INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2014 – 15

MTech CS (2nd Year)

Computational Finance

Date: 18 September 2014 Maximum Marks: 40 Duration: 2 Hours

1. Critically explain the concepts: [5 X 2 10]
   a) Dominant strategy
   b) Capital Asset Pricing Model

2. Assuming $V_0 > 0$, the discounted return is
   $$R_n^* = [S_n^*(1) - S_n^*(0)]/S_n^*(0) \text{ for } n = 1,\ldots,N$$
   Show that
   (a) $G^* = \sum_{n=1}^{N} H_n S_n^*(0) R_n^*$
   (b) $R_n^* = \frac{R_n - R_0}{1+R_0}$ for n = 1,...,N.
   (c) Q is a risk neutral probability if and only if $E_Q[R_n^*] = 0$ for n = 1,....,N.
   [4 + 3 + 5 = 12]

3. In the two period model, explicitly solve the Consumption Investment problem for the utility function $u(w) = \frac{1}{\gamma} w^{\gamma}$ where $\gamma < 1$. Show that the Lagrange Multiplier
   $$\lambda = v^{-(1-\gamma)}\{E[(L/B_1)^{-\frac{\gamma}{1-\gamma}}]\}^{(1-\gamma)}$$
   the optimal attainable wealth
   $$W = \frac{v(L/B_1)^{-1/1-\gamma}}{E[(L/B_1)^{-\gamma/1-\gamma}]}$$
   and the optimal objective value is $E[u(W)] = \lambda v/\gamma$.
   Compute the relevant expressions and solve for the optimal trading strategy when N = 1, K = 2, r = 1/9,
   $S_0 = 5$, $S_1(\omega_1) = 20/3$, $S_1(\omega_2) = 40/9$ and $P(\omega_1) = 3/5$. [5 + 4 + 3 + 6 = 18]

NAME :

ROLL NO :

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2014
Course Name: M.Tech. In Computer Science
Subject Name: Computer Architecture

Date: 19.09.2014 *Maximum Marks: 40* *Duration 3 hours*

Answer Question 0 and any 3 from the rest

Question 0: Potpourri: **3+2+2+2+1 = 10 points**

A. What can be changed in ISA, compiler, and microarchitecture to eliminate false register dependencies (output and anti, or write-after-read, write-after-write dependencies) if at all possible, in each of the three levels above? Describe one disadvantage of each approach

ISA
Approach:
Disadvantage:

Compiler
Approach:
Disadvantage:

Micro-architecture
Approach:
Disadvantage:

B. Among the following sets of two concepts, circle the one that makes programmer's task harder and **at the same time** micro-architect's (or hardware designer's) task easier Interpret programmer broadly as we did in class, e.g. a compiler writer is also a programmer

- Reduced instruction set versus complex instruction set
- VLIW versus superscalar execution
- More addressing modes versus less addressing modes
- Unaligned access support versus alignment requirements in hardware

C. Assume you have a machine with a **4-entry** return address stack. A code executing on this machine has **six** levels of nested function calls, each of which end with an appropriate return instruction. What is the return address prediction accuracy for this code?

D. Say that we were to take a processor architecture designed as big-endian and change it to little-endian. Assume that your processor supports the x86 instruction set on a load-store architecture, and a memory with a word-level interface. Can you identify two instructions from the x86 subset which will be affected by the endian-ness change?

E. The VAX ISA has an instruction **INSQUE ptr1, ptr2** which causes the node pointed to by **ptr1** to be inserted into a double linked list immediately after the node pointed to by **ptr2** We say the doubly linked list is a _____ in the VAX ISA.

1. Design for Performance (1 X 10 = 10 Points)

You are a programmer at a large corporation, and you have been asked to parallelize an old program so that it runs faster on modern multicore processors.

(a) You parallelize the program and discover that its speedup over the single-threaded version of the same program is significantly less than the number of processors. You find that many cache invalidations are occuring in each core's data cache. What program behavior could be causing these invalidations (in 20 words or less)?

(b) You modify the program to fix this first performance issue. However, now you find that the program is slowed down by a global state update that must happen in only a single thread after every parallel computation. In particular, your program performs 90% of its work (measured as processor seconds) in the parallel portion and 10% of its work in this serial portion. The parallel portion is perfectly parallelizable. What is the maximum speedup of the program if the multicore processor had an infinite number of cores?

(c) How many processors would be required to attain a speedup of 4?

(d) In order to execute your program with parallel and serial portions more efficiently, your corporation decides to design a custom heterogeneous processor.

- This processor will have one large core (which executes code more quickly but also takes greater die area on-chip) and multiple small cores (which execute code more slowly but also consume less area), all sharing one processor die.
- When your program is in its parallel portion, all of its threads execute **only** on small cores
- When your program is in its serial portion, the one active thread executes on the large core
- Performance (execution speed) of a core is proportional to the square root of its area
- Assume that there are 16 units of die area available. A small core must take 1 unit of die area. The large core may take any number of units of die area n^2, where n is a positive integer
- Assume that any area not used by the large core will be filled with small cores.

How large would you make the large core for the fastest possible execution of your program?

What would the same program's speedup be if all 16 units of die area were used to build a homogeneous system with 16 small cores, the serial portion ran on one of the small cores, and the parallel portion ran on all 16 small cores?

Does it make sense to use a heterogeneous system for this program which has 10% of its work in serial sections?

YES **NO**

Why or why not?

(e) Now you optimize the serial portion of your program and it becomes only 4% of total work (the parallel portion is the remaining 96%). What is the best choice for the size of the large core in this case?

What is the program's speedup for this choice of large core size?

What would the same program's speedup be for this 4%/96% serial/parallel split if all 16 units of die area were used to build a homogeneous system with 16 small cores, the serial portion ran on one of the small cores, and the parallel portion ran on all 16 small cores?

Does it make sense to use a heterogeneous system for this program which has 4% of its work in serial sections?

YES **NO**

Why or why not?

2. Branch Prediction and Dual Path Execution [1+3 X 3 =10 points]

Assume a machine with a 7-stage pipeline. Assume that branches are resolved in the sixth stage. Assume that 20% of instructions are branches.

(a) How many instructions of wasted work are there per branch misprediction on this machine?

[ ] instructions.

(b) Assume N instructions are on the correct path of a program and assume a branch predictor accuracy of A. Write the equation for the number of instructions that are fetched on this machine in terms of N and A. (Please show your work for full credit.)

(c) Let's say we modified the machine so that it used *dual path execution* like we discussed in class (where an equal number of instructions are fetched from each of the two branch paths). Assume branches are resolved before new branches are fetched. Write how many instructions would be fetched in this case, as a function of N. (Please show your work for full credit.)

(d) Now let's say that the machine combines branch prediction *and* dual path execution in the following way:

A branch confidence estimator, like we discussed in class, is used to gauge how confident the machine is of the prediction made for a branch. When confidence in a prediction is high, the branch predictor's prediction is used to fetch the next instruction; When confidence in a prediction is low, dual path execution is used instead.

Assume that the confidence estimator estimates a fraction C of the branch predictions have high confidence, and that the probability that the confidence estimator is wrong in its high confidence estimation is M.

Write how many instructions would be fetched in this case, as a function of N, A, C, and M (Please show your work for full credit.)

3. Value Prediction [2+2+2+2+2=10 points]

In class, we discussed the idea of value prediction as a method to handle data dependences. One method of value prediction for an instruction is "last-time prediction." The idea is to predict the value to be produced by the instruction as the value produced by the same instruction the last time the instruction was executed. If the instruction was never executed before, the predictor predicts the value to be 1. Value prediction accuracy of an instruction refers to the fraction of times the value of an instruction is correctly predicted out of all times the instruction is executed.

Assume the following piece of code, which has four load instructions in each loop iteration, loads to arrays x, y, z, t:

```
initialize integer variables c, d, e, f to zeros
initialize integer arrays x, y, z, t

for (i=0; i<1000; i++) {
    c += x[i];
    d += y[i];
    e += z[i];
    f += t[i];
}
```

Assume the following state of arrays before the loop starts executing:

- x consists of all 0's
- y consists of alternating 3's and 6's in consecutive elements
- z consists of random values between 0 and $2^{32} - 1$
- t consists of 0, 1, 2, 3, 4, ..., 999

a) What is the value prediction accuracy of the aforementioned predictor for the four load instructions in the program?

load of x[i]:

load of y[i]:

load of z[i]:

load of t[i]:

b) Can you design a predictor that can achieve higher accuracy for the prediction of $x[n]$?

YES NO

If so, explain your design.

c) Can you design a predictor that can achieve higher accuracy for the prediction of $y[n]$?

YES NO

If so, explain your design.

d) Can you design a predictor that can achieve higher accuracy for the prediction of $z[n]$?

YES NO

If so, explain your design.

e) Can you design a predictor that can achieve higher accuracy for the prediction of $t[n]$?

YES NO

If so, explain your design.

4. Branch Prediction [2+2+6=10 points]

Assume the following piece of code that iterates through a large array populated with **completely (i.e., truly) random** positive integers. The code has four branches (labeled B1, B2, B3, and B4). When we say that a branch is *taken*, we mean that the code *inside* the curly brackets is executed.

```
for (int i=0; i<N; i++) {  /* B1 */
    val = array[i];        /* TAKEN PATH for B1 */
    if (val % 2 == 0) {    /* B2 */
        sum += val;        /* TAKEN PATH for B2 */
    }
    if (val % 3 == 0) {    /* B3 */
        sum += val;        /* TAKEN PATH for B3 */
    }
    if (val % 6 == 0) {    /* B4 */
        sum += val;        /* TAKEN PATH for B4 */
    }
}
```

(a) Of the four branches, list all those that exhibit *local correlation*, if any.

(b) Which of the four branches are *globally correlated*, if any? Explain in less than 20 words.

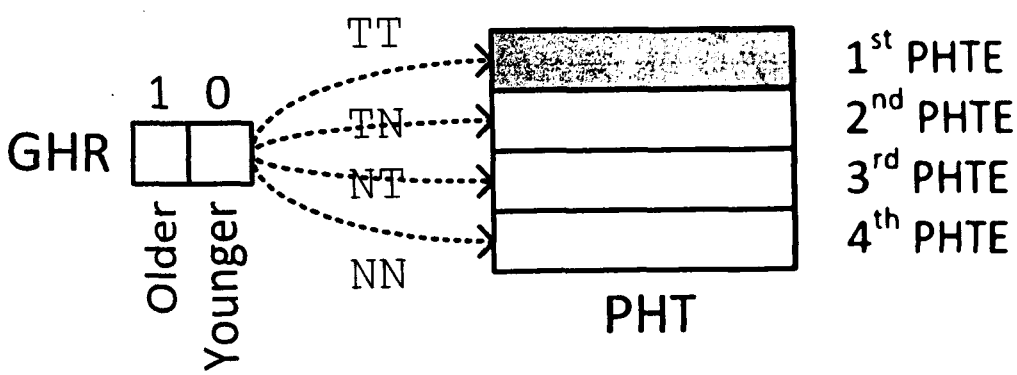
Now assume that the above piece of code is running on a processor that has a global branch predictor. The global branch predictor has the following characteristics.

- Global history register (GHR): 2 bits.
- Pattern history table (PHT): 4 entries.
- Pattern history table entry (PHTE): 11-bit signed saturating counter (possible values: -1024–1023)
- Before the code is run, all PHTEs are initially set to 0.
- As the code is being run, a PHTE is incremented (by one) whenever a branch that corresponds to that PHTE is taken, whereas a PHTE is decremented (by one) whenever a branch that corresponds to that PHTE is not taken.

(c) After 120 iterations of the loop, calculate the **expected** value for only the first PHTE and fill it in the shaded box below. (Please write it as a base-10 value, rounded to the nearest one's digit.)

Hint. For a given iteration of the loop, first consider, what is the probability that both B1 and B2 are taken? Given that they are, what is the probability that B3 will increment or decrement the PHTE? Then consider...

Show your work.

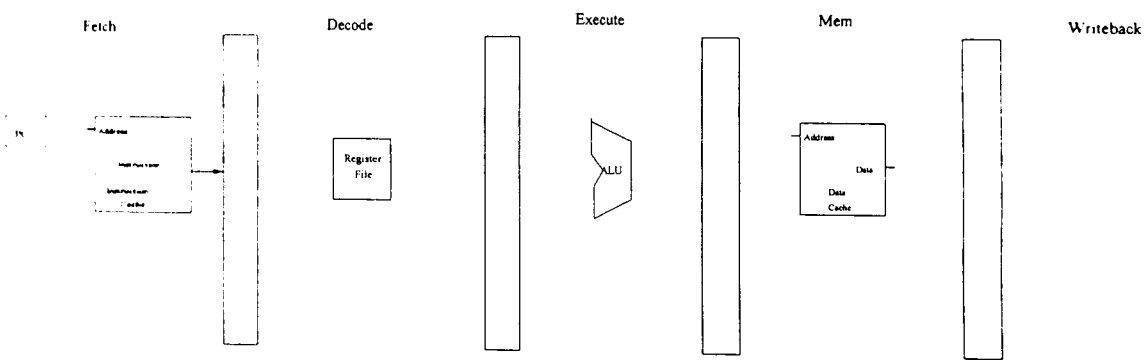


5. Fine-Grained Multithreading (1+1+1+1+1+1+1+1+2=10 Points)

Consider a design "Machine I" with five pipeline stages: fetch, decode, execute, memory and writeback. Each stage takes 1 cycle. The instruction and data caches have 100% hit rates (i.e., there is never a stall for a cache miss). Branch directions and targets are resolved in the execute stage. The pipeline stalls when a branch is fetched, until the branch is resolved. Dependency check logic is implemented in the decode stage to detect flow dependences. The pipeline does not have any forwarding paths, so it must stall on detection of a flow dependence.

In order to avoid these stalls, we will consider modifying Machine I to use fine-grained multithreading.

(a) In the five stage pipeline of Machine I shown below, clearly show what blocks you would need to add in each stage of the pipeline, to implement fine-grained multithreading. You can replicate any of the blocks and add muxes. You don't need to implement the mux control logic (although provide an intuitive name for the mux control signal, when applicable).



(b) The machine's designer first focuses on the branch stalls, and decides to use fine-grained multithreading to keep the pipeline busy no matter how many branch stalls occur. What is the minimum number of threads required to achieve this?

Why?

(c) The machine's designer now decides to eliminate dependency-check logic and remove the need for flow-dependence stalls (while still avoiding branch stalls). How many threads are needed to ensure that no flow dependence ever occurs in the pipeline?

Why?

A rival designer is impressed by the throughput improvements and the reduction in complexity that FGMT brought to Machine I. This designer decides to implement FGMT on another machine, Machine II. Machine II is a pipelined machine with the following stages.

Fetch	1 stage
Decode	1 stage
Execute	8 stages (branch direction/target are resolved in the first execute stage)
Memory	2 stages
Writeback	1 stage

Assume everything else in Machine II is the same as in Machine I.

(d) Is the number of threads required to eliminate branch-related stalls in Machine II the same as in Machine I?

YES NO (Circle one)

If yes, why?

If no, how many threads are required?

(e) What is the minimum CPI (i.e., maximum performance) of each thread in Machine II when this minimum number of threads is used?

(f) Now consider flow-dependence stalls. Does Machine II require the same minimum number of threads as Machine I to avoid the need for flow-dependence stalls?

YES NO (Circle one)

If yes, why?

If no, how many threads are required?

(g) What is the minimum CPI of each thread when this number of threads (to cover flow-dependence stalls) is used?

(h) After implementing fine grained multithreading, the designer of Machine II optimizes the design and compares the pipeline throughput of the original Machine II (without FGMT) and the modified Machine II (with FGMT) both machines operating at their maximum possible frequency, for several code sequences. On a particular sequence that has no flow dependences, the designer is surprised to see that the new Machine II (with FGMT) has lower overall throughput (number of instructions retired by the pipeline per second) than the old Machine II (with no FGMT). Why could this be? Explain concretely.

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Mid Semestral Examination:(2014-2015)

MTech C.S. 2nd Year

Digital Signal Processing

Date: 20.9.2014 Maximum Marks: 60 Duration: 2 hours

Note: The marks add up to 72. The maximum you can score is 60. The exam is open-book, open-notes. Use of calculators is permitted.

1. A causal (equals zero for $t < 0$) exponential continuous-time signal $x(t) = 2e^{-3t}$ is sampled using a sampling period $T_s = 1$. Determine

   (a) The z-transform of the corresponding discrete-time signal.

   (b) Does the signal have a valid Discrete Time Fourier Transform (DTFT) ? Justify your answer.

   [6+4]

2. Let $x[n] = 0.7(1 + [-1]^n)u[n]$ where $u[n]$ is the unit step sequence. Obtain the z-transform, using the z-transform properties and sketch its poles and zeros. [10]

3. The z-transform of the impulse response of a causal system is

$$H(z) = \frac{z^{-2}}{(1 - z^{-4})(1 - 0.3z^{-1})}$$

   Determine its impulse response. Also state and justify if the corresponding system can be stable. [8+2]

4. Let $x[n] = n(0.5)^n u[n]$. Denoting the DTFT of $x[n]$ by $X(e^{j\omega})$, compute, without actually evaluating $X(e^{j\omega})$, the inverse DTFT of the following:

   (a) $e^{j3\omega}X(e^{j\omega})$

   (b) $X(e^{j(\omega+0.6\pi)})$

   (c) $\frac{dX(e^{j\omega})}{d\omega}$

   (d) $Im\{X(e^{j\omega})\}$

   [5+5+5+5]

5. Consider the system represented by the difference equation $y[n] = 0.5y[n-1] + x[n+1] + x[n-1]$ assuming that $y[-1] = 0$ and $x[n]$ is causal, determine if the system is linear, time-invariant, causal and stable. [3+3+3+3]

6. A bandlimited continuous-time signal $x(t)$ containing a maximum frequency of 1 kHz. is sampled at twice the Nyquist rate to produce $x[n]$. This signal passes through the non-linear system represented by the inout-output relationship:

$$y[n] = x[n]^2$$

   (a) Determine the highest frequency contained in $y[n]$.

   (b) You must now convert $y[n]$ to a continuous-time signal $y(t)$ by passing it through a D/C converter. What should be the sampling frequency used in the converter so that the highest frequency contained in $y(t)$ and $x^2(t)$ be the same?

[5+5]

Indian Statistical Institute
Mid-Semestral Examination: 2014
Course Name: M. Tech. in Computer Science
Subject Name: Mobile Computing

Date: 20-09-2014 Maximum Marks: 60 Duration: 2 hours 30 minutes

Instructions: You **may** attempt **all** questions which carry a total of **66** marks. However, the maximum marks you can score is only **60**.

1. State the frequency assignment problem in cellular network. Represent the frequency assignment problem in terms of a classical vertex coloring problem. $3 + 3 = 6$

2. State the perturbation-minimizing frequency assignment problem (PMFAP). Explain with an example the unforced assignment (UA) and the forced assignment with rearrangement (FAR) operations used to solve the PMFAP. Explain how an algorithm that solves the PMFAP can also be used to solve the frequency assignment problem. $4 + (2 + 5) + 4 = 15$

3. State the differences between horizontal handover and vertical handover. Briefly explain the following RSS based horizontal handover decision strategies: i) relative RSS with hysteresis, and ii) relative RSS with hysteresis and threshold. Explain an adaptive lifetime based vertical handover decision strategy. $4 + (3 + 3) + 5 = 15$

4. Give some reasons why routing in MANET is intrinsically different from traditional routing in infrastructure based networks. State the differences between proactive and reactive routing protocols. Briefly explain the destination-sequenced distance vector routing protocol for MANET. $4 + 3 + 8 = 15$

5. Write the formula for computing the individual throughput obtained by the STAs from their respective associating APs under the random polling access MAC scheduling in WLAN. Assume that the interference matrix $A = (a_{jj'})$ and the data rate matrix $R = (r_{i,j})$ are known and k orthogonal frequency channels are available. Assume that the aggregate throughput is the logarithmic sum of the individual throughput obtained by the STAs from their respective associating APs. Write an integer programming formulation of the problem that jointly 1) selects an appropriate subset of APs, 2) finds a conflict-free frequency assignment of the selected APs, and 3) finds an association between the STAs and the selected APs. The objective of the integer programming is to maximize the aggregate throughput under the random polling access MAC scheduling. $3 + 12 = 15$

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination

M. Tech (CS) - II Year, 2014-2015 (Semester - III)

Multidimensional Search and Computational Geometry

Date : 22.09.2014 Maximum Marks : 75 Duration : 3.0 Hours

Note: Answer as much as you can, but the maximum you can score is 75.

(Q1) Let P be a set of n points in the plane. Design an efficient algorithm to compute a simple polygon that has all the points of P as its vertices. [5]

(Q2) Prove that the smallest perimeter polygon P containing a set of points $\mathcal{P}$ is convex. [3]

(Q3) Given a set of points P_1 and another set of points P_2. Let y_1 and y_2 be the maximum y-coordinates of the point set P_1 and P_2, respectively. Draw the line segment $\overline{y_1 y_2}$. Are all the points in $P_1 \cup P_2$ going to lie below the line segment $\overline{y_1 y_2}$? If yes, prove it, else show a counter-example. [2]

(Q4) Given a set $\mathcal{L}$ of n lines, no two of which are parallel, let $\mathcal{H}$ be the set of points generated by the intersection of these n lines. $|\mathcal{H}| = O(n^2)$. Now, find out a tight asymptotic upper bound on the number of points that might lie on the convex hull of $\mathcal{H}$. [5]

(Q5) What is the dual of a set of points inside a given triangle with vertices p, q and r? [5]

(Q6) a) Given two x-monotone polygonal chains P and Q of n_1 and n_2 sides, respectively, give a bound on the maximum number of intersections that might occur between the edges of P and Q? [2]

b) Given two monotone polygonal chains P and Q of n_1 and n_2 sides with different directons, give a bound on the maximum number of intersections that might occur between the edges of P and Q? [3]

[2+3=5]

(Q7) An *orthogonal polygon* is one whose edges are either aligned (i.e., parallel) with the horizontal or the vertical coordinate axis. Prove or disprove, with a counterexample, the following statement. *The number of vertices in an orthogonal polygon will always be even.* [5]

[Hints: What can be the values of the interior angles of an orthogonal polygon?]

(Q8) Let $\mathcal{P}$ be a simple polygon of n vertices with $r \geq 1$ reflex vertices.

(i) Draw the sketch of a simple polygon of n vertices where r is maximized. [3]

(ii) Prove that $\mathcal{P}$ can be partitioned into at most $r+1$ convex pieces. [7]
[Hints: Can you use induction?]

(iii) Show that r guards are occasionally necessary and always sufficient to see inside $\mathcal{P}$. [5+5]

[3+7+(5+5)=20]

(Q9) You are given a set of n lines that intersect a circle C. Describe an $O(k + n \log n)$ time algorithm that reports all the pair of lines that intersect within the circle C, where k is the number of intersections within the circle. See Figure for an illustration. [15]

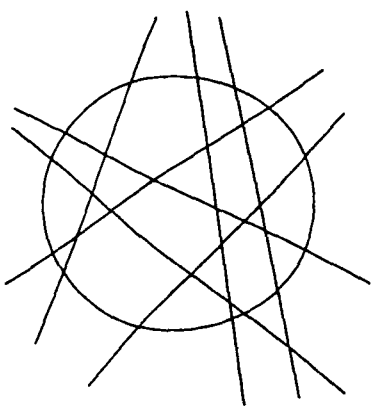


Figure 1: Intersection of lines and circles

(Q10) Let $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$ be n points on the plane. You are allowed to preprocess $\mathcal{P}$ such that you can answer a *range counting query*. In a *range counting query*, we are interested only in the number of points that lie in a range as opposed to range query where we look for the co-ordinates of the points that lie in that range. Mention efficient preprocessing and query algorithms to tackle *range counting query*. Also mention the space complexity. [10]

[Hints: Note that you need not report the points, you need only to count! So, the range query that we studied needs some modification.]

(Q11) Show that the average number of Voronoi vertices in a Voronoi cell is at most 6. [5]

(Q12) Let P be a set of n points in the plane. A point $q \in P$ is said to be the nearest neighbour of a point $p \in P$ if q is the closest to p among all points in $P \setminus p$. Design an efficient algorithm that finds the nearest neighbour of each and every point in P. [10]

Data Mining - Fall 2014
M. Tech CS II Mid-Sem Examination

Monday, September 22, 2014
Time: 2:30 hours
Full marks: 60

Please use the printouts of the slides for reference. No electronic devices should be used. Strictly no talking to others. For any explanation, ask only the course instructor. Partial credits will be given for unsuccessful attempts if the approach is described clearly and is potentially promising.

1. Consider the following set of points $S = \{(x, y) | x, y \in \mathbb{Z}, -5 \leq x, y \leq 5\}$, classified (labeled) into two classes. If x and y both are odd, or both are even, then the point is labeled white, otherwise it is labeled blue, as shown in Figure 1.

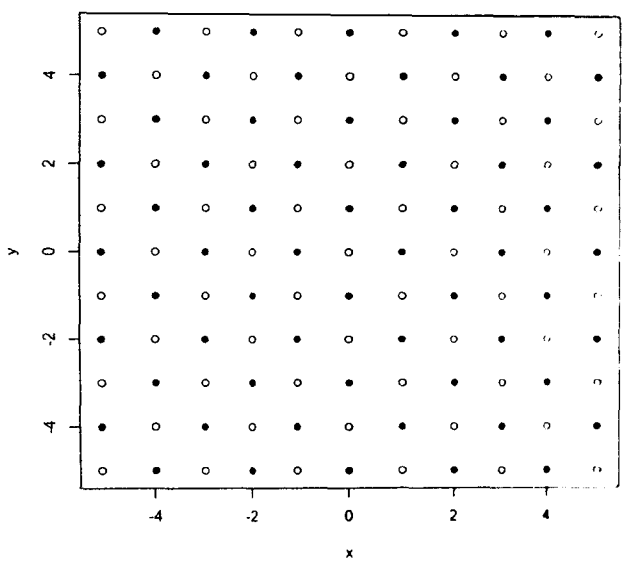


Figure 1: Scatter plot of sample data for Problem 1

(a) Suppose the label of the point $(0, 0)$ is lost and it needs to be labeled (classified) using some classifier. If a k-NN classifier (with Euclidean distance) is used, for some $k \in \{1, 3, 5, 7\}$, will it be classified with the right label? Explain your answer.

(6 marks)

(b) Instead of the natural Euclidean distance measure, can we define any other distance measure, for which a 3-NN classifier be able to classify the point $(0, 0)$ correctly? If your answer is yes, then

define such a distance measure and show why the classification will be correct. If your answer is no, prove or justify that given any distance measure a 3-NN classifier will not be able to classify the point correctly.

(8 marks)

2. Consider the data points shown in Figure 2, where each point is labeled white or blue. Assume that all the white points are strictly inside the box defined by the four points $\{(1,0),(1,1),(2,1),(2,0)\}$ and all the blue points are strictly inside the box defined by the four points $\{(-1,0),(-1,1),(-2,1),(-2,0)\}$ but the points are very close to the boundaries of the respective boxes.

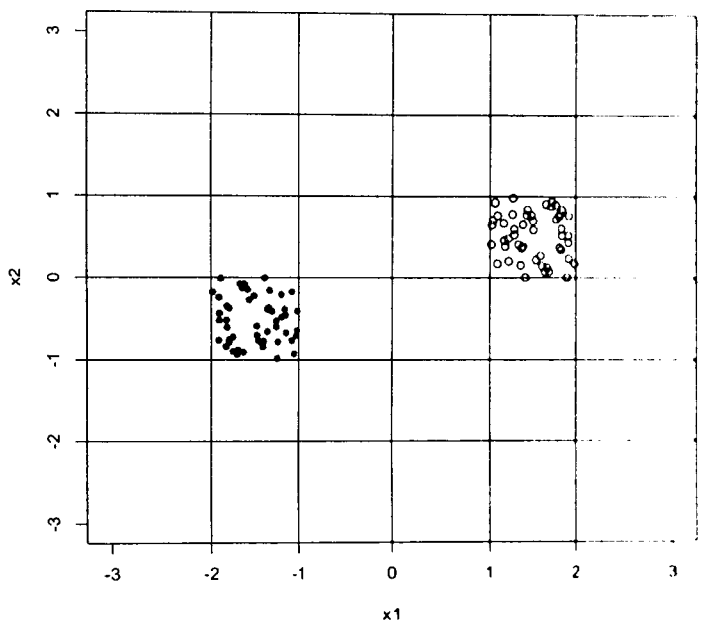


Figure 2: Scatter plot of sample data for Problem 2

(a) Draw a linear classifier (in other words, a separation line) which separates the two classes with 100% accuracy. Draw another linear classifier which does not separate the two classes with 100% accuracy.

(4 marks)

(b) Consider a *random* line L and let the L be a used as the separation line for a linear classifier for these two classes of points. Calculate the probability that L would be able to separate the two classes with 100% accuracy.

(10 marks)

3. Using a scatter plot, show an example of a dataset such that a decision tree can classify the data with 100% accuracy with not a very large decision tree, but a linear classifier cannot. For convenience, let the data points be two dimensional, with both dimensions being independent variables, and labeled with two classes, colored black or blue and white, as has been shown in Figure 1, or Figure 2.

(4 marks)

4. A survey shows that the unemployment rate among adults in India is 3.8%. Interestingly, the rate is 10% among graduates. According to the survey, India's official unemployment rate is 3.8% with urban unemployment at 5.1% and rural at 3.5%. Unemployment is higher among women than among men: 6.7% for women as against 2.8% for men.

   Having learnt this, suppose you meet a graduate man who lives in a village, and you do not know whether he works or not. Estimate the chance that he is unemployed.

   (7 marks)

5. Suppose in a market basket scenario, the following association rules are detected with a minimum support 25% and a minimum confidence 80%:

$$\{A, B\} \to \{C\}$$

$$\{B, C\} \to \{D\}$$

   What can you say about the minimum support of the itemset $\{A, B, C, D\}$?

   (4 marks)

6. Suppose we consider a string as a *set* of characters (removing duplicate characters), and define the Jaccard similarity of two words by the usual Jaccard similarity of the set representation of the words. For example, the words "example" and "explanation" would be considered as the sets {e,x,a,m,p,l} and {e,x,p,l,a,n,t,i,o,n}. Give an example (with justification) of three words w, x, y with the following condition, or prove that there cannot be any such three words of positive length. The condition is that w is closer to x than it is to y by the Jaccard similarity, but y is closer to w than x is, by edit distance. In other words:

$$J(w, x) > J(w, y).$$

   But

$$ed(w, y) < ed(w, x)$$

   where $J(a, b)$ is the Jaccard similarity and $ed(a, b)$ is the edit distance between two words a and b.

   [Recall that the edit distance of a string b from a string a is defined as the minimum number of insertions and deletions of single characters required to obtain b from a. Also, note that Jaccard similarity is a similarity; greater the value, closer the words are. The edit distance is a distance, greater the value further the words are.]

   (7 marks)

7. In the space $\{0,1\}^d$ of d-dimensional bit vectors, the Hamming distance of two bit vectors x and y is defined as the number of positions in which they differ. With this distance measure defined, for each $i = 1, 2, \ldots, d$, we define a function $f_i : \{0,1\}^d \to \{0,1\}$, where $f_i(x)$ is defined as the ith bit of x. Show that the family $\mathbb{F} = \{f_1, f_2, \ldots, f_d\}$ is a $(d_1, d_2, 1 - \frac{d_1}{d}, 1 - \frac{d_2}{d})$-sensitive family of hash functions for any given Hamming distances d_1 and d_2 such that $0 < d_1 < d_2 < d$.

   (10 marks)

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2014-15(First Semester)
Course Name: M.Tech. (CS) 2nd Year
Subject Name: Natural Language Processing

Date: 23.09.2014 Maximum Marks: 50 Duration: 2.5 Hrs

NOTE: There are questions of 60 marks and the maximum you can get is 50

1. Consider the following bi-gram language models.

   (a) **Language Model 1 :** $V = \{$"*the*", "*dog*"$\}$.

   $p($"*the*"$|START) = p($"*dog*"$|$"*the*"$) = p(STOP|$"*dog*"$) = 1$.

   All the other conditional probabilities are set to 0.

   (b) **Language Model 2 :** $V = \{$"*the*", "*a*", "*dog*"$\}$.

   $p($"*the*"$|START) = p($"*a*"$|START) = 0.5$.

   $p($"*dog*"$|$"*a*"$) = p($"*dog*"$|$"*the*"$) = p(STOP|$"*dog*"$) = 1$.

   All the other conditional probabilities are set to 0.

   START and *STOP* are usual pseudo words to indicate beginning and ending of a sentence respectively.

   Now, consider a test sentence "**the dog**". Which of the above two language model gives lower perplexity on this test sentence?

   [8]

2. The vocabulary set V of a language consists of m distinct words ($m > 1$). There are two special pseudo words *START* and *STOP* which denotes beginning and ending of a sentence respectively. To build a (word level) n-gram language model with the vocabulary set V and the pseudo words *START* and *STOP*, every possible n-gram must satisfy the following three properties:
   For any n-gram $< w_1, w_2, \ldots, w_n >$,

   (a) $w_1 \in \{START\} \cup V$

   (b) For $i = 2$ to $(n-1)$, $w_i \in \{START\} \cup V$. But any w_i ($2 \leq i \leq n-1$) can be *START*, iff for $j = (i-1)$ to 1, $w_j = START$ $\forall j$.

   (c) $w_n \in \{STOP\} \cup V$.

   Calculate the total number of distinct possible n-grams for this language model.

   [12]

3. There is a Google plug-in in the web that can detect language from a single line of text (say, consisting of at least 8-10 words). It covers about 80 languages. The application gives a probability score for the detected language and sometimes when the score is not sufficiently high for a single language, it gives more than one detection result with corresponding probability.

   If you are asked to develop this application what would be your method. Your resource requirements should be stated clearly and you should also mention how the resource requirements can be easily met up. Steps of your algorithm should be properly explained so that coding of your algorithm (written in pseudo code) would be straight forward. High-level concepts for which coding is not readily realisable/understandable will not be given any credit. However, you need not write code in any particular language.

   Assume your application name is `detectlang`. One should be able to use your application as

   ```
   detectlang testfile.txt
   ```

   Results should be in this format:
   `[En 0.9997]` or `[Fr 0.45678; Ro 0.33564]`, etc. where `En`, `Fr`. or `Ro` are language codes for English, French, Romanian, and so on.

4. Assume that we want to train a tri-gram HMM tagger on a training set with the following two sentences.

   (a) "The dog saw the cat" => $D\ \ N\ \ V\ \ D\ \ N$

   (b) "The dog saw the saw" => $D\ \ N\ \ V\ \ D\ \ N$

   Assume that we estimate the parameters of the HMM using maximum likelihood estimation and there are two pseudo tags START and STOP which are incapable of emitting words.

   Consider the test sentence "**the cat saw the saw**"

   (a) Draw the automata for the HMM and estimate the values of its parameters.

   [10]

   (b) Find the joint probability of the test sentence and the most probable tag sequence for that sentence.

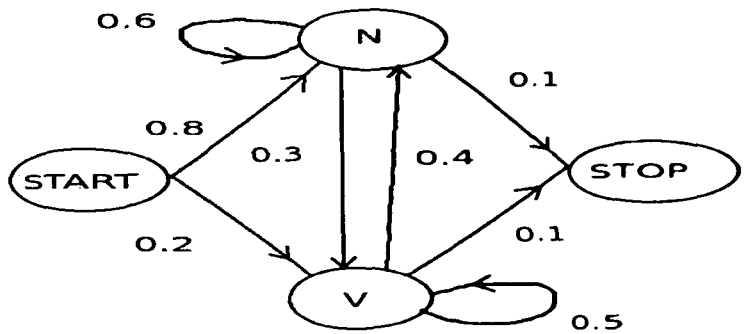
   [5]

5. Consider the bi-gram HMM shown below. There are two tags N and V which can emit words. Two special tags START and STOP are also there which are incapable of emitting words. Let there are only three words "I", "You" and "He". The word emission probabilities are given below.

$p(\text{"}I\text{"}|N) = 0.2, p(\text{"}You\text{"}|N) = 0.4$ *and* $p(\text{"}He\text{"}|N) = 0.4$

$p(\text{"}I\text{"}|V) = 0.5, p(\text{"}You\text{"}|V) = 0.4$ *and* $p(\text{"}He\text{"}|V) = 0.1$

The observation sequence is **"He I He"**



Calculate the most probable tag sequence by Viterbi algorithm.

[10]

Indian Statistical Institute

M. TECH. (CS) 2 Year : 2014-2015
Mid-Semester Examination
Subject: Information and Coding Theory
Date: 23/09/2014 Time: 2 hours Marks 60

Note: Notations used are as explained in the class.

1. Prove that for a binary symmetric channel with crossover probability $p < 1/2$, the maximum likelihood decoding rule is the same as the nearest neighbour decoding rule. 5

2. Define Hamming distance between two vectors. Prove that the Hamming distance is a metric. 5

3. Suppose the parity check matrix of a $[n, k]$ linear code is $[A|I_{n-k}]$. Show that its generator matrix is $[I_k|-A^T]$. 5

4. Construct a single-error correcting linear code with $n = 10$. Exhibit two codewords. 5

5. Describe a systematic encoding of cyclic codes. Give a non-systematic encoder of RS codes. Describe the majority logic decoding of RS codes. 10

6. Prove that the dual of an MDS code is an MDS code. Also prove that the dual of a BCH code may not be a BCH code. 5

7. Let C be an $[n, k, d]$ code over $GF(q)$. Prove that the following statements are equivalent.

   (a) C is MDS.

   (b) Every k columns of a generator matrix G are linearly independent.

   (c) Every $n-k$ columns of a parity check matrix H are linearly independent. 10

INDIAN STATISTICAL INSTITUTE

Periodical Examination: (2014 – 2015)

M.Tech. (CS) II Year

Parallel Processing: Architectures and Algorithms

Date: 24/09/2014 Total Marks: 68 Duration 3 hrs

NOTE: You may answer all questions but maximum marks attainable is 60

1. a) With schematic diagrams mention the important differences between the following pairs of computer architectures:
   i. SIMD and MISD
   ii. PRAM and MIMD
   iii. CUDA and PRAM.

   b) Show that the CRCW model of PRAM with N processors can always be simulated on the EREW model having N processors with an $O(\log N)$-fold increase in the processing time.

   [3 · 3 · 5 14]

2. a) Derive the expressions for upper bounds on Speed-up given by Amdahl's law and Gustafson's law respectively, mentioning the difference between the two.

   b) For a given program, 80% of the code is executable simultaneously by 20 processors. The rest of the code is to be executed sequentially. Find the maximum speed-up achievable using i) Amdahl's law and ii) Gustafson's law respectively. Which bound is more achievable if the workload is scalable?

   [(2 · 2) · (2 · 2 · 1) 9]

3. a) A Bit-Permute-Complement (BPC) permutation P: $x_3 x_2 x_1 x_0 \rightarrow x_2 x_3 x_0 x_1$, is to be realized on a 16 · 16 Omega network with 2 × 2 switches. Is the permutation admissible in a single pass in the network? Justify your answer. Show the conflict graph. What is the minimum number of passes required to route P in the network? Find the corresponding partitioning of the paths.
   If an extra stage is added to the network, does it make P admissible? Justify your answer.

   b) Draw the block diagram of a 3-stage non-blocking N×N Clos' network using (n · m) switches at the input stage. Prove that (2n-1) middle-stage switches are sufficient for non-blocking operation of the network.

   [(2 · 2 · 1 · 2 · 1 · 2) · 6 16]

4. Answer in brief:
   a) Given m sets of numbers, each containing n elements: $S_j = \{x_{j1}, x_{j2}, \ldots, x_{jn}\}$, $1 \le j \le m$, n is a power of 2, it is required to find the sum of each set: $SUM_j = \sum_k x_{jk}$, $1 \le j \le m$, $1 \le k \le n$. Develop a parallel architecture that may produce the results in $(\log n + m - 1)$ steps. How many processors are required and what will be the interconnection network? Mention the input data distribution and the delivery of output.

   b) In an N-node hypercube H_n, ($N=2^n$), if each node has a single packet to broadcast to all other nodes. Develop an algorithm that each node should follow to complete the whole procedure. How many communication rounds will be required to complete the broadcasting?

   [8 · 8 16]

5. Given a large array A of n integers ($x_0, x_1, x_2, \ldots, x_n$), write a kernel function in OpenCL that computes the prefix sum ($y_0, y_1, y_2, \ldots y_n$), such that:
   $y_0 = x_0$, $y_1 = x_0 + x_1$, $y_2 = x_0 + x_1 + x_2$, ..., $y_n = x_0 + x_1 + \ldots + x_n$.

   [13]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : (2014-2015)

M.Tech.(CS) II Year

Advanced Database Theory & Applications

Date : 24.09.2014 **Maximum Marks : 50** **Duration : 2 Hours**

1. Consider a database organized in terms of the following hierarchy of objects :

   a) The database itself is an object (DB) and it contains two files F1 and F2.
   b) Each file contains 1000 pages. The pages are identified as P_1 .... P_{2000}. So F1 has pages from P_1 to P_{1000} and F2 has pages from P_{1001} to P_{2000}.
   c) Each page contains 100 records. Each record is identified as $P_i : j$ where P_i is the page identifier with j as the record number within the page.

   Multiple-granularity locking protocol is used with S,X,IS,IX and SIX locks. Locking facility is available for the entire database or at file-level, page-level or record-level. Two transactions T1 and T2 want to execute the following operations :

   T1 : read records $P_{1200} : 98$ through $P_{1205} : 2$

   T2 : Delete pages P_{500} through P_{520} (consider delete as equivalent to write).

   Indicate the different types of locks to be allocated to each node for executing the two transactions separately.
   If T2 arrives while T1 is in execution, examine whether the two transactions can be executed concurrently?

   (5+5=10)

2. Three transactions T1, T2 and T3 are executed concurrently according to the following schedule

   T1 : R(X), T3 : R(Y), T2 : W(X), T3 : W(Y), T3 : R(X), T1 : R(Y), T2 W(Y)

   Where, R(Q) = read data item Q.
   W(Q) = write data item Q.

   TS(T) represents the time-stamp of any transaction T. The given schedule is executed using timestamp ordering protocol where a transaction is rolled back for any time conflict (Thomas Write Rule has not been considered). If TS(T1) > TS(T2) > TS(T3), what would be the status of the three transactions at the end of the schedule and what would be the value of the read and write timestamps of the different data items. Consider the initial value of all the read and write timestamps to be less than TS(T3).

   (4x5=20)

3. In a distributed database environment an organization maintains a database of its suppliers for supply of office and project related items. The projects are executed at different locations The relations maintained for this purpose are:

   Supplier (sid, sname, slocation, turnover)
   Item (ino, itype, iname, make)
   Catalog (sid, ino, unit_price)
   Project (pno, pname, budget, plocation, fagency)

Primary keys of the relations are underlined. Besides the unique id, Supplier relation maintains the name and location of operation (slocation) of each supplier. Average annual turnover of each supplier is also kept. While ino is a unique item number against each item, item type (itype) can be either office-item or project-item. Item name (iname) and manufacturer (make) of each item are also kept in Item relation. Catalog provides the unit_price of each item against each supplier. Each project is identified by a unique project number (pno). Name of the project (pname), its budget, location where it is executed (plocation) and the name of the funding agency (fragency) are also kept in the Project relation.

In the present status of the database, the projects are getting executed at Delhi, Bangalore and Kolkata where Kolkata is the Head-quarter. The entire database is maintained at Kolkata. Other project sites maintain only the horizontal fragments of the relations relevant to their activities. The organization has adopted the following guideline for placing orders to its suppliers:

- Orders for office-items are always placed to the local suppliers from the project location.
- Orders for project-items with unit_price less than or equal to Rs.5 lakhs can also be placed locally to the local suppliers.
- Order for any project-item costing more than Rs.5 lakhs is placed from the Head-quarter.

Following the above guidelines, find the horizontal fragments (both primary and derived) that need to be stored at different project sites.

(20)

-x-

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2014-2015

M. Tech. (CS) 2nd Year

Artificial Intelligence

Date: 25.09.2014 Maximum Marks: 60 Duration: 2.5 hours

Answer all questions in brief.

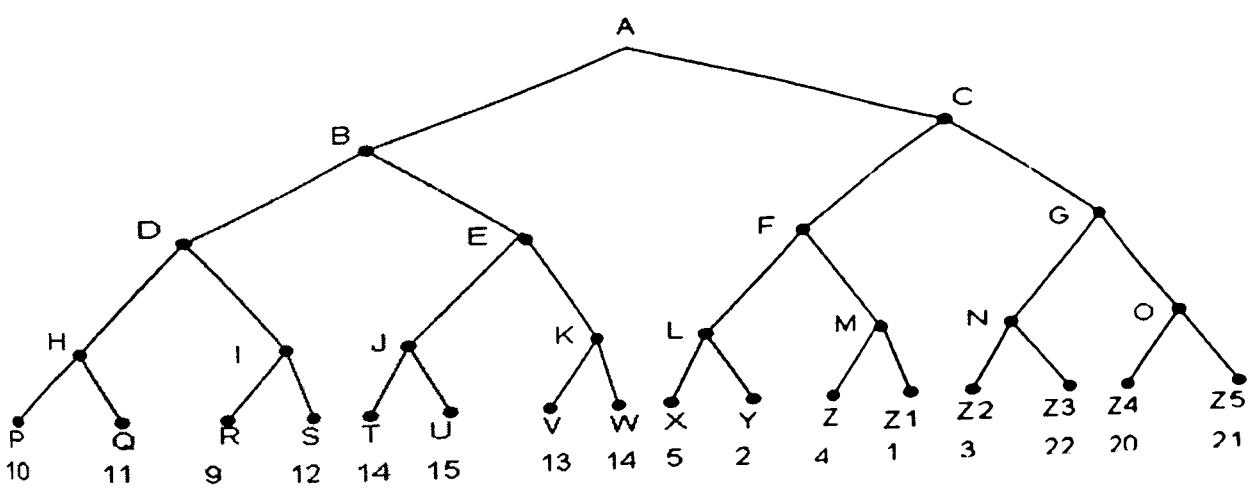
1. In *farmer-fox-goose-grain* puzzle, a farmer wishes to cross a river taking his fox, goose, and grain with him. He can use a boat which will accommodate only the farmer and one possession. If the fox is left alone with the goose, the goose will be eaten. If the goose is left alone with the grain it will be eaten. Draw a state space search tree for this puzzle using left-bank and right-bank to denote left and right river banks, respectively. [10]

2. Describe the *depth-first iterative deepening* algorithm. Prove that it is asymptotically optimal among brute-force tree searches in terms of time, space, and length of solution. [4 + (3 + 2 + 1) = 10]

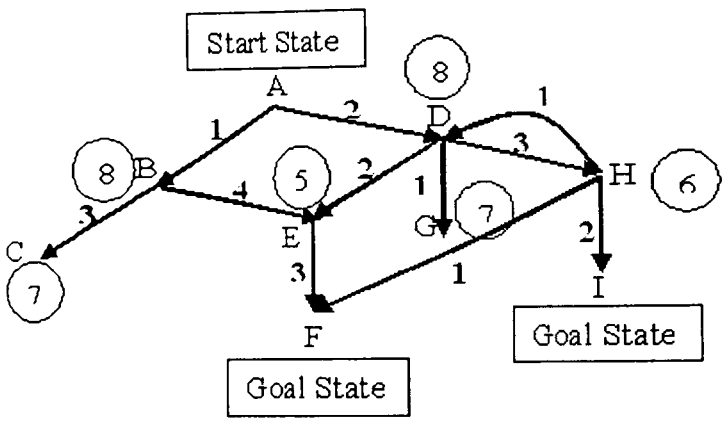
3. Define monotone property of a heuristic. Prove that any monotonic heuristic is admissible. Prove that the set of states expanded by algorithm A^* is a subset of those examined by breadth first search. [3 + 4 + 3 = 10]

4. What do you mean by *ridge* and *plateau*? What is the difference between *simulated annealing* and *steepest ascent hill climbing* approach? Describe the crossover and mutation operators of genetic algorithm. [1 + 1 + 4 + 4 = 10]

5. Perform the *minimax* search procedure on the game tree shown below in which the static scores are all from the first player's point of view and MAX is allowed to move first. Perform the left-to-right α-β pruning procedure on this tree and show how many nodes can be pruned away. [4 + 6 = 10]



6. Execute the *uniform cost search* and *best first search* algorithms on the following search graph, and show the solution path, along with its cost and list the expanded nodes for each case (each node of the graph is represented by a letter and the encircled value is the heuristic evaluation of the corresponding node, while the bolded numerical value represents the actual length of the path between two nodes). [5 + 5 = 10]



Indian Statistical Institute

Mid-Semester Examination (2014-2015)

M.Tech. (CS) II

Advanced Algorithms for Graph and Combinatorial Optimization Problems

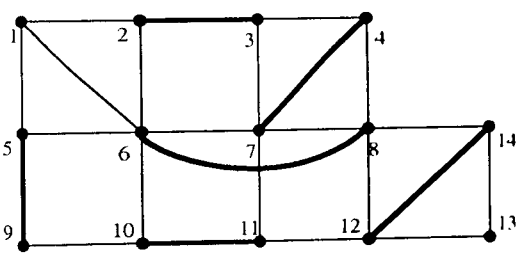
Date: September 25, 2014 Maximum Marks: 60 Time: 2 hours

Answer as much as you can. The maximum you can score is 60 marks. Marks allotted to each question are indicated within square brackets near the right margin.

1. (a) Formulate the maxflow problem as an instance of linear programming.
   (b) Suppose $G = (V, E)$ is a given network with a positive integer capacity c_e on each edge e, a source vertex $s \in V$ and a sink vertex $t \in V$. You are also given an integer maximum flow function f from s to t in G defined by f_e on each edge $e \in E$. Now, we pick a specific edge $\hat{e} \in E$ and increase its capacity by 1 unit. Give an $O(V + E)$ algorithm to update the maximum flow. Justify its correctness. Will your algorithm work if $c_{\hat{e}}$ had been decreased by 1 unit? [3 + (3 + 2 + 2) = 10]

2. In Goldberg-Tarjan's preflow-push algorithm for finding the maxflow in a network with n nodes and m arcs, prove that the number of (i) saturating pushes is $O(nm)$, and (ii) non-saturating pushes is $O(n^2m)$. [5 + 5 = 10]

3. Define a *blossom* of a graph with respect to a matching M for G. For the matching shown below, trace the steps to augment it. [2 + 5 = 7]



4. Given a chordal graph G, present an efficient algorithm to obtain a valid vertex coloring for G. How many colors does your algorithm need? Analyze the time complexity of your algorithm. [5 + 2 + 2 = 9]

5. (a) Are bipartite graphs transitively orientable? Justify your answer.
   (b) Present an algorithm to obtain a maximum clique of a comparability graph. Justify its correctness. [3 + (5 + 4) = 12]

6. Let G be a graph having n vertices $\{v_1, v_2, \ldots, v_n\}$. Then the Mycielski graph $\mu(G)$ of G has
   - the set of vertices $\mu(V) = \{v_1, v_2, \ldots, v_k, u_1, u_2, \ldots, u_k, w\}$, and
   - the set of edges $\mu(E) = E(G) \cup \{(u_i v_j) | (v_i v_j) \in E(G)\} \cup \{(u_i, w) | i = 1, 2, \ldots n\}$.

   The k^{th} Mycielski graph of G is defined as $\mu^k(G) = \mu(\mu^{k-1}(G))$ where $\mu^1(G) = \mu(G)$.

   a. If $G = K_2$, the complete graph on two vertices, then draw $\mu(G)$.

   b. Show that $\mu^k(G)$ is triangle free for all $k \geq 1$.

   c. Prove that $\chi(\mu^k(G)) = k + 2$ for all $k \geq 1$. What is $\omega(\mu^k(G))$?

[2+4+(4+2)=12]

7. a. Give an $O(n)$ time algorithm for determining whether or not a given non-increasing sequence of n integers $d_1, d_2, \ldots, d_n$, such that $n - 1 \geq d_1 \geq d_2 \geq \cdots \geq d_n \geq 0$, is graphic. Prove that your algorithm is correct.

   b. Give an example of two non-isomorphic split graphs having the same degree sequence.

[(3+3)+2 = 8]

8. Let $\mathcal{F}$ be a family of intervals on a line such that no interval contains another. Show that none of the left endpoints coincide. Devise an efficient method to construct a family $\mathcal{F}'$ of unit intervals (intervals of length 1) such that the intersection graphs of $\mathcal{F}$ and $\mathcal{F}'$ are isomorphic.

[3+4=7]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2014-15

Course Name: M. TECH CS - II

Subject: Computer Graphics

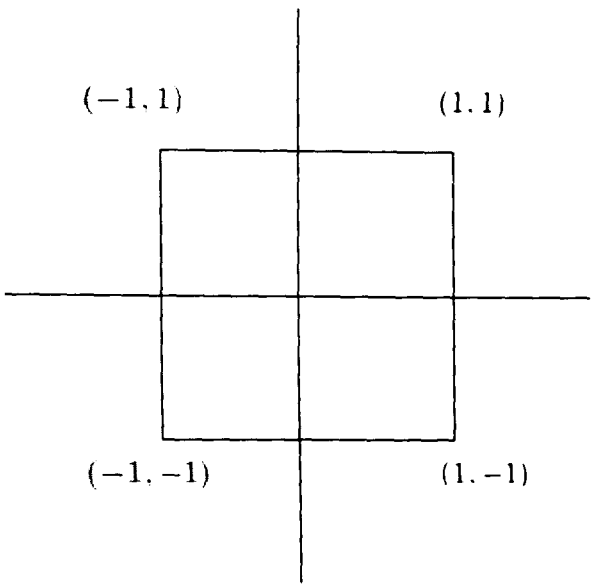
Date: 25.09.2014

Maximum Marks: 60

Duration: 2 hrs.

Answer as much as you can.

1. a) Here is a test pattern in the plane:



Draw its image after being transformed by each of the following (individually, not cumulatively) and give brief explanations for each transform.

i) $\begin{bmatrix} \cos 60° & \sin 60° & 0 \\ -\sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 0 & 1.5 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4+3+3 = 10)

(b) Determine the form of a transformation matrix for a reflection about an arbitrary line with equation $y = mx + b$. (5)

2. a) Given a clipping window P(0, 0), Q(30, 0) R(30, 20), S(0, 20), use Sutherland-Cohen algorithm to determine the visible portion of the line A(10, 30) and B(40, 0). Explain your steps. (9)

b) You are standing on the middle of the railroad tracks when you see that a train is approaching in the distance. You hold your thumb out in front of one eye at arm's length and notice with interest that your thumb exactly covers the width of the train. You know that your arm is one

meter long and tour thumb is 2 cm. wide. Looking down, you judge that the railroad tracks are 1.5 m. apart, and you see that the train's width extends exactly to the width of the tracks.

i) How far away is the train?
ii) If the train is travelling with 50 km/h. how many seconds do you have to get out of the way? (2+2 = 4)

c) Consider a raster system with resolution of 640 by 480. What is the access time per pixel for a display controller that refreshes the screen at a rate of 60 frames per second? (2)

3. a) Consider the line which starts at (4, 13) and ends at (22, 3).

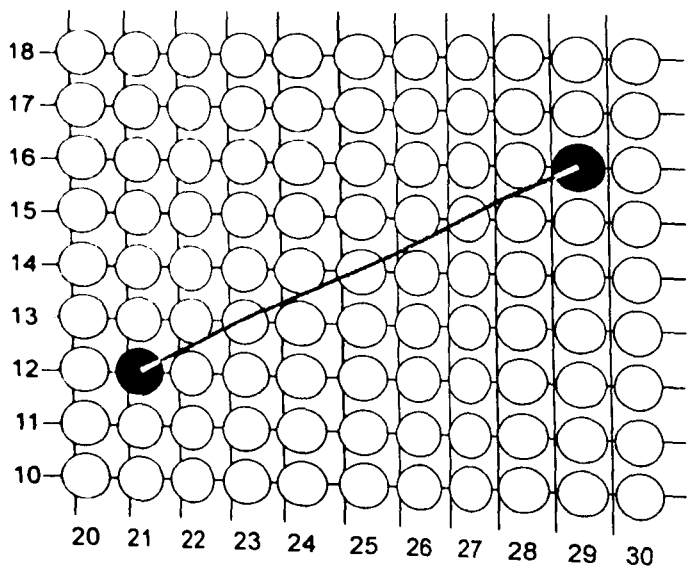
i) How many pixels will there be in this line?
ii) With the DDA (Digital Differential Analyzer) algorithm, what will be the amount added to the secondary component each time through the loop (incrementing value)? Explain. (2+3 = 5)

b) Derive the window to viewport mappings for W = (0, 100, 50, 200) and V = (10, 50, 0, 60). (3)

c) Suppose an RGB raster system is to be designed using a 10 inch by 12 inch screen with a resolution of 150 pixels per inch in each direction. If we want to store 8 bits per pixel in the frame buffer, how much storage (in bytes) do we need for system the frame buffer? (3)

d) Calculate a 4×4 matrix to rotate points about a vector A = [1 -2 1] by 60 degrees. Show your steps. (4)

4. Illustrate the steps of the Bresenham line drawing algorithm for a line going from (21,12) to (29,16). Refer to the following pixel grid (Fig. 1) and fill out the table shown below. Show all your calculations. Here p_k refers to the decision parameter at the k-th step. (15)



k	p_k	(x_{k+1}, y_{k+1})
0		
1		
2		
3		
4		
5		
6		
7		
8		

5. A cubic Bézier curve has the following mathematical form:

$$C(t) = \sum_{i=0}^{3} P_i b_i(t)$$

a) Give explicit formulae for $b_0(t)$, ..., $b_3(t)$.

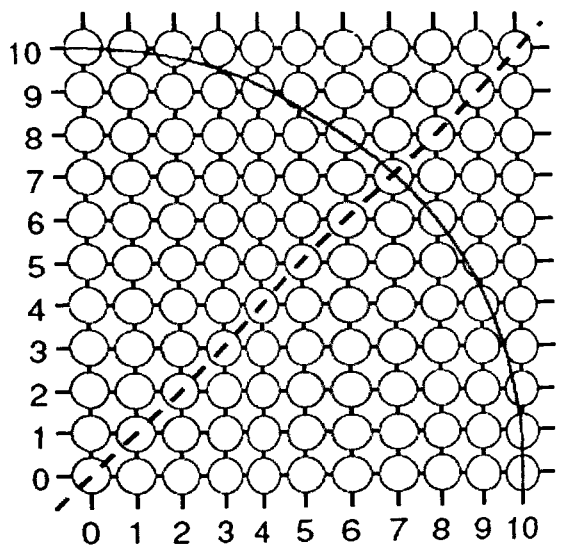
b) Show that $b_i(t) \geq 0$ for $0 \leq t \leq 1$.

c) Show that $\sum_{i=0}^{3} b_i(t) = 1$ for all t.

d) Suppose **R** is a rotation matrix. Show that $\mathbf{R}C(t)$ is given by a Bézier curve that has control points $\mathbf{R}P_i(t)$.

(4+3+3+5 = 15)

6. Demonstrate the mid-point circle algorithm by drawing the first quadrant of a circle centred at (0, 0) with radius 10. Use the pixel grid shown in Fig. 2 and fill out the Table below. Show all your calculations for the first 6 steps while filling up the Table. (15)



k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0				
1				
2				
3				
4				
5				
6				

Indian Statistical Institute

Semester-1 2014-2015

M.Tech.(CS) - Second Year

Mid-semester Examination (26 September, 2014)

Subject: Compiler Construction

Maximum marks: 40 Total marks: 43 Duration 2.5 hrs.

Please keep your answers brief and to the point.

1. A *tags table* for a given program, say `xyz.cmm`, is a file that contains information about the locations (i.e., line numbers) of all identifiers used in `xyz.cmm`.[1] Your goal in this question is to use `(f)lex` to create a *tags table generator* (TTG) that will take as input any program written in a hypothetical language C-- as input, and will generate a tags table for this program. Assume that

   - the input is a correct C-- program;
   - the syntax of C-- declarations is similar to that in C, i.e., declarations consist of a type name, followed by a comma-separated list of identifiers that is terminated by a semi-colon (;).
   - the tags table consists of one line per identifier, containing the following 3 fields in order identifier name, identifier type, and line number in the original source file in which the identifier is declared; the return type of a function is regarded as its type;
   - the only permissible types in C-- are `char`, `int`, and `float`; these type names are reserved keywords;
   - comments in C-- are either enclosed within `/*` and `*/`, or start with `//` and continue till the end of the line;
   - C-- does not permit any preprocessor directives or `typedefs`, nor does it permit any parameters / arguments to be passed to functions.

   For this question, you may ignore all issues related to the scope of variables. If necessary, you may make additional (reasonable) assumptions. **Clearly state any assumptions you make.**

   Your TTG should

   (a) echo to standard output a copy of the input program **with all comments removed.**

   (b) write the tags table to a file called `TAGS` located in the current directory.

   15

P.T.O

[1] Tags tables may be used by IDEs to assist a programmer to write / browse code.

2. Recall that the following grammar was discussed as an example of a **non-SLR** grammar.

$$S \to L = R \qquad S \to R \qquad L \to *R \qquad L \to \mathbf{id} \qquad R \to L$$

Now consider the following grammar G (uppercase letters denote non-terminals):

$S \to L\ \mathbf{id}$ $\quad S \to \mathbf{id}\ R\ *$ $\quad S \to *L *$ $\quad S \to *R\ \mathbf{id}$
$L \to \mathbf{id}$ $\quad R \to \mathbf{id}$

(a) Prove that G is unambiguous.

(b) Is G an $LL(1)$ grammar? Justify your answer.

(c) Construct an equivalent grammar G' that is $LL(1)$. Justify your answer.

(d) Construct the canonical collection of sets of $LR(1)$ items for G.

(e) Compute the $FOLLOW$ sets for the non-terminals in G.

(f) Show that G is not an SLR grammar. **You <u>do not</u> need to construct the canonical collection of sets of $LR(0)$ items for G.**

(g) Hence construct an example of an $LR(1)$ grammar for which the SLR parsing table has both shift-reduce and reduce-reduce conflicts (not necessarily in the same row / column of the parsing table).

[3 + 1.5 + 4 + 8 + 3 + 4.5 + 4 = 28]

Indian Statistical Institute
Semister-I 2014-2015
M.Tech.(CS) - Second Year
Mid-term Examination (26 September, 2014)
Subject: Patter Recognition and Image Processing

Total marks: 80 **Maximum marks:** 60 **Duration:** 2 **Hrs**

Answer maximum of 60 marks (any part of any question). Simple calculators can be used.

1. (a) Describe the following classifiers with their advantages and disadvantages.
   (i) Bayes classifier. (**5 Marks**)
   (ii) Minimum distance classifier. (**5 Marks**)
   (iii) K-Nearest Neighbour (KNN) classifier. (**5 Marks**)
   (b) Training patterns for two classes are given as in Figure 1. Classify an unknown pattern $(5, 3)$ using K-NN (for $K = 1, 3, 5, 7$) classifier. (**5 Marks**)

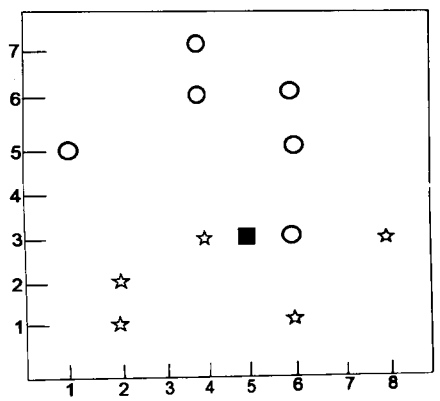


Figure 1: Training patterns ($\circ$ – *class* 1, $\star$ – *class* 2) of two classes.

2. (a) Define the Sum-of-Squared-Error clustering criterion to cluster n samples into k clusters, and interpret the criterion. (**4 Marks**)
   (b) Consider an application of k-means clustering algorithm to the following two dimensional data (Figure 2) for $k = 2$ (using Euclidean distance measure). Start with the two cluster initial means as $m_1 = (1, 3)$ and $m_2 = (4, 3)$.
   (i) What are the means and the clusters after the first iteration? (**3 Marks**)
   (ii) What are the final cluster means and the clusters after convergence of the algorithm? (**5 Marks**)
   (c) Explain the differences between $k-$means and $k-$medoids clustering algorithms. (**3 Marks**)
   (d) What are the advantages of hierarchical clustering over partitioning based clustering? Explain different agglomerative clustering methods. (**5 Marks**)
3. (a) Define variance-covariance matrix. Explain the differences between feature extraction and selection. (**5 Marks**)

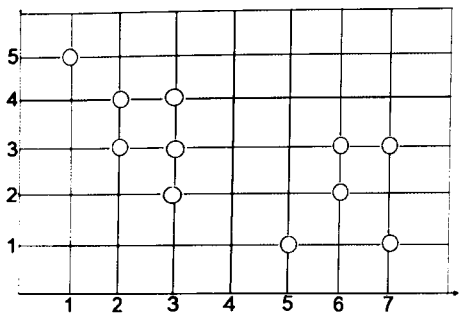


Figure 2: Two dimensional data for $k-$means.

(b) Find the first and second principal component vectors for the data $\{(1,1),(1,2),(2,1),(2,3),(3,2),(3,3),(4,4),(4,5),(5,4),(5,5)\}$. **(15 Marks)**

4. (a) Explain the differences between filter and wrapper approaches. **(3 Marks)**

(b) Consider a dataset in which every pattern is represented by a set of 10 features. The goal is to identify a subset of 5 features or less that gives the best performance on this dataset. How many feature subsets would be considered (i.e., the number of times the criterion function will be invoked) by each of the following feature selection algorithms before identifying a solution?

(i) Exhaustive search. **(3 Marks)**

(ii) Sequential Forward Selection (SFS). **(3 Marks)**

(iii) Sequential Backward Selection (SBS). **(3 Marks)**

(c) For the following objective function $J(x)$, perform an SFS to select 3 features from 5 features.

$$J(x) = 2x_1x_2+3x_1+5x_2-6x_1x_2x_5+7x_2x_4+9x_2x_5+4x_1x_4-2x_1x_2x_3x_4+5x_4x_2x_5+3x_3x_2x_5,$$

where x_k's are indicator variables that determine if the k^{th} feature has been selected ($x_k = 1$) or not ($x_k = 0$). **(8 Marks)**

INDIAN STATISTICAL INSTITUTE

First Mid-Semestral Examination: 2014-15

Subject Name : **Cryptography** Date : 29/09/14

Course Name : M.Tech. (CS) I yr. Maximum Score: 80 Duration: 3 Hours

Note: Attempt all questions. Marks are given in brackets. Total score is 92. But maximum you can score is 80. Use separate page for each question.

1. [6+6 = 12] Let $\mathbf{E} = (KG, f, g)$ be a deterministic symmetric-key encryption scheme ($K_e = K_d$) with message space $\mathcal{M}$ and ciphertext space $\mathcal{C}$.

(a) Suppose $\mathcal{M} = \mathcal{C}$. Is (KG, g, f) always an encryption scheme? Justify your answer.
(b) Is the condition $\mathcal{M} = \mathcal{C}$ required? Justify.

2. [4+6=10] Solve the following anagram: "HISSESAYIT". So anagram can be viewed as a ciphertext of a plaintext. Formally define an encryption scheme which corresponds to anagram mechanism.

3. [6+6+6 = 18] Let $X, Y \in_R S$ and $f : S \to S$ be a computable function. Prove or disprove the following statements:

(a) $\Delta(f(X), f(Y)) \leq \Delta(X, Y)$.
(b) For all algorithm A there exists B such that $\Delta_A(f(X), f(Y)) \leq \Delta_B(X, Y)$.
(c) For all algorithm A, $\Delta_A(f(X), f(Y)) < \Delta_A(X, Y)$.

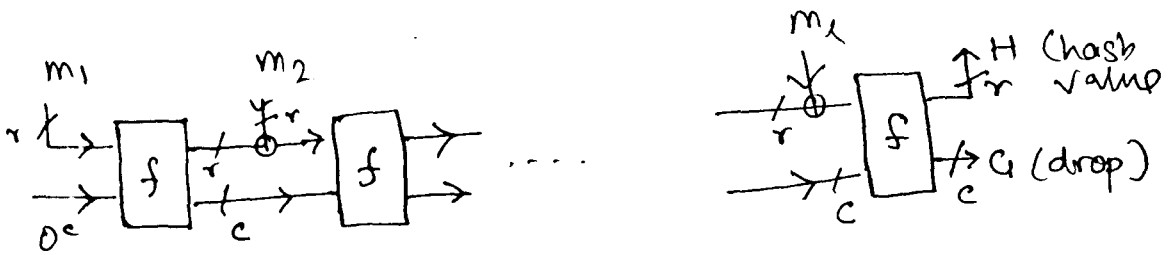
4. [4+4+4+6 = 18] Suppose KG is same as the key-generation of RSA-encryption scheme. We define a new probabilistic encryption $E((e, N), m|r) = (r^e \bmod N,\ r \cdot m \bmod N)$ where $r \in_U \mathbb{Z}_N^*$ (the set of all integers modulo which are relatively prime to N) is the random coin.

(a) Write down it's decryption algorithm.
(b) Does the decryption algorithm always correctly decrypt.
(c) Given a ciphertext C of a unknown message x, re-encrypt the message x.
(d) Show that the encryption scheme has one-way security under the hardness assumption of RSA.

6. [5 + 5 =10] Suppose $E : \{0,1\}^{2n} \times \{0,1\}^n \to \{0,1\}^n$ is a function such that for all $k \in \{0,1\}^n$, $E(k, \cdot)$ and its inverse are efficiently computable permutations over $\{0,1\}^n$. We define a compression function f such that $f(h, m) = E(m, m \oplus h) \oplus m$. Find a collision and preimage attack for f.

5. (10 + 10 = 20) Describe a Preimage and 4-multicollision attack in complexity about $O(2^{c/2})$ on the sponge hash function based on a permutation f over $\{0,1\}^{r+c}$. Assume that any message M is of the form $(m_1, \ldots, m_l)$ where $|m_i| = r$. The r bit hash value H is defined as

$$(H, G) = f(\cdots f(f(\overline{m_1}) \oplus \overline{m_2}) \cdots), \quad \overline{x} = (x, 0^c), x \in \{0,1\}^r$$



Sponge hash function

Hint. f is permutation. Both f and f^{-1} are efficiently computable. (for preimage attack)

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : (2014 - 2015)

Course Name : M. Tech. (CS) **Year : 2nd year**

Subject Name : Neural Networks & Applications

Date : September 29, 2014 **Maximum Marks : 50** **Duration : 2 hrs**

Answer all the questions.

1. Show geometrically how a single node perceptron model of artificial neural networks classifies a set of two dimensional patterns distributed in linearly separable two classes. [25]

2. Describe the McCulloch-Pitts model of a neuron of an artificial neural network. [10]

3. Consider a two input XOR problem in the domain of pattern classification. Show how a multilayer perceptron model of artificial neural networks deals with this classiciation. Clearly show the possible values of the parameters of the model, assuming step function as the activation function of the neurons. [15]

INDIAN STATISTICAL INSTITUTE

First End-Semestral Examination: 2014-15

Subject Name : **Cryptology** Date: 29th Nov 2014
Course Name : M.Tech. (CS) II yr. Maximum Score: 50 Duration: 3 Hours

Note: Attempt all questions. Marks are given in brackets. Total score is 58. But maximum you can score is 50. Use separate page for each question.

1.[4 + 6 = 10] State RSA signature scheme and show its correctness. Given any arbitrary message m, write down a forging algorithm with the access of a signing oracle, for signing the message m. Note that the forger can not submit m to the signing oracle.

2.[6] Let $e_k : \{0,1\}^n \to \{0,1\}^n$ be a blockcipher, $k \in \{0,1\}^n$. We define

$$f(m,h) := e_{m \oplus h}(m \oplus h) \oplus h.$$

Find a preimage attack of f in complexity $2^{n/2}$.

3.[4 + 6 =10] Extend the definition of Multiline hash over $(\{0,1\}^n)^{\leq \ell}$ mapping to $\{0,1\}^n$ Prove that it is 2^{-n}-U hash.

4.[3+3 + 6 =12] Define Integrity and privacy advantage of an authenticated encryption scheme with the access to both the encryption and decryption oracles. Let $e_k(tw, \cdot)$ be a tweakable blockcipher. Construct an integrity attack for $E_K(m_1, \ldots, m_r) = (C_1, \ldots, C_{r+1})$ where $C_i = E_K(i, m_i)$ for all $1 \leq i \leq r$ and $C_{r+1} = E_K(0, \oplus_i m_i)$.

5.[3 + 5 =8] Let $pad(m_1, \ldots, m_r) = (m_1, \ldots, m_r, \oplus_i m_i)$. Show that it is not a prefix-free padding. Further construct a forgery attack of CBC-MAC with the above padding rule.

6.[6 + 6 =12] Show that message recovery is hard for an ideal random permutation. State the reduction algorithm which shows that a symmetric key encryption is message recovery secure whenever it is SPRP.

Indian Statistical Institute

Semester-1 2014-2015

M.Tech.(CS) - Second Year

End-semester Examination (1 December, 2014)

Subject: Computer Architecture

Maximum marks: 50 Duration 3 hrs.

Please keep your answers brief and to the point.

Answer any 5 questions out of 7. Each question carries 10 marks.

1. (a) Consider an Instruction Set Architecture (ISA) in which all instructions are 32 bits, there are 32 general purpose registers (GPRs), and all immediate values are 16-bits. Any instruction uses either two GPRs, or one GPR and one immediate value as operands. Compute the maximum number of instructions that your ISA can support. [3]

   (b) Companies A and B both manufacture processors supporting the same ISA. A's processors are 4 times faster than B's processor on commonly used benchmarks. B's engineers find that 66% of the total time for these benchmarks is spent in waiting for L1 data cache misses. They decide to redesign the data cache hierarchy. Explain why this is not a good decision. [2]

   (c) Recall that the average memory access time is determined by the hit time, miss rate and miss penalty. Briefly explain (in 2-3 lines each) what impact the following have on these three quantities: (i) cache block size; (ii) number of banks in the cache / memory. In what other ways do these two parameters affect performance? [3]

   (d) Compare the hypercube and 2-D mesh topologies for an N-node interconnect based on cost, latency, and bandwidth. [2]

2. (a) Assume a VLIW processor with three integer units (X, Y, Z), one multiply unit (M), and two load-store units (LS0, LS1). ALU instructions have a latency of 1 cycle, multiply instructions have a latency of 5 cycles, and loads have a latency of 2 cycles. One branch can execute per cycle and executes in the Z pipeline. The following code has been bundled assuming the EQ scheduling model. What is the value of R12, R13, and R14 after this code executes? Keeping the register names unchanged, reschedule this code assuming the LEQ model. Why is the LEQ model more flexible? [2+2+1=5]

```
{ADDI R9, R0, 9; ADDI R10, R0, 10;}
{ADDI R6, R0, 6; ADDI R8, R0, 8; ADDI R5, R0, 5;}
{LW R6, 0(R7); LW R8, 4(R7);}
{ADDI R12, R6, 1; ADDI R13, R8, 2;}
{MUL R7, R6, R9;}
{MUL R5, R8, R10;}
{LW R14, 8(R7);} // Assume that 8(R7) contains the value 0x1
{ADD R15, R16, R17;}
{ADD R14, R14, R5;}
{SUB R19, R18, R22;}
{ADD R5, R7, R5;}
```

(b) Rewrite the following code assuming the instruction set has been augmented with conditional move instructions, *movz* and *movn*. Assume that the branch mis-prediction penalty is 10 cycles and that the branch to *forward* is random and data-dependent. Does it make sense to predicate? The instructions *movz* and *movn* have the following semantics as described below: [3+2=5]

```
movz rd, rs, rt     if ( R[rt] == 0 ) then R[rd] <- R[rs]
movn rd, rs, rt     if ( R[rt] != 0 ) then R[rd] <- R[rs]
```

```
Code Sequence:
ADDI R6, R0, 1
ADDI R3, R0, 50
loop: LW R8, 0(R9)
      BEQZ R8, forward
      ADD R12, R15, R8
      SUB R24, R24, R12
      J done
forward:
      ADDI R24, R24, 10
done:
      SUBI R3, R3, 1
      BNEZ R3, loop
```

3. Consider a program running on a vector processor with the following latencies for various instructions:

- VLD and VST: 50 cycles for each vector element; fully interleaved and pipelined
- VADD: 4 cycles for each vector element (fully pipelined)
- VMUL: 16 cycles for each vector element (fully pipelined)
- VDIV: 32 cycles for each vector element (fully pipelined)
- VRSHF (right shift): 1 cycle for each vector element (fully pipelined)

Assume that:

- The machine has an in-order pipeline
- The machine supports chaining between vector functional units
- In order to support 1-cycle memory access after the first element in a vector, the machine interleaves vector elements across memory banks. All vectors are stored in memory with the first element mapped to bank 0, the second element mapped to bank 1, and so on.
- Each memory bank has an 8KB row buffer
- Vector elements are 64 bits in size
- Each memory bank has two ports (so that loads / stores can be active simultaneously), and there are two load / store functional units available.

(a) What is the minimum power-of-two number of banks required in order for memory accesses to never stall? Assume a vector stride of 1.

(b) The machine (with as many banks as you found above) executes the following program (assume again vector stride set to 1):

```
VLD V1 <- A
VLD V2 <- B
VADD V3 <- V1, V2
VMUL V4 <- V3, V1
VRSHF V5 <- V4, 2
```

It takes 111 cycles to execute this program. What is the vector length?

(c) If the machine did not support chaining (but could still pipeline independent operations), how many cycles would be required to execute the same program? Explain your answer.

(d) The number of banks is now reduced by a factor of 2 from the number of banks you found in part (a) above. Since loads and stores might stall due to bank contention, an arbiter is added to each bank so that pending loads from the oldest instruction are serviced first. How many cycles does the program take to execute on the machine with this reduced-cost memory system with chaining?

(e) We now want to design the second generation of the vector processor. The aim is to build a multi-core machine in which 4 vector processors share the same memory system. The number of banks is scaled by 4 to match the memory system bandwidth to the new demand. However, when we simulate this new machine design with a separate vector program running on every core, we find that the average execution time is longer than if each individual program ran on the original single-core system with $\frac{1}{4}$ the number of banks. Explain why could this happen. What change in the shared memory hierarchy can we make in order to alleviate this problem? [2 × 5 = 10]

4. (a) We define the *SIMD* utilization of a program run on a GPU as the fraction of SIMD lanes that are kept busy with active threads during the run of a program. The following code segment is run on a GPU. Each thread executes a single iteration of the shown loop. Assume that the data values of the arrays A, B and C are already in vector registers, so there are no loads and stores in this program. Notice that there are 4 instructions in each thread. A warp in the GPU consists of 64 threads, and there are 64 SIMD lanes in the GPU.

```
for (i = 0; i < 1024768; i++) {
   if (A[i] > 0) {
      A[i] = A[i] + C[i];
      B[i] = A[i] + B[i];
      C[i] = B[i] + 1;
   }
}
```

(i) How many warps does it take to execute this program?

(ii) Is it possible for this program to yield a SIMD utilization of 100%? If yes, explain what should be true about the arrays A, B and C for the SIMD utilization to be 100%? If not, explain your answer. [2+3=5]

(b) A four-processor shared-memory system implements the MESI protocol for cache coherence. For the following sequence of memory references, show the state of the line containing the variable **a** in each processor's cache after each reference is resolved. Each processor starts out with the line containing **a** invalid in their cache. [5]

(i) P0 reads **a** (ii) P1 reads **a** (iii) P2 reads **a** (iv) P3 writes **a** (v) P0 reads **a**

5. (a) Explain the working principle of Tomasulo's algorithm with an example.

(b) Explain how the non-speculative Tomasulo algorithm resolves the following classes of hazards: **RAW, RAR, WAW, WAR.** [8+2=10]

6. (a) Consider a system that has physically addressed instruction and data caches, each of which has 8-byte cache lines and is 1KB in size. Physical addresses are 16 bits. The following loop is executed on this machine.

```
for (x = i = 0; i < 256; i++) {
    x = x + A[i] + B[i] * C[i];
}
```

A, B and C are arrays of 16-bit integers, with the following starting addresses: 0x3000, 0x4200, 0x5400.

(i) If the cache is direct-mapped, what is the hit ratio?

(ii) The cache is now made n-way associative, keeping the overall size and cache line size fixed. Calculate the minimum value of n for which the hit ratio improves. What is the improved hit ratio for this value of n? [3+5]

(b) Briefly explain the difference between victim buffers, write buffers and stream buffers. [2]

7. For the following, make reasonable assumptions where necessary, but please clearly state your assumptions.

(a) Suppose you are evaluating the Retention-Aware Intelligent DRAM Refresh (RAIDR) technique proposed by Liu et al. (2012) for a 2 GB DRAM chip with 8 KB row size. Assume that the retention times for the rows are distributed as follows: 0.125% of the rows need to be refreshed every 64 ms, 6.25% need refreshing every 128 ms, and the rest are refreshed every 256 ms. Assume that the controller keeps track of the 1st bin using a 256 byte bloom filter, and the 2nd bin using a 1024 byte bloom filter. How much energy can be saved by using RAIDR instead of the default (uniform) refresh policy? Neglect the energy consumption of the bloom filters and related circuitry. Also assume that the hash functions make uniform use of the bloom filter. [7]

(b) Suppose you are trying to determine the settings used by the power management software on your laptop. Assume that the 8 GB DRAM on your laptop consumes 1.6 W of power when idle. Also assume that it takes 0.5 nJ and 2.56 μJ to read / write a row of size 64 bytes from / to DRAM, and your SSD hard-disk respectively. If hibernation simply involves copying the contents of DRAM to the swap partition, how long should a system be idle to benefit from hibernating? [3]

Indian Statistical Institute

End-Semester Examination (2014-2015)

M.Tech. (CS) II

Advanced Algorithms for Graph and Combinatorial Optimization Problems

Date: 02.12.2014 Maximum Marks: 100 Time: 3 hours

Answer as much as you can. The maximum you can score is 100 marks. Marks allotted to each question are indicated within square brackets near the right margin. This paper has two pages.

1. Let $G_n = (V, E)$ be a graph with the set of vertices $V(G_n) = \{1, 2, \ldots, n\}$, and the set of edges $E(G_n) = \{(a, b) | (a + b)$ is divisible by 2 or 3 or 5$\}$.
   (a) Draw the graph G_{10}.
   (b) Prove that the graph G_∞ is perfect. [3 + 9 = 12]

2. (a) Let G be a 3-regular plane graph having n vertices, in which every vertex lies on one face of length 4, one face of length 6, and one face of length 8.
   (i) Determine the number faces of each length in terms of n.
   (ii) How many faces does G have in all?
   (b) Give a linear time algorithm to check whether two maximal outerplanar graphs are isomorphic. [(4 + 2) + 8 = 14]

3. Consider the *cardinality maximum cut* problem: given an undirected graph $G = (V, E)$, find a cut in G which partitions V into sets V_1 and V_2 such that the number of edges in the cut is maximum. For any subset $A \subset V$ and a vertex $v \in V$, $d(v, A)$ denotes the number of edges between v and any vertex in A. Show that the following is a 2-factor approximation algorithm for this problem:
   begin
   1. Initialize $V_1 := v_1$; $V_2 := v_2$; /*v_1 and v_2 are any two vertices in V*/
   2. **for** $v \in V - \{v_1, v_2\}$ **do if** $d(v, V_1) \geq d(v, V_2)$ **then** $V_2 := V_2 \cup \{v\}$ **else** $V_1 := V_1 \cup \{v\}$;
   3. Output V_1 and V_2.
   end [10]

4 For a given 3-SAT formula with n variables and k clauses, suppose each variable is independently assigned 0 or 1 with probability 1/2 each. Then, show that for any instance of 3-SAT, there is a truth assignment that satisfies at least a 7/8 fraction of all clauses. [10]

5 (a) Give a greedy algorithm for the vertex colouring a given undirected graph G with $\Delta + 1$ colours, where Δ is the maximum degree of a vertex in G.

(b) Using the above algorithm, design an algorithm for colouring a given 3-colourable undirected graph having n vertices with $O(\sqrt{n})$ colours. [6+8=14]

6 For a given set S of n strings formed with a finite alphabet Σ:

(a) define the prefix graph P_S for S and show that the minimum weight cycle cover (i.e., a collection of vertex disjoint cycles which cover all vertices) of P_S can be computed in polynomial time;

(b) using an algorithm for finding set cover, sketch an algorithm to find the shortest superstring that contains, for each string $s_i \in S$, both s_i and its reverse s_i^R as substrings.

[(2+8)+10=20]

7 For the bin packing problem,

(a) give an instance for which the First-Fit algorithm gives $\frac{5}{3} \cdot OPT$;

(b) present an asymptotic PTAS along with its approximation factor;

(c) give its ILP formulation and LP relaxation.

[6+8+(4+2)=20]

8 (a) Give an f-factor approximation algorithm for the set cover problem where f is the maximum number of sets in which any element occurs. Justify the correctness of your guarantee.

(b) Why may dual-fitting be necessary to obtain an approximate solution to an optimization problem by using LP-duality?

(c) For the max-flow problem, why does an algorithm based on LP give an optimal solution?

[(3+3)+3+3=12]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2014 - 2015)

Course Name: M. Tech. (CS) **Year: 2nd year**

Subject Name: Neural Networks & Applications

Date: December 03, 2014 **Maximum Marks: 100** **Duration: 3 hrs**

Answer all the questions.

1. a) What is the main use of the Kohonen's Self-Organising Feature Map.
   b) Describe its architecture.
   c) Elaborate how the model vectors of this artificial neural network are updated.
   d) How are the input data points represented after convergence of this artificial neural network?
   e) Explain the relationship between the above representation of input data points and the size of this artificial neural network. [1 + 5 + 10 + 1 + 3 = 20]

2. a) Calculate the weight matrix for a Hopfield network to store following two patterns: $[1\ -1\ 1\ -1]^T$ and $[-1\ -1\ -1\ 1]^T$.
   b) A pattern $[-1\ -1\ 1\ -1]^T$ is presented to the above network, and then the nodes of the network are updated until a steady state is reached. What is the final state of the network? [10 + 10 = 20]

3. Derive how Oja's model of principal component analysis network extracts the first two principal components of a data set. Assume that the model converges in a finite number of iterations. [20 + 5 = 25]

4. Consider the problem of pattern classification to be performed by a multi layer perceptron model of artificial neural networks with only one hidden layer of nodes. The updation rule for weight w_{ji} between a j^{th} node in the hidden layer and an i^{th} node in the input layer is given by

$$\Delta w_{ji} = \eta \delta_j x_i$$

where δ_j is associated with j^{th} hidden node and x_i is input to the i^{th} input node. Derive an expression for δ_j in terms of desired and actual outputs of the multi layer perceptron, and its some other weights. [20]

Please turn over

5. a) A perceptron with a unipolar step function has two inputs with weights $w_1 = 0.5$ and $w_2 = -0.2$, and a threshold $\theta = 0.3$ (θ can therefore be considered as a weight for an extra input which is always set to -1). For a given training example $\mathbf{x} = [0, 1]^T$, the desired output is 1. Does the perceptron give the correct answer (that is, is the actual output the same as the desired one)?

b) The above perceptron is trained using the learning rule $\Delta\mathbf{w} = \eta (d - y)\,\mathbf{x}$, where $\mathbf{x}$ is the input vector, η is the learning rate, $\mathbf{w}$ is the weight vector, d is the desired output, and y is the actual output. What are the new values of the weights and threshold after one step of training with the input vector $\mathbf{x} = [0, 1]^T$ and desired output 1, using a learning rate $\eta = 0.5$? [5 + 10 = 15]

INDIAN STATISTICAL INSTITUTE

Semestral Examination : 2014 – 15

MTech CS (2nd Year)

Computational Finance

Date: 4 December 2014 Maximum Marks: 50 Duration: 3 Hours

The paper carries 52 marks. Attempt ALL questions. The maximum you can score is 50.

1. Critically explain the concepts: [3 X 4 = 12]
   (i) Martingale
   (ii) State price density
   (iii) Stopping time

2. a) Define the following option contracts:
   (i) Lookback
   (ii) Barrier
   (iii) Chooser

   For each of them, state the payoff function carefully, explaining all notation.

   [3 X 4 = 12]

3. In the two period model, explicitly solve the Consumption Investment problem for the utility function $u(w) = \ln w$. Compute the relevant expressions and solve for the optimal trading strategy when
   $N = 1, K = 2, r = 1/9, S_0 = 5, S_1(\omega_1) = 55/9, S_1(\omega_2) = 40/9$ and $P(\omega_1) = 4/5$.
   (The notation are as used in class) [10]

4. Suppose $S_0 = 12$, $T = 3$, $r = 0$, $u = 1.5 = 1/d$ are the parameters for a Binomial model. Compute the prices of the following options:
   i) Asian Call option with exercise price = 10
   ii) Up-an-Out Barrier Call with Barrier = 20 and exercise price = 15
   iii) American Call option with exercise price = 16. [3 X 6 = 18]

INDIAN STATISTICAL INSTITUTE

M.Tech (Computer Science)

Second Year, First Semester, 2014

Pattern Recognition and Image Processing

Time: 3 hours

04·12·14

Answer a maximum of six questions. Maximum marks one can score is 100.

(1)

I. Consider two image subsets S_1 and S_2 as shown below. Let V= {1} be the set of grey-level values required to define connectivity. Determine the set of pixels in S_1 and S_2 which are

a) 4-connected,

b) 8-connected and

c) m-connected.

Determine whether S_1 and S_2 are adjacent.

S_1					S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	1	1	1
0	0	1	1	1	0	0	1	1	1

II. Equalize the histogram of the following 8 x 8 image. The image has grey levels in the range [0...7].

4	4	4	4	4	4	4	0
4	5	5	5	5	5	4	0
4	5	6	6	6	5	4	0
4	5	6	7	6	5	4	0
4	5	6	6	6	5	4	0
4	5	5	5	5	5	4	0
4	4	4	4	4	4	4	0
4	4	4	4	4	4	4	0

(9+6=15 Marks)

(2)

I. Find the resultant image of the following 4x4 image after applying (use zero padding)

1	2	4	5
5	2	5	2
1	1	3	6
2	4	6	7

a) Mean filter

b) Median filter

c) Laplacian filter

d) Robert's gradient, and

e) Sobel's gradient.

(P.T.O)

II. Find the Fourier transform of the sequence f(0)=2, f(1)=3, f(2)=4, f(3)=4. Then calculate the inverse Fourier transform and compare the result with the original sequence. Draw the Fourier spectrum. (10+5=15 Marks)

(3)

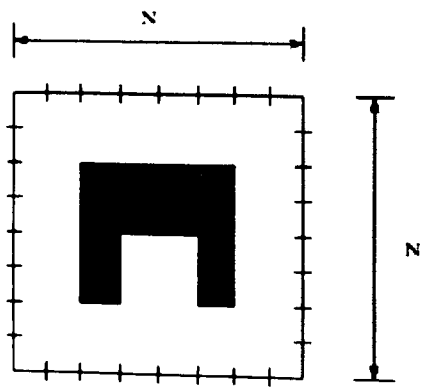
I. A source has 8 symbols a_i, i=1... 8, whose probabilities of occurrences are 0.6, 0.2, 0.08, 0.06, 0.02, 0.02, 0.01 and 0.01. Construct Huffman code for the same.

II. Explain: (a) Run length coding, and (b) Block truncation coding with examples. Discuss their advantages and disadvantages. (7+8=15 Marks)

(4)

I. Explain: (a) Global, (b) Otsu's and (c) Multiple histogram thresholding methods for image segmentation.

II. Segment the image shown below using split and merge procedure. Let P (R_1) = TRUE if all pixels in R_1 have the same intensity. Show the quad tree corresponding to your segmentation.



(6+9=15 Marks)

(5)

I. Given one dimensional data set {1, 5, 8, 10, 2}, use the agglomerative clustering algorithm with complete linkage (Euclidean distance) to establish a hierarchical grouping relationship. What are the clusters at each level?

II. Describe the DBSCAN clustering algorithm.

III. Explain the importance of initialization of centres in K-means clustering using examples. (10+6+4=20 Marks)

(6)

I. Describe the method of Principal Component Analysis. What are the advantages and disadvantages of it?

II. Explain Branch and Bound searching strategy with its advantages and disadvantages with respect to feature selection.

III. Describe "Plus-L, minus-R" Selection (LRS) strategy with its advantages and disadvantages.

IV. What is "curse of dimensionality"? (5+5+5+5=20 Marks)

I. **Given the following parameters of a two-variate normal distribution, find the Bayes' decision boundary and explain its significance. Assume equal prior probabilities for the classes.**

$$\mu_1 = [3, 2]^T \, , \mu_2 = [7, 4]^T \text{ and } \Sigma_1 = \Sigma_2 = 2I.$$

Symbols have their usual meaning.

II. **Explain the term "non-linear classifier" with examples.**

III. **Differentiate between supervised and unsupervised learning.**

(10+5+5=20 Marks)

INDIAN STATISTICAL INSTITUTE

End Semestral Examination

M. Tech (CS) - II Year, 2014-2015 (Semester - III)

Multidimensional Search and Computational Geometry

Date : 04.12.2014 Maximum Marks : 100 Duration : 3.5 Hours

Note: The question paper is of 130 marks. Answer as much as you can, but the maximum you can score is 100.

(Q1) You are given two sets of infinite straight lines $\mathcal{L}_1$ and $\mathcal{L}_2$ of size n_1 and n_2 respectively, with both n_1 and n_2 being $O(n)$ and $n_1 + n_2 = n$. Any line $l_{i1} \in \mathcal{L}_1$ is of the form $y = m_1 x + c_{i1}$ and any line $l_{i2} \in \mathcal{L}_2$ is of the form $y = m_2 x + c_{i2}$, i.e. any line in a particular set is parallel to all other lines of that set with different intercepts. Let $\mathcal{P}$ be the set of points generated by the intersection of these n lines. Surely, $|\mathcal{P}| = O(n^2)$. Design and analyse an efficient algorithm to compute the convex hull of $\mathcal{P}$. [5]

(Q2) Consider the Voronoi vertices $Q = \{q_1, q_2, \ldots, q_m\}$ corresponding to the Voronoi diagram $\mathcal{VD}(P)$ of a set of sites $P = \{p_1, p_2, \ldots, p_n\}$ where no four points are cocircular. The maximum empty circle $C_P(q_i)$ centered at each Voronoi vertex q_i contains three sites on its boundary. Let $X = \bigcup_{i=1}^{m} C_P(q_i)$. Now, prove or disprove the following statement: X *covers the union of the bounded faces of* $Vor(P)$ [10]

(Q3) You are given the Voronoi diagram $\mathcal{VD}(S)$ of a set S of n points. Now, you have to delete a point $p \in S$ and compute the Voronoi diagram of the set of points $S \setminus p$. Design an efficient algorithm and analyse it. Try to make your analysis output sensitive. [10]

[Hints: Recomputing the Voronoi diagram for the point set $S \setminus p$ makes little sense. You may look at the dual of the Voronoi diagram.]

(Q4) Given a convex polygon, a single guard is enough to guard the entire polygon. Prove or disprove with a counterexample the following statement. *You can never guard a non-convex polygon with a single guard.* [5]

(Q5) Let $\mathcal{T}(S)$ be the trapezoidal decomposition of a set S of non-crossing segments inside a bounding box R in the plane. $\mathcal{T}(S)$ is obtained by drawing two vertical extensions from every endpoint p of a segment in S, one extension going upwards and one going downwards. The extensions stop when they meet another segment of S or the boundary of R.

Let s be a new segment not crossing any of the segments in $\mathcal{S}$. Prove that a trapezoid $\Delta \in \mathcal{T}(\mathcal{S})$ is also a trapezoid of $\mathcal{T}(\mathcal{S} \cup \{s\})$ if and only if s does not intersect the interior of Δ. [5+5=10]

(Q6) Let $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ be n non-vertical line segments on the plane, and their projections on the x-axis are $\{[x_1, x_1'], [x_2, x_2'], \ldots, [x_n, x_n']\}$, respectively. $\mathcal{L}_S$, the lower envelope of $\mathcal{S}$ is the pointwise minimum of the line segments s_i, $i = 1 \ldots n$ and is defined as

$$\mathcal{L}_S(x) = \min_{1 \le i \le n} s_i$$

(i) Now, consider the following statement. $\mathcal{L}_S(x)$ follows a $(n, 2)$ Davenport-Schinzel sequence. If yes, prove it; else give a counterexample. [8]

[**Hints:** Recall that an $(n, 2)$ Davenport-Schinzel sequence will not have a subsequence like $\ldots s_i \ldots s_j \ldots s_i \ldots s_j \ldots$.]

(ii) Design and analyze an algorithm for finding $\mathcal{L}_S(x)$. [12]

[8+12=20]

(Q7) A Rectangular Intersection Graph (*RIG*) G = {V,E} for a set $\mathcal{R}$ of n axis-parallel rectangles is defined as follows. For each rectangle $r_i \in \mathcal{R}$, we assign a vertex v_i of G; an edge $e \in$ E is assigned between vertices v_i and v_j iff $r_i, r_j \in \mathcal{R}$ $(i \neq j)$ intersect. You can assume that none of the rectangles in $\mathcal{R}$ is completely enclosed within another rectangle in $\mathcal{R}$.

Design and analyze an efficient output sensitive algorithm to form the desired *RIG*.

[10]

[Hints: You can possibly use plane sweep and associated data structures. Can you design an $O(n \log n + k)$ time algorithm, where k is the number of pairs of rectangles that intersect.]

(Q8) Let $\mathcal{P}$ and $\mathcal{R}$ be two polygons with n and m vertices respectively. Find out the complexity of the Minkowski sum $\mathcal{P} \oplus \mathcal{R}$ under the following two cases: (i) one polygon is convex and the other is non-convex; (ii) both polygons are non-convex. Prove your result. [6+9=15]

[You can assume results proved in class.]

(Q9) Given a set $\mathcal{P}$ of n point obstacles and a disk robot $\mathcal{R}$ of radius r with its start and target positions. Your problem is to find out if there exists a collision free path of $\mathcal{R}$ among $\mathcal{P}$. Try to characterize the path, if it exists and then try to devise an algorithm.

[10]

(Q10) The farthest neighbor Voronoi diagram of a set S of points in $\mathbb{R}^2$, denoted by $\mathcal{VDF}(S)$, is the decomposition of $\mathbb{R}^2$ into maximal connected regions so that the farthest point

of S from any point within each region (under the Euclidean metric) is the same. Show that $\mathcal{VDF}(S)$ in the plane is a tree. [10]

[**Hints:** Can the farthest neighbor Voronoi diagram have bounded Voronoi cells?]

(Q11) State and prove the crossing lemma for a graph G with v vertices and e edges. [10]

(Q12) Find out at most how many incidences can there be among a set $\mathcal{P}$ of n points and a set $\mathcal{C}$ of m unit circles on the plane. [10]

[**Hints:** You can proceed as in the case of point line incidences, but here you can have multi-edges. You have to get rid of them to apply crossing lemma.]

(Q13) Find out a tight bound on the maximum number of unit distances determined by a set $P = \{p_1, \ldots, p_n\}$ of n points on a line. Can you find a construction of a set of points that achieves this bound? [3+2=5]

INDIAN STATISTICAL INSTITUTE

Final Examination:(2014-2015)

MTech C.S. 2nd Year

Digital Signal Processing

Date: 5.12.2014 Maximum Marks: 100 Duration: 3 hours

Note: The marks add up to 112. The maximum you can score is 100. The exam is open-book, open-notes. Use of calculators is permitted.

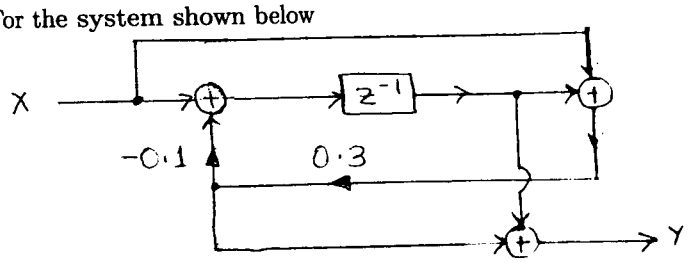
1. You have to compute the DFT (Discrete Fourier Transform) $X[k], k = 0, 1, 2, 3$, of a length 4 sequence $\{x_0, x_1, x_2, x_3\}$ using a radix-2, decimation-in-time algorithm with inputs in bit-reversed order and outputs in normal order. Let $\underline{x} = [x_0\ x_1\ x_2\ x_3]^T$ and $\underline{X} = [X_0\ X_1\ X_2\ X_3]^T$.

   (a) Draw the flow graph.

   (b) Label the intermediary nodes as v_0, v_1, v_2, v_3 and let $\underline{v} = [v_0\ v_1\ v_2\ v_3]^T$ so that $\underline{v} = A_1 * \underline{x}$, $\underline{X} = A_2 * \underline{v}$ and $\underline{X} = A_3 * \underline{x}$ where A_1, A_2 and A_3 are all 4×4 matrices. Determine A_1, A_2 and A_3.

   [5+(5+5+5)]

2. For the system shown below



   (a) Determine the system function $H(z)$ and sketch $|H(e^{j\omega})|$.

   (b) Assuming a two's complement fixed-point representation with a wordlength of 4 bits (B=3), calculate the output noise variance.

   [5+10]

3. For a system function

$$H(z) = \frac{z^{-2} - 0.7z^{-1} + 0.12}{1 - 0.7z^{-1} + 0.12z^{-2}}$$

give

(a) A Direct Form II implementation.

(b) A Parallel Implementation.

(c) A cascade implementation of two all-pass systems.

[5+5+5]

4. The following system function was obtained through impulse invariance *i.e.* by sampling an analog impulse response and then applying the z-transform.

$$H(z) = \frac{2}{1 - e^{-1.3}z^{-1}} + \frac{5}{1 - e^{-2.0}z^{-1}}$$

Determine the analog impulse response if the sampling interval T used was 0.5s . [7]

5. A transmission channel has the system function

$$\frac{(2.2 + 5z^{-1})(1 - 3.1z^{-1})}{(1 + 0.81z^{-1})(1 - 0.62z^{-1})}$$

Design a causal and stable system that can be connected at the receiving end in order to compensate for the magnitude distortion caused by the channel. [13]

6. Consider the sequence

$$x[n] = \begin{cases} 2 & 0 \leq n \leq 3 \\ 0 & 4 \leq n \leq 5 \\ 4 & 6 \leq n \leq 9 \end{cases}$$

Let $X[k]$ denote the samples of $X(z)$(z-transform of the length-10 sequence $x[n]$) evaluated on the unit circle at eight equally spaced points *i.e.*

$$X_8[k] = X(z)|_{z=e^{j2\pi k/8}} \quad k = 0, \cdots, 7$$

Determine the IDFT (8-point) of $X_8[k]$ without actually evaluating $X_8[k]$ [10]

7. Consider the causal sequence $x[n] = (-0.7)^n u[n]$ with z-transform $X(z)$. Determine the inverse z-transform of

(a) $z^{-2}X(z^3)$

(b) $z^{-1}X(z^{-1})$

[5 + 5]

8. A sequence of length 100 is to be filtered by a system with impulse response of length 20. Determine the number of DFTs and IDFTs needed to compute the linear convolution using 32 point DFTs for

   (a) Overlap and add

   (b) Overlap and save

[8 + 8]

9. A Type 2 real-coefficient FIR filter has zeros at 1, -1, 0.5 and $0.8 + j$. Determine the locations of the rest of the zeros (for the lowest possible order) and the corresponding system function. [5 + 5]

Indian Statistical Institute
Semestral Examination: 2014
Course Name: M. Tech. in Computer Science
Subject Name: Mobile Computing
Maximum Marks: 100

Date: 6-12-2014

Duration: 3 hours

Instructions: You **may** attempt **all** questions which carry a total of **110** marks. However, the maximum marks you can score is only **100**.

1. (a) What are the important features that an ideal sensor networks should have? 6

   (b) Explain the difference between the following two fault detection techniques in wireless sensor networks: self-diagnosis and cooperative diagnosis. 4

   (c) Write an approximation algorithm for the following minimum relay node placement problem in wireless sensor networks: given a set of sensor nodes S in a region and a uniform communication radius d, the problem is to place a set of relay nodes R such that the whole network G is connected. The objective of the problem is to minimize $|R|$ where $|R|$ denotes the number of relay nodes in R. Extend this approximation algorithm such that the network G becomes 2-*connected*. 10 + 10 = 20

   (d) Describe how clusters are formed and energy-usage among the nodes are balanced in low-energy adaptive clustering hierarchy (LEACH) routing protocol in wireless sensor networks. 10

2. (a) Compare the direct spectrum sensing and indirect spectrum sensing techniques in cognitive radio networks. 6

   (b) How direct spectrum sensing is achieved using local oscillator detection and closed loop power control in cognitive radio networks? 4 + 4 = 8

   (c) How indirect spectrum sensing is achieved using matched filter detection in cognitive radio networks? 4

   (d) Briefly describe the opportunistic and concurrent spectrum access models in cognitive radio networks. 4 + 4 = 8

   (e) How does Fixed Spectrum Access (FSA) policy contribute to the spectrum scarcity? 5

3. (a) What are the factors that make vertical handover more challenging than the horizontal handover? 5

   (b) Let R_{AP} and R_{BS} be the maximum achievable downlink data rate for an user connected with $WLAN$ and $WCDMA$ respectively. Let γ_{AP} and γ_{BS} be the received $SINR$ at the user end when associated with $WLAN$ and $WCDMA$ respectively. Find the relationship between γ_{AP} and γ_{BS} which $R_{AP} = R_{BS}$. Describe an $SINR$ based vertical handoff strategy which is based on the relationship between γ_{AP} and γ_{BS}. 5 + 6 = 11

   (c) What is an active set in the context of vertical soft handover? Describe the method for active set selection used in the context-aware vertical soft handoff algorithm (CVSH) for heterogeneous wireless networks. 4 + 10 = 14

   (d) Let P_j be the total transmitting power of $BS\ j$, P_{ij} be the transmitting power of $BS\ j$ to user i, C_{ij} be the channel gain between user i and $BS\ j$, α be the orthogonality factor, and N be the thermal noise power. Express γ_{ij}, the $SINR$ received by user i from $BS\ j$, in terms of P_{ij}, C_{ij}, α and N. 5

   (e) Compare the vertical handoff strategies based on Received Signal Strength (RSS) and Signal to Interference and Noise Ratio ($SINR$) respectively. 5

Indian Statistical Institute

Semester-1 2014-2015

M.Tech.(CS) - Second Year

End-semester Examination (8 December, 2014)

Subject: Compiler Construction

Maximum marks: 50 Total marks: 54 Duration 3 hrs.

Please keep your answers brief and to the point.

1. Consider the following Syntax-Directed Definition (SDD). 20 marks

$E \rightarrow T\ C$	{ C.t = T.tlist; C.f = T.flist; E.tlist = C.tlist; E.flist = C.flist; }
$C \rightarrow$ **or** $M\ T\ C_1$	{ bp(C.f, M.place); C1.t = C.t + T.tlist; C1.f = T.flist; C.tlist = C1.tlist; C.flist = C1.flist; }
$C \rightarrow \varepsilon$	{ C.tlist = C.t; C.flist = C.f; }
$T \rightarrow F\ R$	{ R.t = F.tlist; R.f = F.flist; T.tlist = R.tlist; T.flist = R.flist; }
$R \rightarrow$ **and** $M\ F\ R_1$	{ bp(R.t, M.place); R1.t = F.tlist; R1.f = R.f + F.flist; R.tlist = R1.tlist; R.flist = R1.flist; }
$R \rightarrow \varepsilon$	{ R.tlist = R.t; R.flist = R.f; }
$F \rightarrow$ **id**$_1$ **relop id**$_2$	{ F.tlist = 1; F.flist = 0; }
$M \rightarrow \varepsilon$	{ M.place = nextlocation(); }

(a) Is this SDD S-attributed? Is it L-attributed? Justify your answers in 1-2 lines each. (You will not get any credit unless you can correctly justify your answers.) 2

(b) Convert the SDD into a form suitable for bottom-up parsing by using marker non-terminals. *Avoid introducing unnecessary marker non-terminals.* 4

(d) Consider a modified LR parser for your SDD in (b) that uses a value stack and a state/symbol stack to evaluate semantic actions while parsing a given input string. Re-write the semantic actions for the non terminals E and C in terms of elements of the value stack You may assume that multiple attributes of a single grammar symbol can be stored in the same element of the value stack. [5]

(e) Compute the $FOLLOW$ set for R. [2]

(f) Now suppose we want to write a recursive, **top-down, predictive translator** for the given SDD. Assuming that all quantities involved in the SDD are integers, write C code for the function corresponding to R. You should use valid C syntax. You may assume that the procedures for the other non terminals are available to you. [6]

2. Consider the following C function: [34 marks]

```
static void dot_prod (int *A, int *B, int num, int prod)
    {
        int i, j;
        prod = 0;
        for (i = 0; i < num; i++)
            prod += A[i] * B[i];
        return;
    }
```

(a) Translate the executable statements in the above program into **three-address code**. [6]
(Do NOT perform any optimization at this stage)

(b) Write down the Quadruple representation of the above three-address statement in (a) [6]

(c) Generate machine code for the above three-address code in (a) where the target machine is a byte addressable one with four bytes to a word and n general purpose registers, R0, R1, ..., Rn-1. It has two-address instruction of the form:

```
op    source, destination
```

in which op is op-code, and source and destination are data fields. It has the following op-codes:

```
MOV
ADD
SUB
INC   (INC a, increments a)
CMP   (CMP x, y sets the condition code to positive if x > y)
CZ<   (jump to z if the condition code is negative)
```

For handling indexing and pointer type operations in three-address statements, assume that the current location is available on the stack (ST) that grows from low address to high address and the stack pointer (Si) points to the beginning of an activation record. [10]

(d) Optimize your code in (a) by using whichever of the following techniques are applicable: constant folding, global common sub-expression elimination, copy propagation, dead code elimination, code motion, and induction variable elimination. [12]

INDIAN STATISTICAL INSTITUTE

First-Semester Examination: 2014-2015

M. Tech. (CS) 2nd Year

Artificial Intelligence

Date: 8.12.2014 Maximum Marks: 100 Duration: 3 hours

Answer any *ten* questions. All questions carry equal marks.

1. Consider a sliding block puzzle with the following initial configuration:

W	W	W	B	B	B	E

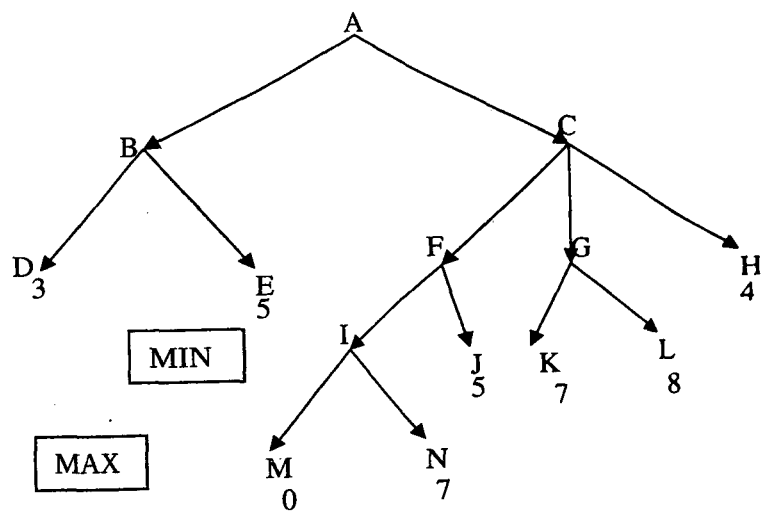
There are three white tiles (W), three black tiles (B), and an empty cell (E). The puzzle has the following moves:

a) A tile may move into an adjacent empty cell with unit cost.

b) A tile may hop over at most two other tiles into an empty cell with a cost equal to the number of tiles hopped over.

The goal of the puzzle is to have all the black tiles to the left of all the white tiles without regard for the position of the empty cell. Define the problem as a state space graph problem and find a sequence of moves that will transform the initial configuration to a goal configuration. What is the cost of the solution? [4 + 6 = 10]

2. Perform the *minimax* search procedure on the game tree shown below in which static scores are all from the first player's point of view and MAX is allowed to move first. Perform the left-to-right and right-to-left α-β pruning procedure on this tree and show how many nodes can be pruned. Discuss why a different pruning occurs. [3 + (3 + 3) + 1 = 10]



3. Define the constraint satisfaction problem and solve the following cryptarithmetic problem:

```
  S E N D
+ M O R E
-----------
M O N E Y
-----------
```

[2 + 8 = 10]

4. Answer the following:
   a) Show that $(\exists x)\ (P(x) \wedge Q(x)) \rightarrow (\exists x)\ P(x) \wedge (\exists x)\ Q(x)$ is valid whereas the converse $(\exists x)\ P(x) \wedge (\exists x)\ Q(x) \rightarrow (\exists x)\ (P(x) \wedge Q(x))$ is not.
   b) Prove using semantic tableaux approach that the following sentences are mutually consistent. "All Indian citizens who are adult have right to vote in election. Mary is an Indian citizen and has voting right. Mary is an adult." [6 + 4 = 10]

5. Answer the following:
   a) Prove that if α is a logical consequence of a set of premises $\sum$, then there is a tableau proof of α from $\sum$.
   b) Prove that a clause C is a logical consequence of a set of clauses S if and only if the set $S' = S \cup \{\sim C\}$ is unsatisfiable. [5 + 5 = 10]

6. Consider the following set of sentences. "Mary will get her degree only if she registers as a student and passes her examination. She has registered herself as a student. She has passed her examination." Prove that "she will get a degree" using both
   a) semantic tableaux approach; and
   b) resolution refutation method. [5 + 5 = 10]

7. Answer the following:
   a) Write a program in Prolog for pre-order traversal of a binary tree. The traversal method stores the elements of the tree in a list.
   b) Explain with example the differences between red cut and green cut in Prolog. [6 + 4 = 10]

8. Answer the following:
   a) Write a program in Prolog for merging two ordered lists.
   b) Describe the difference between the following two codes (i) and (ii) written in Prolog when the goal query is "grandfather(james, X)":

(i)	(ii)
grandfather(X,Y):- father(Z, Y), father(X, Z). father(james, robert). father(mike, william). father(william, james). father(robert, hency).	grandfather(X,Y):-father(X, Z), father(Z, Y). father(james, robert). father(mike, william). father(william, james). father(robert, hency).

[5 + 5 = 10]

9. What are the main features of an expert system? Explain with examples the forward and backward chaining approaches in a rule based expert system. [2 + 4 + 4 = 10]

10. Answer the following:
   a) Discuss the Bayes' theorem for the probabilistic reasoning.
   b) Suppose an initial observation S_1 confirms some hypothesis h with the belief MB = 0.3. The second observation S_2 performs the same hypothesis h with the belief MB = 0.5. Find the certainty factor of the hypothesis h using two observations S_1 and S_2. [5 + 5 = 10]

11. Describe the following:
   a) Dempster-Shafer theory of evidence; and
   b) Simulated annealing approach.

[5 + 5 = 10]

12. What is decision tree? Define information gain. Consider the following decision table and calculate the information gain for four condition attributes, namely, *Credit history*, *Debt*,

Collateral, and *Income*, considering *Risk* as decision attribute. Explain which condition attribute will be selected first, based on information gain.

Credit history	*Debt*	*Collateral*	*Income*	*Risk*
bad	high	none	$0 to $15K	high
unknown	high	none	$15K to $35K	high
unknown	low	none	$15K to $35K	moderate
unknown	low	none	$0 to $15K	high
unknown	low	none	over $35K	low
unknown	low	adequate	over $35K	low
bad	low	none	$0 to $15K	high
bad	low	adequate	over $35K	moderate
good	low	none	over $35K	low
good	high	adequate	over $35K	low
good	high	none	$0 to $15K	high
good	high	none	$15K to $35K	moderate
good	high	none	over $35K	low
bad	high	none	$15K to $35K	high

[2 + 2 + 4 + 2 = 10]

13. Let $I = <U, A>$ be a decision table, where $U = \{x1, ..., x7\}$ is a nonempty set of finite objects, the universe, and $A = C \cup D$ is a nonempty finite set of attributes. Here, $C = \{A1, A2\}$ and D are the set of condition and decision attributes, respectively.

U	*A1*	*A2*	*D*
x1	16-30	50	yes
x2	16-30	0	no
x3	31-45	1-25	no
x4	31-45	1-25	yes
x5	46-60	26-49	no
x6	16-30	26-49	yes
x7	46-60	26-49	no

In the context of rough set theory, explain the following with the above example data:

a) lower and upper approximations of decision attribute,
b) boundary region of decision attribute, and
c) degree of dependency and significance of a condition attribute.

[(2 + 2) + 2 + (2 + 2) = 10]

INDIAN STATISTICAL INSTITUTE
Semester Examination : (2014-2015)
M.Tech.(CS) II Year

Advanced Database Theory & Applications

Date: 09.12.2014 **Maximum Marks: 50** **Duration: 2.5 Hours**

Note: Answer all questions

1. An association formed by some of the faculty members of a university runs a few boarding houses near the university campus for providing accommodation to the students. Each boarding house has a manager, who is a Faculty of the university. The association maintains a database using relational model. The schema is:

   Boarding_house (bh_name, address, phone_no, manager_name)
   Room (boarder_no, name, room_no, room_type, seat_rent)
   Student (boarder_no, name, dept, course, year, student_type)

   The university also maintains a student database using relational model again. The schema is

   Student (roll_no, st_name, course, dept_name)
   PG (roll_no, subject)
   RF (roll_no, stipend, emp_no)
   Dept (dept_name, budget, phone_no, building_no)
   Faculty (emp_no, name, specialisation, designation, basic_salary)

   The university has two types of students; post-graduate(PG) and research-fellow(RF). Accordingly, student_type attribute in the Student relation of the Association schema can have only two values The emp_no attribute associated with the relation RF in the University schema indicates the supervisor of a research fellow.

   Design a global conceptual schema using relational model to create a multidatabase. Show the entire synonym/homonym/hypernym table for appropriate linking among the two databases and thus derive the global schema.

   (15)

2. Three sites at Kolkata, N.Delhi and Mumbai participate in the execution of global transactions. Kolkata originates a transaction T1 and N.Delhi and Mumbai participate. N.Delhi also originates a transaction T2 where Kolkata and Mumbai participate. After Kolkata and N.Delhi send <prepare T1> and <prepare T2> respectively to all participating sites, both the sites go out of order. If both of them recover after sometime and the systems at all the sites follow two-phase commit protocol, what action the systems would take at Kolkata and N.Delhi for both the transactions if,
   a) Kolkata recovers before N.Delhi
   b) N.Delhi recovers before Kolkata.

   (5+5=10)

3. In a distributed database environment an educational institute maintains a database with the following relations :

   Faculty (emp-no, name, dept-name, campus, designation)
   Student (roll-no, st-name, dept-name, course, year, campus)
   Stipend (roll-no, st-name, amount, course, year)
   Personnel (emp-no, name, address, basic-salary)

The institute has three campuses at Delhi, Bangalore and Kolkata where Kolkata is the Head-quarter. The entire database is maintained at the Head-quarter. While the relation 'Personnel' maintains data about all the employees of the institute, the relation 'Faculty' maintains data about the faculty members only. So, the attribute 'name' in 'Faculty' is a subset of the same attribute in 'Personnel'. For 'Faculty' and 'Personnel' relations 'emp-no' is the primary key while in 'Student' and 'Stipend' relations 'roll-no' is the primary key. Two queries are given below, which are executed at all the three campuses. Find the appropriate hybrid fragments (considering both horizontal and vertical fragments) to be distributed among the different campuses so that they will be sufficient to answer the queries. The queries are:

i) Find all faculty members with their name, address and designation for basic-salary > 50000.

ii) Find the total amount to be disbursed per month as stipend among the students at each campus.

(15)

4 Two relations R1 and R2 are placed at two different sites S1 and S2. R1 and R2 have a common attribute 'a' which is the primary key of R2. The relation R2 is also much larger than R1. R1 and R2 are to be joined and the result is to be kept at S1. Provide an appropriate joining strategy so that entire R1 or R2 are never transferred from one site to the other.

(10)

-x-

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2014 – 2015)

M.Tech. (CS) II Year

Parallel Processing: Architectures and Algorithms

Date: 10/12/2014 Total Marks: 112 Duration: 3 hrs

NOTE: You may answer all questions but maximum marks attainable is 100.

1. Given two 2×2 matrices A and B, show the program graph that computes the product matrix C = A × B, and the sum S of the four elements of C.
   Consider the operations 'multiply' and 'addition' as two fine grains.
   For an arbitrary processor, it is given that the addition operation needs 10 CPU cycles, and the multiply operation takes 100 CPU cycles. The interprocessor communication delay is 200 cycles.
   (i) Show the scheduling of the fine grain program using maximum number of processors required to utilize the maximum software parallelism existing in the program. Find out the speed-up and utilization.
   (ii) Can you use grain packing to optimize the number of processors that improves the speed-up and utilization? If yes, compare the results with those obtained in (i) and justify.
   [6 + 6 + 8 20]

2. (a) A parallel program is designed for a parallel machine with *(2N-1)*-nodes interconnected by a binary tree interconnection *(N=2^n)*. If the program is executed on a parallel machine with *2N*-nodes interconnected by a hypercube interconnection, will there be any slow down? Assume the nodes are of same computing speed Justify your answer.

   (b) Define a *pancake* graph with each node of degree *(n-1)*, using the group-theoretic model of Cayley graph Prove that the graph is undirected.
   Find one shortest path from a node (2 4 1 3) to a node (3 4 2 1) on a 24-node pancake graph, showing the respective generators in each step.
   [10 + (3 + 4 + 3) 20]

3. Describing briefly the Fast Fourier Transform (FFT) technique, show how the FFT of an input sequence of length 8 can be computed on butterfly network. Indicate the parallel time complexity of the algorithms in the following two cases :
   (i) when values of the required powers of ω are supplied to the processors a priori.
   (ii) when the processors need to compute the powers of ω while executing the algorithm.
   Here ω is an n^{th} ($n = 2^k$) primitive root of unity, i.e., $\omega = e^{2\pi i/n}$, $i = \sqrt{(-1)}$.
   [8 + 5 + 5 18]

4. Describe the Batcher's odd-even merge sort algorithm for sorting a sequence of elements. Prove its correctness by using 0-1 principle.
   Draw a sorting network for sorting 10 elements by Batcher's odd-even merge sort technique.
   [7 + 5 + 8 20]

5. a) Describe a parallel algorithm for the problem of finding all-pair shortest paths in a weighted graph which will run on a cube connected SIMD computer.

P.T.O

b) In a hypercube of degree n, how many distinct paths of length $k \leq n$ exist between two nodes u and v, if the shortest distance between the two is of length k? Paths are distinct if they differ in at least one intermediate node. Also, find the number of edge-disjoint paths between u and v. Justify your answer.

[16+5=21]

6 What is the output of the OpenCL kernel code given below?
Is there any option to optimize this kernel to get better speed up avoiding conflicts in shared memory? If yes, explain the procedure and justify your answer.

Input data : idata (a matrix in row major order)
Output data : odata (Resultant matrix)
Dimension of the matrix: width x height

```
__kernel void program(__global float *odata, __global float* idata, int width, int height)
{
   unsigned int xIndex = get_global_id(0);
   unsigned int yIndex = get_global_id(1);

   if (xIndex < width && yIndex < height)
   {
      unsigned int index_in  = xIndex + width * yIndex;
      unsigned int index_out = yIndex + height * xIndex;
      odata[index_out] = idata[index_in];
   }
}
```

[5+2+6 = 13]

Data Mining - Fall 2014
M. Tech CS II End-Sem Examination

Time: 3 hours
Full marks: 50

11.12.2014

The question paper carries 58 marks. Answer as much as you can. The maximum you can score is 50. The last page, which contains two figures for you to draw on them, should be attached with your answer sheet.

1. Consider the sample data points shown in Figure 1. The points belong to two classes, one set is marked in red round shapes and another set is marked in rectangular black shapes. For a linear SVM draw the following on Figure 4, which is a copy of Figure 1.

   (a) Mark the set of possible support vectors for the SVM

   (b) Draw a reasonable separation line for the SVM

   (c) Show the margin that is maximized by SVM

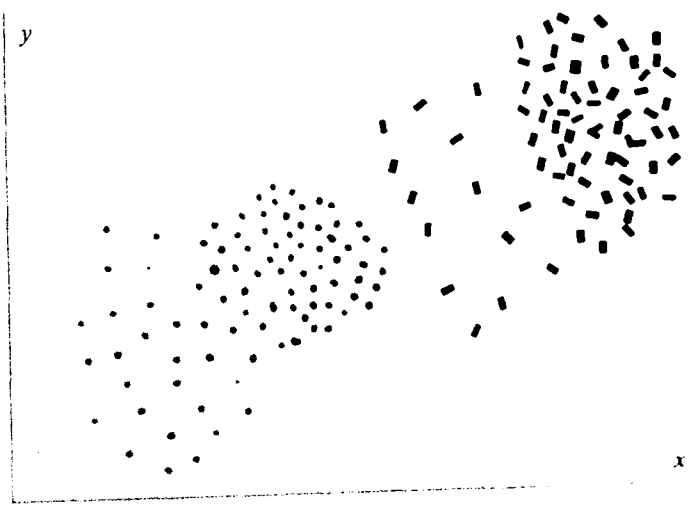


Figure 1: A set of sample data points, belonging to two classes.

(3 + 2 + 2 = 7 marks)

2 Suppose you are given the name A of an entity that belongs to some category C. Come up with an algorithm to find a few names of other entities belonging to the same category. You can use a web search engine such as Google which allows you to search with any *text query*, get the *list of results* and their *snippets* as shown by the search engine, but you should not visit any result page and use the full text of that page. The structure of no such page is known beforehand. You have to stay within the search engine's main page and cannot click on any of the results to read a full document.

For example, the given entity can be *football*, which happens to be the name of a *sport*. Your algorithm should output a few names of other *sports*. Similarly, the given entity could be *tomato*, which is a *vegetable*. Then your algorithm should output the names of few other *vegetables*. Note that the category is not given beforehand. Also, even if there is some webpage which lists the names of several sports including football, your algorithm should not try to find such a page and mine the content of that page.

While it would be easier to try a few approaches using Google hands on, right now you will have to depend on your intuition. While evaluating, however, your approach would be tried hands on to verify to what extent it works.

(5 marks)

3 Deb can decide to go to the institute by one of three modes of transportation, car, a combination of metro and bus, or train. The car is the most comfortable choice, but is costly, and because of high traffic, there is a 40% chance he will be late. If he goes by metro + bus, the it costs less but the probability of being late is still 40%. The train is usually the fastest, the probability of being late is only 20%, but is terribly crowded.

Suppose you know that Deb happens to use the car 50% of the time, takes the train 30% of the time and uses the metro and bus combination for the rest of the days. If you see him coming late some day, what is the probability that he came by car that day?

(5 marks)

4 Design MapReduce algorithms to perform the following. Write the *map* and *reduce* algorithms explaining why they would work.

(a) From many documents containing integers determine the largest integer.

(b) Let R and S be two relations with the same schema. Compute the difference $R - S$.

(5 + 5 = 10 marks)

5 Consider the NRA algorithm for aggregating lists $L_1, \cdots, L_m$ where the resulting list L is the union of all the individual lists L_i, for $i = 1, \ldots, m$ and the score of one item d in L is the *sum* of its scores in each of the individual lists. Explain whether the NRA approach would work if the aggregation function is (a) *maximum*, (b) *multiplication*, instead of sum. To show NRA would work with an aggregation function, explain how the bounds would be computed and how those would increase or decrese in subsequent steps. If NRA would not work with an aggregation function accordingly explain why.

From the above, what property of the aggregation function do you find as *essential* for NRA to work? For the whole problem, if needed, you may use some example of your own, or the one in the slides.

(4 + 4 + 2 = 8 marks)

6. Let $G = (V, E)$ be a social network graph with $|V| = 100$. Assume that a given clustering of G consists of two disjoint communities (in other words, subset of nodes) V_1 and V_2, of 40 and 60 nodes respectively, and there is only one edge going between V_1 and V_2, as shown in Figure 2.

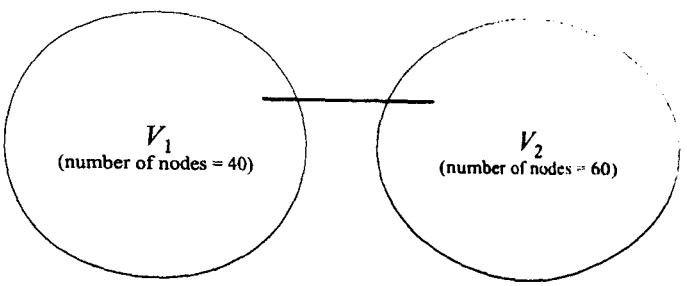


Figure 2: A diagram representing the graph G

Suppose a, b, c, $d \in V$ are four distinct nodes and suppose the neighborhood profiles of the four nodes are as follows:

$$\begin{aligned} N(a) &= \{32, 60, 61, 75, 100, \ldots\} \\ N(b) &= \{10, 40, 41, 65, 100, \ldots\} \\ N(c) &= \{15, 64, 100, \ldots\} \\ N(d) &= \{25, 74, 100, \ldots\} \end{aligned}$$

From this information, what can you infer about the positions of the nodes a, b, c and d in Figure 2? Justify your answer. Along with your explanation, show the positions of a, b, c and d on Figure 5 which you are going to attach with your answer sheet.

(2 × 4 = 8 marks)

7. Let $S = \{s_1, s_2, \cdots\}$ be a stream of IP addresses (represented as 4 byte non-negative integers) and let Z be the set of all non-negative integers less than or equal to $2^{64} - 1$ (in other words, 64-bit integers). Let $h : S \to Z$ be a hash function which supposedly hashes any element of S to some 64-bit integer with equal probability.

Now, if you are given that the minimum hash value of $h(s)$ seen so far is $M \in Z$, then would you be able to provide some estimate (in terms of M) for the number of distinct elements seen so far in S?

(5 marks)

8. In this problem, we will examine the union and intersection of two bloom filters. Let S_1 and S_2 be two sets of elements and let $h_1, \ldots, h_k$ be a set of hash functions using which we construct bloom filters for S_1 and S_2. As you know, the bloom filter $B(S)$ for a set S is the bit vector where the m-th bit is 1 if and only if for some hash function h_i and some element $a \in S$, $h_i(a) = m$. Given two bloom filters, we can produce new bloom filters by bitwise OR or AND operations. Figure 3 shows an example of two such bloom filters $B(S_1)$ and $B(S_2)$ for two sets S_1 and S_2, and also two more bloom filters obtained by bitwise OR and AND operations of the two bloom filters.

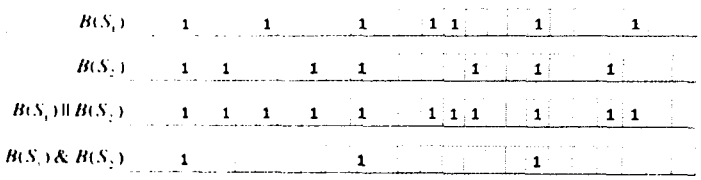


Figure 3: Example of bloom filters obtained by bitwise OR and AND operations.

Prove the following.

(a) The bitwise OR operation of $B(S_1)$ and $B(S_2)$ produces the same bloom filter which we would obtain by directly constructing a bloom filter from $S_1 \cup S_2$. In other words

$$B(S_1)||B(S_2) = B(S_1 \cup S_2)$$

(b) However, the bitwise AND operation of $B(S_1)$ and $B(S_2)$ does not produce the same bloom filter which we would obtain from $S_1 \cap S_2$. In other words,

$$B(S_1)\&B(S_2) \neq B(S_1 \cap S_2)$$

(5+5 = 10 marks)

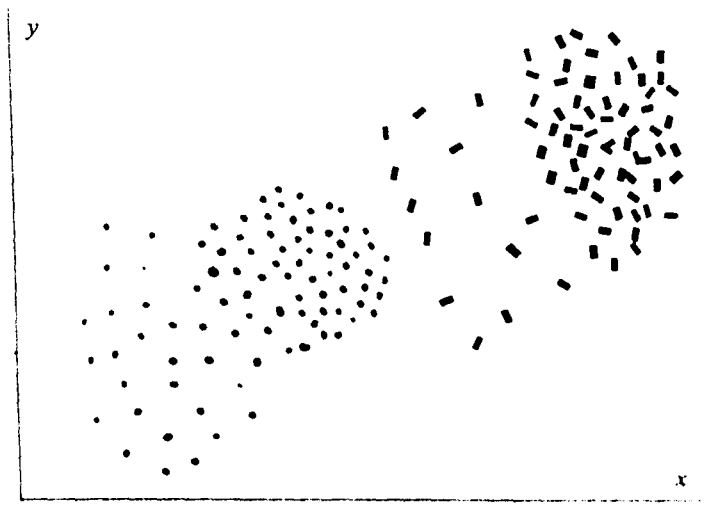


Figure 4: A set of sample data points, belonging to two classes (Problem 1).

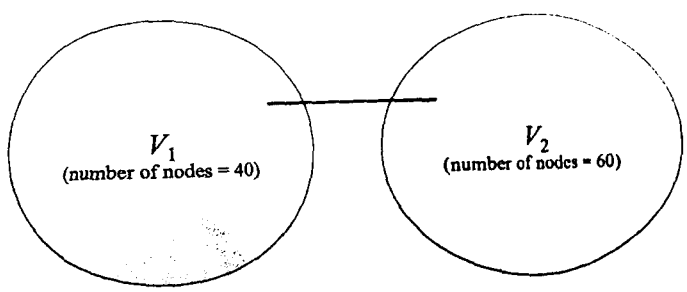


Figure 5: A diagram representing the graph G (Problem 6).

INDIAN STATISTICAL INSTITUTE

First Semestral Examination (2014)

M.Tech. (Computer Science) Second Year

Natural Language Processing

Date: 12.12.2014 **Time: 2 h 30 min** **Maximum Marks: 50**

Notes. Answer all questions. [Open book/note/Internet examination]

Q1. A simple PCFG is given below.

Grammar	Prob		
S → NP VP	0.8		
S → Aux NP VP	0.1	+	1.0
S → VP	0.1		
NP → Pronoun	0.2		
NP → Proper-Noun	0.2	+	1.0
NP → Det Nominal	0.6		
Nominal → Noun	0.3		
Nominal → Nominal Noun	0.2	+	1.0
Nominal → Nominal PP	0.5		
VP → Verb	0.2		
VP → Verb NP	0.5	+	1.0
VP → VP PP	0.3		
PP → Prep NP	1.0		

Lexicon

Det → the | a | that | this
0.6 0.2 0.1 0.1

Noun → book | flight | meal | money
0.1 0.5 0.2 0.2

Verb → book | include | prefer
0.5 0.2 0.3

Pronoun → I | he | she | me
0.5 0.1 0.1 0.3

Proper-Noun → Houston | NWA
0.8 0.2

Aux → does
1.0

Prep → from | to | on | near | through
0.25 0.25 0.1 0.2 0.2

As the grammar is not in CNF, consider the conversion of the above grammar in CNF. Production rules for S are changed in the following set of productions which are now in CNF:

S → NP VP ?
S → X1 VP ?
X1 → Aux NP ?
S → book | include | prefer
? ? ?
S → Verb NP ?
S → VP PP ?

[Production rules corresponding to the other non-terminals are not shown here but other than the above rules, there is no more productions for S in the CNF.]

The question marks (8 such) in the above CNF indicate the corresponding rule probabilities. Compute these probabilities from the original grammar so that probability of any string generated by the CNF version of the grammar remains same as that of the one given by the original grammar. Justify your answer (word of caution: writing the probabilities only will NOT give you any credit).

[12]

Q2. Consider the following English sentence:

BLEU is designed to approximate human judgement at a corpus level, and performs badly if used to evaluate the quality of individual sentences.

[Students NOT comfortable with Bengali please go to PART B]

PART A:

When the above English sentence is sent to two different English-Bengali machine translation systems and their outputs are given below:

System 1: ব্লেউ বানানো হয়েছে কিছু বাক্যের সমষ্টিগত ভাবে গুণাগুণ বিচারের ধারণা পাওয়ার জন্য , আলাদা ভাবে এক একটি বাক্যের জন্য এটি অনুপযুক্ত।

System 2: ব্লেউ বানানো হয়েছে মানুষের রায় কর্পাসের উপর অনুমান করার জন্য এবং খারাপ প্রদর্শনী করবে যদি একটি বাক্যের উপর পরীক্ষা করা হয়।

Evaluate the BLEU score for both the systems using the following Gold Reference.

Gold: ব্লেউ তৈরী হয়েছে কর্পাস-এ মানুষের মূল্যায়ন কে যথাযথ অনুমান করার জন্য প্রতেকটি বাক্যের গুণমান পৃথক ভাবে যাচাই করার ক্ষেত্রে এটি খারাপ ফলাফল দেয়।

PART B:

When the above English sentence is sent to two different English-Hindi machine translation systems and their outputs are given below:

System 1: ब्लेउ एक कोष के पर मानव मूल्यांकन अनुमानित है । यह अलग-अलग वाक्यों की मान का मूल्यांकन करने के लिए इस्तेमाल अगर बुरी तरह से करता है ।

System 2: ब्लेउ एक कोष के स्तर पर मानव मूल्यांकन करने के लिए लगभग तैयार है , और व्यक्तिगत वाक्यों की गुणवत्ता का मूल्यांकन करने के लिए लागू है जब खराब प्रदर्शन कर रहा है ।

Evaluate the BLEU score for both the systems using the following Gold Reference.

Gold: ब्लेउ एक कार्पस के पर मानव निर्णय करने के लिए तैयार है , और अलग-अलग वाक्यों की गुणवत्ता का मूल्यांकन करने के लिए अच्छा परिणाम देना नहीं है ।

[Note: **Consider up to 3-grams only in computing the required BLEU score**]

[10]

Q3. Gender information (masculine, feminine, or neutral) of each antecedent is very vital for pronominal anaphora resolution. Consider the following discourse:

Putum has a sister named Katum and she is eight years old. Recently, Putum and Katum got a pet dog, Jonjo and it is two years old. Jonjo is very happy in its new house. Putum talks everyone about his new playmate. Katum too is very happy with her new pet.

There are three proper nouns namely, Putum, Katum and Jongo which play the role of antecedents for resolving the pronouns appearing in the above discourse.

Assuming that you apply **EM algorithm** for finding the gender information for the antecedents, derive the following probabilities after the **first pass of the EM is executed**:

Prob(masculine | Putum) = ?; Prob(feminine | Putum)=?; Prob(neutral | Putum)=?

Prob(masculine | Katum) = ?; Prob(feminine | Katum)=?; Prob(neutral | Katum)=?

Prob(masculine | Jonjo) = ?; Prob(feminine | Jonjo)=?; Prob(neutral | Jonjo)=?

Note that initially all these probabilities are 1/3.

[Hint: You may use IBM Model-1 to compute the probabilities using EM. STOP after first pass.]

[20]

Q4. Idioms are integral part of any language. Assume a sufficiently large language corpus is divided into nine parts (r_1, r_2, ..., r_9). You observe the numbers of idioms in three parts (i.e. r_i, r_j and r_k) are 200, 100 and 300. So the observed mean($\overline{n}$) is 200. For distribution of idioms, let p_i denote the relative probability of observing idioms in r_i and n_i denote the number of idioms observed in r_i. Find one possible distribution (write down the values of p_i's and n_i's) of the idioms in the nine parts which is consistent with the following two constraints and the distribution should have **maximum entropy** over the other possible distributions:

$$\text{Constraint 1: } \overline{n} = \sum_{i=1}^{9} n_i p_i \text{ ; Constraint 2: } \sum_{i=1}^{9} p_i = 1$$

Justify your answer.

[Note: three n_i values are given.]

[15]

Indian Statistical Institute

Semester Examination (2014-2015)
M. TECH. (CS) 2 Year
Subject: Information and Coding Theory
Date: 15/12/2014 Time: 3 hours Maximum Marks: 100

Note: Notations used are as explained in the class.

1. For a pair of random variables (X, Y), prove that $H(X,Y) = H(X) + H(Y|X)$. Here $H(X)$ denotes the entropy of the random variable X. [8]

2. Prove that $I(X;Y) \geq 0$ where equality holds if and only if X and Y are independent. [7]

3. Let $\mathcal{X} = \{1,2,3\}$ and $X_1, X_2, X_3 \ldots$ be a time invariant Markov chain where X_is take values from the set $\mathcal{X}$. The transition matrix is

$$P = \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}.$$

Find the stationary distribution of the Markov chain. [8]

4. Let (X, Y) have the following joint distribution:

Y \ X	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

Find $H(X), H(Y), H(X|Y), H(Y|X)$ and $I(X;Y)$. [13]

5. State and prove the data processing inequality. [10]

6. Prove that for any instantaneous code over an alphabet of size D, the codeword lengths $l_1, l_2, \ldots, l_m$ must satisfy the inequality

$$\sum_i D^{-l_i} \leq 1.$$

Conversely, given a set of codeword lengths that satisfy this inequlity, there exists an instantaneous code with these word lengths. [15]

7. Prove that the Huffman code is optimal with respect to the expected length. [15]

8. Define channel capacity, noisy typewriter and the set of jointly typical sequences. [9]

9. State the channel capacity theorem and give an outline of the proof. [15]

INDIAN STATISTICAL INSTITUTE
End-Semestral Examination: 2014 – 15

Course Name: M. TECH CS - II

Subject: Computer Graphics

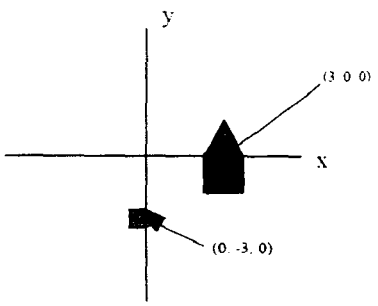
Date: 17.12.2014

Maximum Marks: 100 Duration: 3 hrs.

Answer as much as you can.

1. a) Calculate a 4 × 4 matrix to rotate points about the vector $A = [1\ 1\ 1]'$ by 90°. (5)

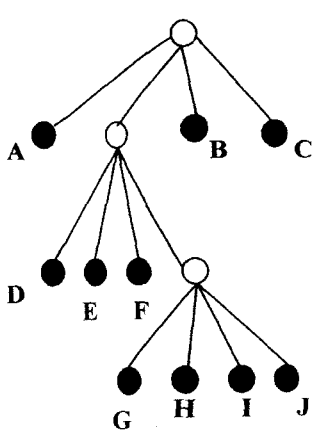
b) Refer to the following figure. Calculate a chain of 4 × 4 matrices that, when post-multiplied by the vertices of the house will translate and rotate the house from (3, 0, 0) to (0, -3, 0). The transformation must also scale the size of the house by half. (6)

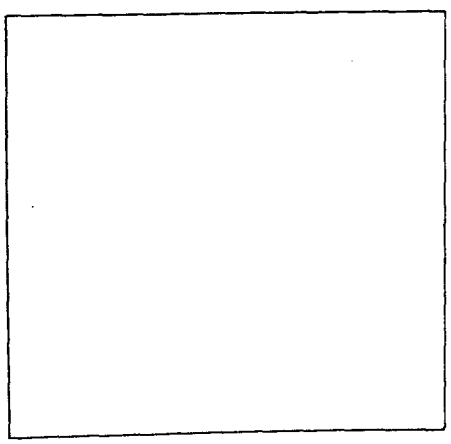


c) "S" is a scaling that contracts any shape by a factor of 1/3 along the x-axis, expands a shape by a factor of 7 along the y axis, and leaves things unchanged along the z-axis. What is the 4×4 matrix representation for S? (4)

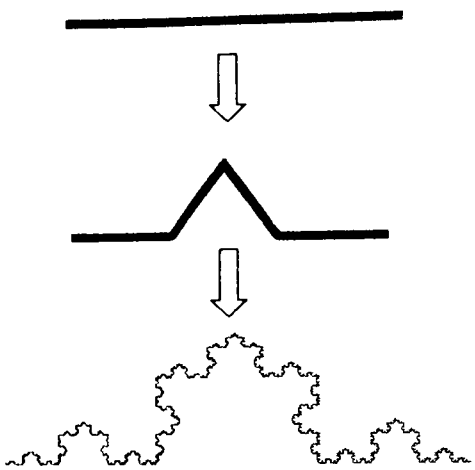
d) Are the transformation operations of uniform scaling ($S_x = S_y$) and rotation commutative? If yes, prove your answer. If no, give a counter-example. (5)

2. a) The tree below represents a quadtree subdivision of a square. The leftmost branch is the upper left quarter, the next branch is the upper right, the third branch is the lower left, and the rightmost branch is the bottom left of a region. The leaf nodes are labelled A – J. Subdivide the square below to indicate the subdivision of the quadtree. Label each portion of the square with the appropriate label (A – J) from the tree. (5)

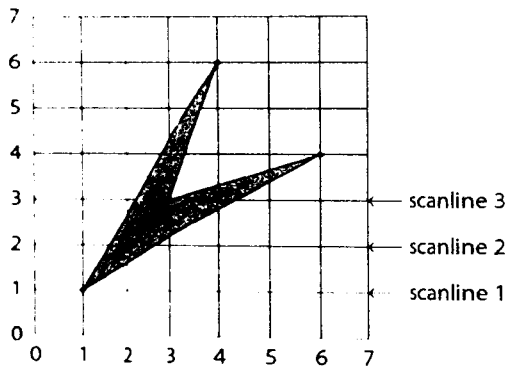




b) What do you mean by fractal dimension? Calculate the dimension of the Koch Curve, formed as shown below and explain your steps: (2+3 = 5)



c) In the following, we want to apply the scan line algorithm for rasterization of random polygons in 2D to this example:



i) Write down the complete edge table (ET) that is used by this algorithm.

ii) Write down all entries in the active edge table (AET) when the current scanline is at position 1, 2 and 3 respectively (cf. image). (4+6 = 10)

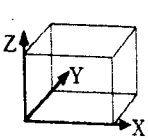
3. a) Given a camera position ***P***, a vector normal to the image plane ***N***, and an up vector ***Vup***, describe how to convert a point W in world coordinates to a point in camera coordinates. Provide your final answer in the form of one (or a product of many) transformation matrix. As a hint note that the origin in camera coordinates is located at ***P*** and the world coordinate axes must be rotated to align with the camera's coordinate axes. (6)

b) Develop a scan conversion algorithm to draw an ellipse whose major axis is oriented at an angle specified by the user. Explain your steps. (14)

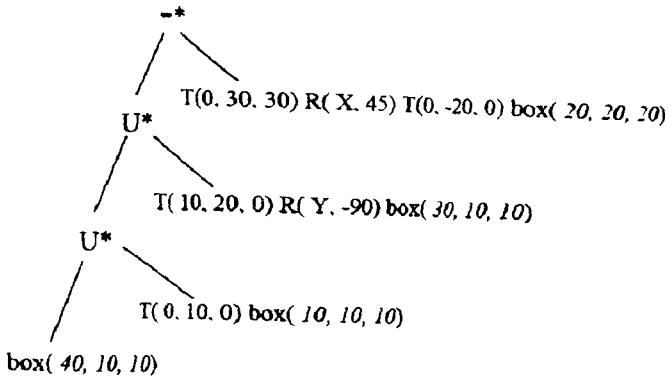
4. a) Consider the following two CSG (Constructive Solid geometry) trees. Do they represent the same shape (give reason, including a sketch of each operation shown in the trees)? The only primitive used is a rectangular solid, as shown below. In the tree, rotation is denoted by R (axis, angle), and translation by $T(x, y, z)$. The operators have their usual meaning in the trees. (10)

CSG Primitive:

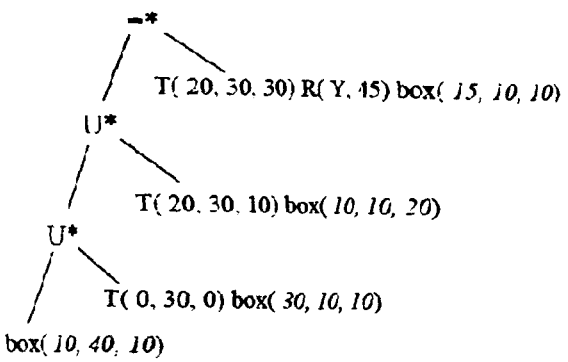
box(*x_length, y_length, z_length*)



Tree 1:



Tree 2:



(b) Suppose a Bézier curve $C(u)$ is defined by the following four control points in the XY-plane: $P_0 = (-2, 0)$, $P_1 = (-2, 4)$, $P_2 = (2, 4)$ and $P_3 = (2, 0)$.

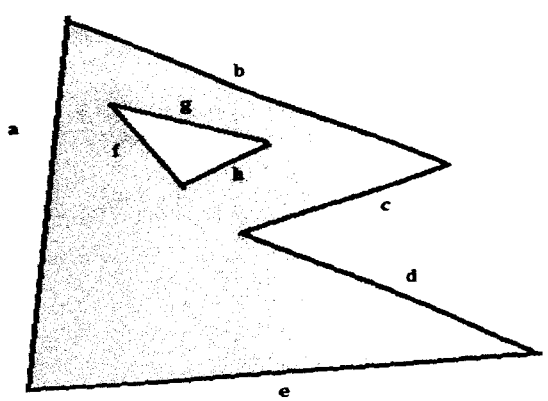
i) What is the degree of $C(u)$?
ii) What are the new control points if the degree of the original curve is increased by one?

(3+4 =7)

(c) Two cubic Bezier curves, with control points P_0, P_1, P_2, P_3 and Q_0,Q_1,Q_2,Q_3 respectively, are joined together at $P_3 = Q_0$. What must be true for them to meet with C^1 continuity and why? (3)

5. (a) Define the term depth buffer (z buffer) and explain how a depth buffer can be used to solve the hidden surface problem without explicitly computing polygon intersections. (10)

(b) Below is a concave polygon with a hole. Draw a BSP tree with suitable explanations that represents this polygon. The letter appearing near each edge is the label you should use for that edge in your tree. State what 'in'/'out' convention you are using, and label the leaves of your tree appropriately. (10)



P.T.O

6. (a) Consider a scene that contains 5 collinear points, A, B, C, D, and E, where collinearity is defined in 3D coordinates. Suppose a perspective projection is carried out on the camera image.

(i) Will the order of the projected points (along the projected line) always be the same in the camera image? If yes, argue why this must be the case. If no, provide a counterexample.

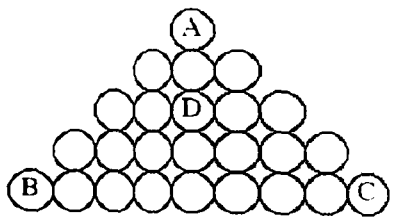
(ii) Prove that under perspective projection the projected points are collinear in the 2D image plane. (3+6 = 9)

(b) In the Cohen and Sutherland clipping algorithm, it is required to clip a number of lines against a screen area starting at (100, 100) and opposite corner at (200, 200). Calculate the end point region codes for the following lines:

(240, 0) to (150, 150)
(50, 150) to (150, 50)
(150, 120) to (180, 120)
(150, 250) to (150, 50).

Mention the lines that will be further clipped and why. (4+3 = 7)

(c) Assume a Gouraud shading model for the triangle below. Pixel A has been assigned the color (100,100,100), pixel B is color (80, 50, 50) and pixel C is (70, 90, 70). What color should be assigned to pixel D? Explain. (4)



7. (a) The colours *D*, *E*, and *F* are defined in terms of the *R*, *G*, and *B* primaries as:

$D = 1.0R + 0.5G$
$E = 1.0R - 0.5B$
$F = 1.0R$

(i) What are *D*, *E*, *F* values that produce an equivalent stimulus to the eye as $R = 0.0$, $G = 1.0$, and B 1.0?

(ii) Write the equations that map from three values in *DEF* colour space into *RGB* colour space.

(iii) Can the colours D, E, F be considered as primaries? Justify your answer.

(3+4+3=10)

(b) What do you mean by image morphing? Explain the steps involved with the warping part of a morphing algorithm with a suitable example. (3+7 = 10)

8. Write short notes on any 2 of the following: (10×2 = 20)

(a) Key-frame Animation.

(b) Phong Specular Reflection Model.

(c) HSV Colour Model.

(d) A-buffer Algorithm.

INDIAN STATISTICAL INSTITUTE

Periodical Examination

M. Tech (CS) - II Year (Semester - II)

Topics in Algorithms and Complexity

Date : February 20, 2014 Maximum Marks : 60 Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 60

1. Describe the link list implementation of the *perfect skip list*, where in each level, the even numbered elements are copied to the next level. Analyze the query time complexity for searching a key value in your implementation of the skip list, and the space required for maintaining your data structure.

   If each element in a layer is copied to its next higher level with probability p, then analyze the expected search time in that skip list. [6 + 4 + 6 = 16]

2. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n points in $\mathbb{R}^2$. The objective to identify a pair of points p_α, p_β such that $d(p_\alpha, p_\beta) = \min_{\forall i,j} d(p_i, p_j)$, where $d(\ldots)$ is the Euclidean distance between a pair of points.

   Show that a randomized algorithm for this problem can be designed with expected time complexity $O(n)$. [10]

3. Consider the problem of testing equality of two n bit numbers. Show that using the communication of $O(\log n)$ bits, one can report the correct result with high probability. You must derive the probability of *false positive* and *false negative* replies by your proposed scheme. [10]

4. Max-SAT problem is defined as follows: Given a boolean formula in CNF over n variables $x_1, x_2, \ldots, x_n$, and a weight w_i for each clause C_i, $1 \leq i \leq m$, find the truth assignment to variables such that it maximizes the sum of weights of the satisfied clauses. A simple randomized algorithm is designed as follows: set the variable $x_i = 1$ with probability $\frac{1}{2}$, and $x_i = 0$ with probability $\frac{1}{2}$. Show that if W is the sum of weights of the satisfied clauses then $E(W) = \frac{1}{2}OPT$, where OPT is the optimum solution.

   Describe a method of derandomization to achieve a deterministic $\frac{1}{2}$-factor approximation algorithm for the Max-SAT problem. [8 + 8 = 16]

5.(a) Let $G = (V, E)$ be an undirected graph with n vertices. We label the vertices in V randomly with the set of integers $\{1, 2, \ldots, n\}$. Now, we obtain a directed graph G' by assigning direction of an edge (i, j) as $i \rightarrow j$ if $i < j$; otherwise the direction is $j \rightarrow i$.

   Prove that if G has a path P of length k, then P is also a path in G' with probability $\alpha = \frac{2}{(k+1)!}$.

(b) Now consider the following algorithm:

repeat steps 1 and 2 $t = \frac{1}{\alpha}$ times
1 Assign directions to the edges of G randoly to get G'.
2 Find longest path in G'.
end (**repeat**)
If the path length is less than k in each time,
then report **failure** to get a path of length k
else report **success**
end

Prove that the above algorithm finds a path of length $k = O(\frac{\log n}{\log \log n})$ (if one exists) in randomized polynomial time.

[8+8=16]

INDIAN STATISTICAL INSTITUTE
M. Tech.(Computer Science) II Year, 2014-15
Periodical Examination
Advanced Pattern Recognition

Date: February 21, 2015 Maximum Marks: 60 Duration: 150 minutes

Note: Answer all the questions.

1. Derive the decision rule that minimizes the cost of misclassification for a 2-class classification problem when the prior probabilities and the class conditional probability density functions are known. [8]

2. Let $\underline{X}' = (X_1, X_2, X_3, X_4)$ be a random vector with dispersion matrix Σ, where $\Sigma = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{pmatrix}$. Find two principal components of $\underline{X}$. [10]

3. Let $p_1(x) = e^{-x}\ ; x > 0 = 0\ ; x \le 0$ and $p_2(x) = 2e^{-2x}\ ; x > 0 = 0\ ; x \le 0$ be probability density functions for classes 1 and 2 respectively with prior probability for class 1 as P. Find the Bayes decision rule for the classification problem, and find its error probability. [5+5]

4. Write short notes on Probabilistic separability based feature selection criteria. [5]

5. Describe the single linkage clustering algorithm. [5]

6. (a) Describe k-nearest neighbour based density estimation procedure.
(b) Derive the k-NN decision rule using the above density estimation procedure.
(c) Describe an algorithm for reducing the size of the training sample set for k-NN decision rule. [5+5+6=16]

7. Find the expression for probability of misclassification of Bayes decision rule for 2-class classification problem where the distribution for each class is multivariate normal, and the dispersion matrices are same. [6]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2014 – 15

Course Name: M. TECH CS - II

Subject: Computer Vision

Date: 23. 02. 2015

Maximum Marks: 60 Duration: 2 hrs.

1. Explain and establish that the eigen-decomposition of the matrix:

$$\mathbf{C} = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}$$

and subsequently thresholding of the eigenvalues constitute a sound basis for a corner finding algorithm. I_x and I_y stand for the following partial derivatives in the formula for **C**:

$$I_x = \frac{\partial I(x,y)}{\partial x} \quad \text{and} \quad I_y = \frac{\partial I(x,y)}{\partial y}$$

where I(x,y) is the image intensity at the pixel located at (x, y). W in the formula for **C** is the local window over which the summations are carried out. (15)

2. a) Give (with suitable explanations) a 3 × 3 filter that approximates a Gaussian function.

b) Given that the Gaussian at pixel position (x, y) is given by: $g(x, y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}}$, show that the Laplacian of Gaussian (LoG) is given by:

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2 - \frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

c) Show that the average value of the Laplacian operator described above is zero.

(3+4+8 = 15)

3. a) An original high resolution image G_0 is used to generate Gaussian and Laplacian pyramids. $G_{l,k}$ and $L_{l,k}$ are the Gaussian and Laplacian pyramid images at level l, expanded k times respectively.

i) How is $G_{3,3}$ different from G_0?

ii) Show how to re-generate $G_{1,1}$ using $G_{3,3}$ and the Laplacian pyramid images only.

(3+4 = 7)

b) You are using k-means clustering in color space to segment an image. However, you notice that although pixels of similar color are indeed clustered together into the same

clusters, there are many discontinuous regions because these pixels are often not directly next to each other. Describe a method to overcome this problem in the k-means framework. (5)

c) Compare the Canny edge detector and the Laplacian-of-Gaussian (LoG) edge detector for each of the following questions:

i) Describe each operator in terms of the order of the derivatives that it computes.

ii) What parameters must be defined by the user for each operator? (1+2 = 3)

4. a) The SIFT descriptor is a popular method for describing selected feature points based on local neighborhood properties so that they can be matched reliably across images. Assume feature points have been previously detected using the SIFT feature detector.

(i) Describe the main steps of creating the SIFT feature descriptor at a given feature point

(ii) Name three (3) scene or image changes that the SIFT descriptor is invariant to (i.e. relatively insensitive to).

(iii) We learned that SIFT features have a number of invariances, but they are not invariant to arbitrary changes in the scene. Name three different invariances that are not addressed by SIFT features. Put differently, what changes to a scene would make a SIFT feature detector fail? (5+3+3 =11)

b) Suppose we have two classes, each class containing a set of points. We run LDA and PCA to produce a 1D linear subspace to approximate the points. We find that LDA and PCA produce the same 1D subspace. Give an example of a set of points that would produce this result. You can show the set by a simple sketch also. (4)

5. In vision and machine learning one frequently encounters optimization problems of the form: $J(x) = \max \dfrac{x^T P x}{x^T Q x}$, where $x \in \mathbb{R}^n$ and P, Q are two symmetric $n \times n$ matrices.

a) The numerator (and also denominator) $x^T P x$ is a quadratic function in the components of x. Show that the gradient of this function is simply the vector $2Px$. Will this result hold if P is not symmetric – explain.

b) Show that $J(x)$ is constant under scaling i.e. $J(x) = J(sx)$ for any $s \in \mathbb{R}$.

c) Show that the optimization problem described in earlier parts is equivalent to the following constrained optimization problem:

$$\max\ x^T P x, \text{ such that } x^T Q x = 1.$$

Using Lagrange multipliers show that the critical values of this optimization problem are the generalized eigenvalues for the two symmetric matrices P and Q. Note that λ is a generalized eigenvalue of P and Q if there exists a non-zero vector x such that $Px = \lambda Q x$.

(5+4+6 = 15)

6. a) Consider a face database with M face images each of size $N \times N$. Suppose matrix A contains the M face vectors after mean subtraction. For computing the eigenfaces it is generally impractical to compute the eigenvectors of the matrix AA^T which is of size $N^2 \times N^2$. So what alternative measure can be suggested to compute the meaningful eigenvectors with fewer amounts of computations? Give mathematical justification to your answer. (7)

b) Describe the basic steps of the mean shift clustering algorithm with reference to image segmentation. (8)

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2014-15 (Fourth Semester)

Course Name: M.Tech. in Computer Science
Subject Name: Distributed Computing Systems

Date: 23.02. 2015 *Maximum Marks: 40* *Duration: 2 hours*

Instructions:

Answer all questions. All parts of a question must be answered in the same place. This is an open book examination.

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1. Indicate whether the following statements are True / False provided with an brief explanation.

[5 X 2 = 10 marks]

   (a) The NetChange algorithm uses more messages in the worst case for stabilizing following a node failure as compared to the number of messages for stabilizing following a node recovery.

   (b) A wave algorithm is better than a traversal algorithm, when some information needs to be sent to all nodes quickly.

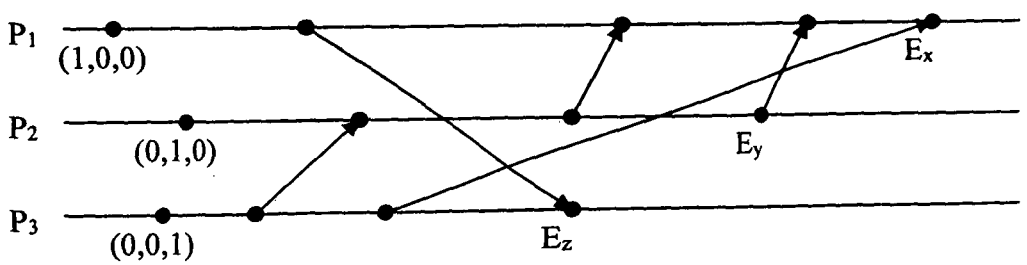
   (c) Roucairol-Carvalho algorithm for mutual exclusion guarantees fairness.

   (d) While the distributed version of the Floyd-Warshall algorithm is in execution, temporary cycles may be created in a routing path between some source and destination.

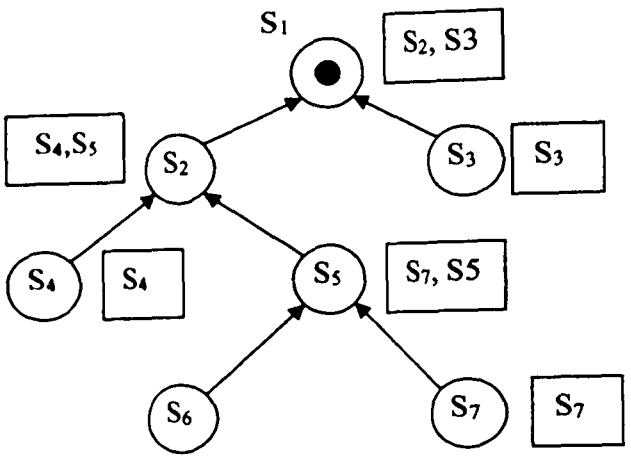
   (e) Vector clocks, like Lamport's clock, define a partial order among events.

2. In the following space-time diagram, vector clocks are being used to timestamp the events. The initial timestamps are given. Write the timestamps of the events E_x, E_y, and E_z.

[3 marks]



3. The figure shows a state of Raymond's tree based algorithm for mutual exclusion. S_1 currently holds the token. The request queue at each node is shown beside the node. For example, at S_5, there is a pending request from S_7 followed by one of its own.



(a) Which sites are requesting?

(b) In what sequence will the requesting sites enter the critical section?

(c) Which of these represent possible sequences in which the requests were made? Answer Yes/No for each of the following.

(i) S_7, S_3, S_4, S_5

(ii) S_4, S_3, S_7, S_5

(iii) S_4, S_3, S_5, S_7

[1 + 4 + 3 = 8 marks]

4. Which of the following algorithms assume FIFO channels? For the ones which require FIFO channels, you must indicate what may happen if the channels are not actually FIFO.
   (a) Ricart-Agrawala Algorithm for Mutual Exclusion
   (b) Chandy-Lamport's Algorithm for Global State Recording
   (c) Birman-Schiper-Stephenson protocol for Causal Ordering of messages

[3 X 3 = 9 marks]

5. (a) The k-exclusion problem is similar to the mutual exclusion problem, but here at most k nodes can be in the critical section at the same time, for some predefined constant k. Design an algorithm to achieve k-exclusion in a distributed system. Argue in brief why no more than k nodes can be in the critical section at the same time in your algorithm. Analyze the message complexity (per critical section entry) of your algorithm.

(b) Explain the types of failures (such as deadlock, starvation, and others) that can happen under the following situations:
   i) The channels do not satisfy FIFO delivery in Lamport's mutual exclusion algorithm
   ii) A control message is lost in Huang's termination detection algorithm
   iii) A node S_i crashes (shuts off) in Maekawa's algorithm

[4 + 6 = 10 marks]

Indian Statistical Institute

M. TECH. (CS) 2 Year : 2014-2015
Mid-Semester Examination
Subject: Computational Complexity
Date: 24/02/2015 Time: 2 hours Marks:60

Note: Notations used are as explained in the class.

1. For each of the following pairs of functions f, g determine whether $f = o(g)$ or $g = o(f)$. If $f = o(g)$ then find the first number n such that $f(n) < g(n)$:
   (a) $f(n) = n^2$, $g(n) = 2n^2 + 100\sqrt{n}$.
   (b) $f(n) = \sqrt{n}$, $g(n) = 2^{\sqrt{\log n}}$. [10]
2. Write a short note on the Church-Turing (CT) thesis and the **strong** form of the CT thesis. [6]
3. State the Cook-Levin theorem and give an outline of the proof. [16]
4. Prove that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{NP} = \mathbf{coNP}$. [8]
5. Prove that there exist oracles A, B such that $\mathbf{P}^{\mathbf{A}} = \mathbf{NP}^{\mathbf{A}}$, and $\mathbf{P}^{\mathbf{B}} \neq \mathbf{NP}^{\mathbf{B}}$. [10]
6. Prove that for every $i \geq 1$, if $\mathbf{\Sigma}_i^p = \mathbf{\Pi}_i^p$ then $\mathbf{PH} = \mathbf{\Sigma}_i^p$; that is, the hierarchy collapse to the ith level. [10]

Indian Statistical Institute

Advanced Image Processing

M.Tech.(CS)-II Year: 2014-15

Full marks: 60 Time: 2 Hours

Date: 24.02.2015

Answer **any six** questions. All questions carry equal marks.

1. a) Write down the image coordinate of a point object (X, Y, Z) using a pin-hole camera model. Suggest a linear transformation in the form of matrix multiplication to arrive at this image coordinate formula.

   b) Prove that due to perspective projection a straight line in 3D is always mapped to a straight line in 2D. [6+4=10]

2. a) State three basic principles of photometric model of image formation.

   b) Derive the equation:

$$g(x, y) = \int_{\infty}^{\infty} \int_{\infty}^{\infty} h(x - \alpha, y - \beta) f(\alpha, \beta) d\alpha d\beta$$

   All terms have their usual meaning. [3+7=10]

3. (a) State and prove the convolution theorem.

   (b) Prove that the origin of the Fourier transform of an image $f(x, y)$ can be moved to the center of its corresponding $N \times N$ frequency square by multiplying $f(x, y)$ by $(-1)^{x+y}$. [(2+5)+3=10]

4. a) Describe the Hotelling transform and show that it is optimal in the least-square-error sense.

   b) Calculate the sequency of each column of Hadamard matrix of order 8. [(5+2)+3=10]

5. Describe the thresholding method proposed by N. Otsu. How do you extend this method to obtain multiple thresholds? [7+3=10]

6. Describe the Fast Fourier Transform algorithm and discuss its computational complexity. [8+2=10]

7. (a) Define (i) principal axis of an image and (ii) bi-linear interpolation.

(b) If m_{ij} denotes the (i,j)th central moment of an image f, and θ represents the slope of the principal axis, then prove that $\tan 2\theta = \frac{2\overline{m}_{11}}{\overline{m}_{20}-\overline{m}_{02}}$.

[(2+2)+6=10]

8. (a) Write short notes on image registration.

(b) Define mutual information. How do you compute mutual information between two images, which are co-registered. [5+(2+3)=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: (2014-2015)
M.Tech C.S., 2nd Year

Advanced Digital Signal Processing

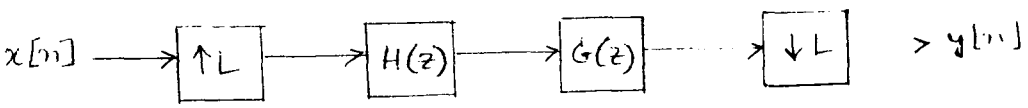
Date: 25.2.2015 Maximum Marks: 60 Duration: 2 hours

Note: The marks add up to 73. The maximum you can score is 60. The exam is open-book, open-notes. You are permitted to use calculators.

Number of question papers required: 3

Questions:

1. Show that the structure given below is an LTI system. Also obtain the condition which the product $H(z)G(z)$ should satisfy if the system has to be an identity system. [5+5]

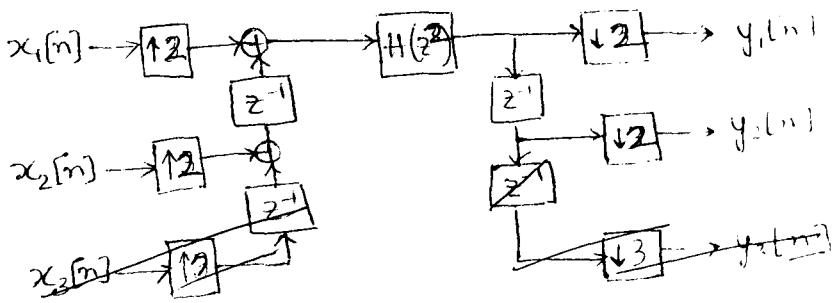


2. For the multirate system shown below

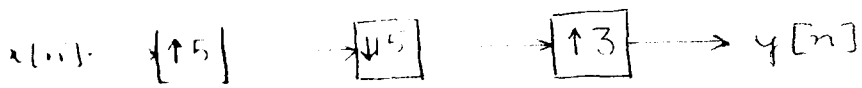
   (a) Determine if the system is time invariant.

   (b) Determine the transfer function from each input to output.

7+(5+5)
~~[5+(4+4+4)]~~



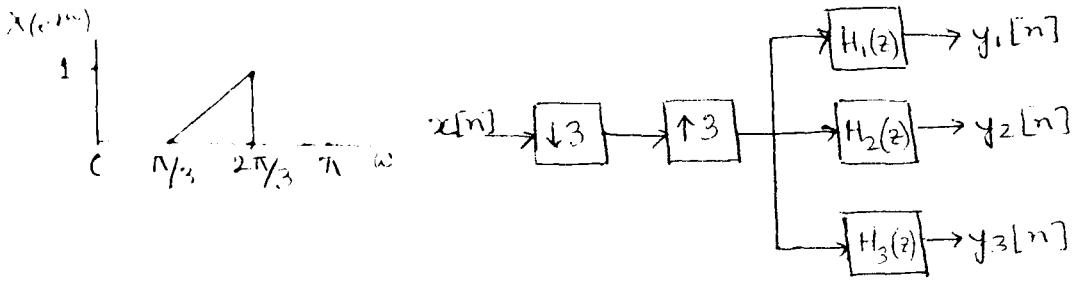
3. Obtain an expression for the output in terms of the input, for the structure given below:



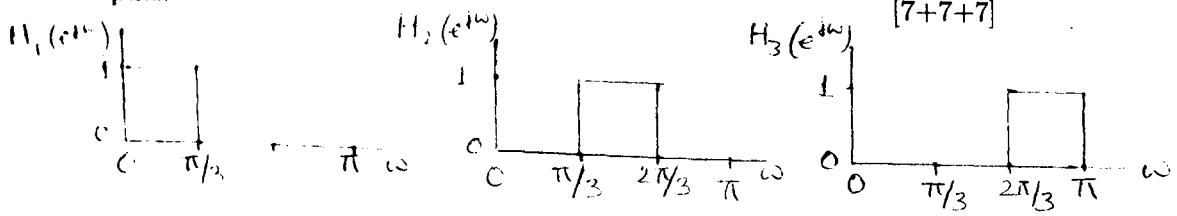
[10]

4. Design a two-stage decimator which will reduce the sampling rate from 60 kHz to 3 kHz. The decimation filter is to be designed as an equiripple filter with a passband edge at 1.2 kHz and passband and stopband ripples of 0.02 and 0.01 respectively. Compare the computational complexity with that of a single stage design. [10+5]

5. A real signal $x[n]$ has the Fourier Transform shown below on the left.



It passes through the multirate structure shown above on the right, with $H_1(z)$, $H_2(z)$ and $H_2(z)$ are zero-phase, real-coefficient lowpass, bandpass and highpass filters respectively with frequency responses as shown below. Sketch the Discrete-Time Fourier Transforms of the outputs. [7+7+7]



INDIAN STATISTICAL INSTITUTE

MID SEMESTRAL EXAMINATION
M.TECH.(CS) II YEAR

OPTIMIZATION TECHNIQUES

Date: 25.2.2015 Maximum marks:30 Duration: 90 minutes

The paper contains 40 marks. Answer as much as you can, the maximum you can score is 30.

1. Let $G = (V, E)$ be an undirected simple graph. Let $f : E \to \mathbb{R}$ be the weight function of edges. Formulate the following problems on graph G as a linear programming problem.

   (a) Single source shortest path problem.

   (b) Single pair shortest path problem.

   [10]

2. **Lemma.** *(**Projection Lemma**) Let $X \subset \mathbb{R}^n$ be a non-empty closed convex set, and $y \notin X$. Then there exists $x_0 \in X$ with minimum distance from y. Moreover, for all $x \in X$, we have $(y - x_0)^T (x - x_0) < 0$.*

   Now prove the following theorem.

   Theorem. *Let $X \subset \mathbb{R}^n$ be a non-empty closed convex set, and let $y \notin X$. Then there exists a hyperplane $H = \{x \in \mathbb{R}^n : a^T x = \beta, a \in \mathbb{R}^n, \beta \in \mathbb{R}\}$ that separates y from x.*

   [10]

3. Let $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$. Prove that

$$Ax = b, x \geq 0$$

   has a solution if and only if

$$A^T y \geq 0, y^T b \leq 0$$

   has no solution. [10]

4. Prove that the following lp problem is infeasible.

$$\begin{aligned} \min \quad & x_1 + 2x_2 + x_3 \\ s.t. \quad & x_1 + x_2 + x_3 = 6 \\ & 2x_1 + 3x_2 + x_3 = 8 \\ & 2x_1 + x_2 + 3x_3 = 0. \end{aligned}$$

   [10]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination:(2014-2015)

M.TECH (CS) II YEAR

Subject Name: Quantum Information Processing and Quantum Computation

Maximum Marks: 30 Duration: 2 hours Date: 26/02/15

Answer any three of the following four questions

1. a) Let the state of a spin 1/2 particle is $|0\rangle$ which is eigen state of σ_z. Show that the results of any spin measurement along any direction in x-y plane is completely random.

b) Consider the following linear operator A acting on a two dimensional Hilbert space.;

$$A = \frac{1}{2}(I + n.\sigma)$$

where $n.\sigma = \sum n_i\sigma_i$ $(i = x, y, z)$, n is a vector in R^3, σ's are Pauli matrices and I is identity operator .
i) Under what condition A is a positive operator .
ii) Under what condition A is a projection operator.

c) Show that the projection operator $|0_x >< 0_x|$ can be expressed as

$$|0_x >< 0_x| = \frac{1}{2}[I + \sigma_x]$$

where $\sigma_x|0_x >= |0_x >$ 3 + 5 + 2

2. a) Consider following two states $|0\rangle$ and $|\psi\rangle = \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$, where $|0 >$ and $|1 >$ are two orthogonal vectors. Show that these two states can not be reliably cloned.

b) Consider the following Unitary operator acting on two systems, each associated with two dimensional Hilbert space;

$$U|0\rangle \otimes |0\rangle = |0\rangle \otimes |0\rangle, U|0\rangle \otimes |1\rangle = |0\rangle \otimes |1\rangle$$

$$U|1\rangle \otimes |0\rangle = |1\rangle \otimes |1\rangle, U|1\rangle \otimes |1\rangle = |1\rangle \otimes |0\rangle$$

Show that by using this unitary operator U as physical operation in quantum mechanics, one can create an entangled state from a suitably chosen product state of two qubits.
c) Consider a Swap operator U_s which acts in the following way;

$$U_s|\psi\rangle \otimes |\phi\rangle = |\phi\rangle \otimes |\psi\rangle$$

for all possible states $|\psi\rangle$, $|\phi\rangle$. Then show that U_s can not be written as

$$U_s = U_1 \otimes U_2$$

where U_1 and U_2 are acting on particle 1 and particle 2 respectively.
d) Show how the swap operation (gate) can be realized by using combinations of the Unitary operation (C-Not gate) defined above (in (b)).

$4+2+2+2$

3. a) Consider the following state of two spin-1/2 system A and B;

$$|\psi >_{AB} = \frac{1}{\sqrt{3}}(|0 >_A |0 >_B + \sqrt{\frac{2}{3}}|1 >_A |1 >_B)$$

where $|0\rangle$ and $|1\rangle$ are eigen states of σ_z. Show that the state is entangled. Find the marginal density matrix of system A.
b) Consider a two qubits product state $|\psi\rangle_A \otimes |\phi\rangle_B$ and the following four Unitary operators;

$$\{\sigma_0(\equiv I), \sigma_x, \sigma_y, \sigma_z\}$$

Show that the four states $\{\sigma_i \otimes I|\psi\rangle_A \otimes |\phi\rangle_B\}$ where $i = 0, x, y, z$ can not form an orthogonal basis? What is the necessary condition so that the set $\{\sigma_i \otimes I|\xi\rangle_{AB}\}$, $|\xi\rangle_{AB}$ being a two qubits state, can form an orthogonal basis?
c) Discuss its implication in quantum dense coding.

$3+2+5$

4. a) Consider a normalized qubit state ;

$$|\psi\rangle = a|0\rangle + b|1\rangle)$$

where a and b are complex numbers.
Then show that

$$\frac{1}{2}I = \frac{1}{4}\sum_{i=0,1,2,3} \sigma_i|\psi >< \psi|\sigma_i$$

where σ_0 is the identity operator.
b) What is the resources required in quantum teleportation? Describe how an unknown quantum state of a qubit can be teleported to distant laboratory.
c) Argue how, in quantum teleportation, no-cloning principle is respected.
3 + 6 + 1

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2014-2015 M. Tech. (CS) Second Year

VLSI Testing and Fault-tolerance

Date: 26.2.2015 Maximum marks = 20 Credit: 20%

Time: 3 hours

Name:_____ Roll No.:_____

Instructions (Read carefully)

A. This is an **OPEN BOOK/OPEN NOTES** exam. Answer all questions; partial credit may be given for incomplete/incorrect answers.

B. Total points = 25; maximum score = 20.

C. You may write your answer on the test booklet.

---

1. (10 points) Consider the following circuit-under-test (CUT) in Figure 1 with 5 Boolean inputs x_1, x_2, x_3, x_4, x_5, which consists of two 2-input NAND gates, one XOR gate, one 2-input OR gate, one 2-input AND gate and one 2-input NOR gate producing a Boolean function F. Some lines in the circuit are labeled as l_1, l_2, l_3, ..., l_9.

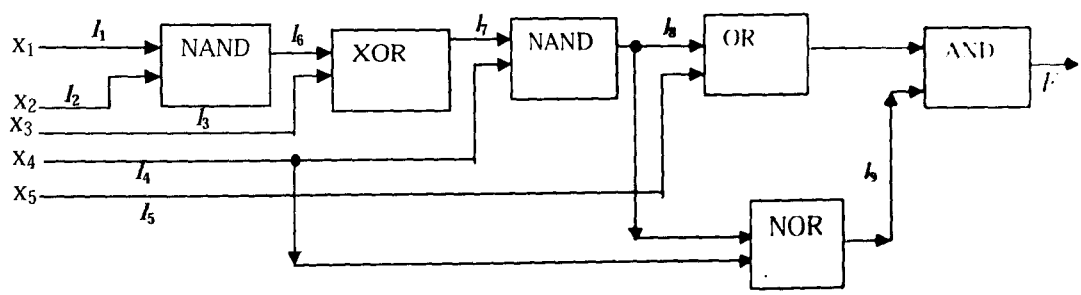


Figure 1. A circuit-under-test (CUT)

(a) Write down the output Boolean function F in terms of the primary input variables.

(b) Determine a test vector (t) that detects the fault l_6 stuck-at 1.

(c) Show by critical path tracing which other faults are detectable by the vector t as obtained in the earlier step.

(d) Ignore the faults in the interior of XOR blocks and consider single stuck-at 0 and 1 faults only on other lines. Mark the checkpoints and find a complete test set T that gives 100% fault coverage. Justify your argument.

(e) Check whether the fault l_2 stuck-at 0 is syndrome testable.

(1+1+2+3+3)

2. (4 points) Referring to the CUT of Figure 1, please answer the following questions.

(a) Compute the Boolean difference (dF/dl_6) of the function F with respect to line l_6, and determine all the test vectors that are capable of detecting l_6 stuck-at 1.

(b) An OR bridging fault between two lines a and b in a CUT changes their corresponding functional values $f(a)$, $f(b)$ to $\{f(a) + f(b)\}$ each. Consider an OR-briging falut f_b between l_3 and l_6. Derive a test vector for f_b. Justify your argument. (2+2)

3. (5 points) Consider the CUT as shown in Figure 2, with three inputs A, B, C and two outputs S and Y.

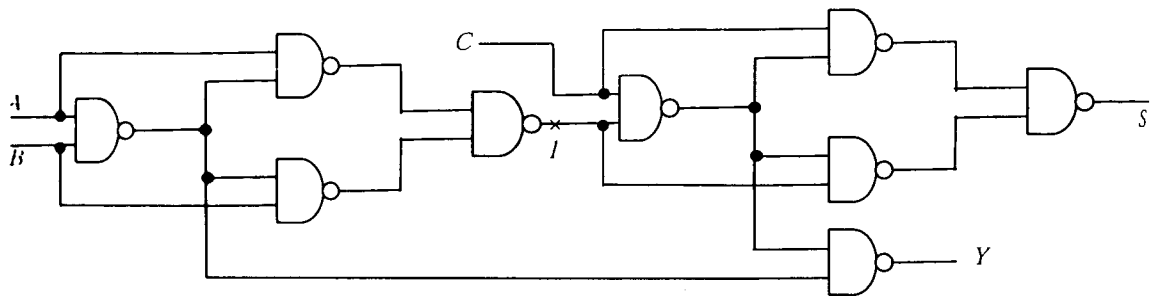


Figure 2. A CUT

(a) Modify the circuit of Fig. 2 (by adding extra logic or extra input/outputs) such that (i) it retains the original behavior of the circuit under functional mode and (ii) for every line L in the modified circuit, all paths from L have equal inversion parity to at least one of the outputs. Your modification should aim at minimum additional cost/delay.

(c) Two inputs I_1 and I_2 to a subcircuit of a combinational circuit are said to be independent if there exists no common signal line in that feeds I_1 and I_2. Show how this CUT can be decomposed into minimum number of sub-circuits such that in each sub-circuit, its inputs are logically independent. How such decomposition help in test generation? (2+3)

3. (3 points) An LFSR-based pseudorandom test pattern generator (TPG) has 4 parallel output lines y_1, y_2, y_3, y_4, that produce a sequence of 4-bit test vectors, where in every vector, each bit has equal probabilities of 0's and 1's. A CUT is given which has 5 primary inputs. However, these 5 inputs require test vectors where 0's should appear with probabilities: {1/2, 3/4, 1/16, 5/8, 3/4} respectively. Design the additional logic for producing such weighted random patterns. (3)

4. (3 points) Consider the sequential circuit as shown below, in which two clocked D flip flops are on feedback paths.

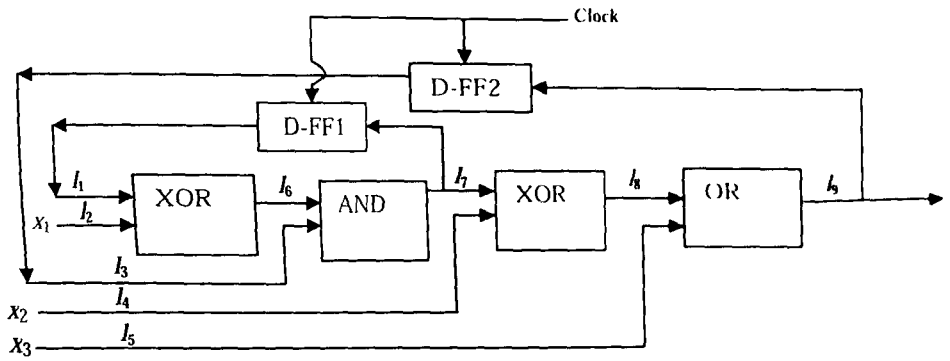


Figure 3. A sequential CUT

Insert a scan chain and show how in the modified circuit that consists of a scan in, a scan out and a functional/test mode selector line, the fault l_3 stuck-at 1 can be tested via scan in and scan-out mechanism. (3)

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination of Second Semester (2003-2004)

M.TECH.(CS) II YEAR

Topics in Algebraic Computation

Date: 27.02.2015 Maximum marks: 60 Duration: two and half hours

Note : The paper contains 80 marks. Answer as much as you can. The maximum you can score is 60.

1. (a) Let

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 2 & 1 & 1 \\ 0 & -2 & 1 & -1 \end{pmatrix}$$

Obtain the upper Hessenberg form of A and hence obtain the characteristic polynomial of A.

(b) Let $p(x)$ be an irreducible polynomial over a field. Show that any matrix which has $p(x)$ as its characterisitic polynomial is similar to the companion form of $p(x)$.

(10 + 10 = 20)

2. (a) Let $M(n)$ be the time required to multiply two $n \times n$ matrices. Briefly describe the idea behind an $O(M(n))$ time algorithm to obtain the inverse of an invertible $n \times n$ matrix.

(b) Use Karasuba's algorithm to perform binary multiplication of 189 and 210.

(10 + 10 = 20)

(a) Let $u, v \in R^n$ where R is an integral domain with identity having a principal n-th root of unity ω. Write a pseudo code for in-place computation of the Discrete Fourier Transform of u and v requiring $O(n \log n)$ time.

(b) Let $u(x) = 3x^3 + 5x^2 - 2x + 1$ and $v(x) = x^3 - 4x^2 + x - 2$. It is desired to multiply $u(x)$ and $v(x)$ over the integers using the FFT algorithm. Determine a suitable ring for doing this.

(10 + 10 = 20)

3. Given a rational number a/b with a, b integers and $b > 0$, show that it is possible to find integers $a_0, \ldots, a_n$ such that

$$\frac{a}{b} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \frac{1}{a_n}}}}.$$

Provide an algorithm for performing this expansion and obtain an upper bound on n.

(10)

4. Is it possible to find a polynomial of degree 4 which is irreducible over the integers but is factorisable both modulo 2 and modulo 5? Justify your answer.

(10)

$$sim(q, d) = \frac{q^T A d}{||q||\ ||d||}$$

Would this represent a valid retrieval model? If yes, explain the intuition behind it. If no, explain why will such a model be not useful at all. You may use examples and/or programs if necessary.

(7 marks)

4. For simplicity, suppose you have posting lists with only docids (no scores, no position information).

   (a) Implement a variable byte encoding scheme `vbe4` with 4 bit units: 1 bit for indicating if it is the last unit, 3 bits for the payload. Also implement the standard variable byte encoding `vbe8` with 8 bit (1 byte) units: 1 bit for indicator, 7 bits for payload.

   You programs should take a posting list in a file with docids separated by space. For example:

   ```
   docid docid docid ...
   ```

   You may use the file:

   `http://www.isical.ac.in/~debapriyo/teaching/ir2015/data/posting`

   as an example input file. Your programs should write the encoding in an output file. For readability, both the input and output should be in ascii text, and not in raw bytes. For example, if your encoding of the posting list is 1000100111001110 simply write that out as text in the output file.

   (6 marks)

   (b) Can you determine a criterion when `vbe8` may work better than `vbe4` and vice versa?

   Consider the Reuter's collection. Which of the above two encoding schemes will perform better for compressing the posting lists generated from this collection? Will it be the same for all posting lists of this collection? Analyze empirically if necessary.

   (4 + 6 = 10 marks)

5. Suppose PageRank is computed using the standard formula with teleporting. That is, at the k-th iteration, the distribution vector v_k is computed as:

$$v_k = \beta M v_{k-1} + (1 - \beta)\frac{e}{n}$$

where n is the numer of pages, M is the transition matrix, β is the probability of following a link instead of teleporting, e/n is the n-vector with all entries equal to $1/n$. The computation is carried out until v_k converges.

Suppose there is an existing page q with PageRank $PR(q)$. You add a new page p, and add a link from q to p. What would be the PageRank of p? Explain your answer.

(4 marks)

6. If a web graph has three identical strongly connected components which are disjoint from each other, explain what would be the PageRanks of the pages of those components.

(3 marks)

7. Implement an algorithm which takes a directed web graph as input and determines if the graph has any spider trap. Assume that the input graph is given to you as a text file, each line consisting of two integers i and j separated by a space, indicating that there is an edge from node i to node j (link from page i to page j), such as below:

```
1 3
2 8
45 3
23 1
```

Your algorithm should take the input filename as an argument and simply output yes if a spider trap is present, no if not. You should explain the algorithm in your answerscript or in a readme.

(7 marks)

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech (CS) - II Year (Semester - II)

Topics in Algorithms and Complexity

Date : April 20, 2015 Maximum Marks : 100 Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 100

1. Consider the problem of *randomized bucket sorting* of $n = 2^m$ input integers from the universe $U = [0, 2^k)$, $k \geq m$, as follows: Create n buckets. Classify an element in the j-th bucket if the content of its leftmost m bits is j. Next, observe that concatenation of the sorted sequence in the content of the buckets is the sorted sequence of the given input.

   Suggest an appropriate data structure for storing the content of each bucket so that the space requirement is $O(n)$.
   Suggest an appropriate algorithm for sorting the content of each bucket such that the expected time complexity of your algorithm is $O(n)$. Justify the time complexity of your algorithm
   Give an instance where the worst case time complexity of your proposed algorithm will not be linear.

   [4 + 10 + 4 = 18]

2. Let U be an universe of n elements, and $S_1, S_2, \ldots, S_m$ are subsets of U. Show that if each member in U is colored with *red* and *blue* with probability $\frac{1}{2}$ each, then the discrepancy $D(S_i) = \max_i\{\text{number of reds in } S_i - \text{number of blues in } S_i\}$ satisfies

$$Probability[D(S_i) \leq \sqrt{12n \log m},\ \forall\ i = 1, 2, \ldots, m] \geq 1 - \frac{1}{m}$$

   [15]

3. Consider the MAXCUT problem for a graph $G = (V, E)$, $|V| = n$ and $|E| = m$, where each vertex is chosen in partition A or in B with probability $\frac{1}{2}$ each. Show that the expected cut size is equal to $\frac{m}{2}$.

   Use derandomization to propose a deterministic scheme to improve the expected cut size. Analyze the time complexity of your algorithm.

   [5 + 10 + 5 = 20]

4. Consider the *colorful path problem* where a graph $G = (V, E)$ and a vertex $v \in V$ are given. Each vertex in V is colored with one of the integer colors $\{1, 2, \ldots, k\}$. The

objective is to test the existence of a path of length k from the vertex v in G such that no color on the path repeats.

Suggest a randomized polynomial time algorithm that reports a colorful path from v (if one exists) with high probability. Analyze the time complexity and failure probability of your algorithm.

[7+5+6=18]

5. Use Lovasz Local Lemma to prove that any k-SAT formula in which no variable appears in more than $\frac{2^k}{4k}$ clauses, has a satisfying assignment.

[12]

6. Consider an infinite random walk on a two dimensional integer grid Z^2 staring from $(0,0)$. In a move from grid location (i,j), it goes to the grid locations $(i-1,j),(i+1,j),(i,j-1),(i,j+1)$ with equal probability. Show that the expected number of visits to the grid location $(0,0)$ is unbounded.

[12]

7. Consider the problem of testing whether a polynomial $P(x_1, x_2, \ldots, x_n)$ is a zero polynomial or not. In other words, for all inputs whether the value of the polynomial is 0 or not.

   We assign random values $(r_1, r_2, \ldots, r_n)$ to $(x_1, x_2, \ldots, x_n)$, where r_i's are chosen from a finite set S. If $P(r_1, r_2, \ldots, r_n) = 0$, we report YES, otherwise NO.

   Show that, if $P(x_1, x_2, \ldots, x_n)$ is not a zero polynomial, then the probability of reporting YES is less than $\frac{d}{S}$, where d is the degree of the polynomial $P(x_1, x_2, \ldots, x_n)$.

[15]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2014-15

Subject Name : **Advanced Cryptology** Maximum Score: 60

Course Name : M.Tech. (CS) II yr. Date: 20th April Duration: 3 Hours

Note: Attempt all questions. Marks are given in brackets. Total marks is 70 but you can score maximum 60. Use separate page for each question.

Problem 1. Show that (i) one-way function exists if pseudorandom generator exists and (ii) pseudorandom generator exists if pseudorandom function exists. [6+6=12]

Problem 2. (i) Let $f_K : \{0,1\}^n \to \{0,1\}^n$ be a uniform random function. Distinguish the following keyed function

$$F_{K_1,K_2}(x,y) = f_{K_1}(x) \oplus f_{K_2}(y). \quad x,y \in \{0,1\}^n$$

against a uniform random function from $2n$ bits to n bits, where K_1 and K_2 are independently chosen.

(ii) Describe Luby-Rackoff (LR) round function. Show that three rounds of LR is not strong pseudorandom permutation. [6+8=14]

Problem 3. Define Block-wise universal hash over $3n$ bits where each block is n-bit long. State Hash-then-Encrypt construction. Show that it is pseudorandom permutation (PRP) by computing the interpolation probability. [3+3+10=16]

Problem 4. Define UOWHF and provide a construction of a UOWHF $H : \{0,1\}^n \to \{0,1\}^{n-1}$ using a one-way permutation. State the reduction algorithm (no need to analyze).[4+6+6=16]

Problem 5. Construct a variable length unforgeable signature scheme using UOWHF and also a fixed length unforgeable signature scheme. Justify why it is unforgeable. [4+8=12]

INDIAN STATISTICAL INSTITUTE

Final Examination: (2014-2015)
M.Tech C.S., 2nd Year

Advanced Digital Signal Processing

Date: 21.4.2015 Maximum Marks: 100 Duration: 3 hours

Note: The marks add up to 116. The maximum you can score is 100. The exam is open-book, open-notes. You are permitted to use calculators.

Questions:

1. An AR(2) process $s[n]$ is generated as

$$s[n] = -0.6s[n-1] - 0.2s[n-2] + v[n]$$

where $v[n]$ is a white noise sequence with variance 0.36. Let a signal $x[n]$ be produced as $x[n] = s[n] + w[n]$, where $w[n]$ is a white noise sequence (uncorrelated with $v[n]$) with variance 1.

   (a) Determine the autocorrelation sequence of $x[n]$ using the Yule-Walker equations.

   (b) Obtain the parameters of a Wiener filter of length 2 which can be used to obtain an estimate of $s[n]$ from $x[n]$. Repeat your calculations for a Wiener filter of length 3.

   (c) Determine the minimum mean squared error for each of the Wiener filters and comment on the relative magnitudes.

[8+(7+7)+(4+4+2)=32]

2. Consider the Mexican hat wavelet

$$g(x) = (1-x^2)e^{-\frac{1}{2}x^2}$$

Can this be the scaling function of a multiresolution analysis? Justify your answer. [10]

3. Compute the time-frequency window of

$$g(t) = e^{\frac{-t^2}{2}}$$

[10]

4. An ARMA process is generated by the difference equation

$$x[n] = 0.6x[n-1] + w[n] - 0.7w[n-1]$$

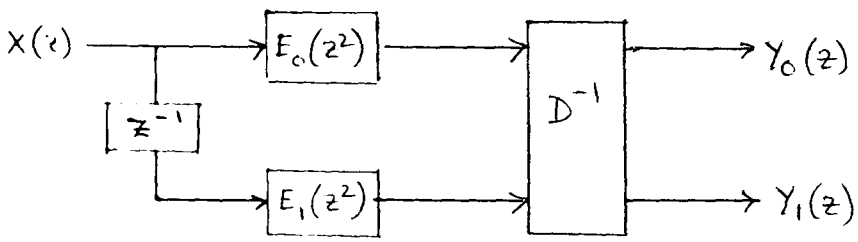
where $w[n]$ is a zero-mean white noise process with variance 1. Determine

(a) the coefficients of a minimum phase system that will act as the whitening filter, and

(b) the power density spectrum of $x[n]$.

[10+5=15]

5. For the two-channel analysis filter bank shown below, determine the transfer functions $H_0(z) = Y_0(z)/X(z)$ and $H_1(z) = Y_1(z)/X(z)$ where $E_0(z) = 1 - 0.5z^{-1}$ and $E_1(z) = 2 - 0.7z^{-1}$. D^{-1} is the 2×2 IDFT matrix. [10]

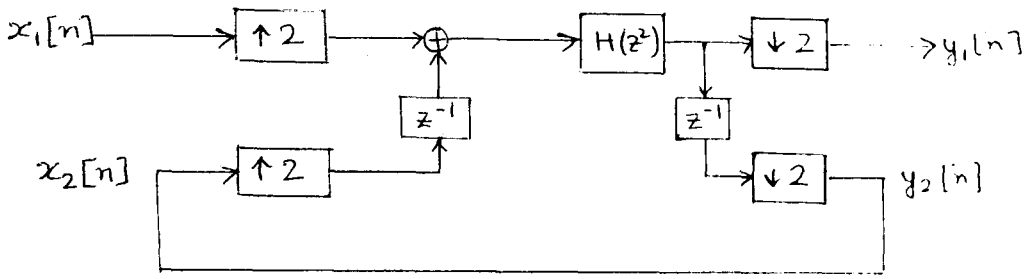


6. The highpass filter $H_1(z)$ of a two-channel perfect reconstruction filter bank is given by

$$H_1(z) = a + bz^{-1} + cz^{-2} + dz^{-3}$$

Determine the expressions for the other filters *i.e.* $H_0(z)$, $G_0(z)$ and $G_1(z)$. [10]

7. Determine the transfer function of the multirate structure shown below. Is it time-invariant? Justify your answer. [10+5]



8. Consider a signal

$$x[n] = cos^2(\frac{\pi n}{2})u[n]$$

Its Time-dependent Fourier Transform $X[n, \lambda)$ is defined as

$$X[n, \lambda) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\lambda m}$$

Using a rectangular analysis window

$$w[n] = \begin{cases} 1 & \text{if } 0 \leq n \leq 9 \\ 0 & \text{otherwise} \end{cases},$$

let $X[n, k] = X[n, 2\pi k/5)$ for $0 \leq k \leq 4$. Determine

(a) $X[0, k]$ for $0 \leq k \leq 4$.

(b) $\sum_{k=0}^{4} X[n, k]$ for $0 \leq n < \infty$.

[7+7]

INDIAN STATISTICAL INSTITUTE

Semestral Examination:(2014-2015)

M.TECH (CS) II YEAR

Subject Name: Quantum Information Processing and Quantum Computation

Maximum Marks: 60 Duration: 2.5 hours Date: 21.04.2015

Answer any four of the following six questions

1. Consider the following three qubits state shared between Alice, Bob and Charlie stationed at distant laboratories;

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]$$

$|0\rangle$ and $|1\rangle$ form an orthogonal basis in two dimensional Hilbert space.
i) Show that the reduced state between Bob and Charlie is separable.
ii) Show that Alice can help to create the Bell state $|\phi^+\rangle$ between Bob and Charlie where all of them are allowed to do local operation and classical communication.
iii) If Alice shares another state

$$|\phi\rangle_{AD} = a|00\rangle + b|11\rangle, |a|^2 + |b|^2 = 1$$

with Dick then show that this state can be prepared between Bob and Charlie by local operation and classical communication.

[3+4+8]

2. a) Consider a function $f : \{0,1\}^n \rightarrow \{0,1\}$
Let there is a unitary gate U_f which acts in the following way;

$$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$$

where $x \in \{0,1\}^n$ and $y \in \{0,1\}$
Show that

$$U_f|x\rangle\frac{1}{\sqrt{2}}[|0\rangle - |1\rangle] = (-1)^{f(x)}|x\rangle\frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$$

b) Consider a function $f : \{0,1\}^n \rightarrow \{0,1\}$, where the function f is either constant or balanced ($f(x) = 0$ for half of the possible input values). Describe the quantum algorithm by which the function can be shown to be either constant or balanced without calculating the function at various points.

[5+10]

3. Consider a 2 to 1 function $f : \{0,1\}^n \rightarrow \{0,1\}^n$. The function has a period given by n-bit string a: that is
$f(x) = f(y)$ iff $y = x \oplus a$
a) Discuss how hard it is to find the period a in the classical world.
(b) Show that there is a quantum algorithm by which the period can be found in polynomial time.

[3+12]

4. a) Consider a copy of the following n-qubit state;

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x.y} |y\rangle$$

How do you estimate the phase x where $x \in \{0,1\}^n$?
b) Again consider another copy of the following n-qubit state;

$$|\phi\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} e^{2\pi i \omega y} |y\rangle$$

where $\omega \in (0,1)$ and y is a binary encoding of the integer y taking values from 0 to $2^n - 1$. Obtain a good estimation of the phase ω.
c) Using the solution of the last problem provide a sketch of the quantum algorithm for factoring large numbers.

[2+8+5]

5. The function $f : \{0,1\}^n \rightarrow \{0,1\}$ is such that;
$f(x) = 1$, for $x = \omega$ and $f(x) = 0$ for $x \neq \omega$
a) In the classical world, how many queries are required for finding ω?
b) Show how a quantum algorithm provides a quadratic speed up for this search problem.

[3+12]

6. a) Show how phase error in computational basis can be converted to bit error.

b) Consider the following two nine-qubit orthogonal states

$$|0\rangle_L = \frac{1}{2\sqrt{2}}[|000\rangle + |111\rangle][|000\rangle + |111\rangle][|000\rangle + |111\rangle]$$

$$|1\rangle_L = \frac{1}{2\sqrt{2}}[|000\rangle - |111\rangle][|000\rangle - |111\rangle][|000\rangle - |111\rangle]$$

Show that using this code, any one qubit error can be corrected.

[3 + 12]

Indian Statistical Institute

Advanced Image Processing

M.Tech.(CS)-II Year, 2014-2015

Full marks: 100 Time: 3.5 Hours

Date: 23.04.2015

Answer any **ten** questions. All questions carry equal marks.

1. State and prove the correlation theorem. Show that the Fourier transform of the autocorrelation function of $f(x)$ is its power spectrum. [7+3=10]

2. (a) How do you rotate an image $f(x, y)$ using Hotelling transform?

   (b) Prove that the Fourier transform of an image $f(x, y)$ is rotated by an angle θ if $f(x, y)$ is rotated by the same angle. [5+5=10]

3. (a) Define reconstruction by dilation.

   (b) How are minima of a function detected? Describe watershed segmentation algorithm starting from minima as markers. [2+(2+6)=10]

4. (a) Define dilation and erosion. Prove that they are dual operators.

   (b) Prove that open operation is idempotent. [5+5=10]

5. Derive the expression of Wiener filter for image restoration using minimum mean-square estimation approach. [10]

6. Consider the following block of gray levels:

9	8	2	1
7	6	2	3
8	4	3	6
4	2	7	8

   Calculate the compressed and reconstructed representation of the block using Block Truncation Coding. Calculate PSNR and bpp. [8+2=10]

7. Consider the following digital signal: [1, 2, 4, 1, -1, -2, -1, 1]. Construct the tree wavelet expansion of this signal using the following scaling filter: [0.4830, 0.8365, 0.2241, -0.1294]. [10]

8. Consider the following block of gray levels:

0	0	1	1
2	2	3	3
0	0	1	1
0	2	2	2

Construct the gray level co-occurrence matrices for angle $\theta = 0°, 45°, 90°, 135°$, considering unit pixel distance, and compute the angular second moment for each case. [(4x2)+2=10]

9. Consider the block of gray levels of Question 8. Encode the above gray levels with strings of 0's and 1's based on Huffman coding. Calculate the average code-word length. [8+2=10]

10. Compute the time dispersion and spectral bandwidth of the following Gaussian signal: $f(t) = e^{-\frac{t^2}{2\sigma^2}}$. Prove that the signal $f(t)$ achieves the minimum of the uncertainty inequality. You may use the following two results:

$$(i) \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}; \text{ and } (ii) \int_{-\infty}^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

[(4+4)+2=10]

11. (a) Describe the HSI model for color image processing.

(b) Write down the expressions for converting colors from RGB to HSI models and HSI to RGB models.

(c) What is pseudocolor image processing? [2+(3+3)+2=10]

12. Write short notes on *any two* of the following:

(a) Active contour; (b) Image fusion; (c) JPEG compression; (d) Digital watermarking; (e) Fast inverse wavelet transform. [5+5=10]

Information Retrieval - Spring 2015
M. Tech CS II End-Sem Examination

23-04-15

There are 13 questions, each carrying 4 marks. The maximum you can score is 50.

Questions 1 - 3: Consider the average precision vs recall graph of three methods in a given IR setting Figure 1. For the sake of *simplicity*, assume that for any particular query, the precision vs recall graph of the methods are expected to be exactly as shown in their average precision vs recall graph.

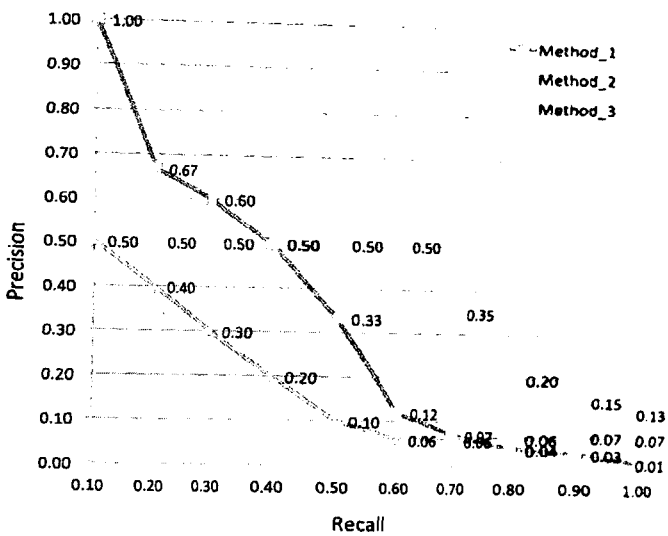


Figure 1: Average precision vs recall graph of three methods

1. Suppose q is a query with exactly 10 relevant documents. What would be the precision@10 achieved by *Method_1* on q?
2. Assume that a query q has exactly 10 relevant documents. For which of the given methods (one, more than one, or none), the second ranked document is relevant to q?
3. For a query such as `tendulkar date of birth`, which among the given methods would you use? If the user is performing a literature survey with a query `relevance feedback methods` which among the given methods would you prefer? Justify your answers.

Questions 4-6: Suppose A is a 5×6 term - document matrix, representing a dataset with 5 terms and 6 documents. Assume that the term weights of A are simply the term frequencies. That is, if $A(i,j) = 4$ then the i-th term occurs 4 times in the j-th document. The singular value decomposition $A = U\Sigma V^T$ of A, is given as below:

$$\begin{pmatrix} 0.2 & 0.5 & 0.2 & 0 & -0.8 \\ 0.2 & 0.7 & 0.4 & 0 & 0.6 \\ 0.6 & 0.1 & -0.2 & -0.7 & 0 \\ 0.7 & 0.2 & -0.1 & 0.7 & 0 \\ 0.1 & 0.5 & 0.9 & -0.1 & 0 \end{pmatrix} \begin{pmatrix} 5.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \end{pmatrix} \begin{pmatrix} -0.4 & -0.1 & -0.7 & -0.6 & -0.2 & -0.2 \\ 0.1 & 0.4 & 0.2 & -0.1 & 0.3 & -0.8 \\ -0.2 & 0.7 & -0.2 & 0 & 0.5 & 0.4 \\ -0.7 & -0.3 & 0.4 & -0.1 & 0.4 & 0.1 \\ 0.4 & 0 & 0.4 & -0.8 & 0 & 0.3 \\ 0.4 & -0.3 & 0.4 & 0.3 & -0.6 & -0.6 \end{pmatrix}$$

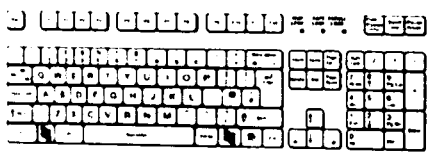
Suppose the 5 terms are $t_1, \cdots, t_5$, with $t_1 = obama, t_3 = modi$, and $t_5 = kolkata$. One among t_2 and t_4 is *india*, the other is *usa*.

4. Under reasonable assumptions, identify t_2 and t_4, or explain why it cannot be done.

5. If the documents are numbered $d_1, \cdots, d_6$ in the order represented by the matrices U, Σ and V^T, which (one, more than one, or none) documents are likely to be about Obama's visit to India while Modi is the prime minister?

6. If possible, estimate the frequency of the term *kolkata* in d_2, or explain why it cannot be done.

Questions 7-8: A user is typing queries very fast using a standard English keyboard, such as one shown in Figure 2. When the user ends up typing a seemingly meaningless query `omfos`, two methods *Method 1* and *Method 2* come up with suggestions as shown in Figure 2.



Method 1

omfos
infosys
infospace

Method 2

omfos
india

Figure 2: A keyboard layout (left) and suggestions by two methods (right).

7. From the suggestions produced, what is likely to be the approach of *Method 1*? Explain.

8. From the suggestion produced by *Method 2*, what can you say about its approach? Explain.

Questions 9-10: We know that if $\phi : \mathbb{R}^n \to \mathbb{R}^m$ maps n dimensional data points to m dimensions, and $K : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is a function such that $K(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i) \bullet \phi(\mathbf{x}_j)\ \forall \mathbf{x}_i, \mathbf{x}_j \in \mathbb{R}^n$, then K works as a valid SVM kernel. By definition, we say that K is a *valid kernel* if for any arbitrary finite subset $\{\mathbf{x}_1, \cdots, \mathbf{x}_k\} \subset \mathbb{R}^n$, the $k \times k$ matrix $K(i,j) = K(\mathbf{x}_i, \mathbf{x}_j)$ is positive semidefinite (Recall: A symmetric $k \times k$ real valued matrix A is called positive semidefinite if for all $\mathbf{v} \in \mathbb{R}^k, \mathbf{v}^T A \mathbf{v} \geq 0$).

9. Suppose K_1 and K_2 are two *valid* SVM kernels. Show that $K_1 K_2$ is also a *valid* SVM kernel.

10. Is $K(\mathbf{x}, \mathbf{y}) = -||\mathbf{x}|| \cdot ||\mathbf{y}||$ a valid SVM kernel? Explain.

11. Suppose the users $u_1, \cdots, u_5$ have liked the movies $m_1, \cdots, m_6$, as given by the following matrix, where the entries indicate the corresponding user has liked (= 1), disliked (= −1), seen but not rated (= 0) or not seen (= blank) the movie.

	m_1	m_2	m_3	m_4	m_5	m_6
u_1				1	0	1
u_2	0	1	1		1	
u_3	1	0				1
u_4				1	−1	0
u_5	−1	0	1		1	
u_6			1			

Suppose user u_6 watches m_3 as his/her first movie and likes it. As a recommender system, what would be the second movie you would suggest to u_6? Take any reasonable approach, but explain your approach.

Questions 12-13: You have a database with records of people (names, locations, age, brief profile, etc). You have a search engine library (e.g. Lucene) which offers field based indexing and field based searching, as well as wildcard prefix queries such as `ind*`, `quer*`. However, the prefix query may perform very slowly or even crash if the number of completions of the prefix becomes very high, as it needs to fetch the posting lists for all of those completions and merge / intersect them. For example, the wildcard query `s*` may generate too many terms matching the prefix `s` and the system may perform very slow in such cases.

12. You have to build a search engine which would allow users to enter any query and the engine should return a person's record as a result if the query words are present anywhere in the person's record. Describe how (say, in which fields) would you index the data and how would you process the query.

13. You have to implement an instant prefix search based on names. That is, as the user types each character of a query, the engine must return the name, location and age of persons whose name match the prefix typed so far by the user. Keeping the limitation of the search engine library on wildcard queries in mind, describe how would you implement the specified feature.

INDIAN STATISTICAL INSTITUTE

End-Semester Examination: 2014-2015 M. Tech. (CS) Second Year

VLSI Testing and Fault-tolerance

Date: 23.4.2015 Maximum marks = 100 Credit: 50%

Time: 3 hours

Name:_____ Roll No.:_____

Instructions (Read carefully)

A. This is an **OPEN BOOK/OPEN NOTES** exam. Answer all questions; partial credit may be given for incomplete/incorrect answers.

B. Total points = 110; maximum score = 100.

---

1. (a) A text message consists of a sequence four symbols: #, $, %, @. The following binary codes are used to encode them during transmission:

# :	00000
$:	01011
%:	11100
@:	10111

The transmission channel is noisy and it may corrupt the binary stream. We claim that such a coding mechanism is capable of correcting 1-bit flip in every symbol received at the receiver's end. Justify the claim (i.e., indicate how decoding can be done unambiguously even in the presence of a single-bit error), or show a counterexample.

(b) The alphabet of a message consists of four words: {0100, 0111, 1001, 1110}. Encode them using Hamming code so that they are capable of correcting one bit error in each code word that may occur during transmission. Explain how the received message can be decoded at the receiver's end. (10 + 10)

2. Choose *one answer* from the choices provided below and *write a line* to justify your answer. (4 × 5)

(a) What is the total number of collapsed single stuck-at faults in a ten-input AND gate?

- ☐ 10
- ☐ 11
- ☐ 20
- ☐ 22
- ☐ None of the above

(b) What is the minimum number of single stuck-at faults guaranteed to be detected in an irredundant combinational circuit by a random binary (0, 1) vector?

- ☐ Number of primary inputs
- ☐ Number of primary outputs
- ☐ Twice the number of primary inputs
- ☐ Twice the number of primary outputs
- ☐ Half of all single stuck-at faults in the circuit

(c) What is the exact number of single stuck-at faults that are activated (controllable) in a combinational circuit by a random binary (0, 1) vector?

- ☐ Number of primary inputs
- ☐ Number of primary outputs
- ☐ Twice the number of primary inputs
- ☐ Twice the number of primary outputs
- ☐ Half of all single stuck-at faults in the circuit

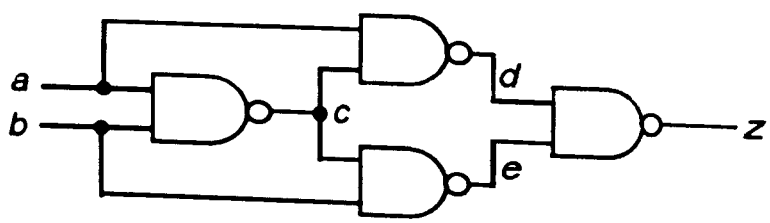
(d) What is the number of transition delay faults in a circuit?

- ☐ Number of paths in the circuit
- ☐ Same as the number of collapsed single stuck-at faults
- ☐ Same as the number of uncollapsed single stuck-at faults
- ☐ Number of gates in the circuit
- ☐ Number of flip-flops in the circuit

3. (a) Show the scheme of Illinois Scan Architecture and discuss the test application scheme.

(b) Consider an Illinois-scan architecture with 10000 flip-flops, 10 scan chains, and 1000 test patterns for detecting stuck-at faults. Out of these 1000 test patterns, 900 vectors are applied in broadcast mode and the rest 100 vectors are applied in serial mode. Assume that functional inputs and outputs are also included in the scan. Determine the test time assuming a clock frequency of 100 MHz (for both scan shifting and functional purposes).

4. Consider the following circuit: (8 + 12)



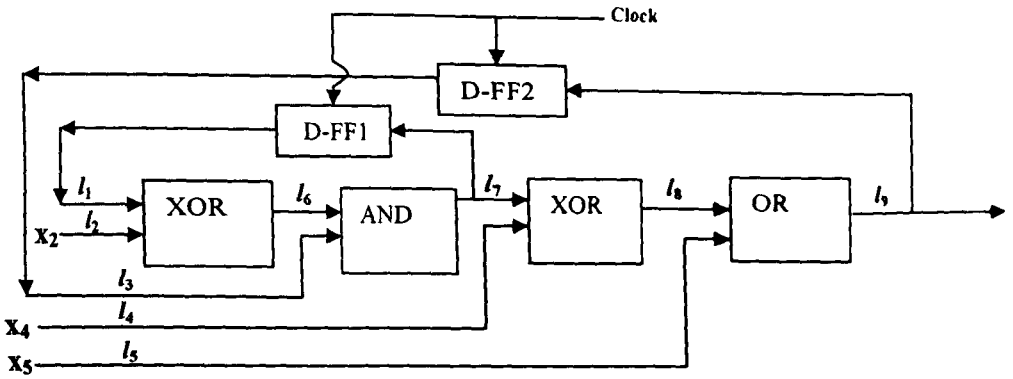
(a) How many path-delay faults can occur in this circuit? List them.

(b) Choose a path-delay fault in the circuit and determine a test t. Indicate whether t is robust or non-robust.

(c) Determine a test for the transition fault $0 \rightarrow 1$ at the primary input line a.

(8+ 6 + 6)

5. Consider the sequential circuit as shown below, in which two clocked D flip-flops are on feedback paths.



Assume both the FFs are initialized to 0. Determine a test for a slow-falling (1 →0) transition fault at line l_6 assuming launch-on-capture (LoC) scheme. Show how the scan chain is to be inserted and explain the procedure of observing the error. (12 + 8)

6. You are given a single-output combinational circuit C, which contains an odd number of true minterms. Assume that C is being tested using an exhaustive test set (i.e., by applying all input combinations), whose response stream is compressed to a single parity bit. Prove that all single/multiple stuck-at faults at all primary input and output lines of C will be tested by this scheme. (10)

Indian Statistical Institute

M. TECH. (CS) 2 Year : 2014–2015
Semester Examination
Subject: Computational Complexity

Date: 24/04/2015 Time: 3 hours Marks: 100

Note: Notations used are as explained in the class.

Prove the following:

(a) A coin with $\Pr[Head] = 1/2$ can be simulated by a probablistic TM with access to a stream of ρ-biased coins in expected time $O(\frac{1}{\rho(1-\rho)})$. [8]

(b) A coin with $\Pr[Head] = \rho$ can be simulated by a probablistic TM in expected time $O(1)$ provided the i-th bit of ρ is computable in poly(i) time. [8]

Let $L \subseteq \{0,1\}^*$ be such that there exists a polynomial time probabilistic TM M satisfying for every $x \in \{0,1\}^*$:
(1) if $x \in L$, then $\Pr[M(x) = 1] \geq n^{-c}$ for some $c > 0$ and
(2) if $x \notin L$, then $\Pr[M(x) = 1] = 0$.
Prove that for every $d > 0$ there exists a polynomial time probabilistic TM M' such that for every $x \in \{0,1\}^*$,
(1) if $x \in L$, then $\Pr[M'(x) = 1] \geq 1 - 2^{-n^d}$ and
(2) if $x \notin L$, then $\Pr[M'(x) = 1] = 0$. [12]

Prove that $\mathbf{DIP} = \mathbf{NP}$. [10]

Define the class $\mathbf{IP}$. Provide an interactive proof for graph nonisomorphism (GNI) problem. [10]

Define the complexity classes $\#\mathbf{P}$, $\mathbf{FP}$ and $\mathbf{PP}$.
Prove that $\mathbf{PP} = \mathbf{P}$ if and only if $\#\mathbf{P} = \mathbf{FP}$. [6 + 9 = 15]

Prove that, if #CYCLE has a polynomial time algorithm, then $\mathbf{P} = \mathbf{NP}$. [12]

Prove that, #SAT is #P-complete. [12]

Define the class $\mathbf{PCP}(r(n), q(n))$. Prove that the language GNI is in $\mathbf{PCP}(poly(n), 1)$. Also prove that $\mathbf{PCP}(r(n), q(n)) \subseteq \mathbf{NTIME}(2^{O(r(n))}q(n))$. [4 + 5 + 4 = 13]

INDIAN STATISTICAL INSTITUTE

M. Tech. (CS) II Year (2014-15), II semester

Semestral Examination

ADVANCED PATTERN RECOGNITION

Date: 28 · 04 · 15 Duration: 195 minutes Maximum Marks: 100

Note: This paper carries 102 marks. Answer as many questions as you can.

1. (a) Describe a density based clustering algorithm for data sets.
   (b) Describe any two feature selection algorithms when a criterion function for feature selection is given. [4+8=12]

2. Suppose you have two 2-dimensional normal populations $N(\mu_1, \Sigma)$ and $N(\mu_2, \Sigma)$ where $\mu_1' = (0.0,0.0), \mu_2' = (1.0,2.0), \Sigma = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 2 \end{pmatrix}$. Let the prior probabilities of the populations be 0.5 and 0.5.

   (a) Find the Mahalanobis distance between the two populations.
   (b) Find the Bayes decision rule for separating the two populations. Also find its probability of misclassification in terms of standard normal probabilities. [5+(3+7)=15]

3. (a) Describe crossover and mutation operations in Genetic Algorithms.
   (b) Describe a way of tackling the problem of clustering using genetic algorithms. [(7+4)+6=17]

4. Let $x_1' = (0.0,0.0), x_2' = (1.0,0.0), x_3' = (0.0,1.0), x_4' = (1.0,1.0)$ and $x_5' = (2.0,2.0)$. Let there be two classes and let the first two points belong to the class 1, and the other two belong to class 2. Let $x + y = 0.5$ be the starting straight line for Perceptron algorithm. Let $\lambda = 0.1$ be the given learning rate. Apply Perceptron algorithm and find the discriminating straight line between the classes. [15]

5. (a) Describe the optimization criterion for Fuzzy C means algorithm.
   (b) Describe Fuzzy C-means algorithm. [5+5=10]

6. (a) Describe the basic steps of gradient descent optimization algorithm.
   (b) Let $f(x, y) = 4x^2 - 12xy + 11y^2 - 10$ where x and y are real numbers. Minimize f by using gradient descent technique. [3+12=15]

7. Let the prior probabilities in a c-class classification problem be $P_1, P_2, ..., P_c$. Show that the misclassification probability for the Bayes decision rule is $\leq 1 - \max\{P_1, P_2, ..., P_c\}$. [6]

(P.T.O.)

8. Write short notes on the following.
   (a) k-fold cross validation
   (b) VC dimension
   (c) Support vectors

[4+4+4=12]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION
M.TECH.(CS) II YEAR

OPTIMIZATION TECHNIQUES

Date: 29.4.2015 Maximum marks:100 Duration: 180 minutes

The paper contains 110 marks. Answer as much as you can, the maximum you can score is 100. The notations are usual notations followed in the class

1. A subset C of $\mathbb{R}^n$ is called a convex cone if for any $x, y \in C$ and any $\alpha, \beta \geq 0$, one has $\alpha x + \beta y \in C$. For any subset X of $\mathbb{R}^n$, define

$$X^* \stackrel{def}{=} \{y \in \mathbb{R}^n | x^T y \leq 1 \text{ for each } x \in X\}$$

   (a) Let P be any polyhedron. Prove that P^* is a polyhedron. [10]

   (b) Prove that, P contains the origin if and only if $(P^*)^* = P$. [10]

2. (a) Let $P = \{x | Ax \leq b\}$ be a nonempty polytope. Prove that $\max\{c^T x | Ax \leq b\}$ is attained by a vertex of P. [8]

   (b) Prove that $\max\{c^T x | Ax \geq b\} = \min\{y^T b | y \leq 0; y^T A = c^T\}$ (assuming both sets are non empty). [12]

3. (a) Prove that a k-regular bipartite graph has a perfect matching if $k \geq 1$. [5]

   (b) Derive that a k-regular bipartite graph has k disjoint matchings. [10]

4. Prove that in a matrix, the maximum number of nonzero entries with no two entries in the same line (i.e. row or column) is equal to the minimum number of lines that include all nonzero entries. [10]

5. Let $G = (V, E)$ be an undirected graph with nonnegative edge weights. Let S, the senders and R, the receivers, be disjoint subsets of V. The problem is to find a minimum cost subgraph of G such that for every receiver $r \in R$, there is at least one sender $s \in S$ such that there is a path connecting r to s in the subgraph. Give a factor 2 approximation algorithm that runs in polynomial time. (Hint: Consider introducing an additional vertex to the graph, and try using the approximation algorithm of the steiner tree problem.) [15]

6. Find the dual of the following problem

$$\begin{aligned} &\min x^T x + 1 \\ &s.t.\ Ax = b. \end{aligned}$$

   where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$. [10]

7 (a) Consider a linearly separable classification problem with three points x_1, x_2, x_3 with tags y_1, y_2, y_3 respectively:

$$\begin{aligned} x_1 &= -1 & y_1 &= 1 \\ x_2 &= 0 & y_2 &= -1 \\ x_3 &= 1 & y_3 &= -1 \end{aligned}$$

Derive the support vectors and the equation of the decision boundary. [8]

(b) Consider the previous points with $y_3 = 1$. If we use the kernel $k(x_1, x_2) = (\langle x_1, x_2 \rangle + 1)^2$, prove that all three vectors are the support vectors. [12]

INDIAN STATISTICAL INSTITUTE
End-Semester Examination: 2014-15 (Second Semester)

Course Name: M.Tech. in Computer Science, Second Year
Subject Name: Distributed Computing Systems

Date: 29. 04. 2015 *Maximum Marks: 100* *Duration: 3 hours*

Instructions:

*Answer **all** questions from Part A and any **three** from Part B.*

Part-A: *Answer all questions*

1. Indicate whether each statement below is true or false with brief justification.
   (a) The one-phase algorithm of Ho-Ramamoorthy may detect phantom deadlocks
   (b) In order to break a deadlock in the OR-request model, we must abort at least one process from each cycle of the wait-for dependency graph.
   (c) There can be no deterministic algorithm for leader election in anonymous networks.
   (d) If the loyalty of the initiator is known, then Byzantine agreement can be reached in one round
   [4 X 4 = 16]

2. The clockwise order of process ids in a ring is 8 – 2 – 3 – 6 – 11 – 5 – 7 – 1 – 9 – 10 – 12 – 4. Indicate the set of processes remaining in contention at the end of each round if the Hirschberg Sinclair algorithm is executed to elect the process having maximum id as the leader. Assume all the processes start together and the system is synchronous.

ROUND-1	
ROUND-2	
ROUND-3	
ROUND-4	

[4]

3. The vector timestamps of some events are given below.
   TS (e_1) = ⟨3, 9, 2⟩ TS (e_2) = ⟨2, 1, 2⟩ TS (e_3) = ⟨5, 7, 5⟩
   TS (e_4) = ⟨6, 7, 7⟩ TS (e_5) = ⟨5, 3, 9⟩ TS (e_6) = ⟨6, 3, 9⟩
   Which events are concurrent with e_3? [4]

4. Some of these sequences of records can be present in the log of a site, when the 2-phase commit protocol is in use. Which ones are possible?
   (a) <no T>, <commit T> (b) <ready T>, <abort T>
   (c) <ready T>, <commit T> (d) <commit T>, <abort T> [4]

Part-B: *Answer any three questions*

1.

(a) Analyze the truth of this claim: *In the OR-request model, a deadlocked process must belong to one or more directed cycles in the wait-for graph.* Your answer must begin with True/False and then provide the justification/counter-example.

(b) What will be the impact of a message loss in Chandy et al.'s diffusion computation based deadlock detection algorithm for the OR-request model. Will it miss a real deadlock? Will it report a phantom deadlock? Justify your answer.

(c) A word's anagram is a different word that is written with the same letters (e.g. sail and lisa). The goal is to discover all the anagrams in a large volume of text. The final output must group all words that are anagrams of each other and must not include words that have no anagrams.

Write the pseudo codes for the *map* and the *reduce* functions that implement a solution to the above problem. [8 + 8 + 8 = 24]

2.

(a) Explain how a solution to the Byzantine agreement problem can be used for a solution to the problems of consensus and interactive consistency.

(b) Byzantine agreement starts with the commander sending its value to the other generals. After successful termination of the agreement protocol, the loyal generals want to reach a consensus on the loyalty of the commander. Can they reach a correct decision on this, given that the traitors will try to prevent this consensus?

(c) Consider the leader election algorithm which uses extinction on waves. For each of the following cases present a brief justification indicating whether the case is possible if a message is lost in the algorithm:

i. A wrong leader is elected (that is, not the one with the best id)
ii. No leader is elected
iii. Two groups of processes elect two different leaders

[8 + 8 + 8 = 24]

P.T.O

3.

(a) Present an abstract outline of the synchronous distributed Gallager-Humblet-Spira (GHS) algorithm for finding a minimum spanning tree, stating the notions of fragments, conditions for joining fragments, and methods by which least cost outgoing edges are identified. The detailed pseudo-code is not needed.

(b) In the GHS algorithm, it was assumed that all edge weights are unique. Explain why this assumption was necessary.

(c) Why does the GHS algorithm not allow a fragment of higher level to join with a fragment of lower level?

[12 + 6 + 6 = 24]

4.

(a) Consider the following check-pointing scheme: Every application process takes a checkpoint in volatile storage (main memory) right before every receive event. Right before a send event, the latest checkpoint in the volatile storage is flushed to the stable storage first (but no new checkpoints are taken), and then the send is done. Will the latest checkpoint in stable storage for each process form a consistent checkpoint for the entire system? Justify clearly.

(b) Consider a system where nodes can fail only by crashing. Further suppose that there is a bound T on the maximum message delay in the system. A node that recovers after a fault first informs all other nodes of its recovery before it participates in mutual exclusion algorithm again (assume this is done, do not worry how). Can you design a mutual exclusion protocol for this system that can tolerate any number of crash failures?

[12 + 12 = 24]

5.

(a) In a distributed system, each node holds a positive integer value. A particular node x wishes to find out all values in the system that are greater than 10. Sketch a simple protocol by which x can find this out.

(b) Explain how the two-phase commit protocol handles the following failure scenario: the coordinator crashes after writing the COMMIT record and before writing the COMPLETE record. (Do not explain the entire two-phase commit protocol!)

[12 + 12 = 24]

INDIAN STATISTICAL INSTITUTE

Second-Semestral Examination: 2014 – 15

Course Name: M. TECH CS - II

Subject: Computer Vision

Date: 05.05.2015

Maximum Marks: 100

Duration: 3 hrs.

1. a) An ideal pinhole camera has a focal length of 5mm. Each pixel is 0.02mm x 0.02mm and the principal point is at pixel (500, 500). Pixel coordinates start at (0, 0) at upper left corner of the image.

   i) What is the 3×3 intrinsic camera calibration matrix K for this camera?

   ii) Assuming that the camera coordinate system is coincident with the world coordinate system and the origin of the world coordinate coincides with the pinhole, what is the 3×4 extrinsic rigid body transformation between the camera coordinates and world coordinates?

   iii) Combining the results from the above two questions find the 2D projection on the image plane of a 3D point (100, 150,800).

   (6+2+4 = 12)

   b) Under what conditions will a line viewed with a pinhole camera have its vanishing point at infinity? (3)

   c) A scene point at coordinates (400,600,1200) is perspectively projected into an image at coordinates (24,36), where both coordinates are given in millimeters in the camera coordinate frame and the camera's principal point is at coordinates (0, 0, f) (i.e., $u_0 = 0$ and $v_0 = 0$). Assuming the aspect ratio of the pixels in the camera is 1, what is the focal length of the camera? (Note: the aspect ratio is defined as the ratio between the width and the height of a pixel) (5)

2. a) Consider a camera with intrinsic parameter matrix:

$$\mathbf{K} = \begin{bmatrix} 300 & 0 & 300 \\ 0 & 300 & 200 \\ 0 & 0 & 1 \end{bmatrix}$$

and the complete camera matrix as:

$$\mathbf{P} = \begin{bmatrix} 300 & 0 & 300 & 300 \\ 0 & 300 & 200 & -400 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

Suppose we add a new camera $\mathbf{P}'$ with the same orientation as that of camera **P**. The camera centre of this second camera is located at [3, 0, 2] (an inhomogeneous point in $\tilde{x}'$), and it has a focal length that is one-third that of camera **P**.

i) What is the camera center for the first camera (**P**) in inhomogeneous coordinates?

ii) Compute the camera matrix for $\mathbf{P}'$.

iii) Compute the epipole in each camera, expressed in inhomogeneous coordinates.

iv) Are the epipolar lines in the first camera parallel to one another? Justify your answer.

(3+4+4+3 = 14)

b) Compare the Canny edge detector and the Laplacian-of-Gaussian (LoG) edge detector for each of the following aspects:

i) Which of these operators is/are isotropic and which is/are non-isotropic?

ii) Describe each operator in terms of the order of the derivatives that it computes.

iii) Which detector is more likely to produce long, thin contours? Briefly explain.

(2+2+2 = 6)

3. a) In the eight-point algorithm we need to solve an equation of the form:

$$A\vec{x} = \vec{0}$$

where $\vec{x}$ is a vector of size 9×1 and the matrix A has rank 8. Prove that one solution can be the eigenvector of the matrix $A^T A$ with the smallest eigenvalue. (10)

b) In a stereo system, made of the cameras P and P',

i) If F be the fundamental matrix of the camera-pair (P, P'), then what is the fundamental matrix for (P', P)?

ii) If for a point x in the first image, the corresponding epipolar line is Fx, then what is the epipolar line corresponding to x' in the second image?

iii) What are the left and right null-spaces of matrix F? (3+3+4 = 10)

4. a) What do you mean by "motion field" and "optic flow"? What is the essential difference between them? (3+2 = 5)

b) Consider estimating the translational component of optical flow given two images: $I(\vec{x},t)$ and $I(\vec{x},t+1)$. You can assume that these images have been dropped so that we only need to estimate a constant displacement between these patches.

i) What do you mean by the brightness constancy constraint?

ii) Given an initial guess $\vec{u}_0$ for the displacement so that $I(\vec{x}+\vec{u}_0,t+1) \approx I(\vec{x},t)$, what are the linearized brightness constancy constraints that can be used to update the displacement? Derive a linear system of equations for the update, say $\vec{v}$.

iii) How can a robust estimator $\rho(e)$ be applied to the linearized brightness constancy constraints? In particular, write out the objective function to be minimized by the displacement update $\vec{v}$.

iv) Derive an iteratively reweighted least squares algorithm for minimizing your objective function in part (iii) above.

(3+4+3+5 = 15)

5. a) i) Discuss the adaptive background subtraction algorithm for detecting moving objects from videos.

ii) "The background update formula in this algorithm can be essentially treated as an exponential moving average IIR filter" – justify or nullify this statement.

iii) What advantages does this algorithm offer over the simple background subtraction?

(5+3+3 = 11)

b) With a clear flow diagram, discuss the persistent frame differencing algorithm. (9)

6. a) i) Briefly describe the RANSAC procedure for fitting a line through a number of points.

ii) Show that the number of samples N to choose for running a RANSAC algorithm can be found by solving the following equation:

$$1-\left(1-(1-e)^{s}\right)^{N} = p$$

where e = probability that a point is an outlier, s = number of points in a sample, N = number of samples (we want to compute this), and p = desired probability that we get a good sample. (7+5 = 12)

b) In eigenface approach, performing PCA directly on the covariance matrix of the images is very often computationally infeasible. If small, say 100 x 100, greyscale images are used, each image is a point in a 10,000-dimensional space and the covariance matrix S is a matrix of $10{,}000 \times 10{,}000 = 10^8$ elements. PCA will require you to compute the eigenvalues of this large matrix. Suggest a procedure to tackle this problem in the eigenface algorithm and justify your proposal mathematically. (8)

7. Consider the factorization approach for Structure From Motion (SFM) detection. Consider N points and F frames.

a) Starting from the basic assumptions, show that the problem of SFM can be expressed in the form of the following matrix equation:

$$W = M.S,$$

where W is a $2F \times N$ centered measurement matrix, M is the $2F \times 3$ motion matrix including parameters for camera rotation, and S is the $3 \times N$ structure matrix containing the 3D scene points.

b) What is the rank of the matrix W and why?

c) Briefly outline how you can use Singular Value Decomposition (SVD) approach for finding the matrix S.

(9+3+8 = 20)

8. Write comprehensive short notes on any two of the following: (10×2=20)

a) Image Mosaicing

b) SIFT

c) Bag of Words

d) Mean shift segmentation

INDIAN STATISTICAL INSTITUTE

Semestral Examination of Second Semester (2014-2015)

M.TECH.(CS) II YEAR

Topics in Algebraic Computation

Date: 05/05/2015 Maximum marks: 100 Duration: 3 hours

Note: The paper contains 120 marks. Answer as much as you can. The maximum you can score is 100.

1. In the following $\mathbb{F}$ is a field and I is an ideal of $\mathbb{F}[x_1, \ldots, x_n]$.
   (a) For $n \geq 1$, show that every finite subset of $\mathbb{F}^n$ is an affine variety.
   (b) Using graded LEX ordering of monomials, divide $x^3 - x^2y - x^2z + x$ by $f_1 = x^2y - z$ and $f_2 = xy - 1$.
   (c) Show that for varieties V and W, $V \subseteq W$ if and only if $I(V) \supseteq I(W)$.
   (d) Show that for any variety V, $I(V)$ is a radical ideal.
   (e) Let I be a monomial ideal and $f(x) \in I$. Show that $f(x)$ can be written as an $\mathbb{F}$-linear combination of monomials in I.
   (f) If I is an ideal of $\mathbb{F}[x_1, \ldots, x_n]$, show that $V(I)$ is an affine variety.

   $(6 \times 5 = 30)$

2. (a) Find the reduced Groebner basis generating the following ideal:

   $$\langle x^3y^2 + 2xy^2, x^2y^2 + x^2y, xy^3 + xy \rangle$$

   (b) Let G and $\tilde{G}$ be two minimal Groebner basis for the same ideal. Show that $LT(G) = LT(\tilde{G})$.
   (c) Let $\mathbb{F}$ be a field and I be an ideal of $\mathbb{F}[x_1, \ldots, x_n]$ and let G be a Groebner basis of I with respect to the LEX order where $x_1 > x_2 > \cdots > x_n$. Then show that for every $0 \leq \ell \leq n$, the set

   $$G_\ell = G \cap \mathbb{F}[x_{\ell+1}, \ldots, x_n]$$

   is a Groebner basis of the ℓ-th elimination ideal of I.

   $(15 + 5 + 10 = 30)$

3. (a) Let $\{b_1, \ldots, b_n\}$ be a basis for a lattice Λ. Show that $\mathcal{P}(b_1, \ldots, b_n) \cap \Lambda = \{0\}$.
   (b) For any full rank lattice Λ and a measurable set S with $\text{vol}(S) > \det(\Lambda)$, show that there exist two distinct points $z_1, z_2 \in S$ such that $z_1 - z_2 \in \Lambda$.
   (c) Show that the Hermite normal form of a matrix with integer entries is unique.
   (d) Let D be the dual basis of a lattic basis B. Show that $\mathcal{L}(B)^* \subseteq \mathcal{L}(D)$.

(c) Let $f(x)$ be a monic polynomial of degree d. Show how to formulate the problem of finding a root of the equation $f(x) \equiv 0 \bmod N$ for $x \leq B < N$ into a lattice problem.

$(7 \times 5 = 35)$

4 (a) In the LLL-algorithm suppose that $b_1, \ldots, b_{k-1}$ is LLL-reduced; for the vector b_k, the coefficient $\mu_{k,k-1} < 1/2$; and the Lovasz condition holds for the vectors b_k and b_{k-1}. Show how to modify b_k such that the size condition holds for $\mu_{k,j}$ with $1 \leq j \leq k-2$. Justify your answer.

(b) Provide a geometric description of the nearest plane algorithm.

(c) Define GapCVP_γ problem. Given oracle access to GapCVP_γ, show how to solve GapSVP_γ.

$(10+5+10 = 25)$