INDIAN STATISTICAL INSTITUTE

Mid-Semester examination: 2015-16

Course Name: M.Tech (QR & OR) 1st Year (E & S Streams)

Subject: Operations Research-I

Date of Exam: **31.08.15** **Total Marks: 60** **Duration:** 2 hrs.

Answer as many as you can. Maximum you can score is 60.

1. Define the following:
   a) Homogeneous system of linear equations
   b) Euclidean Space
   c) Linear independence of vectors
   d) Rank of a matrix
   e) Basis

[10]

2. A fuel manufacturing company wants to mix two fuels (A and B) for its trucks to minimize cost. It needs no fewer than 3,000 litre to run its trucks during the next month. It has a maximum fuel storage capacity of 4,000 litre. There are 2,000 litre of fuel A and 4,000 litre of fuel B available. The mixed fuel must have an octane rating of no less than 80. The octane rating is the weighted average of the individual octanes, weighted in proportion to their respective volumes. Fuel A has an octane of 90 and costs Rs. 80 per litre. Fuel B has an octane of 75 and costs Rs. 60 per litre. Formulate this product mix problem as a LPP.

[10]

3. (a) Show that the set of vectors $\boldsymbol{a}^1 = (2,1,4)$, $\boldsymbol{a}^2 = (1,-1,2)$ and $\boldsymbol{a}^3 = (3,1,-2)$ form a basis in E_3.

   (b) Prove that if the set of all feasible solutions K of a LPP is a Convex Polyhedron, then at least one extreme point must be obtained.

[4+6=10]

4. Consider the following financial problem. Find the optimal solution using the Simplex method. What evidence indicates that an alternate optimal solution exists?

Maximize Return on Investment = $2X_1 + 3X_2$

Subject to $6X_1 + 9X_2 \leq 18$

$9X_1 + 3X_2 \geq 9$

$X_1, X_2 \geq 0$

[8+2=10]

5. What is the role of an artificial variable in Simplex algorithm? When and how an improved b.f.s is constructed from a current b.f.s? Under what conditions infeasibility and degeneracy are observed in simplex tableau?

[2+5+3=10]

6. Determine an initial b.f.s by Minimum Cost method and test for the optimal solution for the following T.P.

		Distrn. Centres				
		D1	D2	D3	D4	Supply
Plants	P1	19	30	50	10	7
	P2	70	30	40	60	9
	P3	40	8	70	20	18
	Demand	5	8	7	14	**34**

[10]

7. Draw the network and estimate ES, EF, LS, LF, Slack, Critical Path and project completion time for the following activities whose completion times are shown below, including predecessor relationships.

Activity	**Activity Time**
1-2	8
1-3	12
1-4	7
2-3	3
2-4	5
2-5	14
3-5	7
4-5	4
4-6	2
5-6	6
6-7	10

[10]

INDIAN STATISTICAL INSTITUTE

Mid Semester Examination: 2015 – 16

Course Name: M Tech (QROR), 1st Year

Subject Name: Quality Management and Systems

Date: 01.09.2015 Maximum Marks: 60 Duration: 2 hours

Note: Answer all questions

1. What are the eight different dimensions of quality identified by Garvin? Explain each of the dimensions briefly. [16]
2. Suppose an organization does not have many dissatisfied customers. Does it mean that the organization has many satisfied customers and consequently does not face a major threat of losing market share? Explain from the perspective of quality management. [5]
3. What is the difference between quality of design and quality of conformance? Explain, with examples, from the perspective of different views of quality. In your opinion which aspect of quality is likely to be more important and why. [9 + 5 = 14]
4. What are the major differences between product quality and service quality? Explain why control of quality of service is likely to be difficult. [6 + 4 = 10]
5. "As we moved from craft to mass production system, the production organization became function rather than process driven." Do you agree with the statement? Explain from the perspective of evolution of quality management. [5]
6. What are the different constituents of cost of quality? Give examples of each of the constituents for an organization engaged in delivering services. [10]

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Course Name: M.Tech. (QROR) **Year: 1st year**

Subject Name: Programming Techniques and Data Structures

Date: 03·09·2015 **Maximum Marks: 70** **Duration: 2 hrs 30 min**

Answer all questions.

1. (a) Giving one example of each, distinguish between linear and non-linear data structures.
   (b) Provide a detailed comparison between array and linked list.
   [5+5=10]

2. Write a C program for searching a given number in a list of 100 numbers stored in an array. Derive the expression for the average number of comparisons required for searching a number in a list of *n* numbers. [10+5=15]

3. (a) What is stack? How is it different from queue? What do you mean by stack overflow?
   (b) In a mathematical expression involving parenthesis, checking whether the parentheses are nested correctly is important. Write the two important conditions that need to be checked for this purpose. Giving an example, show how stack is useful for this purpose.
   [3+3+2+4+4=16]

4. (a) Given the following expression in infix.

   9+3-6/3+7

   Write its prefix and postfix equivalent assuming the precedence of operators as per the BODMAS rule.
   (b) Using struct, write the declaration of a queue in C that uses an array. Write the condition that indicates an empty queue. [3+3+2+2=10]

5. (a) Write the output of the following C code segment:

```
void main()
{
        int x=15,y=3;
        x= x++ + y++;
        y= ++x + ++y;
        printf("x=%d \t y=%d",x,y);
}
```

P.T.O

(b)

```
#include<stdio.h>
void main()
{
        int i=10;
        while (--i)
                printf("%d\t",i);
        printf("\n");
}
```

(c) Let A[10][15] be an array of integers (2 bytes) stored in column major form, with A[0][0] at address 105. What will be the address of A[5][7]? [3+3+3=9]

6. In C, define the structure of a node in a singly linked list. Write an algorithm to print the average of the numbers stored in a singly linked list. What is the advantage of using doubly linked list over singly linked list. [3+5+2=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2015-16

Course Name: M. TECH. (QROR) I Yr.

Subject Name : Electrical and Electronics Engineering

Date : 03.09.15 Maximum Marks : 50 Duration : 2Hrs

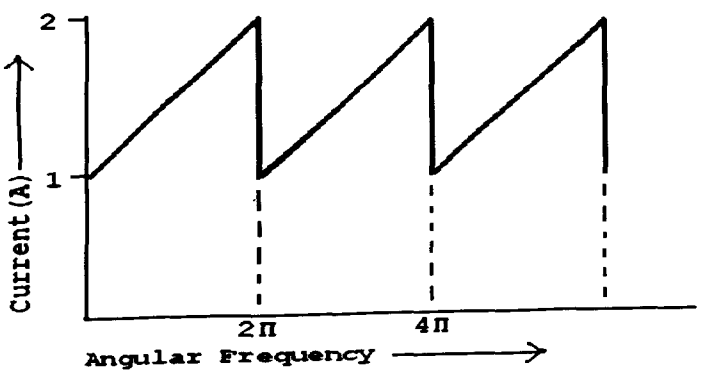
Answer any 5 questions.

1. a) Show that, for an AC voltage the angular frequency $w=2\pi/T$ where, T is the time period of one complete cycle.

   b) Explain with a diagram how AC voltage can be generated for a rectangular coil having N turns rotating in a uniform magnetic field with an angular velocity of w radian/second. [2+8=10]

2. Find the series resonance frequency for a circuit having a capacitor (C), inductor (L) and a resistor (R) connected in series with an A.C. voltage V_0e^{jwt}. [10]

3. a) State Kirchhoff's voltage law.
   b) Calculate the average and r.m.s. value of current epresented in figure below. [2+8=10]



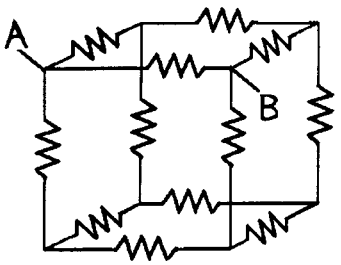
4. a) Explain the maximum power transfer theorem.

   b) Suppose, three resistances R_1 , R_2 , and R_3 are connected in delta formation and the delta formation is equivalent to a star formation comprising of resistances x, y, and z. Find R_1 , R_2 , and R_3 individually in terms of x, y, and z. [5+5=10]

5. Explain the operational principles of an OP-AMP to be used as an adder and differentiator. [5+5=10]

P T O

6. A cube is formed by joining equal wires, each of resistance 1 Ohm. The cube is shown in figure below. Calculate the equivalent resistance between the points A and B. [10]



7. a) Derive the binary equivalent of the decimal number 75.25

b) Show that an OR gate can be realized with diodes, batteries and resistances.

d) Subtract 01110 from 10011 using 1's complement method.
[2+5+3=10]

Indian Statistical Institute

Mid-Semestral Examination : 2015-16

M-TECH(QR&OR) -- 1st YEAR (E - STREAM)

PROBABILITY-1

{Answer all the questions}

Date: 02.09.15 **Full marks: 100** **Time: 2 hours**

[Symbols have their usual meaning]

1. a) Express μ_r in terms of μ_r'.
   b) Let $X \sim Bin(10, 1/3)$. Find its mean, variance, mode.
   c) State and prove Poincare's theorem.

   [4+6+10=20]

2. a) A person is paying Rs 10/ for each participation of the following game:
   He is drawing 2 cards from a deck. He gets Rs 20 if he draws 2 black queens. He gets Rs 30 if he draws 2 kings, He gets Rs 30 if he draws 1 red ace and another black ace. Otherwise he gets nothing. What is his expected gain?
   b) Arrange the following quantities in increasing order of magnitude with proper equality or inequality sign between them
   $P(A)$, $P(A)+P(B)$, $P(A \cup B)$ and $P(A \cap B)$
   c) There are three chests, each having 2 drawers. Each drawer of first chest contains one gold coin. Each drawer of second chest contains one silver coin. Third chest contains one gold coin in one drawer and one silver coin in another. A chest is open at random. A drawer is open and it is found to contain a gold coin. What is the probability that the coin in the other drawer is also a gold coin?

   [8+4+8=20]

3. a) If you permute the word "STATISTICS" in all possible ways what is the probability that the three "S" s will come together.
   b) State and prove Bonferroni's inequality.
   c) Consider r indistinguishable balls randomly distributed in n cells. What is the probability that exactly m cells remain empty?

   [8+12+10=30]

4. Assignments.

   [30]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2015 -16

Course name: M. Tech (QR&OR), Ist Year

Subject Name: Statistical Methods – I

Date: 04.09.15 Maximum Marks: 100 Duration: 3 hours

Note: Answer all the questions

1. Find the correct answer. Also comment briefly on all the options to justify your choice.
   (a) How might the standard deviation (S) of a normal distribution be greater than the mean?
   i. S is given by a square root, and the square root is larger than the fraction.
   ii. In a normal distribution, the variance must equal the mean.
   iii. If some scores are negative, the mean could be very small despite a large S.
   iv. The median would have to be less than the skew.
   (b) Suppose the mean score of the final examination is 24 (of 40), with a standard deviation of 1.5. If you get a 21, how well do you do (relative to the rest of the class)?
   i.Very poorly--perhaps the lowest score.
   ii. Not well, but somewhere in the C's (lowest grade is E).
   iii. OK--about average.
   iv. Nicely--better than the median.
   (c) The correlation between a person's hair length and his score on the midterm is very nearly zero. If your friend has a crew cut (short hair), your best guess as to what he got on the midterm is
   i. The standard deviation of scores on the midterm.
   ii. The mean minus the standard deviation.
   iii. The mean plus the standard deviation.
   iv. The mean score.

P.T.O

(d) There is a low (but real) negative correlation between the amount of ra in a given summer and the amount the summer before. In the absence any information except that this summer is wetter than usual, you a asked to guess next summer's rain. Your best guess:

i. Somewhat more than the average summer rainfall.

ii. The average summer rainfall.

iii. Somewhat less than the average summer rainfall.

iv. The standard deviation of the rainfall.

[8x4 = 32]

2. Each member of a sample of 3,750 women aged 30-39 was measured fo unaided distance vision in both right and left eyes. The results are presente in the following table:

Right Eye	Left Eye				TOTAL
	Highest grade	Second grade	Third grade	Lowest grade	
Highest grade	750	130	60	35	975
Second grade	125	775	210	40	1150
Third grade	60	180	885	100	1225
Lowest grade	20	40	90	250	400
TOTAL	955	1125	1245	425	3750

For a randomly selected person from the population sampled as above, find the estimate of the following probabilities:

(a) The left eye will fall into the third grade of unaided distance vision.

(b) The left eye will have the highest grade given that the right eye has the lowest grade

(c) The left eye will have the highest grade and the right eye will have the lowest grade

(d) Answer the above questions assuming that the vision of the two eyes is independent. Compare the results and offer your comments.

[3 + 3 + 3 + 12]

P.T.O

3. Calculate the covariance and correlation coefficient between the two variables

X	Y
2	4
3	1
5	0

[3 + 3]

4. Let X = # of articles packed in a bag. The probability distribution of X is given below.

X	p(x) = Pr. (X = x)
49	0.05
54	0.2
55	0.2
56	0.35
57	_____

(a) Complete the probability distribution.
(b) Find the probability of getting more than 50 articles in a bag.
(c) What is the expected number of articles in a bag?

[3 + 5 + 5]

5. A survey of registered voters (of India) is taken to estimate the proportion that plan to vote in a forthcoming election. In a sample of size n = 800, 100 say they plan to vote.
   (a) Specify the population of interest, the parameter of interest and its point estimate.
   (b) Can the sampled population be considered as a substitute of the target population?

[2 + 2 + 4 + 2]

6. According to the census conducted ten years ago, the mean family size of a country was 3.3 people. Presently in a survey of 1200 families the observed mean and standard deviation of family size are found to be 3.09 people and 1.10 people respectively.

P.T.O

(a) Find the standard error (s) of the estimate of current mean family size (m).

(b) Assuming m ± 2s gives the 95% confidence interval of the estimate of mean, compute the confidence interval. Does the interval contain the census figure? Explain.

[5 + 2 + 3]

7. Write short notes on the following

(a) Box plot

(b) Correlation ratio

[2x4 = 8]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2015-16

Course Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: Operations Research-I

Date of Exam : 16-11-2015 **Max Marks: 70** **Duration: 3 hrs.**

Assignment: 30 marks
Question no. 7 is compulsory. Answer as many as you can.

1. (a) Explain the physical interpretations of a dual problem considering its objective function, dual variables, dual constraints and its coefficients.

   (b) Prove that If x^* is any feasible solution (f.s.) of primal and w^* is any f.s. of dual such that $c'x^* = b'w^*$, then x^* is the optimal solution of primal and w^* is the optimal solution of dual.

   [7+5=12]

2. As a Project Manager, find the following

   a) Critical Path and expected project completion time;
   b) Least Cost schedule and the corresponding project cost;
   c) 36 days schedule and the corresponding project cost.

   Given: Indirect cost per day = Rs. 175/-

Activity Node	Normal duration (days)	Crash duration (days)	Cost (Rs.)	Crash Cost (Rs.)
(1,2)	8	6	1800	2200
(2,3)	16	11	1500	2200
(1,4)	12	9	2400	3000
(3,5)	14	9	1800	2400
(4,5)	15	14	800	2000
(5,6)	10	8	2000	4000

   Comment on the solutions obtained in b) and c)

   [4+4+4=12]

3. Define *inventory, inventory system, reorder point, order level, safety stock, service level* along with different cost components of an inventory system. Mention the basic differences between (t_p, S) policy and (t, S_p) policy.

[8+4=12]

4. (a) Derive the optimal order quantity for quantity discount model after stating the most important assumptions and underlying constraints.

   (b) A digital printing has an annual demand for portable hard disk of 1,400. The cost of a hard disk is Rs. 400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is Rs. 25 per order. If the company orders in quantities of 300 or more, it can get a 5% discount on the cost of the hard disks. Should the company take the quantity discount? Assume the demand is constant.

[5+7=12]

5. Define a Markov process in terms of a stochastic process. In general, what type of Markov process a Queue model is and why? Derive steady-state solution for p_n of a Poisson queue stating all its assumptions.

[2+3+7=12]

6. Find the expression for p_n and p_0 for $(M/M/m/\text{FCFS}/K/\infty)$. Prove that letting $K \rightarrow \infty$ and $\lambda/(m\mu) < 1$, the expression for p_n and p_0 yields the results for $(M/M/m/\text{FCFS}/\infty/\infty)$.

[9+3=12]

7. Nokia sells and services several brands of home appliances. Past sales for a particular model of Smartphone have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.35	0.25	0.15	0.05

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.10	0.40	0.50

Based on cost considerations as well as storage space, the company has decided to order 10 of these each time an order is placed. The carrying cost is Rs. 50 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 4,000 per stock out. The company has decided to place an order whenever there are only 2 smartphones left at the end of the week. Simulate 10 weeks of operation for Nokia with currently 5 units in inventory.

What would be the weekly carrying cost under this situation?

[12+3=15]

INDIAN STATISTICAL INSTITUTE
First-Semester Examination : 2015-16
M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY -- 1

Note : Answer any FIVE questions

Date: 18. 11.15 **Full marks:100** **Time: 3 hours**

1. a) State and prove Central Limit Theorem due to Lindberg and Levy.
   b) Explain the application of central limit theorem in the field of SQC.
   c) Let $X \sim$ Poisson(m). Show that $P(X \geq 3m) \leq 1/4m$. (State the result you have used.)

   [12+3+5=20]

2. a) The quality assurance manager of a bolt manufacturing company knows that as per contract his client will tolerate a maximum of 2% rejection with respect to diameter of the pipes. He has taken a set of 100 observations on diameter of the pipes and estimated the average as 30 cm and the standard deviation as 3 cm. Where should he set his upper specification limit of the diameter of the pipes so that he can satisfy his client (assume that the distribution of the diameter follows Normal distribution and specification for diameter is having only upper specification limit)?
   b) Probability of having a male child is 1/3. How many female children can be expected before the first male child ?
   c) Suppose that the number of telephone calls received by an operator from 11 AM to 11-30 AM follows a Poisson distribution with $\lambda=5$. Find the probability that in the next 3 days he will receive a total of one call.

   [10+4+6=20]

3. a) Two absent minded roommates A and B forget their umbrellas in some way or another. A always takes umbrella when he goes out, while B forgets to take umbrella with probability 1/2. Probability that each of them forgets his umbrella at any shop is 1/4. After visiting 3 shops they return home. Find the probability that: i) they have only one umbrella after their return. ii) B has lost his umbrella given that there is only one umbrella after their return.
   b) A card from a pack of 52 cards is lost. From the remaining cards, two cards are drawn and are found to be spades. Find the probability that the missing cards is also a spade.

   [12+8=20]

4. a) Let $X \sim N(0,1)$ and $Y^2 \sim \chi^2$ with degree of freedom n and X and Y^2 are independently distributed. Let $T = X/\sqrt{(Y^2/n)}$. Find the p.d.f of T.
   b) Let $(x_1, x_2, \ldots, x_n)$ be a sample of size n drawn from $N(\mu, \sigma^2)$. Let $\bar{x}$ and s^2 be the corresponding sample mean and the sample variance. Obtain the sampling distribution of $\bar{x}$ and s^2 and show that they are independent.

   [8+12=20]

P. T. O

5. a) Let $X_i \sim$ iid exp (λ), $i = 1(1) n$
Let $X_{(1)} = \text{Min}\ (X_1, \ldots, X_n)$
$X_{(n)} = \text{Max}\ (X_1, \ldots, X_n)$
Find the dist^n of $X_{(1)}$ and $X_{(n)}$. Explain its application in the field of SQC.
b) Let ρ be the correlation co-efficient between two random variables X and Y. Prove that $-1 \leq \rho \leq 1$.

[10+10=20]

6. a) Let $A_1, A_2, \ldots\ldots, A_r$ be r events not necessarily mutually exclusive. Find the probability of occurrence of exactly m events ($m<r$).
b) A student takes a multiple choice test consisting of two problems. The first one has 4 possible answers of which one is correct and the second one has 7 possible answers of which one is correct. The student selects one answer at random for each question and gets 5 score if it is correct otherwise he gets 0. Let X be the score of the student. Find E(X) and Var(X).

[10+10=20]

INDIAN STATISTICAL INSTITUTE

M. Tech (QR - OR) 1st Year (S Stream)
Session: 2015-2016
SEMESTER EXAMINATION
Subject: Engineering Drawing

Date of Exam:18.11.15 | Max. Marks : 60 | Time : 3:00 hrs

Note: (a) Answer question No.1 (compulsory) and any other three questions.
(b) Write your Name and Roll no. at one corner of the drawing sheet.
(c) Marks allotted to each question are indicated in the bracket.

1. Sketch a sectional front view and the side view of a Strap Joint with Gib and Cotter. Use suitable dimensions to complete the drawing. 18]

2. A square pyramid, base 40 mm side and axis 70 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the ground and a corner contained by that edge is on the ground. Draw its projections. [14]

3. A pentagonal pyramid, base 30 mm side and axis 65 mm long, has its base horizontal and an edge of the base parallel to the V.P. A horizontal section plane cuts it at a distance of 25 mm above the base. Draw its front view and sectional top view. [14]

4. Show by means of sketch any four of the following thread forms.
   a) Whitworth
   b) Seller
   c) Buttress
   d) Knuckle
   e) Square [14]

5. Show by sketch a pair of mating spur gear and a pinion and also indicate any four of the following six parameters in the figure.
   a) Working depth
   b) Addendum
   c) Dedendum
   d) Fillet radius
   e) Circular pitch
   f) Whole depth

6. The pictorial drawing of a machine part is given below. Draw the top view and the front view of it. Insert all the dimensions in the views. Use third angle projection method. Ref.Figure-1. [14]

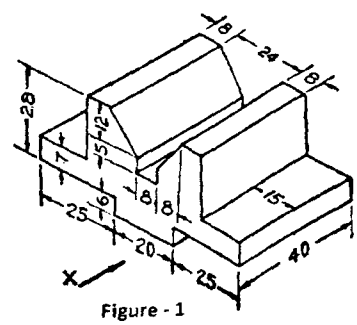


Figure - 1

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2014-15

Course Name: **M. Tech. (QR&OR)**

Subject Name: **Quality Management & Systems**

Maximum Marks: 100 Duration: 3 hours

Date: 20 November 2015

Note: There are two groups. Use separate answer sheet for each group.

Group-A: Maximum Marks: 60

Note: Answer any three questions from this group.

1. What are the eight different steps of six sigma? Explain each of the steps briefly.
[8 + 12 = 20]

2. What are the different barriers towards the achievement of quality? Briefly explain the steps.
[20]

3. Answer the following
   (a) Explain the concepts of internal and external customers with examples.
   (b) Six sigma is a customer focused, process driven, quantitative methodology for quality improvement. Explain briefly each of these three terms i.e., customer focused, process driven, and quantitative.
   (c) Explain the concepts of search, experiential, and credence quality.
[5 + (3+3+3) + (2+2+2) = 20]

4. Answer the following
   (a) Explain the concepts of importance and performance in the context of customer satisfaction.
   (b) Explain briefly the concept of value chain.
   (c) The quality of service is often measured using the RATER dimensions. Explain each of the constituents of the RATER dimension briefly.
[(2+2) + 6 + 10 = 20]

Group-B: Maximum Marks: 40

Note: Answer all the questions from this group.

1. Development of ISO 9001: 2008 Quality Management System may be viewed as a P-D-C-A cycle. Elaborate the concept with explanation.

[8]

2. Write short note on any four of the following terms.
   (a) Accuracy and Bias
   (b) Emergency Preparedness
   (c) Significant Environmental Aspects
   (d) 4T of Hazard Control
   (e) Disposition of Nonconforming Product
   (f) Training Effectiveness

[4 x 4 = 16]

3. Following are the objective evidences of an internal audit conducted according to both ISO 9001: 2008 QMS and ISO 14001: 2004 EMS. For any four objective evidences, identify whether the observation is related to QMS or EMS and provide your explanation and justification on being classified as a nonconformance or not.
   (a) Waste records show that waste has removed only once per week on 4 occasions during last three months instead of twice per week as stated in the procedure PR-6.8.
   (b) The foreman in the Machine Shop was referring to revision 03 of the drawing GT1653/13, but was not sure whether that is the latest revision or not.
   (c) EMS Objectives are found to be measured routinely, but no improvement is observed.
   (d) Testing procedure specifies that the temperature to be controlled within 40 ± 2°C. A calibrated thermometer of increment 1°C is used for this purpose.
   (e) Cleaning of solvent drums has been identified as having Significant Environmental Impact, but no Operating Procedure has been developed.
   (f) Material Safety Data Sheet was not found for the 3 new raw materials, which have been introduced recently.

[4 x 4 = 16]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

Course Name: M.Tech. (QROR) **Year: 1st year**

Subject Name: Programming Techniques and Data Structures

Date: 23·11·2015 **Maximum Marks: 100** **Duration: 3.00 hrs**

Answer all questions.

1

(a) Consider the following numbers: 9614, 5882, 4409, 1825. Find the 2-digit hash address of each number using (i) the division method, with m=97; (ii) mid-square method; (iii) folding method without reversing; and (iv) folding method with reversing.

(b) What is collision in the context of hashing? Mention one advantage and one disadvantage of using linear probing for collision resolution.

(c) Let there be 11 memory locations in a table, and suppose a file F consists of 8 records Z, Y, X, E, D, C, B, and A with the following hash addresses:

Record:	Z,	Y,	X,	E,	D,	C,	B,	A
H(k):	1,	5,	11,	4,	11,	2,	8,	4

Show how the file F appears in memory if linear probing is used for collision resolution. Find the average number S of probes for a successful search and the average number U of probes for an unsuccessful search. [(3x4)+(2+4)+(3+4)=25]

2

(a) What is an AVL tree? What is its advantage? Create an AVL tree by inserting the following numbers in the order in which they are given:

17 25 19 23 75

Draw figure for each step.

(b) Explain threaded binary tree, demonstrating its usefulness with a small example.

[(3+2+5)+5=15]

3. Illustrate the output returned in each pass of bubble sort to sort the numbers 3, 2, 4, 1, 5 in increasing order. Modify bubble sort so that it stops when no interchanges are needed. In terms of time complexity, how does bubble sort compare with quicksort? [7+3+5=15]

P.T.O

4 (a) Given the pre-order and inorder sequences, draw the resultant binary tree and write its post-order traversal.

Preorder :	A	B	D	G	H	E	I	C	F	J	K
Inorder :	G	D	H	B	E	I	A	C	J	F	K

(b) Write an algorithm that recursively finds the depth of a binary tree. [(4+2)+4=10]

5. Write a program that reads a text file (input.txt), and outputs another file (output.txt) where each sentence in input.txt appears on a separate line. [10]

6. Define a Binary Search Tree (BST). Show the binary search tree after each insertion operation starting from an empty tree: 30, 17, 62, 90, 81, 46, 40. Show the tree after deletion of 90.

[3+5+2 =10]

7. Write shorts notes on any three of the following:
   a. Abstract data types
   b. Circular linked list
   c. Union versus structure data type
   d. Array representation of binary tree
   e. Breadth first search tree traversal algorithm [3x5=15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2015-16

M. Tech. (QR & OR), 1st Year, E Stream

Subject: Statistical Methods I

Date: 26/11/2015 **Duration:** 3 hours

Note: This paper carries a total of 110 marks. Answer as many questions as you can. But the maximum you can score is 100.

1. Let $x_i > 0$ and $p \in [-\infty, +\infty]$. Then the generalized mean M_p is defined as

$$M_p(x_1, x_2, \ldots, x_n) = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^p \right)^{\frac{1}{p}}.$$

   (a) State the values of p for which M_p reduces to geometric mean, harmonic mean and minimum respectively.
   (b) Prove the result corresponding to the geometric mean.
   (c) Plot the function $M_p(x, 1)$, where $x \leq 1$ for the arithmetic, geometric and harmonic mean and hence indicate the relationship among the three.

[3 +5 + (6+2)]

2. Consider a daily record that gives the number of undersized, O.K. and oversized pieces produced by a machine.
   (a) Name the variable and identify the type of data for the following two cases: (i) our interest lies only on the oversized pieces and (ii) we are interested in all the three categories.
   (b) Name the type of data we will have in each category if the numbers are transformed into percentages (assume total daily production is 250).
   (c) Name the most appropriate measures of center for the two cases in (a) above.

[(2+3) +2+2]

3. Consider a random sample $(x_1, x_2, \ldots, x_n)$ from an infinite population with mean μ and variance σ^2.
   (a) Define an estimator.
   (b) Distinguish between an unbiased and an asymptotically unbiased estimator.
   (c) Give an example of asymptotically unbiased estimator of μ (other than the sample mean).
   (d) An estimator is consistent if it is asymptotically unbiased and its variance $\rightarrow 0$ as $n \rightarrow 0$. Hence show that the sample mean is a consistent estimator.

[2 + 3 + 3 + 4]

4. Suppose you want to estimate the mean of a normal population whose variance is 10. How large your sample should be to have approximately 95% confidence in your estimate if the desired margin of error is 0.4?

[8]

5. On the last three Sundays a salesman sold 2, 1, and 0 new cars respectively.

P.T.O

(a) Which of the following sentence is descriptive and which one is inferential:

(i) The salesman averaged a sale of one new car during the past three Sundays.

(ii) The maximum number of cars that the salesman is able to sell on a Sunday is 2.

(b) What is the problem with the following statement?

"The salesman sold no car on last Sunday because he fell asleep inside one of the cars"

[2 + 3]

6. State whether the following statements are true or false. Give justifications.
   (a) The standard deviation and the standard error are both used to summarize the spread of a given set of data.
   (b) The 95% confidence interval for the mean is the interval that covers roughly 95% of the observations.

[3 + 3]

7. Write an algorithm for generating 100 random numbers from a Binomial distribution with parameters (50, 0.2).

[10]

8. Consider the following observations on a variable that can take only positive values. The mean and standard deviation of the observations are 4.8 and 4.1 respectively.

0.01	0.4	1.5	2.1	2.8	4.0	5.1	6.3	8.2	11.2
0.02	0.6	1.6	2.1	2.8	4.0	5.1	6.3	8.5	12.2
0.1	0.7	1.6	2.2	2.9	4.0	5.2	6.5	8.6	12.8
0.1	0.8	1.7	2.2	2.9	4.1	5.3	6.5	8.9	13.0
0.1	0.8	1.8	2.2	3.0	4.1	5.4	6.8	9.0	14.4
0.1	0.8	1.8	2.5	3.2	4.4	5.4	7.1	9.1	14.5
0.1	0.8	1.8	2.6	3.3	4.8	5.9	7.1	9.2	14.6
0.3	0.8	1.9	2.6	3.3	4.9	6.0	7.5	9.3	15.0
0.4	0.9	1.9	2.7	3.5	5.0	6.1	7.8	9.3	15.5
0.4	1.3	2.0	2.7	3.9	5.0	6.2	8.0	10.7	16.3

   (a) Construct an histogram of the data
   (b) Select an appropriate distribution for the data. Give justifications.
   (c) Construct a probability plot to examine whether the data supports your choice of the distribution. Construct the probability plot corresponding to the mid points of the frequency distribution obtained in (a). Also merge the adjacent classes to have at least five observations in each class.

[7+3+14]

9. A simple random sample of 30 households is drawn from a city area containing 14848 households. The numbers of persons per household in the sample are as follows.

5	6	3	3	2	3	3	3	4	4
3	2	7	4	3	5	4	4	3	3

P.T.O

4	3	3	1	2	4	3	4	2	4

Compute the probability that the estimate of total number of people in the area is within ±10% of the true value.

[10]

10. Write short notes on the following. [Use the following format – Definition, when to use with examples, comparison with other similar concept/tool, special feature, if any.]
    (a) Maximum likelihood estimation
    (b) Rank correlation coefficient

[5+5]

INDIAN STATISTICAL INSTITUTE

First-Semester Examination : 2015-16

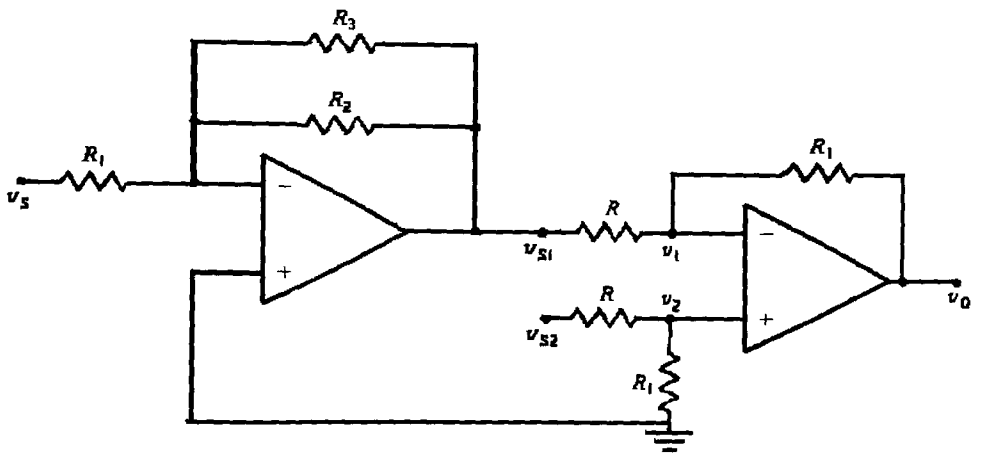
Course Name: M. TECH. (QROR) I Yr.

Subject Name : Electrical and Electronics Engineering

Date : 26.11.15 Maximum Marks : 96 Duration : 3Hrs

Answer any 6 questions.

1. a) Explain the operational principles of an OP-AMP to be used as an integrator.
b) For the circuit, given below, find the output voltage v_0 for v_S=1volt, v_{S2}=0.5volt, R_1=2kOhms, R_2=8kOhms, R=2kOhms and R_3=8kOhms. [6+10=16]



2. a) Show that for an AC voltage the angular frequency w=2π/T where T is the time period of one complete cycle.
b) An inductor (L) is connected in series with a resistance (R) and a capacitor (C) is connected in parallel with the series LR. If an AC voltage V_0e^{jwt} is now applied across this circuit then determine the condition for resonance and the angular frequency at resonance. [3+(10+3)=16]

3. a) Derive the output of a PID (proportional+integral+derivative) control action for a step type error signal. Explain with a figure how the output changes with error signal and time.

b) A PI(proportional+integral) controller is used for controlling a certain process. The settings are as follows: K_p (gain)=2%, P_o=40%, Reset rate =2%/min and E_p (error signal)=4t+6 where t represents time. What will be the controller output in percentage after 2 minutes? [(6+4)+6=16]

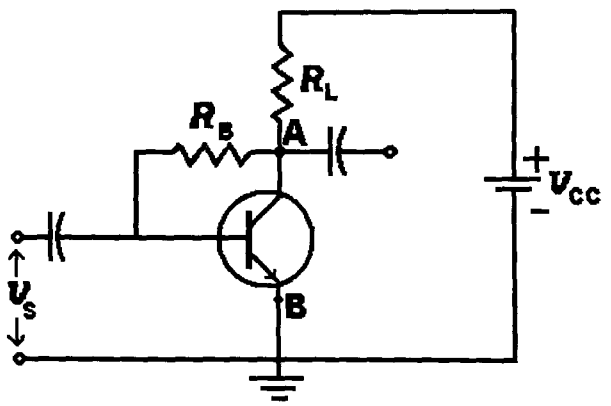
P.T.O

4. What is a transformer? Draw the equivalent circuit of a transformer and show that in ideal situation the ratio of the primary to the secondary voltage is equal to the primary-to-secondary turns ratio. [2+(4+10)=16]

5. State and prove Thevenin's theorem. [4+12=16]

6. a) Draw the circuit diagrams of amplifiers using a p-n-p transistor in common-base, common-emitter and common collector modes of operation.

   b) An n-p-n transistor, shown in the figure below, is used in common-emitter amplifier mode with $\beta=49$, $V_{CC}=10V$ and $R_L=2k$ Ohms. If a 100k Ohms resistor, R_B, is connected between the collector and the base of the transistor, then calculate the quiescent collector current and the collector to emitter voltage drop between points A and B. [(3+3+3)+(3+4)=16]



7. Draw the hybrid-parameter equivalent circuit of a transistor in common base mode. Calculate the current gain and input resistance in terms of the hybrid parameters. [4+(6+6)=16]

8. Explain the operation of a full adder with circuit diagram and truth table. Show that a 3 bit addition can be realized with adders. [12+4=16]

9. a) Derive the binary equivalent of 25.75.
   b) Subtract 01100 from 10011 using 2's complement method and verify the result by doing the same subtraction using 1's complement method.
   c) Show how an AND gate can be realized using diodes and resistances.
   d) Construct an OR gate with AND and NOT gates only. [2+5+5+4=16]

INDIAN STATISTICAL INSTITUTE
Mid -Semester Examination: 2015 – 16
M. Tech (QROR), E-Stream, Semester II
Statistical Methods – II

Date: 22.02.2016 Maximum Marks: 60 Duration: 2 Hrs.

Note: Answer at most six (6) questions.

1. a) Define Type I and Type II error.
   b) Define power of a test. Show graphically, that increases in sample size increases power of a test.

[4 + (2+4)=10]

2. Morning walk time along a certain route assumed to follow normal distribution with mean of 40 min and standard deviation of 1.2 min. A random sample of 9 days walking time is as given below:

   36.5, 35.4, 38.9, 41.7, 36.7, 39.5, 37.6. 38.5, 40.1

   i) Can it be concluded from this data that average time taken is less than 40 min?
   ii) Find the probability of type II error, when average time taken for the walk has become 39.5 min. Use $\alpha = 0.05$.

[6 + 4 =10]

3. a) Define and explain
   i) Chi square test for independence, and
   ii) Chi square test for homogeneity.

b) A company operates four machines in three shifts each day. From production records, following data on number of breakdowns are collected.

Shifts	Machines			
	1	2	3	4
A	41	20	12	16
B	31	11	9	14
C	15	17	16	10

Test the hypothesis that breakdowns are independent of shift. Use α=0.05

[(2+2)+6=10]

4. Write down the linear statistical model involving two factors and their interaction, where first factor has a levels, second factor has b levels and each experimental combination is replicated n times.

   Decompose total variability in such a data set into different possible components of variation and hence find the expression for the 'sum squares for interaction'.

   [2+(2+6)=10]

5. Primer paints are applied to aluminum surfaces by two methods: dipping and spraying. The purpose of the primer is to improve paint adhesion, and same parts can be primed using either application method. The process engineering group responsible for this operation is interested in learning whether three different primers differ in their adhesion properties. An experiment was performed to investigate the effect of primer type and application method on paint adhesion. For each combination of primer type and application method, three specimens were painted, then finish paint was applied, and the adhesion force was measured. The data from this experiment are shown below.

Primer Type	Dipping	Spraying
1	4.0, 4.5, 4.3	5.4, 4.9, 5.6
2	5.6, 4.9, 5.4	5.8, 6.1, 6.3
3	3.8, 3.7, 4.0	5.5, 5.5, 5.0

   Analyse the data suitably and give your conclusion. Use α=0.05.

   [10]

6. Consider the simple linear regression model $y_i = b_0 + b_1 x_i + \varepsilon_i,\ \ i = 1, 2, \ldots, n$ with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$ with the errors ε_i uncorrelated. Show that

   a) $E(SS_R) = \sigma^2 + b_1^2 S_{xx}$ and

   b) Variance of the i-th residual e_i is given by

$$V(e_i) = \sigma^2 \left[1 - \left(\frac{1}{n} + \frac{(x_i - \bar{x})^2}{S_{xx}} \right) \right]$$

   [5+5=10]

7. A chemical engineer in an attempt to explore the relationship between the purity of the oxygen produced in chemical distillation process (y) and the percentage of hydrocarbons present in the main condenser of the distillation unit (x) collected 20 pairs of observations. Summary quantities were

$$\sum x_i = 23.92 \quad \sum y_i = 1843.21 \quad \sum x_i^2 = 29.29$$
$$\sum y_i^2 = 170044.53 \quad \sum x_i y_i = 2214.66$$

a) Calculate the least square estimates of the slope and intercept.
b) Find an estimate of the error variance.
c) Find a 95% confidence interval for the intercept.

[6+2+2=10]

INDIAN STATISTICAL INSTITUTE

M. Tech. (QR OR), I Year
Session: 2015-16, Semester II
Mid-Semestral Examination

Subject : Elements of Stochastic Process

Date : 23.02.2016 **Time : 2 hours** **Maximum Marks : 70**

Notes:

(i) Unless stated otherwise, the abbreviation "M.C." will mean a discrete time parameter Markov Chain with stationary transition probabilities.
(ii) The notations used in this paper have their usual meanings.
(iii) Answer all the questions.

(1) Define the following:

(a) Stochastic Process
(b) Markov Chain
(c) Transition Matrix
(d) First Passage Probabilities

(4 × 5)=[20]

(2) Consider a M.C $\{X_n | n \in \{0,1,2 \ldots\}\}$ with state space I, initial distribution $\{\pi(i) | i \in I\}$ and one-step transition matrix $P = \left((p_{ij})\right)_{i,i\in J}$. Show That:

$$P(X_0 = i_0, X_1 = i_1, \ldots., X_n = i_n) = \pi\,(i_0) p_{i_0 i_1} p_{i_1 i_2} \ldots . p_{i_{n-1} i_n}$$

[15]

(3) Consider the following model for the movement of molecules between two urns, known as the Ehrnfest model:

M molecules numbered 1,2,....M are distributed among two urns. At each time point, one of the molecules is chosen at random, and is removed from its urn and placed in the other urn.

Let X_n denote the number of molecules in the first urn after the n th exchange.

Show that $\{X_n | n \in \{0,1,2, \ldots.\}\}$ Is a M.C and find its one step transition matrix.

[15]

(4) Stat and prove the Chapman – Kolmogorov Equation for a M.C. (You can assume the fact that $P^{(m)} = P^m$ for m=0,1,2,.....) [8]

(5) Show that
$p_{ij}^{(n)} = \sum_{k=0}^{n} f_{ij}^{(k)} p_{jj}^{(n-k)}$ for n=0,1,...and i, j$\in I$ [12]

INDIAN STATISTICAL INSTITUTE
Mid- Semester of Second Semester Examination: 2015-16

Course Name : **M. TECH (QR-OR)-I**
Subject Name : **MECHANICAL ENGINEERING**
Date: **Maximum Marks: 30** **Duration: 1hr 30 min**
Note, if any :

Answer any ***three*** questions.

1. a) Neatly sketch the stress-strain diagram of a ductile material. Also explain it.
   b) Write the differences between hot working and cold working processes. 5 + 5

2. a) How are seamless tubes manufactured? Explain with illustrative sketch(es).
   b) What is spinning operation? With the help of a neat sketch explain this process.
   c) Sketch to show 2 high and 3 high rolling mills. 4 + 3 + 3

3. a) Discuss the deep drawing process.
   b) Draw a neat labeled sketch of a drawing die and state the functions of each part.
   c) How are coins manufactured? 4 + 4 + 2

4. a) Give an idea about tube bending operation.
   b) What are forward and backward extrusion processes?
   c) Discuss about the lubrication in extrusion process. 2 + 5 + 3

INDIAN STATISTICAL INSTITUTE
Mid-Semester of Second Semester Examination: 2015-16
Course: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: SQC

Date of Exam: 24/02/2016 **Max. Marks: 100**

Duration: 3 hrs.

Answer All Questions.

1) Choose the correct alternative out of a, b, c, d and e wherever applicable.

A. A process in a state of statistical control means

I. Only common causes are present
II. Only special causes are present
III. The product will always meet its specifications
IV. The process is stable and only random or inherent sources of variation are present in the process

a) I and II above
b) II and III above
c) III and IV above
d) IV and I above
e) None of the above

B. The rational subgrouping for the control chart means

a) A subgroup that is free from the assignable causes as much as possible
b) A subgroup that represents the homogeneous conditions as much as possible
c) A small group of consecutively produced parts from a production process
d) All of the above
e) To form subgroups by randomly taking samples from the entire production

C. If a process is out of control, the theoretical probability that five consecutive points on an $\overline{X}$ chart will fall on the same side of the mean is

a) $(1/2)^4$
b) $2(1/2)^4$
c) Unknown
d) $1/2(1/2)^4$
e) $(1/2)^5$

D. You are monitoring a critical characteristic which is in a state of statistical control for a long period of time. However, the inherent variation or common cause variation in the process is bigger than the allowable tolerance on the characteristic. The least favorable action that you should take is

a) Live with the process as is and a certain level of defectives and implement a containment plan as it is not economical to change the process
b) Stop monitoring the characteristic as it is not capable

c) Change the system, i.e., change the current manufacturing process to reduce the inherent variation
d) Convince the engineer to open or expand the tolerance and make it realistic based on the data from process capability if there are no adverse consequences to the customer

E. A product characteristic is monitored using control charts, and the quality engineer finds that the process is not in a state of statistical control. Which one of the following actions is not preferable?

a) A root cause analysis of the process parameters should be done to identify and remove the special causes of variation and bring the process in the state of statistical control
b) Analyze the data of individuals to see if the process meets the tolerance limits even with some special causes present
c) Perform a capability study whether the process is in statistical control or not and predict the capability
d) Do not give a very high priority to quality improvement efforts if the process meets the specifications, and if there are no adverse consequences

F. A process has multiple machines producing the same part, e.g., a multicavity molding machine, multiple spindles, etc. Which of the following is not advisable for the different batches produced from each separate machine?

a) All batches should be mixed and samples should be taken at random to monitor the process on a single control chart
b) Each subgroup in the case of a multicavity machine comes from its unique population of that cavity alone and all successive subgroups are taken from the same cavity only
c) Each cavity is treated separately and has its own control chart
d) All of the above

G. The type A OC curve

a) Provides the probability of acceptance of each lot and assumes that each lot is representative of an infinite number of lots produced under similar conditions
b) Provides the probability of acceptance for an individual lot that comes from finite production conditions that cannot be assumed to continue in the future
c) Is a more widely applicable OC curve than type B
d) None of the above

H. The steeper the OC curve, the

a) Less protection for both producer and consumer
b) Lower the AQL
c) More protection for both producer and consumer
d) Smaller the sample size

I. An operation requires shipments from a vendor of small lots of fixed size. The attribute sampling plan used for receiving inspection should have its OC curve developed using

a) The binomial distribution
b) The Poisson distribution
c) The Gaussian (normal) distribution
d) The hypergeometric distribution

J. For a sampling plan, which of the following is true?

a) The consumer's risk is the probability that a bad lot will be rejected
b) The consumer's risk is the probability that a good lot will be accepted
c) The consumer's risk is the probability that a bad lot will be accepted
d) The consumer's risk is the probability that a good lot will be rejected

K. The probability of accepting material produced at an acceptable quality level is defined as

a) β
b) α
c) AQL
d) $1-\beta$
e) $1-\alpha$

L. The maximum percent defectives that can be considered satisfactory as a process average defines

a) The term AQL (acceptable quality level) as used in sampling inspection
b) The term average quality limit
c) The term AOQL, i.e., average outgoing quality limit
d) The quality level

M. The cause and effect or Fishbone diagram can be classified as:

a) Cause enumeration, dispersion classification & tree diagram types
b) Cause enumeration, dispersion classification & activity network diagram or arrow diagram types
c) Dispersion classification, process classification & cause enumeration types
d) Process classification, affinity diagram & tree diagram types

N. Based on their function, the check sheets can be classified as:

a) Process decision program checks, defective item checks, defect location checks, defective cause checks, check-up confirmation checks
b) Defect location checks, defective item checks, defective cause checks, production process distribution checks, check-up confirmation checks
c) Affinity checks, check-up confirmation checks, production process distribution checks, defective cause checks, defect location checks
d) Defect location check, defective item checks, defective cause checks, interrelationship checks, check-up confirmation checks

O. Variation in a process exists due to:

a) Chance causes only
b) Assignable causes only
c) Either chance cause or assignable cause
d) Both chance and assignable causes

[15×2 = 30]

2) The data shown in the Table below are $\bar{x}$ and R values for 24 samples of size $n = 5$ taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345).

a) Set up $\bar{x}$ and R charts on this process. Does the process seem to be in statistical control? If necessary, revise the trial control limits.

b) If specifications on this diameter are 0.5030 ± 0.0010, find the percentage of nonconforming bearings produced by this process. Assume that diameter is normally distributed.

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	34.5	3	13	35.4	8
2	34.2	4	14	34.0	6
3	31.6	4	15	37.1	5
4	31.5	4	16	34.9	7
5	35.0	5	17	33.5	4
6	34.1	6	18	31.7	3
7	32.6	4	19	34.0	8
8	33.8	3	20	35.1	4
9	34.8	7	21	33.7	2
10	33.6	8	22	32.8	1
11	31.9	3	23	33.5	3
12	38.6	9	24	34.2	2

[15+5 = 20]

3) An Automobile manufacturer wishes to control the number of nonconformities in a subassembly area producing manual transmissions. The inspection unit is defined as four transmissions, and data from sixteen samples (each of size 4) are shown in the table below.

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	1	9	2
2	3	10	1
3	2	11	0
4	1	12	2
5	0	13	1
6	2	14	1
7	1	15	2
8	5	16	3

a) Set up a control chart for nonconformities per unit.

b) Do these data come from a controlled process? If not, assume that assignable causes can be found for all out-of-control points and calculate the revised control chart parameters.

c) Suppose the inspection unit is redefined as eight transmissions. Design an appropriate control chart for monitoring future production.

[10+5+5 = 20]

4) A double-sampling plan with $n_1 = 50$, $c_1 = 2$, $n_2 = 100$, $c_2 = 6$ has been proposed. If the incoming lots have fraction non-conforming $p = 0.05$, what is the probability of acceptance on the first sample? What is the probability of final acceptance? Calculate the probability of rejection on the first sample.

[3+12+5 = 20]

{Assignment = 10}

INDIAN STATISTICAL INSTITUTE

Second Mid-Semester Examination (2015 – 2016)

Course Name : **M.Tech (QR & OR)**

Subject : **Industrial Engineering and Management**

Date :

Maximum Marks : **40**

Duration : **90 minutes**

Question Paper

Answer *all* questions

1. The network and durations given below shows the normal schedule for a project. You can decrease (crash) the durations at an additional expense. The Table given below summarizes the time-cost information for the activities. Find crashing time and cost of the project. **[10]**

Activity	Normal Time	Normal Cost (INR)	Crash Time	Crash Cost (INR)
A	3	50	1	20
B	16	80	2	40
C	10	60	1	30
D	11	50	4	25
E	8	100	2	30
F	5	40	1	30
G	6	70	0	00

P.T.O

2. The desired daily output for an assembly line is 500 units. The assembly line operates for a period of 420 minutes a day. The process involves the tasks A, B, C, D, E, F, G, H, I, J, and K. Balance the assembly line and calculate the cycle time and efficiency of the assembly line. **[15]**

Task	**Task Time (Seconds)**	**Tasks that must precede**
A	45	----
B	11	A
C	9	B
D	50	----
E	15	D
F	12	C
G	12	C
H	12	E
I	12	E
J	8	F,G,H,I
K	9	J
Total time	195	

3. (a) What is the importance of ergonomic design?
   (b) How is the process and product design improving the productivity?
   (c) Inefficient time is needed in production process. Comment and justify. **[3X5=15]**

INDIAN STATISTICAL INSTITUTE
Mid-Semester of Second Sementer Examination 2015-16

M. Tech(QR & OR) I Year (E & S Stream)

Reliability-I

Date: 26 February, 2016 Total Marks: 60 Duration: 2 hours.

Note: Answer all questions

1. (a) Define the hazard rate of a lifetime random variable and derive its relationship with the reliability function.

   (b) Show that if the conditional reliability of a device is increasing function of age, then the corresponding hazard rate is decreasing, provided the probability density function exists.

   (c) A device has a constant hazard rate λ_1 up to time x_0 (known positive constant), after which the hazard changes to another constant λ_2. Find the reliability of the device at time x.

   [(1+2)+3+4 =10]

2. (a) Consider a lifetime random variable with the reliability function

$$R(x) = \begin{cases} \frac{b-x}{b} & \text{if } 0 < x < b, \\ 0 & \text{elsewhere.} \end{cases}$$

   Find the mean residual life at age x_0.

   (b) Consider a Gamma lifetime model with the probability density function given by

$$f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, x, \alpha, \beta > 0. \tag{1}$$

   Check whether the lifetime distribution is IFR or DFR.

   (c) Let, $F_1(x)$ be a continuous distribution function of a lifetime random variable. For some positive α, define the distribution function $F_2(x)$ such that

$$\overline{F}_2(x) = \left(\overline{F}_1(x)\right)^\alpha,$$

   where $\overline{F}_1(x) = 1 - F_1(x)$ and $\overline{F}_2(x) = 1 - F_2(x)$. Find the relationship between $\lambda_2(x)$ and $\lambda_1(x)$, the respective hazard rate of $F_2(x)$ and $F_1(x)$.

   [4+6+4=14]

3. (a) Describe Type-I, Type-II and Hybrid censoring schemes with schematic diagram. Describe the data for each censoring.

   (b) Assuming the lifetime distribution follows the exponential distribution with probability density function given by

$$f(x;\lambda) = \lambda e^{-\lambda x}, \lambda, x > 0, \tag{2}$$

   construct a $100(1-\alpha)\%$ confidence interval for λ based on Type-II censored data.

(c) Consider the following observed data on seconds to breakdown of 12 items with " + " indicating right censoring.

$$50, 134, 187, 882, 1450, 1470, 2290, 2930, 4180, 15800+, 15800+, 15800+$$

Assuming the lifetime follows exponential distribution with mean λ, compute the estimate of the reliability at time 120.

[(3+3)+8+4 =18]

4. (a) A life testing experiment was conducted with 15 light bulbs. The lifetime in hours of 15 bulbs are observed as follows.

$$50, 58, 61+, 63, 82+, 92+, 98, 102, 102, 105+, 108, 120, 121+, 140, 162.$$

Here, " + " indicates right censoring. Compute the Kaplan-Meier estimate of the reliability function at time 108 along with estimated variance.

(b) Write down the expression of the Nelson–Aalen estimate of the cumulative hazard function with its estimated variance formula. Define the notations properly that you will use.

[(10+5)+3=18]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2015 – 2016
M. Tech (QROR), First Year (E-Stream)

Statistical Methods – II

Date: 18.04.2016 Maximum Marks: 100 Duration: 3 Hours

*Note: **Answer only five questions.***

1. A mechanical engineer is studying the thrust force developed by a drill press. He suspects that the drilling speed and the feed rate of the material are the two important factors. He randomly selects four feed rates and uses a high and low drill speed chosen to represent the extreme operating conditions. He obtains the following results.

Drill Speed	Feed Rate			
	0.015	0.025	0.045	0.065
125	2.70	2.45	2.60	2.75
	2.78	2.49	2.72	2.86
200	2.83	2.85	2.86	2.94
	2.86	2.80	2.87	2.88

   a) Write down the appropriate statistical model and hence derive the expression for expected mean squares for the model components.
   b) Suitably analyze the data and draw your conclusion.
   c) Estimate the applicable variance components.

[7+10+3=20]

2. a) Derive the expression for least square estimators of the parameters of a simple linear regression model involving one independent variable.
   b) A chemical engineer in an attempt to explore the linear relationship between the purity of the oxygen produced in chemical distillation process (y) and the

percentage of hydrocarbons present in the main condenser of the distillation unit (x) collected 20 pairs of observations. Summary quantities were

$$\sum x_i = 23.92 \quad \sum y_i = 1843.21 \quad \sum x_i^2 = 29.29$$
$$\sum y_i^2 = 170044.53 \quad \sum x_i y_i = 2214.66$$

i) Calculate the least square estimates of the slope and intercept.
ii) Find an estimate of the error variance.
iii) Find a 95% confidence interval for the slope.

[8+(4+4+4)=20]

3. a) Two different formulations of primer paint can be used on aluminum panels. The drying time of these two formulations is an important consideration in the manufacturing process. Sixteen panels are selected; half of each panel is painted with primer 1, and the other half is painted with primer 2. The drying times are observed and reported in the following table. Is there any evidence that mean drying times of the two formulations are different? Use the Wilcoxon Signed-Rank test with $\alpha = 0.05$.

Panel No.	Drying Times (Hr.)		Panel No.	Drying Times (Hr.)	
	Formulation 1	Formulation 2		Formulation 1	Formulation 2
1	1.6	1.8	9	1.8	2.0
2	1.3	1.5	10	1.5	1.6
3	1.5	1.5	11	1.6	1.8
4	1.6	1.7	12	1.3	1.6
5	1.7	1.6	13	1.4	1.2
6	1.9	2.0	14	1.6	1.6
7	1.8	2.1	15	1.5	1.7
8	1.6	1.7	16	1.7	1.5

b) Given below is marks obtained by 10 students of class X of a school in English and Mathematics.

English	56	75	45	71	61	64	58	80	76	61
Maths	66	70	40	60	65	56	59	77	67	63

Calculate Spearman's rank correlation statistic and hence show that there exist an association between the variables. [Use $\alpha = 5\%$]

[10+10=20]

4. a) A fast-food chain having four restaurants wanted to compare the services provided by the restaurants. The management of the company hired six raters to evaluate the four restaurants. The raters were asked to rate each restaurants in 1-100 scale with 100 as the best rating. The results of the experiment are displayed in the following table.

Rater	Restaurant			
	1	2	3	4
1	70	61	82	74
2	77	75	88	76
3	76	67	90	80
4	80	63	96	76
5	84	66	92	85
6	78	68	98	86

Use Friedman's test to test the equality of the median service ratings of the four restaurants. Use $\alpha = 0.05$.

b) The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results are shown in the following table:

Circuit Type	Response Time				
1	9	12	10	8	15
2	20	21	23	15	30
3	6	9	8	10	7

Using Kruskal-Wallis procedure to test the hypothesis that the three circuits have same response time. Use $\alpha = 0.01$.

[12+8=20]

5. a) Consider the following multiple linear regression model involving k variables and $n > k$ observations on the response:

$$y_i = \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i, \quad i = 1, 2, \ldots, n \ .$$

Derive the corresponding least square normal equations in matrix form and hence find the least square estimators of regression coefficients $\underset{\sim}{\beta} = [\beta_0, \beta_1, \beta_2, \ldots, \beta_k]'$.

b) Show that the regression coefficients, as obtained above, is an unbiased estimator of the true regression coefficients.

c) Derive the expression for Error Sum of Squares.

[10+4+6=20]

6. a) Write and explain the statistical model for single factor experiment with one covariate.

b) State the assumptions made.

c) A soft drink distributor is studying the effectiveness of delivery methods. Three different type of hand trucks have been developed, and an experiment is performed in the company's engineering laboratory. The variable of interest is the delivery time in minutes (y); however, delivery time is also strongly related to the volume delivered (x). Each hand truck is used four times and following data are obtained. Analyse the data to test that delivery time is not dependent on hand truck type. Use $\alpha = 0.05$.

Hand Truck Type					
1		2		3	
y	x	y	x	y	x
27	24	25	26	40	38
44	40	35	32	22	26
33	35	46	42	53	50
41	40	26	25	18	20

[3+4+13=20]

7. An electronics engineer is interested in the effect on tube conductivity of four different type of coating for cathode ray tubes in a telecommunication system display device. The following conductivity data are obtained.

Coating Type	Conductivity			
1	152	149	140	143
2	134	133	132	127
3	129	127	132	129
4	147	148	144	142

a) Using $\alpha = 0.05$, test the hypothesis that the four coating types results in same conductivity.

b) Using modified Levene's Test check the equality of treatment variances. Use $\alpha = 0.05$.

[8+12 =20]

INDIAN STATISTICAL INSTITUTE
Second Semester Examination: 2015-16

Course Name : **M. TECH (QR-OR)-I**
Subject Name : **MECHANICAL ENGINEERING**
Date: **Maximum Marks: 100** **Duration: 3 hours**
Note, if any :

18.04.2016

Answer any ***five*** questions.
Assume suitable data if necessary.

1. a) Define tolerance. What are unilateral and bilateral tolerances? State and explain the different types of fit in hole basis system. Give necessary sketches.
   b) Explain the different terms of assembly 40 H8/g6.
   c) What is meant by form error in machining? Give some sketches to show form errors of cylindrical surfaces.
   d) Explain the following:
      i) Maximum attainable accuracy
      ii) Economically feasible accuracy. (2 + 2 + 4) + 4 + 4 + 4

2. a) Draw the stress-strain diagram of ductile material and show the important points on it.
   b) Explain the following terms:
      i) True stress
      ii) True strain
      iii) Toughness
      iv) Ductility
   c) Discuss about the elastic and plastic deformations of material. 6 + 8 + 6

3. a) What is HERF process? With suitable examples explain its use.
   b) Discuss the confined and unconfined explosive forming processes.
   c) For an unconfined explosive forming process a charge of 10 N TNT is used. The constant of proportionality for TNT is 4320 and the exponent is 1.15. Plot the peak pressure over the work surface with stand-off distance. 6 + 8 + 6

4. a) Explain why the unconventional machining processes are used.
   b) State about the energy type, mechanics of material removal and energy source for the processes given below:
      Conventional machining, AJM, USM, ECM, ECG, CHM, EDM, EBM, LBM, IBM, PAM.
   c) Give the concept of R_a value of surface roughness. What is lay?
   d) What is a CNC machine? 5 + 8 + 5 + 2

5. a) Define cutting speed, feed and depth of cut in machining operation.
   b) A mild steel plate of length= 500 mm and width= 300 mm is to be shaped in shaping machine with an hss shaping tool. The cutting speed is 18 m/min and the feed is 1

mm/stroke. The quick return ratio is 1.5. Calculate the machining time for one pass. Assume proper approach and over travel.

c) Sketch the following operations indicating the main motions required:
   i) Drilling operation in drilling machine
   ii) Grooving operation in a lathe
   iii) Turning operation in a lathe. 6 + 8 + 6

6. a) What are the possible causes of tool material failure? Explain them.
   b) Discuss the essential properties of an ideal tool material.
   c) Discuss about hss as a cutting tool material.
   d) Write the tool signature of single point turning tool and sketch the tool to show all the relevant features of it. 5 + 5 + 4 + 6

7. a) What is tool life? When operating with roughing cuts on mild steel at 18 m/min, a certain tool gave a tool life of 160 min. between regrinds. Estimate the life of this tool on similar cuts at a speed of 22 m/min. Take the exponent of tool life as 1/8.
   b) Discuss the crater wear and flank wear of tool in metal cutting operation.
   c) Explain the differences between up cut milling and down cut milling processes.
   d) How is a taper expressed? 7 + 5 + 6 + 2

8. Write short notes on (any four): 4 X 5
   i) Drawing die
   ii) Classification of machine tool
   iii) Electro-hydraulic forming
   iv) Selection of lubricant in extrusion process
   v) Grinding operation
   vi) Productivity in metal cutting
   vii) Thread cutting in lathe.

INDIAN STATISTICAL INSTITUTE

Second End-Semester Examination (2015 – 2016)

Course Name : **M.Tech (QR & OR) – I YEAR**

Subject : **Industrial Engineering and Management**

Date : 20/04/2016

Maximum Marks : **60**

Duration : **180 Minutes**

Question Paper

Direction: Question 1 is compulsory. Attempt *Any two* questions from rest.

1. (a) What do you mean by working capital? What are the important elements of working capital?

   (b) Show a typical balance sheet of an organization.

   (c) You have been given the following information for a company:

Summerized statements of financial position on 30th June

	2007		2006	
	Rs 000	Rs 000	Rs 000	Rs 000
Non-current assets		130		139
Current assets:				
Inventory	42		37	
Receivables	29		23	
Bank	3		5	
Equity and liabilities:				
Ordinary share capital		35		35
Share premium account		17		17

Revaluation reserve		10		
Profit and loss account		31		22
Non-current liabilities:				
5% secured loan notes		40		40
8% preference shares		25		25
Current liabilities				
Trade payables	36		55	
Taxation	10		10	

Summerized income statements for the year ended on 30th June

	2007		2006	
	Rs 000	Rs 000	Rs 000	Rs 000
Revenue		209		196
Opening inventory	37		29	
Purchases	162		159	
Closing inventory	42		37	
Finance costs	2		2	
Depreciation	9		9	
Sundry expenses	14		11	
Taxation		10		10
Dividends:				
Ordinary shares:	6		5	
Preference shares:	2		2	

(i) Calculate the liquidity ratios in 2006 and 2007.

(ii) Calculate the length of the cash operating cycle in 2006 and 2007.

(iii) Comment on your results and appraise how effectively the working capital is being managed?

[5+5+20]

2. (a) What is Net Present Value?

(b) What is the importance of NPV?

(c) Sulabh International is evaluating a project whose expected cash flows are as follows:

Year	Cash flow
0	Rs (10,00,000)
1	1,00,000
2	2,00,000
3	3,00,000
4	6,00,000
5	3,00,000

What is the NPV of the project if the discount rate is 12 percent for the first year and rises every year by 1 percent?

[2+3+10]

3. (a) What do you mean by time value of money? Explain the three Es of time value of money.

(b) What is ROCE?

(c) A project requires an initial investment of Rs 800,000 and then earns net cash inflows as follows:

Year	Cash Inflows (Rs 000)
1	100
2	200
3	400
4	400
5	300
6	200
7	150

In addition, at the end of the seven –year project the assets initially purchased will be sold for Rs 100,000.

Determine the project's ROCE using:

(i) Initial capital costs,

(ii) Average capital investment

[(2+3)+2+(4+4)]

4. Write explanatory notes on ***any three*** of the following: **[3X5]**

(a) Objectives of financial management

(b) Annuity

(c) Internal rate of return

(d) Inflation

(e) Uncertainty and risk in financial management

INDIAN STATISTICAL INSTITUTE
Second Semester Examination: 2015-16
Course Name: M. Tech. (QROR) I Year
Subject Name: Statistical Quality Control

Date: 22/04/2016 Maximum Marks: 100 Duration: 3 hours

[All Symbols Have Usual Meaning.]

Answer Any Four Questions

1. a) Consider a normally distributed process with $\mu = 20$ and $\sigma = 2$. The lower and upper specification limits for the associated quality characteristic are $LSL = 8$ and $USL = 32$ respectively. Set up the parameters of an appropriate control chart for monitoring the mean with sample size $n = 4$, if the mean is allowed to drift to the tune of 1.5σ.

   b) A process of interest is in a state of statistical control and generates continuous random measurements for a measured characteristic X from a normal distribution with a known expected value μ and a known standard deviation σ. Consider that the measured characteristic has lower and upper specification limits (LSL and USL respectively). For such a process prove that $C_{pk} = (1-k)C_p$. Where C_p is process potential index, C_{pk} is process performance index and k is the process centering index.

   c) Explain how one can estimate process capability indices for many of the quality characteristics for which it is desirable to have a skewed data.

[8+8+4 = 20]

2. a) A normally distributed process has specifications of $LSL = 75$ and $USL = 85$ on the output. A random sample of 25 parts indicates that the process is centered at the middle of the specification band and the standard deviation $s = 1.5$.
   (i) Find a point estimate of C_p.
   (ii) Find a 95% confidence interval on C_p and comment on the width of this interval.

   b) Assume that the quality characteristic of interest $X \sim N(\mu, \sigma^2)$. Let the lower and upper specifications for X be $[L, U]$ and the potential process capability index of the process is C_p. If the process is operating at the mid-point of the specification limits then prove that the expected proportion of conforming items is given by $2\phi\left(3C_p\right)-1$.

   c) Prove that for an $\bar{X}$ – chart with sample size n and $k\sigma$ control limits, if the mean shifts from the in-control value μ_0 to another value $\mu_1 = \mu_0 + \delta\sigma$, then the probability of non-detection of the shift is $\beta = \phi\left(k - \delta\sqrt{n}\right) - \phi\left(-k - \delta\sqrt{n}\right)$.

[(2+8)+6+4 = 20]

3. In a study to isolate both gauge repeatability and gauge reproducibility, two operators use the same gauge to measure 10 parts three times each. The data are given below.

Measurement Data

Part Number	Operator 1 Measurements			Operator 2 Measurements		
	1	2	3	1	2	3
1	50	49	50	50	48	51
2	52	52	51	51	51	51
3	53	50	50	54	52	51
4	49	51	50	48	50	51
5	48	49	48	48	49	48
6	52	50	50	52	50	50
7	51	51	51	51	50	50
8	52	50	49	53	48	50
9	50	51	50	51	48	49
10	47	46	49	46	47	48

a) Estimate gauge repeatability and reproducibility.

b) Estimate the standard deviation of measurement error.

c) If the specifications are at 50 ± 10, what can you say about gauge capability?

[14+2+4 = 20]

4. Determine an optimum single-sampling rectifying inspection plan that has an *AOQL* of 0.03, lot size of 4000 items and that minimizes the Average Total Inspection (*ATI*) for $p = 0.015$. The values of $y = P_a p_M n$ are given below for different values of c.

Values of $y = P_a p_M n$

c	y
0	0.3679
1	0.8408
2	1.3720
3	1.9460
4	2.5440
5	3.1720
6	3.8100

P_a = Probability of acceptance

p_M = The proportion defective value for which *AOQL* is attained

n = Sample size

c = Acceptance number.

[20]

5. Design different possible double sampling plans without curtailment with $p_1 = 0.03$, $\alpha = 0.05$, $p_2 = 0.12$, $\beta = 0.10$ and $n_2 = 2n_1$ by consulting the following Grubb's table. The symbols have their usual meanings.

Grubb's Table for Double Sampling Plan

Plan	Ratio of proportion defective for low P_a to that for high P_a	c_1	c_2	pn_1	
				$1-\alpha = 0.95$	$\beta = 0.10$
1	14.50	0	1	0.16	2.32
2	8.07	0	2	0.30	2.42
3	6.48	1	3	0.60	3.89
4	5.39	0	3	0.49	2.64
5	5.09	1	4	0.77	3.92
6	4.31	0	4	0.68	2.93
7	4.19	1	5	0.96	4.02
8	3.60	1	6	1.16	4.17
9	3.26	2	8	1.68	5.47
10	2.96	3	10	2.27	6.72

[20]

ASSIGNMENT: [20]

INDIAN STATISTICAL INSTITUTE
Semester Examination: 2015-2016 (Second Semester)

M. Tech (QR & OR) I Year (E & S Stream)

Reliability-I

Date: April 26, 2016 Maximum Marks: 100 Duration: $3\frac{1}{2}$ hours.

Notes: This paper carries 108 marks. The maximum you can score is 100.

1. Write true or false with justification

   (a) The min path sets of a 3-out-of-4 system are the min cut sets of a 2-out-of-4 system.

   (b) If ϕ is a coherent system, then its dual is not necessarily coherent.

   (c) The modular decomposition of a coherent system is unique.

   (d) For a series system with independent components each having constant hazard, the lifetime follows an exponential distribution.

   [4+5+3+4=16]

2. Consider a system with 3 components. The structure function of the system is

$$\phi(\underset{\sim}{x}) = \max\{\min(x_1, x_2), \min(x_1, x_3), \min(x_2, x_3)\},$$

   where x_1, x_2 and x_3 are binary variables representing the state of the components.

   (a) Show that ϕ is a coherent structure.

   (b) Provide a bound for ϕ with justification.

   (c) Suppose that the components are independent with reliabilities p_1, p_2 and p_3, where $p_1 \leq p_2 \leq p_3$. Which component is most important to the system and under what condition?

   (d) Construct a fault tree for the failure of the system. Find the min path sets of the system using the fault tree.

   [7+2+4+(3+4) =20]

3. Consider a coherent system with min path sets $P_1, \ldots, P_p$ and min cut sets $K_1, \ldots, K_k$. Suppose the component reliabilities are $p_1, \ldots, p_n$.

   (a) If the components are independent, show that the system reliability satisfies

$$\sum_{i=1}^{p} \prod_{r \in P_i} p_r - \sum_{i<j} \prod_{r \in P_i \cup P_j} p_r \leq h(\underset{\sim}{p}) \leq \sum_{i=1}^{p} \prod_{r \in P_i} p_r.$$

(b) If the components are associated, show that the system reliability satisfies

$$\max_{1\le r\le p}\prod_{i\in P_r} p_i \le P[\phi(\underset{\sim}{X})=1] \le \min_{1\le s\le k}\coprod_{i\in K_s} p_i.$$

[6+14=2

4. (a) Let T be a continuous liftime random variable with reliability function $R(t)$. Express $R(t)$ in terms of $m(t)$, the mean residual life function. You may assume the expression for $m(t)$ in terms of $R(t)$. Hence aurgue that $m(t)$ uniquely determines the distribution of T.

(b) Consider a lifetime T with hazard rate

$$\lambda(t)=\begin{cases}\lambda_1 & \text{for } 0\le t\le t_1\\ \lambda_2 & \text{for } t_1\le t<t_2\\ \lambda_3 & \text{for } t_2\le t<\infty\end{cases}$$

Find the mean lifetime.

(c) Consider a n-unit cold standby redundant system with perfect switching. The lifetimes of the units are independent and identically distributed exponential random variables. Show that the lifetime distribution of the system is IFR.

[(6+2)+6+6 = 20

5. Consider a coherent system with five components having min cut sets $\{1\}$, $\{2,3\}$ and $\{4,5\}$. Suppose the components are independent and identically distributed with common reliability function $R(t)=e^{-\lambda t}$, $t>0$. Derive the reliability function of the system. Check whether the lifetime distribution of the system is IFR.

[3+7=10

6. A battery has a constant hazard rate λ_1 or λ_2 depending on if it is of Brand 1 or Brand 2 respectively. Consider a lot of batteries from both the brands with proportion of Brand 1 batteries being p $(0<p<1)$. Derive hazard rate for the life of a battery randomly chosen from this lot. Show that the corresponding lifetime distribution is DFR. Derive the expected lifetime of the selected battery.

[2+5+3 =10

7. (a) Consider a 2-unit standby redundant system with a switching system. The standby unit does not carry a load while in standby mode. The lifetime of the primary unit has constant failure rate λ_1. The lifetime of the standby unit has hazard rate λ_2 while in active mode. While in standby mode, the unit 2 has hazard rate λ_2^*. Let p_s be the probability that the switch activates the standby unit, when required. Find the reliability of the system at time t_0.

(b) Consider a system with n components subject to a common stress X with cdf $F_X(x)=1-e^{-\lambda_1 x^\alpha}$. Suppose the strengths of the components are independent and identically distributed with common cdf $F_Y(y)=1-e^{-\lambda_2 y^\alpha}$. The system functions if all the components function. Find the stress-stregth reliability of the system. Assume that stress and strengths are independently distributed.

[6+6=12]

INDIAN STATISTICAL INSTITUTE

M. Tech. (QR OR), I Year
Session: 2015-16, Semester II
Semestral Examination

Subject : Elements of Stochastic Process

Date : 29.04.2016 **Time : 3 hours** **Maximum Marks : 100**

Notes:

(i) Unless stated otherwise, the abbreviation M.C. will mean a discrete time parameter Markov Chain with stationary transition probabilities.

(ii) The notations used in this paper have their usual meanings.

(iii) Answer all the questions.

(1) Define the following with respect to a M.C.

(a) Irreducibility of a M.C.

(b) Essential and Inessential states.

(c) Recurrent and Transient states.

(d) Positive and Null Recurrent states. (4×5)=[20]

(2) Show that the relationship of communication is an equivalence relation. [20]

(3) Prove that Recurrence is a class property. [20]

(4) Show that a state is recurrent if and only if and only if $\sum p_{ii}^{(n)} = \infty$. [20]

(5) State the definition of g_{ii}, and prove that it can take only two values 0 or 1. [20]

Indian Statistical Institute
Second Semester Examination : 2015-16
M. Tech (QR & OR) I Year (E & S Stream)

Sub: Reliability-I
Back Paper

Date: 27/07/2016 Full Marks: 100 Time: 3 hours.

Note: Answer all questions.

1. (a) Consider a coherent system ϕ with n associated components. Let $\rho_1(\underset{\sim}{x}), \ldots, \rho_p(\underset{\sim}{x})$ be the min path structures and $\kappa_1(\underset{\sim}{x}), \ldots, \kappa_k(\underset{\sim}{x})$ be the min cut structures corresponding to ϕ, then show that

$$\prod_{j=1}^{k} P[\kappa_j(\underset{\sim}{X}) = 1] \leq P[\phi(\underset{\sim}{X}) = 1] \leq \coprod_{j=1}^{p} P[\rho_j(\underset{\sim}{X}) = 1].$$

Hence show that if the components are independent with reliabilities $p_1, \ldots, p_n$, then

$$\prod_{j=1}^{k} \coprod_{i \in K_j} p_i \leq h(\underset{\sim}{p}) \leq \coprod_{j=1}^{p} \prod_{i \in P_j} p_i.$$

(b) Consider a 3-out-of-4 system with independent components. Compute the bounds obtained in (a) assuming $p_i = 0.7$, for $i = 1, 2, 3, 4$.

[(10+5)+10=25]

2. (a) Show that the min path sets of a coherent system are the min cut sets of its dual.

(b) Define modular decomposition of a coherent system with an example.

[7+3 =10]

3. (a) Define DFR life distribution. Is Weibull distribution applicable when the lifetime distribution is neither IFR nor DFR?- Justify your answer.

(b) Consider two independent lifetime random variables T_1 and T_2, where T_i follows gamma distribution with shape parameter p_i and scale parameter λ, $i = 1, 2$. If $p_1 = 7/11$ and $p_2 = 6/11$, then check whether the distribution of $T = T_1 + T_2$ is IFR or DFR.

(c) Let T be the lifetime of a unit with constant hazard rate λ. Show that the mean residual lifetime of the unit is equal to its MTTF.

[(2+2)+ 4 + 4=12]

4. Consider a k-unit parallel system with independent components, in which each component's lifetime is Exp(λ). Find the hazard function of the system lifetime. Show that the lifetime distribution of the system is IFR. Derive expected lifetime of the system.

[2+5+3 =10]

5. Consider a 2-unit standby redundant system with a switch. The standby unit carry no load while in standby mode. The lifetime of the primary active unit has constant failure rate λ_1. The lifetime of the standby unit has hazard rate λ_2. The switch activates the standby unit when primary active unit fails. Let p_s be the probability that switch activates the standby when required. Find the reliability of the system. Find the expected lifetime of the system.

[6+4=10]

6. Consider the following right-censored observations.

$$4, 5+, 8, 9+, 12, 14+, 15, 20, 26+, 30+, 34, 45$$

   (a) Obtain the Kaplan-Meier estimate based on the right-censored lifetime data.

   (b) Give an approximate 95% confidence interval for $R(22)$.

   (c) Suppose the pdf of the lifetime distribution is

$$f(t) = \frac{2t}{\sigma^2} e^{-t^2/\sigma^2}, \quad t \geq 0, \quad \sigma > 0.$$

   Derive the maximum likelihood estimate of σ. Compute the maximum likelihood estimate of $R(22)$ along with its standard error.

[8+4+(5+2+4)=23]

7. Suppose that n identical items are put on a life test. The experiment is continued until a pre-fixed number (r) of items have failed. We observed the r ordered failure time. Suppose the lifetime follows exponential distribution with hazard rate λ. Find an unbiased estimator of expected lifetime.

[10]

INDIAN STATISTICAL INSTITUTE

Back paper Examination: 2016 – 17

M. Tech. (QR OR) ; 1 Year ; E-Stream
Subject : Elements of Stochastic Process

Date : 28.07.2016 **Full Marks** : 100 **Time** : 3 hours

(1) Define the following :

(a) Irreducible Markov Chain.
(b) Positive and Null Recurrence.
(c) Mean Recurrence Time of a recurrent state.
(d) First passage probabilities.
(e) Class Property. (5 × 4) = [20]

(2) A meter measuring a sequence of electric impulses records the highest voltage that has passed through it up to any given time. Suppose that the impulse passing through the meter at the n[th] moment is Y_n. Assume that the probability distribution of Y_n is:

$P(Y_n = k) = \frac{1}{M+1}$ for $k = 0,1,2 \ldots \ldots \ldots, M$

Denote by X_n the meter reading at the n^{th} moment.

(a) Is $\{X_n | n = 0,1,2 \ldots..\}$ a time homogeneous Markov Chain? If so, what is its state space?

(b) What is the one step transition probability matrix of $\{X_n | n = 0,1,2 \ldots..\}$? (8 + 17) = [25]

(3) Derive the relationship between the generating functions of the sequences of n-step transition probabilities $\left\{p_{ij}^{(n)}\right\}_{n=0}^{\infty}$ and the first passage probabilities $\left\{f_{ij}^{(n)}\right\}_{n=0}^{\infty}$. [20]

(4) Consider a discrete time parameter time homogeneous Markov Chain with state space I = {1,2,3,4} and one-step transition matrix

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 \\ 1/8 & 7/8 & 0 & 0 \\ 1/2 & 0 & 1/8 & 3/8 \\ 1/3 & 1/3 & 1/6 & 1/6 \end{bmatrix}$$

(a) What are the possible communications among the states?
(b) Find f_{ii}^{*} for i = 1,2,3,4 and hence conclude which states are recurrent and which are transient. (6 + 14) = [20

(5) A Professor continually gives exams to the students. He can give three possible types of exams and his class is graded as either having done well or badly. Let p_i denote the probability that the class does well on a type i exam and assume that p_1=0.3, p_2=0.6, p_3=0.9. If the class does well on an exam, then the next exam is equally likely to be any of the three types. However, if the class does badly, the next exam is always type 1. What proportion of exams are of type i, i=1,2,3? [15