INDIAN STATISTICAL INSTITUTE

Semestral Examination (Backpaper): Semester II (2013-14)

M. TECH. (QR & OR) I Year

Reliability I

24.07.14

??.??.2014 Maximum marks: 100 Time: 3 hours

Answer all the questions. Calculator and RMM Table needed.

(a) Define modular decomposition of a coherent system. By means of example, show that this decomposition is not unique.

(b) If A is a modular set of a coherent sytem ϕ so that

$$\phi(x) = \psi\left(\chi(x^A), x^{A^c}\right),$$

prove that

$$\phi^D(x) = \psi^D\left(\chi^D(x^A), x^{A^c}\right),$$

so that A is also a modular set of ϕ^D.

[(1+3)+5=9]

2. Consider a bridge structure with independent components having component reliabilities as $p_3 = 0.8$ and $p_j = 0.6$ for $j \neq 3$. Compute the tightest possible bounds for the system reliability. [10]

3. Consider a stereo system with five independent components, which is a series structure of (1) Radio and Player in parallel, (2) an Amplifier, and (3) two Speakers in parallel. The Radio and Player both have exponential(α), the Amplifier has exponential(β), and both the Speakers have exponential(γ) life distributions. Find the reliability of the system at time t_0. Prove that the system's life distribution is IFR. [4+6=10]

4. Consider a system with three independent components, which are required to function till time 1000, 2000, and 1500 hours, respectively. The life distributions for the components are exponential($\lambda = 0.001$). Once a component fails, it is immediately replaced by a spare having identical life distribution. Find the minimum number of spares required to ensure the operation of the system with at least 0.95 probability. [9]

5. (a) Give an example of a life distribution which is neither IFR nor DFR.

(b) Let X have a Gamma distribution. Prove if it is IFR or DFR stating the conditions.

(c) Give an example to show that the sum of two DFR random variables is not necessarily DFR.

[3+5+4=12]

6. Consider a parallel system of two components where the load is shared by the components. When both the components are operating, the p.d.f. of life of each component is $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$ and when only one component is operating (the other having failed), the p.d.f. of life of the operating component is $f^*(x) = 2\lambda e^{-2\lambda x}$, $x \geq 0$. Find the expression for the reliability of the complete system. [10]

P.T.O

7. (a) Describe type II censored data. With type II censored data from exponential (λ) life distribution, find an exact 95% confidence interval for reliability at time $t = t_0$. Give an asymptotic 95% confidence interval for the same. Prove that the maximum likelihood estimate of expected life time is unbiased.

   (b) The following are 15 random numbers between 0 and 1.
   .494 .634 .248 .245 .278 .671 .598 .986 .179 .478 .299 .055 .460 .966 .904
   Simulate 25 type II censored observations from exponential ($\lambda = .04$) life distribution with 15 failures, using the above 15 random numbers, giving details of the simulation method. Find the exact and asymptotic 95% confidence interval for reliability at time $t = 40$, as derived in (a) above, based on this simulated data.

   [(2+5+5+5)+(7+3+3)=30]

8. Consider randomly censored life time data on products from two different brands, the ith brand having n_i products under consideration, for $i = 1, 2$. The life time T_i of a product of ith brand has a Weibull distribution with parameters λ_i and p (independent of i). The interest is to test for homogeneity of the two brands. Formulate the null hypothesis. Develop an asymptotic test giving details of the method. [2+8=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester examination: 2014-15

Course Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: Operations Research-I

Date of Exam: **25.08.14** **Total Marks: 75** **Duration:** 2½ hrs.

Answer as many as you can. Maximum you can score is 75.

1. (a) An indigenous mobile manufacturer produces two brands of mobiles. Long-term projections indicate an expected demand of at least 100 brand-I and 80 brand-II mobiles each day. Because of limitations on production capacity, no more than 200 brand-I and 170 brand-II mobiles can be made daily. To satisfy a contract, a total of at least 200 mobiles much be despatched each day. Each brand-I mobile sold, results in a \$2 loss, but each brand-II mobile produces a \$5 profit. Formulate the optimization problem to maximize the net profit.

   (b) Show graphically how many of each brand should be made daily to maximize net profit? What is the value of expected net profit to be maximum?
   [*Mark all extreme points, feasible region, constraint lines and use line of the objective function to find out the optimal solution.*]

   [10+8=18]

2. (a) Show that the set of vectors $a^1 = (2,1,4)$, $a^2 = (1,-1,2)$ and $a^3 = (3,1,-2)$ form a basis in E_3.

   (b) Show that the following system has no solution.

$$4x_1 + 5x_2 + 10x_3 = 7$$
$$-x_1 + 7x_2 + 14x_3 = 3$$
$$3x_1 - 2x_2 - 4x_3 = 5$$

(c) Identify the special situations (redundancy/infeasibility/degeneracy/unbounded) for the following LP problem and show it graphically.

Maximize Profit = $x_1 + 2x_2$

Sub to

$2x_1 + x_2 \leq 20$

$x_1 + x_2 \leq 15$

$x_1 \leq 20$

[5+5+7=17]

3. (a) Write down the general formulation of an Assignment Problem and define the role of decision variables.

(b) Solve the following assignment problem by Hungarian Method (minimize total cost).

The owner's objective is to assign the three projects to the workers in a way that will result in the lowest total cost to the shop. Note that the assignment of people to projects must be on a one-to-one basis. Calculate the lowest total cost f of the project.

	Project		
Workers	P1	P2	P3
Arjun	22	28	12
Bobby	16	20	22
Rajesh	18	24	14

[5+10=15]

4. (a) Write down the general formulation of a transportation problem after explaining supply and demand constraints and the objective function. What do you mean by an unbalanced transportation problem?

(b) Determine an initial b.f.s by Minimum Cost method and compare it with Vogel's Approximation Method (VAM) for the following T.P.

		Distrn. Centres				
		D1	D2	D3	D4	Supply
Plants	P1	19	30	50	10	7
	P2	70	30	40	60	9
	P3	40	8	70	20	18
	Demand	5	8	7	14	34

[12+13 = 25]

5. (a) Explain the purpose of ABC analysis in inventory management. What are the different components of inventory cost and what is their inter-relationship?

(b) What are the basic assumptions of an EOQ model? Derive optimal order quantity, and ordering interval of the EOQ model.

[7+8 = 15]

INDIAN STATISTICAL INSTITUTE

Mid-semestral Exam.

Course Name: M.Tech. (QROR) **Year: 1st year**

Subject Name: Programming Techniques and Data Structures

Date: 26.08.14 **Maximum Marks: 60** **Duration: 3.00 hrs**

Answer all questions.

1. a) Distinguish between linear and non-linear data structures giving one example of each.
   b) Write if the following is linear or non-linear
   (i) 2-d matrix (ii) singly linked list (iii) doubly linked list (iv) binary tree (v) queue.
   (3+5)

2. a) Using dynamic memory allocation, write a small piece of C code to allocate a block of 50 floating point numbers.
   b) A file named numbers.dat contains a number of integers in random order. Write a C code to open the file, read the numbers, compute their mean, write the mean at the end of the numbers in the file, and then close the file. Note that at the end, the file must contain the numbers in the given order followed by their mean. (3+5)

3. a) A two dimensional array Mat[10][12] is stored in the column major form. Derive the expression for indexing the element Mat[4][8].
   b) You need to store a list of positive integer elements, where the list is to be terminated by entering -999. What data structure will you use for storing these elements? Give reasons for your answer. Write a C code for this problem.
   (3+1+2+5)

4. a) Define stack and queue. Explain how can you implement these structures using linked list (do not write the code)?
   b) What do you mean by recursive function call? Explain with an example how stack may be useful for recursive function call.

c) Write an algorithm for checking if a given string of length *n* is a palindrome or not. (2+2+3+3+2+3+6)

5. a) Convert the expression (x+2) * y-10 – (z-2) * (q-5) into postfix and prefix notations.
   b) In computer systems infix expression is converted into postfix before evaluating it. Give reasons for this. (3+3+2)

6. Define a node of a doubly linked list using struct data type in C. What advantage does a doubly linked list have over single linked list? (2+2)

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Mid-Semestral Examination : 2014-15

M-TECH(QR&OR) -- 1st YEAR (E - STREAM)

PROBABILITY-1

{Answer all the questions}

Date: 27.08.14 **Full marks: 60** **Time: 2 hours**

[Symbols have their usual meaning]

1. a) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls at random. Find the probability that he will draw at least one ball of each colour.
   b) Define monotonic sequence of events. Let $\{A_n\}$ be monotonic sequence of events, each belonging to sigma field of events $A \subseteq \Omega$, then prove that,
   $\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$.

   [6+2+12=20]

2. a) Write down the probability density function of Gamma distribution with parameter α and p. Find out its moment generating function, mean and variance.
   b) Fit a negative binomial distribution to the following data.

x	0	1	2	3	4	5
freq	213	128	37	18	3	1

   [10+10=20]

3. a) There are 4 workstations of type A, each having 6 fitters and 3 turners; and 3 workstations of type B, each having 2 fitters and 4 turners. One workstation is selected at random and a person is chosen at random from it. If he is found to be a turner what is the probability that he is working in type A station?
   b) A machine normally makes items of which 4% are defective. An inspector selects a sample of size 10. If it contains no defective item then the lot is accepted. What is the probability that the lot will be rejected?
   c) Suppose A, B, C are independent components of a system with survival probability 0.3, 0.9, 0.9 respectively. The system will survive if the component A and at least one of the components B and C survive. What is the survival probability of the system?

   [8+6+6=20]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : 2014-15(First Semester)

Course Name: M. TECH. (QROR) I Yr.

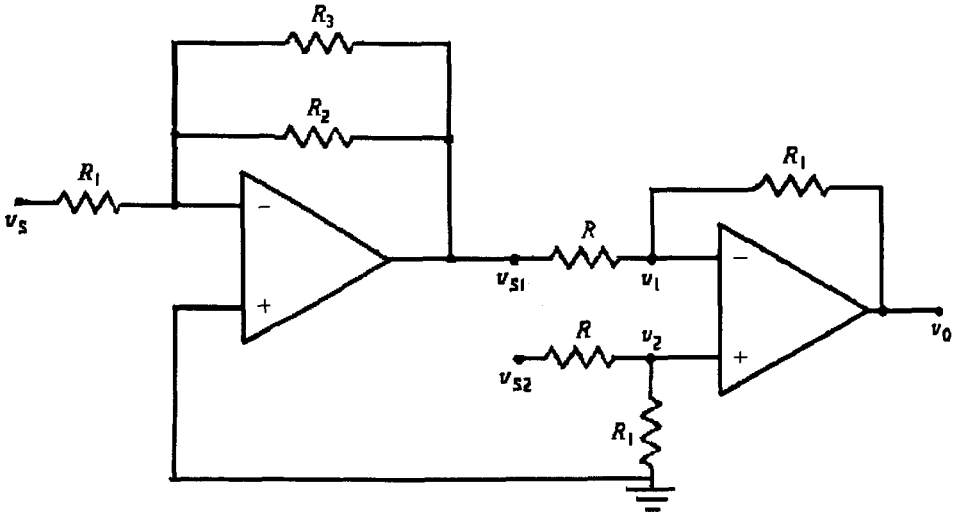
Subject Name : Electrical and Electronics Engineering

Date : 27.08.14 Maximum Marks : 50 Duration : 2Hrs

Answer any 5 questions.

1. What is an OP-AMP? Explain the operations of an OPAMP as an integrator and an adder. [2+4+4=10]

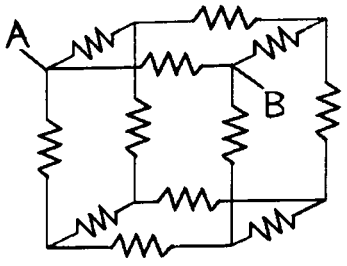
2. For the circuit, given below, find the output voltage v_0 for v_S=1volt, v_{S2}=0.5volt, R_1=2kOhms, R_2=8kOhms, R=2kOhms and R_3=8kOhms. [10]



3. a) If f(t) is a complete response of a circuit, then define the steady state response and the transient response of that circuit. [3]

b) A series RL circuit is excited by a battery of e.m.f. E at time t=0. Find the steady state current and the transient current in that circuit. [4+3]

4. A cube is formed by joining equal wires, each of resistance 1 Ohm. The cube is shown in figure below. Calculate the equivalent resistance between the points A and B. [10]



5. State and prove Thevnin's theorem. [10]

6. a) State Kirchhoff's current and voltage law. [3]

b) Suppose, three resistances R_1 , R_2 , and R_3 are connected in delta formation and the delta formation is equivalent to a star formation comprising of resistances x, y, and z. Find x, y, and z individually in terms of R_1 , R_2 , and R_3. [7]

7. a) Derive the octal equivalent of the decimal number 75.25

b) Show that an OR gate can be realized with diodes, batteries and resistances.

c) Construct an OR gate with AND and NOT gates. [2+5+3=10]

INDIAN STATISTICAL INSTITUTE
M.Tech (QR & OR) 1st YEAR (E & S Streams)
Session: 2014-2016
MIDSEMESTRAL EXAMINATION

Subject: SQC1

Date of Exam: 28/08/2014 **Max. Marks: 100**

Time: 3 hrs.

Group – A (Control Chart)

Answer All Questions.

1. (a) Mention Garvin's 8 dimensions of quality.

   (b) Draw the Kano model for customer satisfaction.

   (c) Draw the ServQual model that was developed by A. Parasuraman, L. L. Berry, and V. A. Zeithaml.

   [8+6+11=25]

2. (a) What is stratification?

   (b) Write in brief about the general principles of how stratification is done.

   (c) What are the different check sheets possible considering their functional aspects?

   (d) Construct a Pareto Chart to identify the "Vital Few" and "Useful Many" defects considering the data given in the following table related to defects on 2000 ceramic trays produced over a six-month period in a factory.

Type of Defect	Frequency
Damaged handles	34
Broken handles	6
Color spots	4
Non-uniform color	147
Missing color	3
Non-flat surface	22
Rough surface	112
Damaged bottom	5
Defective decoration	16
Broken tray	1

   [2+6+6+11=25]

3. A factory manufactures power sockets for use in the refining of oil. The hexagonal inside of the socket is prepared by an electrolytic method, and the resulting diameter of the hexagon is very critical. The quality control engineer has decided to prepare $\bar{X}$ and R charts for this process. Rather than measure every possible diagonal in the socket, one diagonal is selected at random from each socket and recorded. A sample of 5 consecutively produced sockets is taken each day. The average and range of these measures in centimeter are given in the following Table for 22 subgroups collected for a month.

Day	$\bar{X}$	R	Day	$\bar{X}$	R
1	20.813	0.035	12	20.827	0.028
2	20.821	0.021	13	20.808	0.032
3	20.816	0.038	14	20.824	0.037
4	20.810	0.016	15	20.816	0.024
5	20.822	0.028	16	20.818	0.018
6	20.828	0.019	17	20.829	0.033
7	20.814	0.039	18	20.823	0.059
8	20.817	0.025	19	20.813	0.019
9	20.822	0.016	20	20.814	0.023
10	20.809	0.019	21	20.828	0.027
11	20.824	0.037	22	20.811	0.024

(a) Calculate the central lines and trial control limits for both $\bar{X}$ and R charts.

(b) Calculate the revised central lines and control limits for future control purpose for both the charts.

(c) Draw the $\bar{X}$ and R charts and conclude whether the process was in control or not.

(d) Suppose the specifications are (20.80, 20.85). Assuming that the socket dimensions are normally distributed, estimate the fraction of production that does not meet the specifications.

(e) If the process is centered at the mid-specification, how much will be the reduction in fraction of production that does not meet the specifications?

[6+4+8+5+2=25]

4.

(a) A lot contains 100 items, 5 of which do not conform to requirements. If 10 items are selected at random without replacement, then calculate the probability of finding one or less non-conforming item in the sample.

(b) A manufacturer knows that 10% of the items produced in a certain large batch are nonconforming. What is the probability of 2 nonconforming items

occurring in a sample of 5? What is the probability that there are 1 or fewer nonconforming items in the sample?

(c) The average number of paint defects in the bumper area of an automobile is 3. In a randomly selected vehicle, determine the probability of finding 2 or fewer paint defects.

(d) State the central limit theorem.

(e) Why average or mean is preferred to individual observations for constructing control limits for a measurable characteristic?

[5+(3+3)+5+5+4=25]

Indian Statistical Institute

Mid-Semester Examination 2014 – 15

Course Name: M Tech (Q, R & OR), 1st Year **Subject Name: Quality Management**

Date: 29th August 2014 **Maximum Marks: 75** **Duration: 2 hours 30 minutes**

Note: Answer at most 4 questions. However, the maximum you can get is 75 only

1. Provide brief answers to the following.
   a. Explain Kano's model.
   b. What are the eight different dimensions of quality identified by Garvin? Provide brief explanation for each.
   c. Which of the dimensions identified by Garvin are likely to be considered as basic qualities of Kano's model? Explain. [4 + 16 + 5 = 25]
2. Answer the following
   a. How was quality defined by Juran and Crosby?
   b. What are the five different views of quality as defined by Garvin? Explain the views briefly.
   c. Which views do you think the two different definitions represent? Explain.
   d. In the context of quality management how do we define customers? [1 X 2 + 15 + 6 + 2 = 25]
3. Customer satisfaction is multidimensional and needs to be measured as a construct.
   a. Explain the IPA framework for measurement of customer satisfaction.
   b. The importance of different attributes in the context of customer satisfaction is likely to be asymmetric – explain. Explain the insight provided by Juran that 'quality is a moving target' from this perspective.
   c. Explain the concepts of search, experience and credence properties.
   d. Explain briefly the different components of Juran's trilogy. [5 + 4 + 4 + 6 + 6 = 25]
4. Answer the following
   a. What are the different components of cost of quality? Explain briefly. Give two examples of each type of costs.
   b. Draw the cost of quality curves to show how one can look at an optimal level of quality (economic quality perspective).
   c. What is Quality Assurance? How is the concept of cost of quality connected to the concept of Quality Assurance? [6 + 8 + 6 + 2 + 3 = 25]
5. Explain the following briefly
   a. Service differs from products in three major perspectives. What are these perspectives? Explain briefly.
   b. Five major dimensions of service quality have been identified by the latest version of the SERVQUAL model. What are these dimensions? Explain each dimension briefly.
   c. Juran had identified two types of quality – quality of design and quality of conformance. Which one of these types predominantly impact cost and which one predominantly impact sales income? Explain. [6 + 15 + 4 = 25]

INDIAN STATISTICAL INSTITUTE

Semester Examination: 2014-15

M. Tech. (QR & OR), 1st Year, 1st Semester, E Stream

Subject: Statistical Methods I

Date: 01·09·14 **Duration:** 3 hours

Note: This paper carries a total of 106 marks. Answer as many questions as you can. But the maximum you can score is 100.

1. Consider the following as the 100 consecutive observations (coded) made over time on certain characteristic of a product.

Sample No.	Observation	Sample No.	Observation	Sample No.	Observation	Sample No.	Observation	Sample No.	Observation
1	7.5	21	6.4	41	0.9	61	9.5	81	3.2
2	7.8	22	-0.3	42	9.1	62	4.8	82	1.3
3	-0.2	23	8.0	43	6.2	63	0.5	83	9.8
4	6.4	24	7.5	44	0.0	64	9.1	84	4.8
5	8.4	25	0.2	45	8.1	65	6.3	85	0.5
6	0.1	26	6.8	46	7.6	66	0.2	86	9.1
7	4.8	27	8.6	47	0.0	67	7.9	87	6.1
8	9.1	28	0.3	48	6.8	68	7.5	88	0.2
9	1.0	29	4.9	49	8.8	69	0.3	89	8.3
10	3.6	30	9.3	50	0.2	70	7.1	90	7.1
11	10.2	31	0.9	51	5.4	71	8.7	91	-0.3
12	1.9	32	3.6	52	9.7	72	0.3	92	7.0
13	2.2	33	10.2	53	1.4	73	5.3	93	8.4
14	10.4	34	2.3	54	3.9	74	9.5	94	0.1
15	3.4	35	2.5	55	9.9	75	1.2	95	5.3
16	1.2	36	9.6	56	2.2	76	3.6	96	9.6
17	9.6	37	3.6	57	2.5	77	9.7	97	0.8
18	4.7	38	1.5	58	10.1	78	2.2	98	4.0
19	0.7	39	9.6	59	3.1	79	2.7	99	9.7
20	8.8	40	5.1	60	1.1	80	10.0	100	2.0

(a) Draw a run chart (plot of the observations over time) of the first twenty observations and make your observations. Suggest a statistical model for the data (use t and Y to denote the sample number and the observations respectively).

(b) Draw a first order lag plot (scatter diagram of Y_t vs. Y_{t-1}, where Y_t is the t^{th} observation) using the first twenty five observations and offer your comments.

(c) Draw a histogram using all the 100 observations and offer your comments.

(d) Summarize your learning, if any, based on the results obtained in (a)-(c) above. Assume that the patterns seen the run chart and the lag plot holds good for all the observations.

[4 + 5 + 6 + 12 + 8 = 35]

2. Following table gives the tail lengths (in inches) and the weights (in pounds) of ten wolves. Our objective is to predict weight from tail length. The scatter plot shows that the relationship between the two is approximately linear.

Tail Length	10	13	19	19	20	20	23	24	25	27
Weight	79	72	100	116	85	88	100	80	160	120

(a) Develop simple linear regression models both with and without the intercept term.

(b) Compute and then compare the error variances of the two models and hence suggest the model you will choose.

[7 + 7 + 3 + 3 + 3 = 23]

3. Consider the standard notations used in regression analysis.

(a) What is $S_{yy|x} = S_{yy} - (S_{xy})^2 / S_{xx}$?

(b) How can you compute $S_{yy|x}$ in the following situation without using the above formula?

x_1	$y_{11}, y_{12}, y_{13}, y_{14}, y_{15}$
x_2	$y_{21}, y_{22}, y_{23}, y_{24}$

[3 + 6 =9]

4. Prove that the OLS estimate of the slope coefficient β_1 is an unbiased estimate. State clearly the assumptions involved.

[12]

5. Consider the case of SRSWR and prove that the sample mean $\bar{y}$ is an unbiased estimate of the population mean $\bar{Y}$.

[12]

6. Write short notes on the following:

(a) Spearman's rank correlation coefficient

(b) Types of data

(c) Skewness and kurtosis of a distribution

[5 x 3 =15]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: 2014-15

Course Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: Operations Research-I

Date of Exam : 03-11-2014 **Max Marks:** 100 **Duration:** 2½ hrs.

Assignment: 30 marks

Answer any seven questions out of which Q. nos. 1 and 2 are compulsory.

1. Consider the following LPP related to a financial problem. Find the optimal solution using the Simplex method. What evidence indicates that an alternate optimal solution exists?

   Maximize Revenue $= 3X_1 + 4X_2$

   Subject to $2X_1 + 3X_2 \leq 6$

   $3X_1 + X_2 \geq 3$

   $X_1, X_2 \geq 0$

   [8+2=10]

2. Find the optimal solution to the following LPP using duality

   Maximize $(3X_1 + 2X_2)$

   Subject to

   $X_1 + X_2 \geq 1$

   $X_1 + X_2 \leq 7$

   $X_1 + 2X_2 \leq 10$

   $X_2 \leq 3$

   $X_1, X_2 \geq 0$

   [10]

3. If f is a flow in a flow network $G = (V,E)$ with source s and sink t, then prove that the following conditions are equivalent:

   a. f is a maximum flow in G.
   b. The residual network G_f contains no augmenting path.
   c. $|f| = c(S, T)$ for some cut (S, T) of G.

   [10]

4. Define *residual network* G_f, augmenting path p and *cut* (S, T) of a network G. Prove that the value of a flow in a network is the net flow across any cut of the network.

[6+4 = 10]

5. What are the assumptions of a Production order quantity (POQ) model and how it is different from EOQ model? Derive the conditions for which POQ model approaches to EOQ model. Explain the significance of POQ model parameters towards shortage/stock out situation.

[5+3+2=10]

6. State the basic assumptions of (t_p, S) policy. How to determine optimum order level for this model? [Explain clearly all the situations while determining optimum order level and the associated cost components]

[2+8=10]

7. Under the assumption of Poisson arrival and exponential distribution of the service time, find the steady state probability of n persons in the queue system for m service channels.

[10]

8. Characterize a queue model, in general, in terms of type of Markov process and state the reasons. Give some real life examples of encouraging and discouraging queues.

   In a security check point of an airport, arrivals are considered as Poisson with an average time of 3 minutes between one arrival and next. The length of security check is assumed to be exponential with mean 1.5 minutes. i) What is the probability that an arrival has to wait more than 10 minutes? ii) What fraction of time in a shift of 8 hours the security person in the counter will be busy?

[4+3+3=10]

9. Sony sells and services several brands of home appliances. Past sales for a particular model of Smartphone have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.3	0.2	0.15	0.15

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.15	0.35	0.50

Based on cost considerations as well as storage space, the company has decided to order 10 of these each time an order is placed. The carrying cost is Rs. 150 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 4,000 per stock out. The company has decided to place an order whenever there are only 2 washing m/cs. left at the end of the week. Simulate 10 weeks of operation for Sony with currently 5 units in inventory. What would be the weekly carrying cost under this situation?

[7+3=10]

INDIAN STATISTICAL INSTITUTE
First-Semester Examination : 2014-15
M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY -- 1

Note : Answer any FIVE questions

Date: 05.11.14 **Full marks:100** **Time: 3 hours**

[Symbols have their usual meaning]

1. a) State and prove De Moivre's central limit theorem. Explain its application in the field of SQC.
   b) Let X ~ Poisson(m). Show that $P(X \geq 2m) \leq 1/m$. (State the result you have used.)
   [(12+2)+6=20]

2. a) Derive the p.d.f of χ^2 distribution with n degree of freedom. Find a measure of skewness of χ^2 distribution and comment on its shape.
   b) Let X_i ~ independent Negative binomial distribution with parameters r_i and p respectively, i =1,2.
   Let $Y = X_1 + X_2$. Find the distribution of Y.
   [(10+4)+6=20]

3. a) Let X be a random variable with distribution function
   $F_x(x) = P(X \leq x)$
   Then prove that
   i) F is monotonic non decreasing,
   ii) $F(\infty) = 1$,
   iii) $F(-\infty) = 0$,
   iv) F is continuous to the right.
   b) Suppose X is a continuous random variable with p.d.f f(x) and c.d.f F(x). Let Y be another random variable such that Y = F(x). Find the p.d.f of Y.
   c) Let X ~ Bin (n, p). Prove that $P(X \leq k) = \frac{1}{B(n-k,k+1)} \int_0^q z^{n-k-1}(1-z)^k dz$
   [10+5+5=20]

4. a) Let $A_1, A_2, \ldots\ldots, A_r$ be r events not necessarily mutually exclusive. Find the probability of occurrence of exactly m events (m<r).
   b) Explain when do you say that two random variables X and Y are independent? Prove that if X and Y are independent then $\rho_{XY} = 0$. Is the converse true? Justify your answer.
   [12+8=20]

5. a) Two athletic teams A and B play a series of independent games until one of them wins 4 games. The probability of each team winning a game is 1/2. Find the probability that the series will end in at most 6 games.
   b) A and B throw alternately a pair of dice in that order. A wins if he scores 6 points before B scores 7 points. B wins if he scores 7 points before A scores 6 points. If A starts the game what is his probability of winning?

c) At a college entrance examination each candidate is admitted or rejected according to whether he has passed or failed the test. Of the candidates who are really capable 80% pass the test, and of those incapable 25% pass the test. It is known that 40% of the candidates who are appearing in the entrance examination are really capable. Find the proportion of capable college students.

[7+7+6=20]

6 a) The individual monthly wages (in rupees) obtained by a group of people is normally distributed with mean 50000 and standard deviation 15000. If three people are taken randomly from the group what is the probability that exactly two of them are getting monthly salary more than Rs 70000/- ?

b) Suppose that the probability of having a male child is 1/3. How many male children can be expected before the first female child ?

c) What is the probability that at least two students have the same birthday if there are r students in a class?

[10+5+5=20]

INDIAN STATISTICAL INSTITUTE

M. Tech (QR - OR) 1st Year (S Stream)
Session: 2014-2015
SEMESTER EXAMINATION
Subject: Workshop – 1 (Engg. Drawing)

Date of Exam: 5.11.14 Max. Marks : 60 Time : 3:00 hrs

Note: (a) Answer question No.1 (compulsory) and any other three questions.
(b) Write your Name and Roll no. at one corner of the drawing sheet.
(c) Marks allotted to each question are indicated in the bracket.

1. Sketch a sectional front view of a Socket and spigot joint. Use suitable dimensions to complete the drawing. [18]

2. Draw the projections of a hexagonal pyramid, base 50 mm side axis 70 mm long, having its base on the ground and one of the edges of the base inclined at 45° to the V.P. [14]

3. A triangular prism, base 50 mm side and axis 75 mm long, is lying on the ground on one of its rectangular faces with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane, at a distance of 15 mm above the ground. Draw its front view and sectional top view. [14]

4. Show by sketch a pair of mating spur gear and a pinion and also indicate any four of the following six parameters in the figure.

   a) Root diameter
   b) Addendum circle
   c) Dedendum circle
   d) Fillet radius
   e) Pitch circle diameter
   f) Working depth [14]

5. Show the following parts on a sketch of the threaded end of a screw.

   i) Core diameter
   ii) Outside diameter
   iii) Crest
   iv) Flank
   v) Depth
   vi) Pitch
   vii) Nominal diameter [14]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2014-15

Course Name: **M. Tech. (QR&OR)**

Subject Name: **Quality Management & Systems**

Maximum Marks: 100 Duration: 3 hours

Date: 07 November 2014

Note: There are two groups. Use separate answer sheet for each group.

Group-A: Maximum Marks: 60

Note: Answer any four questions out of six. The maximum you can score is 60.

1. Explain briefly the 14 different points identified by Deming for improving the quality of products or services offered by an organization, clearly explaining how these points may be grouped together.

[16]

2. Answer the following
   (a) Explain briefly the difference between 'process oriented' and 'function oriented' approaches for designing organizations.
   (b) According to Maslow human needs may be viewed as a hierarchy. How many levels do the proposed need hierarchy have? Explain the hierarchy briefly.

[5 + (1 + 10) = 16]

3. Answer the following
   (a) Name the eight different steps for carrying out six sigma projects. Names are sufficient; you need not explain the steps.
   (b) The project selection is carried out at the recognize stage and organizations often follow the Prospect – Evaluate – Prove – Close cycle route for selection of projects. Suppose you are a consultant responsible for implementing 'six sigma'. Explain briefly the merits and demerits of getting involved at the different stages of the project selection process.

[8 + 8 = 16]

4. Delivering a quality product or service has many barriers. Explain the seven major barriers that impact the quality of product or service delivered by an organization.

[16]

5. Answer the following
   (a) It is well known that measurement drives human and/or organizational behaviour and consequently it is important to choose measurements appropriately. However, in practice inappropriate measures are often chosen leading to undesirable human and/or organizational behaviour eventually leading to lack of quality. Give two examples of such measurements and their consequent impacts on behaviour.
   (b) Explain Taylor's principles of scientific management. Explain how this leads to functional approach of organizational design.

[8 + (4 + 4) = 16]

6. Answer the following
   (a) What are the different components of cost of quality? Explain each component briefly with examples.
   (b) Explain the concepts of search, experience and credence quality.

[10 + 6 = 16]

Group-B: Maximum Marks: 40

Note: Answer all the questions from this group.

1. You are working in an organization, which decides to implement ISO 14001: 2004. You are in the process of identifying environmental aspects and their associated impacts on all the activities. Suggest a suitable scheme to identify significant environmental aspects.

[12]

2. Write short note on any three of the following terms.
   (a) Repeatability and Reproducibility Error
   (b) Compliance Audit
   (c) Process Map
   (d) Environmental Policy
   (e) Document Control

[4 x 3 = 12]

3. Following are the objective evidences of an internal audit conducted according to both ISO 9001: 2008 QMS and ISO 14001: 2004 EMS. For any four objective evidences, identify whether the observation is related to QMS or EMS and provide your explanation and justification on being classified as a nonconformance or not.
   (a) Waste records show that waste was removed only once per week on six occasions during last six months instead of twice per week as stated in the procedure OpProc-8.
   (b) The foreman in the Machine Shop was referring to revision 02 of the drawing GT16783/10, but was not sure whether that is the latest revision or not.
   (c) In a company it has been noticed that the Quality Manager is also the Computer Project Manager and the Safety Manager is the Management Representative for both EMS and QMS.
   (d) A computer program is used to calculate the output value of the final product characteristic. However, there is no approved documented copy of the program.
   (e) Data for legal requirement, such as ETP (effluent treatment plant) discharge, is not available.

[4 x 4 = 16]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2014-15
Course Name: M. Tech. (QROR) I Year
Subject Name: Statistical Quality Control 1

Date: 10/11/2014 Maximum Marks: 100 Duration: 3 hours

[All Symbols Have Usual Meaning]

Group A (Control Chart): Full Marks 60

Answer Any Three Questions

1. a) What's the role of OC curve in $\bar{x}$ and R charts?

   b) Prove that for an $\bar{x}$-chart if the mean shifts from the in-control value μ_0 to another value $\mu_1 = \mu_0 + \delta\sigma$, then, $\beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n})$.

   c) Prove that ARL = $\frac{1}{1-\beta}$ for an out-of-control process.

   d) What are the assumptions for constructing a sloping $\bar{x}$-R control chart?

   e) Prove that under the assumptions in d) above, the parameters for estimating the central trend line are $\alpha = \bar{\bar{x}}$ and $\beta = \frac{\sum_{i=1}^{m} h_i \bar{x}_i}{\sum_{i=1}^{m} h_i^2}$, where α is the intercept and β is the slope of the straight line equation and h, an odd number, is the revised subgroup numbers with $\sum_{i=1}^{m} h_i = 0$.

   [2+4+4+3+7=20]

2. A paper mill uses a control chart to monitor the imperfection in finished rolls of paper. Production output is inspected for twenty days, and the resulting data are shown below. Use these data to set up an appropriate control chart for nonconformities per roll of paper. Does the process appear to be in statistical control? What center line and control limits would you recommend for controlling current production?

Data on Imperfections in Rolls of Paper

Day	Number of Rolls Produced	Total Number of Imperfections	Day	Number of Rolls Produced	Total Number of Imperfections
1	18	12	11	18	18
2	18	14	12	18	14
3	24	20	13	18	9
4	22	18	14	20	10
5	22	15	15	20	14
6	22	12	16	20	13
7	20	11	17	24	16
8	20	15	18	24	18
9	20	12	19	22	20
10	20	10	20	21	17

[20]

3. In a study to isolate both gage repeatability and gage reproducibility, two operators use the same gage to measure 10 parts three times each. The data are given below.

Measurement Data

Part Number	Operator 1 Measurements			Operator 2 Measurements		
	1	2	3	1	2	3
1	50	49	50	50	48	51
2	52	52	51	51	51	51
3	53	50	50	54	52	51
4	49	51	50	48	50	51
5	48	49	48	48	49	48
6	52	50	50	52	50	50
7	51	51	51	51	50	50
8	52	50	49	53	48	50
9	50	51	50	51	48	49
10	47	46	49	46	47	48

a) Estimate gage repeatability and reproducibility.

b) Estimate part variation.

c) Estimate total study variation.

d) What is your conclusion about gage capability?

[10+5+2+3=20]

4.

a) A company has been asked by an important customer to demonstrate that its process capability ratio C_p exceeds 1.33. It has taken a sample of 50 parts and obtained the point estimate $\hat{C}_p = 1.52$. Assume that the quality characteristic follows a normal distribution. Can the company demonstrate that C_p exceeds 1.33 at the 95% level of confidence?

b) The molecular weight of a particular polymer should fall between 2100 and 2350. Fifty samples of this material were analyzed with the results $\bar{x} = 2275$ and $s = 60$. Assume that molecular weight is normally distributed. Calculate a point estimate of C_{pk} and a 95% confidence interval on C_{pk}.

c) In designing a fraction nonconforming control chart with center line at $p = 0.20$ and three-sigma limits, what is the sample size required to yield a positive lower control limit? What is the value of n necessary to give a probability of 0.50 of detecting a shift in the process to 0.26?

[6+ (3+5) + (3+3) = 20]

Group B (Acceptance Sampling): Full Marks 40

Answer Any Two Questions

5. a) Compute P_a^I, P_r^I, P_a^{II} and P_a for fraction defective $p = 0.05$ in the incoming lot for the double sampling plan $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$, where P_a^I and P_a^{II} represent respectively probabilities of acceptance on the first sample and on the second sample, P_r^I represents the probability of rejection on the first sample and P_a represents the probability of acceptance on the combined samples.

b) Compute the Average Sample Number without curtailment for the double sampling plan in a) for fraction defective $p = 0.05$.

c) Design a double sampling plan by using Grubb's table with $\alpha = 0.05$, $\beta = 0.10$, $p_1 = 0.03$ and $p_2 = 0.14$ and $n_2 = 2n_1$.

Grubb's Table for Double Sampling Plan

Plan Number	$R = p_2/p_1$	c_1	c_2	Pn_1 for	
				$1-\alpha = 0.95$	$\beta = 0.10$
1	14.50	0	1	0.16	2.32
2	8.07	0	2	0.30	2.42
3	6.48	1	3	0.60	3.89
4	5.39	0	3	0.49	2.64
5	5.09	1	4	0.77	3.92
6	4.31	0	4	0.68	2.93
7	4.19	1	5	0.96	4.02

[12+3+5 = 20]

6. a) Draw the *AOQ* curve for the single-sampling plan with $N = 1000$, $n = 45$ and $c = 2$ when the lot goes through a screening inspection process if it is rejected. All the non-conforming units will be replaced by good items. Also find the *AOQL* and the corresponding incoming proportion defective.

   b) For a single-sampling attribute type plan determine the *ATI* for a lot with $N = 10000$, $n = 89$, $c = 2$ and $p = 0.01$.

[18+2=20]

7. Consider the following seven rectifying inspection sampling plans, which have a 0.08 probability of acceptance for a lot percent nonconforming of 12%. Assuming that lots submitted for inspection contain 2% nonconforming, determine the sampling plan for which ATI will be the lowest.

Plan	N	n	c
1	1000	20	0
2	1000	33	1
3	1000	45	2
4	1000	57	3
5	1000	68	4
6	1000	78	5
7	1000	89	6

[20]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

Course Name: M.Tech. (QROR) **Year: 1st year**

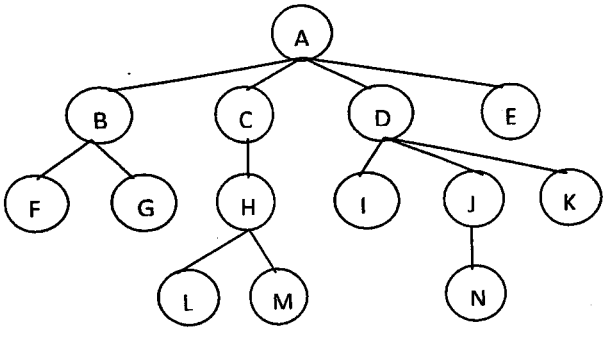
Subject Name: Programming Techniques and Data Structures

Date: 12.11.14 **Maximum Marks:75** **Duration: 3:00 hrs**

Answer all questions.

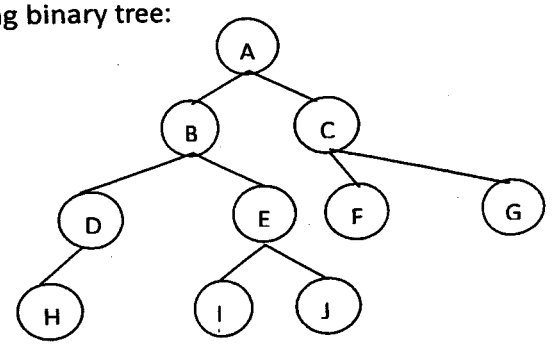
1. a) Write an algorithm to search for a number from an unordered list. What are the best case, worst case and average complexities of this algorithm?
   b) Consider that the numbers are sorted in ascending order. Write an efficient algorithm to search for a number in this list. What is the complexity of this search algorithm?
   (5+3+5+1 = 14)

2. What is a tree? What is the problem associated with the linked representation of a tree? Explain how the left child- right sibling representation solves this problem.



   Show the left child- right sibling representation of the above tree.
   (3+2+2+ 3 = 10)

3. Consider the following binary tree:



   Assuming the array representation for this tree, what is the minimum size of the array required? Give reason for your answer. Show the entire contents of the array representing this tree with adequate explanation.
   (2+3+4 = 9)

4. Write the post-order traversal of the tree shown in Question 3. Write a C function for performing post order traversal (do not write the entire program, but only the post order traversal function).

(3 + 4 = 7)

5. a) Define binary search tree (BST). For a list of *n* numbers, what are the minimum and maximum heights that a BST may have. Explain you answer.
   b) Draw a BST with the following numbers appearing in the given order 6, 8, 7, 2, 5, 1, 9.
   c) Write an algorithm for searching any arbitrary number in a BST.

(4+2+2+3+5=16)

6. A file called Input.txt contains several floating point numbers in an arbitrary order. You are required to read the numbers and save them in another file called Output.txt in the reverse order. Write a C program to perform this task.

(7)

7. Write short notes on any three of the following:
   a) Hashing b) Breadth first search c) Height balanced tree
   d) Complete versus full binary tree e) Bubble sort

(4+4+4=12)

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2014-15

M. Tech. (QR & OR), 1st Year, 1st Semester, E Stream

Subject: Statistical Methods I

Date: 14. 11. 2014 **Duration:** 3 hours

Note: This paper carries a total of 110 marks. Answer as many questions as you can. But the maximum you can score is 100.

1. The specifications for the dimension (X) of a part are given as 40 ± 4. It is assumed that X follows Normal distribution. Also assume that the process is set on target and the process variance is known to be 4. Apart from measuring the dimension (X) using a dial gauge, the parts are also classified as OK-Not OK using Go-No Go gauges. Let Y be the random variable denoting the number of Not OK pieces in a sample of size n.

   a) If we take a sample of size sixteen from the process then find the distributions of X-bar and Y.

   b) Now suppose that the process mean has shifted by one unit. It can be shown that under such a situation, the expected process fraction defective will be approximately 0.073. Our objective is to detect this shift based on a sample of size n = 100. Compute the standardized shift in process mean as

   S_X = (Old mean – New mean)/Standard deviation of sample mean

   Similarly, compute S_Y using the standard deviation of the on-target process.

   (C) Compare S_X and S_Y and comment on the data quality of X and Y.

   [6+6+5]

2. Following table gives the per capita GDP (Gross Domestic Product) and the % of urban population of twelve nations.

Nation	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Per capita GDP	85	100	200	300	500	2000	8000	15000	20000	26000	30000	50000
% of urban population	8	10	12	14	18	25	30	35	40	60	65	75

   a) Draw the scatter diagram using the above observations.

   b) It should be obvious from the plot obtained in (a) that the relationship is non-linear. Make suitable transformation to either of the two or both the variables to obtain a linear relationship. Draw scatter plots to judge the linearity.

   c) Assuming that the linear model satisfies the standard assumptions, state the statistical models using both the original and the transformed variable(s).

[3+6+6]

3. (a) Consider the linear statistical model $Y = \alpha + \beta X + \varepsilon$ that satisfies the standard assumptions. Derive the expression for variance of $\hat{\beta}$.

   (b) For the above linear model when will we have $Cov(\hat{\alpha}, \hat{\beta}) = 0$? Illustrate with an example the advantage of having $Cov(\hat{\alpha}, \hat{\beta}) = 0$.

[7+3+5]

4. Let x-bar and s^2 be the mean and variance respectively of a random sample drawn from a normal population.
   a) Prove that x-bar and s^2 are independent. Offer proper justifications of all the claims made to arrive at the proof.
   b) What are the implications of this result in process control?

[10+4]

5. A sample survey based on stratified random sampling without replacement was conducted to estimate the average age of all the students at the time of their admission. The sampling scheme used and the results of the survey are given below, where the notations used have their usual meaning.

Group	N_h	n_h	$(y\text{-bar})_h$	s_h
B. Stat	60	15	18	0.5
M. Stat	30	15	21	0.7
M. Tech	20	16	24	1.0

   (a) Estimate the average age of all the students.
   (b) Estimate the variance of the estimate obtained in (a) above
   (c) Was the use of stratified random sampling useful for the above purpose? Explain.

[4+6+3]

6. Given $U(0,1)$ and a set S of 100 observations, write an algorithm for drawing 500 random samples with replacement of size 5 from S.

[10]

7. (a) We know that the existence of correlation between two variables does not imply causation. Why?
   b) Find the transformations $f: X \to X'$ and $g: Y \to Y'$, so that we have $r_{XY} = b_{X'Y'}$, where r is the correlation coefficient, b is the slope of the regression line.

[5+5]

8. Write short notes on the following:
   a) Systematic sampling
   (b) Linear congruent generators
   c) χ^2 statistic and its applications
   (d) Finite population correction

[4x4]

INDIAN STATISTICAL INSTITUTE

First-Semester Examination : 2014-15

Course Name: M. TECH. (QROR) I Yr.

Subject Name : Electrical and Electronics Engineering

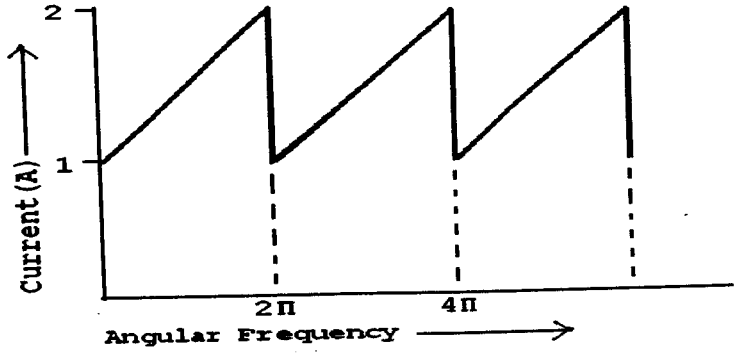
Date : 14.11.2014 Maximum Marks : 96 Duration : 3Hrs

Answer any 6 questions.

1. a) Explain with a diagram how AC voltage can be generated for a rectangular coil having N turns rotating in a uniform magnetic field with an angular velocity of w radian/second.

b) Show that, for an AC voltage the angular frequency $w=2\pi/T$, where T is the time period of one complete cycle.

c) Calculate the average value of current represented in figure below. [7+3+6=16]



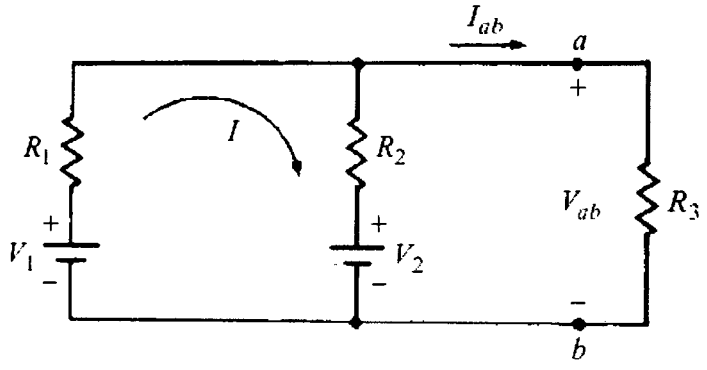
2. Draw the equivalent circuit and write the basic operating equations of a transformer. Show that in ideal situation the ratio of the primary to the secondary voltage is equal to the primary-to-secondary turns ratio. [4+12=16]

3. A DC voltage E is applied across a series RLC circuit. Find the steady state current in the circuit when $(R^2/4L^2) > (1/LC)$ and draw the respective curve of current w.r.t. time. [13+3=16]

4. Define control system, feedback and dead time. Draw the block diagram for a feedback control system and explain its individual blocks. Find the Laplace transform of $\frac{d^2y}{dt^2}$, where y is a function of time. [5+5+6=16]

5. a) State and prove maximum power transfer theorem.

b) For the circuit, given below, V_1 = 12V, V_2 = 6V, R_1 = 4 Ohms, and R_2=6 Ohms. Find the Thevenin equivalent for the network to the left of terminals a,b. Assume that the internal resistances of the batteries are 0. [8+8=16]



6. a) Using OPAMP explain the operations of a differentiator, differential amplifier, non-inverting amplifier and an integrator. [4+4+4+4=16]

7. a) Draw the block diagram of a negative feedback amplifier and derive an expression for closed-loop gain in terms of feedback ratio and transfer gain.

   b) An amplifier with negative feedback has a closed-loop gain of 100. Open-loop gain variation of 10% is expected owing to production limitations. Determine the value of open loop gain and feedback fraction β for which closed-loop gain will only vary by 1%. [7+(4+5)=9]

8. Explain the operation of a full adder with circuit diagram and truth table. [16]

9. Draw the hybrid-parameter equivalent circuit of a transistor in common emitter mode and calculate the current gain and input resistance in terms of the hybrid parameters. [3+6+7=16]

INDIAN STATISTICAL INSTITUTE

Back Paper Examination: 2014-15

M. Tech. (QR & OR), 1st Year, E Stream

Subject: Statistical Methods I

Date: 30.12.2014 **Duration: 3 hours**

Note: This paper carries a total of 118 marks. Answer as many questions as you can. But the maximum you can score is 100.

1. You want to study the stability and the extent of variation with respect to a certain characteristic of all the products produced in 24 hours by a high speed production process. You have decided to use Histogram as the tool for this purpose. Describe your data collection plan and all the steps in detail for constructing the Histogram.

[6+16=22]

2. Following table gives the data on length of service calls in minutes (Y) versus the number of units repaired (X). The repairs made are of the same type.

X	1	2	3	4	4	5	6	6	7	8	9	9	10	10
Y	23	29	49	64	74	87	96	97	109	119	149	145	154	166

(a) Fit the linear model $Y = \alpha + \beta X$ to the data, assuming the errors are normally distributed and have constant variance.

(b) Construct the ANOVA table for the fitted linear model.

(c) Using the model developed as above, construct the 95% confidence interval for the predicted value at X = 8.

[10+7+5=22]

3. Explain the inverse transform method for generating random numbers from a specified distribution using two examples.

[20]

4. Derive the expression for the variance of the estimate of population mean obtained following stratified random sampling without replacement. Describe the situations under which the stratified random sampling will provide a much more precise estimate (of population mean) than simple random sampling.

[14+6=20]

5. Define the terms 'statistic', 'sampling distribution' and 'standard error'. Why do we need to study sampling distributions? Describe the methods for constructing 95% confidence intervals for the Binomial proportion p, when the sample size (n) is small and also for the case when n is large.

[6+3+13=22]

6. Write short notes on the following:

(a) Post stratification

(b) Systematic sampling

(c) Role of simulation models in process improvement

[4x3=12]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2014-15

Course Name: M. Tech. (QROR)

Subject Name: SQC II

Date: 23·02·2015 **Maximum Marks:** 100 **Duration:** 3 hours

Note: Answer any five questions

1. (a) What is a multiple stream process (MSP)? Give two examples of MSP.
   (b) State the assumptions that are necessary for developing a Group control chart for monitoring an MSP.
   (c) What are the statistics plotted in the Group control chart?
   (d) Suppose a process has 6 streams. What will be the one-sided in-control ARL for the event that consecutive five times sample mean of a particular stream is the largest or smallest.
   (e) State four limitations of the Group control chart.

   [4 + 3 + 3 + 4 + 6 = 20]

2. Consider the observations shown in the following table. The target value for the process is 200.

t	y_t	t	y_t	t	y_t
1	200.0	6	174.5	11	174.2
2	215.8	7	151.6	12	143.6
3	195.8	8	174.3	13	163.1
4	191.3	9	166.5	14	172.4
5	185.3	10	159.0	15	160.7

   (a) Set up an integral controller for this process. Assume that the gain for the adjustment variable is $g = 1.2$ and assume that $\lambda = 0.2$ in the EWMA forecasting procedure will provide adequate one-step-ahead predictions.

   (b) How much reduction in variability around the target does the integral controller achieve?

   [17 + 3 = 20]

3. (a) What do you mean by 'Economic Design of $\bar{x}$ Chart'?

(b) The thickness (x) of a printed circuit board is an important quality characteristic and $\bar{x}$ chart is being used for monitoring of its mean. A newly recruited quality manager argued that the currently used parameters of the $\bar{x}$ chart may not be appropriate. These should be selected in such a way that the expected net income per production cycle is maximized.

It is observed that the process standard deviation (σ) remains unchanged. The time required for collection of samples and interpretation of results is proportional to the sample size. Only a single assignable cause of magnitude δ can occur at random according to a Poisson process with an intensity of λ occurrences per hour. Since only a single assignable cause occurs, a fixed time is required for searching and elimination of it. Derive the length of a production cycle under the assumption that the process operation can be allowed to continue in operation during the search for the assignable cause. Assume the necessary process parameters and cost parameters.

[2 + 18 = 20]

4. The tensile strength (x_1) and diameter (x_2) of a textile fibre are two important quality characteristics that are to be jointly controlled. The quality engineer has decided to use n = 10 fibre specimens in each sample. He has taken 20 preliminary samples. The nominal values of these quality characteristics and their sample covariance matrix have been estimated from the analysis of these preliminary samples as follows:

$$\bar{\bar{\mathbf{x}}} = \begin{bmatrix} 115.59 \\ 0.0106 \end{bmatrix} \text{ and } \mathbf{S} = \begin{bmatrix} 1.23 & 0.79 \\ 0.79 & 0.83 \end{bmatrix}$$

(a) Determine the Phase-I and Phase-II control limits of the T^2 control chart that may be used for joint monitoring of the means of the two quality characteristics. Given that $F_{0.001,2,179} = 7.18$.

(b) Determine the control limits of the control chart for generalized variance that may be used to monitor the multivariate dispersion.

(c) Suppose, at a time point t, the values of $\bar{x}_1$ and $\bar{x}_2$ are found to be 115.25 and 0.0104 respectively. On the other hand, at the time point t, the values of S_1^2, S_2^2 and S_{12} are found to be 1.25, 0.87 and 0.80 respectively.

(i) Compute the T^2 value at time point t

(ii) Compute the Generalized variance at time point t

(d) The items manufactured in an industry are packages of food. The lower specification limit of weight marked on each package is L kg. With the aim to ensure that packages meet the lower specification limit, the manufacturing target, T, is set as $L + \delta$. Suppose, n number of packages are manufactured over a period of time among which n_a number of packages are accepted and n_r are the number of packages are rejected. Further, assume that the net selling price of an accepted package is a, the net selling price of a rejected package is r $(r < a)$ and the cost of excess quality per unit in the accepted packages is g. What will be the net income per package?

[4 + 6 + (5 + 2) + 3 = 20]

5. (a) State a graphical approach for detection of autocorrelation in process data.

(b) The data that follow are molecular weight measurements made every 2 hours on a polymer (read down, then across from left).

2048	1935
2025	1948
2017	1966
1995	1954
1983	1970
1943	2039
1940	2015
1947	2021
1972	2010
1983	2012

It is found that the following first-order autoregressive model fit well to the molecular weight data: $x_t = 2005.6 + 0.683x_{t-1} + \varepsilon_t$. Set up an individual control chart on the residuals from this model. Interpret this control chart.

[2 + 18 = 20]

6. Use the following data to set up appropriate short-run $\bar{x}$ and R charts, assuming that the standard deviations of the measured characteristic for each part type are not the same. The nominal dimensions for each part are $T_A = 100$, $T_B = 200$ and $T_C = 2000$.

Sample no.	Part type	Measurements			
		M1	M2	M3	M4
1	A	120	95	100	110
2	A	115	123	99	102
3	A	116	105	114	108
4	A	120	116	100	96
5	A	112	100	98	107
6	A	98	110	116	105
7	B	230	210	190	216
8	B	225	198	236	190
9	B	218	230	199	195
10	B	210	225	200	215
11	B	190	218	212	225
12	C	2150	2230	1900	1925
13	C	2200	2116	2000	1950
14	C	1900	2000	2115	1990
15	C	1968	2250	2160	2100
16	C	2500	2225	2475	2390
17	C	2000	1900	2230	1960
18	C	1960	1980	2100	2150
19	C	2320	2150	1900	1940
20	C	2160	1950	2050	2125

[20]

7. (a) State the definition of 'quality' given by G. Taguchi. Mention two internal noise factors and two external noise factors that are usually responsible for functional variation of a product.

(b) Let m be the ideal value for the length of a component part. The specification of the component part is $m \pm \Delta$ and the loss sustained at the specification terminal points is 'A'. Determine the average loss per product due to variation.

(c) The quality of a product is defined by two characteristics: Brinell hardness number (BHN) and circular diameter. The specifications of these characteristics are:

Hardness in BHN: 250 ± 5

Diameter : 1.0000 ± 0.002 inch

The following BHN measurements were taken:

248	250	249	252	253
249	247	249	250	251
250	249	248	250	251
249	245	246	249	254

The following measurements of the diameter were also taken:

1.0010	1.0020	1.0015	1.0009	1.0019
0.9998	0.9999	1.0020	1.0011	0.9997
0.9980	1.0010	1.0009	0.9996	0.9990
1.0000	1.0013	1.0009	1.0009	1.0009

The loss caused by unacceptable BHN is ₹ 20 and the loss caused by unacceptable diameter size is ₹ 30. What is the total expected loss caused by deviations from target values?

[4 + 6 + 10 = 20]

INDIAN STATISTICAL INSTITUTE

Mid - Semester Examination: 2014 – 15

M. Tech (QROR), E-Stream, Semester II

Statistical Methods – II

Date: 24.02.2015 Maximum Marks: 60 Duration: 2 Hrs.

Note: Answer as much as you can. Maximum possible score is 60.

1. a) State and explain the regularity condition for the Cramer-Rao Inequality.
   b) Prove that, under these conditions, the variance of an unbiased estimator T for $\gamma(\theta)$, a differentiable function involving parameter θ, satisfies the inequality

$$\operatorname{var}(T) \geq \frac{\left[\gamma'(\theta)\right]^2}{E\left[\frac{\partial}{\partial\theta}\log f(x_1, x_2, \ldots, x_n \mid \theta)\right]^2}$$

[5+15=20]

2. a) Let X be distributed as Gamma with shape parameter α and scale parameter β, where both α and β are unknown and positive. Use the method of moments to find the estimator of α and β based on a random sample of size n.

   b) In a random sample of 80 automotive crankshaft bearings, collected from a lot 1000 bearings, 15 bearings have a surface finish that is rougher than the specification will allow. Find the 95% confidence interval on the expected number of bearings in the lot having surface finish inside the specification.

[12+8=20]

3. a) Define and explain
   i) Unbiased Test
   ii) Uniformly most powerful unbiased test.

b) Suppose a random sample of size n is collected from a normal distribution, where both mean μ and variance σ^2 are unknown. Obtain the critical region for testing the following hypothesis

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

using the likelihood ratio approach.

[2+5+13=20]

4. The burning rates of two different solid-fuel propellants used in aircrew escape systems are being studied. It is known that both propellants have approximately the same standard deviation of burning rate and is equal to 3 cm/sec. Two random samples of $n_1 = 20$ and $n_2 = 25$ specimens are tested and mean burning rates are obtained as $\bar{x}_1 = 24$ cm/sec and $\bar{x}_2 = 20$ cm/sec.

   i) Test the hypothesis that both propellants have the same mean burning rate. Use $\alpha = 0.05$.

   ii) What is the P-value of the test in part (i)?

   iii) Construct 95% confidence interval on the difference in means $\mu_1 - \mu_2$, where μ_1 and μ_2 denote the true mean burning rates for the two propellants.

[5+2+3=10]

INDIAN STATISTICAL INSTITUTE

Second Mid-Semester Examination (2014 – 2015)

Course Name : **M.Tech (QR & OR)**

Subject : **Industrial Engineering and Management**

Date : **25/02/2015**

Maximum Marks : **40**

Duration : **90 minutes**

Question Paper

Answer *all* questions

Marks: (2 X 10 + 10X 2 = 40)

1. For multiple choice questions, record your answer in your answer sheet indicating clearly the question number.

A. A manufacturing industry produces 300 spark plugs per shift of 8 hours. If the standard time per piece is 1.5 minute, the productivity would be

a. ¾ b. 5/8
c. 7/16 d. 15/16

B. In what way increase in productivity hels the management?

a. better working conditions
b. to stand better in market
c. job security and satisfaction
d. reduced price of articles

C. High productivity can be achieved by

a. employing more men and machines
b. giving recognition to trade unions
c. adopting proper system of preventive maintenance of machinery
d. increasing the working hours

D. Which of the following methods of increasing production does not come under the preview of productivity?

a. simplification of basic design

b. increasing plant and equipment capacity
c. better utilization of plant and equipment
d. motivation to workers

E. Which is not the objective of plant layout?
a. to make economical use of floor area
b. to minimize material handling
c. to keep machine break down to minimum
d. to maintain flexibility in arrangement of operations

F. All machines and equipment are grouped together at one location according to their functions in
a. process layout
b. product layout
c. fixed position layout
d. hybrid layout

G. Most of the large scale modern industry using automation adopt
a process layout
b. product layout
c. group layout
d. fixed position layout

H. Conveyors are extensively used as material handling equipment in case of
a. process layout
b. product layout
c. group layout
d. fixed position layout

I. For lifting heavy jobs in a shop, use is made of
a. conveyors
b. overhead crane
c. fork lift
d. hoist

J. Check the wrong statement
a. prime cost is greater than factory cost
b. administrative expenses are added to factory cost to obtain the production cost
c. total cost equals the production cost plus selling and distribution expenses
d. income taxes are generally based on gross earnings

2. Crash the following network. **[10]**

Activity	Immediate Predecessor	Normal Time (wk)	Normal Cost (Rs)	Crash Time	Crash Cost (Rs)
A	---	4	8,000	3	11,000
B	A	6	30,000	5	35,000
C	A	3	6,000	3	6,000
D	B	6	24,000	4	28,000
E	D	14	60,000	12	72,000
F	C	5	5,000	4	6,500
G	E&F	2	6,000	2	6,000
H	G	2	4,000	2	4,000
I	G	3	4,000	2	5,000
J	H&I	4	4,000	2	6,400
K	J	2	5,000	2	5,000

3. (a) What are the important factors of plant layout as far as safety is concerned?

   (b) Ineffective time is necessary in plant operation. Comment on it and justify.

   (c) How do you develop wage incentive scheme for different health care personnel?

[3+3+4]

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2014-15

Course Name : M. TECH (QR-OR)-I
Subject Name : MECHANICAL ENGINEERING
Date: 26.02.2015. Maximum Marks: 40 Duration: 1.5 hour
Note, if any :

Answer any *two* questions.
Assume suitable data if necessary.

1. a) What is tool life?
   b) During machining of a steel bar with a turning tool the tool life decreases from 80 min to 20 min while the cutting speed is increased from 60 m/min to 120 m/min. For the same tool find cutting speed at which the tool life will be 40 min.
   c) Discuss the different modes of tool failure. Write the essential properties of an ideal tool material.
   d) What is a machine tool? 2 + 8 + 8 + 2

2. a) What is meant by manufacturing process? Give a block diagram of manufacturing process showing the different inputs and different requirements of output (no explanation is required).
   b) What is meant by break even quantity? For a manufacturing system the fixed cost is Rs. 15,000 and variable cost per piece is Rs. 2. The selling cost is Rs. 5 per piece. Estimate the break even quantity for the system.
   c) Explain the primary cutting motion and feed motion in metal cutting operation.
   d) Define feed in metal cutting operation. 7 + 5 + 5 + 3

3. a) Name five common engineering materials.
   b) Write the assumptions for Merchant's Circle Diagram.
   c) In an orthogonal cutting test with a tool of rake angle 10^0, the following observations are made:

      Cutting ratio=0.3
      Horizontal component of cutting force=1290 N
      Vertical component of cutting force=1650 N

      Calculate
      i) shear angle, ii) coefficient of friction at the chip tool interface, iii) normal force on the shear plane and shear force along the shear plane. 3 + 5 + 12

4. a) Define cutting speed in metal cutting operation. With examples explain how cutting speed depends on work- tool material combination.
   b) Give illustrative sketches to show positive, negative and zero rake angles of cutting tool.
   c) Discuss about HSS and carbide as cutting tool material.
   d) A single point turning with rake angle=10^0 and clearance angle=7^0 is used to turn a MS bar of diameter=50 mm. The tool is set at a height of 2 mm above the job axis. Calculate the new rake angle and clearance angle of the tool. 5 + 4 + 6 + 5

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2014-2015

M.Tech. in QR & OR

Subject : PROBABILITY – II

Date: 26.02.2015 Maximum Marks : 70 Duration : 2 Hours

Notes:

(1) Unless stated otherwise, "M.C." will refer to a discrete time parameter, time homogeneous Markov Chain.

(2) The symbols have their usual meanings.

(1) Define the following:

(a) Stochastic Process.
(b) State space and Parameter space.
(c) Markov property.
(d) Transition probabilities.
(e) First passage probabilities.

(5 x 4) = [20]

(2) Suppose $\{X_n | n\epsilon\{0,1,2 \ldots..\}\}$ is a M.C. with state space I, initial distribution $\Pi = \{\Pi(i) \backslash i\epsilon I\}$ and transition matrix $P = \left((p_{ij})\right)_{i,j\epsilon I}$.

(a) Derive the joint distribution $X_0, X_1 \ldots.$ and X_n.

(b) Using the result obtained in (a), prove that

$P(X_0 = i_0 | X_1 = i_1 \ldots..., X_n = i_n) =$
$P(X_0 = i_0 | X_1 = i_1)$ for all $n \epsilon\{0,1,2..\}$ and $i_0, i_1 \ldots..., i_n \epsilon I$ such that the L.H.S is defined.

(15 + 5) = [20]

(3) Suppose we have two boxes, labeled 1 and 2 and 4 balls, labeled 1,2,3 and 4. Initially, some of these balls are in box 1 and the remainder are in box 2. An integer is selected at random from {1,2,3,4} and the ball labeled by that integer is removed from its box and placed in the other box. This procedure is repeated indefinitely. Assume that the selections are independent from trial to trial. Let X_n denote the number of balls in box 1 after the n^{th} trial (n=1,2....). Show that $\{X_n | n\epsilon\{0,1,2 \ldots..\}\}$ is a M.C. and find its transition matrix.

[10]

(4) (a) State and prove the Chapman – Kolmogorov Equations for a M.C. (You can use the fact that $P^{(m)}=P^m$ for m=0,1,2....).

(b) Show that
$p_{ij}^{(n)} = \sum_{k=0}^{n} f_{ij}^{(k)} p_{jj}^{(n-k}$ for n = 1,2,...and i,jϵI.

(5 + 15 = [20]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2014-15(Second Semester)

Course Name: M. TECH. (QROR) I Yr.

Subject Name: Instrumentation and Computer Engineering

Date: 27.02.15 Maximum Marks: 60 Duration: 2 Hrs 20 min

Answer any 5 questions.

1. Explain the operational procedure and derive the expression of the output voltage for a 4 bit R-2R ladder digital-to-analog (D/A) convertor. [12]

2. Show the classification of input type and passive type transducers with examples. Discuss about the operational procedure of an electromagnetic transducer and rectilinear type capacitor. [5+(4+3)=12]

3. What is the difference between an encoder and a decoder? Explain the operation of a 16 to 4 bit encoder with circuit diagram and truth table. [3+9=12]

4. Draw the circuit diagram of a 4 bit dual-slope analog-to-digital (A/D) convertor and explain its operational procedure. [12]

5. a) Explain the advantages and operational procedure of a PID controller with related equations and output graphs for any one type of error signal.

b) A PI(proportional+integral) controller is used for control of certain process. The settings are as follows. K_p (gain)=2%, P_o=40%, Reset rate =2%/min, E_p (error signal)=4t+6 where, t=time. What will be controller output in percentage after 2minutes? [(2+7)+3=12]

6. What are the advantages of a JK Flip flop over RS flip flop? Explain the operation of a RS flip flop for all possible input combinations. What is race around condition? [2+8+2=12]

7. Explain the operation of a 4 bit asynchronous down counter with circuit and timing diagram. [2+10=12]

8. Explain the operation of a full adder with circuit diagram and truth table. Show that a 3 bit addition can be realized with adders. [9+3=12]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : Semester II (2015-16)

M. TECH. (QR & OR) I Year

Reliability I

Date: 02.03.2015 Full marks: 70 Time: $2\frac{1}{2}$ hours

Notes: Answer all questions. Calculator is allowed.

1. (a) Prove that, for a coherent system ϕ,

$$\phi(\underset{\sim}{x} \amalg \underset{\sim}{y}) = \phi(\underset{\sim}{x}) \amalg \phi(\underset{\sim}{y}), \text{ for all } \underset{\sim}{x} \text{ and } \underset{\sim}{y}$$

if and only if the structure is parallel.

(b) Show that if P is a minimal path set and $Q \subset P$, then Q^c is a cut set.

(c) Prove that the dual of a module of a coherent system ϕ is also a module of the dual ϕ^D.

[7+5+5=17]

2. (a) Let ϕ be a coherent structure of n associated components. Prove that

$$\text{cov}(\phi(X), \Sigma_i^n X_i) \geq \text{var}(\phi(X)).$$

(b) Consider a bridge structure with independent components having component reliabilities as $p_j = 0.6$ for $j \neq 3$ and $p_3 = 0.8$. Compute the tightest possible bounds for the system reliability.

(c) Construct a coherent system with independent components having the vector of component reliabilities $\underset{\sim}{p}$ for which

$$h_\phi(\underset{\sim}{p}) = h_\psi\left(l_{\chi_1}(\underset{\sim}{p}), \cdots, l_{\chi_r}(\underset{\sim}{p})\right) > l_\psi\left(l_{\chi_1}(\underset{\sim}{p}), \cdots, l_{\chi_r}(\underset{\sim}{p})\right).$$

Symbols have their usual meaning.

[5+8+7=20]

3. Consider a coherent system (C, ϕ) with six components $1, 2, \ldots, 6$. Suppose (C, ϕ) has the modular decomposition $(\{1,2,3\}, \chi_1 = x_1 \amalg x_2 \amalg x_3)$, $(\{4\}, \chi_2 = x_4)$ and $(\{5,6\}, \chi_3 = x_5 \amalg x_6)$ with $\phi(x) = \coprod_{i=1}^3 \chi_i$, where x_i is the state of the ith component for $i = 1, 2, \ldots, 6$.

(a) Construct a fault tree for the system.

(b) Find the min cut sets corresponding to the fault tree.

(c) Find the probability of system failure using the min cut sets of the tree (assume that the components are independent).

(d) Compute the structural imprtance of components 1 and 4.

[3+3+6+6=18]

4. (a) Consider a continuous lifetime T with hazard rate $\lambda(t)$. Find the hazard rate of $Y = \ln T$.

(b) Define IFR life distribution.

(c) Consider a lifetime T with hazard rate

$$\lambda(t) = \begin{cases} \lambda_1 & \text{for } 0 \leq t < t_1 \\ \lambda_2 & \text{for } t_1 \leq t \leq t_2 \\ \lambda_3 & \text{for } t \geq t_2. \end{cases}$$

Find the reliability function $R(t)$ and hence obtain the mean lifetime.

[5+2+(4+4)=15]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2014-15

Course Name : M. TECH (QR-OR)-I
Subject Name : MECHANICAL ENGINEERING
Date: 20.04.2015. Maximum Marks: 100 Duration: 3 hours
Note, if any :

Answer any ***five*** questions.
Assume suitable data if necessary.

1. a) What are meant by homogeneous body and isotropic body?
   b) Sketch the typical tension stress-strain curve of ductile material and discuss it.
   c) Explain von Mises' criteria in connection with yielding criteria for ductile materials.
   d) Stress analysis of a spacecraft structural member gives the state of stress shown in Fig. 1. If the part is made from 7075-T6 aluminum alloy with yield stress of 500 MPa, will it exhibit yielding? If not, what is the safety factor? 4 + 6 + 5 + 5

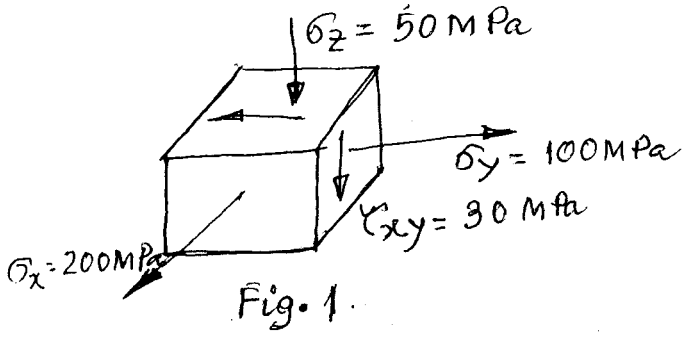


Fig. 1

2. a) Explain resilience and toughness of a material.
   b) Discuss about the measures of yielding and ductility.
   c) A 13 mm diameter tensile specimen has a 50 mm gauge length. The load corresponding to 0.2 percent offset is 6800 kg and the maximum load is 8499 kg. Fracture occurs at 7300 kg. The diameter after fracture is 8 mm and the gauge length at fracture is 65 mm. Calculate i) ultimate tensile strength ii) 0.2 percent offset yield strength iii) breaking stress iv) elongation v) reduction of area. 5 + 7 + 8

3. a) Under what conditions the cold working process is preferred over the hot working process?
   b) With the help of neat sketches explain the three high rolling mill for i) separate work piece ii) same work piece.
   c) Give illustrative sketches to show conventional and reverse drawing operations.
   d) Explain the closed die and open die forging operations. 5 + 5 + 5 + 5

4. a) What are meant by precision and accuracy of an instrument?
   b) Discuss about the effect of alignment in precision measurement.
   c) A dial indicator has a scale from zero to 1 mm and 100 divisions on scale. During a calibration test the following results are obtained.

Calibration length (mm)	Scale value (mm)
0.00	0.00
0.10	0.09
0.20	0.20
0.30	0.29
0.40	0.41
0.50	0.51
0.60	0.61
0.70	0.69
0.80	0.79
0.90	0.91
1.00	1.00

Determine the sensitivity of the dial indicator. Also determine the maximum error i) as a percentage of scale value ii) as a percentage of full scale value. 4 + 6 + 10

5. a) Determine the type of fit for the assembly 20 H7/f8. 20 mm lies in the diameter steps of 18 and 30 mm. The standard tolerance unit, i (in μm) $=0.45\sqrt[3]{D} + 0.001D$, where D is in mm. IT7 = 16i, IT8 = 25i. The upper deviation of 'f' shaft = $-5.5\ D^{0.41}$. Also show the limits of the hole and shaft.
   b) What is limit plug gauge? Explain with the help of a neat sketch.
   c) Discuss about the interchangeability. 12 + 4 + 4

6. a) Discuss about the effects of surface roughness on the performance of machined parts.
   b) Give idea of R_a and R_z values of surface roughness.
   c) The surface finish (R_a value) on the milled surface is not to exceed 6 μm with sampling length of 0.8 mm, machining allowance 0.5 mm and direction of lay parallel. How will you represent it on the working drawing? 10 + 5 + 5

7. a) What is machining accuracy? Give one sketch to show error in form.
   b) One mild steel bar of diameter = 25 mm is turned with an hss tool. The rpm of the job is 330. The depth of cut is 0.5 mm and feed is 0.2 mm/rev. Calculate the volume removal rate during this machining operation.
   c) How is a lathe specified?
   d) Give explanatory sketches to show up cut milling and down cut milling operations. Indicate the main motions required.
   e) Discuss about carbide as cutting tool material. 4 + 4 + 4 + 4 + 4

8. a) A bar of 75 mm diameter is reduced to 73 mm by a cutting tool, while cutting orthogonally. If the mean length of the cut chip is 73.5 mm, find the cutting ratio. If the rake angle is 15^0, what is the shear angle?
   b) Write the specification of a grinding wheel.
   c) Give a classification of machine tool.
   d) Explain the different types of chips formed during machining operation.

5 + 5 + 4 + 6

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2014 – 2015

M. Tech (QROR), First Year (E-stream)

Statistical Methods – II

Date: 20.04.2015 Maximum Marks: 100 Duration: 3 Hours

Note: ***Answer any five questions.***

1. (a) Define the following:

i) level of significance, ii) power of test, and iii) uniformly most powerful test.

(b) A particular type of capacitor has resistance that follows a normal distribution with a standard deviation of 200 mega ohms. The following data shows 8 measurements of the resistance of the capacitors from this population.

536.19 717.47 883.35 640.24 1029.02 505.32 756.86 812.76

The capacitor is said to have a mean resistance of 800 mega ohms. Does the sample suggest that the true mean resistance is different from 800 mega ohms? (Use $\alpha = 0.05$)

(c) Compute the power of the test, if the true mean resistance is 990 mega ohms.

[(2 + 2 + 3) + 8 + 5 = 20]

2. (a) Let X denote the number of defects observed on a large coil of galvanized steel. 75 coils are inspected and numbers of defects observed in each coils are recorded. Following table give the data, in the form of frequency, thus obtained.

Number of defects	1	2	3	4	5	6	7	8
Number of Coils	1	11	8	13	11	12	10	9

Does the assumption of Poisson distribution seem appropriate as a probability model for this data? Use $\alpha = 0.01$.

(b) A researcher claims that at least 10% of all football helmets have manufacturing defects that could potentially cause injury to the user. A sample of 15 helmets revealed that 2 helmets contained such defects. Use exact test to see whether the results of the sample support the researcher's claim? Use $\alpha = 0.01$.

[14 + 6 = 20]

3. An experiment was performed to determine the effect of four different types of tips in a hardness testing machine on the measurement of hardness of a metal alloy. Four specimens of the alloy were obtained and each tip was tested once in each specimen, producing the following data on hardness.

Type of Tip	Specimen			
	1	2	3	4
1	9.3	9.4	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

a) Is there any difference in hardness measurements between the tips?

b) Can it be concluded that tip-wise variations are equal? Use modified Levene test. Use $\alpha = 0.05$, in both the cases.

[9 + 11 = 20]

4. a) Regression method was used to analyze the data from a study investigating the relationship between roadway surface temperature (x) and pavement defection (y). Summary quantities are:

$n = 20, \sum y_i = 12.75, \sum y_i^2 = 8.86, \sum x_i = 1478, \sum x_i^2 = 143215.8, \sum x_i y_i = 1083.67$

i) Calculate the least square estimates of the slope and intercept.

ii) Test the significance of regression using analysis of variance and draw conclusion. Use $\alpha = 0.05$.

b) Assuming, for a multiple linear regression involving p variables, $\hat{b} = \left(X^T X\right)^{-1} X^T y$, where y is a column vector of order n, X is a $[n \times (p+1)]$ design matrix and b is a column vector of order $(p+1)$, define what is known as the 'hat' matrix (H) and prove that

i) H is idempotent, and
ii) H is positive semidefinite.

[12 + (3 + 5) = 20]

5. a) Define and explain nonparametric statistical inference?

b) Write down the advantages and disadvantages of nonparametric inference.

c) The impurity level (in ppm) is routinely measured in an intermediate chemical product. The following data were observed in a recent test:

2.4	2.5	1.7	1.6	1.9	2.6	1.3	1.9	2.0	2.5	2.6
2.3	2.0	1.8	1.3	1.7	2.0	1.9	2.3	1.9	2.4	1.6

It is claimed that the median impurity level is less than 2.5 ppm. State and test the appropriate hypothesis using the Wilcoxon Signed-Rank Test with $\alpha = 0.5$.

[4 + 7 + 9 = 20]

6. a) A manufacturer of a hot tub is interested in testing two different heating elements for his product. The element that produces the maximum heat gain after 15 minutes would be preferable. He obtains 10 samples of each heating unit and tests each one. The heat gain after 15 minutes (in °F) is shown below. Is there any reason to suspect that the second unit is better than the first? Use Mann-Whitney U test with $\alpha = 0.05$.

Unit 1	25	27	29	31	30	26	24	32	33	38
Unit 2	31	33	32	35	34	29	38	35	37	30

b) The total triglyceride level (TGL) and very low-density lipoprotein (VLDL) were measured on each of 15 patients. Data thus obtained are given below.

Patient Number	TGL	VLDL
1	20.19	4.51
2	27.00	6.03
3	51.75	7.98
4	51.36	9.58
5	28.98	7.54
6	21.70	3.96
7	14.40	8.60
8	15.14	5.46
9	50.00	13.03
10	23.73	7.12
11	29.33	8.94
12	19.98	5.85
13	13.28	3.73
14	27.00	6.98
15	14.40	9.32

Calculate Spearman's rank correlation statistic and hence show that there exist an association between the variables. [Use $\alpha = 5\%$]

[10 + 10 = 20]

INDIAN STATISTICAL INSTITUTE

M. Tech. (QR & OR), I Year, E-Stream

Session : 2014 – 2015, Semester II

Semestral Examination

Subject : **Probability II**

Date : 22.04.2015 Time : 3 hours Full Marks : 100

Note : Answer All Questions.

1. Define the following:

   (a) Essential and Inessential States.

   (b) Positive and Null Recurrence.

   (c) Irreducible Markov Chain.

   (d) Class Property.

   (e) Mean Recurrence time of a recurrent state. $(5 \times 4)=[20]$

2. State and prove the Basic Renewal Equation for a Markov Chain. [20]

3. Suppose $i \in I$ where I is the state space of Markov Chain. Show that state i is recurrent if and only if $\sum_{n=0}^{\infty} p_{ii}^{(n)} = \infty$. Interpret the result.

   (Note: You can use Abel's Lemma) (15+5) = [20]

4. Show that Recurrence is a class property. [20]

5. Show that a recurrent state is necessarily essential. [20]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2014-15

Course Name : M. Tech. (QROR)

Subject Name : SQC II

Date : 24.04.15 **Maximum Marks:** 100 **Duration:** 3 hours

Note : Symbols are having their usual meaning

Answer any five questions

1. (a) Describe the procedure of continuous sampling plan, CSP-2, using flow diagram.

   (b) Suppose, $p = 0.02$ is the fraction nonconforming when a process is operating in control. Let a CSP-1 plan with $f = 0.08$ and $i = 100$ is implemented for the process. Find out the following:

   i) The average number of pieces inspected in a 100% screening sequence after a nonconforming item is found.

   ii) The average number of units that will be passed under the sampling procedure before a defective unit is found.

   iii) The average fraction of total manufactured units inspected.

   iv) The average outgoing quality (AOQ).

   v) The average fraction of units passed under the sampling inspection procedure.

   $[5 + 3 \times 5 = 20]$

2. The tensile strength and diameter of a textile fibre are two important quality characteristics that are to be jointly controlled. The quality engineer has decided to use $n = 10$ fibre specimens in each sample. He has taken 15 preliminary samples. The computed sample means, sample variances and covariance in the preliminary samples are shown in the following table:

Sample no.	Sample means		Variance and covariances		
	Tensile strength ($\bar{x}_{1k}$)	Diameter ($\bar{x}_{2k}$)	S_{1k}^2	S_{2k}^2	S_{12k}
1	115.25	1.04	1.25	0.87	0.80
2	115.91	1.06	1.26	0.85	0.81
3	115.05	1.09	1.30	0.90	0.82
4	116.21	1.05	1.02	0.85	0.81
5	115.90	1.07	1.16	0.73	0.80
6	115.55	1.06	1.01	0.80	0.76
7	114.98	1.05	1.25	0.78	0.75
8	115.25	1.10	1.40	0.83	0.80
9	116.15	1.09	1.19	0.87	0.83
10	115.92	1.05	1.17	0.86	0.95
11	115.75	0.99	1.45	0.79	0.78
12	114.90	1.06	1.24	0.82	0.81
13	116.01	1.05	1.26	0.55	0.72
14	115.83	1.07	1.17	0.76	0.75
15	115.29	1.11	1.23	0.89	0.82

Establish a control chart for generalized variance.

[20]

3. (a) Let us consider a quality control system where the process is diagnosed on every 'n' production units by checking a product. If the process is found abnormal, then it is recovered to the normal condition. On the other hand, if the process is found to be normal, the production is continued without any recovering operation. Suppose, loss due to producing one unit of product under abnormal production process is A, cost of a diagnosis is B, adjustment cost is C, average trouble occurrence interval is $\bar{u}$ and the time lag of diagnosis is l. Determine the optimal diagnosis interval for this quality control system.

(b) A manufacturer uses turret lathes to bore the housings of four-way solenoid valves. The cutting tool used is made of HSS (high-speed steel). Two types of defective products occur after the boring is complete. Among all defectives, reworkable defectives occur 20% of time, and non-reworkable defectives occur 80% of time. The cost of reworking a defective housing is ₹6 and the cost of scrapping a non-reworkable defective is ₹50. Diagnosis cost is ₹1.50 and adjustment cost is ₹980. The monthly production is 8500 valves. Two adjustments are performed per month, the tool cost within the adjustment cost is ₹250 and the time lag is 4 units. A new tool with longer life has been introduced in the market with a cost of ₹1200. Find the minimum ratio between the expected life of the new tool and the expected life of the current tool that will make it profitable to use the new tool.

[10 + 10 = 20]

4. (a) The target value of a quality characteristic of a chemical process is y_0. The quality characteristic is measureable and adjustable. After an interval of Δ_t, the value of the quality characteristic is measured and instead of adjusting the whole gap between the measured and the target values, β times the gap is adjusted (β is nonnegative). Determine the optimal value of β.

(b) Describe the procedure (in the form of Worksheet) for calculation of process capability indices from non-normal process data using Clement's Tables for percentile points under the Pearsonian curves.

(c) Write the expressions for the measurement of generalized C_p, C_{pl} and C_{pu}.

[12 + 5 + 3 = 20]

5. (a) Describe one manufacturing operation where DNOM chart is the most appropriate tool for the process monitoring. Explain why the DNOM chart is the most appropriate tool for monitoring the stated manufacturing operation.

(b) Describe the procedure for Skip-lot sampling plan, SkSP-2.

(c) An important quality characteristic of a polymer is its molecular weight (y). It is observed that the disturbances in the molecular weight (i.e. deviation from the target value) can be compensated by making adjustments to the setpoint of the catalyst feed rate (x). It can be assumed that a change in catalyst feed rate (x) will produce all its effects on molecular weight (y) within one period, i.e. $y_{t+1} - T = gx_t$, where T is the target molecular weight and g is a constant called process gain. So the molecular weight can be maintained on target if the value of catalyst feed rate at time t is set in such a way that the disturbance in the molecular weight at time $t+1$ is exactly cancel out. But this cannot be done because the disturbance in the molecular weight at time $t+1$ is unknown at time t. So it is decided to forecast the disturbance of y at time $t+1$ using an EWMA, where $0 < \lambda \leq 1$ is the weighting factor for the EWMA and then, to set the value of x_t at time t in such a way that the forecasted disturbance in the molecular weight at time $t+1$ is appropriately compensated.

Show that under the above model for process adjustment, the actual adjustment that is to be made in x at time t is $-\frac{\lambda}{g}e_t$, where e_t is the error of prediction at time period t.

[(2+3) + 3 + 12 = 20]

6. (a) An $\bar{x}$ chart is used to maintain current control of a process. A single assignable cause of magnitude 2σ occurs, and the time that the process remains in control is an exponential random variable with mean 100 hours. Suppose that sampling costs are ₹0.50 per sample and ₹ 0.10 per unit, it costs ₹5.00 to investigate a false alarm, ₹2.50 to find the assignable cause, and ₹100 is the penalty cost per hour to operate in the out-of-control state. The time required to collect and evaluate a sample is 0.05 hour, and it takes 2 hours to eliminate the assignable cause. Assume that the process is allowed to continue operating during searches for the assignable cause. Evaluate the cost of the arbitrary control chart design $n = 5$, $k = 3$ and $h = 1$.

(b) At present, diagnosis on production process of one part is being done at its assembly process. That is, the production process of this part is being diagnosed by using the information from 100% inspection at its assembly process. Since any defective is found readily at the assembly process, its diagnosis cost is nil. The loss due to a defective part is ₹50. Adjustment cost is ₹5000, time lag of diagnosis is 800 parts, the production during last 3 months was 300000 parts and finally the number of troubles during last 3 months was 20. By the way, management is considering diagnoses immediately after production of parts. In that case, diagnosis cost increases to ₹100 and time lag of diagnosis decreases to 5 parts. Moreover loss due to defective decreases to ₹40 for reduction of supplemental value, and adjustment cost stays same as the present, namely ₹5000.

(a) Discuss the loss or gain for this proposal in which diagnosis are done immediately after production.

(b) As preventive maintenance a periodical tool change with $\overline{u}' = 10000$ parts is introduced where the risk of trouble before a tool change is 0.02. The cost for a tool change is ₹5000. Discuss the loss or gain for both diagnoses systems when this preventive maintenance in introduced.

[10 + (4+6)] = 20]

INDIAN STATISTICAL INSTITUTE
Semester Examination: 2014-2015 (Second Semester)

M. Tech(QR & OR) I Year (E & S Stream)

Reliability-I

Date: 27 April, 2015 Maximum Marks: 100 Duration: 3 hours.

Note: Total mark: 110. Answer as many questions as you can but the maximum you can score is 100.

1. Write true or false with justification

   (a) The hazard rate of a series system with independent components is the sum of its component hazard rates.

   (b) The sum of two independent DFR random variables is not necessarily DFR.

   (c) Let T_i be the lifetime of ith unit of a 2-out-of-3 system, $i = 1, 2, 3$. Then the lifetime of the system is $\min\{\max(T_1, T_2), \max(T_1, T_3), \max(T_2, T_3)\}$.

   (d) If the lifetime has constant hazard rate, then its mean residual life is increasing in time.

   (e) Weibull distribution is an example of neither IFR nor DFR life distribution.

   (f) The survival function corresponding to the proportional hazard model $\lambda(t; Z) = \lambda t \exp(\beta' Z)$ is $S(t; Z) = e^{-0.5\lambda t^2 + \beta' Z}$.

   [4+3+4+3+3+3=20]

2. (a) Consider a continuous lifetime random variable T with reliability function $R(\cdot)$ and cumulative hazard function $\Lambda(\cdot)$. Then, show that

$$R(t + x|t) = \exp\left[-\{\Lambda(t + x) - \Lambda(t)\}\right].$$

   (b) Suppose a discrete lifetime random variable T take on values $t_1, t_2, \ldots,$ with $0 < t_1 < t_2, \ldots, < t_k$. Let λ_i be the hazard at t_i, then derive the expression of $R(t_i)$ in terms of λ_i's.

   (c) Show that the hazard of Poisson distribution is monotone increasing.

   [3+6+6=15]

3. Consider a parallel system of n independent components each having life distribution with constant hazard rate λ.

   (a) Derive the hazard rate of the system lifetime.

   (b) Show that the life distribution of the system is IFR.

   (c) Derive the mean lifetime of the system.

   [4+6+5 =15]

4. (a) Consider a 2-unit standby redundant system with a switching system. The standby unit carry no load while in standby mode. The lifetimes of the units are independently distributed with constant hazard rates λ_1 and λ_2, respectively. The switch functions independently of the units has constant hazard rate λ_s. Find the reliability and MTTF of the system.

(b) Consider a load sharing parallel system with two independent components. The p.d.f. of life of each component is $f(t) = \lambda e^{-\lambda t}$, $t \geq 0$, when both the components are operating, and $f^*(t) = 5\lambda e^{-5\lambda t}$, $t \geq 0$, when only one component is operating (the other having failed). Find the reliability of the system.

[(5+3)+7=15]

5. (a) Describe Type-I and Type-II censoring schemes with data.
   (b) Let D be the number of failures under Type-I censoring. Find the variance of D.
   (c) Write down the drawbacks of Type-I and Type-II censoring schemes.
   (d) What is the difference between truncated and censored data?

[4+2+2+2=10]

6. Consider a life testing experiment in which n identical items are put on a test time $t = 0$ and all the failure times are observed and recorded. Suppose that the lifetime follows exponential distribution with hazard rate λ.
   (a) Derive the maximum likelihood estimate (MLE) of λ.
   (b) Check whether the MLE of λ is unbiased.
   (c) Find an unbiased estimator of mean lifetime.
   (d) Derive an exact 95% confidence interval of λ.

[3+5+2+5= 15]

7. A life testing experiment was conducted with 15 light bulbs. The lifetime in hours of 15 bulbs were as follows:

   50, 58, 61+, 63, 82+, 92+, 98, *102, 102, 105+, 108, 120, 121+, 140, 162.

   (a) Suppose the lifetime follows exponential distribution with hazard rate λ. Find the maximum likelihood estimates of λ, $R(108)$ and median lifetime. Find an asymptotic 95% confidence interval for $R(108)$.
   (b) Find the Kaplan-Meier estimate of $R(108)$ along with estimated variance.

[(3+1+3+3)+(6+4)= 20]

Hint:

(1) PDF of Weibull distribution: $\alpha\lambda(\lambda t)^{\alpha-1}e^{-(\lambda t)^{\alpha}}$.

(2) PDF of Gamma distribution: $\frac{\lambda^k}{\Gamma(k)}t^{k-1}e^{-\lambda t}$.

(3) For a standard normal variable Z, $P[Z \geq 1.96] = 0.025$.

INDIAN STATISTICAL INSTITUTE

Second-Semester Examination: 2014-15

Course Name: M. TECH. (QROR) I Yr.

Subject Name: Instrumentation and Computer Engineering

Date: 29.04.15 Maximum Marks: 96 Duration: 3 Hrs

Answer any 6 questions.

1. What is a content addressable memory? Explain the operation of it with block diagram, hardware implementation and emphasis on match logic. [3+(5+8)=16]

2. a) A compiler designer is trying to decide between two code sequences 1 and 2 and both the code sequences use three different instruction classes A, B, and C. The required CPI (clock cycles per instruction) for A, B, and C, are 1, 2 and 3, respectively. The number of instruction counts for A, B, and C are 2, 1 and 2, respectively, in code sequence 1. The number of instruction counts for A, B, and C are 4, 1 and 1, respectively, in code sequence 2. Which code sequence executes more instructions? Which will be faster? What is the average CPI for each sequence?

   b) A program runs in 20 seconds on a computer which has a 8 GHz clock. A substantial increase in clock rate is possible but that will increase the number of clock cycles by 1.5 times for the same program to run. What clock rate will be required to run that program in 6 seconds. [(3+3+4)+6=16]

3. a) What is a stack? Show the sequence of micro-operations for performing a pop operation in a 64 word register stack.

   b) Find the reverse polish notation for the expression (3x4)+(5x6) and show the related stack operations with diagrams for the same expression. [(3+5)+(3+5)=16]

4. a) What is the decimal value of the 32-bit two's complement (signed integer) number shown below ?
   $1111\ 1111\ 1111\ 1111\ 1111\ 1100\ 1111\ 1100_{two}$

   b) How many bits are allocated for the exponent and the fraction part of a double precision floating point number?

   c) Show the IEEE 754 binary representation of the number -0.75 in 32 bit precision.

d) How exponent underflow can occur in subtracting two numbers using IEEE 754 binary representation and 32 bit precision?

[3+2+7+4=16]

5. a) Explain the operating procedure of a CPU with 7 registers, MUX, ALU and decoder as major components.

b) The binary OPR word for performing XOR operation between contents in A BUS and B BUS in ALU is 01100. Find the 14 bit control word for performing the following operation using seven registers:

Register1 Content ← Register2 Content XOR Register3 Content

[12+4]=16

6. Explain the connection procedure of 4 RAMS (each having capacity of 128 words) and 1 ROM (capacity of 512 words) with CPU for read and write operations. [16]

7. What are the characteristics of a passive transducer? Explain the operation of resistance strain gauge, rotary plate capacitor and A. C. Voltage generator. [3+(4+4+5)=16]

8. a) Draw the block diagram of an automatic control system and define control lag, load variable, manipulated variable, process lag, and dead time.

b) Derive the output of a PID (proportional plus integral plus derivative) control action for a step type error signal. Explain with a figure that how the output changes with error signal and time. [(3+5)+(5+3)=16]

9. How a shift register can store a binary number 11011? Explain the operation with circuit and timing diagram.

[16]

INDIAN STATISTICAL INSTITUTE

Second End-Semester Examination (2014 – 2015)

Course Name : **M.Tech (QR & OR)**

Subject : **Industrial Engineering and Management**

Date : 01.05.2015

Maximum Marks : **60**

Duration : **150 Minutes**

Question Paper

Direction: Question 1 is compulsory. Attempt ***Any two*** questions from rest. All questions carry equal marks.

1. State whether each of the following statements is True (T) or False (F). Also, justify your statement. **[5X4]**

(i) Financial statements are an important source of information to shareholders and stakeholders.

(ii) Financial forecasting is followed by financial planning.

(iii) Cash planning is a part of long-term financial planning.

(iv) Sales and Production Budgets are Capital Budgets.

(v) Capital budgeting decisions are long-term decisions.

2. (a) What is annuity? Prove the following relationship for annuity factor. Notations have their usual meanings.

$$AF = \frac{1-(1+r)^{-n}}{r}$$

(b) Define NPV. State conditions of NPV to select a project.

(c) What do you mean by Equivalent Annual Cost?

(d) A business undertakes high-risk investments and requires a minimum expected rate of return of 17% pa on its investments. A proposed capital investment has the following expected cash flows:

Year	Rs
0	(50,000)
1	18,000
2	25,000
3	20,000
4	10,000

(i) Calculate the NPV of the project if the cost of capital is 15%.

(ii) Calculate the NPV of the project if the cost of capital is 20%.

(iii)Use the NPVs you have calculated to estimate the IRR of the project.

(iv)Recommend, on financial ground alone, whether the project should go ahead.

[(1+4)+(2+3)+3+(2+2+2+1)]

3. (a) What do you mean by risk and uncertainty in relation to financial management? How is sensitivity analysis important, in this context?

(b) An investment of Rs 40,000 today is expected to give rise to annual contribution of Rs 25,000 and annual fixed cost of Rs 10,000 for the next four years; the discount rate is 10%.

Required:

I. Calculate NPV of this investment.

II. Calculate sensitivity of your calculation to the following:

(i) Initial investment

(ii) Contribution

(iii) Fixed costs

(iv) Discount rate

(v) Life of the project.

III. The annual contribution of Rs 15,000 is based on selling one product, with a sales volume of 10,000 units, selling price of Rs 12.50 and variable cost of Rs 10. Calculate the sensitivity margin for:

(i) The sales volume

(ii) The selling price

(c) Show a typical balance sheet of an organization. Explain "asset", in this regard, with example. **[(3+2)+(3+5+2)+(3+2)]**

4. Write short notes on ***any four*** from the following: **[4X5]**

   (a) Value for money

   (b) Impact of inflation on interest rate

   (c) Financial management and its objectives

   (d) WDA and its impact

   (e) Work Method Design

   (f) Importance of human factors in engineering

INDIAN STATISTICAL INSTITUTE
M. Tech(QR & OR) I Year (E & S Stream)

Second Semester: 2014-2015

Back Paper Examination

Reliability-I

Date: 16·07·2015 Full Marks: 100 Duration: 3 hours.

Note: Answer all questions

1. (a) Define min cut set of a coherent system. Express the structure function of a coherent system in terms of its min cut structures.
   (b) Consider a parallel system with n independent components. Show that the component with highest reliability is most important to the system.

   [(2+2)+4 =8]

2. Consider a series system with n associated components. Let T_i be the lifetime of the ith component. Show that

$$P\left[\min_{1\le i\le n} T_i \ge t\right] \ge \prod_{i=1}^{n} P[T_i \ge t].$$

   [15]

3. (a) Let T be a continuous lifetime random variable with hazard rate $\lambda(t)$. Then express pdf of T in terms of $\lambda(t)$ only.
   (b) Suppose $\lambda(t)$ have the form

$$\lambda(t) = \begin{cases} \lambda_0 & \text{for } 0 \le t \le t_0 \\ \lambda_0 + m(t - t_0) & \text{for } t > t_0, \end{cases}$$

   Find the (i) reliability function $R(t)$ and (ii) mean time to failure.
   (c) If the mean time to failure of a CFR component is 50,000 hours, what is the reliability for a 100-hour mission?

   [5+(3+3)+2= 13]

4. (a) Define IFR life distribution.
   (b) Let T be a continuous random variable with pdf

$$f(t) = \frac{2^k t^{k-1} e^{-2t}}{\Gamma(k)}, \qquad k \text{ is integer valued and } t > 0.$$

   Show that the reliability function of T can be expressed as

$$R(t) = \sum_{i=0}^{k-1} \frac{e^{-2t}(2t)^i}{i!}.$$

   Check whether the distribution of T is IFR or DFR.

   [2+(7+4)=13]

5. Consider a k-out-of-n system of independent components each having life distribution with constant hazard rate λ. Derive the reliability function of the system. Hence or otherwise find the mean lifetime of the system.

[7+5=12]

6. Consider a 2-unit standby redundant system with a switch. Suppose unit 1 is the active unit and unit 2 is the standby unit. The standby unit carry weak load while in standby mode. The lifetime of unit 1 has constant failure rate λ_1. The lifetime of unit 2 is exponential with rate λ_2 while in active mode and λ_2^0 while in standby mode. It is known that the switch never fails to activate the standby unit when active unit fails. Assume that all the lifetimes are independently distributed. Find the reliability of the system at time t. Hence or otherwise find the expected lifetime of the system.

[7+5=12]

7. Suppose that n identical items are put on a life test for a pre-fixed time T_0. All the functioning items are censored at T_0. Assume that the underlying life distribution is exponential with mean $1/\theta$. Derive the MLE of θ. Find the asymptotic variance of the MLE of θ. Give an asymptotic 95% confidence interval for θ. Obtain the asymptotic variance of the MLE of $R(t_0)$.

[3+4+2+3=12]

8. Twenty items are put on a test at time $t = 0$ and testing terminated as soon as the 10th failure occurs. The observed failure times (in hours) are

16.0, 17.5, 18.8, 21.0, 31.0, 35.6, 38.8, 41.0, 42.3, 45.0.

Assume that the lifetime distribution follows exponential distribution with mean $1/\lambda$.

(a) Derive the maximum likelihood estimate of λ.

(b) Derive an exact 95% confidence interval for λ.

(c) Find an unbiased estimator of mean lifetime.

[4+8+3=15]