*Sankhyā : The Indian Journal of Statistics*
2013, Volume 75-B, Part 1, pp. 112-135


# Bayesian multiattribute sampling inspection plans for continuous prior distribution

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**Abstract**

This paper is concerned about the economic design of multiattribute sampling scheme for some industry situations encountered during the author's consultancy experience where the inspection is nondestructive, and we take a single sample of size $n$ and inspect for all attributes. Three cases of multiattribute situations are considered: 1) occurrence of defects are mutually exclusive i.e. an item becomes defective due to any one type of defect 2) defect occurrences are independent and 3) defect occurrence are outcome of a Poisson process. It has been shown following an earlier paper that for linear cost functions the expected cost at a given process average can be expressed by the same equation in all the three cases under certain assumptions. In the second part we have considered only the situation where process average is jointly independently distributed. We have used Gamma prior distribution for each one of them, verified this assumption with live data and arrived at specific cost model. We have further demonstrated with examples from real life that it should be possible to locate an optimal MASSP for different alternative acceptance criteria under this setup.

*AMS* (2000) *subject classification.* Primary 62N10; Secondary 62P30.
*Keywords and phrases.* Poisson conditions, beta-binomial distribution, Gamma-Poisson distribution.

## 1 Introduction

Dodge (1950) remarked that "A product with a history of consistently good quality requires less inspection than one with no history or a history of erratic quality." Selection of a plan, therefore should depend upon the purpose, the quality history and the extent of knowledge of the process. This kind of approach resulted in formulation of what is known as Bayesian sampling plans. In this approach we attempt to obtain sampling plans minimizing overall average costs (consisting of inspection, acceptance and rejection cost) with respect to a given prior distribution of the process average.

Dodge–Romig's model may be considered as a Bayesian plan, where cost function is given by the Average Total Inspection (ATI) and the prior distribution is a one point prior with outliers. Hamaker (1951) and Anscombe (1951) are the early contributors to such economic theory of sampling inspection. The linear cost model is most common and there are a number of different ways to formulate it. The most detailed one was proposed by Guthrie and Johns (1959) and further simplified by Hald (1960). Hald has been the major contributor in the field of economic design of acceptance sampling plans. He obtained general solutions for linear cost models under discrete and continuous prior distribution of process average for single attribute. Hald's major emphasis has been on finding asymptotic relationships between $n$ and $c$, and between $N$ and $n$ for a single attribute single sampling plan. Subsequently, there have been considerable amount of published work. See Wetherill and Chiu (1975) for a review.

We note at the outset that the number of elements of the set of characteristics to be verified is unlikely to be just one in most of the practical situations. We may call this as multiattribute inspection as employed for verification of materials procured from outside and further at all stages of production, through semi finished and finished or assembly stages to final despatch to the customers. At all such stages consecutive collections of products called lots, are submitted for acceptance or alternative disposition. We address the problem of design of acceptance sampling plans in such situations. The scope of the present exercise, however, is limited to only attribute type verification of a recognizable collection of countable discrete pieces, called a lot, submitted for acceptance on a more or less continuous basis.

This paper is concerned about the economic design of multiattribute sampling scheme for nondestructive testing for some practical situations encountered during the author's consultancy experience. The primary focus of the present enquiry is to illustrate the effect of different alternative acceptance criteria and to locate an optimal plan in a given set up.

We make use of the general cost models developed for a given process average vector by Majumdar (1980, 1990, 1997) in three cases of multiattribute situations where 1) occurrence of defects are mutually exclusive i.e. an item becomes defective due to any one type of defect 2) defect occurrences are independent and 3) defect occurrence are outcome of a Poisson process.

Based on these considerations we have worked out necessary results for obtaining Bayesian single sampling multiattribute plans for continuous prior distribution of process average under assumption of independence. The assumption of independent prior is found to be corroborated by practical situations as illustrated in this discussion.

## 2 Bayesian cost models in multiattribute situations

The Bayesian multiattribute sampling scheme was considered by Schmidt and Bennett (1972), and further by Case, Schmidt and Bennett (1975), Ailor, Schmidt and Bennet (1975), Majumdar (1980), Moskowitz et al. (1984), Tang, Plante and Moskowitz (1986), and Majumdar (1997). We discuss some of them and examine their relevance in the context of our present enquiries.

i) Case et al. (1975)

This model is developed under following assumptions:

(a) Each attribute is assumed to have its own sample size $n_i$ and an acceptance number $c_i$ for $i = 1, 2, \ldots, r$; $r$ being the number of attributes.

(b) Any item inspected on one attribute may be inspected on all other attributes thus resulting in the total member of items sampled being maximum of $(n_1, n_2, \ldots, n_r)$. Acceptance is realized if and only if number of defects/defectives of ith characteristic observed as $x_i \leq c_i$; $i = 1, 2, \ldots, r$.

(c) The number of items inspected for the $i$th attribute is without exception $n_i$. No screening/sorting is made on the rejected lots.

(d) Irrespective of the lot size, a rejected lot is ‘scrapped’ at a fixed cost.

(e) The sampled items are replaced in the lot by additional items and are taken from a lot of the same overall quality as the sampled lots.

(f) The number of defectives/defects $X_1, X_2, \ldots, X_r$ in the lot are independently distributed.

(g) The number of defectives/defects $x_1, x_2, \ldots, x_r$ in the sample are independently distributed.

Comparison is made (on specific cases) between the optimal plans constructed under (i) assumption of approximate continuous distribution of lot quality and (ii) assumption of discrete prior distribution of lot quality consistent with the resulting continuous distribution of process average. All computations are made using a search algorithm.

ii) Tang et al. (1986)

The basic tenets of this model are:

(a) There are two classes of attributes viz. scrappable and screenable.

(b) For the rejection due to scrappable defects, the cost of rejection is proportional to the lot sizes irrespective of number of defects present in the remainder of the lot.

(c) For rejection due to screenable defects, the cost of rejection is proportional to the number of items screened (inspected) and no fixed cost is incurred.

(d) Whenever a lot is rejected due to a scrappable defect, the remainder of the lot is not at all inspected for any other attribute(s), scrappable or screenable.

(e) In the event the lot is not rejected on scrappable attribute(s) but on screenable attributes(s), the lot is not tested for scrappable attribute(s) at all.

(f) Effectively, the number of defects observed on any member of the screenable set of attributes does not affect the decision (or the cost) in case the number of defects on any member of the scrappable set violates the acceptance criteria. Only when all the members of the scrappable set satisfy the acceptance criteria, the observations on any member of screenable set may affect the decision.

(g) The random variables $X_1, X_2, \ldots, X_r$ are independently distributed.

(h) The random variables $x_1, x_2, \ldots, x_r$ are independently distributed.

(i) Sampling plans for screenable attributes can be obtained by solving a set of independent single attributes models.

A heuristic solution procedure is developed to obtain near optimal multiattribute acceptance sampling plans.

*2.1. Some practical considerations.*

(i) In today's industrial scenario we notice that screening is becoming more and more feasible due to rapid growth in computerized testing and inspection system. In many fields today, the sampling inspection is relevant only for deciding whether to accept or to screen. The basis for differentiating the attributes in these situations depends on their contribution to cost components. In any ease even for a scrappable attribute, one may not be willing to reject the whole lot at a fixed cost, as assumed by Case et al. (1975) or at a cost proportional to the lot size as assumed by Tang et al. (1986), more so, in the situation of nondestructive testing.

(ii) Moreover, the assumption that a lot rejected for a single scrappable attribute is not screened at all for other attributes (scrappable or screenable) may not hold in many situations. For example, a lot rejected for a scrappable attribute like undersize diameter may be screened for a defect like oversize diameter for which one can rework the item. However, this is not to say that the assumptions made in the models described in the earlier section do not hold in some situations.

(iii) There are many practical situations where testing is done on all attributes for all the pieces in a sample. Examples can be cited for finished garment checking, visual inspection of plastic containers, regulatory testing for packaged commodities like biscuits, nondestructive testing for foundry and forged items etc. For testing of components of assembled units, the general practice is to test all components for all the sampled items. Similar practices are considered as practical for screening/sorting the rejected lot. In these cases $n_i = n; i = 1, 2, \ldots, r$. Moreover, in some situations, if we use different sample sizes for different attributes, we may save only testing cost and not on sampling cost, as we may have to draw a sample of size $n$ as the maximum of all $n_i; i = 1, 2, \ldots, r$ to enable testing for all attributes.

(iv) The assumption that a lot rejected for a single scrappable attribute is not screened at all for other attributes (scrappable or screenable) may not hold in many situations.

(v) The defect occurrences in the lot and sample are considered as jointly independent for each attribute. There are many situations where defect occurrences in the lot/sample may be mutually exclusive. This may happen due to the very nature of defect occurrences (e.g. a shirt with a button missing or a shirt with a wrong button), or when defects are classified in mutually exclusive classes e.g. critical, major or minor. It would therefore be necessary to take care of both the situations. Our cost models along with some elementary results as presented in the following section follows Majumdar (1997).

*2.2. A generalized cost model for nondestructive testing.* In our case we take a sample of size $n$ and inspect each item for all the attributes. We observe $x_i; i = 1, 2, \ldots, r$ as the number of defectives on the $i$th attribute. We denote the vector $(x_1, x_2, \ldots, x_r)$ as $\boldsymbol{x}$. We define set A as the set of $\boldsymbol{x}$ for which we declare the lot as acceptable. Let $\overline{A}$ be the complementary set for which we reject the lot. Let $X_i$ denote the number of defectives on $i$th characteristic in the lot, $i = 1, 2, \ldots, r$.

Let the costs be

$$C(\boldsymbol{x}) = nS_0 + \sum_{i=1}^{r} x_i S_i + (N-n)A_0 + \sum_{i=1}^{r}(X_i - x_i)A_i$$

when $\boldsymbol{x} \in A$

$$C(\boldsymbol{x}) = nS_0 + \sum_{i=1}^{r} x_i S_i + (N-n)R_0 + \sum_{i=1}^{r}(X_i - x_i)R_i$$

when $\boldsymbol{x} \in \overline{A}$

The interpretations of cost parameters are as follows: $S_0$ is the common cost of inspection i.e. sampling and testing cost per item in the sample for all the characteristics put together; $x_i S_i$ the cost proportional to the number of defectives of $i$th type in the sample which is the additional cost for an inspected item containing defects of $i$th type.

The cost of acceptance is composed of two parts; $(N-n)A_0$ is cost proportional to the items in the remainder of the lot, and another part $\sum_{i=1}^{r}(X_i - x_i)A_i$ where $A_i$ is the cost of accepting an item containing defective for $i$th attribute. We assume that the loss due to use of defective item is additive over all the characteristics. This means if an item contains more than one defect say for $i = 1$ and 2, the loss will be the sum of the damages for both the characteristics put together. The assumption is reasonably valid under many situations. However, proportion of items containing more than one category of defects will usually be small.

Costs of rejection consists of a part $(N-n)R_0$ proportional to the number of items in the remainder of the lot and another part, $\sum_{i=1}^{r}(X_i - x_i)R_i$

proportional to the number of defective items rejected for all the attributes put together. If rejection means sorting, $R_0$ will give the sorting cost/item for all category of defects put together. The $R_i$ denotes the additional cost for items found with defective of $i$th category (for example, cost of repair) and is additive over different category of defects.

Note that there is only one sample of size $n$ to be taken for inspection for all the characteristics and the cost model given here is only a multiattribute analog of the cost model considered by Hald (1965) in the case of a single attribute.

*2.3. Average costs at a given process average.*

*2.3.1. When defect occurrences are independent.* In this case the probability of getting $x_i$ defectives in a sample of size $n$ from a lot of size $N$ containing $X_i$ defectives, for the $i$th attribute, $i = 1, 2, \ldots, r$, is given by hypergeometric probability as

$$Pr(x_i \mid X_i) = \binom{n}{x_i}\binom{N-n}{X_i - x_i} \Big/ \binom{N}{X_i} \tag{2.1}$$

$X_i$ being the number of defectives of type i in a lot $i = 1, 2, \ldots, r$.

Further,

$$P_r(X_1, X_2, \ldots, X_r) = \prod_{i=1}^{r} P_r(X_i).$$

If the lot quality $X_i$; $i = 1, 2, \ldots, r$ is distributed as binomial, then

$$Pr(X_i) = \binom{N}{X_i} p_i^{X_i}(1-p_i)^{N-X_i}. \tag{2.2}$$

The average cost for lot of size $N$ with $(X_i, \ldots, X_r)$ defects become

$$\sum_{\boldsymbol{x}\in A} C(\boldsymbol{x}) \prod_{i=1}^{r} Pr(x_i|X_i) + \sum_{\boldsymbol{x}\in \overline{A}} C(\boldsymbol{x}) \prod_{i=1}^{r} Pr(x_i|X_i).$$

From (2.1) and (2.2), it can be easily shown that the average cost per lot at a process average $\boldsymbol{p} = (p_1, p_2, \ldots, p_r)$ for the lots of size $N$ is:

$$\begin{aligned} K(N, n, \boldsymbol{p}) = n\left(S_0 + \sum_i S_i p_i\right) &+ (N-n) \\ &\times \left[\left(A_0 + \sum_i A_i p_i\right) P(\boldsymbol{p}) + \left(R_0 + \sum_i R_i p_i\right) Q(\boldsymbol{p})\right], \end{aligned} \tag{2.3}$$

where P($\boldsymbol{p}$) denotes the average probability of acceptance at $\boldsymbol{p}$.

$$P(\boldsymbol{p}) = \sum_{\boldsymbol{x}\in A} \prod_{i=1}^{r} \binom{n}{x_i} {p_i}^{x_i}(1-p_i)^{n-x_i},$$

and $Q(\boldsymbol{p}) = 1 - P(\boldsymbol{p})$.

*2.3.2. When the defect occurrences are mutually exclusive.* In this situation the expression for the probability of observing $(x_1, x_2, \ldots, x_r)$ defective in a sample of size $n$ from a lot of size $N$, containing $(X_1, X_2, \ldots, X_r)$ defectives of types $i = 1, 2, \ldots, r$, will be multivariate hypergeometric as

$$\begin{aligned} &Pr(x_1, x_2, \ldots, x_r \mid X_1, \ldots, X_r) \\ &\quad = \binom{X_1}{x_1}\binom{X_2}{x_2}\cdots\binom{X_r}{x_r}\binom{N - X_1 - X_2 \cdots - X_r}{n - x_1 - x_2 \cdots - x_r} \Big/ \binom{N}{n}. \end{aligned} \tag{2.4}$$

At any process average the joint probability distribution of $(X_1, X_2, \ldots, X_r)$ can be assumed to be multinomial such that

$$\begin{aligned} &P_r(X_1, X_2, \ldots, X_r \mid p_1, p_2, \ldots, p_r) \\ &\quad = \binom{N}{X_1}\binom{N - X_{(1)}}{X_2}\cdots\binom{N - X_{(r-1)}}{X_r} p_1^{X_1}\cdots p_r^{X_r}\left(1 - p_{(r)}\right)^{(N - X_{(r)})}. \end{aligned} \tag{2.5}$$

$X_{(i)} = X_1 + X_2 + \cdots + X_i$ and $p_{(i)} = p_1 + p_2 + \cdots + p_i$; $i = 1, 2, \ldots, r$.

From (2.4) and (2.5) it follows that average cost at $\boldsymbol{p}$ can be expressed as (2.3) replacing P($\boldsymbol{p}$) by

$$\begin{aligned} P(\boldsymbol{p}) = \sum_{x_1, x_2, \ldots, x_r \in A} &\binom{n}{x_1}\binom{n - x_{(1)}}{x_2}\cdots\binom{n - x_{(r-1)}}{x_r} \\ &\times p_1^{x_1}\cdots p_r^{x_r}\left(1 - p_{(r)}\right)^{(n - x_{(r)})}. \end{aligned} \tag{2.6}$$

$x_{(i)} = x_1 + x_2 + \cdots + x_i$ for $i = 1, 2, \ldots, r$.

*2.3.3. Approximation under Poisson conditions.* We use the phrase "Poisson conditions" when Poisson probability can be used in the expressions of type B OC function in the following two situations.

(i) Poisson as approximation to binomial and multinomial

If $p_i \to 0$, $n \to \infty$, and $np_i \to m_i$ then the binomial probability $b(x_i, n, p_i)$ tends to Poisson probability $g(x_i, m_i)$ where

$$g(x_i, m_i) = e^{-m_i}(m_i)^{x_i}/(x_i)!, m_i = np_i, i = 1, 2, \ldots, r.$$

Under this condition, the expression $P(\boldsymbol{p})$ given in equation (2.3) can be modified as:

$$P(\boldsymbol{p}) = \sum_{x_1,x_2,\dots,x_r \in A} Pr(x_1, x_2, \dots, x_r \mid p_1, p_2, \dots, p_r) = \prod_{i=1}^{r} g(x_i, m_i). \tag{2.7}$$

If we also make an additional assumption that $\sum_{i=1}^{r} p_i \to 0$ then the expression $P(\boldsymbol{p})$ given in equation (2.6) can also be modified as (2.7).

(ii) Poisson as an exact distribution and the occurrences of defect types independent

We assume the number of defects for $r$ distinct characteristics in a unit are independently distributed. The output of such a process is called a product of quality $(p_1, p_2, \dots, p_r)$, the parameter vector representing the mean occurrence rates of defects per observational unit. The total number of defects for any characteristic in a lot of size $N$ drawn from such a process will vary at random according to a Poisson law with parameter $Np_i$ for the $i$th characteristics under usual circumstances. Similarly, the distribution of number of defects on attribute $i$, in a random sample of size $n$ drawn from a typical lot will be a Poisson variable with parameter $np_i$. Independence of the different characteristics will be naturally maintained in the sample so that the type B probability of acceptance will be given by the expression of $P(\boldsymbol{p})$ as in equation (2.7). In this situation we will be dealing with defects rather than defectives.

*2.3.4. Generalized cost model.* Thus under Poisson condition described as above, the expression of $K(N, n, \boldsymbol{p})$ is same as (2.3) with

$$P(\boldsymbol{p}) = \sum_{\boldsymbol{x} \in A} \prod_{i=1}^{r} g(x_i, np_i)$$

$$\text{and } Q(\boldsymbol{p}) = 1 - P(p).$$

We have therefore arrived at a model as applicable to a multiattribute situation discussed in the earlier section for $r > 1$. This may be considered as a generalization of the cost model of Hald (1965) for the single attribute situation i.e. for $r = 1$. For discrete prior distribution, Majumdar (1997) had derived the expression of regret function using the above model.

## 3 Bayesian single sampling multiattribute plans for continuous prior distribution of process average under assumption of independence

We obtain the expression for the average cost when the process averages follow independent continuous distributions. In particular, we consider the case when the process average for each attribute can be assumed to follow a gamma distribution. Further, we demonstrate how this expression can be used to compare the expected costs for different acceptance criteria.

*3.1. Cost model for continuous prior.* We have noted that the average costs at $\boldsymbol{p}$:

$$K(N,n,\boldsymbol{p}) = n\left(S_0 + \sum_{i=1}^{r} S_i p_i\right) + (N-n) \times \left[\left(R_0 + \sum_{i=1}^{r} R_i p_i\right) + (A_1 - R_1)\left(\sum_{i=1}^{r} d_i p_i - d_0\right) P(\boldsymbol{p})\right]$$

where, $d_0 = (R_0 - A_0)/(A_1 - R_1)$; $d_i = (A_i - R_i)/(A_1 - R_1)$ for $i = 1, 2, \ldots, r$.

Let $p_i$ be distributed from lot to lot according to the prior distribution $w_i(p_i), i = 1, 2, \ldots, r$ and the $p_i$'s are jointly independent. Then,

$$K(N,n) = nk_s + (N-n)k_r + (N-n)(A_1 - R_1)\int_{p_1}\int_{p_2}\cdots\int_{p_r} \times \left[\left(\sum_{i=1}^{r} d_i p_i - d_0\right) P(\boldsymbol{p}) dw_1(p_1) dw_2(p_2)\cdots dw_r(p_r)\right]$$

where $k_s$ is the average cost of sampling over the prior, i.e.

$$k_s = \int_{p_1}\int_{p_2}\cdots\int_{p_r}\left[\left(S_0 + \sum_{i=1}^{r} S_i p_i\right) dw_1(p_1) dw_2(p_2)\cdots dw_r(p_r)\right]$$

and

$$k_r = \int_{p_1}\int_{p_2}\cdots\int_{p_r}\left[\left(R_0 + \sum_{i=1}^{r} R_i p_i\right) dw_1(p_1) dw_2(p_2)\cdots dw_r(p_r)\right]$$

*3.2. Distribution of process average.*

*3.2.1. General considerations.* The most widely used continuous prior distribution for the process average quality $p_i$ is the beta distribution,

$$\beta(p_i, s_i, t_i) = {p_i}^{s_i-1}(1-p_i)^{t_i-1}/\beta(s_i, t_i), \quad s_i > 0, \quad t_i > 0, i = 1, 2, \ldots, r, \tag{3.1}$$

and $0 < p_i < 1$ where $\beta(s_i, t_i) = \Gamma(s_i)\Gamma(t_i)/\Gamma(s_i + t_i)$.

The mean equals to $E(p_i) = \overline{p_i} = s_i/(s_i + t_i)$ and the variance is $V(p_i) = \overline{p_i}(1 - \overline{p_i})/(s_i + t_i + 1) = (\overline{p_i})^2(1 - \overline{p_i})/(s_i + \overline{p_i})$. We, therefore, can also use the parameters $(\overline{p_i}, s_i)$, instead of the parameters $(s_i, t_i)$.

Corresponding to a beta distribution (3.1) of the single attribute process average of quality $p_i$, the distribution of the lot quality denoted by $X_i$ becomes a beta-binomial distribution:

$$b(X_i, N, \overline{p_i}, s_i) = \binom{N}{X_i}\beta(s_i + X_i, t_i + N - X_i)/\beta(s_i, t_i),$$

$X_i$ nonnegative integer.

It follows that the distribution of the number of defectives $x_i$ in the sample is $b(x_i, n, \overline{p_i}, s_i)$.

Examples of fitting beta-binomial to the observed quality distribution have been given in Hopkins (1955), Hald (1960) and Smith (1965). Chiu (1974) has however demonstrated that beta distribution is not always an adequate substitute for any reasonable prior distribution. He has used the data reported by Barnard (1954) on 226 batches of sizes 24,000–135,000 to substantiate his claim.

For the situation when the process average follows beta-binomial with parameters $\overline{p_i}, s_i, t_i$, we may use the gamma distribution with parameter $\overline{p_i}, s_i$ as an approximation. The gamma distribution in the present context is defined as:

$$f(p_i, \overline{p_i}, s_i)dp_i = e^{-v_i}(v_i)^{s_i-1}dv_i/\Gamma(s_i); \quad v_i = s_i p_i/\overline{p_i}, \quad p_i > 0$$

with mean $E(p_i) = \overline{p_i}$ and the shape parameter, $s_i$. The variance is $V(p_i) = (\overline{p_i}^2)/s_i$.

As pointed out by Hald (1981), this gamma distribution gives fairly accurate approximation to the beta distribution with same $s_i$ when both $\overline{p_i}$ and $\overline{p_i}/s_i$ are small; more precisely, if $\overline{p_i} < 0.1$ and $\overline{p_i}/s_i < 0.2$. Hald (1981) used the gamma distribution to tabulate the optimal single sampling plans. Most of his results are based on assuming gamma as the right prior distribution.

Moreover, corresponding to a beta distribution of the single attribute process average of quality $p_i$, the distribution of the lot quality denoted by

$X_i$ as well as sample quality $x_i$ become a beta-binomial distribution which can similarly be approximated as a gamma-Poisson distribution.

We note that for a single attribute if the process is stable at a given $p_i$ and we count the number of defects per item with reference to the characteristic, we may construct a model for which the number of defects for each unit for the characteristic equals to $p_i$ in the long run. In case of $r$ such characteristics, we assume in addition that the number of defects with reference to different characteristics observed in a unit are jointly independent. The outputs of such a process of $r$ characteristics are called product of quality $(p_1, p_2, \ldots, p_r)$. The vector $(p_1, p_2, \ldots, p_r)$ is also the mean occurrence rate (of defects) vector per observational unit. Dividing the outputs of the process successively into inspection lots of size $N$ each, the quality of the lot expressed by total number of defects for $i$th characteristic will vary at random according to the Poisson law with parameter $Np_i$ $i = 1, 2, \ldots, r.$ and the distribution of defects for the $i$th characteristics defects in a lot of size $N$ drawn from this process will be similarly a Poisson distribution with parameter $Np_i$, $i = 1, 2, \ldots, r$. In this situation if the process average for the $i$th characteristic is distributed as a gamma distribution for the $i$th attribute, the distribution of the lot quality becomes a gamma-Poisson distribution with reference to the $i$th attribute.

The distribution of lot quality $X_i$ for the $i$th characteristic which holds, either approximately or exactly, as the case may be in these two situations (as explained above) can be expressed as:

$$g(X_i, N\overline{p_i}, s_i) = \frac{\Gamma(s_i + X_i)}{X_i!\Gamma(s_i)}\theta_i^{s_i}.(1-\theta_i)^{X_i}; \quad \theta_i = \frac{s_i}{(s_i + N\overline{p_i})}$$

and $X_i$ nonnegative integer.

[Note that we use $g(x, \theta)$ to denote the Poisson distribution term and $g(x, \theta_1, \theta_2, )$ to denote the gamma-Poisson term.]

It follows that the distribution of the number of defectives or defects $x_i$ in the sample follows gamma-Poisson law with probability mass function given by $g(x_i, n\overline{p_i}, s_i)$. Further, there is stochastic independence of $x_1, x_2, \ldots, x_r$. In the sections which follow, we use gamma prior distribution which work either approximately accurately or as exact distributions under different situations as explained.

*3.3. Verification for the appropriate prior distribution.* Table 1 presents the inspection data for 86 lots containing about 25,500 pieces of filled vials of an eye drop produced by an established pharmaceutical company based at Kolkata. Each vial is inspected for six attributes. From the criticality point of view, however, the defects can be grouped in two categories. The first category of defects is due to the presence of foreign matters viz. glass, fiber or impurities, which are critical from the user's point of view. The other category of defects consists of breakage, defective sealing and leakage, reasonably obviously detectable and can be easily discarded by the user.

Since the lot size (25,500) is quite large, the observed lot quality variation approximates almost exactly the distribution of process average. We have, therefore, instead of a gamma-Poisson distribution, fitted a gamma distribution with mean and the variance estimated from the observed data for both types of defects. The results are presented in Tables 2 and 3.

Note that for both the attributes the $\overline{p_i}$'s are less than 0.1 ($\overline{p_1} = 0.01708$; $\overline{p_2} = 0.02302$). The estimated $s_i = \overline{p_i}^2/Var(p_i)$ are 3.4532 and 0.6229 respectively so that $\overline{p_i}/s_i$ are much less than 0.2 justifying the use of gamma distribution as a substitute of beta distribution.

The tables also present the usual $\chi^2$ goodness of fit analysis. It can be seen that the computed $\chi^2$ as a measure of the goodness of fit values are small (they are not statistically significant) enough for both types of attributes to justify the assumption that the prior distributions follow the assumed theoretical gamma distributions.

Further, the scatter plot (see Figure 1) of the observed numbers of defects of the second category against those of the first category exhibits no specific pattern. It would be therefore reasonable to assume that the $p_i$'s are independently distributed in the present context.

It should be pointed out at this stage that the beta and gamma distribution are, however, not always appropriate. For example, we could not fit the gamma or beta distribution in case of quality variation of ceiling fans, garments and cigarettes for which the relevant data were collected. In such cases we will have to take recourse to direct computation of the cost function derived from the empirical distributions as observed. Numerically, the difficulty level in computation will not increase significantly. Nevertheless, since the gamma (or beta) distribution is likely to be appropriate at least in some situations and neat theoretical expressions can be obtained in such cases, we will study the cost functions under such assumptions.

*3.4. Expression of average costs under the assumption of independent gamma prior distributions of the process average vector.*

THEOREM 3.1. *Let each $p_i$ be distributed with probability density function $f(p_i, \overline{p_i}, s_i)$ for $i = 1, 2, \ldots, r$; $p_i$'s are jointly independent; the lot quality $X_i$ is distributed as $g(X_i, Np_i)$ $\forall i$ and $X_i$'s are jointly independent. The optimal*

Table 1: Results of 100% QC checks on 86 lots each of size around 25,500 pieces of eye drop vials. The table presents the number of different type of defects observed

| Day | Glass | Fiber | Impurity | Breakage | Defective Sealing | Leakage | Total |
|---|---|---|---|---|---|---|---|
| 1 | 94 | 357 | 131 | 18 | 20 | 13 | 633 |
| 2 | 82 | 178 | 141 | 22 | 102 | 191 | 716 |
| 3 | 72 | 400 | 115 | 10 | 3 | 89 | 689 |
| 4 | 25 | 66 | 42 | 16 | 42 | 31 | 222 |
| 5 | 37 | 281 | 58 | 5 | 33 | 168 | 582 |
| 6 | 53 | 226 | 78 | 8 | 61 | 142 | 568 |
| 7 | 40 | 123 | 136 | 4 | 30 | 75 | 408 |
| 8 | 74 | 238 | 98 | 16 | 74 | 802 | 1302 |
| 9 | 48 | 348 | 94 | 9 | 59 | 1207 | 1765 |
| 10 | 77 | 544 | 135 | 12 | 3 | 44 | 815 |
| 11 | 58 | 253 | 68 | 11 | 49 | 302 | 741 |
| 12 | 42 | 120 | 29 | 11 | 45 | 28 | 275 |
| 13 | 60 | 406 | 123 | 18 | 62 | 353 | 1022 |
| 14 | 75 | 477 | 154 | 15 | 59 | 505 | 1285 |
| 15 | 60 | 205 | 54 | 16 | 83 | 297 | 715 |
| 16 | 78 | 471 | 91 | 14 | 65 | 296 | 1015 |
| 17 | 69 | 356 | 92 | 11 | 48 | 1471 | 2047 |
| 18 | 48 | 363 | 76 | 6 | 53 | 43 | 589 |
| 19 | 58 | 204 | 86 | 11 | 35 | 211 | 605 |
| 20 | 46 | 171 | 73 | 9 | 34 | 160 | 493 |
| 21 | 49 | 180 | 79 | 7 | 69 | 96 | 480 |
| 22 | 45 | 154 | 83 | 8 | 31 | 85 | 406 |
| 23 | 41 | 155 | 70 | 7 | 36 | 69 | 378 |
| 24 | 36 | 180 | 56 | 8 | 44 | 36 | 360 |
| 25 | 13 | 60 | 43 | 18 | 43 | 66 | 243 |
| 26 | 71 | 297 | 59 | 11 | 19 | 72 | 529 |
| 27 | 48 | 218 | 50 | 15 | 81 | 111 | 523 |
| 28 | 48 | 193 | 83 | 15 | 102 | 117 | 558 |
| 29 | 53 | 180 | 66 | 8 | 92 | 27 | 426 |
| 30 | 46 | 280 | 79 | 18 | 34 | 55 | 512 |
| 31 | 53 | 57 | 45 | 17 | 47 | 95 | 314 |
| 32 | 35 | 140 | 38 | 19 | 128 | 110 | 470 |
| 33 | 43 | 198 | 57 | 22 | 37 | 117 | 474 |
| 34 | 46 | 103 | 33 | 2 | 43 | 171 | 398 |
| 35 | 64 | 305 | 85 | 21 | 197 | 858 | 1530 |

Table 1: (Continued)

| Day | Glass | Fiber | Impurity | Breakage | Defective Sealing | Leakage | Total |
|---|---|---|---|---|---|---|---|
| 36 | 71 | 162 | 70 | 26 | 95 | 365 | 789 |
| 37 | 52 | 270 | 73 | 18 | 198 | 424 | 1035 |
| 38 | 56 | 473 | 110 | 24 | 464 | 841 | 1968 |
| 39 | 44 | 205 | 219 | 19 | 259 | 1807 | 2553 |
| 40 | 61 | 235 | 37 | 44 | 266 | 2257 | 2900 |
| 41 | 69 | 192 | 38 | 15 | 168 | 2025 | 2507 |
| 42 | 170 | 410 | 113 | 29 | 228 | 3521 | 4471 |
| 43 | 117 | 257 | 55 | 22 | 216 | 2887 | 3554 |
| 44 | 50 | 235 | 44 | 14 | 120 | 1967 | 2430 |
| 45 | 81 | 201 | 42 | 13 | 114 | 1594 | 2045 |
| 46 | 44 | 202 | 43 | 16 | 129 | 1768 | 2202 |
| 47 | 50 | 99 | 35 | 15 | 120 | 1752 | 2071 |
| 48 | 67 | 244 | 57 | 16 | 112 | 1631 | 2127 |
| 49 | 70 | 283 | 57 | 11 | 86 | 927 | 1434 |
| 50 | 49 | 260 | 151 | 16 | 108 | 1403 | 1987 |
| 51 | 81 | 1561 | 177 | 17 | 98 | 784 | 2718 |
| 52 | 49 | 676 | 148 | 13 | 88 | 470 | 1444 |
| 53 | 75 | 441 | 69 | 9 | 33 | 60 | 687 |
| 54 | 106 | 564 | 133 | 11 | 110 | 459 | 1383 |
| 55 | 41 | 526 | 55 | 9 | 65 | 502 | 1198 |
| 56 | 72 | 376 | 82 | 11 | 94 | 252 | 887 |
| 57 | 56 | 350 | 62 | 12 | 79 | 478 | 1037 |
| 58 | 90 | 398 | 80 | 33 | 100 | 896 | 1597 |
| 59 | 70 | 120 | 40 | 12 | 55 | 287 | 584 |
| 60 | 58 | 281 | 48 | 15 | 22 | 40 | 464 |
| 61 | 60 | 230 | 61 | 8 | 11 | 88 | 458 |
| 62 | 67 | 204 | 56 | 9 | 49 | 577 | 962 |
| 63 | 58 | 302 | 37 | 5 | 68 | 103 | 573 |
| 64 | 74 | 665 | 104 | 10 | 12 | 85 | 950 |
| 65 | 54 | 400 | 70 | 10 | 64 | 162 | 760 |
| 66 | 45 | 193 | 56 | 5 | 69 | 150 | 518 |
| 67 | 54 | 131 | 62 | 10 | 58 | 164 | 479 |
| 68 | 93 | 406 | 132 | 11 | 86 | 661 | 1389 |
| 69 | 54 | 243 | 82 | 5 | 64 | 82 | 530 |
| 70 | 54 | 144 | 85 | 11 | 46 | 60 | 400 |
| 71 | 53 | 176 | 60 | 10 | 61 | 63 | 423 |
| 72 | 54 | 316 | 75 | 11 | 46 | 69 | 571 |

Table 1: (Continued)

| Day | Glass | Fiber | Impurity | Breakage | Defective Sealing | Leakage | Total |
|---|---|---|---|---|---|---|---|
| 73 | 72 | 245 | 109 | 11 | 49 | 20 | 506 |
| 74 | 54 | 189 | 60 | 32 | 30 | 58 | 423 |
| 75 | 64 | 256 | 109 | 12 | 31 | 59 | 531 |
| 76 | 79 | 480 | 209 | 16 | 45 | 424 | 1253 |
| 77 | 78 | 170 | 70 | 15 | 44 | 219 | 596 |
| 78 | 87 | 617 | 210 | 16 | 42 | 116 | 1088 |
| 79 | 162 | 594 | 122 | 18 | 64 | 452 | 1412 |
| 80 | 65 | 312 | 83 | 29 | 77 | 254 | 820 |
| 81 | 69 | 163 | 79 | 18 | 46 | 205 | 580 |
| 82 | 59 | 300 | 77 | 12 | 56 | 76 | 580 |
| 83 | 52 | 361 | 100 | 16 | 79 | 244 | 852 |
| 84 | 9 | 107 | 78 | 11 | 42 | 73 | 320 |
| 85 | 66 | 182 | 33 | 9 | 43 | 61 | 394 |
| 86 | 53 | 125 | 56 | 4 | 36 | 139 | 413 |

Table 2: Testing goodness of fit of gamma distribution for the observed critical defects in lots containing about 25,500 eye drop vials each

| Number of defects/ unit ($p_1$) | Observed frequency ($O_i$) | Estimated frequency ($E_i$) | $(O_i - E_i)^2/E_i$ |
|---|---|---|---|
| <=0.008 | 7 | 12.3940 | 2.3475 |
| 0.008–0.01 | 4 | 7.7206 | 1.7929 |
| 0.01–0.012 | 15 | 8.4379 | 5.1034 |
| 0.12–0.14 | 13 | 8.4918 | 2.3933 |
| 0.014–0.016 | 9 | 8.0577 | 0.1102 |
| 0.016–0.018 | 9 | 7.3158 | 0.3877 |
| 0.018–0.02 | 6 | 6.4183 | 0.0273 |
| 0.02–0.022 | 4 | 5.4786 | 0.3990 |
| 0.022–0.024 | 5 | 4.5728 | 0.0399 |
| 0.024–0.026 | 4 | 3.7461 | 0.0172 |
| 0.026–0.03 | 3 | 5.4239 | 1.0833 |
| 0.030–0.034 | 3 | 3.3600 | 0.0386 |
| >0.034 | 4 | 4.5827 | 0.0704 |
| Totals | 86 | 86 | 13.811 |

$\overline{p_1} = 0.01708$, $\mathrm{Var}(p_1) = 8.44821\mathrm{E}\text{-}05$, $s_1 = \overline{p_1}^2/\mathrm{Var}(p_1) = 3.4532$

$\mathrm{Prob}\left[\chi^2_{(10)} > 13.811\right] = 0.182$

Table 3: Testing goodness of fit of gamma distribution for the observed visual defects in lots containing about 25,500 Eye drop vials each

| Number of defects/unit | Observed frequency ($O_i$) | Estimated frequency ($E_i$) | $(O_i - E_i)^2/E_i$ |
|---|---|---|---|
| <=0.005 | 25 | 26.2307 | 0.0577 |
| 0.005–0.0075 | 9 | 6.7010 | 0.7888 |
| 0.0075–0.01 | 9 | 5.5052 | 2.2186 |
| 0.001–0.0125 | 5 | 4.6758 | 0.0225 |
| 0.0125–0.015 | 6 | 4.0497 | 0.9393 |
| 0.0125–0.020 | 4 | 6.6984 | 1.0870 |
| 0.020–0.0225 | 3 | 2.8040 | 0.0137 |
| 0.0225–0.03 | 6 | 6.8164 | 0.0978 |
| 0.03–0.0425 | 5 | 7.7194 | 0.9580 |
| 0.03–0.0675 | 5 | 8.1248 | 1.2018 |
| 0.0675–0.08 | 3 | 2.1343 | 0.3511 |
| >0.08 | 6 | 4.5404 | 0.4692 |
| Totals | 86 | 86 | 8.2054 |

$\overline{p_2} = 0.0230$, $\mathrm{Var}(p_2) = 0.000852$, $s_2 = \overline{p_2}^2/\mathrm{Var}(p_2) = 0.0622$

$\mathrm{Prob}\left[\chi^2_{(9)} > 8.205\right] = 0.513$

*plan in this situation for a specified acceptance criteria* $\boldsymbol{x} = (x_1, x_2, \ldots, x_r) \in A$ *is obtained by minimizing the function:*

$$\begin{aligned} K(N,n)/(A_1 - R_1) = nk'_s + (N-n)k'_r + (N-n) \\ \times \left[\sum_{\boldsymbol{x}\in A}\left\{\sum_{i=1}^{r} d_i\overline{p_i}(s_i + x_i)/(s_i + n\overline{p_i}) - d_0\right\}\right. \\ \left. \times \prod_{i=1}^{r} g(x_i, n\overline{p_i}, s_i)\right] \end{aligned} \quad (3.2)$$

*where* $g(x_i, n\overline{p_i}, s_i)$ *is a gamma-Poisson density given by,*

$$g(x_i, n\overline{p_i}, s_i) = \frac{\Gamma(s_i + x_i)}{(x!)\Gamma(s_i)}\theta_i^{s_i}.(1-\theta_i)^{x_i}; \quad \theta_i = \frac{s_i}{(s_i + n\overline{p_i})}$$

*and* $x_i$ *is a nonnegative integer,* $k'_s = k_s/(A_1 - R_1), k'_r = k_r/(A_1 - R_1)$.

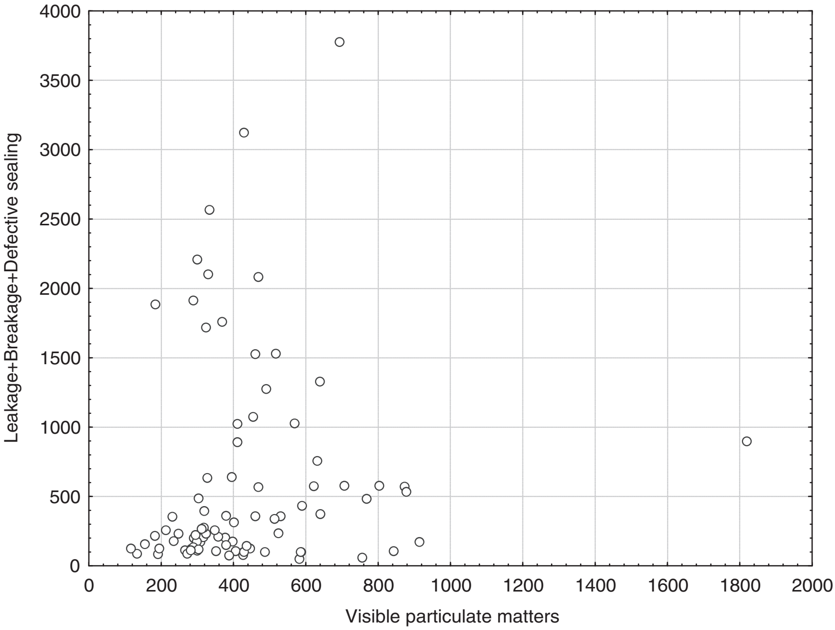


Figure 1: Scatter plots of number of observed defects in 86 lots of eye drop. [X axis: number of occurrences of visible particulate matters. Y axis: Number of occurrences of Leakage, breakage and Defective sealing.] Remark: No pattern of dependence is visible. The two types of defects may be assumed to be independently distributed

Proof. Since,

$$\int_{p_1}\int_{p_2}\cdots\int_{p_r} p_i \prod_{j=1}^{r} g(x_j, np_j) f(p_j, \overline{p_j}, s_j) dp_j$$
$$= \overline{p_i}(s_i + x_i)/(s_i + n\overline{p_i}) \prod_{j=1}^{r} g(x_j, n\overline{p_j}, s_j)$$

we get,

$$\int_{p_1}\int_{p_2}\cdots\int_{p_r} d_i p_i P(\boldsymbol{p}) dw(p_1) dw(p_2)..dw(p_r)$$
$$= \sum_{\boldsymbol{x}\in A} d_i\overline{p_i}(s_i + x_i)/(s_i + n\overline{p_i}) \prod_{i=1}^{r} g(x_i, n\overline{p_i}, s_i).$$

Further,

$$\int_{p_1}\int_{p_2}\cdots\int_{p_r} P(\boldsymbol{p})dw(p_1)dw(p_2)\cdots dw(p_r)$$
$$= \sum_{\boldsymbol{x}\in A}\prod_{i=1}^{r} g(x_i, n\overline{p_i}, s_i).$$

Using the above we get the result.

## 4 Bayesian plans for different acceptance criterion

We consider different acceptance criteria. We may consider the class of plans where we take a sample of size $n$: accept the lot if and only if $x_i \leq c_i; \forall i$. We call these plans as multiattribute single sampling plans (MASSP) of C kind. Note that all MASSP's constructed from MIL-STD-105D (1963) as suggested by the standard are C type plans. Majumdar (1997) introduced a sampling scheme consisting of plans with alternative acceptance criteria: accept if $x_1 \leq a_1; x_1+x_2 \leq a_2; \cdots; x_1+x_2+\cdots+x_r \leq a_r$; reject otherwise. We call this plan an MASSP of A kind. It has been shown that $-PA'_i \geq -PA'_{i+1}$ for $i = 1, 2, \ldots, r-1$ where $PA$ denotes the probability of acceptance for the plan at a given process average and $-PA'_i$ is the absolute value of the slope of the OC function with respect to $m_i$ $[m_i = n * p_i]$ at a given $m = m_1 + m_2 + \cdots + m_r$, assuming $m_i/m$ fixed, $i = 1, 2, \ldots, r$. This property of the A kind MASSP's, therefore, allows us to order the attributes in order of their relative discriminating power. It also follows that if the attributes are ordered in the ascending order of AQL values, then it is possible to construct a sampling scheme ensuring an acceptable producer's risk and also satisfying the condition of higher absolute slope for the lower AQL attribute. Using the set of n.AQL values chosen from MIL-STD-105D, Majumdar (2009) established a MASSP scheme consisting of A kind MASSP's. For the sake of comparison, we also introduce a MASSP of D kind as the one with the following rule: from each lot of size $N$, take a sample of size $n$, accept if total number of defects of all types put together is less than or equal to $k$, otherwise reject the lot. Using the expression (3.2), we may now construct optimal A kind, C kind and D kind plans for a given lot size $N$, cost parameters and the parameters of the prior distributions and compare their relative merit in a given situation.

*4.1. An example.* We demonstrate this with one real life example obtained in respect of plastic containers used for cosmetics. In this case the defects which can be categorized as major type are defects like color variation, improper neck finishing, prominent marks, weak body. The cost of

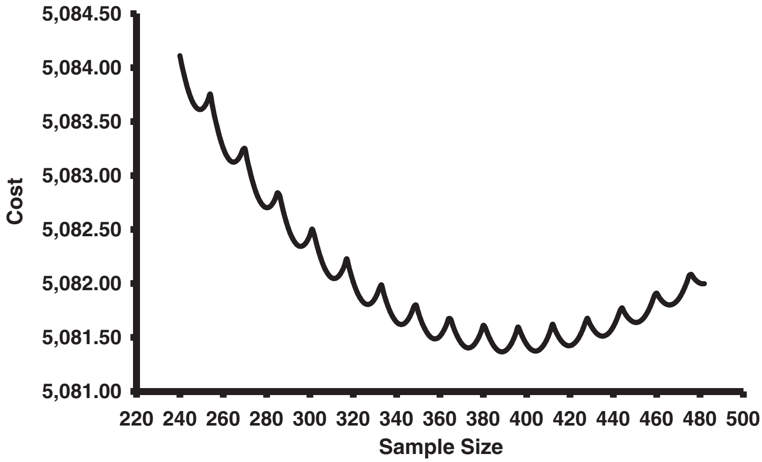


Figure 2: For lot size = 30,000, sample size vs. cost of MASSP D kind

testing is approximately around Rs. 0.06 per unit. The minor type of defects are black spot, shrink marks, less visible parting lines which can be verified at a cost of around Rs 0.14 per unit. (Note that verification for critical defect is less costly.) An item containing the major defects will be discarded at a cost of Rs. 1.50 and an item containing minor defects can be resold to another consumer at a reduced price such that the cost of rejection in this case is around Rs. 0.50. An item containing defects of major category, if found during filling at the consumer's end, costs the producer around Rs. 5.50 and a product containing minor defects will cost the producer around Rs. 3.20. Using the notation of cost model we find that the cost parameters are $S_0 = R_0 = 0.20, S_1 = R_1 = 1.50, R_2 = S_2 = 0.50, A_1 = 5.50$ and $A_2 = 3.20$. From the past data we compute the $\overline{p_1} = 0.0105$, $\overline{p_2} = 0.035$ and standard deviations as 0.0091 and 0.0055 respectively, so that the parameters of the gamma priors $s_1$ and $s_2$ work out to be around 1.2 and 40 respectively. This gives $k'_s = 0.05, d_0 = 0.05, d_1 = 1, d_2 = 0.675$. We now construct optimal A kind, D kind and C kind plans minimizing the cost function $[K(N, n)/(A_1 - R_1)]$. For a lot of size 30,000 the behavior of cost function near the neighborhood of the optimum $k$ and $n$ for the Plan D is presented in Figure 2. For the optimum plan, $k = 29$ and $n = 389$.

For the same lot size, 30,000, the optimum A kind plan has $a_2 = 30$ and $n = 295$. (See Figure 3.) If we vary $a_1$ from 0 to 30, we may see from Figure 4 that the minimum cost is obtained at $a_1 = 9$. For $N = 30{,}000$, the optimal C kind plan (see Figure 5) has sample size $n = 293$ and acceptance numbers

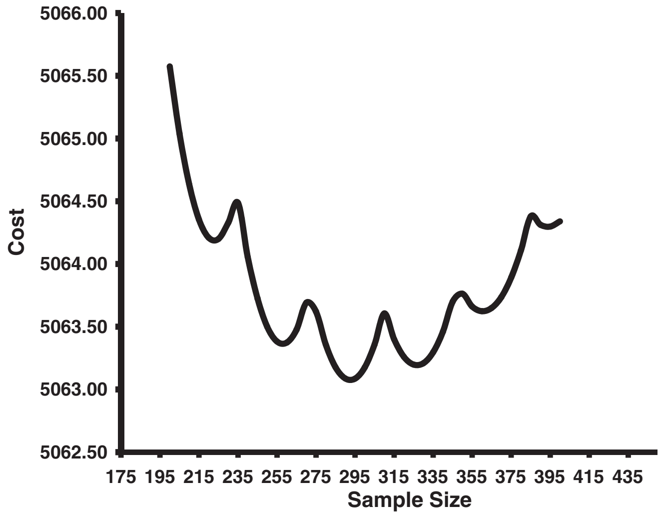


Figure 3: For lot size = 30,000, sample size vs. cost of A kind MASSP

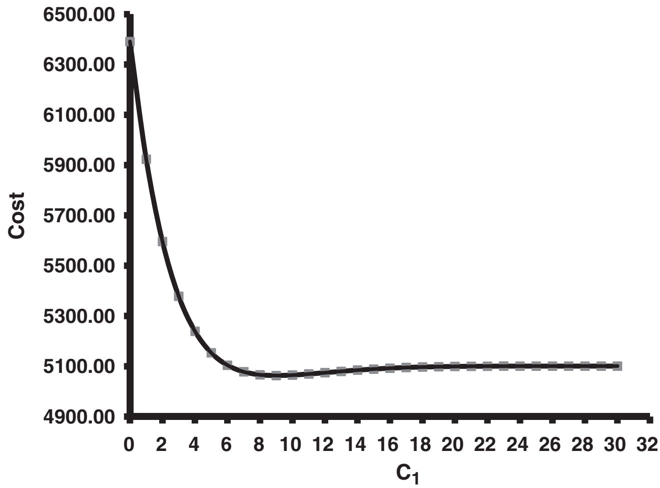


Figure 4: For lot size = 30,000, sample size = 293, $C_2$ = 30 , $C_1$ vs. cost of A kind MASSP

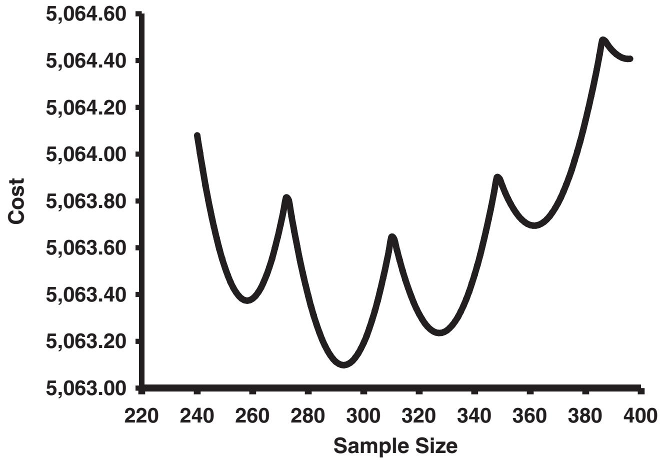


Figure 5: For lot size = 30,000, sample size vs. cost of C kind MASSP

$c_1 = 9, c_2 = 46$. We observe that the optimal A kind plan is cheaper than the optimal C kind plan and the optimal D kind plan. Further the optimal C kind plan is cheaper than the optimal D kind plan in this case.

## 5 Concluding remarks

We have, therefore, demonstrated that it should be possible to obtain an optimal MASSP by the above methods. While doing so, it should be noted that the Bayesian solutions rest on the assumption that the prior distribution is stable and that no outliers occur, so that for small and medium lot sizes the Bayesian plans will often reduce to 'accept without inspection' (Hald, 1981). It is, however, clear that the choice of acceptance criterion does affect the costs of optimal MASSP's. However, in the present chapter we do not attempt at obtaining general theoretical results for choosing the acceptance criteria as in the case of discrete prior distributions.

*Acknowledgement.* I am grateful to Professor A.C. Mukhopadhyay for his valuable suggestions and remarks while preparing the manuscript.

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Paper received: 7 July 2010; revised: 7 March 2012.