INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2014-15

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 8.9.14
Maximum marks: 30
Duration: 2 hours

Q1. Within the framework proposed by Mookherjee (Review of Economic Studies, 1984), provide a set of necessary and sufficient conditions for optimal payments to an agent to solely depend on the rank of his output relative to that of the other agent. Argue rigorously. Make sure to specify the assumptions you are making, and clarify the notation. [15]

Q2. Within the framework proposed by Grossman and Hart (Econometrica, 1983), provide a set of sufficient conditions for optimal payments to an agent to be monotone non-decreasing in his output. Argue rigorously. Make sure to specify the assumptions you are making, and clarify the notation. [15]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: (2014-2015)

MS(QE) II

Selected Topics I

Date: 09.09.2014 Maximum Marks: 40 Duration: 3 hrs.

(1) (a) Define asymmetric weak order for an individual's preference relation P on the product set $X = X_1 \times \ldots \times X_n$. **(2)**

(b) Let $xIy \equiv [not(xPy)\ and\ not(yPx)]$ and $xRy \equiv not(yPx)$. Show that, if P on X is an asymmetric weak order, then both I and R are transitive. **(2+3=5)**

(c) Assume an individual's preference relation P on X is an asymmetric weak order. Then show that for all $x, y, z \in X$:

(I) $xRy \Leftrightarrow (xPy\ or\ xIy)$. **(4)**

(II) (i) $(xPy\ and\ yRz) \Rightarrow xPz$, and (ii) $(xRy\ and\ yPz) \Rightarrow xPz$. **(3+3=6)**

(2) Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{2, 6\}$ and $Pr(\theta = 6) = \frac{1}{2}$. The cost of education is $c(e, \theta) = \frac{e^2}{2\theta}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type θ who chooses education level e and receives wage w. Assume that both worker types earn zero by staying home, that is $r(2) = r(6) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of e^* for each of which we can have a separating equilibrium. Justify your answer. **(8)**

(b) Consider the belief function $\mu^b(e) = \frac{e - \max(0, e-4)}{4}$ for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer. **(9)**

(3) In the two-stage screening game with unknown worker types, show that in any equilibrium (separating or pooling), firms earn zero profits. **(6)**

Indian Statistical Institute
Mid-Semester Examination: 2014-2015
MS(QE) II: 20014-2015
Industrial Organization

Date:10/09/2014 Maximum Marks: 40 Duration: $2\frac{1}{2}$ Hours

Each question carries 8 marks. You may answer all questions.
However, your total score cannot exceed 40.

1. A monopolist faces a market demand $D(p, A)$ where p and A are respectively price and advertisement expenditure. The monopolist chooses both p and A optimally. Given its monopoly pricing behaviour, is the monopoly advertisement level socially optimal?

2. A monopolist faces a single consumer, with demand function q= $a - p$, $a > 0$. The marginal cost of production is 0. The monopolist faces a competitive fringe at price $p_0 < a$ (i.e., a perfect substitute is available at this price). What is the optimal pricing (i) under a linear tariff, (ii) under two-part tariff?

3. Suppose Indian Airlines (AI) is going to provide flight services between city A and city B. During winter and summer the demands for flights are respectively $p_w = 1000 - x_w$ and $p_s = 500 - x_s$. Let the capacity of the airlines determine the maximum number of airplane seats (the airlines intend to acquire). The average cost of an airplane seat is 500 and the cost of flying each passenger is 100. Each airplane seat is expected to give services for 5 years. Find the optimal capacity to be installed and airfares for two seasons when AI decides to (i) maximize profit; (ii) maximize welfare.

4. Consider symmetric n-firm Cournot oligopoly equilibrium. If n goes up, how does it affect the industry profit and social welfare?

5. Your department needs photocopying services. As head of the department you are to decide the following. Either your department will get the services from outside in which case the department will have to pay Re 1 per page, or alternatively, the department can buy a photocopy machine at Rs 50,000/-. The machine is expected to give services for 5 years without disturbances and thereafter it will be obsolete. However, it has operational cost of Rs 500/- per cartridge which photocopies 5000 pages. How do you decide which alternative you will go for? What will be the critical (expected) average number of pages to be photocopied per year?

6. Let the market demand function be $Q = Ap^{-\varepsilon}$, $\varepsilon > 1, A > 0$. The cost function of the monopolist is linear. A consumer at a distance x from the monopolist's trading centre requires to pay tx as transport cost per unit of output to be bought. Under price discrimination what f.o.b. price will the monopolist charge to the consumer? If x goes up, find its effect on the f.o.b. price.

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2014-2015
M.S. (Q.E.), 2nd Year
Econometric Applications I

Date: 10 September 2013 Maximum marks: 100 Duration: 3 hours

[Answer question no. 1 and any **three** from the rest of the questions.]

1. Suppose the urban population is just one-third of the rural population in a country. The average monthly incomes of the bottom 10% population of rural and urban sectors are Rs. 80 and Rs. 100, respectively. The respective Lorenz Ratios are 0.30 and 0.40. Assume that income follows lognormal distribution separately for rural and urban sectors. Calculate the percentage of people with income more than Rs.200/- for the country as a whole. [25]

2. (a) Discuss various types of data that are encountered and the problems with the data that are faced in econometric analysis. Also describe some methods of refining the data.
(b) State Pareto law. How can you graphically test whether a given set of data is coming from a Pareto distribution? Derive Lorenz Curve and Lorenz Ratio of Pareto distribution. [15+10=25]

3. (a) State and explain the scale invariance and the Pigou-Dalton transfer axioms to judge the suitability of a measure of inequality of income. Examine the incomes of the following three groups of persons and arrange the groups in increasing order of their inequalities in the light of these two properties.

Group – I:	10	20	30	40	50	60	70	80	90	100
Group – II:	20	37	57	76	95	114	133	152	172	189
Group – III:	21	21	84	84	105	126	147	147	210	210

(b) Examine whether Coefficient of Variation satisfy Transfer Axiom. [15+10=25]

4. (a) Describe how one can construct Lorenz Curve and hence Lorenz Ratio graphically and numerically given grouped data. Give your comments on the accuracy of the two methods.

(b) Prove that the Lorenz Ratio found by the numerical method is same as that found by the formula using Gini's Mean Difference. [15+10=25]

5. (a) Write down the important steps in deriving Atkinson's measures of inequality based on the Social welfare Function Approach. How can one interpret the unknown parameter in the measure?

(b) Prove that the two parameter lognormal distribution is positively skewed and Leptokurtic. [15+10=25]

6. Write short notes on any two of the following:
   (a) Positive measures of inequality.
   (b) Properties of Lorenz Curve of Lognormal distribution.
   (c) Use of Dummy Variables. [12½+12½=25]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination: (2014-2015)

MS (Q.E.) II Year

Macroeconomics II

Date 15.09.14 Maximum Marks: 40 Duration: 3 hours

Answer all questions

1. Consider Mankiw's model of imperfect competition discussed in the class with the following modifications:

   (a) Apart from leisure L consumers consume two consumption goods, domestic and foreign, denoted by c_d, c_f respectively. The utility function of a consumer is given by $U = \alpha \log c_d + \beta \log c_f + \gamma \log L,\ \alpha + \beta + \gamma = 1.$

   (b) The government, apart from providing direct employment, consumes both domestic and foreign goods, denoted by G_d, G_f respectively.

   (c) The price of the foreign good p_f is given.

   (d) Trade is always balanced in the sense that the value of imports = the value of exports.

   The rest of the model is just like Mankiw's.

   (i) Set up the model of income determination.

   (ii) Derive the balanced budget multiplier when (a) there is an increase in G_d and (b) there is an increase in G_f, both financed by an equal increase in taxes.

   (iii) Find the effect of an increase in the number of firms on output.

   (iv) Find the effect of an increase in the price of the foreign good on the level of output.

   [8+6+3+3]

2. In an infinite horizon model, work out the dynamics of per capita asset accumulation for a small open economy, facing a constant rate of interest in the world capital market.

In this context discuss the problems associated with either a very low or a very high rate of interest.

[10]

3. Assuming a Cobb Douglas production function, analyse how the savings rate behave in an infinite horizon Ramsey model while in transit towards the steady state. [10]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2014-2015

MS (Q.E.) and MStat II Year

Econometric Methods II

Date: 19th September 2014 Maximum Marks 30 Duration 2 hours

All notations are self-explanatory. You can answer any part of any question.

1. Let $x_1, x_2, \ldots\ldots\ldots x_{200}$ are *i.i.d.* with pmf equal to $\Pr[x_i = y] = \frac{e^{-\theta}\theta^y}{y!}$, $y = 0, 1, 2, \ldots$. Now suppose that $\sum_{i=1}^{200} x_i^2 = 1200$. Find the Method of Moment estimator for θ. For this particular distribution, *mean = variance* $= \theta$. [10]
2. Consider the non-linear model $y_i = \beta_1 x_i^{\beta_2} + u_i$, *with* $E[u_i|x_i] = 0$.
   (a) Provide two moment functions that would make it possible to estimate $\beta = ((\beta_1, \beta_2)'$.
   (b) Derive the asymptotic distribution for the MM estimator of β obtained in (a).
   (c) Now suppose that $E[u_i|x_i] \neq 0$, but there exists a $p \times 1$ *vector* z_i $(p = 2)$ *such that* $E[u_i|z_i] = 0$ *and* $E[z_i x_i] \neq 0$. Propose a method of moment estimator for β.

   [3+8+4=15]

3. Consider the linear panel data regression model: $y_{it} = \alpha_i + \beta_1 X_{1it} + \beta_2 X_{2it} + \theta Z_i + \epsilon_{it}, i = 1, 2, \ldots N, t = 1, 2, \ldots . T$. ϵ_{it} satisfies all the ideal conditions. $\alpha_i's$ are unobserved heterogeneity variables and are correlated with only X_{1it}. $Z_i's$ are time invariant observed variable and is such that $E(\alpha_i|Z_i) = 0$. It is also known that $E(\alpha_i|X_{2it}) = 0$. Provide a consistent estimator of θ. Supply all analytical derivations. [10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2014-2015)

MS (Q.E.) II Year

Macroeconomics II

Date: 17·11·14 Maximum Marks 60 Duration 3 hours

Answer *any two* from the first three questions and answer *all the rest*

1. Consider a competitive economy with L commodities, I agents and S states. There are two dates, 0 and 1. Endowments are realized (according to the realization of the state) and consumed in date 1. In date 0, the agents can enter into contingent contracts. Show that for such an economy having complete contingent markets at date 0 for each state and each good is equivalent to having S primary securities (in terms of good 1), one for each state, to be traded at date 0.

[15]

2. A two stage game takes place between employers (E) and the monetary authority (M). In stage one E forms an expected inflation π^e given the (rationally expected) actual rate of inflation π chosen by M in stage two. In stage two, M chooses π given the actual choice of π^e in the previous stage. The objective function of E is $-(\pi - \pi^e)^2$ while that of M is $U(\pi, y) = -c\pi^2 - (y - y^*)^2$, where y is the actual level of output and y^* is the desired level of output of M. Actual output is given by $y = by^* + d(\pi - \pi^e)$. Here b, c, d are positive constant with $b < 1$.

Show that the no commitment (sub-game perfect) equilibrium rate of inflation is sub-optimal compared with the equilibrium where M can pre-commit to its desired rate of inflation.

[15]

3. Consider a two period single commodity financial economy. There are T elementary securities for T possible states of the economy in date 1. In date 0, consumers trade in securities and in date 1, uncertainty is resolved, returns to securities are realized and consumption is undertaken. There are two consumers with endowments:

$$\omega_t^1 = T - 0.5(t-1)$$

$$\omega_t^2 = t$$

$t = 1,2,..T$. Here ω_t^h is the endowment of agent h in state t, $h = 1,2$.

All states take place with equal probabilities which is public knowledge and both consumers are expected utility maximizers with an identical utility function

$$u^h(x) = \ln x,\ h = 1,2.$$

(a) Specify each consumer's optimization problem at period 0.

(b) Derive the equilibrium allocations and prices.

[3+12 = 15]

4. Show that with investments having convex adjustment cost, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest.

Show that if the production function and the investment adjustment cost function are both linear homogeneous then Tobin's marginal q would be equal to the average q.

[10+5]

5. In the Blanchard-Yaari model with cohort *dependent* wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation?

Consider the same model, but now with zero population growth, cohort *independent* wages and open to international asset market with a constant rate of interest. Can you show that the aggregate savings in this model is negatively related to the level of assets?

[Savings = Total income – Total Consumption, where Total income = wage income + interest income on assets. Also assume that the steady state exists in the model.]

[10+5]

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2014-2015
MS (Q.E.) II and MStat II Year
Econometric Methods II/ Econometric Methods

Date: 19.11.2014 Maximum Marks 50 Duration 2 hours

All notations are self-explanatory. This question paper carries a total of 55 marks. You can answer any part of any question. But the maximum that you can score is 50. Marks allotted to each question are given within parentheses.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\epsilon|X) = 0$ and $E(\epsilon\epsilon'|X) = \Omega \neq \sigma^2 I$. Define the sample moment conditions $g = \frac{1}{n}X'e = 0$.
   Let $\hat{\beta}^1_{GMM}$ is obtained by minimizing $g'g$; and $\hat{\beta}^2_{GMM}$ is obtained by minimizing $g'S^{-1}g$, $S = E(gg')$.
   Find both $\hat{\beta}^1_{GMM}$ and $\hat{\beta}^2_{GMM}$. Compare their efficiencies.
   [2+3+5 10]

2. Consider a latent variable modeled by $y_i^* = x_i'\beta + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma^2)$. Suppose we observe only $y_i = \begin{cases} 1, & if\ y_i^* < u_i \\ 0, & if\ y_i^* \geq u_i, \end{cases}$

   where u_i is a known constant for the i^{th} individual.
   a. Derive the regression equation for y_i.
   b. How would you estimate all the parameters including β and σ^2 consistently? Discuss in detail.
   c. Assume that $u_i = u\ \forall i, u$ is known. How would you estimate the estimable parameters for the regression of y_i consistently? Discuss in detail.
   d. Discuss why the usual R^2 is not useful for the regression of y_i on x_i. Define pseudo R^2 in this context.
   [6+5+4+(2+3) 20]

3. Suppose that we observe data $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$, where, $x_i = \frac{i}{n+1}$, for $i = 1,2, \ldots, n$; and . $y_i = \mu(x_i) + \varepsilon_i$. Here $\mu(.)$ is a continuous function on the interval $[0,1]$ and $\varepsilon_1, \ldots\ \varepsilon_n$ are $i.i.d.\ N(0, \sigma^2)$ variables. Consider the local average estimate of $\mu\left(\frac{1}{2}\right)$ obtained by the local averaging over the interval $\left[\frac{1}{2} - \frac{1}{\sqrt{n}}, \frac{1}{2} + \frac{1}{\sqrt{n}}\right]$.
   a. Derive simplified expressions for the bias and the variance of the local average estimate.
   b. Show that the MSE of the estimate tends to zero as the sample size grows.
   [7+8 15]

4. Consider the following panel data model:
   $$y_{it} = \alpha_t + x_{it}'\beta + \varepsilon_{it},$$
   where the x_{it} (k× 1) are time-individual varying regressors. Let $x_t = (x_{1t}, x_{2t}, \ldots\ldots, x_{Nt})'$. Assume that $E[\varepsilon_{it}|x_t, \alpha_t] = 0$, and $E[\alpha_t|x_t] \neq 0$. $\sigma_\varepsilon^2 = Var(\varepsilon_{it})$. Provide a consistent estimator of β. Prove its consistency.
   [5+5 10]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2014-2015)

MS (Q.E.) II Year

International Economics I

Date: 21.11.14 Maximum Marks 60 Duration 3 hours

Answer *any two* from the first three questions and answer *all the rest*

1. Consider a country where two goods, x_1, x_2 are produced with two resources, R_1 and R_2. Technology in each sector is fixed coefficient and exhibits constant returns to scale. Let $a_{11} = 15, a_{12} = 5, a_{21} = 40, a_{22} = 25$ where a_{ij} is the amount of the ith resource required to produce one unit of the jth good, $i = 1,2\,; j = 1,2$. The endowments of the two resources are given as $R_1 = 100, R_2 = 150$. Finally, the residents of the country devote ½ of their income to the purchase of x_1 and the remaining half to the purchase of x_2. Assume perfect competition and free mobility of factors across sectors.
   (a) Determine output levels, factor prices and the relative commodity price in autarkic equilibrium.
   (b) Now suppose the country opens up to trade and faces a relative commodity price of unity in the world market. Assume that the country is small and is unable to affect world prices. Compute output levels and the levels of exports and imports of the two goods under free trade.

[8+7 = 15]

2. (a) In a static framework show that for a competitive economy free trade is better than autarky.
   (c) In a dynamic activity analysis framework with competition and constant returns to scale prove that free trade is superior to autarky.

[3+12=15]

3. Using an appropriate *general equilibrium* model show that welfare under free trade is higher than that under autarky according as the coefficient of relative risk aversion is less than or greater than unity.

[15]

P T O

4. Show that in a two agent setting, Walras stability guarantees that the recipient of a transfer necessarily gains and the transferor loses. Do you observe that the terms involving the income effects (the marginal propensities to consume, of both the transferor and the transferee) do not enter into the final welfare change equation? Provide an explanation for this.

[15]

5. Show how distributional conflict is related to factor endowment ratios and economies of scale.

[15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2014-15
M.S. (Q.E.), 2nd Year, Semester I
Econometric Applications I

Date: 24.11.2014 Maximum marks: 100 Duration: 3 hours

*[Note: Answer **question no. 1** and any **three** from the rest.]*

1. Suppose in a population of 30,000 people there are 5,000 people who are below poverty level (BPL) income and the rest above the poverty line (APL) income. The mean incomes and the Lorenz ratios of these groups, i.e., BPL and APL, are given in the following table.

Population Groups	Mean Income (Rs.)	Lorenz Ratio
BPL	1000	0.28
APL	3000	0.44

Find the overall Lorenz ratio. [25]

[You should derive the associated results.]

2. (a) Suppose X is a size variable having the distribution function $F(.)$. Define the Lorenz Ratio (LR) of X. Show that it can be expressed as

$$LR = 1 - 2\int_0^1 F_1 dF,$$

where the symbols have their usual meanings. Prove that the LR can also be obtained from

$$E|Y_1 - Y_2|/(2M),$$

where Y_1 and Y_2 are i.i.d. having the distribution function $F(.)$ and M is their common mean.

(b) Show, in particular, that if $X \sim LN(\mu,\sigma^2)$ then the $LR = 2\Phi(\sigma/\sqrt{2})-1$. [20+5=25]

[You should derive the results of 'moment distribution' of LN distribution and then derive the LR]

3. (a) Consider the following production function $Q = f(K,L)$ with two factors of production – capital (K) and labour (L). Define elasticity of substitution (σ). Prove that the following expressions for σ are equivalent.

$$\sigma = \frac{R(LR+K)}{KL(R\frac{\partial R}{\partial K} - \frac{\partial R}{\partial L})} = \frac{R(LR+K)}{KL(\frac{d^2K}{dL^2})} = \frac{Q_K Q_L (KQ_K + LQ_L)}{-KLT},$$

where $T = Q_K^2 Q_{LL} + Q_L^2 Q_{KK} - 2Q_K Q_L Q_{KL}$ and R is the MRTS. Other symbols have their usual meanings. Hence prove that σ can be simplified to $\sigma = Q_K Q_L/(QQ_{KL})$, if the production function is homogeneous of degree 1.

(b) State the statistical problems involved in the estimation of Cobb-Douglas production function. [20+5=25]

4. Write a brief account of the treatment of household size in the Engel curve analysis. [25]

5. Write down the criteria for a good measure of concentration in business and industry. Show how these criteria are satisfied by (i) the Herfindahl – Hirschman index and (ii) the Hall and Tideman index. Also discuss why Lorenz Ratio cannot be taken as a measure of concentration in business and industry. [25]

6. Write short notes on any **two** of the following:

(a) Kapteyn's Law of Proportionate Effect and its modification due to Kalecki.
(b) Desirable properties for a measure of inequality.
(c) Treatment of qualitative characteristics in econometric analysis.
(d) Linear Expenditure System. [12½ + 12½ = 25]

[Submit Practical Assignments.]

Indian Statistical Institute
Semestral Examination: 2014-2015
MS(QE) II: 2014-2015
Industrial Organization

Date: 24/12/2014 Maximum Marks: (20+40) Duration: 3 Hours

In the Examination Hall you are to answer only Group B

[Group A: Presentation and Assignments: 20 Marks]

Group B: Answer any FOUR questions : 40 Marks

1. (a) The long-run total cost of a typical firm in a competitive industry is given by the function: $C(Q) = 1200Q - 60Q^2 + Q^3$. The industry demand function is: $P = 375 - 0.025Q$ (all notations are as usual). Find the long-run competitive equilibrium price and number of firms.

   (b) Now suppose that an entrepreneur buys each firm in the industry and then acts as a monopolist. Assume that the monopolist can operate each firm's plant at exactly the same cost as before and, if it likes, it can close down some plants without any additional cost. How many plants should he close down to maximize profit and what will be the corresponding monopoly price of the product?

   [3+7=10]

2. There are two demand functions $D_i = 1 - 2p_i + p_j; i, j = 1,2; i \neq j$. Firm 1 is the incumbent and firm 2 is the entrant. Firm 2's marginal cost is 0. Firm 1's marginal cost is initially ½. By investing I = 0.205, the incumbent can buy the new technology and reduce its marginal cost to 0.

   (i) Consider the following sequence of moves. The incumbent first chooses whether to invest; then the entrant observes the incumbent's investment decision; then the firms compete in prices. Find the equilibrium investment decision of the incumbent.

   (ii) Do you think that the investment decision of the incumbent may be changed if the entrant faces a fixed entry cost?

   [7+3=10]

3. Consider the following model. There are two firms producing two distinct qualities of a product. Firm 1 first decides its product quality q_1 from an interval $[\underline{q},\bar{q}]$; $\bar{q} > \underline{q} > 0$. Then firm 2 decides its product quality q_2 from the interval $[\underline{q},q_1)$. Finally, they simultaneously and independently decide the prices of their product. Consumers with taste parameter θ are assumed to be distributed uniformly over the length of an interval $[\underline{\theta},\bar{\theta}]$; $\bar{\theta} > \underline{\theta}$, and each consumer buys at most one unit of the product. The (net) utility of the θ consumer is given by

$$U(q_i,\theta,p_i) = \theta q_i - p_i \quad \text{if (s)he buys the product of quality } q_i \text{ at price } p_i$$
$$= 0 \quad \text{if (s)he does not buy the product.}$$

Now suppose that in equilibrium the market is fully covered, that is, all consumers buy the product. (i) Show that in equilibrium the principle of maximum product differentiation will hold. (ii) State the condition(s) required for full market coverage in equilibrium.

[8+2=10]

4. Suppose that consumers need to buy two complementary goods (or components) for a composite good. Consider three firms, 1, 2 and 3, producing X, Y and Z, respectively. Let X and Y be complementary to each other. Similarly X and Z are complementary. So the consumers consume either $\alpha = (X,Y)$ or $\beta = (X,Z)$ to complete consumption, that is, for them α and β are two composite goods which are substitutes to each other, because Y and Z are assumed to be two brands of the same product.

Let P_X, P_Y and P_Z be the individual prices of X, Y and Z, respectively. Consumers are only interested in system prices where system prices are: $P_\alpha = P_X + P_Y$ and $P_\beta = P_X + P_Z$. Finally, the demand functions for α and β are linear and given by

$$D^\alpha = a - \gamma_\alpha + \lambda\gamma_\beta \text{ and } D^\beta = a - \gamma_\beta + \lambda\gamma_\alpha$$

where $a > 0$ and the parameter $\lambda \in (0,1)$ denotes the degree of substitutability between the products. Answer the following.

(i) Derive demand for individual products X, Y and Z.

(ii) Firms are assumed to determine the individual prices simultaneously and non-cooperatively. Determine the individual prices and profits of the firms.

[2+8=10]

5. Consider symmetric n firm Cournot oligopoly model. The firms produce homogeneous goods. The market demand function is linear and the average cost of production is constant. (i) Show that all firm cartel can never be stable if $n \geq 3$. (ii) Can a two firm cartel be stable in n firm Cournot oligopoly?

[8+2=10]

6. (i) Present the circular city model (of Salop) to determine the price and number of firms in equilibrium. (ii) Is the number of firms socially efficient?

[7+3=10]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2014-2015)

MS(QE) II

Selected Topics I

Date: 28.11.2014 Maximum Marks: 60 Duration: 3 hrs.

(1) Consider the social aggregation problem, where $A = \{x, y\}$ is the set of alternatives, $|A| = 2$, N is the finite set of agents and $|N| \geq 2$.

(a) Suppose that the social welfare function satisfies symmetry and neutrality, and the number of agents preferring x over y is the same as the number of agents preferring y over x. Then show that the social welfare function necessarily prescribes indifference between x and y.

(b) Define non-triviality and positive responsiveness of a social welfare function.

(c) Can you find a social welfare function that satisfies non-triviality and positive responsiveness but fails to satisfy Pareto? Justify your answer. **(7+2+6=15)**

(2) Consider the social aggregation problem, where A is the set of alternatives, $|A| \geq 3$, N is the finite set of agents and $|N| \geq 2$.

(a) Define 'almost decisiveness' and 'decisiveness'.

(b) A social ordering satisfies *positive responsiveness* if the following holds: If $R_i = R_i' \; \forall \; i \in N \setminus \{j\}$ and either (yP_jx and $xR_j'y$) or (xI_jy and $xP_j'y$), then xRy implies $xP'y$. Show that if a social ordering satisfies positive responsiveness, then 'almost decisiveness' and 'decisiveness' are equivalent. **(2+13=15)**

(3) For the pure public goods problem, define efficiency of decision, dominant strategy incentive compatibility and the class of VCG mechanisms. Show that if a mechanism with efficient decision is dominant strategy incentive compatible, then it has to be a VCG mechanism. **(6+9=15)**

(4) In the landlord-tenant model with hidden action, show that in equilibrium the incentive constraint is binding. **(15)**

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2014-2015) (Back paper)

MS (Q.E.) II Year

International Economics I

Date 30|12|2014 Maximum Marks: 100 Duration: 3 hours

Answer *all* questions.

1. Show that in a short run model of trade with Keynesian unemployment, free trade expands employment and increases consumption in the investment good exporting country and reduces both employment and consumption in the investment good importing country.

[25]

2. Develop a two-good, two-factor dynamic model of trade and show how the static Heckscher-Ohlin results can be generalized to this dynamic set-up.

[25]

3. Explain the mechanics through which capital inflow in Dei's model necessarily leads to welfare improvement. How is this result strikingly different from the Brecher, Diaz-Alejandro result, where the import competing capital intensive sector is tariff protected and the country is small.

[25]

4. Show that in a two factor two good model, with one of the sectors exhibiting increasing returns and the other constant returns to scale, the condition that guarantees positive price-output response also guarantees the validity of the Stolper-Samuelson theorem in its magnification version.

[25]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2014-2015) (Back paper)

MS (Q.E.) II Year

Macroeconomics II

Date 30/12/2014 Maximum Marks: 100 Duration: 3 hours

Answer all questions

1. In the Lucas model show that unanticipated monetary policy leads to business cycles. Can a pre-announced monetary growth solve the problem of cycles?

[20+5=25]

2. Consider a two-period (period 0 and period 1) competitive economy without production. There is a single commodity and two consumers. Each consumer is a price taker. All consumption takes place at date 1 when endowments are realized. There are two possible states at date 1. Let e_s^h denote endowment of consumer h at date s, h = 1,2; s = 1,2. In particular, the endowments are given by

$$(e_1^1, e_2^1) = (3,2) \qquad (e_1^2, e_2^2) = (4,3)$$

At period 0, consumers can buy or sell securities. There are two securities with return structures

$$r^1 = (2,1)$$

$$r^2 = (2,3)$$

State 1 takes place with probability 0.1 and state 2 with probability 0.9. Consumers know these probabilities. Both consumers are expected utility maximizers with an identical utility function

$$U^h(x_s^h) = 10\ln(x_s^h) \quad h = 1,2$$

where x_s^h is the consumption of consumer h in state s.

(a) Specify each consumer's optimization problem.

(b) Derive the equilibrium of this economy.

[5+20=25]

3. In an infinite horizon model, work out the dynamics of per capita asset for a small open economy, facing a constant rate of interest in the world capital market.
In this context discuss the problems associated with either a very low or a very high rate of interest.

[25]

4. Consider a two period OLG model which can potentially accommodate money in equilibrium. Assume that agents live for two periods and are endowed with Y units of good in the first period when they are young and earn nothing in the second period of their life; when they are old. These agents can potentially carry over the first period endowment of goods in the second period, in which case they will get in the second period $\frac{1}{1+\delta}$ times what they carry over from the first period, where $-1 < \delta$
Agents have a two period utility function given by $U = \ln c_t^y + \frac{1}{1+\rho}\ln c_{t+1}^o$, where c_t^y and c_{t+1}^0 denote the consumption of an young agent in period t and the consumption of the agent in period t+1 (when she is old) respectively and $0 < \rho$. The old agent also gets a transfer from the government financed through seigniorage, where the real transfer to the old in period t+1 denoted by $T_{t+1} = \frac{M_{t+1}-M_t}{P_{t+1}}$, where M_{t+1} and

P_{t+1} denote the money supply and price level in period t+1. The government follows a simple monetary rule given by $M_{t+1} = (1 + \alpha)M_t$, with $0 < \alpha$.

Derive the condition on the rate of inflation for which agents would hold money in equilibrium.

Also show that, in such a case where money is held in equilibrium, the rate of inflation is equal to the growth rate of money.

[25]

INDIAN STATISTICAL INSTITUTE
Back paper Examination: 2014-15

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 31.12.14
Maximum marks: 100
Duration: 3 hours

Answer all questions

Q1. Consider the framework proposed by Lazear and Rosen (Journal of Political Economy, 1981). Suppose that workers are risk-neutral and are of two possible types, with the marginal cost of one type being less than that of the other type. Show that a pure price-rationing mechanism cannot induce sorting, and that the equilibrium results in an inefficient allocation of resources. Argue rigorously. Make sure to specify the assumptions you are making, and clarify the notation. [20 + 10]

Q2. Within the framework proposed by Green and Stokey (Journal of Political Economy, 1983), show that as long as the marginal distribution function for the common shock is unchanged, then the principal will always prefer to propose a tournament rather than offer agents individual contracts, if the number of agents becomes very large. Argue rigorously. Make sure to specify the assumptions you are making, and clarify the notation. [40]

Q3. Within the 2-agent framework proposed by Nalebuff and Stiglitz (Bell Journal of Economics, 1983), suppose there are only two possible rewards, one for winning and the other for losing. Characterise the optimal tournament, i.e., derive the agents' optimal effort choice rule, the safe income and the prize. What can you say about the nature of equilibrium as the possible variation in individual level shocks becomes small? Argue rigorously. Make sure to specify the assumptions you are making, and clarify the notation. [20 + 10]

(Back Paper)

Indian Statistical Institute
Semester-II Examination: 2014-2015
MS(QE) II: 2014-2015
Subject: Industrial Organization

Date: ~~2/4/2014~~ 2/01/2015 Maximum Marks: 100 Duration: 3 Hours

Answer ALL the questions

1. Consider the following structure. There are two firms, firm 1 and firm 2. They produce a homogeneous good X and compete in quantities. The market demand function is given by: $p = 1 - x_1 - x_2$. The production requires a key input Y, but only firm 1 has the input production technology to produce at a constant unit cost normalized to zero. Assume that final goods production requires only this input and no other inputs. So firm 1 can produce the final good at zero cost. Firm 2 however can produce the final good only if firm 1 decides to sell inputs to firm 2 and in that case firm 1 will charge an appropriate price per unit of inputs.
   (i) Suppose firm 1 supplies inputs to firm 2 at an appropriate input price. Find the optimal input price and the corresponding market price of the final product.
   (ii) If firm 1 likes, it can foreclose the production of firm 2 by not supplying inputs to it at all. Is it optimal for firm 1 to foreclose firm 2's production? Find the optimal product price in this case.

   [30]

2. Suppose the market demand function is of the form $P = a - bQ; a, b > 0$. Firm 1 uses an old production technology which has marginal cost of Rs 15. Firm 2 uses a modern technology with marginal cost of Rs 10. In the current equilibrium the product price and aggregate output are respectively Rs 16.66 and 8.33. Then how much would firm 1 be willing to pay for the modern technology? Derive the effect on welfare when both firms use modern technology.

   [12+8=20]

3. Consider a linear city of length 1. Consumers are uniformly distributed over the length of the city and each consumer buys exactly one unit of a product supplied by two firms, $i=1,2$. Each consumer is to pay a price for the product and a transport cost proportional to the square of the distance (i.e., td^2, where t is the transportation cost per unit distance and d is the consumer's distance to the selected shop). Assume that there is no production cost and that firm 1 is already located at one edge of the city. Now firm 2 is to decide its location in the city. Finally, they compete in prices. Find the optimal location of firm 2 and the prices the firms will charge for their products.

[20]

4. Suppose that there are n number of firms producing a homogeneous good and competing in quantities. Let k be the number of firms ($k \leq n$) which form a cartel and act as a Stackelberg leader to set quantities before the remaining firms decide quantities simultaneously. Assume that both the market demand function and the cost function are linear, with fixed cost equal to zero. Derive the condition(s) for which the cartel will be stable in the sense that no firms will have any further incentive to enter or leave the cartel.

[30]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2014-15

Course Name: M.S. (Q.E.) II YEAR / M.STAT. II YEAR
Subject Name: Social Choice and Political Economy
Date: 23-2-15 Maximum Marks: 50 Duration: 3 hours

Problem 1. Consider a finite set of alternatives A and a domain $\mathcal{D}$ of preferences over A such that for all $a, b \in A$, there exists $P \in \mathcal{D}$ with $r_1(P) = a, r_2(P) = b$. Is the domain $\mathcal{D}^n; n \geq 2$ a dictatorial domain? Justify your answer.

(10)

Problem 2. Let $\mathcal{S}$ be the domain of single-peaked preferences over a finite set of alternatives $A = \{a_1, \ldots, a_m\}$. Two preference profiles $(P_1, \ldots, P_n), (P'_1, \ldots, P'_n) \in \mathcal{S}^n$ are tops-equivalent if $r_1(P_i) = r_1(P'_i)$ for all $i = 1, \ldots, n$. A social choice function $f : \mathcal{S}^n \to A$ satisfies tops-only property if $f(P_1, \ldots, P_n) = f(P'_1, \ldots, P'_n)$ for all tops-equivalent profiles $(P_1, \ldots, P_n), (P'_1, \ldots, P'_n) \in \mathcal{S}^n$. Prove that any unanimous and strategy-proof rule defined on $\mathcal{S}^n$ satisfies tops-only property.

(10)

Problem 3. Consider a social choice environment with the set of players $N = \{1, 2\}$ and the set of finitely many alternatives $A = \{a_1, \ldots, a_m\}$. Let $\mathcal{P}$ be a domain of preferences over A. For a strategy-proof and unanimous social choice function $f : \mathcal{P}^2 \to A$, the option set of a player $i \in N$ at a preference P_j of the other player $j \neq i$, denoted by $O_i(P_j)$, is defined as

$$O_i(P_j) = \{a \in A : f(P_i, P_j) = a \text{ for some } P_i \in \mathcal{P}\}.$$

Show that if $\mathcal{P}$ is the domain of single-peaked preferences $\mathcal{S}$ over A then $x \in O_i(P_j)$ and yP_jx imply $y \in O_i(P_j)$ for all $i, j \in N$ and $x, y \in A$. Find a domain of preferences over A for which this fact is not true.

(10)

Problem 4. Justify your answer by a proof or a counterexample.
(a) Monotonicity implies strategy-proofness for any domain of preference profiles.
(b) Strategy-proofness and unanimity imply efficiency for any domain of preference profiles.

(10)

Problem 5. Let $\mathcal{S}$ be the domain of single-peaked preferences over a set of alternatives $A = \{a_1, \ldots, a_m\}$. For a preference $P \in \mathcal{S}$, the reversed preference of P, denoted by $\hat{P}$, is defined as $r_i(\hat{P}) = r_{m-i+1}(P)$. Let $\hat{\mathcal{S}}$ be the domain of reversed single-peaked preferences, i.e.,

$$\hat{\mathcal{S}} = \{\hat{P} : P \in \mathcal{S}\}.$$

Justify your answer by a proof or a counterexample.
(a) Every strategy-proof and unanimous rule defined on $\hat{\mathcal{S}}^n$ satisfies tops-only property.
(b) Every strategy-proof and onto rule defined over $(\mathcal{S} \cup \hat{\mathcal{S}})^n$ is dictatorial.

(10)

Indian Statistical Institute
MSQE II
Economic Development
Mid-Term Examination

Date: 25 Feb 2015
Time: 2 hours
Maximum Marks: 40

*Answer **question 1 and any one** from questions 2 and 3*

1. Consider an economy where agents live for two periods. In the first period they can acquire t units of education where t is a choice variable and $t\epsilon[0,1]$. The total cost of acquiring t units of education is $C(t)$ with $C'(t) > 0, C''(t) > 0$. An agent acquiring t units of education in the first period earns $(1-t)w_u$ in the first period and $w_u + t(w_n - w_u)$ in the second period. Assume that $w_n > 2w_u$. Consumption and bequests take place at the end of the second period. An agent chooses his bequest (b) and consumption (c) to maximize his utility given by $U = c^{\alpha}b^{1-\alpha}$. At the beginning of his life an agent is endowed with an inheritance x.

   (a) First assume that there is a perfect credit market where an agent can borrow or lend as much as he wants to and the borrowing rate is equal to the lending rate. Find the level of education t acquired by an agent as a function of his inheritance. Also derive the long term distribution of wealth in the economy, for any given distribution of initial wealth.
   (b) Now assume that the credit market is imperfect, that is, the borrowing rate is greater than the lending rate. Find acquired education t as a function of inheritance and the long term distribution of wealth.

   [10 + 10]

2. Consider a variant of Krugman's history versus expectations model where the return in the increasing returns sector depends not only on how much capital has moved to that sector but also on the state of nature which is observed by the agents with a noise. Show that there is a unique signal x^* such that an agent receiving a signal $x \geq x^*$ must move to the increasing returns sector. What is the implication of this result?

   [17+3]

3. In a model of endogenous growth and electoral democracy where only capital is taxed, show that an increase in the inequality in capital holding leads to a fall in the rate of growth.

   [20]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2014-15

Course name: MSQE II

Subject name: Selected Topics I (Auction Theory)

Date: 27-2-15.

Maximum marks: 30

Duration: 2 hours

Consider the sealed-bid, independent private value auction model. There are N bidders. Values are drawn from the set $[0, \overline{x}]$ according to the continuously differentiable, increasing distribution function F.

(I) Suppose the seller wishes to sell using a first-price auction.

Show that the symmetric equilibrium bid function, assuming bidders with 0 value submit a 0 bid, is $\beta(x) = E(Y_1|Y_1 < x)$, where Y_1 is the highest-order statistic of $N-1$ independent draws from the distribution F, i.e., the highest of the values of $N-1$ bidders. [10]

(II) Show that the equilbrium bid function can also be represented as $\beta(x) = x - \frac{\int_0^x F^{N-1}(y)dy}{F^{N-1}(x)}$. [5]

(III) What is the expected revenue to the seller, assuming bidders follow the bid strategy derived above? [5]

(IV) What happens to expected seller revenue as the number of bidders becomes very large? [5]

(V) Now suppose the seller wishes to sell using a second-price auction.

Propose an asymmetric equilibrium in which all bidders submit different bids. [5]

INDIAN STATISTICAL INSTITUTE

Mid-semestral examination: (2014-2015)

MS(QE) II and MSTAT II

Game Theory II

Date: 02. 03. 2015 Maximum Marks: 40 Duration: 3 hrs.

(1) Consider the problem where a monopolist sells a good to a set of consumers, each of whom has utility function of the form $U(T, q; \theta) = \theta V(q) - T$, with $V(0) = 0$, $V'(q) > 0 > V''(q)$ for all $q > 0$. Profit of the monopolist is given by $\Pi = T - cq$. Derive the first best and the second best mechanisms for the two-type case. **(10)**

(2) Consider a bargaining problem between one buyer and one seller for a single indivisible good. Individual 1 owns the object and individual 2 wants to buy the object. Let θ_1 and θ_2 denote the values of the object to the seller and the buyer respectively. Assume that θ_1 and θ_2 are independent random variables distributed over the intervals $[a_1, b_1]$ and $[a_2, b_2]$ respectively. Assume that $0 < a_i < b_i < \infty$ for $i = 1, 2$. Let $f_i(.)$ denote the probability density function for θ_i $(i = 1, 2)$. Assume that $f_i(.)$ is continuous and positive on its domain $[a_i, b_i]$. Let $F_i(.)$ be the cumulative distribution function. Answer the following questions.

(a) Define Bayesian incentive compatibility and individual rationality of a direct bargaining mechanism (p, x). **(2+2=4)**

(b) Define the "virtual" type of the seller as $z_1(\theta_1) = \theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}$ and that of the buyer as $z_2(\theta_2) = \theta_2 - \frac{1-F_2(\theta_2)}{f_2(\theta_2)}$. If a mechanism (p, x) is incentive compatible and individually rational, then show that

$$\int_{a_2}^{b_2} \int_{a_1}^{b_1} [z_2(\theta_2) - z_1(\theta_1)]\, p(\theta_1, \theta_2) dF_1(\theta_1) dF_2(\theta_2) \geq 0. \quad \textbf{(12)}$$

(3) Consider the problem of dissolving a partnership when there are n agents indexed by $i \in N = \{1, \ldots, n\}$. For each $i \in N$, r_i is the share of the good to be traded to agent i and θ_i is the valuation of the good to agent i and θ_i lies in the interval $[\underline{\theta}, \overline{\theta}]$ for all $i \in N$. We assume that θ_i is private information to agent i and that the interval $[\underline{\theta}, \overline{\theta}]$ from which each agent's type is drawn is common knowledge. More specifically, it is also common knowledge that the valuations are drawn independently from a distribution F with support $[\underline{\theta}, \overline{\theta}]$ and this distribution has positive continuous density.

(a) Define Bayesian incentive compatibility of a trading mechanism (s, t). **(2)**

(b) Show that a trading mechanism (s, t) is Bayesian incentive compatible if and only if for all $i \in N$,

(B1) $S_i(\theta_i)$ is non-decreasing, and

(B2) $T_i(\theta_i') - T_i(\theta_i) = \int_{x=\theta_i'}^{x=\theta_i} x dS_i(x)$ for all $\theta_i, \theta_i' \in [\underline{\theta}, \overline{\theta}]$. **(12)**

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2014-15

Course: Masters in Quantitative Economics Year II

Subject: Political Economy and Comparative Systems

Date: 4 March 2015 **Maximum Marks:** 100 **Duration:** 3 hours

Answer all 3 questions. All questions carry equal marks.

1. Consider a toll road run by a company (C). The demand for the services of this toll road is given by the inverse demand function $p = A - q$, where p is the toll rate (price) charged and q is the size of the population using the road; $A > 0$. C has to first declare a toll charge $\tilde{p}$. Subsequently a popular organization representing road users, D, contests the price declared. Thus, the actual price received by C is $\tilde{p}$ with the probability $(\frac{x_C}{x_C + x_D})$ if $x \equiv x_C + x_D > 0$, and with probability $\frac{1}{2}$ otherwise; C receives nothing (price 0) with the remaining probability; where x_C is the amount spent by C on enforcing the toll charge and x_D is the amount spent by D on evading it. Road usage is given according to the demand function on the basis of the actual price (either $\tilde{p}$ or 0) determined probabilistically as the outcome of the contest. Thus, road usage level is also stochastic *a priori*. The company maximizes its expected profit net of its conflict expenditure x_C, while the users' organization maximizes total expected consumers' surplus net of its conflict expenditure x_D. There are no costs to providing road services.

   (a) How does the (sub-game perfect Nash) equilibrium declared price $\tilde{p}$ compare with that without conflict, i.e., the monopoly price obtaining in the absence of D?

   (b) How do the equilibrium declared price, the probability of its enforcement, and total conflict expenditure respond to an increase in the demand parameter (A)?

2. Consider a population of measure 1, with individual status quo incomes distributed uniformly in the unit interval. Revolutionary coalitions can only obtain if they are of the form $= [x_1, x_2] \cup [x_3, x_4]$ for some set of income levels $\{x_1, x_2, x_3, x_4 | 0 \leq x_4 < x_3 \leq x_2 \leq x_3 < x_1 \leq 1\}$. The regime can impose a punishment $d_i \in [0, I_i]$, i.e. an income loss up to her entire income, on any individual member i of the revolutionary coalition in case the attempt at revolution fails. The probability of success of a revolutionary coalition S is given by $P_S = n_S(1 - m_S)$, where n_S is the size of the coalition S and m_S its mean income. Assume that a revolutionary coalition is formable iff its total expected net income from attempted revolution is at least as much as its total income in the status quo, and that any alternative income distribution offered to the members of a revolutionary coalition by the revolutionary entrepreneur must provide at least as much net expected income from participation to every member as her income in the status quo. The objective of the regime is to minimize the probability of revolution, while that of the revolutionary entrepreneur is to maximize it. The regime first chooses the punishment schedule; the revolutionary entrepreneur subsequently chooses the revolutionary coalition.

(a) Find the equilibrium revolutionary coalition, its probability of success, and the alternative income distribution offered that sustains it.

(b) Now suppose the income distribution changes to the following: some proportion $\mu \in (\frac{1}{2}, 1)$ have a status quo income $I_L \in (0,1)$, while the remaining proportion have a status quo income $I_H \in (I_L, 1)$; $\mu I_L + (1 - \mu) I_H = 1$. Repeat the exercise in (a). Explain how your results change if, given I_L, either μ or I_H changes, with the other variable adjusting according to the budget constraint.

3. Consider the two-period model of democratization discussed in class. Suppose that the model is extended in the following manner. If democratization occurs in period 1, then the elite may engage in a military coup to reverse it in period 2. In case of a military coup in period 2, $(1 - z) \in (0,1)$ proportion of total income for that period is destroyed regardless of whether the coup succeeds. All remaining income reverts to the elite if the coup is successful; all remaining income is equally distributed in the society if the coup fails. An attempted coup succeeds with probability p. A coup cannot occur if either the poor engage in revolution or the elite do not democratize in period 1. Explain how changes in (a) the probability of a coup, p, and (b) the rate of loss due to a coup $(1 - z)$, affect the Markov perfect equilibrium of this game.

Indian Statistical Institute
Mid Semestral Examination: (2014 – 2015)
M.S.(QE) – II year
Econometric Applications II

Date: 04.03.2015 *Maximum Marks – 60* *Duration: 2 hours*

Answer any ***two*** questions.

1. a) Describe the different approaches to the specification of an income distribution.

   b) Suppose the income variate x follows a Gamma distribution with density function

   $$f(x) = \frac{1}{\Gamma\alpha}\ e^{-x}\ x^{\alpha-1}, \quad x \geq 0$$

   Find the Lorenz curve and Lorenz Ratio.

   c) Suppose the distribution function of an income distribution is given by $F(x) = \frac{x^{\alpha}}{x_0^{\alpha} + x^{\alpha}}$. What is the median of the distribution? Why is it called the *sech square* distribution?

   d) Find the Lorenz Curve for the distribution in (c).

   e) Find the 'Failure rate' and 'Proportionate Failure rate' for Pareto distribution.

   [5 + 8 + 6 + 6 + 5 = 30]

2. a) What are 'relative' and 'absolute' measures of income inequality? Give examples.

   b) State and explain the desirable properties that an inequality index should satisfy.

   c) What is an 'intermediate' inequality index? Why is it called a 'centrist' measure? Explain through an example with reference to the 'leftist' and 'rightist' measures.

   d) Given the functional from of the Lorenz Curve $L(F,p) = p - ap^{c_1}(1-p)^{c_2}$, $a > 0,\ 0 < c_1 \leq 1,\ 0 < c_2 \leq 1$, show that it is strictly increasing in p.

   e) Explain how the Kernel density estimation procedure can be viewed as an extension of the Histogram concept.

   [4 + 8 + 8 + 4 + 6 = 30]

3. a) Explain the 'Direct method' and 'Income method' of poverty measurement.

b) Suppose some income is transferred from a poor person to a non poor person, other things remaining the same. What is expected to happen to a poverty measure? [state the axiom you use]. Examine the 'head count ratio' and 'income gap ratio' measures in light of this axiom.

c) In the context of measurement of poverty, state the axioms (i) Replication invariance, (ii) Subgroup consistency and (iii) Decomposability.

d) Consider the income profile $x_1 \leq x_2 \ldots \leq x_n$ and the poverty measure $P = \frac{1}{n}\sum_{i=1}^{q}(1 - \frac{x_i}{z})^{\alpha}$, where q: number of poor; z: poverty line; and $\alpha = 0,1,2$.

Examine the index in light of the axioms in (c) for different values of α.

[4 +5 + 6 + 15 = 30]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2014-15

Course: Master of Science in Quantitative Economics Year II

Subject: Political Economy and Comparative Systems

Date: 24/04/15 **Maximum Marks:** 100 **Duration:** 3 hours

ANSWER ANY TWO QUESTIONS (All questions carry equal marks).

1. Suppose a worker's expected utility is given by:

$$u = (1 - p^d)w - \frac{L^2}{2};$$

where $L \in [0,1]$ is the effort actually provided, w is the wage rate, and p^d is the probability of being dismissed, with $p^d = (1 - L)p^0(s)$, where s is the expenditure on supervisory inputs: $p^0(s) = \frac{s}{1+s}$. In case of dismissal, the worker receives 0 wage. A profit-maximizing capitalist hires exactly one worker to produce an output, using a production technology $Q = \varphi L^{\frac{1}{2}}$, where Q is the level of output, L is the labour effort actually provided by the worker, and φ is some positive productivity parameter. The capitalist is a monopolist who faces the demand function $Q = 2B - p$, where p is the price charged and B is a demand parameter, with $\varphi > 2B, B \geq 1.25$.

(a) Show how the equilibrium supervision cost, wage paid, share of supervision cost in total production cost (i.e., $\frac{s}{w+s}$) and profit change with increase in the productivity parameter φ.

(b) Show how the equilibrium values of the variables specified in part (a) change with a marginal increase in the demand parameter B, given the productivity parameter φ.

P.T.O

_. Consider a society with two income levels: $I_H > I_L > 0$. Average income in this society is $\bar{I}$, total population is n and income class $i \in \{H, L\}$ has n_i members, with $n_L \equiv \rho n$, $\rho \in (0,1)$. Individuals have identical preferences, represented by the utility function $\ln x + \ln y$, where x is the amount of a private good consumed by the individual and y is the total amount of a public good generated by voluntary contributions. Define $I_H \equiv I(1+\theta), \theta > 0$.

(i) Find the range of values of θ for which individual consumption bundles in the Cournot-Nash equilibrium are independent of the income distribution. Show how this range changes with a change in: (a) the total population (n), and (b) the population distribution (ρ). Explain your results.

(ii) Now (a) find the real incomes of both income classes: (b) find the real impact of a balanced budget redistribution proposal to increase all P nominal incomes marginally, and (c) show how this magnitude changes with increases in θ and ρ.

3. Consider a linguistic minority (N) of size $n \in (0, \frac{1}{2})$. This community's cost of assimilation to majority (M) linguistic norms is uniformly distributed in the $[0, k]$ interval: $k \in (0,1)$. For any minority individual i, the return from adopting M's linguistic norms is $\theta_M(1 - c_i)$, where θ_M is the proportion of the population that adopts M norms, and c_i is that individual's cost of assimilation. For such an individual, the return from persisting with N norms is $(1 - \theta_M)$.

(a) Suppose each community comes to be led by a warlord who allocates a fixed proportion of the community's total income, b, to consumption of each community member, and the remaining proportion $(1 - bs_j)$ to war against the other community: where s_j is the size of community j. What happens to total military expenditure as the size of the minority population, n, increases? Assume $b > \frac{k}{2}$ and that the size of the majority population is $(1 - n)$.

(Hint: first find the proportion of the minority community which assimilates in equilibrium and the resultant income distribution.)

(b) Now suppose the state can develop and impose a third set of cultural-linguistic norms, with inputs from both communities. Thus, for community N, the cost of assimilating to this set of norms is uniformly distributed in $[0, kt]$, while it is uniformly distributed in $[0, k(1 - t)]$ for community M, where $k \in (0,1), t \in [0,1]$. Thus, a higher value of t implies that the state's official norms are closer to those of the majority. Find the value of t that maximizes total income in society. How does this value change with changes in the size of the minority?

Semestral Examination

Economic Development I

MSQE II

Maximum Marks: 60

Time: 3 hours

Date: 27.4.2015

Answer any three questions

1. An economy produces a single good using a single factor. The size of the population is normalized to unity. At any period t two technologies are available. In technology A, one unit of output can be produced by $(a - \lambda_{t-1})$ units of input where a is a constant and λ_{t-1} is the proportion of the population using technology A in period t-1. Similarly for technology B, one unit of output can be produced by $(b - (1 - \lambda_{t-1}))$ units of input. The population consists of μ informed and $(1 - \mu)$ uninformed agents. The informed agents at any period t choose that technology which has the lower cost and if the two technologies have the same cost, they choose technology A. A fraction λ_{t-1} of the uninformed agents chooses technology A at period t and the rest chooses technology B.

(a) Write down λ_t as a function of λ_{t-1} and μ.

(b) Show that if $\lambda_0 \geq (a - b + 1)/2$, all agents eventually use technology A, and if $\lambda_0 < (a-b+1)/2$, all agents eventually converge to using technology B, where λ_0 is the initial value of λ.

[7 + 13]

2. Consider an informal insurance arrangement between two individuals A and B. Each individual has an uncertain income stream over an infinite horizon. In particular, for each person, in each period, income can take a value $y > 0$ with probability p $(0 < p < 1)$ and a value 0 with probability (1-p). Every period, each person has a strictly increasing and concave utility function U(y) with $U(0) = 0$. Also the individuals have a common discount rate $r > 0$.

P.T.O

(a) Define the first best insurance contract and show that it is implementable if the rate of discount is sufficiently small.

(b) Show that no insurance contract is implementable if the agents are risk neutral having a common utility function $U(y) = y$. Find the intuition behind the result.

[10 + 10]

3. Give Marshall's argument as to why share tenancy might be an inefficient arrangement. Justify the existence of share tenancy as an optimal contract when the work effort of the tenant cannot be observed by the landlord.

[5+15]

4. Show, in terms of a suitable model, that imperfect property rights might impede pledged collateral to mitigate moral hazard problems in a credit market. What kind of government intervention would be necessary in this context?

[15+5]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2014-2015)

MS(QE) II & MSTAT II

Game Theory II

Date: 21.04.2015 Maximum Marks: 100 Duration: 3 hrs.

Note: Answer all questions.

(1) Define cohesiveness and super-additivity of a game in coalition form. Show that if a game is super-additive, then it is cohesive. Identify a cohesive game which is not super-additive. **(4+4+4=12)**

(2) An expedition of $n \geq 3$ people has discovered treasure in the mountains; each pair of them can carry out one piece. A cooperative game in coalition form that models this situation is v, where

$$v(S) = \begin{cases} |S|/2 & \text{if } |S| \text{ is an even number,} \\ (|S|-1)/2 & \text{if } |S| \text{ is an odd number.} \end{cases}$$

Find the core of this game. (Given any set X, $|X|$ denotes the cardinality of the set X.) **(20)**

(3) Define balancedness of a cooperative game in coalition form. Let v^* be a cooperative game in coalitional form such that $v^*(S)/|S| \leq v^*(N)/|N|$ for all non-empty $S \subseteq N$. Is v^* balanced? Justify your answer. **(5+8=13)**

(4) Define the Weber set. Show that core is a subset of the Weber set. **(5+10=15)**

(5) "If a two-person game has a non-empty core, then the Shapley value belongs to the core". Is the statement correct? Justify your answer. **(10)**

(6) Consider a cooperative game in coalition form v. Define the dual of the game v. Prove or disprove the following statement: The Shapley value of a game coincides with that of its dual game. **(5+10=15)**

(7) Let $N = \{A_1, \ldots, A_n\}$ be the set of players. For each non-empty set $S \subseteq N$, define the unanimity game. Given N, show that the set of all unanimity games forms a basis of the set of all possible cooperative games in coalition form G^N. **(15)**

INDIAN STATISTICAL INSTITUTE

Final Examination: 2014-15

Course name: MSQE II
Subject name: Selected Topics (Auction Theory)
Date: 02.05.2015
Maximum marks: 50
Duration: 3 hours

Answer all questions

Q1. Derive rigorously the symmetric equilibrium bid function in the first-price (sealed-bid, winner-pay) auction, given a common value environment. [15]

Q2. There are two bidders with common valuation for an object. Bidder i's private signal is S_i, where S_i is uniformly distributed on $[0,1]$. The value of the object is $V = S_1 + S_2$.

Consider sealed-bid, winner-pay auctions.

a) Find the symmetric equilibrium bid function and expected seller revenue if a first-price auction is used. [5 + 5]

b) Find the symmetric equilibrium bid function and expected seller revenue if a second-price auction is used. [5 + 5]

Q3. There are two bidders, whose private valuations are independently distributed uniformly on $[0,1]$. The seller announced the following rules.

- There is a reserve price $r \in (0,1)$: no bid below r will be accepted.
- Bidder 1 has to submit a bid $b_1 \in (r,1)$ or withdraw.
- If bidder 1 does not submit a bid, bidder 2 can obtain the item and pay r. If he does not wish to obtain the object, it remains unsold, and the seller gets no revenue.
- If bidder 1 submits a bid, bidder 2 can match it, i.e., win the item and pay b_1. If bidder 2 does not match b_1, given that bidder 1 has submitted a bid, then bidder 1 wins, and pays b_1.

  What r and b_1 should the seller choose to maximise expected revenue? [15]

INDIAN STATISTICAL INSTITUTE

Final-Semestral Examination : 2014-15

Course Name: M.S. (Q.E.) II YEAR / M.STAT. II YEAR
Subject Name: Social Choice and Political Economy
Date: 7.5.15 Maximum Marks: 50 Duration: 3 hours

Problem 1. Justify your answer by a proof or a counterexample.
In one sided matching model
a) there is no mechanism other than Top Trading Cycle (TTC) mechanism that is strategy-proof and individually rational,
b) there is no mechanism other than Top Trading Cycle mechanism that is efficient and individually rational,
c) individual rationality and efficiency imply stability,
d) Fixed Priority Top Trading Cycle (FPTTC) mechanisms are the only strategy-proof and efficient mechanisms.

(20)

Problem 2. Write down the condition on the priorities of the houses under which Fixed Priority Top Trading Cycle mechanism is individually rational.

(5)

Problem 3. Justify your answer by a proof or a counterexample.
a) Every strategy-proof and unanimous random social choice function is a probability mixture (convex combination) of strategy-proof and unanimous deterministic social choice functions.

(10)

Problem 4. Justify your answer by a proof or a counterexample.
a) In every domain 2-cycle monotonicity implies k-cycle monotonicity for all $k = 1, 2, 3, \ldots$
b) In every domain affine maximizer allocation rule satisfies k-cycle monotonicity for all $k = 1, 2, 3, \ldots$
c) In the single peaked domain every implementable rule is affine maximizer.

(15)

Back Paper

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2014-15

13.07.15

Course: Master of Science in Quantitative Economics Year II

Subject: Political Economy and Comparative Systems

Maximum Marks: 100 **Duration:** 3 hours

ANSWER ALL QUESTIONS (All questions carry equal marks).

1. Using the model of competitive wage determination with worker-side moral hazard and costly monitoring discussed in class, identify how, and the exact sense in which, technology choice may turn out to be socially inefficient under capitalist control of the workplace.

2. Using the model due to Dasgupta and Kanbur discussed in class, explain why greater philanthropy by the rich may exacerbate class antagonism against them.

3. Develop a model to explain why relative income equality across groups and the presence of meta-communal identities may *both* be necessary to hold communal tensions in check.

Back Paper

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2014-2015)

MS(QE) II & MSTAT II

Game Theory II

Note: Answer all questions.

Date: 14.07.2015 Maximum Marks: 100 Duration: 3 hrs.

(1) Define the core of a cooperative game in coalition form. Also define a balanced game. Show that a game has non-empty core if and only if it is balanced. **(3+3+24=30)**

(2) Define symmetric games in coalitional form. Find the necessary and sufficient condition for balancedness of a symmetric game. **(3+22=25)**

(3) Define the Shapley value of a cooperative game in coalition form. Find the Shapley value of the three-player majority game. **(5+10=15)**

(4) Show that Shapley value is the only value that satisfies null-player condition, symmetry, efficiency and additivity. **(30)**

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Final-Semestral Examination (Back Paper) : 2014-15

Course Name: M.S. (Q.E.) II YEAR / M.STAT. II YEAR
Subject Name: Social Choice and Political Economy
Date: 15.7.15 Maximum Marks: 50 Duration: 3 hours

Problem 1. Prove that every strategy-proof and unanimous social choice function defined on the unrestricted domain with arbitrary number of players and at least three alternatives is dictatorial.

(15)

Problem 2. Prove that the median voter rules are the only strategy-proof, unanimous, and anonymous social choice functions on the single-peaked domain.

(15)

Problem 3. Show that any probability mixture (convex combination) of strategy-proof deterministic social choice functions is strategy-proof.

(10)

Problem 4. Show that the men proposing Deferred Acceptance (DA) algorithm in the context of two sided matching problem is not strategy-proof for women.

(10)

INDIAN STATISTICAL INSTITUTE

Back paper Examination: 2014-15

Course name: MSQE II
Subject name: Selected Topics (Auction Theory)
Date: 16·07·15
Maximum marks: 100
Duration: 3 hours

Answer all questions

Q1. Derive rigorously the symmetric equilibrium bid function in the second-price (sealed-bid, winner-pay) auction, given a common value environment. [15]

Q2. Derive rigorously the symmetric equilibrium bid function in the third-price (sealed-bid, winner-pay) auction, given an independent private value environment. [15]

Q3. There are two bidders with common valuation for an object. Bidder i's private signal is X_i, where $X_i = S_i + T$, and S_1, S_2 and T are independent and uniformly distributed on $[0, 1]$. The value of the object is $V = X_1 + X_2$.

Consider sealed-bid, winner-pay auctions.

a) Find the symmetric equilbrium bid function and expected seller revenue if a first-price auction is used. [15]

b) Find the symmetric equilbrium bid function and expected seller revenue if a second-price auction is used. [15]

c) Which auction format yields the seller higher expected revenue? [10]

Q4. There are three bidders, with common valuation for an object V. It is known that V is uniformly distributed over $[0, 1]$. Additionally, given $V = v$, for all bidders, signals X_i are independently distributed uniformly over $[0, 2v]$. What is the expected seller revenue from a sealed-bid, winner-pay, second-prie auction in this environment? [30]