INDIAN STATISTICAL INSTITUTE

MID-SEMESTRAL EXAMINATION: (2015-2016)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 07.09.2014 Maximum marks: 40 Duration: 2 Hours

Note: Answer **all** questions.

Note: Throughout, $\mathbb{R}^L$ is the L-dimensional Euclidean space. Let

$$\mathbb{R}^L_+ = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i \geq 0 \text{ for all } 1 \leq i \leq L\}$$

and

$$\mathbb{R}^L_{++} = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i > 0 \text{ for all } 1 \leq i \leq L\}.$$

Q1. Let $\succeq$ be a *preference relation* over a non-empty set of alternatives X. Suppose also that $\succ$ and $\sim$ are the *strict preference relation* and the *indifference relation* respectively associated with $\succeq$.

(i) Show that $\succ$ is transitive. [3]

(ii) Suppose that $\succeq$ is rational and $U : X \to \mathbb{R}$ is a utility function representing $\succeq$, that is, $x \succeq y \Leftrightarrow U(x) \geq U(y)$ for $x, y \in X$. Show that "U is a utility function representing $\succeq$" is equivalent to "$x \succ y \Leftrightarrow U(x) > U(y)$ for any $x, y \in X$". [4]

(iii) Prove or disprove: Suppose that X has only finitely many elements. For any non-empty subset B of X, let $|B|$ denote the number of elements in B. Assume that $\mathscr{B}$ denotes the collection of all non-empty subsets of X. Define choice rules $\mathbb{C}(\cdot)$ on $\mathscr{B}$ by

$$\mathbb{C}(B) := \left\{x \in B : x \succeq y \text{ for all } y \in A \text{ for some } A \subseteq B \text{ with } |A| > \frac{1}{3}|B|\right\}.$$

Then $(\mathscr{B}, \mathbb{C}(\cdot))$ satisfies *WARP*. [3]

(iv) Prove or disprove: If U and V represent $\succeq$ then there is a strictly monotonic function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x) = f(U(x))$. [2]

Q2. Answer **all** questions.

(i) Show that *WARP* is not a sufficient condition to ensure the existence of a rationalizing preference relation. [3]

(ii) Show that for a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfying Walras's law, $WARP$ holds for all compensated price changes if and only if for any compensated price change from (p, w) to $(p', w') = (p',\ p' \cdot x(p, w))$, the following inequality holds:

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0,$$

with strict inequality for $x(p', w') \neq x(p, w)$. [5]

(iii) Suppose that X is a non-empty set of alternatives and $(\mathscr{B}, \mathbb{C}_1(\cdot))$ is a choice structure of X that satisfies $WARP$. Consider the choice structure $(\tilde{\mathscr{B}}, \mathbb{C}_2(\cdot))$, where $\tilde{\mathscr{B}} = \{\mathbb{C}_1(B) : B \in \mathscr{B}\}$. Show that if $(\tilde{\mathscr{B}}, \mathbb{C}_2(\cdot))$ satisfies $WARP$ then $(\mathscr{B}, \mathbb{C}(\cdot))$ must satisfy $WARP$, where $\mathbb{C}(B) = \mathbb{C}_2(\mathbb{C}_1(B))$ for all $B \in \mathscr{B}$. [4]

Q3. Answer **all** questions.

(i) Prove or disprove: Let $(\mathscr{B}, C(\cdot))$ be a choice structure. The *strict revealed preference relation* $\succ^*$ is defined by $x \succ^* y \Leftrightarrow \exists\ B \in \mathscr{B}$ such that $x, y \in B, x \in C(B)$ and $y \notin C(B)$. If $(\mathscr{B}, C(\cdot))$ satisfies $WARP$, then $\succ^*$ is transitive. [2]

(ii) Suppose that X is a non-empty set of alternatives and $(\mathscr{B}, \mathbb{C}(\cdot))$ is a choice structure of X that satisfies $WARP$. Show that $\succ^*$ and $\succ^{**}$ are identical, that is, for any $x, y \in X$, $x \succ^* y \Leftrightarrow x \succ^{**} y$, where $\succ^*$ is the *strict revealed preference relation*, $x \succ^{**} y \Leftrightarrow [x \succsim^* y$ and $y \not\succsim^* x]$ and $\succsim^*$ is the *revealed preference relation*. [4]

(iii) Let $X = \{x, y, z\}$ and $\mathscr{B} = \{\{x, y\}, \{y, z\}, \{z, x\}, \{x, y, z\}\}$. Suppose that $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y, z\}$, $C(\{z, x\}) = \{z, x\}$ and $C(\{x, y, z\})$ is a non-empty subset of X. Does $C(\cdot)$ satisfy $WARP$? Justify your answer. [3]

(iv) Let $(\mathscr{B}, C(\cdot))$ be a choice structure. If a rational preference relation $\succsim$ *rationalizes* $C(\cdot)$ relative to $\mathscr{B}$, then show that $C(B_1 \cup B_2) = C(C(B_1) \cup C(B_2))$ for every $B_1, B_2 \in \mathscr{B}$ such that $B_1 \cup B_2 \in \mathscr{B}$ and $C(B_1) \cup C(B_2) \in \mathscr{B}$. [5]

(v) Show that if a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfies $WARP$ then it is homogeneous of degree zero. [2]

Indian Statistical Institute

M.S.Q.E. 1st Year : 2015–2016
Mid-Semester Examination
Subject: Mathematical Methods

Date: 11/09/2015 Time: 3 hours Marks : 100

Answer Group-A and Group-B on separate answer scripts.

Group-A

1. If A and B are square matrices, then is $AB = BA$? Either prove or give a counter example. [5]

2. If a square matrix has two identical rows, show from definition hat it has determinant 0. [5]

3. Show (using definition only) that the rows of the following matrix are linearly independent
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$
Find its inverse. [5 + 10]

4. For what values of k does
$k.x + y = 1$
$x + k.y = 1$
have no solution, one solution or infinitely many solutions? [10]

5. Solve the following system of liner equations.
$x + 3y + z + 2u = 1$
$2x + 6y + 4z + 8u = 3$
$2z + 4u = 1$ [15]

Group-B

1. Give examples of functions :
   (a) **One-to-one** but not **onto**.
   (b) **Onto** but not **one-to-one**.
   (c) Prove that there exist a **bijection** from $[0, 2]$ to $[0, 10]$. [3 + 3 + 4]

2. If $a_1, a_2, \ldots, a_n$ are any real numbers, then prove that $|a_1 + a_2 + \ldots + a_n| \leq |a_1| + |a_2| + \ldots + |a_n|$. for all $n \geq 1$. [6]

3. Show that $\sqrt{2}$ is not a rational number, however it is a real number. [4 + 10]

4. Prove that the unit interval $[0, 1]$ is not countable.

5. (a) Prove that the union of any collection of open sets is an open set.

   (b) Prove that the intersection of a finite collection of open sets is an open set.

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2015-16

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR / M.MATH. II YEAR

Subject Name: Game Theory I

Date: 25-08-2015 Maximum Marks: 40 Duration: 90 minutes

Problem 1. (a) Let S be the set of pure strategy Nash equilibria of a game G. Under what minimal (most general) condition a correlated strategy q of the form $q(s) > 0$ if and only if $s \in S$ is a correlated Nash equilibrium?
(b) Justify your answer by a proof or a counterexample.
Let G be a zero-sum game with $|S_i| \geq 3$ for all $i \in N$ which does not have any pure strategy Nash equilibrium. Then G does not have a correlated equilibrium that is *not* a mixed Nash equilibrium. (QE1506, AK), (20, 21), (29, 30) (5+5)

Problem 2. Justify your answer by a proof or a counterexample.
Consider a finite extensive form game with imperfect information where the number of edges in the longest path in the game tree is three. Then
(i) Sequential rationality implies consistency,
(ii) Consistency implies sequential rationality,
(iii) Sequential equilibrium is subgame perfect,
(iv) Sequential equilibrium always exists.

(2+3+5+5)

Problem 3. Consider a Bayesian game $G^B = \langle N, (T_i)_{i\in N}, (P_i)_{i\in N}, (A_i)_{i\in N}, (u_i)_{i\in N} \rangle$ where $T_i = A_i = [0,1]$ and $P_i = (U[0,1])^{|N|}$ for all $i \in N$.
(i) Justify your answer by a proof or a counterexample. There exist u_i; $i \in N$ such that a strategy-tuple s^* of the form $s_i^*(x) = x$ for all $i \in N$ is

(a) a Bayesian Nash equilibrium,

(b) unique Bayesian Nash equilibrium.

(ii) Suppose $N = \{1, 2\}$ and $u_i(t, a) = t_i - t_j + a_i - a_j$ for all $i \in N$. What can you say about the Bayesian Nash equilibrium?
(iii) Justify your answer by a proof or a counterexample. Suppose $u_i(t_i, t_{-i}, a) \geq u_i(t_i', t_{-i}, a)$ for all $t_i \geq t_i'$ and a. Then for any equilibrium s^* it must be that $s_i^*(t_i) \geq s_i^*(t_i')$ for all $t_i \geq t_i'$. (5+5+5)

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2015-16

Course Name: MSQE

Subject Name: Statistics

Date: 15/09/2015 Full Marks: 30 Duration: 1 hr

1. A committee consists of 6 professors, 3 readers, 2 lecturers and 2 administrators. A sub-committee of 4 is chosen at random. What is the probability that all the 4 groups are represented in the sub-committee? (5)

2. Suppose that the probability of living to be older than 60 is 0.6 and the probability of living to be older than 70 is 0.2. If a person reaches his/her 60th birthday, what is the probability that he/she will celebrate his/her 70th birthday? (5)

3. Let $G(x) = 1-(1-e^{-x/\theta})^n$ where $x > 0$, $\theta > 0$ and $G(x) = 0$ for $x < 0$ and n is a positive integer. Can G be looked upon as a distribution function of a random variable? If so, find the corresponding probability density. (10)

4. The number of oil tankers arriving at a certain refinery each day has a Poisson distribution with parameter $\lambda = 2$. Present port facilities can service three tankers a day. If more than three tankers arrive in a day, the tankers in excess of three must be sent to another port.
   (a) On a given day, what is the probability of having to send tankers away?
   (b) How much must present facilities be increased to permit handling all tankers on approximately 90 per cent of the days?
   (c) What is the expected number of tankers serviced daily?
   (d) What is the expected number of tankers turned away daily?
   (2+3+3+2=10)

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2015-16
M. S. (Q.E.) .I Year
Basic Economics

Date: 17·09·15 **Maximum Marks: 40** **Duration:** $2\frac{1}{2}$ **Hours**

Answer any four

1 a) Define money and discuss its functions

b) Describe how commercial banks create money.

(5+5)

2 Distinguish between (i) GDP and GNP, (ii) GDP and NDP, (iii) Real income and Nominal income, (iv) Final product and Intermediate product (v) Direct tax and Indirect tax.

(5×2)

3 Using the classical macro-economic model, analyse how a full employment equilibrium is always ensured and why money is neutral in effects. Discuss the effects of an increase in Government expenditure in this model.

(4+4+2)

4 a) What is an IS curve? How does it shift ?

b) Using an IS-LM model , analyse the effects of fiscal policy and monetary policy on the equilibrium level of income and rate of interest.

(3+7)

5 How do you derive the aggregate supply curve and aggregate demand curve in the Keynesian model. Analyse the effect of a wage – cut policy on the4 equilibrium price and output.

(4+4+2)

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INDIAN STATISTICAL INSTITUTE

MidSemestral Examination: (2015-2016)

M. S. (Q.E.) – I Yr.

Computer Programme and Application

Date: 18.09.2015 | Maximum marks: 40 | Duration: 2 hrs

1. Answer any five: [5×2 = 10]
   a) What is the difference between ALU and FPU?
   b) What is the difference between 'single tasking system' and 'single user system'?
   c) What is cache memory?
   d) What are the primitive operations carried out by the CPU of a computer system?
   e) What is system bus? Name different types of system bus.
   f) Convert the decimal number 18 into its binary and hexadecimal representations.
   g) What is called a compiler? How does it differ from an interpreter?

2. Answer any five: [5×2 = 10]

a) What is the use of columns 1-5 of each line of a FORTRAN program?

b) What are the different variable types used in a FORTRAN program?

c) What are the rules for selection of variable names in a FORTRAN program?

d) What will be written due to the following two FORTRAN statements when DAY = 18, RATE = 66.4451827, MTH = 'SEP'

```
35  WRITE (*, 55) 'TODAY', DAY, MTH, '1 USD =', RATE, 'INR'
55  FORMAT (1X, A5, I3, A4, 1X, A7, F6.2, 1X, A3)
```

e) Write a READ and corresponding FORMAT statements to read the following data from an input file. Here, the symbol '^' denotes a blank space.
^2345.62^^1234.5678^^^45.123456789

f) Write FORTRAN statements to print 'INVALID' in case the value in the variable FLAG is 0 and otherwise print 'VALID'

3. Attempt any one of the following two. [5]

a) Read the following FORTRAN code and write what will be printed on the screen?

```
      PROGRAM MAIN
      INTEGER I, I_START, I_END, I_INC
      REAL A(10)

      I_START = 1
      I_END = 10
      I_INC = 2

      A(I_START) = 1
      DO 15 I = I_START, I_END, I_INC
         A(I+I_INC) = A(I)+10
15    CONTINUE

      I_START = I_START+1
      A(I_START) = 1
      DO 25 I = I_START, I_END, I_INC
         A(I+I_INC) = A(I)+10
25    CONTINUE
35    PRINT *, (A(I), I=1, I_END)
      STOP
      END
```

P.T.O

b) What does the following FORTRAN program compute?

```
      PROGRAM MAIN
      INTEGER N_VAL, R_VAL
      REAL VAL, VAL1, VAL2

      N_VAL = 10
      R_VAL = 5
      VAL1 = N_VAL
      VAL2 = R_VAL

      DO 15 I = 1, (R_VAL-1)
         VAL1 = VAL1*(N_VAL-I)
         VAL2 = VAL2*(R_VAL-I)
 15   CONTINUE

 35   PRINT *, VAL1/VAL2
      STOP
      END
```

3. Attempt any one of the following two. [15]

a) A file consists of two matrices and the respective orders as the following example. Write a FORTRAN program to read similar matrices and their orders from an input file, compute their product and write the result into a new file in the same format as the input file. Your program should be able to compute the product of any two matrices of maximum order 100.

```
Inputfile.txt
3 4
1 2 3 4
5 6 7 8
9 0 1 2

4 3
1 2 3
4 5 6
7 8 9
0 1 2
```

b) A file consists of an unknown number of numbers each number occupying a distinct row, i.e., no two numbers are there in the same line. Write a FORTRAN program to find the minimum and maximum of the given numbers and print them in an output file. You should be careful to use minimum storage space, i.e., unnecessary variable declaration should be avoided.

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2015-2016)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 16.11.2015 | Maximum Marks: 60 | Duration: 3 Hours

Note: Answer any 4 questions. Each question is worth 15 marks.

Note: Throughout, $\mathbb{R}^\ell$ is the ℓ-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ be a continuous utility representation of a locally non-satiated preference $\succeq$ over $\mathbb{R}^\ell_+$. Given a $(p, \omega) \in \mathbb{R}^\ell_{++} \times \mathbb{R}_{++}$, the *utility maximization problem* (UMP) of a consumer is the following:

$$\sup\{U(x) : x \in B(p, \omega)\}$$

where $B(p, \omega) = \{x \in \mathbb{R}^\ell_+ : p \cdot x \leq w\}$ is the budget set for (p, w). The *expenditure minimization problem* (EMP) for any given $p \in \mathbb{R}^\ell_{++}$ and any $u > U(0)$ is the following:

$$\inf\{p \cdot x : x \in \mathbb{R}^\ell_+, U(x) \geq u\}.$$

(i) Show that the indirect utility function $v : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}$, defined by

$$v(p, w) = \sup\{U(x) : x \in B(p, \omega)\},$$

is strictly increasing in ω.

(ii) Show that if x is an optimal solution of the UMP when $\omega > 0$, then x is an optimal solution of the EMP when the required level of utility is $U(x)$. Show further that the minimum expenditure level in this EMP is ω. [6+9]

Q2. Answer **all** questions.

(i) Let Y be a convex production set. Show that the additive closure of Y is $\bigcup\{nY : n \geq 1\}$, where $nY = \{ny : y \in Y\}$ for all $n \geq 1$. Find the additive closure of

$$Y = \{(-x_1, -x_2, y) \in \mathbb{R}^3 : x_i, y \geq 0 \text{ for all } i = 1, 2 \text{ and } \operatorname{Sin} y \leq x_1 + x_2\}.$$

(ii) Show that if Y is a closed and convex production set and $-\mathbb{R}^\ell_+ \subseteq Y$, then free disposal holds. [9+6]

Q3. Answer **all** questions.

(i) Take $\ell \geq 3$ and $X = \{(x, y, z, 0, \cdots, 0) \in \mathbb{R}^{\ell} : x, z \geq 0 \text{ and } y < 0\}$. Define $x : \mathbb{R}^{\ell}_{++} \times \mathbb{R}_{++} \to X$ by

$$x(p, w) = \left(\frac{p_2}{p_3}, -\frac{p_1}{p_3}, \frac{w}{p_3}, 0, \cdots, 0\right).$$

Show that x violates the $WARP$.

(ii) Suppose that X is a countable set and $\succsim$ is a rational preference relation over X. Show that there exists a utility function $U : X \to \mathbb{R}$ that represents $\succsim$.

(iii) A consumer has a utility function $U : \mathbb{R}^3_+ \to \mathbb{R}$, defined by $U(x, y, z) = \min\{x, y\} + z$. The prices of the three commodities are given by $p = (p_1, p_2, p_3)$ and the wealth of the consumer is ω. Find the demand correspondence and indirect utility function. [5+5+5]

Q4. Answer **all** questions.

(i) Consider the utility function $U(x) = \sqrt{x}$. Find the certainty equivalence and the probability premium for lotteries

$$L_1 = \left[\frac{1}{2}(16), \frac{1}{2}(4)\right] \text{ and } L_2 = \left[\frac{1}{2}(36), \frac{1}{2}(16)\right].$$

Compare the probability premium of L_1 and L_2.

(ii) Suppose that an agent has an initial endowment $w \in \mathbb{R}^3_+ \setminus \{0\}$ and a preference relation $\succsim$ on $\mathbb{R}^{\ell}_+$ represented by a utility function $U : \mathbb{R}^3_+ \to \mathbb{R}$, defined by

$$U(x, y, z) = \sqrt{x} + \sqrt{y} + y + \frac{z}{1 + z}.$$

If $z > 0$, then verify that $(x, y + z, 0) \succ (x, y, z)$, where $\succ$ denotes the strict preference relation associated with $\succsim$. If a price $p = (p_1, p_2, p_3) \in \mathbb{R}^3_{++}$ satisfies $p_2 = p_3$, then show that the demand set $D(p, \omega, \succ) \subset \{(x, y, 0) : x, y \in \mathbb{R}_+\}$. [6+9]

Q5. Answer **all** questions.

(i) Suppose that $U : \mathbb{R}_+ \to \mathbb{R}$ is a utility function representing the natural order on $\mathbb{R}_+$ and the first derivative of U exists on $(0, \infty)$. Assume that $U(x) \neq -1$ for all $x \in \mathbb{R}_+$. Define a function $V : \mathbb{R}_+ \to \mathbb{R}$ by

$$V(x) = \frac{(1 + U(x))^2 + 1}{1 + U(x)}.$$

Verify whether V is a utility function representing the natural order on $\mathbb{R}_+$.

(ii) Prove or disprove: the preference relation $\succ$ over $\mathbb{R}^2_+$, defined by $(x, y) \succsim (x', y')$ if and only if $xy' \geq x'y$, is monotone.

(iii) Let $\succsim$ be a rational preference relation over $\mathbb{R}^{\ell}_+$. Show that if $\succsim$ is locally non-satiated and weakly monotone then it is monotone. [5+5+5]

Indian Statistical Institute

M.S.Q.E. 1st Year : 2015–2016
Semester Examination
Subject: Mathematical Methods

Date: 19/11/2015 Time: 3 hours Marks : 100

Answer Group-A and Group-B on separate answer scripts.
Notations used are as explained in the class.

Group-A

1. Let $x_1 \geq 2$ and $x_{n+1} = 1 + \sqrt{(x_n - 1)}$, $n \geq 1$. Show that (x_n) is decreasing and bounded below by 2. Find the limit. [10]

2. Show that if (z_n) is a sequence of nonzero real numbers that converges to a nonzero limit z, then the sequence $(\frac{1}{z_n})$ converges to $\frac{1}{z}$. [9]

3. If $f : \mathbb{A} \to \mathbb{R}$ and if c is a cluster point of A, then prove that f can have only one limit at c. [7]

4. Using ϵ-δ method, prove that the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is continuous at every point. [8]

5. State and prove Rolle's Theorem. [8]

6. Use the Mean Value Theorem to prove that $\frac{x-1}{x} < \ln x < x - 1$ for $x > 1$. [8]

Group-B

1. Is the set $\{0, 1, 2, \cdots, 10\}$ with operations: addition and multiplication modulo 11 a field? Justify your answer. [6]

2. (a) Let $V \subseteq \mathbb{R}^3$, $V = \{(x_1, x_2, x_3) : \frac{x_1}{3} = \frac{x_2}{4} = \frac{x_3}{2}\}$. Does V form a vector space over $\mathbb{R}$? If so, what is its dimension?

   (b) Is $\mathbb{R}$ with usual addition and multiplication, a vector space over the rational number $\mathbb{Q}$? If so, what is its dimension?

   (c) Let $\mathcal{P}_n$ be the set of all polynomials with real coefficients and of degree $\leq n - 1$. Show that it is a vector space and following three polynomials form a basis of $\mathcal{P}_3$: $f_1(x) = 1, f_2(x) = x - 2, f_3(x) = (x - 2)^2$. Express $3x^2 - 5x + 4$ as a linear combination of f_1, f_2 and f_3.

   [6 + 6 + 6]

3. Find the determinant of the following $n \times n$ matrix

$$\begin{pmatrix} r & \lambda & \cdots & \lambda \\ \lambda & r & \cdots & \lambda \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \lambda & \lambda & \cdots & r \end{pmatrix}.$$

[6]

4. Find an orthogonal basis for the linear space spanned by the vectors:
   $(1, 0, 1, 1), (1, -1, 1, 0), (1, -1, 1, -1)$

5. Find the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}.$$

INDIAN STATISTICAL INSTITUTE
Semester I Examination: 2015-16
Course: MSQE-1st Year
Subject: Statistics

Date: 23.11.2015 Maximum Marks: 70 Time: 3 hours

Note: Answer all questions. Maximum you can score is 70.

1. (a) There are two drawers in each of three boxes that are identical in appearance. The first one contains a gold coin in each drawer, the second contains a silver coin in each drawer, but the third contains a gold coin in one drawer and a silver coin in the other. A box is chosen, one of its drawers is opened and a gold coin is found. What is the probability that the other drawer too will have a gold coin?

   (b) A man addresses n envelopes and writes n cheques in payment of n bills. If the n bills and n cheques are placed at random in the n envelopes, one bill and one cheque in each envelope, what would be the probability that in no instance would the enclosures be completely correct? (5+7=12)

2. Suppose the duration in minutes of long-distance telephone calls made from a city is found to be a random variable with a probability distribution specified by the distribution function F, given by,

   $$F(x) = 0 \quad for \quad x \le 0$$
   $$= 1-(2/3)\exp(-x/3)-(1/3)\exp(-[x/3]) \quad for \quad x > 0$$

   (a) Check that F represents a distribution function.
   (b) Sketch the distribution function.
   (c) What is the probability that the duration of long-distance call will be between 4 and 7 minutes? (4+2+4=10)

3. (a) Let $S=\sum_{i=1}^{n} X_i$ where $X_1, X_2, \ldots, X_n$ are independent random variables with mean μ and variance σ^2. If $T=\sum_{i=1}^{n} iX_i$, find the covariance and correlation coefficient between S and T.

   (b) Suppose (X,Y) is a bivariate random vector such that Y has an exponential distribution with mean 1, and for every $y>0$, the conditional distribution of X given $Y=y$ is uniform over $(0,y)$. Let $Z=Y-X$. Find the mean and variance of Z. (6+7=13)

4. (a) Let $X_1, \ldots, X_n$ be a random sample from Poisson(θ) distribution and let $\gamma(\theta)=\frac{e^{-\theta}\theta^k}{k!}$ for some fixed positive integer k. Find the minimum variance unbiased estimator for $\gamma(\theta)$.

P.T.O

(b) Suppose X and Y are independently distributed random variables following Geometric(p_1) and Geometric(p_2) distributions. Find the conditional distribution of $X|X+Y=t$ and hence develop a level α test for testing $H_0: p_1 = p_2$ against $H_1: p_1 > p_2$. (6+6=12)

5. (a) Let $X_1, X_2, ..., X_n$ be a random sample distributed independently and identically from a Poisson distribution with mean μ. Suppose one statistician suggests that he can estimate μ using the statistic $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ while another wants to use the statistic $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. Which one will you prefer? Justify your answer.

(b) If $X_1, X_2, ..., X_n$ are i.i.d. observations from a Exponential distribution with p.d.f. $f(x) = \theta e^{-\theta x}, x > 0, \theta > 0$. Determine a likelihood ratio test at level α for $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.
[Use: $2\theta_0 X_i \sim \chi_2^2$ *for each* $i = 1,2,...,n$] (6+7=13)

6. (a) Consider a simple linear regression model:
$y_i = \alpha + \beta x_i + \varepsilon_i,\ i = 1,2,...,n$.
Assume that $\varepsilon_1, ..., \varepsilon_n$ satisfy $E(\varepsilon_i) = 0, V(\varepsilon_i) = \sigma^2$ and $Cov(\varepsilon_i, \varepsilon_j) = \rho\sigma^2$ for some $0 < \rho < 1$, for $1 \le i \neq j \le n$. Prove that, if $\sum_{i=1}^{n}(x_i - \bar{x})^2 \to \infty$, the ordinary least square estimator of β is consistent.

(b) If $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{P} Y$, show that $X_n + Y_n \xrightarrow{L} X + Y$, where symbols have their usual meanings. (7+7=14)

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2015-16
M. S. (Q.E.) .I Year
Basic Economics

Date: 26.11.2015 **Maximum Marks: 40** **Duration: 3 Hours**

Answer any three

1 a) Define Pareto efficiency and derive the conditions of Pareto efficient allocation of resources between two sectors.

b) Explain the role of competitive markets to ensure such an allocation

c) Also explain how externality creates a problem in this context .

(2+6 + 5+7)

2 a) What is a production possibility curve and how do you obtain it in a two sector two factor economy?

b) Analyse how an open economy equilibrium differs from a closed economy equilibrium [use production possibility curve and indifference curve]

c) Also explain alternative conditions for trade between two countries.

(2+4 + 7+7)

3. a) Explain the difference between demand pull inflation and cost push inflation. Discuss the role of various anti-inflationary policies to solve these problems.

b) Explain the concept of steady-state equilibrium growth and then find out the sources of economic growth in that equilibrium.

c) Discuss major features of alternative phases of a trade cycle.

(5+5 + 5+5)

4 Write short notes on any four of the following.

a) Surplus labour.
b) Dual Economy.
c) Low level equilibrium trap.
d) Export promotion vs. Import substitution strategy.
e) Keynesian investment multiplier.
f) Speculative demand for money.
g) Balance of payment equilibrium.

(4×5)

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INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2015-16

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR / M.MATH. II YEAR

Subject Name: Game Theory I

Date: 28.11.2015 Maximum Marks: 50 Duration: 2 hours

Problem 1. The two-player game of Tic-Tac-Toe is played on a 3×3 board. Player 1 starts by putting a cross on one of the nine fields. Next, player 2 puts a circle on one of the eight remaining fields. Then player 1 puts a cross on one of the remaining seven fields, etc. If player 1 achieves three crosses or player 2 achieves three circles in a row (either vertically or horizontally or diagonally) then that player wins. If this does not happen and the board is full, then the game ends in a draw.

(a) Design a pure maximin strategy for player 1. Show that this maximin strategy guarantees at least a draw to him.

(b) Show that player 1 cannot guarantee a win.

(c) What is the value of this game? (10)

Problem 2. An $m \times n$ matrix game $A = (a_{ij})$ is called symmetric if $m = n$ and $a_{ij} = a_{ji}$ for all $i, j = 1, ..., m$.

Prove that the value of a symmetric game is zero and that the sets of optimal strategies of players 1 and 2 coincide. (15)

Problem 3. Consider the following game:

	1	2	3	J
1	$-1,1$	$1,-1$	$1,-1$	$-1,1$
2	$1,-1$	$-1,1$	$1,-1$	$-1,1$
3	$1,-1$	$1,-1$	$-1,1$	$-1,1$
J	$-1,1$	$-1,1$	$-1,1$	$1,-1$

a. Find a completely mixed Nash equilibrium in which each player assigns the same probability to the actions 1, 2, and 3.

b. Show that the equilibrium you found in part a is the only equilibrium of the game.

(15)

Problem 4. Consider the following bimatrix game:

$$\begin{array}{cc} & \begin{array}{cc} L & S \end{array} \\ \begin{array}{c} L \\ S \end{array} & \begin{pmatrix} 2,2 & -1,-1 \\ -1,-1 & 1,1 \end{pmatrix} \end{array}$$

(a) Which payoffs can be reached as limiting average payoffs in subgame perfect equilibria of the infinitely repeated game $G^{\infty}(\delta)$ for suitable choices of δ?
(b) Which payoffs can be reached as limiting average payoffs in Nash equilibria of the infinitely repeated game $G^{\infty}(\delta)$ for suitable choices of δ?
(c) Describe a subgame perfect Nash equilibrium of $G^{\infty}(\delta)$ resulting in the limiting average payoffs $(3/2, 3/2)$. Also give the corresponding restriction on δ.
Hint: Use folk theorem.

(10)

Indian Statistical Institute

Semester I Examination 2015-2016

M. S. (Q. E.) - I year

Subject: Computer Programme and Application

Full Marks: 100 Duration: 3 hrs.

(Answer all questions)

1. a) Write the binary and decimal numbers equivalent to the hexadecimal digit 'A'.

   b) What are the primitive operations performed by a computer system?

   c) What is the role of system bus of a computer system?

   d) What do you mean by the latency of an HDD? How the performance of an HDD may be specified?

   e) Name a type of memory, which is smaller in size but faster in speed than RAM.

   f) Mention if there is any fundamental tradeoff with this type of memory.

   [2+2+2+(1+1)+1+1 = 10]

2. a) What are the different integer types of data supported by C programming language?

   b) What is the void data type in C?

   c) State which ones among the following are valid variable declarations : (a) int abc123; (b) int 123abc; (c) int _abc; (d) int _123; (e) int !abc; (f) int !abc123

   d) The Personnel Department of an office maintains record of various information such as name, address, age and sex of each of its employees. Declare variables suitably to represent this information in a C program in a compact fashion.

   [2+1+3+4 = 10]

3. a) Find which one(s) among the following C programs will throw compilation error and explain the reasons.

i)
```
#include <stdio.h>
extern int MyVar;
int main(){
   printf("%d", MyVar);
   return 0;
}
```

ii)
```
#include <stdio.h>
int main(){
 extern int MyVar;
 printf("%d", MyVar);
 return 0;
}
```

iii)
```
#include <stdio.h>
extern int Myvar ;
int main(void)
{
int MyVar = 10;
return 0;
}
```

b) What will be printed in each of the following two cases?

i)
```
#include <stdio.h>
int main(){
int MyVar=5;
do
   printf("%d ", MyVar);
  while (MyVar-- > 0);
return 0;
}
```

ii)
```
#include <stdio.h>
int main(){
int MyVar=5;
while (MyVar-- > 0)
      printf("%d ", MyVar);
return 0;
}
```

[(3×2) + (2×2) = 10]

4. (i) Write a C program implementing the following algorithm:

Step 1: Read N integers from a file, and store them into an array Arr of size N
Step 2: Set i = 1 and FLAG = 0
Step 3: if Arr[i] > Arr[i+1], swap these two values (Arr[i] and Arr[i+1]) and set FLAG = 1
Step 4: Add 1 to i
Step 5: if i < (N-1), go to Step 3
Step 6: if FLAG = = 1, go to Step 2
Step 7: Print Arr[j] for j=1, ..., N

(ii) If the input N integers are 9, 3, 7, 1, 4, 2, 6, 5, 8, then what will be printed by the program? [8 + 2 = 10]

5. (i) What is the maximum number of cases that may be supported by a C switch statement?
(ii) What does happen if a case of a switch statement does not include a break statement?
(iii) What will be the output of the following C code?

```
#include <stdio.h>
void main(){
   int x = 1;
   if (x--){
      printf("God is truth\n");
      --x;
   }
   else
      printf("%d", x);
```

```
}
```

(iv) What will be the output of the following program?

```c
#include<stdio.h>
void main(){
   int x = 1, y = 2;
   if(--x  && --y)
       printf ("x=%d  y=%d", x, y);
   else
       printf("%d %d", x, y);
}
```

(v) What will be the output of the following program:

```c
#include<stdio.h>
int main(){
    int j;

    for (j=0; ;j++){
       printf("%d ", j);
       if( j >= 3)
           break;
    }

return 0;
}
```

[2+2+2+2+2 = 10]

6. (a) Assuming the availability of library functions isupper() and strlen(), decide what will be returned by the following function for the input string "mdAJMER".

```c
char Guess_Me(char *string) {
     static int i = 0;
     if (i < strlen(string)) {
         if (isupper(string[i])) {
              return string[i];
         }
         else {
              i = i + 1;
              return Guess_Me(string);
         }
     }
        else return 0;
}
```

(b) Suppose A be a 2-dimensional integer array of size 3×3, which contains the following

1	2	3
4	5	6
7	8	9

Find the value of *(*(A+2) + 2).

[5+5 = 10]

7. (i) Write down two library functions commonly used for dynamic memory allocation.
(ii) Write down the differences between them.

(iii) What do you know about 'call by value' and 'call by reference'?

[2+3+5 = 10]

8. Write a C program to print in an output file the product of two matrices provided in an input file as given below. The program should
   (i) declare two double pointers to integers,
   (ii) allocate memory dynamically using the above two pointers for the two matrices,
   (iii) use these memory locations to read the two matrices from the file,
   (iv) the product of the two matrices should be written in an output file without storing the entire output matrix into the main memory (RAM).

```
3   4
1 2 3 4
4 3 2 1
1 0 1 0

4   2
1 1
1 0
2 1
1 2
```

[20]

9. Write a C program to read two strings from the keyboard, concatenate them without using any string library function and print the result. [10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination (Back Paper): 2015-16

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR / M.MATH. II YEAR

Subject Name: Game Theory I

Date: 29·01·16 Maximum Marks: 50 Duration: 2 hours

Problem 1. Consider the following version of prisoner dilemma game.

1/2	C	D
C	4,4	0,6
D	6,0	2,2

(a) Is it true that the outcome (C,C) forever can be a subgame perfect equilibrium of the infinitely repeated prisoners dilemma game? If yes, provide the equilibrium strategies. Find the restriction on the discount factor δ.

(b) What do you think about the other Subgame Perfect Equilibria that are possible in this repeated prisoners dilemma game (use folk theorem)? Plot graphically: (i) The set of subgame perfect Nash Equilibria payoffs, (ii) The set of feasible payoffs.

(c) Consider the following tit-for-tat (TFT) strategy (row player version)
First round: Play C.
Second and later rounds: If the history from the last round is (C,C) or (D,C) play C. If the history from the last round is (C,D) or (D,D) play D.
(i) Is it true that TFT supports (C,C) forever as an NE in the Infinitely Repeated PD? - Justify your answer by a proof.
(ii) Is it true that TFT as an equilibrium strategy is Subgame Perfect?
- Justify your answer by a proof.

(20)

Problem 2. A buyer wants to buy a car but does not know whether the particular car he is interested in has good or bad quality (a lemon is a car of bad quality). About half of the market consists of good quality cars. The buyer offers a price p to the seller, who is informed about the quality of the car: the seller may then either accept of reject this price. If he rejects, there is no sale and the payoff will be 0 to both. If he accepts, the payoff to the seller will be the price minus the value of the car, and to the buyer it will be the value of the car minus the price. A good quality car has a value of 15,000, a lemon has a value of 5,000. (a) Set up the extensive as well as strategic form of this game. (b) Compute the subgame perfect equilibrium or equilibria of this game.

(15)

Problem 3. Let (b, β) be a consistent assessment in an extensive form game Γ. Show that (b, β) is Bayesian consistent.

(15)

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2015-2016)
MS (Q.E.) I Year
Macroeconomics I

Date: 23·02·2016 Maximum Marks 40 Duration 3 hours

Group A

Answer Question 1 and any one of the remaining two questions. Question 1 carries 7 points. Questions 2 and 3 carry 13 points each.

1. Suppose that there is a shock to the simple Solow economy *without* technical progress in the form of a one-time permanent increase in the labour force. (For example, a famine can cause large movements in the labour force across international borders.) What will be the short-run and long-run effects on the rate of growth of y? Prior to the shock, the economy may or may not have been in steady state. (Take the equation $\dot{k} = sy - (n+\delta)k$ for granted. You don't need to derive it.) (7)

2. Consider the following Mankiw-Romer-Weil model of growth with human capital, instead of the simpler formulation discussed in the class. The production function for aggregate output Y is $Y = K^{\alpha}H^{\beta}(AL)^{1-\alpha-\beta}, \alpha > 0, \beta > 0, \alpha+\beta < 1$. H represents the stock (*as well as flow*) of human capital, i.e. education or knowledge, in the same sense in which K is a stock *as well as a flow in* Solow. L represents the labour force (assumed to be equal to population), growing at the exogenously given rate n. Finally, A is total factor productivity, growing exogenously at the rate μ. Physical and human capital depreciate at the same rate δ. They accumulate according to

$$\dot{K} = s_K Y - \delta K. \quad 0 < s_K < 1.$$

and

$$\dot{H} = s_H Y - \delta H. \quad 0 < s_H < 1.$$

Define $\hat{y} = Y/AL$, $\hat{k} = K/AL$ and $\hat{h} = H/AL$.

Solve the model for the path of $y = Y/L$ along the balanced growth path as a function of $s_K, s_H, n, \mu, \delta, \alpha$ and β. (13)

3. Consider the Solow model with exogenous technical progress. Use the notational framework used in the class (with $\delta = 0$), assume that $u(c) = (c^{1-\theta} - 1)/(1-\theta), \theta =$ constant and derive the optimal rate of balanced growth of per capita $c = C/L$. [Hint: Use the Keynes-Ramsey argument.] Assume the aggregate production function to be $Y = K^{\alpha}(AL)^{1-\alpha}$. What will be the value of K/AL when c is growing at the optimal balanced growth rate? What will be the value of the saving rate (s^*) along the optimal balanced growth path? (13)

Group B

Answer all questions

1. In an appropriate new Keynesian model derive the long run multiplier of a balanced budget increase in government expenditure and show that with variety effect absent, such multiplier is smaller than what would obtain in the short run.

(10)

2. Show that in the flexible price equilibrium that obtains in the Blanchard and Kiyotaki model, money is completely neutral. (10)

[Note: You need not derive the Dixit –Stiglitz demand functions, just use them directly.]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
MID-SEMESTRAL I EXAMINATION (2015 – 16)
M.S.(Q.E.) 1st Year

Time Series Analysis & Forecasting

Date: 26·02·16 Maximum Marks: 50 Time: 2 hours

This question paper carries a total of 60 marks. You can answer any part of any question. But the maximum that you can score is 50. Marks allotted to each question are given within parentheses.

1. Examine whether the following statements are TRUE or FALSE or UNCERTAIN. Give brief explanations in support of your answer.

(i) Method of curve fitting is a better method than the method of moving average for obtaining the trend of a time series.

(ii) The assumption of white noise process may hold for non-stationary time series as well.

(iii) The ACF of an AR process may be oscillatory in nature.

(iv) For the following AR (3) process

$$x_t = 2.1 + 1.2x_{t-1} - 0.9x_{t-2} + 0.4x_{t-3} + a_t$$

where $\{a_t\}$ is a white noise process,

$\Phi_{11} = 1.2,\ \Phi_{22} = -0.9,\ \Phi_{33} = 0.4$ and $\Phi_{kk} = 0$ for all $k \geq 4$.

(v) For an ARMA (1, 2) process, the ACF is exponentially decaying in nature for all lags.

[$4 \times 5 = 20$]

2. (a) Discuss what you understand by seasonality in a time series.

(b) Briefly describe a method for obtaining seasonal indices in a trended time series.

[$4 + 6 = 10$]

3. Examine, with derivations, if the following time series $\{x_t\}$ is stationary and invertible

$$x_t + 1.9x_{t-1} + 0.88x_{t-2} = 2 + a_t + 0.6a_{t-1}$$

where $\{a_t\} \sim WN(0,4)$. Find its mean, variance and autocovariances if these exist. Would the conclusions on the two properties of stationarity and invertibility of $\{x_t\}$ remain the same if the coefficients of x_{t-1} and a_{t-1} are interchanged? Give justifications for your answer.

[8 + 3 = 11]

4. Let $\{x_t\}$ be a normal white noise process with mean μ and variance σ^2. Consider the time series $y_t = x_t x_{t-1}$. Determine the mean and the autocovariance function of $\{y_t\}$, and check whether $\{y_t\}$ is weakly stationary. Is $\{y_t\}$ also strongly stationary? Justify your answer.

[6 + 3 = 9]

5. What are sample ACF and sample PACF? Discuss briefly how these are used in correlogram analysis for deciding which of the standard stationary time series models is appropriate for a given time series.

[4 + 5 = 10]

INDIAN STATISTICAL INSTITUTE

MID-SEMESTRAL EXAMINATION: (2015-2016)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 29.02.2016 Maximum marks: 40 Duration: 2 Hours

Note: Answer **all** questions.

Note: Throughout, $\mathbb{R}^\ell$ is the ℓ-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Note: Throughout, $\mathscr{E} = \{I; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in I}\}$ is an economy, where I is the set of agents containing m many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and ω_i are the preference and initial endowment of agent i, respectively. Suppose further that $\succ_i$ and $\sim_i$ are the strict preference and indifference relations associated with a rational preference relation $\succeq_i$ for all $i \in I$. A price is an element of $\mathbb{R}^\ell \setminus \{0\}$. Assume

$\mathscr{W}(\mathscr{E})$: the set of Walrasian equilibrium allocations of $\mathscr{E}$;

$\mathscr{C}(\mathscr{E})$: the core of $\mathscr{E}$;

$\mathscr{P}(\mathscr{E})$: the set of Pareto optimal allocations of $\mathscr{E}$.

Q1. Answer any **five** questions.

(i) Show that the demand set $D_i(p, \omega_i, \succeq_i) \neq \emptyset$ for a price $p \in \mathbb{R}^\ell_{++}$ and an upper semi-continuous rational preference $\succeq_i$.

(ii) Assume that $N = \{1, 2\}$ and $\ell = 2$. Suppose that $\succeq_i$ is represented by a utility function for $i = 1, 2$. Let

$$\begin{cases} \omega_1 = (0, 4), & U_1(x, y) = \sqrt{x} + \sqrt{y}; \\ \omega_2 = (2, 2), & U_2(x, y) = x. \end{cases}$$

Show that $((0, 6), (2, 0))$ is not an Walrasian equilibrium allocation of $\mathscr{E}$.

(iii) If $\succeq_i$ is strictly convex for all $i \in I$, then show that $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{P}(\mathscr{E})$.

(iv) Suppose that $\succeq_i$ is strictly monotone on $\mathbb{R}^\ell_+$ such that everything in $\mathbb{R}^\ell_{++}$ is strictly preferred to anything on the boundary of $\mathbb{R}^\ell_+$ and $p \cdot \omega_i = 0$. Verify whether $D_i(p, \omega_i, \succeq_i) = \emptyset$.

(v) Let $I = \{1,2\}$ and $\ell = 2$. Suppose that the preference relation $\succeq_i$ is represented by a utility function U_i for $i = 1,2$. Given that

$$\begin{cases} \omega_1 = (1,6), & U_1(x,y) = \min\{x,y\}; \\ \omega_2 = (5,0), & U_2(x,y) = \min\{x,2y\}. \end{cases}$$

Find the set of Walrasian equilibrium of $\mathscr{E}$.

(vi) If an allocation is both a Walrasian equilibrium allocation and a quasi-equilibrium allocation, then show that it is a Pareto optimal allocation. $[5 \times 5 = 25]$

Q2. Answer any **two** questions.

(i) If $\{p_k : k \geq 1\} \subseteq \mathbb{R}^\ell_{++}$ satisfies $p_k \to p \in \mathbb{R}^\ell_{++}$, then show that there exists a bounded subset M_i of $\mathbb{R}^\ell_+$ such that the demand set $D_i(p_k, \omega_i, \succeq_i) \subseteq M_i$ holds for each $i \in I$ and $k \geq 1$.

(ii) Let $B_i(p)$ denote the budget set of agent $i \in I$ for a given price p. Suppose that $x \in \mathbb{R}^\ell_{++} \cap B_i(p)$ and $p \cdot y \geq p \cdot \omega_i$ for all $y \in \mathbb{R}^\ell_+$ satisfying $y \succ_i x$. If $\succeq_i$ is continuous and strictly monotone, then show that $p \in \mathbb{R}^\ell_{++}$.

(iii) Show that $\mathscr{C}(\mathscr{E})$ is a compact set. $[2 \times 7.5 = 15]$

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2015-16

MS (QE) I YEAR

Econometric Methods I

Date: 2 March 2015 | Maximum Marks: 100 | Duration: 3 hours

[Note: Answer question no. 1 and any three from the rest of the questions]

1. Data on three-variable linear regression problem $y = b_1+b_2x_2+b_3x_3+e$ yield the following results:

$$X'X = \begin{bmatrix} 33 & 0 & 0 \\ 0 & 40 & 20 \\ 0 & 20 & 60 \end{bmatrix}, X'y = \begin{bmatrix} 132 \\ 24 \\ 92 \end{bmatrix} \text{ and } \Sigma(y-\bar{y})^2 = 150.$$

(a) What is the sample size?
(b) Write down the normal equations and solve for the regression coefficients.
(c) Estimate the standard error of b_2 and test the hypothesis that b_2 is zero.
(d) Compute R^2 and interpret it. Also interpret the values of the regression coefficients.
(e) Predict the value of y given $x_2 = -4$ and $x_3 = 2$.
(f) Comment on the possibilities of any of the regressors being dummy variable.

[1+9+8+6+2+2=28]

2. State the assumptions of Classical Normal Linear Regression Model (CNLRM). Derive the Least Squares (LS) estimator of the regression coefficient vector. Prove that it is BLUE. Also find it's variance-covariance matrix. Give an unbiased estimator of the variance of the regression error and prove that it is unbiased. [4+4+6+4+6=24]

3. (i) Define Coefficient of Determination (R^2).

(ii) If $r_{yj} = \rho$ for all i = 2, 3, …, K, then prove that $R^2 = \{(K-1)\rho^2\}/\{1+(K-2)\rho^2\}$ for K > 1.

(iii) In the linear regression model $y=b_1+b_2x_2+b_3x_3+e$, simplify R^2 to $\frac{r_{y2}^2+r_{y3}^2-2r_{y2}r_{y3}r_{23}}{1-r_{23}^2}$, where r_{yj} is the simple correlation coefficient between y and x_j, j = 2, 3, and r_{23} is the simple correlation coefficient between x_2 and x_3. [2+10+12=24]

4. In a CNLRM set up with $y = X\beta + e$ and $Var(e_t^2) = \sigma^2$ for all t, derive ML estimators $\tilde{\beta}$ and $\tilde{\sigma}^2$ for β and σ^2 respectively. Prove that $T\tilde{\sigma}^2/\sigma^2 \sim \chi^2_{T-K}$. Also show that $\tilde{\beta}$ and $\tilde{\sigma}^2$ are independent. Find an interval estimate of a linear combination of βs. [5+8+4+5=24]

5. Comment of the following statements giving proof if true or counter example if false:

   i. If a variable x is uncorrelated with z, the addition of z to a regression in which x is used as an independent variable will not change either the coefficient of x or the standard error of the coefficient.

   ii. Consider three variables x_1, x_2 and x_3. If r_{12} and r_{23} are significantly close to +1 or -1, then r_{13} will have the sign of the product of r_{12} and r_{23}.

   iii. If there is no intercept in the regression model, the LS regression residuals will not sum to zero. [15+4+5=24]

6. Write short notes on any three of the following:

   (a) Detection of Outlying Observations by Dummy Variables.

   (b) Prediction by Dummy Variables

   (c) Partial correlation coefficient.

   (d) Problems with Economic Data. [8×3=24]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2015-16

MS (QE) I YEAR

Econometric Methods I

Date: 20 April 2016 Maximum Marks: 100 Duration: 3 hours

Note: Answer question 1 and any **three** from the rest of the questions]

1. Suppose you have the following observations on x and y.

x	2	3	1	5	9
y	4	7	3	9	17

(i) Obtain the OLS estimate $\hat{b}$ of the regression coefficient b of the linear equation y = a + bx + e. Find also the standard error of $\hat{b}$.

(ii) If the variance matrix for the disturbances underlying the data is

$$Var(e) = \sigma^2.diag\{0.10, 0.05, 0.20, 0.30, 0.15\},$$

then calculate the GLS estimate $\hat{b}_G$ of the regression coefficient b and its standard error.

(iii) Assuming that the true model is heteroscedastic as in (ii), compute the variance of $\hat{b}$.

(iv) Assuming that the true model is homoscedastic, compute the variance of $\hat{b}_G$. [8+10+10+12=40]

2. Consider the following hypothetical system of simultaneous equations in which the Y variables are endogenous and X variables are predetermined.

$$\begin{aligned}
&Y_{1t} - \beta_{10} && - \beta_{12} Y_{2t} - \beta_{13} Y_{3t} - \gamma_{11} X_{1t} && = u_{1t} \\
&Y_{2t} - \beta_{20} && - \beta_{23} Y_{3t} - \gamma_{21} X_{1t} - \gamma_{22} X_{2t} && = u_{2t} \\
&Y_{3t} - \beta_{30} - \beta_{31} Y_{1t} && - \gamma_{31} X_{1t} - \gamma_{32} X_{2t} && = u_{3t} \\
&Y_{4t} - \beta_{40} - \beta_{41} Y_{1t} - \beta_{42} Y_{2t} && - \gamma_{43} X_{3t} && = u_{4t}
\end{aligned}$$

Examine the Rank and Order conditions of identifiability of each of the above equations. [20]

3. Suppose, in a CLRM set up $y = X\beta + e$, a new observation x_0 is available. Define a BLUE of $x_0'\beta$ and prove it. What will happen if you want to predict the value of y corresponding to x_0? [20]

4. Define "perfect" and "absence of" multicollinearity and discuss their consequences. What is meant by the term multicollinearity? Suggest some strategies to solve the problem of multicollinearity. [8+2+10=20]

5. Outline the salient features that distinguish each of the following estimation methods in the context of simultaneous equations model: (a) OLS, (b) ILS, (c) 2SLS and (d) LIML. [20]

6. Discuss and compare different estimation procedures in a regression model with first order autocorrelated errors. [20]

7. Write short notes on any two of the following:
   (a) Interaction terms in regression and use of dummy variables.
   (b) Durbin-Watson Test [10×2=20]
   (c) IV estimation in Errors-in-Variables Models.

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
SECOND SEMESTER EXAMINATION: 2015-16
M.S. (Q.E.) 1st Year
Time Series Analysis and Forecasting

Date: 23.04.2016 Maximum Marks: 100 Time: 3 hours

*[Answer any **FIVE** questions. Marks allotted to each question are given in parentheses.]*

1. (a) Derive the conditions for weak stationarity of a time series from the definition of strong stationarity. Also obtain the conditions for stationarity of an AR (2) process in terms of its parameters.

   (b) Suppose $X_t \sim \text{AR}(1)$ with root α and $Y_t \sim \text{AR}(1)$ with root $-\alpha$ and X_t and Y_t are independent. Further, variances of the white noise terms of these two processes are the same. Show that Z_t defined as $Z_t = X_t + Y_t$ follows a special AR (2) process where the term with the first lag is absent.

   $[\overline{6+6}+8 = 20]$

2. (a) Find the h-step ahead forecast of $\{X_t\}$ where $\{X_t\}$ is given by
   $$X_t = \theta + \Phi X_{t-1} + a_t, \quad a_t \sim WN(0, \sigma^2) \text{ and } |\Phi| < 1.$$
   Further show that as $h \to \infty$, the variance of the resulting forecast error becomes $\sigma^2 / (1-\Phi^2)$.

   (b) Discuss how out-of-sample forecasts are obtained by rolling window method. Does rolling window method generate more efficient forecast than those of recursive window method? Give justifications for your answer.

   $[\overline{6+4}+\overline{6+4} = 20]$

3. (a) Discuss how the ADF test may be used to conclude that the underlying trend of a time series is DSP or TSP or both.

   (b) Describe briefly the KPSS test. Also bring out the distinctive features of this as well as the ADF tests.

   $[10+\overline{6+4} = = 20]$

4. (a) Let $\{X_t\}$ be a stationary time series with mean zero, and $\{Y_t\}$ be a new time series defined as $Y_t = a + bt + s_t + X_t$ where s_t is a seasonal component with period 12 and a and b are constants. Check if $\Delta\Delta_{12} Y_t$ is stationary or not.

(b) Discuss the nature of unit roots of a quarterly time series.

(c) Describe the HEGY test for detecting the presence of seasonal and non-seasonal unit roots in a quarterly time series data.

[5+5+10 = 20]

5. (a) Describe how the presence of unit roots in a time series can be appropriately tested when the series is assumed to have a structural break at a known time point.

(b) Describe the Bai-Perron procedure for testing the presence of multiple structural breaks in a time series.

[10 + 10 = 20]

6. (a) State and prove the standard properties of spectral density function of a stationary time series.

(b) Suppose that a time series $\{X_t\}$ is represented by

$$X_t = \sqrt{2}\sum_{j=1}^{q} \sigma_j \cos(\lambda_j t - \gamma_j)$$

where $\gamma_1, \gamma_2, \ldots, \gamma_q$ are independent random variables, each being uniformly distributed in the interval $[0, 2\Pi]$, and σ_j and $\lambda_j, j = 1, 2, \ldots, q$, are constants. Show that $\{X_t\}$ is a weakly stationary process.

[12+ 8= 20]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2015-16

Course Name: M.S. (QE) I Year & M.S. (QE) II Year

Subject Name: Environmental Economics

Date: April 26, 2016 **Maximum Marks: 60** **Duration: 3 hours**

A. Answer *any four* of the following: 5x4 = 20

1. A builder proposes a skyscraper that would block sunlight to the neighboring houses. The building would have net benefits to the builder of Rs.100,000.00. The neighbors, who use some solar heating, would face reduced property values and increased heating costs totaling Rs.80,000.00.
   (a) The law clearly stipulates that the neighbors have the right to solar access. Is a welfare-improving exchange possible? What do you expect the outcome to be?
   (b) Suppose the neighbors' association hires an attorney to negotiate at a fee of Rs.25,000.00. Is a welfare-improving exchange possible? What do you expect the outcome to be?
   (c) How, if at all, would the outcome be different if the builder had the right to construct the skyscraper, even if it blocked solar access?
   (d) Now suppose that the builder has the rights, and the costs of the lawyer (still Rs.25,000.00) belong to the neighbors. Is a welfare-improving exchange possible? What do you expect the outcome to be?
   (e) Compare and comment on the outcomes of cases (a) – (d). [1+1+1+1+1]

2. Consider a simple economy with one consumer who maximizes net utility from driving. The benefit from driving is (μs), where s is the size of the car, and $\mu > 0$ is a constant. The cost to the driver is (βs^2), and the damage to the road is (λs^3), ($\beta > 0$ and $\lambda > 0$ are constants).
   (a) What size of car is chosen by this driver in the absence of any regulation?
   (b) What is the socially optimal size of the car?
   (c) Compare these two and interpret the differences.
   (d) Design a toll system that induces the driver to choose the socially optimal car size. [1+1+1+2]

3. Suppose the newly elected member of the Mayor Council in charge of municipal solid waste management in Kolkata is approaching you for your expert opinion on the two alternative solutions to the following problem:
   (a) In the local community there is a strong social norm for not dumping household waste in the neighbor's doorstep. When someone is caught dumping solid waste by violating this norm he is compelled to sweep the community pavements in

broad daylight and in terms of opportunity cost of time, the length of pavement to be cleaned is proportional to the cost of transporting the illegally disposed waste to the landfill.

(b) The new member of the Mayor Council thinks that this informal arrangement is not effective. To ensure efficient outcome he wants to install a guard in the locality to keep eye on proper waste disposal and in case of detection of any illegal dumping wants to impose a flat fine of Rs.100.00.

Compare option (a) and (b) critically and recommend the one that is more likely to be effective. [5]

4. Consider two thermal power plants, each emitting 20 units of SO_2 into the environment making a total of 40 units in the region. The Pollution Control Board sets a standard of 20 units for the region. The abatement cost function of the polluters are as follows:
Plant I: $C_1 = 10 + 0.75(SO_2)^2$, Plant II: $C_2 = 5 + 0.5(SO_2)^2$.
   (a) Suppose the PCB allocates abatement responsibility equally such that each plant must abate 10 units of SO_2. What will be the total abatement cost of the region?
   (b) If the government, following the advice of the PCB, imposes an emission fee of Rs.16.00 per unit of SO_2 emission beyond the allowable limit of 10 units per plant how much of pollution each plant will abate?
   (c) If instead, the government used a tradable pollution permit system, what permit price would achieve a cost-effective allocation of abatement?
   (d) Compare the "allocation of abatement" suggested by options (a), (b) & (c). Which one would you prefer from the stand point of economic efficiency? Explain. [1+1+1+2]

5. For each of the following, does the revenue recycling effect tend to increase, decrease, or have no effect on optimal emissions? Also, for each does the tax-interaction tend to increase, decrease, or have no effect on optimal emissions? Finally, for each effect, let E solve MC (E) = MD (E). Is optimal emission greater than, less than, or equal to E, or is the answer ambiguous?
   (a) An emissions standard for SPM (Suspended Particulate Matter);
   (b) A gasoline tax to reduce SPM emissions;
   (c) A permit market on SPM where the permits are initially given away free of charge. [1+2+2]

B. <u>Answer *any five* of the following:</u> 6x5 = 30

6. Two companies require identical skills and training from their workers. Both employ 10,000 people. On average, Safety First has one worker fatality per year, while Safety

Second has two worker fatalities per year. Jobs at Safety First pay Rs.50,000.00/year, while jobs at Safety Second pay Rs.50,500.00/year.

(a) Why do these jobs with identical requirements pay different salaries? Justify on the basis of the information presented here.

(b) What is the risk for a worker of a fatal accident at each company? What is the pay premium associated with the higher risk?

(c) The value of a statistical life is the difference in wage divided by the difference in risk. What is the value of a statistical life for workers with these skills and training? [2+2+2]

7. The Forest Service is deciding how to allocate its recreational funding. It can increase fishing opportunities in an area (for instance, by stocking), or it can build more hiking trails. It presents several options in a survey:

Option	Fish Stocked (Hundreds of Fish)	Hiking Trails (Miles)	Cost (Rs.)
A	1	9	50
B	3	2	100
C	3	9	125

(a) In the first round, the survey respondent was asked to rank each option without regard to cost; in the next round, the survey respondent was given the cost information and then asked to rank the options while considering those costs.

(b) In the first round, one respondent ranked alternative C as best, and ranked A as tied with B. Plot the different combinations on a graph, with Fish Stocked on the horizontal axis and Miles of Hiking Trails on the vertical axis. Draw indifference curves that reflect this set of preferences. Do these indifference curves reflect more is better, slope downward, and not cross, as indifference curves should?

(c) Based on the comparison of alternatives A and B, what is this respondent's trade-off between fish stocking and hiking trails? In other words, if he were to give up one hundred fish stocked, about how many miles of hiking trails would he need to get in compensation? What does this comparison tell you about how this respondent values fish stocking compared to hiking trails?

(d) When the respondent was presented with the cost information, he changed his ranking so that A was best, C was second-best, and B was least preferred. Does the fact that A and B are no longer equally preferred mean that the respondent is inconsistent in his preferences? Why or why not?

(e) Compare alternatives A and C. What can you say about how much the respondent is willing to pay for three hundred additional fish stocked when there are already one hundred stocked? [1+1+1+1+2]

8. If a consumer buys non-recycled paper when recycled paper is available at the same price, the consumer reveals that she prefers the non-recycled paper; in addition, she would be worse off if non-recycled paper were not available.

(a) Suppose the price of recycled paper is higher than the price of non-recycled paper. A consumer buys the non-recycled paper. Would requiring that all paper be recycled make her worse off? Why or why not?

(b) Suppose the price of recycled paper is higher than the price of non-recycled paper. A consumer buys the recycled paper anyway. Is this consumer irrational? Why or why not? Is she made worse off by the presence of non-recycled paper? Why or why not?

(c) A university is considering requiring the use of recycled paper, although it is more expensive than recycled paper. Students are either like the consumer in (a), and would buy cheaper non-recycled paper if it is available, or like the consumer in (b), who buy more expensive recycled paper. What is the total effect on student well-being if recycled paper is required—that is, does consumers' surplus increase or decrease?

(d) What is the effect of greater availability of choices on consumer well-being? That is, in economic modeling, do additional choices make people better off or worse off? Why? [1+1+2+2]

9. (a) SB resides in Delhi and she plans a trip to Keoladeo National Park (KNP) which is 100 kilometers away from Delhi. The entry fee in the park is Rs.100.00 and it takes 3 hours to travel from Delhi to KNP. The transport cost is Rs.10.00 per kilometer. She likes to spend 6 hours inside the park. SB works in an MNC for Rs.2, 92,000.00 per annum. Calculate her revealed willingness to pay for the visit. (In your calculation if you make any assumption, then specify it clearly).

(b) Suppose you are interested in applying Hedonic Price Theory to assess the effect of environmental degradation on house rent where in the presence of rent control only the information on nominal rent is observable and no information is available on the lump-sum down payment. Suggest a method to arrive at the correct estimation of actual rent. [3+3]

10. Suppose a consumer cares for both the composite good X and the health stock H, where H is sensitive to both the amount of medical care (M) and the quality of environment (α). Both X and M are marketed goods with unit price p_x and p_m, respectively. The consumer enjoys earned income $w.T_w$, w being the market wage rate and asset income A. Moreover, she makes her optimal choice subject to both the income and time budget constraints. Time (T) is spent for work (T_w), for consumption of X (T_x), for consumption of M (Tm) and may be in illness (T_L).

(a) Formulate the model.

(b) Derive an expression for consumers' marginal willingness to pay (WTP) for an improvement in α.

(c) What problem would you encounter in estimating this expression empirically?
(d) Suggest a way out. [1+2+1+2]

11. Answer the following questions with reference to a contingent valuation survey:
(a) Explain the notion of Closed-ended-referendum.
(b) How would you estimate the average maximum willingness to pay from the bid responses collected in this case?
(c) Comment on the importance of randomization of the asked bid in applying this estimation technique successfully.
(d) What are the commonly encountered biases from which your estimation may suffer?
[1+2+1+2]

C. <u>Answer *any two* of the following</u>: 2x5 = 10

12. Consider two countries A & B between which trade has opened up involving two goods X & Y, where X is a dirty good and Y is a clean good and country A has (natural) comparative advantage in X whereas country B has the same in Y. Assume that the pollution externality for dirty good X originates during the process of production.
(a) Decompose the effect of trade on the production of X in country A into scale, composition and technique effect.
(b) Show how your answer will change if the pollution externality is experienced at the level of consumption of X instead. [3+2]

13. Differences in environmental regulations entail a pattern of international specialization in which dirty industries have a tendency to gravitate towards the South.
(a) How do you argue theoretically in favor of this claim?
(b) Do empirical findings corroborate this claim? Suggest a suitable method to examine this claim. [3+2]

14. Consider the statement that "the transboundary pollution is difficult to control" and briefly comment on the suitability of each one of the following lines of action to control the problem.
(a) Victim country levying tariffs on imports from the country generating the pollution.
(b) Linking international agreements between the two countries on totally unrelated issues to control the pollution problem.
(c) Forge international agreements on pollution control like different protocols.
[1½+1½+2]

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2014-2015
M.S. (Q.E.) I, II Years and M. Math. II Year
Game Theory II

Date: 29.04.2016 Maximum Marks: 100 Time: 3 hours

Note: Answer Parts (A) and (B) in separate answer scripts. Clearly explain the symbols you use and state all the assumptions you need for any derivation. The paper carries 110 marks. You may attempt any part of any question. The maximum you can score is 100.

A

1. Define a voting game. Establish a necessary and sufficient condition for non-emptiness of the core of such a game in terms of a blocker. (10)

2. Consider the problem of allocating costs for providing some service to a set of customers. Assume that the following conditions hold: (a) all non-users of the service do not pay for it but all users should be charged equally; (b) the total cost of using the service is the sum of capital and operating costs, and (c) the service provider will recover the entire cost from the customers. Clearly demonstrate that there is a unique solution to this cost recovery game. (12)

3. Identify the relation between the nucleolus and the kernel of a coalition form game for the grand coalition structure by giving necessary preliminaries. (10)

4. State and prove the Bondareva-Shapley theorem by defining all necessary concepts. (16)

5. Formulate a bankruptcy situation, where there are $n > 1$ claims against an estate and the sum of the claims exceeds the estate's worth, as a coalition form game. Show that such a game is convex. (10)

P.T.O

6. Let $N = \{A_1, A_2, A_3\}$ be a set of 3 firms producing a homogenous output whose price function is $10 - x_{A_1} - x_{A_2} - x_{A_3}$, x_{A_i} being the output of firm A_i. The maximum output a firm can produce is 3. The cost function of firm A_i is $(1 + x_{A_i})$. The worth of any non-empty coalition $S \in 2^N$ is defined as $v(S) = \max_{x_{A_i}, A_i \in S} \min_{x_{A_j}, A_j \notin S} \sum_{A_i \in S} x_{A_i}(10 - x_{A_1} - x_{A_2} - x_{A_3}) - \left(\sum_{A_i \in S}(1 + x_{A_i})\right)$. Determine the numerical value of worth of each non-empty coalition. Also identify the set of core elements. (20)

7. (a) Show that the solution to the two-person bargaining problem satisfying the Nash axioms exists and is unique. (7)

   (b) When do you say that a marriage matching is stable? (5)

B

1.(a) Show that for a weighted majority game, the problem of finding the number of swings for a particular player can be reduced to SUBSET SUM problem. Hence, determine the time complexity of finding the number of swings for a particular player.

(b) Suppose that the Gale-Shapley "men propose" algorithm is modified whereby in a round each unmatched man proposes to the second most preferred woman in his current list. Will this modified algorithm also result in a stable matching? Explain your answer.

© Assuming that the "men propose" strategy results in the best partners for men show that it results in the worst partners for women. (8+6+6)

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2015-2016)

MSQE I

Microeconomic Theory II

Date: 02. 05. 2016 Maximum Marks: 60 Duration: 2 hrs 30 minutes

Note: Answer Group A and Group B in **separate** answer scripts.

Group-A

Note: Answer **all** questions.

Q1. Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{a_1, a_2\}$, $0 < a_1 < a_2 < \infty$ and $Pr(\theta = a_2) = \frac{1}{2}$. The cost of education is $c(e, \theta) = \frac{e^2}{2\theta}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type θ who chooses education level e and receives wage w. Assume that both worker types earn zero by staying home, that is $r(a_1) = r(a_2) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of e^* for which we can have a separating equilibrium. Justify your answer. [8]

(b) Consider the belief function

$$\mu^b(e) = \frac{e - \max\left\{0, e - \sqrt{2a_1(a_2 - a_1)}\right\}}{\sqrt{2a_1(a_2 - a_1)}}$$

for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer. [10]

Q2. In the two-stage screening game with unknown worker types, show that in any equilibrium (separating or pooling), firms earn zero profits. [7]

Q3. Consider the landlord-tenant market model where the effort level of the tenant is neither observable nor verifiable. Derive the second best contract. [15]

Group-B

Note: Answer **all** questions.

Note: Let $\mathbb{R}^\ell$ denote the ℓ-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Suppose that $\mathscr{E}_r$ is the r-fold replicated economy of $\mathscr{E}$ and $\succeq_i$ is continuous, convex and strictly monotone for all $i \in N$, where $N = \{1, \cdots, n\}$ is the set of agents of $\mathscr{E}$. Let

$$x = (x_{11}, \cdots, x_{1r}, x_{21}, \cdots, x_{2r}, \cdots, x_{n1}, \cdots, x_{nr})$$

be a Walrasian allocation of $\mathscr{E}_r$. Show that the allocation $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ of $\mathscr{E}$, defined by

$$\hat{x}_i = \frac{1}{r}\sum_{j=1}^{r} x_{ij},$$

is a Walrasian allocation of $\mathscr{E}$. [6]

Q2. Let $\succeq$ be a preference relation over $\mathbb{R}^\ell_+$. Prove or disprove the following statements. In case you wish to disprove the statement, give an example to show that the statement is incorrect.

(a) Assume $\succeq$ is continuous and $x \succ \omega$ implies $p \cdot x \geq p \cdot \omega$ for $p, x, \omega \in \mathbb{R}^\ell_+$. If $p \cdot \omega > 0$ and $x \succ \omega$, then $p \cdot x > p \cdot \omega$. [4]

(b) Assume $\succeq$ is continuous and $x \succ \omega$ implies $p \cdot x \geq p \cdot \omega$ for $p, x, \omega \in \mathbb{R}^\ell_+$. If $p \cdot \omega = 0$ and $x \succ \omega$, then $p \cdot x > p \cdot \omega$. [4]

Q3. Consider an economy with two agents and two commodities. The utilities and initial endowments of two agents are given by

$$\begin{cases} U_1(x, y) = x + y; \\ U_2(x, y) = \min\{x, y\}, \end{cases}$$

and

$$\begin{cases} \omega_1 = (2, 1); \\ \omega_2 = (1, 2). \end{cases}$$

Find the set of Pareto optimal and weak Pareto optimal allocations. [6]

Indian Statistical Institute
MSQE I and II
Economic Development I
Semestral Examination

Maximum Marks 60

Time 3 hours

Date 4.5.2016

Answer question 1 and any two from the rest.

1. Consider risk neutral borrowers and lenders. The lenders are perfectly competitive and earn zero profits in equilibrium. Each borrower has two alternative projects, a risky project and a safe project. Each project requires one unit of capital, the opportunity cost of which is $\rho > 0$ to the lender. A safe project yields an income Y_s with certainty. A risky project yields Y_r with probability p and zero with probability $(1-p)$. We assume that

$$Y_r > Y_s = \rho > pY_r$$

The lender offers a contract specifying the gross rate of interest r which the borrower may or may not accept. The reservation payoff of each borrower is zero. There is limited liability so that in case the yield is zero, the borrower pays nothing. Finally assume that the lender cannot influence the project choice of the borrower.

(a) What is the socially optimal project choice?
(b) Suppose the lender lends on the basis of individual liability. Show that in equilibrium the borrower will always choose the risky project.
(c) Now suppose the lender lends on the basis of joint liability to a group of two borrowers. In particular, a borrower is liable to pay not only his own interest but also the interest of his defaulting partner in case he has enough funds to do so. Find the equilibrium project choice when choices are made non-cooperatively.
(d) Suppose in (c) above project choices are cooperatively made. Show that the equilibrium outcome is socially optimal.

[2+3+7+8 = 20]

2. Two political parties are engaged in a political competition. There are two types of voters: non-strategic voters who vote on the basis of their perception about the realized

P.T.O

state of the economy and strategic voters who vote for personal benefits and for the party they choose to be affiliated with. Show that the higher is the proportion of the strategic voters, the lower is the level of effort expended by the ruling party.

[20]

3. Show how reciprocity can be a basis of informal insurance. Under what circumstances would this informal insurance differ from the first best?

[17+3 = 20]

4. A trader-cum-money lender enters into an interlinked contract with a farmer according to which the farmer takes a production loan from the trader-cum-money lender at a contracted rate of interest and sells the output to him at a contracted price. Show that the optimal contract exhibits a subsidized rate of interest and will be Pareto efficient. Give the economic reason behind the subsidization of the rate of interest.

[15+5 = 20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2015-2016)
MS (Q.E.) I Year
Macroeconomics I

Date: 06.05.2016 Maximum Marks 60 Duration 3 hours

Group A
Answer both questions. Each question carries 15 points.

1. Consider the Arrow Learning by Doing model, with the restriction that $\alpha = 1$ and $\delta = 0$.
(a) Show that this model cannot accommodate a balanced growth path unless the rate of population growth is zero. (4)
(b) What will be the only value of the interest rate that the model will permit? (4)
(c) Calculate the balanced growth rate for the model for the utility function $u(c) = \ln c$. (4)
(d) Show the solution in a diagram using the concepts of demand and supply rates of growth. [Do not derive the demand rate of growth function. Simply assume it to be the one corresponding to the utility function $u(c) = \ln c$.] (3)

2. Solve for the equilibrium value of s_R, the share of the labour force engaged in the research sector in the Romer-Jones model. In answering this question, you need not work out the entire model. Instead you may assume the (correct) solutions for w_R, w_Y , P_A and π. Also, $g_{A=\frac{\tau L_A}{A}}$, i.e. research workers do not internalise either the "standing on the shoulder" effect or the "stepping on the toe" effect. (15)

Group B
Answer all questions

1. Show that in an OLG model, introducing a 'pay as you go' pension scheme will reduce the steady state per capita capital stock.
What would be the effect of such a pension scheme on the steady state welfare? (15)

P.T.O

2. a) In the Blanchard-Kiyotaki model; show that a coordinated reduction in all prices and wages, beginning from a situation of monopolistically competitive equilibrium, will raise real profits and also the utility.

b) Consider an economy with the representative agent having the utility function:

$$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}, 0 < \alpha, \gamma < 1$$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0 < \rho < 1$ and c_i is the consumption of the i^{th} variety. L is the labour supply, P is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. M is the money balances (and suppose M_0 is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where w is the money wage rate and π is the economy wide profits and T is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \leq F$$
$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

Y_i is the output of i^{th} variety and L_i is the labour employed in the production of the i^{th} variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed n).

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where G takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and g_i is the government consumption of the i^{th} variety.

(ii) What would be the effect of such an increase in government expenditure on P?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ($M = M_0$).]

(10+5)