute

First Midsemestral Examination, 2010-11

B. Stat II year
Probability III

Date: September 10, 2010 | Maximum Marks: 20 | Time: 2 hours

Note that the paper contains 24 marks

1. Define the support of a univariate cdf $F(x)$. Show that if $\{x_n\}$ belong to the support of F and $x_n \to x$ then x also belongs to the support of F. [7]

2. Show that a bivariate cdf $F(x, y)$ of a pair of random variables (X, Y) is continuous if and only if the marginal univariate cdf's of both X and Y are continuous. [7]

3. Let U have a U$(0, 1)$ distribution. Define the following recursion process for $n = 1, 2, \ldots$:

$$
\begin{aligned}
2U &= [2U] + R_1 \\
2R_1 &= [2R_1] + R_2 \\
\ldots & \ldots \ldots \\
2R_n &= [2R_n] + R_{n+1} \\
\ldots & \ldots \ldots
\end{aligned}
$$

where $[x]$ is the largest integer less than or equal to x and the remainders $R_1, R_2, \ldots, R_n, \ldots$ are defined through the recursion process. Show that $[2U], [2R_1], [2R_2], \ldots, [2R_n]$ are iid Bernoulli random variables for any $n \geq 1$. [10]

Indian Statistical Institute

Second Semester Examinations, (2010-2011)

B. Stat (Hons.), Second Year

C and Data Structures

Date : November 19, 2010 Maximum Marks : 100 Time : 3 Hours

***Attempt all questions**. The paper carries a total of 115 marks. Maximum you can score is 100. Figures in the right margin indicate the marks on the different parts of a question.*

1. Let P and Q point to the root nodes of two heaps, having equal height, stored using pointers. Write a function in C, which takes P and Q as inputs and returns a pointer pointing to a heap consisting of the union of nodes of the given heaps. [20]

2. We are given an array of integers $A[1..n]$, such that , for all i, $1 \leq i < n$, we have $|A[i] - A[i+1]| \leq 1$. Let $A[1] = x$ and $A[n] = y$, such that $x < y$. Design an efficient search algorithm to find j such that $A[j] = z$ for a given value z, $x \leq z \leq y$. Justify the correctness of your algorithm. Derive the maximal number of comparisons to z, that your algorithm makes. [12+4+4=20]

3. The *Fibonacci numbers* are defined by the following recurrence relation:
   $F(1) = 1, F(2) = 1, F(\text{n}) = F(n-1) + F(n-2)$ $(n > 2)$.
   Prove that every integer $n > 2$ can be written as a sum of at most $\log_2 n$ Fibonacci numbers. Design an algorithm to find such a representation for a given integer n. [12+8=20]

4. Write a C function to delete a node (specified by its value) from a threaded binary search tree. [20]

5. What are the techniques used for handling collision in hashing? Explain primary clustering and secondary clustering with examples. Derive an estimate for the cost of deleting a record in a hash table using pseudo-random probing. What are the problems faced in deleting a node from a hash table? How do you handle them? [5+6+5+4=20]

6. Write a function in C to insert a node in a B-tree. [15]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2010-11

B.Stat. (Hons.) II Year
Analysis III

Date: 23.11.10 **Maximum Marks: 60** **Duration: $3\frac{1}{2}$ Hours**

(Note : There are 5 questions carrying 80 marks. Maximum one can score 60)

1. Let $A = \{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } 0 < y < x^2\}$
   (a) Show that every straight line through (0, 0) contains an interval around (0, 0) which is in $\mathbb{R}^2$ - A.

   (b) Define f : $\mathbb{R}^2 \to \mathbb{R}$ *by* $\begin{aligned} f(x) &= 0 \text{ if } x \notin A \\ &= 1 \text{ if } x \in A \end{aligned}$

   For h $\in \mathbb{R}^2$ *define* $g_h : \mathbb{R}^2 \to \mathbb{R}$ *by* $g_h(t) = f(th$, $t \in \mathbb{R}$. *Show that each* g_h *is continuous at 0, but f is not continuous at (0, 0).*

   [3+7 = 0]

2. *(a) Let* $f: U \subset \mathbb{R}^n \to \mathbb{R}$ *be a function which satisfies the condition : there exists* $M > 0$ *such that* $|f(a) - f(b)| \leq M\|a - b\|$ for all a, b $\in$ U.
   Show that f can be extended uniquely to a continuous function on $\overline{U}$.

   (b) Let $f: \mathbb{R}^p - \{0\} \to \mathbb{R} (p \geq 2)$ *be a continuously differentiable function whose partial derivatives are uniformly bounded,*
   $|D_i f(x)| \leq M > 0$ for all $x = (x_1, x_2, ..., x_p) \neq (0, 0, ..., 0)$, for $i = 1, 2, ..., p$. *Show that f can be extended to a continuous function on all of* $\mathbb{R}^p$. *(Hint : First show that* $|f(y) - f(x)| \leq pM\|y - x\|$ for x, y $\in \mathbb{R}^p - \{0\}$).

   [8+5 = 13]

3. *(a) Let* $f: \mathbb{R}^n \to \mathbb{R}$ *be a continuously differentiable function where* $n \geq 2$. *Show that f cannot be one – one.*

   (b)Let $g_1, g_2, : \mathbb{R}^2 \to \mathbb{R}$ *be continuously differentiable and suppose* $D_1 g_2 = D_2 g_1$.
   Let $f: \mathbb{R}^n \to \mathbb{R}$ *be defined by*

$$f(x,y) = \int_0^x g_1(t,0)\,dt + \int_0^y g_2(x,t)\,dt.$$

   Show that $D_1 f(x, y) = g_1(x, y)$ & $D_2 f(x, y) = g_2(x, y)$.

   [14 + 6 = 20]

P. T. O.

4. (a) Let $f : [0, 1] X [0, 1] \to \mathbb{R}$ be defined by

$$F(x, y) = 1 - \frac{1}{q} \text{ if x is rational} = \frac{p}{q} \text{ in lowest terms and y rational}$$
$$= 1 \quad \text{otherwise}$$

Show that f is integrable and $\int\limits_{[0,1]X[0,1]} f = 1$

Examine whether the equality $\int\limits_{[0,1]X[0,1]} f = \int_0^1 \left(\int_0^1 f(x, y)dy \right) dx$ holds.

(b) Let A be a subset of $\mathbb{R}^n$. When do you say A has (i) measure 0, (ii) content 0?

Show that if $C \subset \mathbb{R}^n$ *has content 0, then* $C \subset A$ *for some closed rectangle A with* $\int_A \chi_c = 0$.

(c) If $f : A \to \mathbb{R}$ *is a non-negative function where A is a closed rectangle in* $\mathbb{R}^n$ *and* $\int_A f = 0$, show that $\{x \in A : f(x) \neq 0\}$ has measure 0. (Hint : Prove that $\left\{x \in A : f(x) > \frac{1}{m}\right\}$ has content 0 for all $m \geq 1$).

[(10 + 4) + (2 + 2 + 5) + 9 = 32]

5. Find the maximum value of $\left|\sum_{k=1}^{n} a_k x_k\right|$, *if* $\sum_{k=1}^{n} x_k^2 = 1$ by using Lagrange's multiplier method.

[5]

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INDIAN STATISTICAL INSTITUTE
First Semester Examination: (2010-2011)
BStat II
Economics II

Date: 26 .12.2010 Maximum Marks: 100 Duration: 2 hrs.

Part I (20 marks): Answer any one question

(1) Consider a linear city of unit length (the Hotelling model), with a unit measure of consumers distributed uniformly over it. Any consumer consumes 0 or 1 units, and gets 0 from consuming 0 units, and $u > 0$ from consuming 1 unit. She faces a transport cost of t per unit distance. Suppose there are 4 vendors. Show that it is an equilibrium (Nash) for two vendors to locate at 0.25 and two to locate at 0.75. Is this the only Nash equilibrium? **(15+5)**

(2) There is a competitive industry with many potential firms. All have the same cost function $c(q) = 4 + q^2$.
Industry demand is $Q = 1000 - 5p$.
Given profit-maximising behaviour and simultaneous production choices by price-taking firms, what n, the number of firms, is consistent with long-run or 0-profit equilibrium? Also derive output price and industry output in equilibrium. **(15+5)**

Part II (40+40 marks): Answer any two question

(1) There are two firms who compete as in Cournot, i.e., there is a single, homogeneous, divisible good; each firm can produce any amount it wishes, making simultaneous production decisions, to maximise profits. The market demand function is $p = A - Q$, where $Q = q_1 + q_2$ is aggregate output, where q_i is firms i's output.
The cost functions are $c_1 = (q_1)^2 + q_2$, $c_2 = (q_2)^2$.
What quantities do the firms choose in equilibrium?
What is the efficiency impact of imposing a tax of t per unit of output on any one or both firms? **(20+20)**

(2) Suppose a non-divisible good is being sold through a sealed-bid winner-pay second-price auction, i.e., the highest bidder wins the good and pays the second-highest price. If there are multiple bidders submitting the highest bid, then one of them is picked randomly as the winner, and the winner's bid is the price. There are three bidders. Any bidder's valuation for the good is Rs. 100, 200 or 300 with

equal probability.

Show that a strategy profile where all bidders bid their own valuations is a Nash equilibrium.

Suppose the seller sets a minimum or a reserve or floor price of Rs. 150. So any bidder bidding less than 150 will never win the auction, and the good remains unsold if all bids are less than 150. If the highest bid is no less than 150 and the second-highest bid is less than 150, then the highest bidder wins and pays 150. Of course, the floor does not bind if the second-highest bid is no less than 150.

What is the expected revenue to the seller? **(20+20)**

(3) Suppose a principal can hire an agent. If the agent puts in effort a=0, the principal obtains a gross profit $x = a + e$, where e is uniformly distributed on $[0, 1]$ and non-observable. The agent's payoff is $[((w^{(1-b)})/(1-b)) - ((a^2)/2)]$, where $0 < b < 1$, and w is the wage the principal pays to the agent. The principal's payoff is $x - w$. The agent has an outside option $u = 0$.

Suppose a is contractible. What contract does the principal offer, and what effort does the agent take? What is the principal's expected payoff?

Suppose a is not contractible, but x is contractible. What contract does the principal offer, and what effort does the agent take? What is the principal's expected payoff? **(20 + 20)**

Indian Statistical Institute

Semester Examination (B. Stat-II, Biology I, Year-2010) 26/11/11

Answer any five; All questions carry equal marks; Full marks = 50; Time = 2.5 hours

1. (a) Nucleotide compositions of three different viruses were found to be: (a) 35% A, 35% T, 15% G and 15%C; (b) 35% A, 15% T, 20% G and 30% C; (c) 35% A, 30% U, 30% G and 5% C. Mention the physical nature of the nucleic acid present in these viruses.

(b) Distinguish between DNA and RNA with respect to their chemical natures and structures.

(c) In a couple, husband is albino and wife has an albino sibling. What is the risk of their baby to be an albino? (Albinism is an autosomal recessive disease and parents of the wife are normal).

[3+3+4]

2. (a) Define, with examples, mitotic and meiotic cell division in human. If two cells with genotypes (A/a) and (A/a, B/b) undergo mitotic and meiotic cell divisions, respectively, what will be the genotypes/gene compositions in the resultant diploid and haploid cells with respect to the above-mentioned alleles? [5]

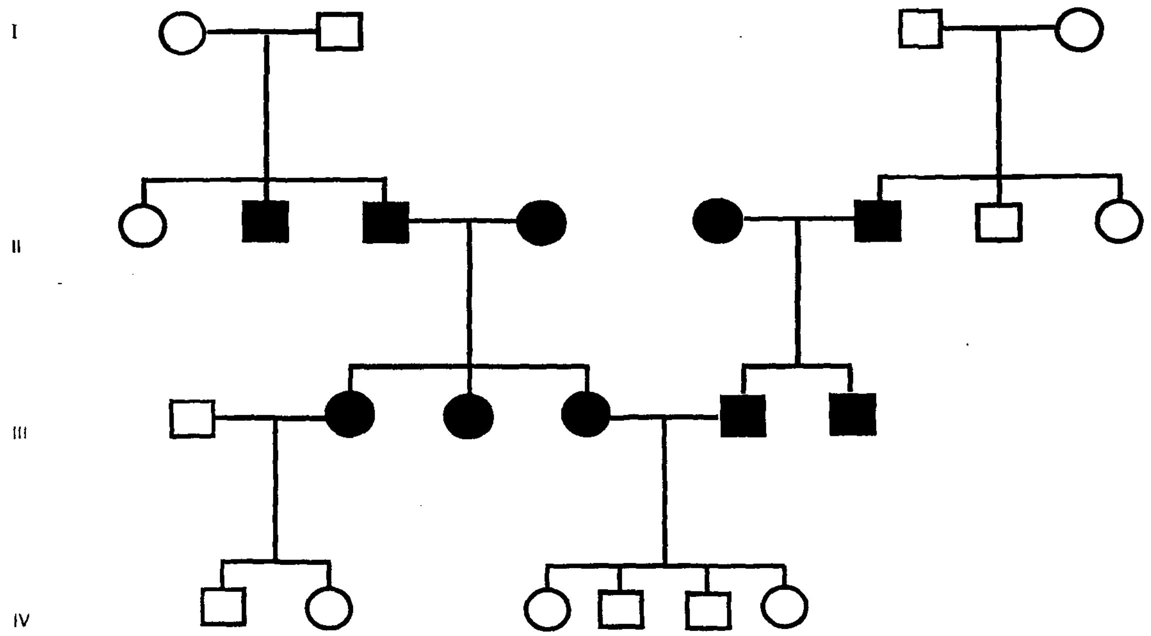
(b) Explain why two brothers, from the same parents, are not genetically identical. What is the chance that they would be genetically identical (monozygotic twins should be excluded)? [5]

3. If four babies are born on a given day. What are the chances that (a) number of boys and girls will be equal (b) all four will be girls and (c) at least one baby will be girl? What combination of boys and girls among four babies is most likely? [10]

4. Two plants with white flowers, each from true-breeding lines, were crossed. All first generation (F1) plants had red flowers. When these FI flowers were intercrossed they produced a F2 consisting of 177 plants with red flowers and 142 plants with white flowers. Propose an explanation for the inheritance of flower color in these plant species and a biochemical pathway for flower pigmentation indicating gene/s involved. [10]

5. A man with X-linked color blindness marries a woman with no history of color blindness in her family. The daughter of this couple marries a normal man and their daughter also marries a normal man. What is the chance that the last couple will have a child with color blindness? If the last couple already had a child with color blindness, what is the chance that their next child will be color blind? Draw a pedigree and answer the questions. [10]

6.



In the above-mentioned pedigree, one deaf couple, each of them homozygous for recessive mutations, had children with normal hearing. Propose an explanation how the disease alleles/genes are transmitted in this family. (Empty circles and squares are normal hearing individuals, black-filled circles and squares are deaf individuals) [10]

7. (a) Write short notes phenylketonuria and alkaptonuria [5]

(b) Polymerase chain reaction and cell culture experiments could be used for synthesis of huge amount of DNA. What are the distinctive features between these two experiments? [5]

INDIAN STATISTICAL INSTITUTE

Final Examination, 1^{st} Semester, 2010-11

Statistical Methods III, B.Stat 2^{nd} Year

Date: *November 29, 2010* Time: $3\frac{1}{2}$ hours

This paper carries 55 marks. Attempt all questions. The maximum you can score is 50.

1. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a continuous distribution with unknown c.d.f. F. For any real a, define $\widehat{F}_n(a)$ as:

$$\widehat{F}_n(a) = \frac{\# \ X_i\text{s} \leq a}{n}$$

(*This is called the empirical distribution function.*)

   (a) Show that $\widehat{F}_n(a)$ is the m.l.e. as well as the UMVUE of $F(a)$.

   (b) Examine whether $\widehat{F}_n(a)$ is consistent for $F(a)$. [7 + 3]

2. Assume that the number of times an individual has defaulted in his credit card payment is distributed as Poisson with mean λ. A survey is carried out among n randomly selected individuals (irrespective of whether they possess credit cards or not) as to their default history. Of course, those who do not possess credit cards would answer the survey question as zero. Thus, the probability that the recorded number of default cases is zero would be higher than that under a Poisson distribution. (*This is known as a zero-inflated Poisson distribution*). Given the frequency distribution of the recorded number of default cases, describe an EM algorithm to estimate λ and the proportion, p, of individuals in the population possessing credit cards. Show all the computational steps clearly. [10]

3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from an exponential distribution with mean θ. Consider a test for $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$ which rejects H_0 if and only if $\overline{X} > k$. If the size of the test is α, determine the value of k in terms of the quantiles of some known distribution. Draw a rough sketch of the power function of the above test.

P.T.O.

If n is large, show that the power of the above test at $\theta = \theta_1 (> \theta_0)$ can be approximated by $\Phi\{\frac{\sqrt{n}(\theta_1 - \theta_0) - \theta_0 z_\alpha}{\theta_1}\}$ where Φ and z_α are the c.d.f. and the $(1\text{-}\alpha)^{th}$ quantile, respectively, of a standard normal variate. [10]

4. It is believed that Vitamin B12 levels are lower in people consuming a vegetarian diet compared to those consuming a non-vegetarian diet. 8 people in the vegetarian group and 12 people in the non-vegetarian group were randomly sampled from a population and their Vitamin B12 levels measured. Assuming that Vitamin B12 levels have a normal distribution in each of the two groups, the m.l.e.s of the mean and the standard deviation of Vitamin B12 levels in the vegetarian group were found to be 135.7 pmol/L and 12.6 pmol/L, respectively; while those in the non-vegetarian group were found to be 154.2 pmol/L and 13.4 pmol/L, respectively.

   (a) Construct a 95% equal tail confidence interval for the ratio of the variances of Vitamin B12 levels in the two groups.

   (b) Stating your assumptions clearly, test at level 0.01 whether the mean Vitamin B12 level among vegetarians is indeed lower than that among non-vegetarians. [5 + 5]

5. (a) Consider the usual regression model:

$$y_i = \alpha + \beta x_i + e_i; \; i = 1, 2, \ldots, n$$

   where, x_is are fixed and e_is are i.i.d. N(0,σ^2). Show that the likelihood ratio test for $H_0 : \beta = \beta_0$ vs $H_1 : \beta \neq \beta_0$ rejects H_0 if and only if $\frac{|\widehat{\beta} - \beta_0|}{\sqrt{\Delta^2 - \widehat{\beta}^2}}$ is large, where $\widehat{\beta}$ is the least squares estimator of β and $\Delta^2 = \frac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{\sum_{i=1}^{n} (x_i - \overline{x})^2}$.

   (b) Suppose we want to estimate the difference in the proportions of students in B.Stat 1st Year and B.Stat 2nd Year having less than 75% attendence using an asymptotic 95% equal tail confidence interval. What is the minimum <u>combined</u> sample size required such that the error in estimation based on the confidence interval is at most 0.05? [8 + 7]

Indian Statistical Institute

First Semestral Examination, 2010-11

B. Stat II year
Probability III

Date: December 1, 2010 | Maximum Marks: 100 | Time: 3 hours

This is an open notes examination. Note that the paper contains 110 points. **Five extra points are reserved for brief, to the point answers and neatness of the answerscript.**

1. Consider $S \subset \mathbb{R}$ satisfying $S \cap (x, \infty) \neq \emptyset$ for every $x \in \mathbb{R}$. Let $g : S \longrightarrow \mathbb{R}$ be a bounded, nondecreasing function. Is the extended function $G : \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$G(x) = \inf_{y \in S \cap (x, \infty)} g(y),$$

for $x \in \mathbb{R}$, non-decreasing and right-continuous? Justify your answer.

[10]

2. Let $\{F_n\}$ be a sequence of cumulative distribution functions (cdf's) on $\mathbb{R}$. Show that there is a subsequence $\{F_{n_k}\}$ such that $F_{n_k}(x) \to G(x)$ at each continuity point x of some G which is a non-decreasing, right-continuous function defined on $\mathbb{R}$, satisfying $0 \leq G \leq 1$.

[20]

3. Show that if a characteristic function of a cdf satisfies $|\phi(t)| = 1$ for all $|t| \leq \frac{1}{T}$, for some $T > 0$, then the corresponding cdf is degenerate.

[20]

4. Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent random variables defined on the same probability space. Suppose that the distribution functions F_n of $S_n = X_1 + X_2 + \ldots + X_n$, $n \geq 1$, converge weakly to another cdf F. Show that $\{S_n\}$ is Cauchy in probability.

[20]

5. Let $Z_1, Z_2, \ldots, Z_n$ $(n > 3)$ be a sequence of iid standard normal random variables. Define $S_k^2 = Z_1^2 + Z_2^2 + \ldots + Z_k^2$, for $1 \leq k \leq n$. Derive the joint distribution of

$$\left(\frac{S_k^2}{S_{k+1}^2} : 1 \leq k \leq n-1\right)$$

and S_n^2. [20]

6. Let $X_1, X_2, \ldots, X_n$ be iid random variables with symmetric distribution, that is, $X_1 =_d -X_1$. Show that

$$\Pr\left(|X_1 + X_2 + \cdots + X_n| \geq \max_{1 \leq i \leq n} |X_i|\right) \geq \frac{1}{2}.$$

[20]

INDIAN STATISTICAL INSTITUTE
First Semester Back paper Examination: 2010-11

B. Stat. (Hons.) II Year
Analysis III

Date:12.01.11 **Maximum Marks: 100** **Duration: 3 Hours**

Answer all questions.

1. (i) If A is a non-empty closed subset of $\mathbb{R}^n$ and $x \notin A$, prove that there exists $d > 0$ such that $\|y - x\| \geqq d$ for all $y \in A$, where $\| \bullet \|$ stands for the Euclidean norm $\mathbb{R}^n$.

   (ii) If A is closed and B is a compact subset of $\mathbb{R}^n$, and $A \cap B = \phi$, prove that there is $d > 0$ such that $\|y - x\| \geqq d$ for all $y \in A$ and $x \in B$.

   (iii) Give a counter example in $\mathbb{R}^2$ *for (ii) if* A and B are closed but neither is compact.

   (iv) Let $\{C_n, n \geqq 1\}$ be a family of connected subsets of $\mathbb{R}^n$. Suppose $C_n \cap C_{n+1} \neq \phi$ for every $n \geqq 1$. Show that $\bigcup_{n \geqq 1} C_n$ is connected.

   (v) Let $U \subset \mathbb{R}^2$ be an open set. For each $x \in U$, let $C(x)$ denote the largest connected subset of U containing x ($C(x)$ is called the connected component of x contained in U). Show that each $C(x)$ is open in $\mathbb{R}^2$. (Hint : open discs are connected).

[3+6+5+5+6=25]

2. (i) Given two real- valued functions f_1, f_2, defined and having derivatives in the open interval $(a,b) \subset \mathbb{R}$. For each $x = (x_1, x_2)$ in the 2-dimensional rectangle.

$$S = \{(x_1, x_2) : a < x_i < b, i = 1,2\}, \text{ define } f(x) = f_1(x_1) + f_2(x_2).$$

   Show that f is differentiable at each point of S.

   (ii) Consider the functions defined on $\mathbb{R}^2$ by the following formulas.

   (a) $$f(x, y) = \frac{xy}{x^2 + y^2} \text{ if } (x,y) \neq (0,0), \qquad f(0,0) = 0$$

   (b) $$f(x,y) = \frac{x^2 y^2}{x^4 + y^4} \quad if\ (x, y) \neq (0,0), \quad f(0,0) = 0$$

   In each case show that the partial derivatives $D_i f$, $D_2 f$ exist for every $(x, y) \in \mathbb{R}^2$ and find these derivatives.

   Explain why the functions described in (a) & (b) are not differentiable at $(0,0)$.

(iii) Let f and g be real-valued functions defined on $\mathbb{R}$ with continuous second derivatives f'' and g''. Define

$$F(x, y) = f(x + g(y)) \text{ for each } (x,y) \text{ in } \mathbb{R}^2.$$

Find expressions for all partial derivatives of F of first and second order in terms of the derivatives of f and g. Verify the relation

$$(D_1F)(D_{1,2}F) = (D_2F)(D_{1,1}F)$$

[8+12+10=30]

3. (i) Let $f : \mathbb{R}^n \to \mathbb{R}$ with $n \geqq 2$ be a continuously differentiable function. Show that f is not one-one.

(ii) Given a decreasing sequence of real numbers $\{G(n)\}$ such that $G(n) \to 0$ as $n \to \infty$. Define a function f on $[0,1]$ in terms of $\{G(n)\}$ as follows : $f(0) = 1$; $f(x) = 0$ if x is irrational ; if x is the rational m/n (in the lowest form), then $f(m/n) = G(n)$. Compute the oscillation $O(f, x)$ at each $x \in [0,1]$ and deduce that f is integrable on $[0,1]$.

Let f be defined as above with $G(n) = \frac{1}{n}$. Let $g(x) = 1$ if $0 < x \leqq 1$, $g(0) = 0$. Show that the composite function h defined by $h(x) = g(f(x))$ is not Riemann integrable on $[0,1]$, although both f and g are Riemann integrable on $[0,1]$.

[15+(9+6)=30]

4. (i) Let $A \subset \mathbb{R}^n$ be a closed rectangle and $f : A \to \mathbb{R}$ be integrable.

Show that $|f| : A \to \mathbb{R}$ is integrable and $\left| \int_A f \right| \leqq \int_A |f|$.

(ii) Find the maximum value of $\left| \sum_{k=1}^{n} a_k x_k \right|$, $\sum_{k=1}^{n} x_k^2 = 1$, by using

(a) the Cauchy-Schwarz inequality.
(b) The Lagrange's multiplier method.

[5 + (5+5) = 15]

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INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : Semester II (2010-2011)

B. Stat 2nd Year

Statistical Methods IV

Date: 21. 2. 11 Maximum marks: 45 Time: 2 hours.

Note: Answer all questions. Maximum you can score is 45.

1. Let $\mathbf{X}_{n\times p}$ be a data matrix from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and $\Gamma_{n\times n}$ be an orthogonal matrix with the last row $\sqrt{n}$ times the unit vector. Suppose $\mathbf{Y} = \Gamma\mathbf{X}$. Denote the rows of $\mathbf{Y}$ by $Y_1, Y_2, \ldots, Y_n$. Show that

   (a) $Y_i, i = 1, 2, \ldots, n$ are independent. [3]

   (b) Y_n follows $N_p(\sqrt{n}\boldsymbol{\mu}, \boldsymbol{\Sigma})$. [3]

   (c) $Y_i, i = 1, 2, \ldots, n-1$ are i.i.d. $N_p(0, \boldsymbol{\Sigma})$. [5]

   (d) Express $\bar{X}$ in terms of Y_n only, and S in terms of $Y_i, i = 1, 2, \ldots, n-1$. [4]

   (e) Find the distribution of $\bar{X}$ and S, and show that they are independent. [5]

2. Suppose $\mathbf{X}_{n_1\times p}$ and $\mathbf{Y}_{n_2\times p}$ are two data matrices from $N_p(\boldsymbol{\mu}_1, \boldsymbol{\Sigma})$ and $N_p(\boldsymbol{\mu}_2, \boldsymbol{\Sigma})$ respectively. Define the Mahalanobis distance between the two sample means based on the pooled sample variance-covariance matrix and derive its distribution under the condition $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$. [10]

3. Suppose $\boldsymbol{X} \sim \mathrm{N}_p(\boldsymbol{\mu}, \sigma^2\boldsymbol{I}_p)$, where σ is a known positive quantity. Derive the likelihood ratio test for the hypothesis $\mathrm{H}_0 : \boldsymbol{\mu}'\boldsymbol{\mu} = 1$ against $\mathrm{H}_1 : \boldsymbol{\mu}'\boldsymbol{\mu} \neq 1$. [15]

INDIAN STATISTICAL INSTITUTE

B. Stat II Year 2010-2011

Mid-Semestral Examination

Subject : Demography & SQC & OR

Date : 22.02.2011 Full Marks : 100 Duration : 3 hrs.

Instruction : Begin each group on a separate answer-script.

Group A : SQC & OR (Maximum Marks : 50)

Note : This group carries 53 marks. You may answer as much as you can, but the maximum you can score is 50.

1. Two alloys, A and B are made from four different metals, I, II, III and IV, according to the following specifications:

Alloy	**Specifications**	**Selling price ($)/ton**
A	At most 80% of I	200
	At least 30% of II	
	At least 50% of IV	
B	Between 40% & 60% of II	300
	At least 30% of III	
	At most 70% of IV	

The four metals, in turn, are extracted from three different ores with the following data:

Ore	**Max. Quantity (tons)**	**Constituents (%)**					**Purchase Price ($)/ton**
		I	**II**	**III**	**IV**	**others**	
1	1000	20	10	30	30	10	30
2	2000	10	20	30	30	10	40
3	3000	5	5	70	20	0	50

How much of each alloy should be produced to maximize the profit. Formulate the problem as a LP model.

[20]

2. Write down the dual of the following problem:

P : Max $z = 4x_1 + 5x_2$

Subject to

$$3x_1 + 2x_2 \leq 20$$
$$4x_1 - 3x_2 \geq 10$$
$$x_1 + x_2 = 5$$
$$x_1 \geq 0;\ x_2 \text{ is unrestricted in sign}$$

[10]

3. Write the following LP into a standard form:

Max $z = x_1 + 2x_2 - x_3$

Subject to the following constraints:

$$x_1 + x_2 - x_3 \leq 5$$
$$-x_1 + 2x_2 + 3x_3 \geq -4$$
$$2x_1 + 3x_2 - 4x_3 \geq 3$$
$$x_1 + x_2 + x_3 = 2$$
$$x_1 \geq 0;\ x_2 \geq p;\ x_3 \text{ is unrestricted in sign.}$$

Mention the range of p in the standard LPP.

[8+2=10]

4. The system $Ax = b, x \geq 0$ is given by

$$x_1 + x_2 - 8x_3 + 3x_4 = 2$$
$$-x_1 + x_2 + x_3 - 2x_4 = 2$$

Find

a) a nonbasic feasible solution

b) a basic solution which is not feasible

c) a BFS which corresponds to more than one basis matrix. Write all the corresponding basis matrices.

d) a solution which is neither basic nor feasible.

[3+3+4+3=13]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: (2010 - 2011)

B. STAT, II YEAR

Demography

Date: 22, February, 2011 **Maximum Marks:** 30 **Duration:** 1.5 hrs

Note: (i) Desk calculators are allowed in the exam, (ii) Symbols and notations have their usual meaning.

Answer the following questions

1. Suppose $l_0 = 1000, l_1 = 800$, $l_2 = 650$, $l_3 = 400$, $l_4 = 150$, $l_5 = 50$, $l_6 = 20$ and $l_7 = 0$ are given for a life table population. Calculate life expectancy of this population.

[10]

2. a) Given an autonomous differential equation $f(N) = \frac{dN}{dt}$, where N is the population density. Show that $n(t) = n(0)exp\left\{ \frac{df(N)}{dN}\Big|_{N*} \right\} t$, where $n(t)$, $n(0)$ are perturbations at time t and at time 0, and N^* is equlibrium point.

b) Suppose a population model is described by following equations:

$\frac{dN_1}{dt} = -aN_1N_2$, $\frac{dN_2}{dt} = aN_1N_2 - bN_2$, $\frac{dN_3}{dt} = bN_2$.

Initial conditions are $(N_1(0), N_2(2), N_3(0)) = (N_{1,0}, N_{2,0}, N_{3,0})$. Here total population N is divided into three sub-populations N_1,N_2,N_3 respectively, i.e. $N = N_1 + N_2 + N_3$. Note that the rate of change in the sub-population N_3 in the above set of equations is not dependent on N_3. a and b are positive constants. If $N(t)$ represent a population at time t and $N(0)$ represent population at time 0, then show that

$$N_2(t) \quad = \quad N_1(0) - N_1(t) + \frac{b}{a}ln\left(\frac{N_1(t)}{N_1(0)}\right) + N_2(t).$$

c) Can we produce an analytical solution to the population model in 2 (b) ? Please justify.

d) Suppose *exponential growth rate* from Population *at time 0,* i.e. P(0) to Population *at time t, i.e.* P(t) is 0.003 *and* $P(0) = 1250$ *millions,* project the value of $P(t)$ in *millions? Mention any two disadvantages of such a projection method?*

$[\mathbf{5 + 5 + 3 + 5}]$

3. a) If $l_x = 640\sqrt{100 - x}$, which of the following statements is true? Briefly Justify. (takak nearest integer if necessary)

i) The probability of survival from birth to age 51 is 0.8.

ii) At age 51, l_x is decreasing at the rate of 35 lives per unit time.

iii) At age 35, l_x is decreasing at the rate of 40 lives per unit time.

iv) None of the above

b) In a life table, the probability that an individual aged 90 will survive for two years i 0.3 and the probability that an individual aged 90 will survive for one year is 0.5. Then th probability that an individual aged 91 will die within one year is (Briefly Justify)

i) 0.6

ii) 0.3

iii) 0.4

iv) 0.35

[2 + 2]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2010-11 (Second Semester)

B. STAT. II YEAR
Elements of Algebraic Structures (Math 103)

Date : 24·02·11

Duration : $3\frac{1}{2}$ Hours
Total Marks: 100
Maximum Marks : 90

1. Let G be a finite group.

   (a) Without using Lagrange's Theorem, show that there exists a positive integer N such that $a^N = e, \forall\, a \in G$. [7]
   (b) If N is a positive integer such that $a^N = e, \forall\, a \in G$, what is the relation between $o(G)$ and N? [3]
   (Here you can use Lagrange's Theorem.)

2. (a) Show that a subgroup of a cyclic group is cyclic. [7]
   (b) Hence or otherwise, show that if a cyclic group has subgroups of order m and n, then it has a subgroup of order $\text{lcm}(m, n)$. [5]

3. Let $G = \langle a \rangle$ be the infinite cyclic group.

   (a) How many generators does G have? [4]
   (b) What is $Aut(G)$? [4]
   (Give reasons for your answers.)

4. State and prove Cauchy's Theorem for finite abelian groups. [7]

5. Show that any group G, with $o(G) = 6$, which is not cyclic, is isomorphic to S_3. [8]

6. Let $G = (\mathbb{R}, +)$, $N = (\mathbb{Z}, +)$, $S^1 = \{z \in \mathbb{C} |\ |z| = 1\}$, with complex multiplication. Show that $G/N \cong S^1$. [5]

7. (a) Prove that S_n is non-abelian $\forall n \geq 3$. [3]
   (b) Prove that $S_n = \langle (1, 2), (1, 2, ..., n) \rangle$. [7]

P.T.

8. (a) Define an even permutation. [2]

(b) Which of the following permutations are even: [2+2]
$\sigma = (1,2)\ (1,3,5,4)\ (4,6)\ (3,2,6)$
$\tau = (2,3)\ (3,5,4)$

(c) Which of the following elements are conjugate in S_6: [6]

$\sigma = (1,2)\ (1,3,5,4)\ (4,6)\ (3,2,6)$

$\tau = (2,3)\ (1,6,4)$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 2 & 6 & 3 & 1 \end{pmatrix}$$

$$\omega = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 6 & 1 & 2 \end{pmatrix}$$

9. Let $D_{2n} = \langle x, y | x^2 = e = y^n,\ xy = xy^{-1} \rangle$.

(a) Prove that $N = \langle y \rangle$ is normal in D_{2n}. [3]
(b) Prove that $D_{2n}/N \cong \mathbb{Z}/2\mathbb{Z}$. [4]
(c) Find the centre of D_{2n}. [7]

10. Let G be a group of order 117.

(a) Show that G has a normal subgroup of order 13. [6]
(b) Show that if G has a normal subgroup of order 9, then G is abelian. [8]

Indian Statistical Institute

Mid-Semester Examination, 2010-11 (Second Semester)

B.Stat

Physics II

Date : 28.02.11/Maximum Marks 30 Duration 2 hrs

Answer Each Group In Separate Sheets

Group A

ANSWER ANY THREE QUESTIONS. ALL QUESTIONS CARRY 5 MARKS.

1) Find the work done during isothermal transformation of one mole of an ideal gas from density ρ_1 to ρ_2 at temperature T.

Derive the van der Waals equation of Corresponding States and discuss its significance. [2+3=5]

2) Find the efficiency η of a Carnot cycle operating between temperatures T_1 and T_2 with $T_2 > T_1$. How does it conform to Kelvin's postulate of the Second Law of Thermodynamics?

Show that for van der Waals gas the energy can depend on volume. [3+2=5]

3) Derive the law $pV^{\kappa} = constant$ for an ideal gas undergoing adiabatic transformation. $\kappa = C_p/C_V$.

Show that $\oint \frac{dQ}{T} = 0$ for a reversible cyclic transformation. How does it get modified for an irreversible transformation?

Show that for a system $S = \kappa ln\pi + c$ where κ, c are constants. S denotes entropy and π is related to the probability of being in a particular microstate. [3+2=5]

4) Using Clapeyron equation find the amount of pressure that is to be applied to change the boiling point of water by two degrees Kelvin. using the information given below:

Latent heat of vaporization = $540 cal./gm$; specific volume of vapor = $1677 (c.m.)^3/gm$;specific volume of water = $1 (c.m.)^3/gm$; T = $371^0 K$.

Show that for a fluid obeying the ideal gas equation of state its energy can depend only on temperature. [3+2=5]

5) Explain qualitatively the modifications introduced in van der Waals equation of with reference to the ideal gas. Draw and discuss features of the van der Waals isotherms.

Calculate the entropy for the van der Waals gas. [2+3=5]

Group B

ANSWER ANY THREE QUESTIONS

1. In photoelectric emission from a given target. the velocity of the emitted electrons is 10^6 m/s when light of wavelength 2.5×10^{-7} meter is used. Calculate (a) the velocity of emitted electrons with light of wavelength 5×10^{-7} meter (b) work function of the target in electron volt. **(5)**

2 (a). If the maximum energy imparted to an electron in Compton scattering is 45 keV. what is the wavelength of the incident photon ? **(3)**

(b). According to Bohr model. how many revolutions will an electron make in the $n = 2$ state of hydrogen before dropping to the $n = 1$ state if the life-time in the excited state is 10^{-8} s ? **(2)**

3 (a). Show that the de Broglie wavelength of a particle is approximately same as that of a photon with the same energy when the energy of the particle is very large compared to its rest energy. **(3)**

(b). Suppose the uncertainty in the momentum of particle is equal to the particle's momentum. How is the minimum uncertainty in the particle's location related to its de Broglie wavelength ? **(2)**

4. A particle is confined to a one dimensional line of length L. Using uncertainty principle. obtain the ground state energy of the particle. **(5)**

5. In a hydrogen atom. an electron moves from one allowed orbit to another. Determine the correction to the wavelength of an emitted photon when the recoil kinetic energy of the hydrogen nucleus is taken into account. **(5)**

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination (2010-11) (Second Semester)
B.Stat (Hons.) – II
Biology – II
Date..25/02/11.........Maximum Marks: 30 Duration: 2:00 hrs.

(Answer any three)

1. What is Moisture Availability Index? Draw a suitable rice crop calendar with the following data

Week No.	***22***	***23***	***24***	***25***	***26***	***27***	***28***	***29***	***30***	***31***	***32***	***33***	***34***	***35***	***36***	***37***	***38***	***39***	***40***
Rainfall (mm) at 0.5 Prob.	***5***	***0***	***15***	***10***	***9***	***37***	***18***	***35***	***18***	***20***	***98***	***142***	***95***	***80***	***15***	***32***	***2***	***0***	***0***
PET (mm)	***45***	***42***	***33***	***37***	***33***	***25***	***22***	***20***	***22***	***20***	***19***	***17***	***19***	***20***	***25***	***28***	***31***	***33***	***34***

3+7

2. Name the different meteorological variables that are related to crop production. Write down the names of apparatus use to measure those parameters.
4+6
3. Classify rice and rice culture depending on eco-geographical situation. Briefly describe the cultural practices associated with rainfed lowland rice cultivation.
5+5

4. Critically highlight the variation in yield of *Aman* rice and *Boro* rice. Calculate the quantity of VC, Urea, Single Super Phosphate and Muriate of Potash (KCL) required for 1 ha. rice crop to supply the nutrient requirement of 160 kg N, 80 kg P_2O_5 and 80 kg K_2O per hectare. 50% of required N should be given through VC. 4+6

5. Write in brief :

- Intercropping and mixed cropping
- Micro-climate and phyto-climate
- Cropping system and farming system
- Distribution of rainfall

10

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination :2011
Course : B- Stat II Year 2011
Subject : Economics II (Macroeconomics)

Date : 25.02.11

Duration : 150 minutes Maximum Marks 40

Answer all questions.

1 (a) Derive the identity

$$NS \equiv I + X - M + NFY + NFT_r$$

and hence argue whether the investment, in this identity is gross or net.

(b) Suppose in an economy under flexible exchange rate regime, the net inflow of foreign loan to the domestic sector is Rs. 25,000 cr. In addition the foreigners purchased shares of domestic companies worth Rs.10,000cr. Domestic economic agents didn't buy any foreign financial asset. Difference between *NNP* and private disposable income is Rs.50,000 cr. It is given that,

$NFT_r = 0$, $G =$ Rs. 40,000 cr., personal saving = Rs.30,000 cr, undistributed corporate profit = Rs.50,000cr. and the government pays Rs. 20,000 cr. to households as interest on outstanding loan.

Compute the values of capital account balance, current account balance and net investment . (12+15=27)

2. Consider a closed economy without government. Suppose in a given period the households of the economy purchased goods and services worth Rs. 20,000 cr. Firms purchased goods worth Rs.10,000cr. And added to stock. However, all the firms together produced goods and services worth Rs.35,000. They therefore could not sell Rs.5,000 worth of goods and had to hold them involuntarily in their inventory.
What is firm investment, I_F in this case?
Does the identity $GDP \equiv C + I$ hold in this case? (6)

P.T.O

2. Consider the police force of a country. Its job is to maintain law and order of the country .Suppose that in a given year it spent Rs.10 cr. as interest on its outstanding loan to households, Rs.20 cr. as wages and salaries, Rs.10 cr. on stationeries, fuel, power and acquired buildings,Rs.30 cr. on equipments and cars.

What is the *GVA* of this sector?
Did it con tribute anything to *G* ? Explain.
Did it contribute anything to transfer payment? Explain. (6)

3.Consider a closed economy with government activities. Suppose that in a certain year the private and public sector enterprises produced goods and services worth 10cr. i.e their total *GVA* in that year was Rs.10cr.
In the same year they sold goods and services worth Rs. 8cr to households And Rs. 5 cr. to govt. Besides, the govt. sector paid Rs. 10cr. as wages and salaries and 1cr.as interest to households
What is the aggregate gross firm investment in the given year? Explain.
Calculate the GDP of the economy, using spending approach. (6)

Mid-Semester Examination:2010-11(Second Semester)
Course name:B.Stat-11(Second Year)
Subject Name: Economic and Official Statistics

28.02.11
Date:[redacted] Maximum Marks:100 Duration:3 hours

Q1.Define an Index Number .Show that both Laspeyers and Paasches Index Numbers are biased. Why Fisher's Index is called an Ideal index? State the axioms under the axiomatic theory of price indices.

[20]

Q2.Discuss different stages of a survey operation with reference to computation of consumer price index number .Discuss different kinds of error ,an index number is to suffer. Indicate the stages where non sampling and sampling error are likely to occur. What is the measure of sampling error ? Discuss some methods of controlling error due to non response with reference to computation of any index number.

[25]

Q3.Justify the statement" The Indian Statistical System is largely decentralized". "It has many problems but also has many achievements and much promise"—can you state some of the achievements with criticisms? Explain why is the present system unable to fulfill the need of the data with respect to decentralized planning?

[15]

Q4.What are the considerations that lead to the set up of National Statistical Commission? State the fivefold remedial approach made by the Commission.

[15]

Q5.What are the sources of statistical data? Name the statistical agencies with their functions.State fundamental principles of official statistics adopted by United Nations Statistical Commission.

[25]

INDIAN STATISTICAL INSTITUTE
End-Semester Examination: 2010-11 (Second Semester)

B. STAT. II YEAR
Elements of Algebraic Structures (Math 103)

Date : 02.05.11

Duration : $3\frac{1}{2}$ Hours
Total Marks: 106
Maximum Marks : 100

Note: Rings will be assumed to be commutative and contain unity.
$\mathbb{Z}$: the ring of integers.
$\mathbb{Q}$: the field of rational numbers
$\mathbb{F}_p$: the prime field of characteristic p.

1. (a) Find the minimal polynomial over $\mathbb{Q}$ for $a+bi$, where $a, b \in \mathbb{Z}$, $b \neq 0$. [4]
(b) Determine the splitting field for $x^4 + 4$ over $\mathbb{Q}$ and compute its degree over $\mathbb{Q}$. [6]

2. (a) Let F be a field such that $char(F) \neq 2$. Let D_1, $D_2 \in F$ such that neither D_1 nor D_2 is a perfect square in F. If D_1D_2 is not a perfect square in F, then show that $[F(\sqrt{D_1}, \sqrt{D_2}) : F] = 4$. [7]
(b) Prove that the ring $\mathbb{F}_2[x]/(x^3+x+1)$ is a field but $\mathbb{F}_3[x]/(x^3+x+1)$ is not. [6]

3. Let I, J be ideals of a ring R such that $I + J = R$.
(a) Prove that $IJ = I \cap J$. [5]
(b) Hence or otherwise, prove the following: Let $m, n \in \mathbb{Z}$ be such that $(m, n) = 1$. Then for any $a, b \in \mathbb{Z}$ the system of equations $x \equiv a(\text{mod } m)$, $x \equiv b(\text{mod } n)$. has a solution. [6]

4. (a) Let F be a finite field of characteristic p. Prove that $|F| = p^n$, for some positive integer n. [5]
(b) Prove that $(f(x))^p = f(x^p)$, for any $f(x) \in \mathbb{F}_p[x]$. [5]

5. Let R be a ring. $x \in R$ is called nilpotent if some power of x is zero.
(a) The nilradical N of a ring R is the set of its nilpotent elements. Prove that N is an ideal of R. [5]
(b) Prove that if $x \in R$ is nilpotent then $1 + x$ is a unit. [5]

P.T.O

6. (a) Prove that the ring $\mathbb{Z}[i]/(1+2i)$ is isomorphic to $\mathbb{Z}/5\mathbb{Z}$. [8]
(b) Show that for any field F such that $char(F) = 0$, there exist infintely many monic irreducible polynomials in $F[x]$. [5]

7. Let p be a prime.
(a) Show that $x^{p-1} + x^{p-2} + ... + x + 1$ is irreducible in $\mathbb{Z}[x]$. [6]
(b) Prove that $x^p - x + 1$ is irreducible and separable over $\mathbb{F}_p$. [8]

8. Let K be a field extension over F, $\alpha \in K$. If $F(\alpha) = F(\alpha^2)$, show that α is algebraic over F. [6]

9. (a) Prove that a group of order 100 has a normal subgroup of order 50. [10]
(b) Prove that if P is a Sylow p-subgroup of a group G such that H is a subgroup of G containing P, then P is a Sylow p-subgroup of H. [4]

10. Determine the greatest common divisor of $a(x) = x^3 + 4x^2 + x - 6$ and $b(x) = x^5 - 6x + 5$ over $\mathbb{Q}[x]$ and write it as a linear combination of $a(x)$ and $b(x)$ in $\mathbb{Q}[x]$. [5]

The End

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: (2010 - 2011)

B. STAT, II YEAR
Demography & SQC & OR

Date: May 4, 2011 **Maximum Marks: 80** **Duration: 3 hrs**

Group A: Demography (Maximum Marks: 30)

Instructions: (i) Begin each group on a seperate answer-script. (ii) Desk calculators are allowed in the exam. (iii) Symbols and notations have their usual meaning. (iv) RMMR Tables will be provided in the exam.

Answer the following questions

1. Consider a Lotka-Volterra predator-prey population model as given below:

$$\frac{dN_1}{dt} = rN_1 - hN_1N_2$$
$$\frac{dN_2}{dt} = \beta hN_1N_2 - mN_2$$

where N_1 is a prey population, N_2 is a predator population, r is a growth rate, the probability that, upon meeting a prey, the predator will successfully kill it will be represented by h, β represents the conversion efficiency of prey biomass into predator biomass and it is assumed that predators die at random with a probability m. What are the two equilibrium points ? Describe the behaviour of the system using the eigenvalue approach. What is the Jacobian matrix at coexistence equilibrium ? What are eigenvalues for the coexistence situation?

[1+3+2+2=8]

2. a) Suppose $l_x = s(x).l_0$ and $s(x) = \frac{1}{10}(100-x)^{1/2}$, where $s(x)$ is the survival function and $l_0 = 1000$. Construct the life table columns q_x, T_x and e_x^0 for ages $x = 0, 1, 2, 3$. b) Can we approximate Net Reproduction Rate from Total Fertility Rate? Explain.

[6+2=8]

3. a) Using the Makeham's Law of Mortality i.e. $\mu_x = A + BC^x$ (with A, B and C as constants) derive an expression for the number of survivors at age x in a life table.

b) In a *birth - death process, when* $\lambda < \mu$, prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$, where P_n is the probability that there are n individuals in the system at any point of time,λ is the birth rate and μ is the death rate.

[3+5=8]

4. a) Explain briefly the procedure of population projection by the method of Leslie's matrix.

b) Deduce a dynamic model for understanding growth in the one-sex age structured population.

c) What is Lotka's stable population model?

[3+3+3=9]

INDIAN STATISTICAL INSTITUTE

B. Stat. (Hons.) II: 2010 – 2011
Second Semestral Examination
Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | 6 May 2011

Economic Statistics and Official Statistics

(Answer question no. 1 and any ***four*** *from the rest. Marks allotted to each question are given within brackets.)*

1. Suppose monthly per capita expenditures (x) of persons in a country follow Pareto law with inequality parameter $\nu = 2.0$ and the threshold parameter $c = 10$.
   (i) Find the Elteto and Frigyes measures.
   (ii) Suppose all the persons with income less than Rs. 20 have migrated to some other country. Recalculate the Elteto and Frigyes measures. [10+10=20]

2. State Pareto law. Give your comments on the universality of Pareto law stating the evidence for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? Derive the formulae for Lorenz curve and Lorenz ratio of Pareto distribution. [20]

3. Write down the important steps in deriving Atkinson's measures of inequality based on the Social Welfare Function Approach. How can one interpret the unknown parameter in the measure? [20]

4. State Pigou-Dalton principle of Transfer of Income. Examine CV, LR and RMD in the light of the above principle. [20]

5. Write a brief account on treatment of household size in Engel curve analysis. [20]

6. Describe in detail the economic and statistical criteria for choosing an Engel Curve. [20]

7. Define Index Number. Why is the choice of base period and commodities so important while constructing a Consumer Price Index Number? Define chain base index numbers. What are the merits and demerits of chain base index number compared to the fixed base index number? Examine the situation when a chain base index number would be effectively same as a fixed base index number. [2+6+2+6+4=20]

8. Write short notes on any **two** of the following:
   (i) Sen's poverty measure.
   (ii) Cobb Douglas production function
   (iii) Three-parameter lognormal distribution.
   (iv) Properties of Lorenz Curve. [10+10=20]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2010-2011)

B. Stat 2nd Year

Statistical Methods IV

Date: 9. 5. 11 — Maximum marks: 100 — Time: 3 hours.

Note: Answer all questions. Maximum you can score is 100.

You may use any result proven in class after stating the results clearly in the proper place

You may use calculators.

1. (a) Suppose $\mathbf{X}$ follows $N_4(0, \mathbf{I})$. Find the distribution of

$$\frac{\sqrt{2}(X_1X_3 + X_2X_4)}{X_3^2 + X_4^2}$$

[10]

(b) Suppose $\mathbf{X}$ follows $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)^T$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{bmatrix},$$

where ρ is such that $\boldsymbol{\Sigma}$ is positive definite.

Derive the conditional distribution of (X_1, X_2) given X_3. [10]

2. Consider a random vector $\mathbf{X}$ which has the $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution where $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ (p.d.) are unknown. The problem is to test the hypothesis that $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_0$ (known) against $\boldsymbol{\Sigma} \neq \boldsymbol{\Sigma}_0$ on the basis of a random sample.

(a) Derive the Likelihood ratio test (LRT) statistic for this test. [10]

(b) Derive the Union-Intersection test statistic for the same test, and compare it with the LRT statistic found in part (a). [15]

3. (a) Let $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$ be such that $Y_i, i = 1, \ldots, n$ are independent $N(0,1)$ variables. Suppose $\mathbf{Y}'A\mathbf{Y}$ and $\mathbf{Y}'B\mathbf{Y}$ have Chi-square distribution. Show that a necessary and sufficient condition that they are independently distributed is $AB = 0$. [10]

(b) Suppose $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_k$ are symmetric, idempotent matrices such that

$$\mathbf{C}_1 + \ldots, + \mathbf{C}_k = \mathbf{I}.$$

Show that $\mathbf{C}_i\mathbf{C}_j = 0$, for $i \neq j$. [10]

[P. T. O.]

4. (a) Let $X_1, \ldots, X_n$ be i.i.d. observations from a density f. Find the density function of the sample range. Derive this density function exactly when f is Unif$[0, 1]$. [10+5]

(b) Consider a 2x2 contingency table. Define the odds ratio, and derive an asymptotic 95% confidence interval for this quantity. [15]

5. A researcher considered three indices measuring severity of heart attacks. The values of these indices for 40 heart-attack patients arriving at a hospital emergency room produced the following sample covariance matrix

$$\mathbf{S} = \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ 63 & 80.2 & 55.6 \\ 71.0 & 55.6 & 97.4 \end{bmatrix}$$

Make an appropriate statistical test for testing whether the populatin multiple correlation coefficient between the first index and the others is 0. [15]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2010-2011
Course Name: B. Stat.-II yr.
Subject Name: Biology II

Date: 11.5.11. Maximum Marks: 50 marks Duration: 3 hours

Answer Five Questions

Part A

(This question is compulsory)

1. Write down a competition model between two important species (Paddy - Legume intercropping) with proper agricultural justification. Find out agriculturally feasible equilibrium and discuss the nature of the persistence / extinction of the species. [The agricultural justification(s) of each of the conditions are to be given] (10)

Part B

(Question number 1 and 2 are compulsory. Answer 2 questions from the rest.)

1. (a) Charles Darwin's cousin, who was knighted by the Queen of England and an early pioneer of Statistics, coined the term, 'Eugenics'. What is the name of this scientist? Define Eugenics.

   (b) A well known geneticist, Davenport studied various ethnic groups and races and concluded that on the basis of genetics the Germans ranked highest in quality such as leadership, humor, generosity, sympathy and loyalty. Italians and Irish ranked lowest in most of these traits. Jews were highest in obtrusiveness and all of these were said to be genetically template. Do you agree with such kind of Statistical analyses?

   (c) At the beginning of World War I, IQ tests were first instituted in the USA. What was the major reason behind implementation of such a test?

   (d) We humans have been doing breeding on plants and livestock for at least 10,000 years. Define plant breeding. What are the aims and objectives of plant breeding?

   (e) The pace with which human brain has evolved over the last half million years, and more recently the last 200,000 years, has been so frighteningly rapid that the evolution of human cognitive functions and sensory perceptions can only have happened through the actions of a small number of genes. Do you agree with this statement? Why?

   (f) Among Orthodox Ashkennazi Jews of New York, between 2-3% of the population carries an allele for Tay-Sachs disease which is phenotypically silent in the heterozygous stage but in the homozygous condition leads to a devastating condition resulting in death during the first years of birth. Now, before two young people decide to marry, they carry out a test to find out whether they are heterozygous for the Tay-Sachs allele. If one says that we are practicing a subtle form of eugenics, do you agree? If a time comes where part of the marriage contract states that the bride or groom want a sample of that each other's buccal swab or some lymphocytes to check out what kind of DNA he or she has, then what is going to happen? Some Biologists say that the truth must come out and that everything that can be learned should be learned and we will learn how to digest it and then we will learn how to live with that. Do you think we should go ahead and try to find genes for functions like manic depression which affects 2-3% of the population worldwide or study on a trait which was discovered in a family of Netherlands where a very specific grammatical defect (in a way they assembled the syntax of the sentences) was linked with a certain allele of a certain gene. Give answers on the basis of logic based reasoning.

(1+1+1+2+2+3)

[Source: Eric Lander, Robert Weinberg and Claudette Gardel, 7.012, Introduction to Biology, Fall 2004 (MIT, USA). http://ocw.mit.edu]

2. (a) What is the three point test cross? Answer with a schematic diagram.

(b) In the Chinese primrose, slate color flower (s) is recessive to blue flower (S); red stigma (r) is recessive to green stigma (R); and long style (l) is recessive to short style (L). All three genes involved are on the same chromosome. The F1 of a cross between two true breeding strains, when test crossed, gave the following progeny:

PHENOTYPE	NUMBER OF PROGENY
slate flower, green stigma, short style	27
slate flower, red stigma, short style	85
blue flower, red stigma, short style	402
slate flower, red stigma, long style	977
slate flower, green stigma, long style	427
blue flower, green stigma, long style	95
blue flower, green stigma, short style	960
blue flower, red stigma, long style	27
Total	3000

(a) What were the genotypes of the parents in the cross of the two true-breeding strains?
(b) Make a map of these genes, showing gene order, and distance between them.
(c) Derive the coefficient of coincidence for interference between these genes. (1+3+2+2+2)

3. What is a germ plasm or polar plasm? Write a short note on Germ plasm theory. What is the definition and scope of Agricultural / Farming system biodiversity? Write a short note on α-, β- and γ diversity. What are the major causes for loss of biodiversity? How loss of biodiversity can be linked with food and nutrition security of the country? (1+1+2+2+2+2)

4. Define the role of plant nutrition in plant growth and development. What are the major processes important to plant nutrition? Write short notes on the functions of six macro- and six micro-nutrients in plant nutrition. (2+2+3+3)

5. Write short notes with examples on the following- (a) determinants for plant breeding, (b) different mode of reproduction in plants, (c) sexual groups for plant breeding, (d) natural selection and (e) artificial selection. Write a comparative analysis on the major characteristics and breeding methods of self- and cross-pollinated crops.
(5+2+3)

6. Outline with texts and sketches, the procedure for genetic engineering in plant. What is *Agrobacterium tumifaciens*? How does it transfer foreign DNA into plant? Give examples of the three model organisms used for genetic engineering studies worldwide. What are the approximate sizes of their genomes? (3+1+1+3+2)

7. What are the three major classes of signaling events during the development of animal zygote? What is a morphogen gradient? Define properties of stem cell with examples. What is a half mouse embryo? What are the major stages of early development in fruit fly, *Drosophila melanogaster*? (2+1+2+1+4)

Indian Statistical Institute

Semester Examination

Course - B Stat II Year (2011)

Economics II (Macroeconomics)

Date - 11.5.11

Time:2.5 Hours

Answer any three questions from the following

Full Marks : 60

1. a) Explain the concept of paradox of thrift in the simple Keynesian model.

   b) Suppose in a simple Keynesian model planned consumption is a proportional function of NDP denoted by Y. Marginal propensity to invest with respect to Y is 0.3. Start with an initial equilibrium situation. Suppose the saving function shifts parallely downward by 3 units and following this, saving in the new equilibrium is found to increase by 9 units. Derive the new consumption function. 10+10=20

2. Consider a simple Keynesian model for an open economy without any government activity. Suppose the income earners in the economy are classified into two groups, namely, Group 1 and Group 2. Group1 earns 800 units of income, while Group 2 earns Y-800. The average consumption propensity of Group1 is 0.8, while the average propensity to import of the same group is 0.4. Average propensity to consume of Group 2 is 0.5, and Group 2 does not use imported goods. It is also given that $I = \bar{I} = 400$, which is met entirely from domestic production. It is also given that $X = \bar{X} = 200$. (a) Find out the equilibrium level of Y. (b) Suppose that there takes place a transfer of 100 units of income from Group 2 to Group 1. At the same time a ceiling is imposed on import at 250 units. Find out the new equilibrium Y. (c) How does the answer to (b) change if the import ceiling is raised to 400 units? 7+7+6=20

3. (a) A farmer in a given year produced wheat and sold it to a miller for Rs.20,000. The farmer used Rs.2000 worth of seeds carried over from the previous year's production. The miller converted it into flour and sold the flour to a baker for Rs.24000. The baker held one third of the flour bought in stock and converted the rest into bread, which he sold in the market for Rs.20000. Find the respective GVAs of the farmer, miller and the baker.

   b) Firms in an economy produced GDP of Rs.35 crore. Out of this total production goods of Rs. 20 crore were purchased by households. Firms on the other hand purchased goods of Rs.10 crore and added them to stock. The remaining part of GDP of Rs.5 crore could not be sold. What is the gross investment of firms? Derive the GDP of the economy by expenditure method.

c) A retired person lives in his own house and keeps his savings in post office savings bank account. Does he contribute anything to the GDP of the economy? 9+6+5=20

4. a) What is high powered money? In what different forms is high powered money held in the economy? Describe the process of generation of high powered money following the central bank giving a loan to the government. b) Consider a system where there is no currency and CRR is 25 per cent. Banks are fully loaned up. What is the change in money supply following an increase in high powered money by 100 units, when excess demand for credit in the system at the time the increase in high powered money took place was 290 units. c) How does your answer to b) change, if the excess demand for credit were 500 units instead? 10+6+4=20

Indian Statistical Institute

Second Semester Examination : 2010-11

B.Stat 2nd year

Physics II

11.5.11.

Date : Maximum Marks 60 Duration 3 hrs

Answer each group in separate sheets

Group A

Answer any THREE questions. All questions carry 10 (TEN) marks.

(1) a) Consider a system in equilibrium and in thermal contact only with a heat reservoir. Define the concept of temperature and derive the canonical distribution law (that is the probability of finding the system in a particular microstate r).

b) Compute the Partition Function for a classical ideal gas and find its equation of state.

(4+6=10)

(2) For the Maxwell velocity distribution,

$$f(\vec{v})d^3\vec{r}d^3\vec{v} = n(\frac{m}{2\pi kT})^{\frac{3}{2}}e^{-\frac{m\vec{v}^2}{2kT}}d^3\vec{r}d^3\vec{v},$$

where the symbols have their usual meanings, calculate

a) root mean square velocity,

b) most probable velocity.

(5+5=10)

(3) Consider a substance having N_0 non-interacting spin-1/2 atoms per unit volume. The system is placed in a constant magnetic field $\vec{H}$ and is in thermal equilibrium at

temperature T.

a) Find the mean magnetic moment of the system per unit volume in the direction of $\vec{H}$.

b) Sketch the mean magnetic moment vs. magnetic field graph and discuss the features briefly.

(7+3=10)

(4) For a canonical distribution at temperature T,

a) Find the expression for energy dispersion $\overline{(E-\bar{E})^2}$ in terms of the Partition Function.

b) Find the expression for average pressure.

c) Consider an ideal gas at temperature T in presence of gravity. The gas particles have mass m. Calculate the probability of finding a paricle at a height h above the ground.

(3+2+5=10)

(5) a) Find the specific heat of a solid, consisting of N_a (Avogadro number) atoms, from classical considerations. The solid atoms are allowed to vibrate about their equilibrium positions.

b) What was the problem with this theoretical result?

c) How did the Einstein model of specific heat for solid solve this problem?

(2+1+7=10)

(6) a) Find the possible states for a system of three particles to be distributed in two states obeying the following statistics:

i) Bose-Einstein (indistinguishable particles),

ii) Fermi-Dirac (indistinguishable particles),

iii) Maxwell-Boltzman (distinguishable particles).

b) In each case find the probability of finding all three particles in a single state.

c) Draw a schematic graph of energy level vs. occupation number for $T = 0, T \to \infty$ and some intermediate T for a fermion system and discuss briefly the features.

(3+3+4=10)

Group B

Answer any three questions

1.(a) The de Broglie wavelength of an electron is $0.15A$. Compute the phase and group velocities of the de Broglie wave. Find also the kinetic energy of the electron. The rest mass of electron is $0.511\ MeV/c^2$. **(2+2)**

(b) In a Compton scattering experiment a photon is scattered from an electron at rest. The Compton wavelength shift is observed to be three times the wavelength of the incident photon and the photon scatters at angle 60^o. Find the (1) wavelength of the incident photon, (2) energy of the recoiling electron and (3) the angle at which the electron scatters. **(2+2+2)**

2.(a) Rydberg constant for hydrogen is $1.09678 \times 10^7 m^{-1}$ and for ionized helium is $1.09722 \times 10^7 m^{-1}$. It is given that the mass of helium nucleus is four times heavier than the hydrogen nucleus. Calculate the ratio of electron mass to proton mass. **(5)**

(b) For hydrogen, show that when $n >> 1$ the frequency of an emitted photon in a transition from nth level to $(n-1)$th level equals the classical rotational frequency. **(5)**

3.(a) A particle is moving in a potential

$$\begin{aligned} V &= 0, \quad -a/2 < x < a/2 \\ &= \infty, \quad \text{elsewhere} \end{aligned}$$

Find the energy eigenvalues and the corresponding wave functions. **(5)**

(b) Show that the energy of a simple harmonic oscillator can not be lower than $\hbar\omega/2$ without violating the uncertainty principle. **(5)**

4.(a) An electron with kinetic energy 25eV moves from left along the x axis. The potential energy is given by

$$\begin{aligned} V &= 0, \quad x \leq 0 \\ &= 20\, eV, \quad x > 0 \end{aligned}$$

Find the reflection (R) and transmission (T) coefficients. Also find the probability of finding the particle in the region $x > 0$ when the particle moves from left along the x axis with kinetic energy 15eV. **(4+2)**

(b) Explain what is meant by population inversion. **(4)**

5.(a) Identify the decays associated with the reactions

(1) ${}^{12}{}_{5}B \rightarrow {}^{12}{}_{6}C + e^- + \bar{\nu}$,

(2) ${}^{12}{}_{7}N \rightarrow {}^{12}{}_{6}C + e^+ + \nu$

(3) $e^- + {}^{7}{}_{4}Be \rightarrow {}^{7}{}_{3}Li + \nu$

and obtain the Q values for each of these reactions in terms of parent and daughter rest masses. **(2+2+2)**

(b) What is a superconductor? Explain the difference between type 1 and type 2 superconductor. For mercury, the critical temperature at which superconductivity ensues with zero applied magnetic field is $4.15K$. The critical applied magnetic field at which superconductivity will not take place at any temperature is $0.041T$. Find the applied magnetic field that will stop superconductivity at $2.2K$. **(1+1+2)**

6.(a) Write down the expressions for creation and annihilation operators for a linear harmonic oscillator. Using these operators find the normalized ground state and the first excited state wave functions. **(1+1+1+2)**

(b) Explain nuclear fusion and nuclear fission with suitable examples. **(5)**

INDIAN STATISTICAL INSTITUTE

Back Paper Examination : Semester II (2010-2011)

B. Stat 2nd Year

Statistical Methods IV

Date: 01/07/11 Maximum marks: 100 Time: 3 hours.

You may use any result proven in class after stating the results clearly in the proper place
You may use calculators.

1. Let $\mathbf{X}_{n \times p}$ be a data matrix from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and $\Gamma_{n \times n}$ be an orthogonal matrix with the last row being $\frac{1}{\sqrt{n}}$ times the vector with each entry 1. Suppose $\mathbf{Y} = \Gamma \mathbf{X}$. Denote the rows of $\mathbf{Y}$ by $Y_1, Y_2, \ldots, Y_n$. Show that

   (a) $Y_i, i = 1, 2, \ldots, n$ are independent. [3]

   (b) Y_n follows $N_p(\sqrt{n}\boldsymbol{\mu}, \boldsymbol{\Sigma})$. [3]

   (c) $Y_i, i = 1, 2, \ldots, n-1$ are i.i.d. $N_p(0, \boldsymbol{\Sigma})$. [5]

   (d) Express $\bar{X}$ in terms of Y_n only, and S in terms of $Y_i, i = 1, 2, \ldots, n-1$. [4]

   (e) Find the distribution of $\bar{X}$ and S, and show that they are independent. [5]

2. Suppose $\mathbf{X}_{n_1 \times p}$ and $\mathbf{Y}_{n_2 \times p}$ are two data matrices from $N_p(\boldsymbol{\mu}_1, \boldsymbol{\Sigma})$ and $N_p(\boldsymbol{\mu}_2, \boldsymbol{\Sigma})$ respectively. Define the Mahalanobis distance between the two sample means based on the pooled sample variance-covariance matrix and derive its distribution under the condition $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$. [10]

3. Consider a random vector $\mathbf{X}$ which has the $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution where $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ (p.d.) are unknown. The problem is to test the hypothesis that $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ (known) against $\boldsymbol{\mu} \neq \boldsymbol{\mu}_0$ on the basis of a random sample.

   (a) Derive the Likelihood ratio test (LRT) statistic for this test. [10]

   (b) Derive the Union-Intersection test statistic for the same test, and compare it with the LRT statistic found in part (a). [15]

4. Let $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$ be such that $Y_i, i = 1, \ldots, n$ are independent $N(0,1)$ variables. Show that $\mathbf{Y}'A\mathbf{Y}$ will have Chi-square distribution if and only if A is idempotent. [15]

5. Let $X_1, \ldots, X_n$ be i.i.d. observations from a density f. Find the density function of the sample range. Derive this density function exactly when f is exponential with mean (λ). [10+5]

[P. T. O.]

6. A researcher considered three indices measuring severity of heart attacks. The values of these indices for 40 heart-attack patients arriving at a hospital emergency room produced the following sample covariance matrix

$$\mathbf{S} = \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ 63 & 80.2 & 55.6 \\ 71.0 & 55.6 & 97.4 \end{bmatrix}$$

Make an appropriate statistical test for testing whether the population partial correlation coefficient between the first and the second indices after adjusting for the third is 0. [15]

INDIAN STATISTICAL INSTITUTE

B. Stat. (Hons.) II: 2010 – 2011
Second Semestral Back Paper Examination
Economic Statistics and Official Statistics

Duration: 3 hours   Maximum marks: 100   Date: 01.07.11

[Answer all questions. Allotted marks are given in brackets [] at the end of each question.]

1. Write short notes on any **four** of the following:

   (a) Millennium Development Goals
   (b) Functions and composition of NSSO
   (c) Index of Industrial Production.
   (d) Law of Proportionate Effect.
   (e) Measures of poverty.
   (f) Fixed base indices vs. Chain base indices. [10+10+10+10=40]

2. Write down the desirable properties of a measure of inequality. Give examples of some positive measures of inequality. Derive Atkinson's measure of inequality based on Social Welfare Function Approach. Do you think it is a superior measure to Positive Measures of inequality? Give reasons for your answer. [5+5+7+3=20]

3. Is it true that the demand for a commodity will always increase if the mean income increases and the inequality of income decreases for a given group of people? Explain your answer assuming a suitable form of the Engel curve and a specific income distribution. [20]

4. Define Index Number. Why is the choice of base period and commodities so important while constructing an Index Number? How are the data collected and combined for estimating a price index? Write down Fisher's tests for an index number formula. Show that Fisher's Ideal Index Number formula satisfies these tests. [1+5+8+2+4=20]