INDIAN STATISTICAL INSTITUTE

Analysis 3 : B. Stat 2nd year
Mid Semester Examination: 2015-16
September , 2015.

Maximum Marks 40 Maximum Time 2:30 hrs.

(1) $f : \mathbb{R}^n \to \mathbb{R}$ is called homogeneous of degree $m \in \mathbb{Z}$ if $f(tx) = t^m f(x)$ for all $t \in (0, \infty)$ and $x \in \mathbb{R}^n$. If f is homogeneous of degree m and differentiable then prove that $\sum_{i=1}^{n} x_i \frac{\partial f}{\partial x_i}(x) = mf(x)$, for all $x \in \mathbb{R}^n$. [4]

(2) Let $SO(n) = \{A \in M(n, \mathbb{R}) \mid AA^t = I_n, \det(A) = 1\}$. Prove that $SO(n)$ is a compact subset of $M(n, \mathbb{R})$. [4]

(3) Prove that there exists $\epsilon > 0$ such that for any $Y \in M(n, \mathbb{R})$ with $\|Y - I_n\|_{op} < \epsilon$ there is an $X \in M(n, \mathbb{R})$ such that $X^3 = Y$. [4]

(4) Consider the function $f : M(n, \mathbb{R}) \to M(n, \mathbb{R})$ defined by $f(A) = AA^t$. Prove that f is differentiable everywhere and compute the rank of $Df(I_n)$. [4]

(5) Let $f : U(\subset \mathbb{R}^n) \to \mathbb{R}$ be C^1 with $\nabla f(x_0) \neq 0$ for some $x_0 \in U$. Prove that there exists an open set $U_0 \subset U$ containing x_0 and a diffeomorphism $\varphi : U_0 \to V$, $V \subset \mathbb{R}^n$ such that $f \circ \varphi^{-1}(v_1, v_2, \ldots, v_n) = v_1$ for all $(v_1, v_2, \ldots v_n) \in V$. [6]

(6) Let $S = \{A \in M(2, \mathbb{R}) \mid \det(A) = 1\}$ and view it as a subset of $\mathbb{R}^4$. Use the method of Lagrange multiplier to find $d(0, S)$ where d denotes the Euclidean metric on $\mathbb{R}^4$. [6]

(7) let $f : U(\subset \mathbb{R}^n) \to \mathbb{R}$ be a C^2 function and $a \in U$ be a stationary point of f. If all the eigenvalues of the Hessian matrix $H(a)$ are negative then prove that f has a local maximum at a. [6]

(8) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = xye^{-(x^2+y^2)}$. Identify the points of local maximum, local minimum and the saddle points. Also find the points of absolute maximum and absolute minimum. [8]

INDIAN STATISTICAL INSTITUTE

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B. STAT II YEAR

Probability III

Midsem. Exam. / Semester I 2015-16

Date: September 3, 2015

Time: 2 hours/ Maximum Score: 30

1. (3+7=10 marks)

(a) Let X be a random variable with mean 1, and variance 4. Find a suitable upper bound for $P(X \leq -5)$.

(b) Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent Geometric random variables, such that $X_i =$ number of trials required to get first success, and $E(X_i) = i+1$. Show that

$$\frac{S_n}{E(S_n)} \to 1 \quad \text{in probability, as } n \to \infty,$$

where $S_n = X_1 + \cdots + X_n$.

2. (3+4+4=11 marks)

(a) Find whether the following are the characteristic function of some random variable. Justify your answer briefly.

(i) $(1+\cos t)/2$, (ii) $1/(1+ct^2)$, $c > 0$, a constant.

(b) Let X and Y be two independent random variavles following $Unif(-1,1)$ distribution. Then the characteristic function of $X+Y$ can easily be found as $(\sin t/t)^2$. Let Z and W be two random variavles with joint density function given by

$$f(z,w) = \begin{cases} (1/4)[1 + zw(z^2 - w^2)], & \text{for } |z| < 1, \ |w| < 1, \\ 0, & \text{otherwise.} \end{cases}$$

It is given that the characteristic function of $Z+W$ is also $(\sin t/t)^2$. Find the density function of $Z+W$.

3. (4+4+4=12 marks)

(a) Find the values of α such that SLLN holds for the following sequence of independent random variables.

(i) $P(X_n = \pm\alpha^n) = (2/3)^{n+1}$, and $P(X_n = 0) = 1 - (2/3)^{n+1}$.

(ii) $P(X_n = \pm n^\alpha) = (1/(2n))$, and $P(X_n = 0) = 1 - (1/(2n))$.

(b) Let $\{X_j\}_{j\geq 1}$ be a sequence of independent random variables such that $X_j \sim$ *Normal* with mean zero and variance $= j^{1/4}$. Let $S_n = X_1 + \cdots + X_n$. Justifying your answer, can you find out whether $S_n/n^{3/4} \to 0$ almost surely, as $n \to \infty$?

All the best.

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2015-2016)

B. Stat II

Elements of Algebraic Structures

September 4, 2015 Maximum marks : 30 Duration : 2 hours 15 minutes

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1. (a) Let G be a group and H, K be subgroups. Prove that HK is a subgroup if and only if $HK = KH$. [4]

   (b) Let G be a group and H, K be subgroups such that $H \subseteq N_G(K)$, where $N_G(K)$ is the normalizer of K in G. Prove that HK is a subgroup of G. [2]

   (c) Let G be a group and H, K be subgroups such that $H \subseteq N_G(K)$. Prove that $H \cap K$ is a normal subgroup of H. Prove further that [6]

$$\frac{HK}{K} \simeq \frac{H}{H \cap K}$$

2. (a) Let G be a group and H be a subgroup of G of index 2. Prove that, for any $a \in G$ we have $a^2 \in H$. Prove further that H is normal in G. [2 + 2]

   (b) If τ is any cycle of length r in the group S_n then show that $o(\tau) = r$. Let σ be another cycle of length s such that τ, σ are disjoint. Prove that $o(\tau\sigma) = \operatorname{lcm}(r, s)$. [2+3]

   (c) Prove that the group A_4 does not have a subgroup of order 6. [3]

3. (a) Let G be a group, $x \in G$ and a be a positive integer. Assume that x is of finite order, say, $o(x) = n$. Prove that $o(x^a) = \frac{n}{\gcd(n,a)}$. [3]

   (b) Let G be a cyclic group of order n generated by x and a be a positive integer. Show that x^a is a generator of G if and only if $\gcd(n, a) = 1$. [2]

   (c) Let G be a cyclic group of order n. Show that for each positive integer a dividing n, there is a unique subgroup of G of order a (namely, the cyclic subgroup generated by $x^{\frac{n}{a}}$). [4]

   (d) Let n be a positive integer. Prove the following identity

$$n = \sum_{d|n} \varphi(d),$$

   where φ is the Euler φ-function. [3]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2015-16

Course Name : B. STAT. II Year (First Semester)

Subject Name : Physics I

Date : 05/09/2015 **Maximum Marks :** 40 **Duration :** $2\frac{1}{2}$ hours

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Answer any FOUR questions. All questions carry equal marks.

1. (a) What do you mean by an electric dipole?
(b) Find the electric field at any arbitrary point P due to an electric dipole. How the field strength gets modified when the point P is placed (i) on the axis of the dipole, and (ii) on the perpendicular bisector of the dipole? 2+(4+2+2)

2. (a) The potential of a certain charge configuration is expressed by $V = 2x + 3xy + y^2$ volts. Find the electric field at a point $(5, 2)$. Determine the acceleration experienced by an electron along the X-direction.
(b) Explain why the electric field does not exist inside a conductor under electrostatic condition.
(c) Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Evaluate the electric field at the centre of the pentagon. (3+2)+2+3

3. (a) Establish Gauss's law of electrostatics from Coulomb's law.
(b) An infinitely long cylinder of radius R is charged with a linear charge density λ. Calculate the electric field E at a distance r from the axis of the cylinder using Gauss's theorem for (i) $r > R$, (ii) $r = R$ and (ii) $r < R$. Draw the variation of E with r. 2+(3+1+2+2)

4. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.
(b) Determine the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
(c) If $\vec{A} = \vec{r}/r$, find grad div $\vec{A}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 4+4+2

5. (a) If $\vec{F} = \vec{\nabla}\phi$, where ϕ is a single-valued and has continuous partial derivatives, show that the work done in moving a particle from one point P_1 in this field to another point P_2 is independent of the path

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joining the two points.

(b) If $\phi = 2xyz^2$, $\vec{F} = xy\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$, evaluate the line integral $\int_C \vec{F} \times d\vec{r}$.

(c) Consider $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$. Determine the surface integral $\int_S \vec{F}.\hat{n}dS$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

3+3+4

INDIAN STATISTICAL INSTITUTE

Mid- Semestral Examination: 2015-16

Course Name: B. STAT. II YEAR
Subject Name: Molecular Biology

Date: 03/09/205 Maximum Marks: 40 Duration: 2 hrs 30 mins

Note: Answer any five out of first seven questions, and any five from Question no. 8 to 14. For Multiple choice questions only first attempted five will be considered. For descriptive questions best five will be considered.

Multiple choice questions: Answer any five out of seven questions (Each question carries 2 marks)

1. An investigator would be able to distinguish a solution containing RNA from one containing single stranded DNA by
   a) heating the solution to 82.5 °C and measuring the absorption of light at 260nm.
   b) comparing the Tm of each solution.
   c) measuring the absorption of light at 260nm.
   d) none of the above.
2. Which of the following enzymes unwind short stretches of DNA helix immediately ahead of a replication fork?
   a) DNA polymerase.
   b) Single-stranded binding proteins.
   c) Helicase.
   d) Topoisomerase.
3. In Polymerase chain reaction, the synthesis of DNA occurs in
   a) 3' to 5' direction.
   b) 5' to 5' direction.
   c) 3' to 3' direction.
   d) 5' to 3' direction.
4. In a Polymerase chain reaction, by mistake, ddATP was added in the reaction mixture instead of dATP. What would be the consequence?
   a) No DNA synthesis would occur.
   b) Normal DNA synthesis would occur.
   c) Synthesis would always stop at the position at which first A was incorporated.
   d) Synthesis would terminate randomly regardless of the nucleotide incorporated.
5. DNA replication
   a) is a semiconservative process.
   b) involves one origin of replication in eukaryotes.
   c) involves multiple origin of replications in prokaryotes.
   d) none of the above.
6. During DNA replication, Okazaki fragments used to elongate the
   a) lagging strand towards the replication fork.
   b) leading strand toward the replication fork.

c) lagging strand away from the replication fork.

d) leading strand away from the replication fork.

7. A Ramachandran plot describes, for a particular amino acid, the sterically permissible angles for

a) rotation about the C_α-C bond.

b) rotation about the N-C_α.

c) both a) and b) .

d) none of the above.

Descriptive questions: Answer any five out of seven questions (Each question carries 2X3 =6 marks)

8. a) It is a common observation that antiparallel strands in a β-sheet are connected by short loops, but that parallel strands are connected by α-helices. Why do you think this is?

b) Why proline is sometimes called as "helix breakers"?

9. a) Establish the Henderson-Hasselbalch equation.

b) You need to produce a buffer solution that has a pH of 5.27. You already have a solution that contains 10.0 mmol (millimoles) of acetic acid. How many millimoles of sodium acetate will you need to add to this solution? The pK_a of acetic acid is 4.75.

10. a) What are the main differences in leading and lagging strand synthesis during DNA replication?

b) Assuming that there were no time constraints on replication of the genome of a human cell, what would be the minimum number of origins that would be required? If replication had to be accomplished in a an 8 hours S-phase and replication fork moved at 50 nucleotides/second, what would be the minimum number of origins required to replicate the human genome? Assume that the human genome comprises a total of 6.4×10^9 nucleotides on 46 chromosomes).

11. a) What are the major differences between Topoisomerase I and Topoisomerase II.

b) What would you expect to happen if dideoxycytidine triphosphate (ddCTP) were added to a DNA replication reaction in large excess over the concentration of deoxycytidine triphosphate (dCTP)? Give your reasoning.

12. a) Explain why DNA is negatively charged?

b) DNA isolated from a bacteria contains 25% A, 33%T, 22%C and 20%G. Do these results strike you as peculiar? Why or why not? How might you explain these values?

13. a) What are the differences between α-helix and β-sheet protein conformations?

b) Briefly explain how telomeres are replicated?

14. a) The diploid human genome comprises 6.4×10^9 bp. What will be the length of DNA in a human cell? How the DNA of that length can fit inside a nucleus of 6 μM diameter?

b) What will be the complementary DNA sequence of ATTGGCCATGACGATG?

Indian Statisctical Institute

Mid-Semester Examination: 2015-16

Course Name: B Stat Second Year

Subject Name: Economics I (Microeconomics)

Date: 02.09.2015 **Maximum Marks: 40** **Duration: 2.5 Hours**

1.a. Explain how it is possible for the consumer to rank the members of his consumption set in accordance with his preference on the basis of the different axioms of the classical theory of consumer behaviour. **[10]**

b. Suppose $U = U(x_1, x_2)$ $x_1 > 0, x_2 > 0$ is a well behaved utility function of a consumer depicting strictly convex preference and diminishing marginal rate of substitution between the two goods on the part of the consumer. Suppose $p_1 = 1, p_2 = 2$ and $W = 40$. Now consider the following two government schemes: (i) The government makes a lump sum payment of $\bar{S} > 0$ to the consumer to augment his budget. (ii) The government gives to the producers a unit subsidy on the sales of x_1 at the rate s $(0 < s < 1)$ so that, while the sellers charge the consumer $p_1 - s$ per unit of x_1 sold, they actually receive p_1 ($(p_1 - s)$ from the consumer plus s from the government) per unit of x_1 sold. The government chooses s in such a manner that the total amount of subsidy it pays to the producers on account of their sales to the consumer under consideration is exactly $\bar{S}$. The consumer is found to choose the combination $(x_1 = 40, x_2 = 10)$ under the unit subsidy scheme, Scheme (ii). Compute the value of $\bar{S}$. **[10]**

2. a. Derive and explain the Slutsky equation. **[8]**

b. Suppose $U = x_1 + x_2$ is the utility function of a consumer. The price of x_2 and the consumer's budget are fixed at $\bar{p}_2$ and $\bar{W}$ respectively. Derive the Walrasian and Hicksian demand functions for good 1 for every $p_1 < \bar{p}_2$. Compute $\frac{\partial x_1^*}{\partial p_1}$ and $\frac{\partial h_1}{\partial p_1}$ (again for every $p_1 < \bar{p}_2$) and, hence, express $\frac{\partial x_1^*}{\partial p_1}$ in this example in terms of the Slutsky equation. **[12]**

INDIAN STATISTICAL INSTITUTE

Semestral Examination 2015-2016

B. Stat II Year

Elements of Algebraic Structures

Date: November 16, 2015 Maximum marks: 60 Duration: 4 hours

Group A (*Answer all questions*)

1. (a) Let R be a PID. Show that every non-zero prime ideal of R is maximal. [4]

   (b) Let R be a PID. Let $a \in R$ be irreducible. Show that a is a prime element. [4]

   (c) Prove that the ideal $\langle 2, 1+\sqrt{-5} \rangle$ is not a principal ideal of $\mathbb{Z}[\sqrt{-5}]$. [4]

2. (a) Let R be a ring and I, J, K be proper ideals of R. It is given that $K \subseteq I \cup J$. Prove that K must be contained in one of I, J. [3]

   (b) Show that $\langle 1+2i \rangle$ is a prime ideal in $\mathbb{Z}[i]$. [5]

   (c) Give an example of a subring of $\mathbb{Z}[X]$ which is not a Unique Factorization Domain (give complete justification). [4]

Group B (*Answer any two questions:* 6×2)

3. Let G be a group. For $a \in G$ define $\sigma_a : G \longrightarrow G$ by $\sigma_a(x) = ax$. Show that σ_a is an element of the group $S(G)$ (the set of all bijections on G). Now prove that G is isomorphic to a subgroup of $S(G)$.

4. Let G be a group of order $p^r m$, where p is a prime. Let P_1, P_2 be two Sylow p-subgroups of G. Prove that P_1 and P_2 are conjugates of each other.

5. Let G be a finite group and H be a subgroup of G such that $|H| = p^n$ for some prime p and $n \geq 1$. Prove that $[N(H) : H] \equiv [G : H] \pmod{p}$.

Group C (*Answer any two questions:* 6×2)

6. Prove that the polynomial $X^2 - 3$ is irreducible over the field $\mathbb{Q}(\sqrt{2})$. Determine the degree of the field extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.

7. Let $F \subset K \subset L$ be fields such that L is algebraic over K and K is algebraic over F. Prove that L is algebraic over F.

8. Let n be a positive integer and p be a prime. Show that there exists a field consisting of p^n elements.

[P.T.O]

Group D (*Answer any two questions:* 6×2)

9. Let G be a group of order 12. Show that either G has a unique Sylow 3-subgroup or G is isomorphic to A_4.

10. Consider the multiplicative group $S^1 = \{z \in \mathbb{C} : |z| = 1\}$. Show that S^1 is isomorphic to the group $\mathbb{R}/\mathbb{Z}$.

11. Determine the irreducibility of the following polynomials in $\mathbb{Q}[X]$:

    (a) $X^4 + 1$

    (b) $X^3 - 3X - 1$

12. Determine the splitting field of the polynomial $X^3 - 2$ over $\mathbb{Q}$.

INDIAN STATISTICAL INSTITUTE

Final- Semestral Examination: 2015-16

Course Name: B. STAT. II YEAR

Subject Name: Molecular Biology

Date: 18.11.2014 **Maximum Marks: 50** **Duration: 3 hrs**

Note: Answer any five from Group A, and any five from Group B. In Group A, only first attempted five answers will be considered. In Group B, best five answers will be considered.

Group A

Multiple choice questions: Answer any five out of seven questions (Each question carries 2 marks)

1. The first mRNA codon to specify an amino acid is generally

   (a) TAG

   (b) UGG

   (c) AUG

   (d) UAG

2. Human blood type is determined by I^A, I^B and I alleles. The I^A and I^B alleles are codominant, and the i allele is recessive.

   The possible phenotype for human blood group are type A, type B, type AB and type O. Type A and B individuals can be either homozygous or heterozygous.

   A woman with type A blood and a man with type B blood could potentially have offspring with which of the following blood types?

   (a) type A

   (b) type B

   (c) type AB

   (d) type O

   (e) All of the above.

3. A geneticist isolates a gene for a specific trait under study. She also isolates the corresponding mRNA. Upon comparison, the mRNA is found to contain 1,000 fewer bases than the DNA sequence. Did the geneticist isolate the wrong DNA?

(a) yes, mRNA is made from a DNA template and should be the same length as the gene sequence

(b) yes, the mRNA should contain more bases than the DNA sequence because bases flanking the gene are also transcribed

(c) no, the final mRNA contains only exons, the introns were removed

(d) no, the mRNA was partially degraded after it was transcribed

4. Transcription and translation of a gene composed of 300 nucleotides would form a protein containing no more than ___ amino acids.

(a) 100

(b) 150

(c) 300

(d) 50

5. Consider a group of 100 individuals (50 couples), all of whom carry a recessive disease allele. If 200 children were born to these couples, what percentage of the children would, theoretically, be carriers like their parents?

(a) 0

(b) 25

(c) 50

(d) 75

6. When a heterozygous genotype results in a phenotype that is intermediate between the two homozygous conditions, this type of inheritance is referred to as ________.

(a) complete dominance

(b) incomplete dominance

(c) codominance

(d) epistasis

7. A geneticist crosses a plant with red flowers to a plant with white flowers. The offspring include plants with red flowers (1/4), pink flowers (1/2), and white flowers (1/4). Which allele is dominant?

(a) red

(b) white

(c) neither, they are incompletely dominant

(d) neither, they are codominant

Group B

Answer any five out of the following seven questions

(Each question carries 8 marks)

8. (a) An RNA polymerase is transcribing a segment of DNA that contains the sequence

5′-GTAACGGATG-3′
3′-CATTGCCTAC-5′

If the polymerase transcribes this sequence from left to right, what will the sequence of the RNA be? What will the RNA sequence be if the polymerase moves right to left?
(b) Briefly explain tRNA charging? **[2+2+4]**

9. (a) Do you expect the following mRNA would be translated? (b) Where could translation begin in a mRNA? (c) In general, can a ribosome bind at more than one site on mRNAs? Note: AUG is the first codon.

5'- GGCCAGGAGGCUUCCAUGCGAUUGUUCAAGUGACA-3'

(d) What would be the result (in terms of the produced protein) if RNA polymerase initiated transcription one base upstream of its normal starting point and if so, what would be the scenario? What would be the result (in terms of the produced protein) if translation began one base downstream of its normal starting point and if so, what would happen? **[1+1+1+5]**

10. In humans, two abnormal conditions, cataracts in the eyes and excessive fragility in the bones, seem to depend on separate dominant genes located on different chromosomes. A man with cataract and normal bones, whose father had normal eyes, married a woman free from cataract but with fragile bones. Her father had normal bones. What is the probability that their first child will (a) be free from both abnormalities; (b) have cataract but no fragile bones; (c) have fragile bones but not cataracts; (d) have both cataracts and fragile bones? **[2+2+2+2]**

11. (a) Why is Mendel's Second Law of Genetics called the Law of Independent Assortment?

(b) The hair-form gene shows incomplete dominance. There are two alleles, curly and straight. The heterozygote has wavy hair. A man with dark (dominant), curly hair marries a woman with light, straight hair. Their daughter, who happens to have dark hair, marries a man with light, wavy hair. Answer the following questions about this dark-haired daughter and her family. (i) Draw a Punnett's square for this marriage, and predict the phenotypic ratio among the offspring of the daughter and her husband. (ii) What is the chance that they will have a child with hair just like his or her father's? **[2+6]**

12. (a) Why multiple "origin of replication" is needed for replication of entire human genome?

(b) Why transcription and translation occur simultaneously in bacterial cell but not in human cell?

(c) What are the differences in leading and lagging strand synthesis during DNA replication? **[2+3+3]**

13. (a) In cats, black color is dominant to a special, temperature-sensitive albino gene which produces cats with dark legs, faces and tails (Siamese cats). A short haired (dominant) Siamese colored female is bred to a long-haired black male. They have eight kittens: 2 black, short-haired; 2 black, long-haired; 2 Siamese, short-haired; and 2 Siamese, long-haired. What were the genotypes of the two parents?

(b) A test cross is used to determine if the genotype of a plant with the dominant phenotype is homozygous or heterozygous. (i) If the unknown is homozygous, what will be the phenotype of all offspring of the test cross? (ii) If the unknown is heterozygous, what will be the phenotype of all offspring of the test cross?

[4+2+2]

14. Consider the properties of two hypothetical genetic codes constructed with four common nucleotides: A, C, G and T.

(a) Imagine that one genetic code is constructed so that pairs of nucleotides are used as codons. How many different amino acids could such a code specify?

(b) Imagine that the other genetic code is a triplet code; that is, it uses three nucleotides to specify each amino acid. In this code, the amino acid specified by each codon depends only on the composition of the codon-not the sequence. Thus, for example, CCA, CAC, and ACC, which all have the composition C_2A (two C and one A, would encode the same amino acid. How many different amino acids could such a code specify?

(c) Would you expect the genetic codes in (a) and (b) to lead to difficulties in the process of translation, using mechanisms analogous to those used in translating the standard genetic code?

[1+3+4]

INDIAN STATISTICAL INSTITUTE

First-Semester Examination : 2015-16

Course Name : B. STAT. II Year (First Semester)

Subject Name : Physics I

Date : 18|11|2015 **Maximum Marks : 60** **Duration : 3 hours**

Answer any SIX questions. All questions carry equal marks.

1. (a) A ring of radius a contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance x from the centre.
(b) Three charges, each equal to Q, are placed at the three corners of a square of side l. Determine the electric field at the fourth corner.
(c) In a circuit 20 C of charge is passed through a battery in a given time. The plates of the battery are maintained at a potential difference of 15 V. How much work is done by the battery?
4+4+2

2. (a) A point charge Q is fixed at the point $(0, y)$ and another equal charge is fixed at the point $(0, -y)$. A point charge q can move along the X-axis. Determine the value of x for which the total force on q due to the charges Q is maximum.
(b) How can you explain that free isolated magnetic poles do not exist?
(c) Establish Ampere's circuital law when a material is conducting as well as magnetized.
5+2+3

3. (a) A current of 2 A exists in a wire of cross-sectional area 1 mm^2. If each cubic meter of the wire contains 6×10^{28} free electrons, find the drift speed of the electrons.
(b) A charge of 2 μC moves with a speed of 2×10^6 m/s along the positive X-axis. A magnetic field $\vec{B}$ of strength $(0.2\hat{j} + 0.4\hat{k})$T exists in space. Determine the magnetic force acting on the charge.
(c) A circular coil of radius 1.5 cm carries a current of 1.5 A. If the coil has 25 turns, find the magnetic field at the centre.
4+3+3

4. (a) Using Biot-Savart law find the magnetic field at the centre of a regular n-sided polygon, carrying a steady current I. From it, estimate the field at the centre of a circular loop carrying the same current.
(b) Consider an infinitely long straight wire of radius a carrying a current I. Using Ampere's circuital law calculate the magnetic field B for (i) $r > a$ and (ii) $r < a$.
(4+1)+(3+2)

5. (a) Consider a curve, passing through two end points (x_1, y_1) and (x_2, y_2), is rotating about Y-axis. Using variational principle find the curve which on revolving about the axis form a geometry of minimum surface area.
(b) Assume a system composed of two masses m_1 and m_2 suspended over a frictionless pulley of radius a and connected by a flexible string of constant length l. Find (i) the Lagrangian function and (ii) the equation of motion.
5+(2+3)

P T O

6. (a) Construct the Hamiltonian of a compound pendulum from its Lagrangian function. From this Hamiltonian, determine the canonical equations of motion.

(b) Consider the Lagrangian

$$L = \frac{1}{2}m\left(\dot{x}^2 - \omega^2 x^2\right) e^{\gamma t}$$

for the motion of a particle of mass m in one-dimension (x). The constants m, γ and ω are real and positive. Find the canonical momentum and from this construct the Hamiltonian function.

(2+3)+(2+3)

7. (a) Show that the transformation

$$q = \sqrt{2P}\sin Q$$
$$p = \sqrt{2P}\cos Q$$

is canonical.

(b) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 + \cos\theta)$$

where $0 \leq \theta \leq 2\pi$.

Find (i) the Lagrangian function and (b) the equation of motion. Neglect the friction between the bead and the wire.

4+(3+3)

Indian Statistical Institute
First Semester Examination 2015-2016
Course Name: B Stat Second Year
Subject Name: Economics I

Date: 18/11/2015 Maximum Marks: 60 Duration: 2.5 Hours

Answer all questions

1a. The cost function of a competitive firm is given by $C(q) = TVC(q) + F$, where $F > 0$ is a constant and $TVC(q)$ (total variable cost) is upward sloping and strictly convex in q with $TVC(q) = 0$. Now, consider two cases: Case 1 and Case 2. In Case 1, $C(0) = 0$ and in Case 2, $C(0) = F$. Draw the AVC, AC and MC schedules in a diagram and explain. Hence, derive the competitive firm's supply curves in the above-mentioned two cases.

b. The cost function of a representative firm in a competitive industry is given by $C = q^2 + 1 \forall q \geq 0$. Market demand function is given by $P = 100 - Q$. Derive the long run industry equilibrium. [13+7]

2. Consider two interdependent firms with identical cost functions producing a homogeneous good. The cost function is such that the marginal cost function is linear. Market demand function is also linear. If firm 1 behaves as the Stackleberg leader and firm 2 as the Stackleberg follower, their output levels are found to be $q_1 = 45$ and $q_2 = 22.5$. If firm 1 were a monopolist, its output level would have been 45. Compute the Cournot output levels of the two firms. [20]

3. Two interdependent firms in an oligopoly market are found to produce differentiated products. The cost functions of the two firms are given by $C_1 = 2.5q_1^2$ and $C_2 = 25q_2$. Market demand functions of the two firms are $p_1 = 100 - 2q_1 - q_2$ and $p_2 = 95 - 3q_2 - q_1$ respectively. These firms are players in a kinked oligopoly model. The price and quantity combinations that are found to prevail in the market are $p_1 = 70, p_2 = 55$ and $q_1 = q_2 = 10$. Derive the kinked demand function for each firm and, hence, explain why the above mentioned price and quantity combinations will prevail despite changes in cost conditions of the firms, when the change in the cost of production remains within a certain range. [20]

INDIAN STATISTICAL INSTITUTE

Statistical Methods III

Semestral Examination

B II, Semester I, 2015-16

Date: 20.11.2015 Time: 3 hours

[Total points 100. Answer as many as you can.]

1. Does the nature of the background sound have an effect on the learning of a student? Is there any difference between the effects of a constant background sound, an unpredictable sound and no sound at all? An experimenter randomly divided twenty-four students into three groups of eight. All students study a passage of text for 30 minutes. Those in group 1 study with background sound at a constant volume; those in group 2 study with noise that changes volume periodically and those in group 3 study with no sound at all. After studying, all students take a 10 point multiple choice test over the material. Their scores are as given below.

Group	Test Scores							
Constant Sound	7	4	6	8	6	6	2	9
Random sound	5	5	3	4	4	7	2	2
No Sound	2	4	7	1	2	1	5	5

Perform a one way analysis of variance test at level $\alpha = 0.05$ to test whether the means of the three groups are the same. Clearly state the necessary assumptions. [15]

2. Derive the influence functions of the mean functional and the median functional and show that the first one is an unbounded function and the second one is a bounded function. [20]

3. Suppose that $X_1, X_2, \ldots, X_n$ represent a random sample from a Poisson(λ) distribution.

   (a) Describe the likelihood ratio type, Wald type and score type tests for the hypothesis $H_0 : \lambda = \lambda_0$ against the alternative $H_0 : \lambda \neq \lambda_0$.

   (b) Suppose that we have sample size $n = 36$, and $\sum_{i=1}^{n} X_i = 189$. Perform the tests described in item (a) to test for the hypothesis $H_0 : \lambda = 5$ against $H_1 : \lambda \neq 5$ at level $\alpha = 0.05$. [5+10=15]

4. Under appropriate assumptions, find the seasonally adjusted values for the following time series using an appropriate method. (That is, find the values of the series after the seasonal component have been removed).

Year	Quarter	Series
1	I	28.4
	II	33.8
	III	37.7
	IV	32.6
2	I	32.7
	II	37.0
	III	41.3
	IV	36.5
3	I	36.0
	II	41.3
	III	45.1
	IV	40.9
4	I	40.6
	II	33.2
	III	37.4
	IV	32.2

[15]

5. (a) Suppose that in a statistical decision problem the loss function is $L(\delta, \theta) = (\delta - \theta)^2$, i.e. loss is squared error loss. Show that if $T(X)$ is an unbiased estimator of $\tau(\theta)$, such that $Var(T(X)) > 0$, then $T(X)$ is not a Bayes estimator for any prior.

   (b) Suppose that $X|\Theta$ has a Bernoulli distribution with parameter Θ, and we have a discrete prior $\pi(\cdot)$ on Θ given by $\pi(\Theta = 0) = 1/3$ and $\pi(\Theta = 1) = 2/3$. Show that the posterior distribution is given by $\pi(\Theta = 0|X = 0) = 1$ and $\pi(\Theta = 1|X = 1) = 1$. Also show that $E(\Theta|X) = X$, which is an unbiased estimator. Why does this not violate the result in part (a)? [10+10=20]

6. Assignments. [15]

B.Stat. II / Probability III

Final Exam. / Semester I 2015-16

Date - October 24, 2015 / Time - 3 hours

Maximum Score - 50

NOTE : SHOW ALL YOUR WORK TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.

1. (a) (4 marks) Let X_i follow $N(\mu_i, \sigma_i^2)$, for $i = 1, 2$. Find the Lévy distance between the two distributions, $N(\mu_i, \sigma_i^2)$, $i = 1, 2$, in terms of their means and variances.

   (b) (4 marks) Let f and $\{f_n\}$ be a sequence of nonnegative function from $\mathcal{R}$ to $\mathcal{R}$, with $\int_{\mathcal{R}} f_n(x)dx = 1 = \int_{\mathcal{R}} f(x)dx$. Show that $f_n \to f$ pointwise implies $\int_{\mathcal{R}} |f_n(x) - f(x)|dx \to 0$.

   (c) (4 marks) Show that $\cos^{3n}(t/5n)$ is a characteristic function for each integer $n \geq 1$, even when $n \to \infty$. Calculate its limit as $n \to \infty$ and identify the limiting random variable,

2. Let X_1, X_2, X_3 be three random variables defined on a probability space $(\Omega, \mathcal{F}, P)$.

   (a) (2 marks) Define the conditional expectation of X_1, given X_2 and X_3, i.e., $E(X_1|X_2, X_3)$.

   (b) (4 + 4=8 marks) When (X_1, X_2, X_3) follow a multivarate Normal with the mean vector $\mu = (\mu_1, \mu_2, \mu_3)$ and the covariance matrix Σ. Calculate $E(X_1|X_2, X_3)$ and then $E(E(X_1|X_2, X_3)|X_3)$. Check whether your findings fits with your definition in (2a).

3. Let $\{X_n\}$ be a sequence of independent random variables with $P(X_n = \pm n^\alpha) = (1/(2n))$, and $P(X_n = 0) = 1 - (1/(n))$.

   (a) (5 marks) Find the values of α such that SLLN might hold for the above sequence of independent random variables.

   (b) (5 marks) Find the values of α such that CLT might hold for appropriately normalized sequence of random variables formed from the above independent sequence of random variables.

4. Write 'TRUE' or 'FALSE' and justify your answer.

   (a) (3 marks) Let $X_n \to X$ in distribution, then $X_n - X \to 0$ in distribution.

   (b) (4 marks) Let $\{X_n\}$ be a sequence of independent random variables such that, $P(X_n = \pm 2^n) = (1/3)^{n+1}$ and $P(X_n = 0) = 1 - 2(1/3)^{n+1}$. Then $S_n/n \to 0$, in probability, where $S_n = X_1 + \cdots + X_n$.

   (c) (3 marks)
   Let X and Y be two random variables defined on a probability space $(\Omega, \mathcal{F}, P)$ with finite expectation. Let $E(XI_A) = E(YI_A)$ for each $A \in \mathcal{F}$. Then $X = Y$ almost surely.

   (d) (4 marks) Let $\{X_n\}$ be a sequence of i.i.d. random variables with mean zero and finite second moment σ^2. Then $n(\cos(\bar{X}_n) - 1)^2/\sigma^2 \to \chi_1^2$ in distribution.

P. T. O

5. Arrival of customers in a big supermarket (open for 24 hour) is assume to follow a Poisson process with intensity parameter λ. Assume after arrival each customer decides to buy a product with probability $0 < p < 1$ independently of others.

   (a) (3 marks) Show that the customers who buy a product follow a Poisson process. Find its intensity parameter.

   (b) (3 marks) Calculate the joint distribution of first k arrival of customer who buy a product given there total $n(> k)$ customer at time t.

   (c) (4 marks) Calculate the probability that there are at least k customers who buy a product before there are total $n(> k)$ customers in the shop.

All the best.

INDIAN STATISTICAL INSTITUTE

Analysis 3 : B. Stat 2nd year
End Semester Examination: 2015-16
November 27, 2015.

Maximum Marks: 60 Maximum Time: 3 hrs.

Answer all the questions. But maximum you can score is 60.

1.a) Let f be a radial, compactly supported continuous functions on $\mathbb{R}^n$. Prove that the function
$$h(x) = \int_{\mathbb{R}^n} f(x-y)e^{(1+\|y\|^2)}dy,$$
is also a radial function on $\mathbb{R}^n$. [4]

1.b) If f is a continuous, radial function on $\mathbb{R}^n$ which is homogeneous of degree α with α negative then prove that $f(x) = 0$ for all $x \in \mathbb{R}^n$. [6]

2.a) For $t > 0$, define
$$\begin{aligned} \phi(x,t) &= x, \quad 0 \le x \le \sqrt{t}, \\ &= -x + 2\sqrt{t}, \quad \sqrt{t} \le x \le 2\sqrt{t}, \\ &= 0, \quad x > 2\sqrt{t}. \end{aligned}$$

If $t < 0$ the define $\phi(x,t) = -\phi(x,|t|)$ for all $x \in \mathbb{R}$. Prove that ϕ is continuous on $\mathbb{R}^2$ and evaluate $\frac{\partial \phi}{\partial t}(x,0)$ for all $x \in \mathbb{R}$. [6]

2.b) If
$$f(t) = \int_{-1}^{1} \phi(x,t)dx,$$
then is it true that
$$f'(0) = \int_{-1}^{1} \frac{\partial \phi}{\partial t}(x,0)dx.$$
Justify your answer. [6]

[P.T.O]

3.a). Let γ be any circular arc with initial point $(0,0,0)$ and final point $(1,1,1)$. Evaluate the integral

$$\int_\gamma \omega,$$

where $\omega = 3x^2dx + 2yzdy + y^2dz$. [4]

3.b) Evaluate the integral

$$\int_\gamma (2y + \sqrt{1+x^5})dx + (5x - e^{y^2})dy,$$

where $\gamma = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 4\}$. [4]

4.a) Let $E \subset \mathbb{R}^3$ be an open set and $F : E \to \mathbb{R}^3$ be a vector field. If there exists a C^2 function $\phi : E \to \mathbb{R}$ such that $\nabla\phi = F$ then prove that $Curl\ F = 0$. [4]

4.b) If $F(x,y,z) = (x^2, xy, 1)$ then does there exist any C^2 function $\phi : \mathbb{R}^3 -> \mathbb{R}$ such that $\nabla\phi = F$ on $\mathbb{R}^3$? Justify your answer. [4]

5.a) Let $E = \mathbb{R}^3 \setminus \{0\}$ and

$$\omega = \frac{x\ dy \wedge dz + y\ dz \wedge dx + z\ dx \wedge dy}{(x^2+y^2+z^2)^{3/2}}.$$

Prove that ω is a closed form in E. [4]

b) Consider the 2-surface

$$T(u,v) = (\sin u \cos v, \sin u \sin v, \cos u), \quad u \in [0,\pi], v \in [0, 2\pi],$$

and evaluate

$$\int_T \omega.$$

[4]

c) Prove that ω is not exact and T is not boundary of any 3-chain of class C^2 in E.

[4]

6.a) If ω is a closed 1-form of class C^2 in a convex open set $E \subset \mathbb{R}^n$ then prove that ω is exact.

[8]

6.b) Let $\phi : \mathbb{R}^2 \setminus \{0\} -> \mathbb{R}^2$ is given by $\phi(u,v) = (u^2 - v^2, 2uv) = (x,y)$ and

$$\omega = \frac{x}{x^2+y^2}dy - \frac{y}{x^2+y^2}dx.$$

If ω_ϕ denotes the pullback of ω under ϕ then prove that $\omega_\phi = 2\omega$. [4]

INDIAN STATISTICAL INSTITUTE

Analysis 3 : B. Stat 2nd year
Back Paper Examination: 2015-16

14.01.2016

Maximum Marks: 100 Maximum Time: 3 hrs.

(1) For $x \in \mathbb{R}^n$ define $\|x\|_2 = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$. If $\|\ \|$ denotes a norm on $\mathbb{R}^n$ then prove that there exist positive real numbers c_1 and c_2 such that $c_1\|x\| < \|x\|_2 < c_2\|x\|$ for all $x \in \mathbb{R}^n$. hence prove that if $\|\ \|'$ is another norm on $\mathbb{R}^n$ then there exist positive real numbers c_1 and c_2 such that $c_1\|x\| < \|x\|' < c_2\|x\|$ for all $x \in \mathbb{R}^n$. [12]

(2) If $E \subset \mathbb{R}^n$ is open and $f : E \to \mathbb{R}^m$ is differentiable at $x_0 \in E$ then prove that the derivative is unique. [6]

(3) Given $A \in GL(n, \mathbb{R})$ define $f : M(n, \mathbb{R}) \to M(n, \mathbb{R})$ by $f(X) = AXA^{-1}X$ for all $X \in M(n, \mathbb{R})$. Prove that f is differentiable on $M(n, \mathbb{R})$ by evaluating the derivative. [6]

(4) Using the method of Lagrange multipliers to prove that

$$(a_1 a_2 \ldots a_n)^{1/n} < \left(\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n}\right)^{1/2},$$

for n nonnegative real numbers $a_1, a_2, \ldots a_n$. [10]

(5) Let $E = \mathbb{R}^2 \setminus \{0\}$ and consider the 1-form

$$\omega = \frac{x}{x^2+y^2}dx - \frac{y}{x^2+y^2}dy.$$

on E. Prove that ω is closed but not exact. [10]

[P.T.O]

(6) Fix $b > a > 0$ and define $\phi(r,\theta) = (r\cos\theta, r\sin\theta)$, $r \in [a,b]$ and $\theta \in [0,2\pi]$. If $\omega = x^3 dy$ then show directly (without using Stoke's theorem) that

$$\int_\phi d\omega = \int_{\partial\phi} \omega.$$

[12]

(7) Let $\phi : \mathbb{R}^2\setminus\{0\} \to \mathbb{R}^2$ is given by $\phi(r,\theta) = (r\cos\theta, r\sin\theta)$, $r \in (0,\infty)$, $\theta \in [0,2\pi]$ and $\omega = dx \wedge dy$. If ω_ϕ denotes the pullback of ω under ϕ then prove that $\omega_\phi = r\, dr \wedge d\theta$. [6]

(8) Let $D = \{(x,y) \mid x^2 + y^2 \leq r^2\}$ where r is a given positive real number. Consider the parametric representation of the upper hemisphere $\phi(x,y) = (x, y, \sqrt{r^2 - x^2 - y^2})$, $(x,y) \in D$. If $P = (0,0) \in D$ then calculate the normal $N(P)$ and the tangent plane T_p. [6]

(9) Evaluate the integral

$$\int_D e^{\frac{y-x}{y+x}} dxdy,$$

where D is the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$. [6]

(10) Consider the integral

$$\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy.$$

(a) Evaluate the integral when C is the circle $x^2 + y^2 = 1$ oriented counterclockwise. [4]

(b) Use Greens theorem and part (a) to evaluate the integral when C is the ellipse $x^2 + \frac{y^2}{4} = 1$. [6]

(11) Use the divergence theorem to evaluate the surface integral

$$\int_S F.n\, dA,$$

where $F(x,y,z) = (x + y^2, y + z^2, z + x^2)$, S is the upper hemisphere $\{(x,y,z) \mid x^2 + y^2 + z^2 = 1, z > 0\}$ and n is the unit normal. [10]

(12) Let S be the upper hemisphere $\{(x,y,z) \mid x^2 + y^2 + z^2 = 1, z > 0\}$. If $F(x,y,z) = (x^3e^y, -3x^2e^y, 0)$ then evaluate the integral

$$\int_S F.n\, dA,$$

where n is the unit normal. [6]

INDIAN STATISTICAL INSTITUTE

Semestral Examination 2015-2016 (Back Paper)

B. Stat II Year

Elements of Algebraic Structures

19/01/16 Date: Maximum marks: 100 Duration: 3 hours

Answer all questions: 10×10

1. Let G be a group of order p^r for some prime p. Prove that there is a chain of subgroups

$$(1) = H_0 \subset H_1 \subset H_2 \subset \cdots \subset H_n = G \text{ such that}$$

H_i is normal in H_{i+1} for $0 \le i \le n-1$ and $|H_i| = p^i$ for $0 \le i \le n$.

2. Prove that a group of order p^2 is abelian, where p is a prime.

3. Let G_1, G_2 be cyclic groups of order m, n respectively, where m, n are relatively prime. Prove that the direct product $G_1 \times G_2$ is cyclic of order mn.

4. Let G be a group and H, K be two normal subgroups of G such that $H \cap K = \{1\}$. Prove that HK is a subgroup of G and $HK \simeq H \times K$.

5. Let I, J be ideals of a ring R and P be a prime ideal of R. Assume that $IJ \subset P$. Prove that either $I \subset P$ or $J \subset P$.

6. Let R be a ring. An element $a \in R$ is called nilpotent if $a^k = 0$ for some $k \ge 1$. Consider the set $\mathfrak{n}$ of all nilpotent elements in R. Prove that $\mathfrak{n}$ is an ideal of R. Show further that the quotient ring $S = R/\mathfrak{n}$ does not have any nilpotent element.

7. Let R be a PID. Show that, for any ascending chain of ideals $I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots$, there is $n \ge 1$ such that $I_n = I_{n+r}$ for all $r \ge 1$.

8. Let F be a field and $p(X) \in F[X]$ be an irreducible polynomial. Show that there is a field K containing an isomorphic copy of F such that $p(X)$ has a root in K.

9. Let $F \subseteq K$ be fields such that $[K : F]$ is finite. Show that K is algebraic over F. Give an example of an algebraic extension of $\mathbb{Q}$ which is not finite.

10. Compute the degree of the splitting field of $X^4 + 4$ over $\mathbb{Q}$.

INDIAN STATISTICAL INSTITUTE

Statistical Methods III

Backpaper Examination

B II, Semester I, 2015-16

Date: 21.01.2016 Time: 3 hours

[Total points 100. Answer as many as you can.]

1. In a comparison of the cleaning action of four detergents, 20 pieces of white cloth were first soiled with India ink. The cloths were then washed under controlled conditions with 5 pieces washed by each of the detergents. Unfortunately three pieces of cloth were lost in the course of the experiment. Whiteness readings made on the 17 remaining pieces of cloth, are shown below.

Detergent

A	B	C	D
77	74	73	76
81	66	78	85
61	58	57	77
76		69	64
69		63	

Test the null hypothesis of no difference between the means of the brands as regards the whiteness measure after washing. Use level of significance $\alpha = 0.05$. Clearly state all the assumptions. [20]

2. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a Bernoulli(p) distribution. We want to test $H_0 : p = p_0$ against the not equal to alternative.

   (a) Describe the form of the likelihood ratio type, Wald type and score type tests for these hypotheses.

   (b) Suppose that we have sample size $n = 50$, and $\sum_{i=1}^{n} X_i = 27$. Perform the tests described in item (a) to test for the hypothesis $H_0 : p = 0.5$ against not equal to alternative at level $\alpha = 0.05$. What are your decisions in each case? [8+12=20]

3. Suppose that X_1, X_2 are i.i.d. Uniform$(\theta, \theta+1)$. We want to test $H_0 : \theta = 0$ versus $H_1 : \theta > 0$. Consider the following tests:

$$\phi_1(\text{Test 1}): \quad \text{Reject } H_0 \text{ if } X_1 > 0.95$$
$$\phi_2(\text{Test 2}): \quad \text{Reject } H_0 \text{ if } X_1 + X_2 > C$$

(a) Find the value of C so that the tests ϕ_1 and ϕ_2 have the same size.

(b) Using the value of C obtained in part (a), determine which of the two tests (between ϕ_1 and ϕ_2) have greater power at $\theta = 1$. [8+12=20]

4. The following time series provides the volume of beer production in Australia in relevant units over a period of four years. Under suitable assumptions, find the seasonally adjusted values for this time series using an appropriate method.

Year	Quarter	Series
1	I	257.45
	II	260.10
	III	262.83
	IV	264.68
2	I	265.41
	II	264.65
	III	262.46
	IV	260.40
3	I	261.26
	II	262.98
	III	266.18
	IV	269.23
4	I	270.51
	II	271.46
	III	272.17
	IV	274.01

[20]

5. Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from $N(\theta, \sigma^2)$ population, and suppose that the prior distribution on θ is $N(\mu, \tau^2)$, where σ^2, μ and τ^2 are known.

(a) Find the joint distribution of $\bar{X}$ and θ.

(b) Find the marginal distribution of $\bar{X}$.

(c) Find the posterior distribution of θ given the data. [6+7+7=20]

B.Stat. II / Probability III

Back Paper Exam. / Semester II 2015-16

Time - 3 hours

18.01.16 Maximum Score - 100

NOTE : SHOW ALL YOUR WORK TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.

1. (a) (6 marks) Let X_i be two random variables such that $P(X_i = j) = p_{ij}$, for i=1,2, and $j = 1, 2, 3$ with $p_{1j} = 1/3$ for all j and $(p_{21}p_{22}, p_{23}) = (2/5, 1/5, 2/5)$. Find the Lévy distance between the two distributions.

   (b) (6 marks) Let X and Y be two independent random variavles following $Unif(-1,1)$ distribution. Then the characteristic function of $X+Y$ can easily be found as $(\sin t/t)^2$. Let Z and W be two random variavles with joint density function given by

$$f(z,w) = \begin{cases} (1/4)[1 + zw(z^2 - w^2)], & \text{for } |z| < 1, \; |w| < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   It is given that the characteristic function of $Z+W$ is also $(\sin t/t)^2$. Find the density function of $Z+W$.

   (c) (6 marks) Let X and Y be two random variables defined on a probability space $(\Omega, \mathcal{F}, P)$ with finite expectation. Show that, $E(XI_A) = E(YI_A)$ for each $A \in \mathcal{F}$. Show that $X = Y$ almost surely.

2. Let Y_1, Y_2, Y_3 be three random variables defined on a probability space $(\Omega, \mathcal{F}, P)$.

   (a) (4 marks) Define the conditional expectation of Y_1, given Y_2 and Y_3, i.e., $E(Y_1|Y_2, Y_3)$.

   (b) (8 + 8=16 marks) When (Y_1, Y_2, Y_3) follow a Dirichlet distribution with parameters $(\alpha_1, \alpha_2, \alpha_3)$ Calculate $E(Y_2|Y_1, Y_3)$ and then $E(E(Y_2|Y_1, Y_3)|Y_1)$. Check whether your findings fits with your definition in (2a).

3. Let $\{X_n\}$ be a sequence of independent random variables with $P(X_n = \pm\beta^n) = (1/3)^n$, and $P(X_n = 0) = 1 - 2(1/3)^n$.

   (a) (10 marks) Find the values of β such that SLLN might hold for the above sequence of independent random variables.

P T O

(b) (10 marks) Find the values of β such that CLT might hold for appropriately normalized sequence of random variables formed from the above independent sequence of random variables.

4. Arrival of customers in two big supermarkets (open for 24 hour) I and II are assume to follow independent Poisson process with intensity parameters λ_1 and λ_2.

   (a) (6 marks) Calculate the probability that at time t the number of customers in the supermarket I is bigger than the number of customers in supermarket II.

   (b) (6 marks) Given that at time t supermarket I and II together have m customers what is the probability that the supermarket I has $k(< m)$ customers?

   (c) (8 marks) What is the probability that at time t, the total number of customers in two supermarkets together is m? Would the total arrival of customers in two supermarkets together follow a Poisson process? Justify your answer. Given that they together have m customers at time t. Find the joint distribution of their arrival times.

5. Write 'TRUE' or 'FALSE' and justify your answer.

   (a) (5 marks) $X_n \to X$ in distribution iff $E(g(X_n)) \to E(g(X))$ for every continuous and compactly supported function g.

   (b) (6 marks) Let $\{X_n\}$ be a sequence of independent and nonnegative random variables such that, $S_n/n \to 0$ in probability, where $S_n = X_1 + \cdots X_n$. Then $S_n/n \to 0$ a.s.

   (c) (5 marks)

   $\phi_n(t) = \cos^{7n}(t/8n)$ is a characteristic function for each integer $n \geq 1$, even when $n \to \infty$.

   (d) (6 marks) Let $\{X_n\}$ be a sequence of i.i.d. random variables with finite fourth moment. Then $\sqrt{n}(s^2 - \sigma^2) \to N(0, c^2)$, where s^2 and σ^2 are sample and population variance, respectively and c is a constanct depending the moments of X.

All the best.

Indian Statistical Institute
Back Paper Examination 2015-2016
Course Name: B Stat Second Year
Subject Name: Economics I

Date: 02/02/16 Maximum Marks: 100 Duration: 3 Hours

Answer the following questions

1a. Consider the optimisation problem, max $\max_{\{x_i\}} U(x_1, x_2, \ldots\ldots, x_n)$, $U_i > 0,\ x_i \geq 0\ \forall\, i$, s.t. $\sum_i r_i x_i = w,\ w > 0, r_i > 0\ \forall\, i$. Suppose the utility function is strictly quasiconcave. Derive the optimality condition for the following two cases: (i) When there is interior solution and (ii) When there is corner solution. Explain your answers with the help of diagrams.

b. Suppose x and y are two commodities in the consumption basket of an individual consumer. Let the marginal utility of x be given by $mu_x = 40 - 5x$ and marginal utility of y by $mu_y = 30 - y$. Prices of x and y are Rs.5 and Re.1 respectively. The budget of the consumer is Rs.40. (i) Derive the optimum consumption bundle the consumer will choose. (ii) How will your answer to (i) change if the consumer's budget falls to Rs.10? [25]

2a. Suppose $q = f(\underline{x}), f_i > 0, f(0) = 0$ is the production function of a perfectly competitive firm and it is strictly concave; $\underline{x}$ denotes the vector of inputs. Show that the cost function $C = C(q)$ derived from the above production function has the following properties: (i) $C(q) > 0\ \forall q > 0$, (ii) $C(q)$ is strictly increasing in q and (iii) $C(q)$ is strictly convex in q.

b. Suppose the market demand function in a perfectly competitive industry is given by $P = 100 - Q$. Suppose the market price changes from 5 units to 10 units. Compute the change in the consumer surplus. [25]

3a. A monopolist is found to sell his output in two different markets at two different prices. The demand curves in the two markets are $P_1 = 40 - 2.5q_1$ and $P_2 = 18 - 0.25q_2$ respectively. The monopolist's marginal cost of production is 10 units. Compute the profit maximising outputs and prices in the two markets.

b. A monopolist faces a market demand function $P = 40 - 0.5Q$, while the total cost function of the monopolist is $TC = 300 + 2Q$. The monopolist practises perfect price discrimination. What will the monopolist's output and profit be? [25]

4. Discuss how the price and outputs are determined in a two-firm Cournot model. [25]

INDIAN STATISTICAL INSTITUTE

Mid-semester of Second Semester Examination: 2015-2016

B. Stat. (Hons.) 2nd Year

Statistical Methods IV

Date: February 22, 2016 Maximum Marks: 35 Duration: 1 and 1/2 hours

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• This question paper carries 38 points. Answer as much as you can. However, the maximum you can score is 35.

• You should provide as much details as possible while answering a question. You must state clearly any result stated (and proved) in class you may need in order to answer a particular question.

---

1. (a) Suppose $\boldsymbol{X}_{ij}$, $j = 1, \ldots, n_i$, $i = 1, 2$, are two sets of p-dimensional observations. Let $\bar{\mathrm{X}}_i$ and $\mathbf{S}_i$ $(i = 1, 2)$ be the sample mean vector and sample dispersion matrix, based on $\boldsymbol{X}_{ij}$, $j = 1, \ldots, n_i$. Also, let $\bar{\mathrm{X}}$ and $\mathbf{S}$ be the sample mean vector and sample dispersion matrix, based on all the $n_1 + n_2$ $\boldsymbol{X}_{ij}$'s. Prove and interpret the following identity. [6+4=10]

$$n\mathbf{S} = n_1\mathbf{S}_1 + n_2\mathbf{S}_2 + \frac{n_1 n_2}{n_1 + n_2}(\bar{\mathrm{X}}_1 - \bar{\mathrm{X}}_2)(\bar{\mathrm{X}}_1 - \bar{\mathrm{X}}_2)^{\mathrm{T}}. \tag{1}$$

(b) Suppose now that $\boldsymbol{X}_{ij}$, $j = 1, \ldots, n_i$, $i = 1, 2$, are i.i.d. $\mathrm{N}_p(\boldsymbol{\mu}_i, \Sigma)$, where $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2 \in \mathbb{R}^p$ are unknown and $\boldsymbol{\Sigma}$ is an unknown p.d. matrix. Also, let all the $\boldsymbol{X}_{ij}$'s be independent. Assume that $n_i > p$ for $i = 1, 2$. We wish to develop a test for the hypothesis $\mathrm{H}_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$.

(i) Suggest, with reasons, a suitable test statistic based on (1) above. Also, describe, with reasons, the critical region. [Your statistic should be one whose null distribution can be expressed in terms of standard distributions.]

(ii) Obtain the null distribution of your test statistic or of a suitable transformation of it. [(4+3)+9=16]

2. Suppose $\alpha := \boldsymbol{d}^{\mathrm{T}}\mathbf{M}^{-1}\boldsymbol{d}$, where $\boldsymbol{d} \sim \mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\mathrm{M} \sim \mathrm{W}_p(\boldsymbol{\Sigma}, m)$, $m > p + 1$ and $\boldsymbol{\Sigma}$ is p.d., are independent. Let $\lambda := \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$. Show that

$$\mathrm{E}(\alpha) = \frac{p + \lambda}{(m - p - 1)}. \qquad [12]$$

* * * * * ** *Best of Luck!* * * * * **

Indian Statistical Institute

Mid-semester of Second Semester Examination: 2015-16

Course Name: BSTAT II

Subject Name: Economic and Official Statistics

Date: 23/2/2016 **Maximum Marks: 30** **Duration : 1 hour**

Note , if any: ***All the Questions are compulsory . For weightage calculation, marks obtained will be determined based on maximum marks as 15***

1. Explain what do mean by Factor Reversal test in Index Number Theory .Show how an index number satisfies such a test. 6

2. In the following table you are given two index number series. Splice them on the base assuming 1984=100 to complete the series. 6

year	Old Price index number (base 1975=100)	New Price Index number(base 1984=100)
1980	151.5	
1981	173.7	
1982	168.2	
1983	166.8	109.8
1984	167.1	as per assumption
1985		112.3

3. Write Short Notes on the following: a) Harmonised Commodity Description and Coding System (HS) b) Purchasing Power Parity c) Key Educational Statistics Published by MHRD, Government of India. 6*3=18

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2015-2016

Course Name: B.Stat. (Hons.) 2nd Year

Subject Name :Economic and Official Statistics and Demography

(Answer Group A and Group B on two separate answer booklets)

23-02-2016

Group B

(Answer as much as you can. Maximum you can score is 30)

Maximum Marks: 30 Time : 1 Hours

1. What do you understand by neo-Malthusianthinking. Explain briefly the statement "population pressure contributed to (induced) technological development and thus led to a rise in per capita income and production". Describe the population growth pattern in the post-independence period and explain how the neo-Malthusian thinking persisted in the period. [1 + 4 + 5 = 10]

2. Explain the logic behind the construction of the Whipple index (W). State the assumption behind the use of Sex Ratio Score (SRS) for measuring accuracy of data on population age distribution. Write the mathematical expressions for SRS along with explanations for the symbols. [4 + 2 + 4 = 10]

3. Given that the quality of age data in most developing countries is poor suggest a method of adjustment for the age distribution of a developing nation which is done by comparing with the age distribution of a suitable stable population, assuming it as a standard. [10]

4. Following data refers to the male populations of three countries A, B and C.
   (a) Calculate the crude death rate for each country.
   (b) Using the population of A as the standard, calculate the standardized death rates for B and C.
   (c) Comment on your results.

Age Group	Country A		Country B		Country C	
(in years)	Population ('000)	No. of Deaths	Population ('000)	No. of Deaths	Population ('000)	No. of Deaths
0 - 4	1,767	11,832	1,857	5,179	150	860
5 - 14	3,062	1,390	3,372	2,300	286	132
15 - 24	2,430	2,816	3,123	6,646	243	322
25 - 44	4,101	9,690	3,724	12,702	294	614
45 - 64	2,755	36,581	1,587	15,441	134	925
65 +	1,129	70,138	478	27,034	51	2343

(3 + 5 + 2 = 10)

=== END===

INDIAN STATISTICAL INSTITUTE

Mid-Semester Second Semester Examination: (2015-16)

B. Stat II Year

Agriculture

Date 24/02/16 Maximum Marks 30 Duration Three hours.

(Attempt all questions)

(Number of copies of the question paper required Five)

1 Name the weather parameters related to crop production? Write down the names of the apparatus used for measuring/estimating different weather components with their units. Calculate the amount of water to receive by Potato crop grown in 2 ha. of land if the rainfall was 54 mm. 3+4+3

2 Define Moisture Availability Index (MAI)? Draw a suitable rice calendar with the following data

2+6

Week No.	***22***	***23***	***24***	***25***	***26***	***27***	***28***	***29***	***30***	***31***	***32***	***33***	***34***	***35***	***36***	***37***	***38***	***39***	***40***
Rainfall (mm) at 0.5 Prob.	***15***	***0***	***5***	***21***	***27***	***29***	***35***	***64***	***72***	***20***	***77***	***98***	***93***	***67***	***35***	***30***	***12***	***0***	***0***
PET (mm)	***31***	***42***	***33***	***29***	***23***	***33***	***32***	***26***	***22***	***25***	***23***	***27***	***31***	***33***	***39***	***31***	***21***	***33***	***34***

3 Differentiate between: 4 x 3

a Decade rainfall and weekly rainfall
b Phytoclimate and microclimate
c Season and weather
d Evapotranspiration and Potential Evapotranspiration

INDIAN STATISTICAL INSTITUTE

Mid-Semester of Second Semester Examination: 2015-16

Course Name: B.Stat Second Year

Subject Name: Economics II

24.02.2016 Maximum Marks- 40 Duration: 2 Hours

1a. Suppose a retired person lives in his own house. The monthly imputed rent of the house is Rs.15000. He receives a pension of Rs. 45,000 per month. He has kept a part of his savings in a savings scheme in a post office and the rest as deposits in a bank. He receives from the post office and the bank interest incomes of Rs.12,000 and Rs.18,000 respectively per annum. Compute the person's contribution to (i) personal income and (ii) national income.

b. For producing current period output firm B incurs the following costs: It purchases goods worth Rs.50,000 from firm A, holds half of it in inventory and uses the other half as raw material for current production. It pays out Rs.40,000 in wages half of which is paid to a labour contractor. It pays Rs.35000 as interest to a bank, Rs.5,000 as interest on bonds sold to households, Rs.15,000 as rent to households, Rs.5,00,000 as dividend to households, Rs.20,000 in profit tax, Rs.75,000 in net indirect taxes and donates Rs.10,000 to Ramakrishna Mission. Its undistributed profit is Rs.2,00,000.

(i) Compute the intermediate input cost incurred by firm B, the value of output

produced by firm B and the value added of the same firm.

(ii) Compute firm B's contribution to national income.

(iii) Compute firm B's contribution to personal income. [10+15]

2a. How do you define aggregate domestic saving (S)? What are the components of S? Can you show if saving exceeds investment in the domestic economy, then the rest of the world investment exceeds the rest of the world saving by the same amount?

b. Suppose in an economy NNP is 1600 units, private disposable income is 1000 units, government budget deficit is 20 units, consumption is 850 units, trade deficit is 10 units, net factor income from abroad is 30 units, net transfer earning from abroad is nil and depreciation is 50 units. Using the information given above, answer the following questions:

(i) How large are S and investment (I)?

(ii) How large is government spending? [7+8]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2015-16

Course Name : B. STAT. II Year (Second Semester)

Subject Name : Physics II

Date : 24|02|2016 **Maximum Marks : 40** **Duration : $2\frac{1}{2}$ hours**

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1. (a) In a thermodynamic process how can you relate the following three quantities - amount of heat transferred, amount of change in the internal energy and the amount of work done on the system. Justify your answer with physical explanation.
(b) Rewrite your expression for a closed cycle and explain.
(c) For a mole of ideal gas at $T = 0^0C$, calculate the work done (in Joules) in an isothermal expansion from V_0 to $10V_0$ in volume.
(d) For an ideal monoatomic gas initially at $T_i = 0^0C$, and volume V_0, find the final temperature T_f (in ^0C) when the volume is expanded to $10V_0$ reversibly and adiabatically.
[2+1+(3+2+2)]

2. (a) Suppose you have a system with a finite number of particles distributed in different non-degenerate energy levels (total energy being U) which is thermally isolated from the rest of the environment. Show that the number of particles n_i in a non-degenerate energy level ϵ_i is given by $n_i = e^{-\alpha-\beta\epsilon_i}$ where α and β are two constants.
(b) Show that the entropy is given by $S = Nkln\sum_i(\frac{\epsilon_i}{kT}) + \frac{U}{kT}$ where $\beta = \frac{1}{kT}$.
[8+2]

3. (a) Suppose you have a 6×6 checkerboard. Each square of the board represents a localised spin $1/2$ particle with 2 possible orientations. How many possible microstates are there for each of the macrostates with 13 spins up and 18 spins down?
(b) Two identical blocks of metal, one at 80^0C and the other at 4^0C, are put in thermal contact.
(i) What is the total change of entropy when equilibrium is established? (Assume that the heat capacity of each block over the temperature

range is constant and neglect the volume change.)
(ii) What is the maximum amount of work that can be extracted from the hot block ?

[(2+2)+(3+3)]

4. A thermally insulated container is originally divided into two halves by a thermally insulated partition. Find the total change in entropy for the following two processes:
(a) Initially one half is occupied by n moles of an ideal gas at temperature T_0. Then the partition is removed without doing any work.
(b) Initially each half of the container is occupied by n moles of two different ideal gases at temperature T_0. Then the partition is removed without doing any work. Justify your answers.
(c) State the Carnot's theorem. From the non-decrease of entropy proof the theorem.

[3+3+(2+2)]

5. a) State Faraday's law of electromagnetic induction and express it in differential form.
(b) Calculate the induced emf in a rectangular wire in the plane of an infinitely long wire carrying a current $I(t)$.
(c) What is the basic difference between an induction generator and a motor?

[4+4+2]

Introduction to Markov Chains
B. Stat. IInd Year Semester 2
Indian Statistical Institute

Mid-semestral Examination
Time: 2 Hours Full Marks: 35
Date: February 25, 2016

This is an OPEN NOTE examination. You may use any handwritten notes that you have brought with you, but you are not allowed to share them. Also no notes in printed, photocopied, cyclostyled or any other form (including electronic storage) is allowed.
Calculators and any other computing devices are strictly not allowed.

1. Decide whether the following statements are True or False, with appropriate justifications:
   (a) If a transition matrix $\boldsymbol{P}$ is irreducible, then so is $\boldsymbol{P}^2$.
   (b) If a finite state space Markov chain has unique stationary distribution, then it must be irreducible.
   (c) If i is a positive recurrent state and $i \rightsquigarrow j$, then j is also a positive recurrent state.
   [$3\times 3 = 9$]

2. For a simple symmetric random walk in dimension 1, find the expected number of visits to the state 2016 in a cycle starting from the state 0 and continuing till the first return to the state 0. [6]

3. Consider a $D \times D$ matrix P with $\delta = \min_{i,j} p_{ij} > 0$ and $\sum_j p_{ij} = 1$. Show that, for all n, we have
   (a) $\min_{i,j} p_{ij}^{(n)} \le \min_{i,j} p_{ij}^{(n+1)} \le \max_{i,j} p_{ij}^{(n+1)} \le \max_{i,j} p_{ij}^{(n)}$;
   (b) $\min_l p_{lj}^{(n)} + \delta \left(\max_l p_{lj}^{(n)} - \min_l p_{lj}^{(n)}\right) \le p_{ij}^{(n+1)} \le \max_l p_{lj}^{(n)} - \delta \left(\max_l p_{lj}^{(n)} - \min_l p_{lj}^{(n)}\right)$;
   (c) $\max_{i,j} p_{ij}^{(n)} - \min_{i,j} p_{ij}^{(n)} \le (1-2\delta)^n$.

   <u>**Hence**</u> prove that there exists a probability vector $\boldsymbol{\pi} = \{\pi_i\}$ with all coordinates positive, such that, for all n and i,j, we have $p_{ij}^{(n)} \to \pi_j$ geometrically uniformly in i,j. [3+4+3+4=14]

4. Let $\boldsymbol{P}$ be a transition matrix with finitely many transient states and an invariant measure ν. Show that $\nu_i = 0$ for all transient states i.
   Give an example of an irreducible transient transition matrix $\boldsymbol{P}$, which has a nonzero invariant measure. [3+3=6]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination

B. Stat. - II Year (Mid-Semester - II)

Discrete Mathematics

Date : 26/02/16 Maximum Marks : 60 Duration : 2:30 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. (i) Prove that if $A = R(k, m-1)$ and $B = R(k-1, m)$ are both even then $R(k, m) \leq A + B - 1$.

   (ii) Find, with proof, the value of R(4,3).

[10+10=20]

2. Suppose that G is a triangle-free simple n-vertex graph such that every pair of nonadjacent vertices has exactly two common neighbors.

   (i) Prove that G is regular. (A graph is regular if degrees of all its vertices are same.)

   (ii) Given that G is regular of degree k, prove that the number of vertices in graph G is $1 + \binom{k+1}{2}$.

[20]

3. Prove that if $G_1, G_2, \ldots G_k$ are pairwise intersecting subtrees of a tree G, then G has a vertex that belongs to all $G_1, G_2, \ldots G_k$. [10]

4. If chromatic number of a graph G is k but chromatic number of every proper subgraph of G is less than k then G is k-critical. Prove that if H is k-critical then minimum degree of any vertex of H is greater than or equal to $k-1$.

5. Suppose $a_1 < a_2 < \ldots < a_k$ are distinct positive integers. Prove that there is a simple graph with $a_k + 1$ vertices whose *set* of distinct vertex degrees is $a_1, a_2, \ldots, a_k$. (Hint: use induction on k to construct such a graph.) [20]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2015–2016

B.Stat. (Hons.) 2nd Year. 2nd Semester

Statistical Methods IV

April 22, 2016 | Maximum Marks: 65 | Duration: 3 hours

- This question paper carries 70 points. Answer as much as you can. However, the maximum you can score is 65.
- You should provide as much details as possible while answering a question. You must state clearly any result stated (and proved) in class you may need in order to answer a particular question.

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1. Suppose $\boldsymbol{X} = (X_1, \ldots, X_p)^{\mathrm{T}} \sim \mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} \in \mathbb{R}^p$ and $\boldsymbol{\Sigma}$ are both unknown. We wish to test the hypothesis

$$\mathrm{H}_0 : X_1, X_2, \ldots, X_p \textit{ are independent} \quad \text{against} \quad \mathrm{H}_1 : \mathrm{H}_0 \textit{ is false},$$

based on n i.i.d. realizations of $\boldsymbol{X}$, denoted by $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$. Denote by $\mathbf{R}$, the sample correlation matrix. Let λ denote the likelihood ratio test (LRT) statistic for testing H_0 against H_1.

(a) Show that $-2\log\lambda = -n\log(\det \mathbf{R})$.

(b) Find, with reasons, the asymptotic distribution of $-2\log\lambda$ under H_0. [7+5 = 12]

2. Consider i.i.d. observations $X_1, \ldots, X_n$ from a Cauchy distribution with *unknown* location parameter θ and *unknown* scale parameter σ, $\theta \in \mathbb{R}, \sigma > 0$. We wish to estimate σ.

(a) Argue that a suitable estimator of σ is given by R_n, the sample semi-interquartile range.

(b) Find the asymptotic distribution of $n^{1/2}(\hat{R}_n - \sigma)$.

(c) Obtain the variance stabilising transformation for the distribution in (b) and describe how you can use it to obtain an approximate $100(1-\alpha)\%$ confidence interval for σ.

[4+6+(3+3) = 16]

[P.T.O.]

3. Suppose a multiple linear regression model is being modelled with **independent** observations denoted by $Y_1, \ldots, Y_n$ and two non-random co-variates taking values $(x_1, \ldots, x_n)$ and $(z_1, \ldots, z_n)$. Assume that the $n \times 3$ matrix

$$\mathbf{X} := \begin{pmatrix} 1 & x_1 & z_1 \\ 1 & x_2 & z_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & z_n \end{pmatrix}$$

is of full column rank. Suppose that $\mathrm{Var}(Y_i) = \sigma^2$ for $i = 1, \ldots, n$. We are interested in testing if the contribution of the covariate z is significant.

(a) Argue that

$$T := \left|\mathrm{Cov}(Y, z) - \frac{\mathrm{Cov}(x, z)}{\mathrm{Var}(x)}\mathrm{Cov}(Y, x)\right|$$

is expected to be large when the contribution of the covariate z is significant.

(b) Suggest a suitable test statistic based on T.

(c) Assuming normality of the Y_i's obtain the null distribution of your test statistic.

[10+4+4 = 18]

[Note. You may assume the following.

The set of $n \times 1$ vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, defined by

$$\mathbf{v}_1 := (1\ 1\ \cdots\ 1)^{\mathrm{T}}, \quad \mathbf{v}_2 := (x_1 - \bar{x}\ x_2 - \bar{x}\ \cdots\ x_n - \bar{x})^{\mathrm{T}},$$
$$\mathbf{v}_3 := \left(z_1 - \bar{z} - \frac{\mathrm{Cov}(x, z)}{\mathrm{Var}(x)}(x_1 - \bar{x})\ \cdots\ z_n - \bar{z} - \frac{\mathrm{Cov}(x, z)}{\mathrm{Var}(x)}(x_n - \bar{x})\right)^{\mathrm{T}},$$

is an orthogonal basis of $\mathcal{C}(\mathbf{X})$. Moreover, $\{\mathbf{v}_1, \mathbf{v}_2\}$ is an orthogonal basis of the submatrix of $\mathbf{X}$, *formed by its first two columns.*

You may also assume distribution theoretic facts about quadratic forms of a multivariate normal vector.]

4. Suppose we have a Type II censored sample from $\mathrm{Exp}(\theta, \sigma)$ population, $\theta \in \mathbb{R}$, $\sigma > 0$, both *unknown*. The number of items put on test is n and only the first r order statistics $X_{(1)} \leq \cdots \leq X_{(r)}$ are observed.. We wish to estimate θ.

(a) Find the MLE of (θ, σ), denoted by $(\hat{\theta},\ \hat{\sigma})$.

(b) Find the distribution of $(\hat{\theta},\ \hat{\sigma})$.

(c) Discuss how you can use (b) to obtain a $100(1 - \alpha)\%$ confidence interval for θ.

[8+10+6 = 24]

[Note. You may assume distribution theoretic facts about exponential spacings.]

Indian Statistical Institute

Second Semester Examination: 2015-16

Course Name: BSTAT II Year

Subject Name: Economic and Official Statistics

Date: 26 April, 2016 **Maximum Marks: 50** **Duration:2 hrs**

Note : Answer three questions from Group A and two questions from Group B

Group A

1. Define a true Index number. Show that Laspeyres and Paasche's Index numbers provide one bound each for two different true index numbers of prices. 3+7 =10
2. State and explain the desirable properties of an Engel Curve. 10
3. Show how pooling of Cross section and Time series data is possible to estimate a linear form of Engel Function. 10
4. State the generalised form of the two factor CES production function and derive the marginal products of inputs and its elasticity of substitution. 2+4+4=10

Group B

1. Explain how the Index number for wholesale prices in India is computed (state the details of the weighting scheme in this context). 10
2. State the poverty and inequality indicators used by World Bank. 10
3. State and explain the Monetary Policy instruments of Reserve Bank of India. 10
4. Mention the major findings of AIDIS conducted by NSSO in 2013. 10

INDIAN STATISTICAL INSTITTUE, KOLKATA

Second Semestral Examination 2015-16

B.Stat (Hons.) II Year

Subject: Economic & Official Statistics and Demography

*(**Instruction**: Answer Group A and Group B on separate answer scripts. Standard notations are followed.)*

Group B: Demography & Economic Statistics

Date: 26.04.16 | Maximum Marks: 50 | Duration: 2 hours

(Answer as much as you can.)

1(a) Define/ Explain the following.

i) Late Neo-natal Mortality Rate

ii) Demographic Dividend

iii) Parity Progression Ratio

iv) Maternal Mortality Ratio

(b) Explain step by step how a life table is constructed from the m-type mortality rates to obtain life expectancy at some age. (1+2+2+1+4 = 10)

2(a) Express ${}_nL_x$ in terms of l_x, ${}_na_x$ and ${}_nd_x$.

(b) Why q_x cannot be calculated directly? Show how it gets estimated.

(c) For age groups with width of n years, derive an equation that relates q-type mortality rate with m-type mortality rate. (2 + 1 + 4 + 3 = 10)

3. Describe the procedure of calculating sample size in NFHS-4. Write a note on the Chandrasekhar-Deming method, followed in Sample Registration System. (5+5 = 10)

4. What is the purpose of El-Badry's procedure? Explain how the purpose is served. All symbols must be defined clearly. (1 + 9 = 10)

5. Define Gross Reproduction Rate (GRR). In what way is it different from Total Fertility Rate (TFR) ? How is GRR related to TFR ? Define and explain Net Reproduction Rate (NRR) Prepare a self-explanatory tabular format which allows calculation of TFR, GRR and NRR.

(1.5+1.5+1+2+4 = 10)

6. Write down the important properties of a Lorenz curve with all used symbols fully explained. Show how Lorenz ratio is related to the Gini Mean Difference.

(5+5 = 10)

END

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2015 – 2016)

B. Stat II Year

Agricultural Science

Date 29.04.16 Maximum Marks 50 Duration 3:00 hours .

(Attempt any five questions)

(Number of copies of the question paper required 5)

1. Write in brief about different types of rice. Briefly describe the cultural practices associated with rice grown under SRI. 3+7

2. What are the elements essential for plants? Briefly mention the criteria for the essentiality of plant nutrients. Calculate the quantity of VC, Urea, SSP and KCL required for 1.5 hectare potato crop to supply the nutrient requirement of 200 kg N, 120 kg P_2O_5 and 100 kg K_2O per hectare. Note that 50% of required N should be given through VC.
2+2+6

3. Describe the suitable agro-techniques for rice nursery bed preparation in SRI. Estimate the expected yield of rice grain in t/ha from the following data:
(i) Average no. of tillers/hill –85, (iii) Average no. of effective tillers/hill –73, (iv) Average no. of grain panicle 160, (v) Average panicle length -15 cm, (vi) Average no. of unfilled grain/panicle –30, (vi) Test weight -28 g.
4+6

4. Write the differences between: 2.5 x 4
   a) Manures and Fertilizers
   b) Inter cropping and mixed cropping
   c) Soil texture and soil structure
   d) Macro and micro nutrients

5. Write short notes on any two of the following : 5 x 2
   a) Available soil moisture
   b) Weather forecasting
   c) Drought

6. Write short notes on any five of the following : 2 x 5
   a) Monsoon onset
   b) Potential Evapo-transpiration
   c) Cup counter anemometer
   d) Reproductive stages in rice
   e) Permanent wilting point
   f) Field capacity
   g) Capillary water

INDIAN STATISTICAL INSTITUTE

Semestral Examination : 2015-16

Course Name : B. STAT. II Year (Second Semester)

Subject Name : Physics II

Date : 29|04|2016 **Total Marks : 50** **Duration : $2\frac{1}{2}$ hours**

PART-A

Answer any FIVE questions

1. Let the energy of a system be function of f generalized co-ordinates and f generalized momenta. If the system is in equilibrium at the absolute temperature T, show that the mean value of each independent quadratic term in the energy is given by $\frac{1}{2}kT$, where k is the Boltzmann constant.
[5]

2. Imagine that a composite system A consists of two weakly interacting systems A_1 and A_2. If the entropy of a single system is defined as $S = -k\sum_r P_r \ln P_r$, then show that the entropy is simply additive i.e., $S(A) = S(A_1) + S(A_2)$. Here P_r is the probability of a system being found in the rth state.
[5]

3. Consider a system of N identical but distinguishable particles at temperature T and total energy E. Each of the particles has two energy levels 0 or $\epsilon > 0$ where the upper level is g fold degenerate and the lower level is non-degenerate. Using the theory of micro-canonical ensemble find the occupation number n_+ and n_0 corresponding to the upper and lower energy levels, respectively.
[5]

4. Consider a substance which contains N_0 magnetic atoms per unit volume and is placed in an external magnetic field $\mathbf{H}$. Assuming that each atom has spin $1/2$ and an intrinsic magnetic moment μ, find the mean magnetic moment of such an atom. Discuss the limiting behaviors.
[3+2]

5. A system, in contact with a heat reservoir at temperature $T = \frac{1}{k\beta}$, consists of two particles. Each of the particles can be in anyone of the three quantum states of respective energies of 0, ϵ and 2ϵ.

(a) Write an expression for the partition function Z if the particles obey BE statistics.

(b) What is Z if the particles obey FD statistics?

[2.5+2.5]

6. Using $p_i = \frac{e^{-\beta E_i}}{Z}$ in the Gibbs form for the entropy, show that the Helmholtz free energy is given by $F = -kT\ln(Z)$, where Z is the canonical partition function.

[5]

(If not specified otherwise, the symbols have their usual meanings.)

PART-B

Answer any FIVE questions

1. Consider two long parallel wires, each having radius a, separated by a distance d ($d > a$) in air. The wires carry the same current I in the opposite directions. Find the self-inductance L per unit length of the system. Under which condition this self-inductance becomes negligible?

[4+1]

2. (a) What do you mean by 'coefficient of coupling' of two circuits?

   (b) A coil of self-inductance 100 mH is connected in series with another coil of self-inductance 170 mH. The effective inductance of the combination is found to be 70 mH. Evaluate the coefficient of coupling.

[1+4]

3. A dc voltage of 80 V is switched on to a circuit containing a resistance of 5 Ω in series with an inductance of 20 H. Calculate the rate of growth of current at the instant when the current is 6 A.

[5]

4. Using Maxwell's electromagnetic field equations show that

$$\text{div}\left(\vec{E} \times \vec{H}\right) = -\frac{1}{2}\frac{\partial}{\partial t}\left(\vec{E}.\vec{D} + \vec{B}.\vec{H}\right) - \vec{J}.\vec{E}$$

where $\vec{B} = \mu\vec{H}$ and $\vec{D} = \epsilon\vec{E}$ for a linear medium, and the symbols carry their usual meanings. Give physical interpretations of the terms on the two sides of the above equation.

[3+2]

5. (a) Explain the concept of displacement current.

   (b) Write Maxwell's equations for vacuum and derive the wave equation for the electric field in vacuum.

[1+(2+2)]

6. Show that for a plane electromagnetic wave propagating in free space, the electric field vector and magnetic field vector are mutually perpendicular.

[5]

END OF THE QUESTION PAPER

Indian Statistical Institute
End-Semester Examination 2016
Course Name: B.Stat Second Year
Subject Name: Economics II (Macroeconomics)

Date of Examination: 29.04.2016 **Maximum Marks – 60** **Duration: 2.5 Hours**

Answer all questions

1.a. Consider a simple Keynesian model. Define the concept of aggregate planned saving. Show that aggregate planned saving is always equal to aggregate actual investment irrespective of whether the system is in equilibrium or in disequilibrium.

b. Consider an IS-LM model where aggregate planned investment (I) is a function of the rate of interest r alone and $\frac{\partial I}{\partial r} = -50$. Now, a shift in the money demand function (such that at every (Y, r) demand for real balance exogenously increases by 1 unit) causes the LM curve to shift horizontally and vertically by -4 units and $.016$ units respectively. Again, given other things, a shift in the consumption function is found to increase the equilibrium level of Y by 500 units. Using the information given above, derive (i) the slope of the LM curve, (ii) the change in the level of aggregate planned investment from the initial to the final equilibrium resulting from the aforesaid shift in the consumption function. (All relations in the model are linear).

[10+10=20]

2. Consider a Simple Keynesian Model for an open economy without government activities. Suppose 20 per cent and 60 per cent of aggregate planned consumption expenditure and aggregate planned investment expenditure respectively are spent on imported goods. Suppose the marginal propensity to consume and marginal propensity to invest (net) with respect to NDP (Y) are 0.8 and 0.3 respectively. Given the above information, answer the following questions under the assumption that all the relations in the model are linear:

a. Compute the marginal propensity to import with respect to Y and check if the equilibrium is stable.

b. Compute the autonomous expenditure multiplier in this model.

c. Suppose starting from an initial equilibrium situation there takes place parallel upward shifts in the saving and investment functions by 5 units and 10 units respectively. Compute the resulting change in aggregate planned saving from the initial to the new equilibrium.

[7+3+10=20]

3. Consider an IS-LM model given by the following equations:

$$C = 40 + 0.75Y, G = 60, I = 300 - 50r, L = 0.25Y - 62.5r \text{ and } \frac{M}{P} = 500.$$

a. Compute the equation of the IS curve and that of the LM curve

b. Derive the states of the goods market and the credit market at $\left(Y = 2500, r = \frac{1}{5}\right)$.

c. Drawing upon the assumption regarding the relative speeds of adjustment of Y and r in disequilibrium situations in the IS-LM model, derive the change in the level of aggregate planned investment that occurs before there takes place any change in Y starting from an initial situation where $\left(Y = 2500, r = \frac{1}{5}\right)$. [6+6+8=20]

Introduction to Markov Chains
B. Stat. IInd Year Semester 2
Indian Statistical Institute

Semestral Examination

Time: 3 Hours Full Marks: 50

Date: May 3, 2016

This is an OPEN NOTE examination. You may use any handwritten notes that you have brought with you, but you are not allowed to share them. Also no notes in printed, photocopied, cyclostyled or any other form (including electronic storage) is allowed. Calculators and any other computing devices are strictly not allowed.

1. Let (X_n) be a Markov chain with transition matrix P, with all diagonal entries strictly less than 1, on state space S. Define τ_n inductively as follows: $\tau_0 = 0$, $\tau_1 = \inf\{n \geq 1 : X_n \neq X_0\}$ and $\tau_{n+1} = \inf\{i \geq \tau_n : X_i \neq X_{\tau_n}\}$. Put $Y_n = X_{\tau_n}$. Show that this forms a Markov chain. Calculate its transition matrix. Discuss irreducibility in terms of the original chain. If the original chain is positive recurrent, use its stationary distribution and the diagonal entries of P matrix to give a criterion for positive recurrence of the new chain. [2+2+4+5=13]

2. In a state, there are m major parties. A voter of the party j will remain loyal and vote for the same party in the next election with probability p_j. If he changes party, he will choose one with equal probability from the remaining $(m-1)$ parties. Show that the party chosen by a voter in n-th election form a Markov chain and give the transition matrix. Show that the long run proportion of support for the party j is given by
$$\pi_j = \frac{(1-p_j)^{-1}}{\sum_{l=1}^{m}(1-p_l)^{-1}}.$$
[2+2+6=10]

3. Show that the total variation distance to probability vectors π and η on a finite state space S is given by $1 - \sum_{i \in S}(\pi(i) \wedge \eta(i))$. [6]

4. If T is a stop time, show that so is T^2. [4]

5. In a shooting competition, there are N targets to be hit. A competitor shoots every time and hits one of the targets at random, including the ones already hit. Let $X_n^{(N)}$ denote the number of targets not hit out of original N after n-th attempt. For each fixed N, show that $\{X_n^{(N)}\}$ is a Markov chain, find its transition matrix and classify the states. Let T_N be the time required to reach the absorbing state starting with N targets. Show that $T_N/N\log N \xrightarrow{\text{P}} 1$. (You may compare and draw analogy with any chain taught in the class, but you must give the proofs of all the steps involved.) [2+2+3+10=17]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

B. Stat. - II Year (Semester - II)

Discrete Mathematics

Date : 06.05.12 Maximum Marks : 100 Duration : 3:30 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1. Answer the following: [4*9=36]

   a. Let G be a graph with n vertices and m edges. Give upper and lower bounds on the number of triangles in G.

   b. Let G be a graph with $n \geq 4$ vertices. What is the upper bound on the intersection number of graph G?

   c. Suppose n points are distributed on a plane and they are in general position. Give the bounds on n in order to ensure the existence of empty convex pentagon formed using those points on the plane as vertices. Instead of empty pentagon, if we consider empty heptagon then, write down the bound on n?

   d. Let G be a connected graph other than a clique or an odd cycle. Suggest a bound on the chromatic number of G in terms of degree of vertices graph G.

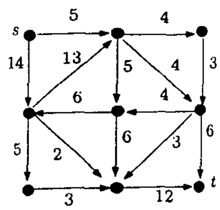
   e. State Kuratowski's Theorem for planar graph.

   f. Let A be a set with n elements. What is the maximum number of subsets of A that can be formed such that the size of each subset is odd and number of common elements for every pair of such subsets is even.

   g. Does there exist a graph which is not a tree and whose degree sequence is $< 5, 3, 2, 1, 1, 1, 1, 1, 1 >$?

   h. G is a planar graph. In a planar drawing of G, every face is a 4-cycle. Find the number of faces in G in terms of number of vertices.

   i. Find the maximum value of a flow from s to t of the following network with edge capacity as indicated.



P.T.O

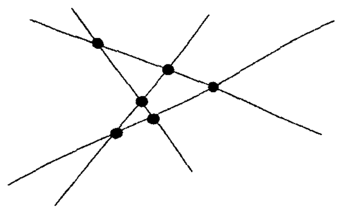
2. The Ramsey number $R_k\{s_1, s_2, \ldots, s_k\}$ is the minimum number n such that any coloring of the edges of K_n with k colors contains a clique of size s_i in color i, for some i. Prove that $R_k\{s_1, s_2, \ldots, s_k\}$ is finite for any $s_1, s_2, \ldots, s_k > 1$. [10]

3. Show that there exist graphs having arbitrarily large chromatic number even though they do not contain K_3. [15]

4. Find a solution for $a_n = 2a_{n1} + 3^n$ with $a_1 = 5$. [10]

5. Let $\tau(G)$ denote the number of spanning trees of a graph G. Prove that if edge $e \in E(G)$ is not a loop, then $\tau(G) = \tau(G - e) + \tau(G.e)$. [15]

6. Given a set of lines in the plane with no three meeting at a point, form a graph G whose vertices are the intersection of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. [15]



7. Prove that for any graph G on n vertices, not containing a 4-cycle,

$$E(G) < \frac{1}{4}(1 + \sqrt{4n-3})n.$$

[15]

Indian Statistical Institute
Back Paper Examination 2016
Course Name: B.Stat Second Year
Subject Name: Economics II (Macroeconomics)
Date of Examination: **Maximum Marks – 100** **Duration: 3 Hours**

Answer all questions

1. Consider the following problem of national income accounting: For producing current period output firm B incurs the following costs: It purchases goods worth Rs.50,000 from firm A, holds half of it in inventory and uses the other half as raw material for current production. It pays out Rs.40,000 in wages half of which is paid to a labour contractor. It pays Rs.35000 as interest to a bank, Rs.5,000 as interest on bonds sold to households, Rs.15,000 as rent to households, Rs.5,00,000 as dividend to households, Rs.20,000 in profit tax, Rs.75,000 in net indirect taxes and donates Rs.10,000 to Ramakrishna Mission. Its undistributed profit is Rs.2,00,000.

(i) Compute the intermediate input cost incurred by firm B, the value of output produced by firm B and the gross value added of the same firm.
(ii) Compute firm B's contribution to national income.
(iii) Compute firm B's contribution to personal income. **[9+8+8=25]**

2.a) Explain the concept of paradox of thrift in the simple Keynesian model.

b) Suppose in a simple Keynesian model planned consumption is a proportional function of NDP denoted Y. Marginal propensity to invest with respect to Y is 0.3. Start with an initial equilibrium situation. Suppose the saving function shifts parallely downward by 3 units and following this, saving in the new equilibrium is found to increase by 9 units. Derive the new consumption function. (All relations are linear). **[12+13=25]**

3. Consider a simple Keynesian model for a closed economy without government. At GDP (Y) = 1000, producers have to sell 20 units from their stock to meet the customers' demand fully. It is given that the equilibrium level of GDP is 1040. Find out the impact of an increase in autonomous expenditure on the equilibrium level of Y, assuming the expenditure function to be linear. **[25]**

4. Explain the impact of an increase in the supply of real balance in the IS-LM model. Show in a diagram how GDP and interest rate adjust from the initial equilibrium to the new equilibrium.

[25]

INTRODUCTION TO MARKOV CHAINS
B. STAT. IIND YEAR SEMESTER 2
INDIAN STATISTICAL INSTITUTE

Backpaper Examination

Time: 3 Hours Full Marks: 100

Date: August 1 , 2016

This is an OPEN NOTE examination. You may use any handwritten notes that you have brought with you, but you are not allowed to share them. Also no notes in printed, photocopied, cyclostyled or any other form (including electronic storage) is allowed.
Calculators and any other computing devices are strictly not allowed.
You need to justify all your answers and claims.

1. Prove or disprove with justification: A finite state irreducible Markov chain, whose transition matrix has at least one diagonal entry strictly positive, is ergodic. [10]

2. Let $\{r_i\}_{i=0}^{\infty}$ be a probability vector, that is, $0 \leq r_i \leq 1$ and $\sum_{i=0}^{\infty} r_i = 1$. Consider a transition matrix P on the state space $S = \{0, 1, 2, \ldots\}$ with $p_{0,i} = r_i$ and $p_{i,i-1} = 1$ for $i \geq 1$. Show that P is irreducible. Classify the chain as transient, null recurrent or positive recurrent based on the probability vector $\{r_i\}$. [5+15=20]

3. A particle moves on a circle through points marked (in a clockwise order) as 0, 1, 2, 3, 4. At each step it moves to its neighbouring point - to the right with probability p and to the left with probability $1-p$. Let X_n denote its location after n steps. Show that $\{X_n\}$ is a Markov chain and find its transition matrix. Find the Cesaro limit of the powers of the transition matrix. [8+12=20]

4. Let $\boldsymbol{P}$ be a transition matrix of a Markov chain on a state space S with stationary distribution π and let $\rho(\mu, \nu)$ be the total variation distance between the probability measures μ and ν. Further denote the space of all probability distributions on S by $\mathcal{P}$. Show that, for all t,
$$\sup_{\mu \in \mathcal{P}} \rho(\mu \boldsymbol{P}^t, \pi) \leq \sup_{\mu, \nu \in \mathcal{P}} \rho(\mu \boldsymbol{P}^t, \nu \boldsymbol{P}^t).$$
[15]

5. If S and T are stop times, show that so is $S + T$. [15]

6. Let $\{X_t\}$ be a simple symmetric random walk on a cycle with n vertices. Let σ be the first time $\{X_t\}$ visits all the vertices. Show that X_σ is uniformly distributed over all but the first vertex. Now consider a coin with probability of Head as $1/n$. If the coin lands up Head, define τ to be 0, and else define it to be σ. What is the distribution of X_τ? Are τ and X_τ independent? (Hint: What happens when $\tau = 0$?) Is τ a uniform time? [10+4+4+2=20]

4. What is the purpose of El-Badry's procedure? Explain how the purpose is served. All symbols must be defined clearly.

[1 + 9 = 10]

5. The table below gives the parity progression ratios for a number of recent birth cohorts in a country.

(i) Assuming that no woman in any of these birth cohorts had a fifth child, calculate

(a) the proportion of women in each birth cohort who had exactly 0, 1, 2, 3 and 4 children,

(b) the total fertility rate for women in each birth cohort,

(ii) Comment on your results.

Calendar years of birth	Parity Progression Ratios			
	0 -1	1 – 2	2 – 3	3 – 4
1931-33	0.861	0.804	0.555	0.518
1934-36	0.885	0.828	0.555	0.489
1937-39	0.886	0.847	0.543	0.455
1940-42	0.890	0.857	0.516	0.416
1943-45	0.892	0.854	0.458	0.378
1946-48	0.885	0.849	0.418	0.333

[(6 + 3) + 1 = 10]

6(a) Write how Newton's halving formula is used for tackling errors due to inaccurate age reports or faulty enumeration.

(b) Construct the table for estimation of TFR, GRR and NRR.

[5 + 5 = 10]

==END==

Back Paper

INDIAN STATISTICAL INSTITTUE, KOLKATA

Second Semestral Examination 2015-16

B.Stat (Hons.) II Year

Subject: Economic & Official Statistics and Demography

Group B: Demography & Economic Statistics

Date: 03-07-2016 Maximum Marks: 50 Duration: 2 hours

(Instruction: Answer Group A and Group B on separate answer scripts. Standard notations are followed.)

Group B: Demography

(Answer all questions.)

1. Answer the following.

i) The equation

$q_x = 2m_x / (2 + m_x)$

shows how the m-type and q-type mortality rates are related to one another. Derive a similar equation for the more general case of an age group of width n years.

ii) The standardized death rate for the town A was 1.23 when the population of town B was used as the standard. What does this tell you about mortality in A to that in B.

iii) Show that the crude birth rate in a stationary population corresponding to a life table is equal to $(1/e_0)$ where e_0 is the life expectation at birth. [4 + 2 + 4 = 10]

2(a) Express ${}_nL_x$ in terms of l_x, ${}_na_x$ and ${}_nd_x$.

(b) Why q_x cannot be calculated directly? Show how it gets estimated.

(c) If the crude birth rate in a country remains constant over a number of years, but the general fertility rate increases steadily, what does this tell you about the country's population? [3 + 1 + 3 + 3 = 10]

3. Describe a method of evaluating birth and death registration using age distribution and child survivorship data. [10]

P.T.O

Indian Statistical Institute

Second Semester Examination: 2015-16 (Back Paper)

Course Name: BSTAT II Year

Subject Name: Economic and Official Statistics

Date: 03/07/,2016 **Maximum Marks: 50**

Duration: 2 hrs

Note, if any : Answer three questions from Group A and two question from Group B

Group A

1. Derive the projected demand for the future assuming that the Engel curve remain invariant over time (use Engel Curve of Double Log form). 10
2. State the Homogeneity Hypothesis in Engel curve analysis (for effects of variation household size) and state its implications. 4+6=1
3. Derive the profit maximisation conditions of a producing firm (assuming th producers maximise net revenue subject to constraint imposed on productio function, existence of perfect competition in factor market and absence of prefe competition in output market). 1
4. State and explain the type of errors observed while constructing an index number. 1

Group B

1. State the major findings of the 70th round survey carried out by NSSO for Situatio Assessment of Agricultural Households. 1
2. Explain the Type of financial market data published by Reserve Bank of India. 1
3. State the expected outcomes of ISSP Project (6 major outcomes). 1
4. Write short note on Big data in Official Statistics. 1

INDIAN STATISTICAL INSTITUTE

Backpaper Examination: 2015–2016
B.Stat. (Hons.) 2nd Year. 2nd Semester
Statistical Methods IV

Date: August 05, 2016 Total Marks: 100 Duration: 3 hours

- You should provide as much details as possible while answering a question. You must state clearly any result stated (and proved) in class you may need in order to answer a particular question.

---

1. Suppose $X = (X_1, X_2, X_3)^{\mathrm{T}} \sim \mathrm{N}_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Write $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)^{\mathrm{T}}, \boldsymbol{\Sigma} = ((\sigma_{ij}))_{3\times 3}$.
   (a) Show that $\mathrm{E}(X_1X_2X_3) = 0$, when $\boldsymbol{\mu} = \mathbf{0}$.
   (b) Show, using (a) or otherwise, that $\mathrm{E}(X_1X_2X_3) = \mu_1\mu_2\mu_3 + \mu_1\sigma_{23} + \mu_2\sigma_{31} + \mu_3\sigma_{12}$. [6+5 = 11]

2. Suppose $\mathbf{M} \sim \mathrm{W}_p(\boldsymbol{\Sigma}, n),\ n \geq p+2$. Show that

$$\mathrm{E}(\mathbf{M}^{-1}) = \frac{\boldsymbol{\Sigma}^{-1}}{n-p-1}.$$ [13]

3. Suppose $\mathbf{X}_i$ is an $n_i \times p$ ($n_i \geq p+1$) data matrix from $\mathrm{N}_p(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$, $i = 1, 2$. The parameters $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2 \in \mathbb{R}^p$, and $\boldsymbol{\Sigma}$ is a positive definite matrix. Let $n \stackrel{def}{=} n_1 + n_2$. Assume that $\mathbf{X}_1$ and $\mathbf{X}_2$ are independent. Consider the problem of testing

$$\mathrm{H}_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 \ \text{ against } \ \mathrm{H}_1 : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2.$$

   (a) Obtain the LRT statistic, denoted by λ, of this testing problem.
   (b) Obtain the null distribution of λ or of a suitable equivalent of λ. [8+8 = 16]

4. Suppose $X_1, \ldots, X_n$ is a random sample from a Cauchy population with *unknown* location parameter θ, $\theta \in \mathbb{R}$. We wish to estimate θ by $\hat{\theta}_n := (\hat{\xi}_{p,n} + \hat{\xi}_{1-p,n})/2$, where $\hat{\xi}_{p,n}$ and $\hat{\xi}_{1-p,n}$ are the p-th and $(1-p)$-th sample quantiles, respectively. Find the asymptotic distribution of $\hat{\theta}_n$. [12]

[P.T.O.]

5. Suppose $y \sim N_n(X\beta, \sigma^2 I_n)$, where X is an $n \times (p+1)$ matrix with fixed (non-random) entries and all entries in its first column are 1, $\beta \equiv (\beta_0, \beta_1, \beta_2, \ldots, \beta_p)^T \in \mathbb{R}^{p+1}$, $\sigma > 0$. Assume that the rank of X equals $p+1$. Suppose we wish to test the hypothesis $H_0 : \beta_1 = \cdots = \beta_p = 0$ against $H_1 : H_0$ *is false*. Let J_n denote the $n \times n$ matrix with all entries equal to 1, and let $H := X(X^T X)^{-1} X^T$.

(a) Argue that a suitable test for testing H_0 against H_1 is the following: reject H_0 if F is large, where

$$F := \frac{y^T(H - n^{-1}J_n)y/p}{y^T(I_n - H)y/(n-p-1)}.$$

(b) Find the null distribution of F In (a) above. [10+9 = 19]

6. Suppose $X_1, X_2, \ldots$ are i.i.d. $N(\theta, 1)$ variables, where $\theta \in \mathbb{R}$. Consider the problem of estimating the parametric function $\psi(\theta) := P_\theta(X_1 \leq c_0)$, where c_0 is a known constant.

(a) Find the MLE of $\psi(\theta)$, denoted by $\hat{\psi}_n$, based on $X_1, \ldots, X_n$.

(b) Show that for suitable $\mu \in \mathbb{R}$ and $\sigma > 0$, to be obtained by you, the asymptotic distribution of $n^{1/2}(\hat{\psi}_n - \mu)$, as $n \to \infty$, is normal with mean zero and variance σ^2.

[3+7 = 10]

7. Suppose that the lifetimes $T_1, \ldots, T_n$ are independent and follow an exponential distribution with location parameter 0 and scale parameter σ, $\sigma > 0$. Let a Type 2 censoring scheme be employed where only the r smallest lifetimes $t_{(1)} \leq \cdots \leq t_{(r)}$ are observed, r being a specified integer between 1 and n.

(a) Show that the maximum likelihood estimate (MLE) of σ is given by $\hat{\sigma} = T/r$, where $T := \sum_{i=1}^{r} t_{(i)} + (n-r)t_{(r)}$.

(b) Show that $2T/\sigma \sim \chi^2_{2r}$.

(c) Discuss how you can obtain a $100(1-\alpha)\%$ confidence interval for σ which is based on T. [6+7+6 = 19]

[Note. You may assume distribution theoretic facts about exponential spacings.]
