INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2013-2014

MS (Q.E.) and MStat II Year

Econometric Methods II

Date: 06/09/2013 Maximum Marks 35 Duration 2 hours

All notations are self-explanatory. You can answer any part of any question.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic but endogenous. Assume that all other CLRM (ideal conditions) assumptions hold. Let Z be a matrix of instruments for X. Number of columns in Z may be more than that of X.
   a. Find the GMM estimator of β. Show that the GMM estimator is consistent.
   b. Find the GMM estimator of β when data is heteroscedastic.

   [(5+5) +5=15]

2. Suppose we have a model $y_i = \theta \exp(x_i'\beta) + \epsilon_i$, where $E[\epsilon_i|x_i] = 0$, for $i = 1, \ldots, n, where\ \beta$ is a $p \times 1$ vector of parameters and θ is a scalar parameter. Assume that the regressors, x_i, do not include an intercept.
   a. Suggest a $p \times 1$ moment function $g_1(y_i, x_i, \theta, \beta)$ such that $E[g_1(y_i, x_i, \theta, \beta)] = 0$.
   b. Suggest another $p \times 1$ moment function $g_2(y_i, x_i, \theta, \beta)$ such that $E[g_2(y_i, x_i, \theta, \beta)] = 0$.
   c. Show how to combine moment functions in (a) and (b) to get the most efficient estimator for $\delta = [\theta, \beta']'$, say $\hat{\delta}$. [5+5+10=20]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2013-2014
M.S. (Q.E.), 2nd Year
Econometric Applications I

Date: 02 September 2013 Maximum marks: 100 Duration: 3 hours

[Answer question no. 1 and any **three** from the rest of the questions.]

1. Suppose in a population of 30,000 people there are 5,000 people whose incomes (X) are below a certain level of income C. The mean incomes and the Lorenz ratios of the groups of people with incomes less than C and not less than C are given in the following table.

Population Groups	Mean Income (Rs.)	Lorenz Ratio
$X < C$	1000	0.28
$X \geq C$	3000	0.44

Find the overall Lorenz ratio. [25]

[You should derive the associated results]

2. Define Pareto Distribution. State and prove its properties. [2+23=25]

3. Define three-parameter lognormal distribution. Derive its quantiles, coefficient of variation, measures of skewness and kurtosis and Lorenz Ratio. Also discuss some estimation procedures for this distribution. [2+15+8=25]

4. What do you mean by concentration of firms in an industry? Write down the axioms proposed by Hall and Tideman in this context. Define Absolute Concentration Ratio (ACR), Herfindahl-Hirschman index (HHI) and Hall and Tideman index (HTI). Prove that HHI and HTI satisfy all these axioms. How is the measure of concentration different from a measure of inequality? [2+5+3+10+5=25]

5. Write down the important steps in deriving Atkinson's measures of inequality based on the Social welfare Function Approach. How can one interpret the unknown parameter in the measure? [20+5=25]

6. Write short notes on any two of the following:
   (a) Desirable properties for a measure of inequality.
   (b) Properties of Lorenz Curve of Lognormal distribution.
   (c) Kapteyn's Law of Proportionate Effect and it's modification due to Kalecki.
   (d) Positive Measures of Inequality [12½ + 12½ = 25]

INDIAN STATISTICAL INSTITUTE

Mid-semestar Examination: (2013-2014)

MS(QE) II

Game Theory II

Date: 09. 09. 2013 Maximum Marks: 40 Duration: 3 hrs.

(1) Consider the non-excludible pure public goods problem under incomplete information. Answer the following questions.
   (a) Define efficiency of a decision. **(2)**
   (b) Define dominant strategy incentive compatibility and feasibility of a mechanism. **(4+2=6)**
   (c) Find a mechanism which, given efficiency of decision, satisfies dominant strategy incentive compatibility and feasibility. Justify your answer. **(10)**

(2) Let $N = \{1, \ldots, n\}$ be the finite set of agents and $A = \{x, y, z \ldots\}$ be the finite set of alternatives with $n \geq 2$ and $|A| \geq 3$. Assume that preferences are strict (that is, there is no indifference). Answer the following questions.
   (a) Define a social choice function. **(2)**
   (b) Define unrestricted domain, full range property and unanimity. **(2+2+2=6)**
   (c) Show that if a social choice function satisfies unrestricted domain and unanimity, then it also satisfies the full range property. **(4)**
   (d) Define strategyproofness. **(2)**
   (e) Show that if a social choice function satisfies unrestricted domain, strategyproofness and full range property, then it also satisfies unanimity. **(8)**

Supplementary Examination

INDIAN STATISTICAL INSTITUTE

Mid-semestar Examination: (2013-2014)

MS(QE) II

Game Theory II

Date: 10. 10. 2013 Maximum Marks: 40 Duration: 3 hrs.

(1) Consider the non-excludible pure public goods problem under incomplete information. Show that a mechanism satisfies efficiency of decision and dominant strategy incentive compatibility if and only if it is a Vickrey-Clarke-Groves (or VCG) mechanism. **(20)**

(2) State and prove the Gibbard-Satterthwaite theorem for the two agent case by giving all the relevant definitions. **(20)**

Mid Term Examination September 2013

MSQE II

Economic Development

Time 2 hours

Maximum Marks: 40

Date : 11.9.13

Answer both questions.

1. Developing a model of speculative attack on an over-valued exchange rate show that a small distortion in observing the state of the economy leads to a unique equilibrium involving coordination of the speculators. Do we get uniqueness if the distortion is not small?

[15+5=20]

2. Discuss the Galor-Zeira model of growth and inequality. How does the conclusion of the model change if the borrowing rate is equal to the lending rate?

[15+5=20]

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: (2013-2014)
MS (Q.E.) II Year
International Economics I

Date: 13.09.13 Maximum Marks 40 Duration 3 hours

Group A

Answer all questions

1. Examine the validity of any two of the following statements in the context of a 2x3 specific factor model.
   a) If all the factors of production grow at equal rate, factor prices also change at the same rate.
   b) Walras law is always valid.
   c) Production possibility curve slopes negatively but is not convex to the origin

   (2 x 5 =10)

2. Answer any two of the followings, considering 2x2 HOS model.
   a) What do you mean by factor intensity reversal? Is Rybczynski theorem valid in this case- Explain.
   b) Suppose that labour endowment is infinitely large. Discuss how you can explain trade between two countries.
   c) State and prove the factor price equalization theorem and discuss its implications.

   (2 x 5 = 10)

Group B

Answer all questions

1. Show that in a two agent setting, with the equilibrium being Walras' stable, the recipient of a transfer is always a gainer.

   (10)

2. Considering a 2 country, 2 commodity trading world, show that imposing an ad- valorem export tax is the same as imposing an ad-valorem import tariff when the government

redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax. Report if you find anything interesting in the result.

[Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2013-2014)
MS (Q.E.) II Year
Macroeconomics II

Date: 16.09.13 Maximum Marks 40 Duration 3 hours

Use separate booklets for group A & B

Group A

Answer any two questions

1. If an asset i is bought currently at a price p_i and *sold later* at a *price* q_i, the rate of return (r_i) is given by: $r_i = (q_i - p_i)/p_i$. This rate equals the rate of return on a riskless asset (r_f) in a world of certainty in which case the asset's price is given by:

(A) $$p_i = \frac{q_i}{1+r_f}$$

In an uncertain world, however, q_i is random and the CAPM says that, if $E(q_i)$ is the mean value of the asset's later price, $E(r_M)$ is the mean (rate of) return on an (efficient) market portfolio M with variance σ_M^2, and r_f is the rate of return of a risk-free asset, then

(B) $$\frac{E(q_i) - p_i}{p_i} = r_f + \beta_{iM}\,[E(r_M) - r_f]$$

(**a**) Derive the expression for β_{iM} from a mean-variance (optimization) exercise in a world with a risk-free asset and interpret the relation (B).

(**b**) How does the relation (A) then get modified in an uncertain world? Can you interpret it?

[*Hint for* (b): Both the numerator and denominator will change. In particular, the denominator will have an extra term. Find that term and interpret.]

[6 + 4] = [10]

2. (a) Suppose, rates of return of two risky assets 1 and 2 are **perfectly negatively** correlated and their standard deviations are σ_1 and σ_2. Show that a portfolio P having w proportion of fund invested in asset 1 and the rest in asset 2 have a rate of return with standard deviation given by

$$\sigma_P = |w\sigma_1 - (1-w)\sigma_2|$$

Suppose, $\sigma_1 = 0.2$ and $\sigma_2 = 0.8$, and mean rates of returns of the two assets are: $\mu_1 = 0.4$ and $\mu_2 = 0.9$. Can you find a particular portfolio of these two assets which behaves exactly like a risk-free asset yielding a *particular* rate of return with certainty? Explain.

(b) Write a note on ***any one*** of the following topics:

(i) *Arbitrage pricing* theory with two factors; (ii) *Equity-premium puzzle*.

[3 + 7] = [10]

3. Consider CAPM with only risky assets. Two **frontier** portfolios P and Q will satisfy

$$Cov(r_P, r_Q) = \frac{C}{D}\left\{E(r_P) - \frac{A}{C}\right\}\left\{E(r_Q) - \frac{A}{C}\right\} + \frac{1}{C},$$

where $A = e'V^{-1}\mathbf{1}$, $B = e'V^{-1}\mathbf{1}$, $C = \mathbf{1}'V^{-1}\mathbf{1}$, $D = BC - A$, e is the vector of assets' *mean* returns, $\mathbf{1}$ is the *sum vector* and V is the *variance-covariance matrix* of the assets' returns (which is *positive definite*). **Assume this result**.

(a) Find the variance of rate of return of the portfolio P (i.e, σ_P^2) and the expected rate of return of the portfolio $ZC(P)$ $\{E(r_{zc(P)})\}$, where $ZC(P)$ is the *zero covariance* portfolio of the portfolio P.

(b) Show that for any portfolio Q (*not necessarily* a frontier portfolio),

$$E(r_Q) = E(r_{zc(P)}) + \beta_{QP}\,[E(r_P) - E(r_{zc(P)})],$$

(c) Suppose the portfolio Q mentioned in **(b)** above has the same expected rate of return as that of the frontier portfolio P: $E(r_Q) = E(r_P)$. Show that $Cov(r_P, r_Q) = \sigma_P^2$ and, **as a consequence**, the correlation coefficient of r_P and r_Q lies in $(0,1]$.

[2 + 4 + 4] = [10]

Group B

Answer all

1) a) Show that the initial period consumption in the Ramsey model is a function of the initial asset and the present discounted value of lifetime wage earnings.

b) Using that expression, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital market.

In this context discuss the problems associated with either a very low or a very high rate of interest. [3+7] = [10]

2) a) Show that all paths other that the convergent one in the Ramsey model would either violate the transversality condition or one of the other necessary conditions for an optimum.

Show that the No Ponzi condition together with optimality delivers what is required by the transversality condition .

b) Assuming a Cobb-Douglas production function, work out the transitional dynamics of the savings rate in the Ramsey model. [5+5] = [10]

Supplementary Examination

\INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: (2013-2014)
MS (Q.E.) II Year
International Economics I

Date: 21.10.13 Maximum Marks 40 Duration 3 hours

Group A

Answer all questions

1. Consider the 2x2 HOS model and then analyse the validity of the following statements.
   a) If capital stock and labour endowment grow at equal rates, then at constant product prices, level of outputs of two sectors also grow at same rate.
   b) In a small open economy, an increase in capital stock, given the labour endowment, raises the wage rate and lowers the rental rate on capital. (5x2)

2. Consider an otherwise identical 2x3 specific factor model with the following modification:
   i) Production functions in all the sectors are of fixed coefficient type.
   ii) Labour endowment varies positively with the wage rate.
   Analyse the effect of an increase in sector specific capital stock on the factor prices. (10)

Group B

Answer all questions

1. Show that a tariff on imports might end up depressing the domestic price of the importables. (10)

2. Considering a 2 country, 2 commodity trading world, show that imposing an ad-valorem export tax is the same as imposing an ad-valorem import tariff when the

PTO

government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax. Report if you find anything interesting in the result.

[Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.] (10)

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2013-2014

MS (Q.E.) II and MStat II Year

Econometric Methods II/ Econometric Methods

Date: 13.11.13 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 110 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\epsilon_i|X_i) \neq 0$ but there are available instruments Z with $E(\epsilon_i|Z_i) = 0$ and $V(\epsilon_i|Z_i) = \sigma_i^2$, where $dim(Z) > dim(X)$. We consider the GMM estimator $\hat{\beta}$ that minimizes
$G_N(\beta) = [\frac{1}{N}\sum Z_i(Y_i - X_i'\beta)]'W_N[\frac{1}{N}\sum Z_i(Y_i - X_i'\beta)]$.
   a. Show that OLS estimator of β is inconsistent.
   b. Suggest an Instrumental variable estimator for β using the entire vector of instruments.
   c. Show that the estimator suggested in (b) can be viewed as a GMM estimator as defined above.
   d. If errors are homoscedastic what choice of W_N would you use?
   e. If errors are heteroscedastic what choice of W_N would you use?
   f. State how to obtain a consistent estimate of the asymptotic variance of $\hat{\beta}$.

[5 +7+7+7+7+7=40]

2. Consider a latent variable modeled by $y_i^* = x_i'\beta + \varepsilon_i$, with $\varepsilon_i \sim N(0,1)$. Suppose we observe only $y_i = \begin{cases} 1, & if\ y_i^* < u_i \\ 0, & if\ y_i^* \geq u_i, \end{cases}$
where u_i is a known constant for i^{th} individual.
   a. Find $\Pr[y_i = 1|x_i]$.
   b. How would you estimate β consistently? Discuss in detail.
   c. Discuss why the usual R^2 is not useful for the regression of y_i on x_i. Define pseudo R^2 in this context.

[7+6+ (3+4) =20]

3. Consider the following panel data model:

$$y_{it} = \alpha_i + x_{it}'\beta + z_i'\delta + \varepsilon_{it},$$

Where the x_{it} $(k \times 1)$ are both time and individual varying regressors, the $z_i(m \times 1)$ are time invariant regressors. Let $x_i = (x_{i1}, x_{i2}, \ldots, x_{iT})'$. Assume that $E[\varepsilon_{it}|x_i, z_i, \alpha_i] = 0$, and $E[\alpha_i|x_i, z_i] = 0$. Let $\sigma_\alpha^2 = Var(\alpha_i)$, and $\sigma_\varepsilon^2 = Var(\varepsilon_{it})$.

a. Let $c_i = \alpha_i + z_i'\delta$. Find $Var(c_i)$ and compare it with σ_α^2. P.T.O.

b. Explain why estimation of the model by fixed effects transformation will lead to a larger estimated variance of the unobserved effects than if we estimate the model by random effects.
c. Show that both fixed effect estimator and the random effect estimator of β are consistent.
d. Discuss how you will test for the assumption $E[\alpha_i|x_i, z_i] = 0$. [(4+4)+6+8+8=30]

4. Let $y_t = \mu + \varepsilon_t$, $\varepsilon_t = u_t \times (\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)^{0.5}$, $u_t \sim i.i.d\ N(0,1)$, $\alpha_0 > 0, \alpha_1 \geq 0$.
   a. Compute the first and second moments for y_t (i) conditional on y_{t-1} and (ii) unconditional.
   b. Write down the likelihood function based on T number of observations on y_t.
   c. Do you feel MLE estimator of μ will be more efficient than that of OLS estimator? Give the intuitive reason.
   d. How will you test for $H_0: \alpha_1 = 0$.

[(2+4)+5+4+5=20]

Indian Statistical Institute

Economic Development I

Semestral Examination

MSQE, 2013-14

18·11·13

Maximum Marks: 60

Time 3 hours

Answer any three questions.

1. There are two agents with independently and identically distributed income each period over an infinite horizon. In particular, in each period each person's income can take only two values: a value 1000 with probability 4/5 and a value 500 with probability 1/5. The agents have an informal insurance arrangement between them based on reciprocity. Under this arrangement the agent with high-income realization makes a transfer t to the agent with low-income realization and there is no transfer if realized incomes of the two agents are the same. The per period utility function of each agent is given by

$$U(x) = 0 \text{ for } x < 600$$

$$= x \text{ for } x \geq 600$$

where x is net income after transfer. Let r be the common rate of discount.

(a) Find the set of first best transfers.

(b) Find the range of values of r such that the first best *cannot* be implemented.

(c) Suppose income is in terms of paddy, which is storable for a maximum of one period. Now, instead of having reciprocal insurance, each agent stores a part of his income if realization is high so that consumption next period may be supplemented in case income realization is low. Compare the life time utility achievable by an agent under this *savings* option with that under *the informal insurance* option.

[6+6+8=20]

2. Wheat is produced in period 0 and consumed in period 0 and period 1. There are n oligopolistic sellers in the market who can affect the market price and each of whom comes

to the market with a given stock of wheat. Apart from them there is a large number of small price taking sellers, whose total stocks are also given. The small sellers are compelled to sell their stocks in period 0 because of, say, precommitted cash requirements while the large sellers are free to distribute their sales over the two periods so as to maximize profits. Assume that there is no storage cost of holding wheat from one period to another. Demand for wheat is uniform over the two periods.

(a) Setting up an appropriate model show that the price of wheat is higher in period 1 than in period 0.

(b) What happens to the price difference between the two periods if the small sellers are able to carry over stocks from period 0 to period 1 by incurring a storage cost per unit of output? Assume for simplicity that the big sellers do not incur any storage cost.

[14+6=20]

3. Demonstrate, in terms of a growth model, that the policy of maximizing the steady state rate of growth is different from that of maximizing social welfare. Also demonstrate that unequal societies tend to grow at a slower rate when policies are chosen through majority voting.

[15+10]

4. In terms of a suitable model, demonstrate how a big push might be required to take an economy from a bad equilibrium to a good equilibrium.

[20]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2013-2014)

MS(QE) II

Game Theory II

Note: Answer all questions.

Date: 20.11.2013 Maximum Marks: 100 Duration: 3 hrs.

(1) Consider the model of regulating a monopolist and assume that the marginal cost θ of the monopolist is unknown and that it belongs to the interval $[\underline{\theta}, \overline{\theta}]$ where $0 < \underline{\theta} < \overline{\theta}$. Show that a regulatory mechanism $M(\theta) = (q(\theta), t(\theta))$ is incentive compatible if and only if

(a) $q(\theta)$ is non-increasing in θ.

(b) $\pi(\theta) = \pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(x)dx$. **(25)**

(2) Consider the bilateral trading model under incomplete information where the seller's type lies in the interval $[a_1, b_1]$ and the buyer's type lies in the interval $[a_2, b_2]$. Define the "virtual" type of the seller as $z_1(\theta_1) = \theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}$ and that of the buyer as $z_2(\theta_2) = \theta_2 - \frac{1-F_2(\theta_2)}{f_2(\theta_2)}$. Show that if a mechanism (p, x) is incentive compatible and individually rational, then

(a) $p_1(\theta_1)$ is non-increasing,

(b) $p_2(\theta_2)$ is non-decreasing and, more importantly,

(c) $U_1(b_1) + U_2(a_2) = \int_{a_2}^{b_2} \int_{a_1}^{b_1} [z_2(\theta_2) - z_1(\theta_1)]\, p(\theta_1, \theta_2) dF_1(\theta_1) dF_2(\theta_2) \geq 0.$

(30)

(3) Define the core of a coalition form game with transferable utilities. Also define a convex game. Show that for a convex game the core is non-empty. **(3+2+20=25)**

(4) Define the Shapley value of a coalition form game with transferable utilities. **(5)**

(5) "If a two-person game has a non-empty core, then the Shapley value belongs to the core". Is the statement correct? Justify your answer. **(15)**

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2013-2014)
MS (Q.E.) II Year
Macroeconomics II

Date: 29.11.2013 Maximum Marks 60 Duration 3 hours

Group A

Answer **any two** questions

1. An economy with a constant population consists of many infinitely-lived individuals. The social planner maximises the present discounted value of the lifetime utility of the representative individual of the following form:

$E_0[\sum_{t=0}^{\infty} \beta^t \{\ln c_t + \mathrm{b}\,(1 - l_t)\}]$ $\{l_t$: labour at time t ; $0 < \beta < 1\}$

subject to

(i) $c_t + k_{t+1} = y_t = z_t k_t^{\alpha} l_t^{\alpha}$ and, (ii) $\ln z_t = \mu + \ln z_{t-1} + \varepsilon_t$,

where the symbols have usual meanings and k_t, capital stock at time t , depreciates fully after one period and technology shock ($\ln z_t$) follows a random walk with drift μ.

(a) Define the value function $v(k_t, z_t)$ for this problem and explain intuitively why this function must satisfy the following condition known as the *Bellman equation*:

$$v(k_t, z_t) = \max_{c_t, l_t}[\{\ln c_t + b \ln(1 - l_t)\} + \beta E_t\{v(k_{t+1}, z_{t+1})\}].$$

Solve this equation and show that the optimal value of $l_t = \bar{l}$ (a constant) for all t and that the optimal k_{t+1} is proportional to y_t.

(b) Suppose that the economy was having, $\ln z_t = \mu + \ln z_{t-1}$, upto time T and then there is a one-time *positive* productivity shock at time T (i.e. $\varepsilon_T = \varepsilon > 0$ and $\varepsilon_t = 0 \ \forall\, t \neq T$). Derive the time path of $\ln y_t$ for all $t \geq$ T.

[7 + 8] = [15]

2. What does *intertemporal substitution of leisure* mean in the context of a RBC model?

(a) Develop a suitable two-period utility maximization model of a household (with perfect foresight) to show how such a substitution may arise.

(b) If the **wage rate** in the *second period* and (**1+ rate of interest**) **both** rise in the *same* proportion while the wage rate in the first period remains unchanged, would the household increase its labour in the second period and/or in the first period or in no period? Explain. [8+7] = [15]

3. Outline the *basic structure* of a Real Business Cycle (RBC) model. Discuss what is known as *calibration* of a RBC model.

[15]

Group B

Answer all

1. Show that with investments having convex adjustment costs, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest.
   Show that with both the production function and the investment adjustment cost function being linear homogenous the Tobin's marginal q would be equal to the average q.

   (10+5)

2. In the Blanchard-Yaari model with cohort dependent wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation? Explain. (15)

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2013-2014)
MS (Q.E.) II Year
International Economics I

Date: 25.11.2013 Maximum Marks 60 Duration 3 hours

Group A

Answer any two

1. Trade is the engine of economic growth to less developed countries- Examine the validity of this statement in the light of a North- South model. (15)
2. Consider the Spencer-Brander model of export share rivalry in which two firms of two countries play a Cournot-game. Examine the effect of export-subsidy policy on the level of export of both the countries. (15)
3. Formulate the 3x2 specific factor model stating its assumptions. Analyse the effect of a change in the price of a product on the factor prices in this model. (5+10)

Group B

Answer all questions

1. Consider two Ricardian countries which can potentially produce two goods (1 and 2) with consumers having identical preference given by $u = C_1^{\alpha} C_2^{1-\alpha}$, $0 < \alpha < 1$.

   The technological coefficients are

   $a_{L1} = 2$

   $a_{L2} = 4$

   $a^*_{L1} = 10$

   $a^*_{L2} = 5$

   Total labour endowments are

   $L = 80$

   $L^* = 400$

P.T.O

a) Find out the range of α for which both the countries will strictly gain by engaging in trade.

b) Say L and L^* are not given and $\alpha = \frac{1}{4}$. Find the range of $\frac{L}{L^*}$ such that countries remain completely specialized under trade.

[All notations have their usual meaning] (15)

2. Show that in a three-agent setting, a transferor might gain and the recipient lose (transfer paradox) even when the equilibrium is Walras stable. In this context discuss the role of substitution effects towards ensuring normal results. (15)

or

Show how trade driven distributional conflict is related to factor endowment ratios and economies of scale. (15)

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2013-14
M.S. (Q.E.), 2nd Year, Semester I
Econometric Applications I

Date: 28.11.2013 Maximum marks: 100 Duration: 3 hours

*[Note: Answer question no. 1 and any **three** from the rest of the questions. Marks allotted to each question are given at the end of the question]*

1. Using the following data on the Per-capita Expenditure (PCE) compute (i) the Head Count Ratio (H), (ii) the Income Gap Ratio (I) and (iii) the Sen's Index of Poverty (P). Assume that the poverty line was Rs. 20 per 30 days in the base period and the current CPI for agricultural labourers with that base is Rs. 400.

Table 1: The size distribution of population by PCE

PCE (Rs./30 days)	Percentage of population	Average PCE (Rs./30 days)
0 – 30	0.91	25.02
30 – 40	2.48	35.81
40 – 50	5.10	46.17
50 – 60	7.98	55.23
60 – 70	9.75	65.54
70 – 85	15.35	77.45
...	...	...

[25]

2. Define and compare Head Count Ratio, Income Gap Ratio and Sen's Index of Poverty. What are the limitations of Sen's poverty index? Describe some poverty measures, which overcome these problems? [18+2+5]

3. Consider the following production function Q = f(K,L) with two factors of production – capital (K) and labour (L). Define elasticity of substitution (σ). Prove that the following expressions for σ are equivalent.

$$\sigma = \frac{R(LR+K)}{KL(R\frac{\partial R}{\partial K} - \frac{\partial R}{\partial L})} = \frac{R(LR+K)}{KL(\frac{d^2K}{dL^2})} = \frac{Q_K Q_L (KQ_K + LQ_L)}{-KLT},$$

where $T = Q_K^2 Q_{LL} + Q_L^2 Q_{KK} - 2Q_K Q_L Q_{KL}$ and R is the MRTS. Hence prove that σ can be simplified to $\sigma = Q_K Q_L / (QQ_{KL})$, if the production function is homogeneous of degree 1. [25]

4. Describe in detail the economic and statistical criteria of choosing an Engel Curve. [25]

5. Describe how you will estimate Engel elasticity using Specific Concentration Curve. (You should show the derivations of the associated results.) [25]

6. Discuss the cost minimization and profit maximization problems of Cobb-Douglas Production function using and not using Lagrange multiplier method and derive the optimization conditions for both without and with the Lagrange methods. [25]

7. Write short notes on any two of the following:
   (a) Properties of Pareto distribution.
   (b) Treatment of household size in Engel Curve.
   (c) Linear Expenditure System. [12½+12½ =25]

INDIAN STATISTICAL INSTITUTE

MID SEMESTER EXAM: 2013-14

Course Name : MSQE II

Subject Name: SPECIAL TOPICS I

Date: 22/2/2014 **Maximum Marks: 60 pts.** **Duration: 150 minutes**

Note: Provide brief explanation for answers to the multiple choice questions. You can use a calculator and/or Microsoft excel blank sheet. You are not to access the internet or use phone anytime during the exam.

Multiple Choice *(3 x 5 = 15 pts.)*

1. Suppose the market risk premium is currently at 6%. If investors were to become more risk averse, the market risk premium will increase to 8%. What effect does it have on prices of risky financial assets?

a. prices would be unaffected

b. prices would increase

c. prices will decrease

2. Tata Motors have 9% annual coupon bonds that are callable and have 18 years left till maturity. The bonds have a Rs. 1000 par value and their current market price is Rs.1,220.35. However, Tata may call the bonds in 8 years at a call price of Rs.1,060. The YTM and YTC on Tata Motors bonds are 6.84% and 6.05%, respectively.

i. If interest rates are expected to remain constant, what is the best estimate of the remaining time left for Tata Motors bonds?

a. 5 years

b. 8 years

c. 18 years

d. Not enough information to give the answer

ii. If Tata Motors issued new bonds today, what coupon rate must the bonds have, to be issued at par?

a. 9%

b. 6.05%

c. 6.84%

3. Assume that the risk-free rate is 6% and the market risk premium is 5%. Given this information, which of the following statements is CORRECT? *(3 pts.)*

a. An index fund with beta = 1.0 should have a required return of 11%.

b. If a stock has a negative beta, its required return must also be negative.

c. An index fund with beta = 1.0 should have a required return less than 11%.

d. If a stock's beta doubles, its required return must also double.

4. A project can be taken at a later date until firm has more information about demand, costs, prices, or all three. What form of real options can manager use to value this project?

a. Option to Delay

b. Option to Abandon

c. Option to expand

5. Which of the following events is likely to encourage a company to raise its target debt ratio, other things held constant?

a. An increase in the corporate tax rate.

b. An increase in the personal tax rate.

c. RBI tightens interest rates in an effort to fight inflation.

d. The company's stock price hits a new high.

Short Questions *(3 x 4 =12 pts.)*

6. Would you expect a BB rated corporate bond of equal maturity and liquidity to have a higher, lower, or equal yield to maturity than the 5 year AAA-rated bond?

7. If starting from being an all equity firm, the firm decides to take successively larger levels of debt, why would you expect its stock price to first rise, then hit a peak, and then begin to decline?

8. Mathematically, can the required rate on a stock (think CAPM model) be less than the risk free rate? Explain.

9. If you buy a *callable* bond and interest rates decline, will the value of your bond rise by as much as it would have risen if the bond had not been *callable* ?

Problems

10. Zynga Technologies is a relatively young company. Zynga has been wildly successful, but it has yet to pay a dividend. An analyst has forecasted that Zynga is likely to pay its first dividend three years from now. He expects Zynga to pay 2.50 dividend at that time (i.e. D_3 = Rs. 2.50), and believes that the dividend will grow by 30% for the following two years (D_4 and D_5). However, after 5 years, she expects Zynga's dividend to grow at a constant rate of 7% per year. If Zynga's required rate of return is 11.9%, what is Zynga's current intrinsic value of the stock price (i.e., P_0)? *(7 points)*

11. Nk Inc. has 12-year, 10% annual coupon bonds outstanding. The bonds have a par value of Rs. 1,000, a YTM of 9%, and are non-callable?

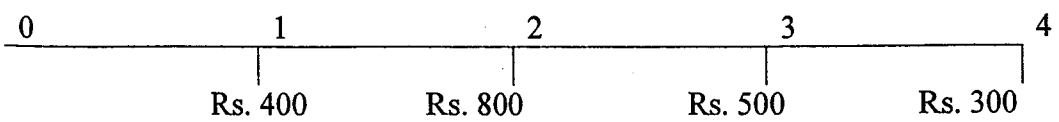
i. What is the current market price of the bonds? *(4 points)*

ii. What is the one-year Current yield (or Coupon yield) on Nk's outstanding bonds? *(1.5 points)*

iii. What is the expected one-year capital gains yield on Nk's outstanding bonds? *(1.5 points)*

12. iFlex solutions has a capital structure that consists of 40% debt and 60% equity. The firm's current beta is 1.50. Company's CFO has been urging the company to issue more debt and repurchase some of its outstanding equity. The CFO's plan would change the firm's capital structure to 60% debt and 40% equity. If the firm carried out its plan and the tax rate remained at 40%, what would its new levered beta? *(6 points)*

13. The cash flows of a project of a firm with a missing initial cost are as follows



The project has the same risk as the firm's average product. While you don't know the project's initial cost, you have been told the project has an IRR of 13.3703% and a WACC of 12.8%. What is the project's NPV? (Hint: You have to find the project's initial cost first) *(6 points)*

14. You own a rental building in Rajarhat and are interested in installing a cooling system. You are faced with the following alternatives:

a. A solar cooling system, which will cost Rs. 1,20,000 to install and Rs. 5,000 a year to run and will last forever (assuming that your building will, too).

b. A electric cooling system, which will cost Rs. 50,000 to install and Rs. 10,000 a year to run and will last twenty years.

c. An oil cooling system, which will cost Rs. 35,000 to install and Rs. 12,000 a year to run and will last fifteen years.

If your discount rate (or opportunity cost) is 10 percent, which of these three options is best for you? *(7 pts.)*

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2013-14

Course Name : MS (QE) Second Year

Subject Name : Econometric Applications II

Date: 24 February 2014 **Maximum Marks:** 50 **Duration:** 2 hours

Note : Answer question 1 and any two from the rest of the questions

1. Write down the general definition of Lorenz Curve as proposed by Gastwirth. Show how you can find Lorenz ratio using this definition by taking separately one discrete and one continuous distribution of income. [2+4+4]

2. What do you mean by failure rate of a distribution? Show how Singh and Maddala derived an income distribution modifying the failure rate. [2+18]

3. Write down different techniques of estimation of parameters of a distribution from a grouped data. How can one measure the goodness of fit of a distribution? [10+10]

4. Write a brief account of non-parametric density estimation of a distribution. [20]

5. Write short notes on any two of the following:
   (a) Properties of Gamma distribution.
   (b) Unit Consistency and Intermediate Inequality Measures.
   (c) Generalized Lorenz Curve. [10+10]

Indian Statistical Institute
Mid-Semester Examination: 2013-2014
MS(QE) II: 20013-2014
Industrial Organization

Date: --28/02/2014 Maximum Marks: 40 Duration: 3 Hours

Answer any FOUR questions

1. There are two groups of consumers. Consumers with taste parameter θ_1 are in proportion μ and those with parameter θ_2 are in proportion $(1-\mu)$. Group i consumers $(i = 1, 2)$ have the following preference:
   $U_i = \theta_i u(q) - T(q)$ if they pay T and consume q units of the monopoly good, and $U_i = 0$ if they do not buy, where $u(q) = \{1-(1-q)^2\}/2$.
   The unit cost of production is $c < \theta_1 < \theta_2$.
   (i) Let $T(q) = pq$ where p is the per unit price of the product. Derive the demand function for each group at price p and the consumer surplus at that price.
   (ii) Let θ denote the (weighted) harmonic mean of θ_1 and θ_2. Derive the aggregate demand at price p in terms of θ. Hence determine the non-discriminating monopoly price.
   (iii) If the monopolist charges (uniform) two-part tariff of the form T(q) = $F + pq$, find the optimal values of F and p.

   [3+3+4=10]

2. Suppose that consumers have a linear demand function and they are located uniformly from distance $x = 0$ to distance $x = 1$. The transportation cost to distance x is $t > 0$.
   (i) Compute the optimal f.o.b. prices when discrimination is allowed.
   (ii) Suppose that transportation is operated through a competitive sector at the same unit cost, t. Compute the optimal uniform (i.e., non-discriminatory) f.o.b. price, assuming that the whole market is served.
   (iii) Do the same exercise of (i) and (ii) if the consumers are located uniformly on the unit length circumference of a circle, and the monopolist is also located at a point on the circumference.

   [3+3+4=10]

3. There are two consumers with demand functions $x_1 = 1 - p$ and $x_2 = 1 - (\frac{p}{2})$. Obviously, consumer 2 is high demand consumer. The monopolist knows these demand functions but does not know who the high demand consumer is and who the low demand consumer.
   (i) If its marginal cost of production is zero, formulate the problem of second degree price discrimination. Then derive the optimal menu pricing.
   (ii) If identities of the consumers were known to the monopolist, what would be the optimal pricing scheme?

[7+3=10]

4. (i) Derive the Dorfman-Steiner condition in the context of the monopoly model with persuasive advertisement.
   (ii) Given that the monopolist chooses advertisement and price optimally, is the advertisement chosen socially optimal?

[4+6=10]

5. Consider a monopoly firm producing two goods A and B at zero cost. A consumer is identified by a pair (θ_1, θ_2) where θ_1 and θ_2 are the consumer's valuations for A and B respectively. Consumers are distributed with density $f(\theta_1, \theta_2)$. Assume that (θ_1, θ_2) is uniformly distributed over the unit square, hence θ_1 and θ_2 are uniformly distributed over [0, 1]. Further assume that when the goods are sold in a bundle, a bundle consists of one unit of each of these two goods, and a consumer's valuation for the bundle is equal to the sum of his separate valuations for the component goods.
   (a) Suppose consumers' valuations for A and B (i.e., θ_1 and θ_2) are independent. Then find the optimal prices under separate selling. Also find the optimal bundle price.
   (b) Suppose θ_1 and θ_2 are perfectly negatively correlated (i.e., $\theta_2 = 1 - \theta_1$). Find the optimal prices under separate selling and the price under bundling.

[4+6=10]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2013-14

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 28.2.14
Maximum marks: 30
Duration: 2 hours

Q1. There are two types of consumers for a monopolist's product, high (H) and low (L), with the proportion of high types denoted by α. The total measure of consumers is 1. If a consumer of type i consumes x amount of the good, and pays f in exchange, her utility is given by

$\theta_i \frac{1-(1-x)^2}{2} - f$

The monopolist has a constant marginal cost of production c.

(a) Suppose the monopolist can distinguish between the two types of consumers, but can only charge a simple price per unit $p_i$to any type. What are monopolist's optimal prices? [5]

(b) Suppose the monopolist cannot distinguish between the types, but can set a lump-sum fee F as well as a price per unit p. What fee and price will it set, assuming it serves both types of consumers? [5]

(c) Suppose the monopolist cannot distinguish between the types, but can set a price-schedule, i.e., a price that depends on the level of purchase. What is its pricing policy? [5]

Q2. A landlord who cannot farm has two cultivable plots of land. There are two types of farmers: type 1 with high production q_1 and type 2 with low production, q_2,with $q_1 > q_2$. The landlord is approached by two tenanats. He knows they are of different types, but does not who is of which type. Each tenant knows her own type. He can offer two possible contracts, one a sharecropping contract with tenant share α, and two a fixed rental contract with rent r.

(a) When is it optimal to offer the same contract to both tenants? [5]

(b) When is it optimal to offer different contracts to the tenants? Which type gets the rental contract? [5 + 5]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2013-14

Course: Masters in Quantitative Economics Year II

Subject: Political Economy and Comparative Systems

Date: 4 March 2014 **Maximum Marks**: 100 **Duration:** 3 hours

ANSWER ALL 4 QUESTIONS. EACH QUESTION CARRIES 25 MARKS.

1. Consider a society consisting of 4 classes: $N = \{1,2,3,4\}$. Total income in this society is $I = 10$, and class incomes are given by $I_i \in [0,10]; \forall i, j \in N, I_i < I_j$ whenever $j > i$. Classes can form coalitions to change the income distribution. In case a revolutionary coalition $S \subset N$ is formed, its probability of success is given by $P_S = \sum_{i \in S} p(I_i)$, where $p(I_i) = \frac{10 - I_i}{40}$; and a revolutionary coalition loses its entire income if the attempt at revolution fails.

(a) First suppose the equilibrium revolutionary coalition is $\{1, 2\}$. What does this imply for the magnitudes of I_4 and I_3? Explain your answer.

(b) Now suppose some foreign country is willing to provide asylum to revolutionaries in case of a failed attempt at revolution. In case of asylum, a revolutionary earns the same in the foreign country as what she would in the status quo in her native country. However, the foreign country is willing to take in refugees only from a single class. The asylum offer, along with the chosen class, is known prior to the attempt at revolution. Which class would a revolutionary entrepreneur prefer, *ex ante*, for guaranteeing foreign sanctuary? Which class would the regime prefer to amnesty and send off to exile? Explain your answers.

2. Suppose a peasant society consists of a single producer. The peasant consumes two goods, leisure (l) and food (f), and has a time endowment of 1. The peasant's preferences are given by $u = (\theta^{-1} f^{\theta} + l)$; where $\theta \in (0,1)$. Food is produced by labour according to the production function: $f = (1-l)^{\beta}$; $\beta \in (0,1]$. Some proportion, t, of the peasant's produce is taken away by warlords. Warlord i has the ability to impose a tax t_i on the producer; $i \in \{1,2,\ldots,n\}$, so that $t = \sum_i t_i$. The warlords choose their tax rates simultaneously and non-cooperatively, while taking the peasant's response into account.

(a) Find the Nash equilibrium level of food production, total tax rate, total tax revenue, and the welfare of the peasant.

(b) Find out how these four values change as the number of warlords increases. Interpret your results.

(c) Also find out how the total tax rate and tax revenue change as θ and β increase. Interpret your results.

(d) Suppose now that the there is only one warlord, who can control the choice of agricultural technology (i.e. choose any $\beta \in (0,1]$). What value of β would the warlord choose? Explain your finding.

3. Suppose two ethnic groups, A and B, are contesting for control over some mineral resource valued at v; $v > 0$. The probability of success of group $i \in \{A, B\}$ in the contest over the mineral resource is given by: $p_i = \frac{L_i}{L_i + L_{-i}}$; where $L_i \in [0,1]$ is the amount of labour allocated by the group to the contest and L_{-i} is the amount of labour allocated to the contest by the other group. Ethnic group i can produce (non-contestable) food according to the production function $D\theta_i^{\alpha}(1 - L_i)$, where $D > 0$ is a parameter representing the fertility of available agricultural land, of which $\theta_i \in [0,1]$ is the proportion controlled by i, and $\alpha > 0$ is a scale variable. The price of food is normalized to unity. Each ethnic group maximizes its total income from production and appropriation.

(a) Find the total labour allocated to conflict and the success probability of ethnic group i in the Cournot-Nash equilibrium.

(b) Show how greater equalization of land-ownership across ethnic groups affects the distribution of the mineral resource and the total labour allocated to conflict.

(c) Assuming equal distribution of land-ownership, show how total social loss due to conflict is affected by an increase in the value of the contestable resource, and an increase in the agricultural productivity parameter D.

(d) Explain what your results in parts (b) and (c) suggest regarding social policy for restraining conflicts in a mineral-exporting economy which faces a sudden boom in international prices for its exports.

4. Consider a society with two income levels: $I_H > I_L > 0$. Average income in this society is $\bar{I}$, total population is n and income class $i \in \{H, L\}$ has n_i members, with $n_L \equiv \rho n$, $\rho \in (0,1)$. The society consists of two communities, A and B: community A contains both H and L individuals, while community B contains only L individuals. The size of community B is ρ_B, $\rho_B \in (0, \rho)$. Individuals belonging to A have identical preferences, represented by the utility function $lnx + lny$, where x is the amount of a private good consumed by the individual, and y is the total amount of a public good generated by voluntary contributions. Individuals belonging to B have preferences given by the utility function $lnx + \ln(y_B + \theta y_A)$, where y_A, y_B constitute the total amounts of the public good contributed be members of communities A and B, respectively, and $\theta \in (0,1)$.

(a) Find the interval of values within which the income ratio $\frac{I_L}{I_H}$ must fall if only H individuals are to make positive contributions to the public good. Show how this interval changes with changes in θ and ρ.

(b) Suppose now that the income ratio $\frac{I_L}{I_H}$ falls within the interval identified in part (a). Find the real incomes of L individuals in both communities, and show how a marginal balanced budget income redistribution from H to L affects real incomes of L individuals; identify the community where L individuals benefit more from the redistribution.

(c) Now suppose all individuals in society make positive contributions to the public good. Explain how L individuals would react to a marginal redistribution proposal.

(d) Explain what your results imply for differences in the degree of radicalization between L individuals across communities.

Indian Statistical Institute
Second Semestral Examination: 2013 –2014
M.S.(Q.E) – II Year/ M.Stat. – II Year
Econometric Applications II

Date: 29.4.2014 Maximum Marks: **100** Duration: 3 hours

Answer any ***four*** questions. All questions carry equal marks.

1. (a) Find the distribution (continuous) underlying the Lorenz curve

$$L(p) = 1 - (1-p)^{1-\frac{1}{a}}, \quad (0 \le p \le 1).$$

(b) Consider two income profiles $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ with $x_1 \le x_2 \le \cdots \le x_n$ and $y_1 \le y_2 \le \cdots \le y_n$.

(i) Define Lorenz dominance of x with respect to y. State the assumptions clearly.

(ii) Why is the ordering of income profiles, generated by the LC comparison, a quasi-ordering? Illustrate with an example.

(c) What do you mean by a "Bistochastic" matrix? Suppose the income distribution $y = (1,6)$ is obtained from a distribution $x = (2,5)$ through income transfer. Show that if we write $y = Ax$, then A can not be Bistochastic.

(d) Show that the proportional failure rate (PFR) for Pareto distribution is constant and that for Lognormal distribution is increasing all through out.

[8+4+4+(2+7)= 25]

2 . (a) State and explain the properties of the Marshallian demand functions.

(b) Consider the Rotterdam model given by

$$w_i d\log q_i = b_i(d\log x - \sum_k w_k d\log p_k) + \sum_j c_{ij} d\log p_j \ , \ i=1,2,..n,$$

where $w_i = \frac{p_i q_i}{x}$ and $c_{ij} = \frac{p_i p_j S_{ij}}{x}$, S_{ij} being the compensated price effects.

(All the symbols have their usual meanings).

What are the restrictions to be imposed on the parameters for the system to satisfy the properties of demand function? Show how the restrictions are obtained.

(c) Show that for the Linear Expenditure System, all commodities are price inelastic, that is, $|\gamma_{ii}| < 1$, γ_{ii} being the non-compensated own price elasticity of item i. State the assumptions clearly.

[8+10+7=25]

3. (a) Describe (briefly) the procedure for estimating the function $m(\cdot)$ using the Nadaraya–Watson kernel weights in the following budget share function of the *h-th* household.

$$w_h = m(\log x_h) + \varepsilon_h, \quad h = 1,2,\ldots,H \quad .$$

(b) Explain the 'Double hurdle model' (with discrete random preference regimes) for modelling 'zero expenditure'.

(c) Given the dynamic model for 'clothing': $q(t) = \alpha + \beta s(t) + \gamma x(t)$, where

$q(t)$: rate of demand at time t

$x(t)$: income during the same time

$s(t)$: inventory of 'clothing' at time t

and assuming that the stock is used up at a constant depreciation rate δ, find the short term derivative of consumption with respect to income.

[10+ 10 + 5 = 25]

4. (a) Describe the Ramsey-Samuelson-Diamond-Mirlees approach of determination of optimal commodity taxation.

(b) Give an example of a bivariate sample selection model with a participation and an outcome equation. How would you estimate the parameters using Heckman's procedure?

[13+12=25]

5. (a) Describe the properties of a production function.

(b) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans. For both input and output oriented technical efficiencies, demonstrate (graphically) the relationship between Debreu-Farrell and Koopmans measures.

(c) Describe the Charnes, Cooper, Rhodes DEA model. State the assumptions clearly.

[5+10+10=25]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2013-14

Course: Masters in Quantitative Economics Year II

Subject: Political Economy and Comparative Systems

Date: 9th May 2014 **Maximum Marks**: 100 **Duration:** 3 hours

ANSWER ALL THREE QUESTIONS (All questions carry equal marks).

1. Consider a linguistic minority (N) of size $n \in (0, \frac{1}{2})$. This community's cost of assimilation to majority (M) linguistic norms is uniformly distributed in the $[0, k]$ interval; $k \in (0,1)$. For any minority individual i, the return from adopting M's linguistic norms is $\theta_M(1 - c_i)$, where θ_M is the proportion of the population that adopts M norms, and c_i is that individual's cost of assimilation. For such an individual, the return from persisting with N norms is $(1 - \theta_M)$.

(a) Find the size of the minority population that assimilates in equilibrium.

(b) Find the total income in this society and the income share of the minority community in equilibrium.

(c) Now suppose that each community is led by a warlord who allocates a fixed proportion of the community's total income, b, to consumption of each community member, and the remaining proportion $(1 - bs_j)$ to war against the other community; where s_j is the size of community j. What happens to total military expenditure in the society as the size of the minority population, n, increases? Assume $b > \frac{k}{2}$.

(d) Lastly, suppose that the state can develop a third set of cultural-linguistic norms, with inputs from both communities. Thus, for community N, the cost of assimilating to this set of norms is uniformly distributed in $[0, kt]$, while it is uniformly distributed in $[0, k(1-t)]$ for community M, where $k \in (0,1), t \in [0,1]$. Thus, a higher value of t implies that the state's official norms are closer to those of the majority. Find the value of t that maximizes total income in society. How does this value change with the size of the minority?

2. Suppose a worker's expected utility is given by:

$$u = A(1-p^d)w - \frac{L^2}{2};$$

where L is the effort actually provided, A, the marginal utility of income, is some positive constant, w is the wage rate, and p^d is the probability of being dismissed, with $p^d = (1-L)p^0(s)$, where s is the amount of supervisory input; $p^0(0) = 0, p' > 0, p'' < 0, p'(0) = \infty$. In case of dismissal, the worker receives zero income. All prices are unity. A profit-maximizing capitalist hires n identical workers to produce an output, using a production technology $Q = ZnL$, where $L \in [0,1]$ is the per worker labor effort actually provided, and Z is some positive productivity parameter.

(a) Find the worker's labor extraction function.

(b) Using your result in (a), and given the level of output, find how an increase in labor productivity (Z), and workers' valuation of income (A), individually affect the capitalist's profit (output net of supervision and labor costs) and the wage bill. Explain your results.

(c) Now suppose the firm is owned by a workers' state, which pays the entire output net of supervision cost to workers as wage, and maximizes the wage income of workers subject to the requirement of producing some given level of output, taking the labor extraction function in (a) as given. Show how an increase in labor productivity (Z), and workers' valuation of income (A), individually affect the wage bill.

(d) Explain what your results in (b) and (c) imply about the claim that a socialist society should discourage conspicuous consumption as a bourgeois vice.

3. Using the model of ethnic conflict developed by Esteban and Ray, identify the key factors that determine the activism response function of a group. Explain how greater aggression in supplying activists, on the part of one group, impacts on the equilibrium activism level for *both* groups.

Indian Statistical Institute

Semestral Examination: 2013-2014

MS(QE) II: 2013-2014

Industrial Organization

Date: 13/05/2014 Maximum Marks: 40 Duration: 3 Hours

Answer any FOUR question

1. Let the consumers with taste parameter θ be distributed uniformly over the length of an interval $[0,1]$. Each consumer is assumed to buy at most one unit of a product X of quality s, where s is a positive real number --- higher s signifies higher quality. The (net) utility of the θ consumer is given by

$$U(s,\theta,p) = s\theta - p \quad \text{if (s)he buys the product of quality } s \text{ at price } p$$
$$= 0 \quad \text{if (s)he does not buy the product.}$$

The cost of producing x units of the product of quality s is: $C(x,s) = \left(\frac{1}{2}s^2\right)x$.

(i) Derive the market demand for the product of quality s, i.e., $P(x,s)$.

(ii) Find the profit maximizing levels of production and quality of the product by a monopolist.

(iii) Find welfare maximizing quantity and quality of the product.

[2+4+4=10]

2. Consider a linear city of length 1. Consumers are uniformly distributed over the length of the city and each consumer buys exactly one unit of a product supplied by two firms, $i = 1,2$. Each consumer is to pay a price for the product and a transport cost proportional to the square of the distance (i.e., td^2, where t is the transportation cost per unit distance and d is the consumer's distance to the selected shop). Assume that there is no production cost and that firm 1 is already located at one edge of the city. Now firm 2 is to decide its location in the city. Finally, they compete in prices. Find the optimal location of firm 2 and the prices they will charge for their products.

[10]

3. Suppose the market demand function is of the form $P = a - bQ; a, b > 0$. Firm 1 uses an old production technology which has marginal cost of Rs 15. Firm 2 uses a modern technology with marginal cost of Rs 10. In the current equilibrium the product price and aggregate output are respectively Rs 16.66 and 8.33. Then how much would firm 1 be willing to pay for the modern technology? Derive the effect on welfare when both firms use modern technology.

[6+4=10]

4. Consider a duopoly producing a homogeneous product. Firm 1 produces one unit of output with one unit of labour and one unit of raw material. Firm 2 produces one unit of output with two units of labour and one unit of raw material. The unit costs of labour and raw material are w and r. The demand is $p = 1 - x_1 - x_2$, where x_i denotes the supply of firm i; the firms compete in quantities.
   (i) Compute the Cournot equilibrium.
   (ii) How is firm 1's profit affected if the price of labour goes up? Give economic intuition of the result.

[5+5=10]

5. Suppose that there are n number of firms producing a homogeneous good and competing in quantities. Let k be the number of firms ($k \leq n$) which form a cartel and act as Stackelberg leaders to set quantities simultaneously before the remaining firms decide quantities simultaneously. Assume that both the market demand function and the cost function are linear, with fixed cost zero. Derive the condition(s) for which the cartel will be stable in the sense that no firms will have any further incentive to enter or leave the cartel. If $n = 10$ and $k = 5$, will the cartel be stable?

[8+2=10]

INDIAN STATISTICAL INSTITUTE

FINAL SEMESTER EXAM: 2013-14

Course Name: MSQE II

Subject Name: SELECTED TOPICS I

Date: 16/5/2014 **Maximum Marks: 90** **Duration: 3 hrs.**

Note: The Exam is out of 100 points but the maximum you can score will be 90. You can use a calculator in this exam. You are not allowed to use phone, laptop or access the internet anytime during the exam. For multiple choice questions, provide a brief explanation to support your choice.

1. In the modified MM model with corporate τ and personal taxes on debt income and equity income (τ_D and τ_E), what would happen to the tax shield if $\tau_D \cong \tau_E$. (5)

2. Show that a firm's total market value is independent of its capital structure (MM proposition) using a no-arbitrage argument. (7)

3. Express the percentage change in bond price,(dP/P), as a non linear function of interest rates, r. (4)

4. Assume that all interest rates in the economy decline from 10% to 9%. Which of the following bonds will have largest percentage increase in price? (4)

   a) A 10-year bond with 10 percent coupon
   b) An 8-year bond with a 9 percent coupon
   c) A 10-year Zero coupon bond
   d) A 1-year bond with a 15 percent coupon.
   e) A 3-year bond with a 10 percent coupon.

5. What is the 1-year forward rate 3 years from now; given a 3-year spot rate of 7.24% and a 4-year spot rate of 8.216% (both continuously compounded annual rates)? (6)

6. In any firm, the balance of power between stockholders and managers is a function of a number of factors – internal as well as external. Events can cause the power to shift towards managers or towards stockholders or leave the balance unchanged. Evaluate how each of the following events would alter the balance of power (3 x 5 = 15)

Add a short explanation for your choice. Clearly state any assumptions you make.

a. The firm decides to expand its board of directors from 11 members to 22 members and allows the CEO to pick the additional directors.

A. Management Power increases
B. Stockholder Power increases
C. No Effect

b. An activist investor manages to get three of his nominees elected to the board of directors at the expense of management nominees

A. Management Power increases
B. Stockholder Power increases
C. No Effect

c. The firm's stock, which is currently followed by no analysts, is added to the list of followed companies at four investment banks

A. Management Power increases
B. Stockholder Power increases
C. No Effect

d. A closely held firm (insiders hold 40% of the 100,000 shares) issues 500,000 new nonvoting shares to the public to raise fresh capital

A. Management Power increases
B. Stockholder Power increases
C. No Effect

e. The holdings of institutional investors in the firm increase at the expense of small individual investors.

A. Management Power increases
B. Stockholder Power increases
C. No Effect

7. Consider the Investment model as in Jensen & Meckling (1976). In time 0, the entrepreneur has R amount of money and invests I in period 0. The entrepreneur needs to issue 1- α of outside equity E (he retains α as own equity). The entrepreneur doesn't commit to invest optimally. The investment function is specified as $V(I) = kI^{1/2}$, where $k > 0$. Setup the entrepreneur's payoff function and the maximization problem subject to constraint that $0 < \alpha < 1$. (8)

8. Write some of the practical implications of the Adaptive Market Hypotheses. (5)

9. XYZ Corporation is a non-dividend-paying stock that is currently priced at Rs.49. An analyst has determined that the annual standard deviation of returns on XYZ stock is 8% and the annual risk-free interest rate on a continuously compounded basis is 5.5%. Assume the 'down' factor to be a reciprocal of the 'up' factor. Calculate the value of a 6-month American call option on XYZ stock with a strike price of Rs.50 using a two period binomial model. (10)

10. Which of the following would most likely to lead to a decrease in a firm's dividend payout ratio? (4)

a. Its earnings become more stable.

b. Its access to the capital markets increases.

c. Its R&D efforts pay off, and it now has more high-return investment opportunities.

d. Its accounts receivable decrease due to a change in its credit policy.

e. Its stock price has increased over the last year by a greater percentage than the increase in the broad stock market averages.

11. Firm M is a mature firm in a mature industry. Its annual net income and net cash flows are both consistently high and stable. However, M's growth prospects are quite limited, so its capital budget is small relative to its net income. Firm N is a relatively new firm in a new and growing industry. Its markets and products have not stabilized, so its annual operating income fluctuates considerably. However, N has substantial growth opportunities, and its capital budget is expected to be large relative to its net income for the foreseeable future. Which of the following statements is CORRECT? (4)

a. Firm M probably has a lower debt ratio than Firm N.

b. Firm M probably has a higher dividend payout ratio than Firm N.

c. If the corporate tax rate increases, the debt ratio of both firms is likely to decline.

d. The two firms are equally likely to pay high dividends.

e. Firm N is likely to have a clientele of shareholders who want to receive consistent, stable dividend income

12. XYZ's projected capital budget is $2,000,000, its target capital structure is 40% debt and 60% equity, and its forecasted net income is $1,000,000. If the company follows a residual dividend policy, how much dividends will it pay and how much new stock must it issue? (6)

13. XYZ's most recent (Free Cash Flow) FCF was $60 million; the FCF is expected to grow at a constant rate of 6%. The firm's WACC is 10%, and it has 15 million shares of common stock outstanding. The firm has $30 million in short -term investments, which it plans to liquidate and distribute to common shareholders via a stock repurchase. The firm has $400 million in debt and $40 million in preferred stock. (3+2+1+1+3= 10)

a. What is the value of operations?
b. Immediately prior to the repurchase, what is the intrinsic value of equity?
c. Immediately prior to the repurchase, what is the intrinsic stock price?
d. How many shares will be repurchased? How many shares will remain after the repurchase?
e. Immediately after the repurchase, what is the intrinsic value of equity? The intrinsic stock price?

14. What does $\rho(X + Y) \leq \rho(X) + \rho(Y)$ represent in characteristics of coherent risk measures, if $\rho(.)$ is any particular risk measure? (6)

a. Subadditivity
b. Monotonicity.
c. Positive homogeneity
d. Translation invariance

Show that the VaR measure using a non-elleptical distribution violates atleast one of these above characteristics.

15. A risk manager is interested in calculating the VaR of an asset that he/she is considering adding to the bank's portfolio. The asset has a current value of $1,000,000 portfolio and the historical standard deviation of daily returns is 0.0011. Calculate the weekly Value at risk (VaR) at 5% significance assuming the asset returns could be approximated using a standard normal distribution. (6)

(Back Paper)

Indian Statistical Institute

Semester-II Examination: 2013-2014
MS(QE) II: 2013-2014
Subject: Industrial Organization

Date: 19/06/2014 Maximum Marks: 100 Duration: 3 Hours

Answer ALL the questions

1. An MNC has two alternative strategies to enter a foreign country. It can directly export goods to that country. In that case it faces a tariff $t > 0$ per unit. Alternatively, it can set up a subsidiary in the foreign country and serve the market. In this case it has a plant set up cost $F > 0$. In either case, the MNC is the only firm to serve the market. The market demand is negatively sloped and the MNC has constant marginal cost of production. Derive the optimal entry strategy of the MNC in terms of F and t.

[20]

2. Consider the following structure. There are two firms, firm 1 and firm 2. They produce a homogeneous good X and compete in quantities. The market demand function is given by: $p = 1 - x_1 - x_2$. The production requires a key input Y, but only firm 1 has the input production technology to produce at a constant unit cost normalized to zero. Assume that final goods production requires only this input and no other inputs. So firm 1 can produce the final good at zero cost. Firm 2 however can produce the final good only if firm 1 decides to sell inputs to firm 2 and in that case firm 1 will charge an appropriate price per unit of inputs.
   (i) Suppose firm 1 supplies inputs to firm 2 at an appropriate input price. Find the optimal input price and the corresponding market price of the final product.
   (ii) If firm 1 likes, it can foreclose the production of firm 2 by not supplying inputs to it at all. Is it optimal for firm 1 to foreclose firm 2's production? Find the optimal product price in this case.

3. There is a competitive fringe consisting of 10 firms. Each firm has the cost function $C(q) = q^2/2$. A dominant firm first sets the price of a product but it can serve only the residual demand. The fringe firms take that price as given and decide their sales. The market demand is $Q = 120 - 5P$ and the dominant firm's marginal cost of production is 4. Find the equilibrium price in the market and sales of the different firms.

[20]

4. Consider Cournot oligopoly with n number of identical firms; $n > 2$. Each firm has constant marginal cost, $b > 0$. The market demand is linear and in inverted form given by, $p = a - \sum_i x_i$; $0 < b < a$. Show that a two-firm (horizontal) merger is never privately profitable but a k-firm merger is profitable if and only if $k/n \geq .0.8$ and $n \geq 5$. [Use $\pi(n) = [(a-b)/(n+1)]^2$]

[30]

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2013-2014
M.S.(Q.E.) I & II year
Theory of Finance I
Back Paper

Date: 16.06.2014 Maximum Marks: 100 Time: 3 hours

Note: Answer Parts A and B in separate answer scripts. You can use a calculator in the examination. The paper carries 110 marks. The maximum you can score s 100.

A

1. State and prove a result which shows that if the relative risk aversion measure is increasing, then the wealth elasticity of demand for the underlying risky prospect is less than one.

(14)

2. Define and graphically explain first order stochastic dominance. Clearly demonstrate its expected utility equivalence.

(1+2+10=13)

3. State and prove a necessary and sufficient condition for completeness of an asset market by giving necessary preliminaries. (13)

4. Show that in a simple two-period binomial model the discounted stock price can be regarded as a martingale under risk neutral probability. (You must state and prove any result that you use here.)

(14)

5. Derive the Black-Scholes-Merton partial differential equation using the argument that the payoff of the derivative security can be exactly replicated using a dynamic portfolio of the underlying asset and a risk-free bond.

(16)

6. (a) Explain the usefulness of writing a covered call and show its payoff function graphically.

   (b) Show that the price of an American call option and the price of a European call option have an upper bound. Establish explicitly the relation between these three quantities.

(3+5)