Essays on Conflict and Organisation Theory

Dushyant Kumar

Thesis submitted to the Indian statistical Institute in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy

Essays on Conflict and Organisation Theory

Dushyant Kumar

June, 2015

Thesis Supervisor : Professor Prabal Roy Chowdhury

Thesis submitted to the Indian statistical Institute in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy

Acknowledgement

Writing acknowledgement for a doctoral thesis is a difficult task, difficult to comprehend. It involves the support and guidance of a really large number of individuals and institutions. I begin by expressing my gratitude to the Indian Statistical Institute, Delhi Centre. It provides a wonderful environment for research work.

I am grateful to my supervisor, Prabal Roy Chowdhury for his constant support and guidance. His contribution goes much beyond this thesis. He has been patient with my wandering academic pursuits all these years. He has always encouraged me to explore new areas.

I would like to express my gratitude to all the faculty members of the Economics and Planning Unit. They have all been really supportive and a source of inspiration through their academic as well as non-academic endeavors.

I would like to thank all my friends particularly the students and hostelers of ISI, Delhi centre. They have been a wonderful support network all these years. I especially thank Mridu, Soumendu, Ridhima, Anup, Abdul, Anuradha, Soham, Narendra, Nit, Nandy and Priyanka.

I am indebted to my family for their constant and unconditional support. The graduate years are immensely taxing for the family, particularly parents. But they (my parents, maa and kaka) have been rock solid behind me, all these years.

Last but not the least, I would like to specially thank Kaki and Babuji. They have introduced and walked me through the world of numbers and alphabets, in some cases literally as the room walls and floors acted as writing pad.

The research fellowship from Indian Statistical Institute is gratefully acknowledged. I am also indebted two anonymous referees for their insightful comments and encouragement.

To Kasrour

Contents

Chapter 1

Introduction

This thesis comprises three chapters on conflict theory and organization theory. All these three chapters are work on applied microeconomic theory. Here we apply the tools of game theory and contract theory to specific issues which has wider applicability. The first two chapters, *chapter 2 and 3*, deal with conflict theory. *Chapter 2* uses a incumbent-rebel setup to explore the conflict game between the government and the rebel. Particularly, here we study the conditions under which it is optimal for the government to increase the development works when faced with an increase in rebellion activities. In *chapter 3*, we extend this analysis to accommodate a signaling game between the government and the people. The government can be of two types: benevolent and non-benevolent. The government's type is it's private information. The government can use development work to signal it's benevolence, to the people. Here we explore the possibility and effect of using development as signal. *Chapter 4* deals with organization theory. In this chapter, we explore and study the organizational structures of a firm. We use a principal-agent framework to work on centralized and delegated contracts. In this chapter, we allow for collusion among agents.

Now we provide description of these chapters in greater detail.

1.1 Conflicts

In this chapter, we study domestic (intra country) conflicts. We limit our analysis to the internal conflicts only. So, here we abstain from issues which are particularly relevant to the inter-state conflicts. These internal conflicts are quite common across the world and quite persistent across the time. Blattmen and Miguel (2010) analyses the data on such internal conflicts, for the period 1960-2006. They show that twenty percent of the countries have experienced at-least ten years of conflicts, with more than 1000 casualties per year. Moreover, these conflicts affect the human life qualitatively as well. Wide presence and continuance of such conflicts also imply that a significant part of economic resources is always engaged in unproductive works, i.e., these conflicts. So these conflicts have huge economic costs.

These internal conflicts are present across the world but some parts or regions are relatively more effected. Latin America, Africa, developing countries in Asia and the Middle East have witnessed relatively more intense conflicts. A significant portion of these conflict-ridden areas are underdeveloped. Hence, often, particularly in popular media, lack of development is attributed as the root cause behind the emergence of these conflicts. A significant portion of popular media, as well as academia, advocates increase in development, in the concerned area, as the solution to such conflicts. They do not approve taking hard actions against such rebels. In this context, we study government's optimal response when faced with a rebellion group.

We are particularly interested in Maoist rebellion problem in India. The origin of the Maoists in India can traced back to 1967. In a village of West Bengal state, Naxalbaari, a small group of farmers revolted against the local landlords. Owning to it's origin place, it is also referred as the Naxalite movement. Through various chain of events, it spread to the various parts of the country, by 1970s and 80s. Currently it is affecting a large part of the country. However in some places, like some parts of Chhatishgarh, West Bengal, Orissa and Bihar, it is a really serious issue. Most of these effected areas are the backward with regard to development and infrastructure. While dealing with the rebels, the government typically faces a dilemma of stick or carrot. The government can either take strict actions like crushing the rebels with police/military power or follow the path of development which eventually might weaken (the support of) the rebels.

We study this in a incumbent-rebel setup. The government is the incumbent who enjoys the state-power. The rebels want to overthrow the government and capture the state-power. The rebels organize rebellion activities (R) and the government can counter this with military deployment (M) and/or development work (D). We use difference-form contest success function to model the conflict game between the government and the rebel.

$$P(R, M) = \frac{1}{1 + e^{M-R}}.$$

Here $P(R, M)$ is the probability of success for the rebel. D does not enter directly into the probability function but it affects the cost of rebellion activities ($C^R(R, D)$). We do not put any condition on $C^R(R, D)$ with respect to D. If the people take D as indicator of the government's benevolence, C^R_D will be positive. However, it is also possible that the people think that the government is doing development just because of the presence of the rebel. In this case C^R_D will be negative.

First we study the existence of pure strategy Nash equilibrium. We show that the issues with regard to the non-existence of pure strategy Nash equilibrium, in difference-form conflict games, is an artifact of very specific cost functions. With difference-form contest success function, we can have peace (full as well as partial) as pure strategy Nash equilibrium. So in equilibrium, it is possible that one side or both sides put zero fighting effort.

We study the conditions under which the government does not find it optimal to increase development when faced with increased rebellion activities. Suppose $C^R_{RD} > 0$, i.e., a increase in development increases the marginal cost of rebellion activities. Consider the case when in equilibrium, the rebellion activities are not too low. Here an exogenous increase in development decreases equilibrium level of rebel activities but increases equilibrium level of military/police deployment by the government. Also, if the rebel gets stronger in cost efficiency sense, the government find it optimal to decrease the equilibrium level of development. These results depends crucially on whether initially the equilibrium level of rebellion activities are too low or too high.

1.2 Conflicts: Incomplete Information

This chapter complements the analysis done in last chapter, *chapter 2*. Here we add a signaling stage (between the government and the people), to the conflict game studied in last chapter. The government can be of two types: benevolent and non-benevolent. The government's type is it's private information. The people supports the government if they think that the government is of benevolent type and does not support the government if they think that the government is of non-benevolent type. First, the signaling game takes place. The government decides on the level of development (D). The people observe the government's decision of D and decides whether to support the government or not. After this, the conflict game between the government and the rebel takes place. People's support is valuable for the government as it increases government's probability of winning the conflict game.

$$P(R, M) = \frac{1}{1 + e^{M-R+x}}.$$

Here $P(R, M)$ is the probability of winning for the rebel. x takes some positive value if the people supports the government, zero otherwise.

We specify the utility functions for the benevolent and non-benevolent type governments in such a way that it provides rationale for the people's behavior. Here, we have restricted the people decision to support the government, to a deterministic rule. However, one can specify an appropriate utility function for the people which can support this deterministic rule as an optimal strategic behavior. This analysis provides a foundation of the rebel's cost function that we have used in *chapter 2*.

We explore both, separating as well as pooling perfect Bayesian equilibrium. We establish the existence of both equilibriums. We find that, in general, there will be over-signaling. So, the equilibrium level of development will be too high. To get rid of multiple equilibria problem, we use intuitive criterion refinement. It eliminate all pooling equilibriums and also all, but one, separating equilibriums. So, intuitive criterion implies that, in the equilibrium, benevolent type government and non-benevolent type government will always be separated. In the surviving separating equilibrium, non-benevolent type does zero level of development and benevolent type does minimum possible level (from the support interval for separating equilibrium) of development. This result is similar to the result in case of Spence's schooling model. However, the intuitive criterion refinement has been criticized. Particularly, when the signal has utility for the other player, the forward induction technique of intuitive criterion refinement seems less convincing.

1.3 Centralization vs. Delegation: A Principal-Agent Analysis

In this chapter, we study two types of organizational structures, namely centralization and delegation. Centralization refers to a contractual relationship where the principal contracts with all the agents directly. Delegation refers to a contractual relationship where the principal contracts with some agents (a proper subset, to be precise) and give them the right to contract with others. Both types of contract are quite commonly used. Here we use a principal-agent framework. In the absence of collusion among agents, the standard result in the literature is that centralized contract outperforms delegated contracts. This is based on the application of *revelation principle*. Here in this chapter, we allow for the collusion among agents.

We first explore the centralized contracts with perfect collusion among agents. We provide a nice way to model collusion among agents in an adverse selection-moral hazard framework. The agents can misreport their types and efforts, in a cooperative way. Moreover they can manipulate their cost-account books also, in a coordinated way. This notion of collusion can potentially result in cost synergies apart from standard coordination benefits. We argue that the principal can also gain from such collusion among agents. We provide a sufficiency condition for it.

For centralization, we provide the first-order conditions which characterize Nash equilibrium outcomes for both with and without collusion case. This approach provides a convenient way to derive Nash equilibrium outcome. It can be applied in many other contexts.

We model and explore the delegated contracts. we derive a sufficiency condition under which delegated contracts outperform centralized contracts.

Chapter 2

Conflicts

2.1 Introduction

Throughout the human civilization, domestic (intra country) conflicts have been present across societies, cultures and countries. These conflicts are extremely common as well as persistent. For the period 1960-2006, twenty percent of the countries have experienced at-least ten years of conflicts with more than 1000 casualties per year (Blattman and Miguel, 2010). It causes huge loss of human life, quantitatively, as well as qualitatively. It results in huge number of casualties, as well as damage to the existing output, resources and infrastructure. The persistent presence of such conflicts across the world means that a significant number of people as well as resources are always devoted to the these unproductive works. Combining all these, such conflicts have huge economic impact.

Such conflicts are more common or intense in some regions. Latin America, Africa, developing countries in Asia and the Middle East have been severely affected from such conflicts for last three decades particularly. During the cold war, most of these conflicts were seen as the conflict between the communist and the capitalist ideologies. However, the end of the cold war has failed to stop or reduce such conflicts. Often poverty and underdevelopment are attributed as the cause for these conflicts. However the causality can be in other direction as well, so we need to be careful. Also historically such conflicts along with external conflicts have been instrumental in (successful) institution building. However, the wide presence of these conflicts warrants thorough analysis, both theoretical as well as empirical. Economics, with the tools and insights available with it's sub-disciplines like- game theory, contract theory, industrial organization, behavioral economics, etc. can offer a good analysis in this area. In economics, we need to realize the importance of the appropriation and defense as a separate economic activity.

South Asia has seen a lot of internal conflicts lately. In India, we have seen rise in left-wing extremism. It is also referred as the Maoist. There are also issues in the north-eastern states. Assam, Manipur and Nagaland have faced many separatist movements. Similar Maoist activities has caused a lot of bloodshed in Nepal, however it seems that it is settled now. Sri Lanka has also experienced huge bloodshed and conflict by the Tamil separatist group LTTE. This was settled by the decisive victory of the Sri Lankan army over LTTE, but it came at the huge cost of human life and other resources. Pakistan is also facing many such problem particularly in Balochistan and Sindh provinces. We are particularly interested in conflicts like the Maoists problem in India. We study it in an incumbent-rebel setup. However the line between the rebel and the terrorist as well as the line between the political instability and such conflict is not that clear.

The origin of the Maoists in India can traced back to 1967. In a village of West Bengal state, Naxalbaari, a small group of farmers revolted against the local landlords. Owning to the origin place, it is also referred as the Naxalite movement. Through various chain of events, it has spread to the various parts of the country by 1970s and 80s. Currently it is affecting a large part of the country. However, the intensity is different in different parts. In some places, like some parts of Chhatishgarh, West Bengal, Orissa and Bihar, this is a very serious issue. These areas might seem quite small when seen relative to the whole country. However time and again, top leaders and officials of the country (even the Prime Minister) have recognized the Maoists as one of the biggest security threats to the country. Most of these affected areas are the backward with regard to the development and infrastructure. While dealing with the rebels, the government typically faces a dilemma of the stick or carrot. The government can either take strict actions like crushing the rebels with police/military power or follow the path of development which eventually might weaken (the support of) the rebels. This choice becomes even more important as a significant portion of the population, media as well as academia believes that a major reason for the rise of such rebel activities is the lack of development. They advocate use of development instead of tough actions to counter such rebels. In this chapter we have studied conditions under which increasing development is an optimal response to such rebellion activities.

We study this in an incumbent-rebel setup. The government is the incumbent, enjoying the state power; the rebel wants to capture the state power through the rebellion activities. To model this contest between the government and the rebel, we need to specify the contest success function. A contest success function (also referred as probability success function) specifies the probability of winning as a function of efforts or resources put up by the players. In the present context, it can also be interpreted as the part of the territory captured by the player. This analysis also depends on the form of probability success function $P(R, M)$ assumed. Here R is the level of rebel activities put by the rebels and M is the level of military/police deployment by the government. For our analysis here, we take R to be expenditure on acquiring arms, by the rebel group and M to be expenditure on military/police deployment in the affected areas, by the government.[1] $P(R, M)$ is the rebels' probability of winning this conflict game. We will be using difference-form contest/probability success function. In the literature, two type of probability functions are

[1]However these R and M can be defined in a more broad way as well. For example, R may include attacking state forces and/or civilians, mobilizing people to revolt, conspiring against the state, obstructing or jeopardizing the functioning of state machinery or industry, etc. Whereas M may include attacking rebels, enacting strict laws, etc.

most commonly used: ratio-form ($\frac{R}{R+M}$) and difference-form ($\frac{1}{1+e^{M-R}}$). It is shown that only these two types of functions satisfy the desirable properties of the probability functions in these kind of conflict games.

Traditionally the ratio-form is more commonly used as there has been some issues with regard to the existence of the pure-strategy Nash equilibrium in the case of the difference-form. However these existence issues are mainly the artifact of the assumption of the linear cost function. We study these existence issues in detail here. Hirshleifer (1989) studies these two type of contest success functions. He concludes that the difference-form is more suitable for conflicts in non-idealized situations, conflicts with frictions. In case of ratio-form, the probability of winning is zero if a player is putting zero fighting effort and other player is putting some positive (no matter how small) fighting effort. This is not true in the case in the difference-form contest success function, even a zero fighting effort result in positive probability of winning. This property is more suited for intra-organizational conflicts. In the present context, this probably separates such domestic conflict from the terrorism. Even in the case of outright win, the government does not crush the rebels absolutely. In the appendix of the chapter, we study both contest success functions in greater detail.

For difference-form contest success function, we study the existence of pure strategy Nash equilibrium, for both linear as well as non-linear cost functions (cost of fighting for the rebel and the government). For symmetric cost functions, pure strategy Nash equilibrium exists if the cost of fighting is relatively high compared to the prize (V). Moreover, in this case, no fighting is the unique pure strategy Nash equilibrium ($R^* = 0, M^* = 0$)). If cost of fighting is relatively low compared to the prize V, we can have pure strategy Nash equilibrium if the cost functions of the rebel and the government are sufficiently different. If the government and rebel value the prize differently (different V's), then this asymmetry has similar effect, as the asymmetry in the cost functions. For a general cost function, we can derive an upper bound on V as a sufficient condition for the existence of the pure strategy Nash equilibrium. Like for symmetric and quadratic cost functions ($C^R = R^2, C^M = M^2$), $V \leq 6/\sqrt{3}$ is the sufficient condition for the existence of pure strategy Nash equilibrium. We can also have multiple Nash equilibriums for more general cost functions.

Further we analyze the effect of changes in the development level on the rebellion activities. Consider the case $C^R_{RD} > 0$, i.e., an increase in development increases the marginal cost of rebellion activities. Suppose there is an increase in development due to some exogenous reason, say through some foreign grant. It decreases the equilibrium level of rebellion activity (R^*). However it might increase the equilibrium level of police and military deployment (M^*) by the government. It happens when rebels are stronger. Another interesting finding is that if the rebel gets stronger, the government might find it optimal to decrease the equilibrium level of development. This is surprising because with $C^R_{RD} > 0$, one might think that increasing D is the best response. We also get the expected result that the equilibrium level of development is positive only if $C^R_{RD} > 0$.

2.1.1 Related Literature

Herschel I. Grossman and Jack Hirshleifer have done several works in late 1980s and early 1990s, which serve as foundation for the conflict theory. There has been many other works, theoretical as well as empirical. But as Blattman and Miguel (2010) has summarized, conflict related works have "stood at the periphery of economics research and teaching." The issue that we study here is closely related to the findings of some of the empirical works. Blattman and Miguel (2010), Iyer (2009) among others have studied the link between development and conflict and have found a close correlation. Collier and Hoeffler (2004) test alternative theories of grievances and greed as the causes of such conflicts. They find that variables like high inequality, less political freedom, ethnic and religious divisions, which are more related to political and social issues, have less explanatory power compared to economic variables related to financing and opportunity of rebellion activities. Bazzi and Blattman (2014) differentiate between factors responsible for conflict onset and the factors responsible for conflict continuation. The development related issues is more related to conflict onset. While analyzing data for Latin America, Bhattacharya and Thomakos (2006) finds that poor trade variables, particularly agricultural exports and fuel imports explains the persistence of domestic conflicts. We could not find any theoretical work focusing on development as an instrument in a conflict setup.

2.2 The Model

The model comprises of two strategic players, the *government* and the *rebel*. There is conflict between the government, the incumbent who enjoys state power/authority, and the rebel, who tries to overthrow the incumbent and capture state power through various means, either constitutional or unconstitutional. The prize over which they fight, i.e. the worth of having the state power, is denoted by V.

The tools for fighting this conflict are rebel activities, denoted by R, for the rebel, and military/police activities, denoted by M, and/or development works in the affected areas, denoted by D, for the government. For our analysis here, we take D as expenditure on infrastructure development works in the affected areas, by the government. Note that R is taken to be expenditure on acquiring arms, by the rebel group and M is taken to be expenditure on military/police deployment in the affected areas, by the government.

We then introduce some notations. Let $P(R, M)$ denote the probability of success of the rebel. Alternatively, $P(R, M)$ can also be interpreted as the part of state power that would be captured by the rebel. Let the rebel's cost function be denoted by $C^R(R, D)$. Finally, the government's cost for military action is denoted by $C^M(M)$, and that for developmental activity is denoted by $C^D(D)$.[2]

We shall carry the following assumption throughout this chapter.

Assumption 1.
(i) $P(R, M)$ is twice differentiable with $0 \leq P(R, M) \leq 1$. Moreover, $P_R(R, M) > 0$, $P_M(R, M) < 0$.
(ii) Let $C^R(R, D)$ be twice differentiable, with $C^R_R(R, D) > 0$ and $C^R_{RR} > 0$.
(iii) Let $C^M(M)$ and $C^D(D)$ be twice differentiable, with $C^M_M(M) > 0, C^M_{MM}(M) > 0$ and $C^D_D(D) > 0, C^D_{DD}(D) > 0$.

Note that the developmental activity D does not directly enter in the contest success function. However, as we shall find shortly, by affecting the rebel's cost of organizing the rebel activities, it will affect the equilibrium choices of both R and M.

At this point we don't impose any restrictions as to how this development work will effect the rebel's cost function, it might increase or decrease it depending on the inference drawn by the population following the development works done by the state. If the people perceive that these development works are merely a response to rebel activities, so as to appease the population, then $C^R_D(R, D)$ will be negative. Whereas if the people perceive that those development works are the indicator of state benevolence, then $C^R_D(R, D)$ is expected to be positive. So both effects are possible.

We shall initially analyze this problem in a complete information framework, allowing for both these possibilities. Later on, we shall introduce uncertainty over the benevolence level of the state, and then solve for perfect Bayesian equilibrium, trying to get a sense of which effect is likely to prevail.

[2] For our purpose, these cost functions are taken to be primitives. Following Jean-Paul Azam, 1995, one can however derive them from endogenously starting from an initial endowments.

We then write down the utility function of both the agents, taken to be risk neutral. The utility function of the government is given by:

$$(1 - P(R, M))V - C^M(M) - C^D(D). \tag{2.1}$$

Whereas the utility function of the rebel is given by:

$$P(R, M)V - C^R(R, D). \tag{2.2}$$

We then turn to writing down the game form:[3]

Stage 1. The government moves first and chooses the D to maximize it's payoff.

Stage 2. The government and the rebel observe the government's choice of D and select M and R, respectively to maximize their payoff.

2.3 Exogenous D Case

For clarity of exposition, in this section, we fix D and then solve for the Nash equilibrium (henceforth NE) of the stage 2 of above one period multi-stage sequential move game. We then perform comparative statics on the level of D so as to get a sense of, particularly, the trade-off between M and D. So for this subsection, the modified game form is:

Stage 1. For a given D, the government and the rebel select M and R, respectively to maximize their payoff.

[3] One can think of the simultaneous move version where both the government and rebel moves simultaneously, the government deciding M and D and the rebel deciding R. The solution of this version/game will be given by following equations:

$$-P_M(R, M)V = C^M_M,$$

$$-C^D_D(.) = 0,$$

and

$$P_R(R, M)V = C^R_R.$$

Assume that the second-order sufficiency conditions for maximization are met, i.e., the matrix of second-order partials are negative semi-definite. These are usual marginal cost equals marginal benefit equations. Due to simultaneous moves, interdependencies are not getting captured here. Since development works do not enter directly in probability function, the state does not have any incentive for the development works (as a counter strategy to the rebel activity).

In the next section, we endogenize D and solve for the sub-game perfect Nash equilibrium (henceforth SPNE) of the above one period multi-stage sequential move game. As mentioned in the introduction, we will be working with the difference-form contest success (probability)function. For the difference-form probability function, $P_{RR} = P_{MM} = -P_{RM}, P_R \geq 0, P_M \leq 0$. (The second-order partials behave differently than that in the case of the ratio-form.)

$$P_{RR} \quad \geq (\leq) \quad 0 \quad as \quad R \quad \leq (\geq) \quad M.$$

As is usual, we solve the rebel's optimization problem and the government's optimization problem, for a given D, simultaneously.

Hence, the *rebel's optimization problem* is given by:

$$\max_{R \geq 0} P(R, M)V - C^R(R, D). \tag{2.3}$$

The interior solution will satisfy the following first order condition:

$$P_R(R, M)V = C^R_R(R, D). \tag{2.4}$$

The following second-order sufficiency condition also need be satisfied for the maximization of the rebel's utility:

$$P_{RR}(R, M)V - C^R_{RR}(R, D) \leq 0. \tag{2.5}$$

Next, the *government's optimization problem* is given by:

$$\max_{M \geq 0} (1 - P(R, M))V - C^M(M) - C^D(D). \tag{2.6}$$

The interior solution will satisfy the following first order condition:

$$P_M(R, M)V = -C^M_M(M). \tag{2.7}$$

Further, the following second-order sufficiency condition also need be satisfied for the maximization of the government's utility:

$$-\frac{\partial P_M(R, M)}{\partial M} V - \frac{\partial C^M_M(M)}{\partial M} \leq 0. \tag{2.8}$$

We can solve equation 2.4 and equation 2.7 to write the optimal level of rebel activity as $R^*(D)$ and the optimal level of military activity as $M^*(D)$. We assume that these also satisfy the 2.5 and 2.8.

We can summarize above findings in the following proposition:

Proposition 1 *Let Assumption 1 hold. Assuming the second order sufficiency conditions are satisfied,* (R^*, M^*) *which satisfy the equations 2.4 and 2.7 constitute the Nash Equilibrium (NE) of the conflict game with an exogenously given* D.

Now using the above first order conditions, we derive $\frac{\partial R}{\partial M}$, $\frac{\partial M}{\partial R}$ to characterize the best response function of the rebel and the government.

From the equation 2.4, we can derive $\frac{\partial R}{\partial M}$:

$$\frac{\partial R}{\partial M} = -\frac{\frac{\partial P_R}{\partial M} V}{\frac{\partial P_R}{\partial R} V - \frac{\partial C_R^R}{\partial R}} \tag{2.9}$$

Here the denominator is non-positive if the second-order (sufficiency) condition 2.5 is satisfied. So,

$$\frac{\partial R}{\partial M} \leq (\geq) 0 \quad \textbf{as} \quad \frac{\partial P_R}{\partial M} \leq (\geq) 0.$$

From the equation 2.7, we can derive $\frac{\partial M}{\partial R}$:

$$\frac{\partial M}{\partial R} = -\frac{\frac{\partial P_R}{\partial M} V}{\frac{\partial P_M}{\partial M} V + \frac{\partial C_M^M}{\partial M}} \tag{2.10}$$

Here again the denominator is non-negative if the second-order (sufficiency) condition 2.8 is satisfied. So,

$$\frac{\partial M}{\partial R} \leq (\geq) 0 \quad \textbf{as} \quad \frac{\partial P_R}{\partial M} \geq (\leq) 0.$$

These are quite intuitive. For the rebel, if an increase in M increases the marginal revenue for the rebel ($P_{RM} > 0$), the rebel will respond to an increase in M with an increase in R. Similarly for the government, if an increase in M increases the marginal revenue for the rebel ($P_{RM} > 0$), the government will respond to an increase in R with an decrease in M. For the difference-form probability success function, $P_{RM} > 0$ if $R > M$ and $P_{RM} < 0$ if $R < M$. In the following subsection, we discuss the existence and nature of the pure strategy Nash equilibrium using the above analysis.

2.3.1 Existence and Nature of the Equilibrium

Here we are particularly interested in the existence of the pure strategy Nash equilibrium. Existence of the pure strategy Nash equilibrium has been an issue with the difference-form contest success function in the literature. However it has been mainly an artifact of the assumed linear cost function. Here we analyze the existence problem for both the linear, as well as non-linear cost function. For the simplicity, in this subsection, we consider the cost function of the rebel as just the function of the rebellion activities $C^R(R)$. This is just to keep the argument regarding the existence of the equilibrium simple.

First, consider the case of the linear cost function. Here we solve for the rebel's best response function, the government's best response can be derived similarly. Suppose the rebel's total cost function takes the form kR where, $k > 0$. The maximum value that the marginal probability of winning for the rebel ($P_R = \frac{e^{(M-R)}}{(1+e^{(M-R)})^2}$) attains, is 1/4. Accordingly we classify three cases: 1. $k > V/4$, 2. $k = V/4$ and 3. $k < V/4$.

Case 1: $k > V/4$

Consider the rebel's optimization problem. For any given M, marginal utility $P_R V - C^R_R$ is negative. This is because the marginal probability ($P_R = \frac{e^{(M-R)}}{(1+e^{(M-R)})^2}$) takes the maximum value of 1/4 at $M = R$. So, for a given M, the first order necessary condition for the interior solution to the maximization exercise is not satisfied at any R. Starting from any positive R, since the marginal utility is negative, decreasing R increases the total utility. This is true for all $R > 0$. Hence, $R = 0$ is the best response for all M. Similar is true for the government's optimization problem as well. Hence, $(R^*, M^*) = (0, 0)$ is the unique pure strategy Nash equilibrium is this case.

Case 2: $k = V/4$

Again, consider the rebel's optimization problem. For any given M, marginal utility $P_R V - C^R_R$ attains a maximum value of zero at $R = M$. So, at $R = M$, the first-order necessary condition for the maximization is satisfied. At $R = M$, the second order sufficiency condition is satisfied as well. So, all points along the 45-degree line satisfy the first order necessary condition as well as the second order sufficiency condition for the interior solution. But we know that as R increases, the utility becomes negative. The benefit from the rebellion activity is bounded by the value V on the upper side while the cost of the rebellion activities increases without any bound on the upper side

$$\lim_{R \to \infty} \{P(R, M)V - C^R\} = -\infty$$

. However, for any M, the rebel's utility form putting zero rebellion activity is always positive and

$$\lim_{M \to \infty} \{P(R = 0, M)V - C^R\} = 0$$

So, there must exist some point of M at which the rebel find it beneficial to have zero rebellion activity rather than $R = M$ and we move to the corner solution from the interior solution. A comparison between the rebel's utility at $R = M > 0$ and $R = 0$ shows that this is true for all $M > 0$. Hence, once again $R = 0$ is the best response for all M. Similar is true for the government's optimization problem as well. Hence, once again $(R^*, M^*) = (0, 0)$ is the unique pure strategy Nash equilibrium in this case.

The *figure 2.1* shows both the cases i.e. for $k \geq V/4$:

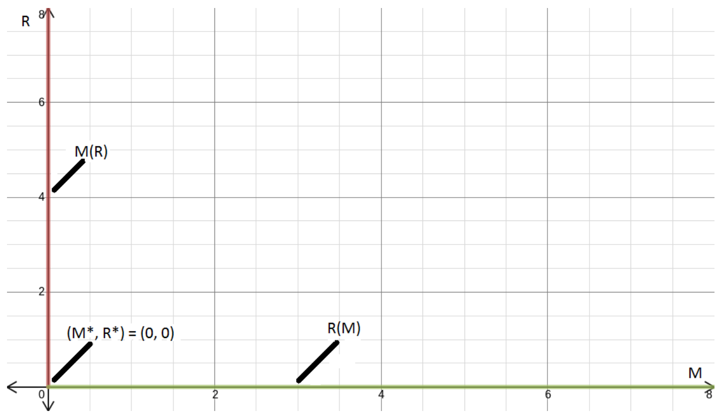


Figure 2.1: k greater than V/4

Case 3: $k < V/4$

Once again, consider the rebel's optimization problem. Now, at $M = 0$, if the rebel respond with $R = 0$, it's utility is $V/2$. Also at $(M = 0, R = 0)$, the marginal utility of the rebel is positive. So, an increases in R increases the rebel's utility. The rebel keeps on increasing R till the marginal utility becomes zero. Lets denote this point by τ. So, now for any given M, $R = M + \tau$ satisfies the first order necessary condition as well as the second order sufficiency condition for the maximization of the rebel's utility. But once again, we know that as R keeps on increasing, the rebel's utility becomes negative after some point. So, there must exist some point of M at which the rebel finds it beneficial to have zero rebellion activity rather than $R = M + \tau$ and we move to the corner solution from the interior solution. For the symmetric case, this discontinuity in the rebel best response function $(R^*(M))$ takes place at a point to the right of intercept of the government's best response function $(M^*(R))$ to the M-axis. Here by symmetry, we mean same cost function. The *figure 2.2* shows this case.

In this case, there does not exist any pure strategy Nash equilibrium (we will still have mixed strategy Nash equilibrium). However, if we have sufficient asymmetry in the cost functions (or in the values attached to the state authority, V), we will have the pure strategy Nash equilibrium.

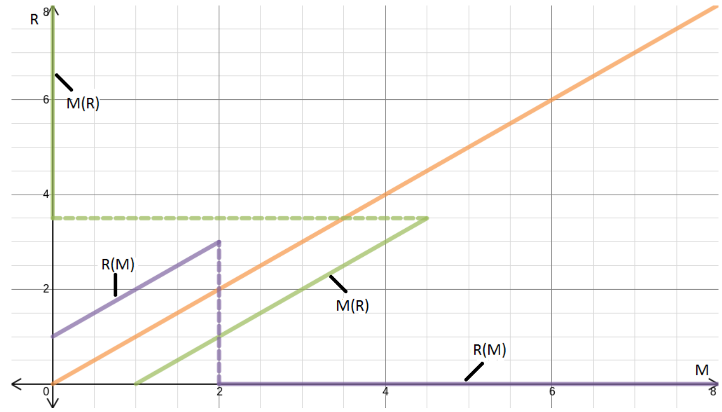


Figure 2.2: k less than V/4

Now, consider the general case of the non-linear cost function. We assume that $C_R^R(R=0)=0$. First, consider the rebel's response function. At $M=0$, if the rebel chooses $R=0$, it's marginal utility is positive. Hence, the rebel increases R to increase it's utility. So, the rebel's best response function starts at $(R^* > 0, M=0)$. For the $R > M$ case, the second order sufficiency condition 2.5 is always satisfied. So the rebel's response function is positively slopped in $R > M$ side, $\frac{\partial R}{\partial M} > 0$. It attains the maximum at $R = M$ line. In $M > R$ zone, the second order sufficiency condition is no longer always satisfied. In this zone, $P_{RR} > 0$, so for the second order sufficiency condition to be met, $P_{RR}V$ must be lower than C_{RR}^R. The function $P_{RR} = -\frac{e^{(M-R)}(1-e^{(M-R)})}{(1+e^{(M-R)})^3}$ has a global maximum at $1/6\sqrt{3}$. So, if the C_{RR}^R function has a lower bound away from zero, we can put a maximum limit on V such that $P_{RR}V - C_{RR}^R < 0$. For example, consider the quadratic cost function $C^R = R^2$. Here, $V < 6\sqrt{3}$ ensures that the second order sufficiency condition is met for all the solutions to the first order condition 2.4. Lets denote such upper bound of V by $\bar{V}$. For $V < \bar{V}$, $\frac{\partial R}{\partial M} < 0$ in the $M > R$ zone. Also, for all $M > 0$, we have $R^* > 0$. Similarly, we can derive the response function of the government. The *figure 2.3* shows the response functions for $V = 8, C^R = 0.5R^2, C^M = 0.5M^2$.

For the asymmetric case $V = 8, C^R = R^2, C^M = 0.5M^2$, we have asymmetric equilibrium where $R^* < M^*$ and $P^* < 1/2$ (*figure 2.4*).

Notice that $V < \bar{V}$ is a sufficient condition for the existence of the interior pure strategy Nash equilibrium here, not the necessary one. We can have intersection between the rebel's response function and the government's response function even without it. Also, if the rebel is cost efficient relative to the government, we have $R > M$ in the equilibrium and if the government is cost efficient relative to the rebel, we have $M > R$ in the equilibrium. This is

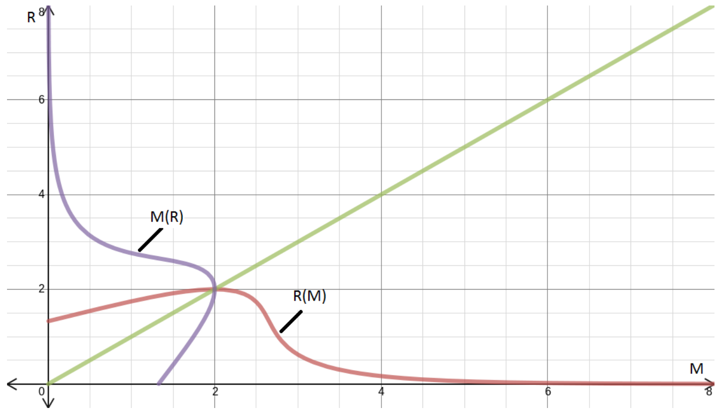


Figure 2.3: symmetric, quadratic cost functions

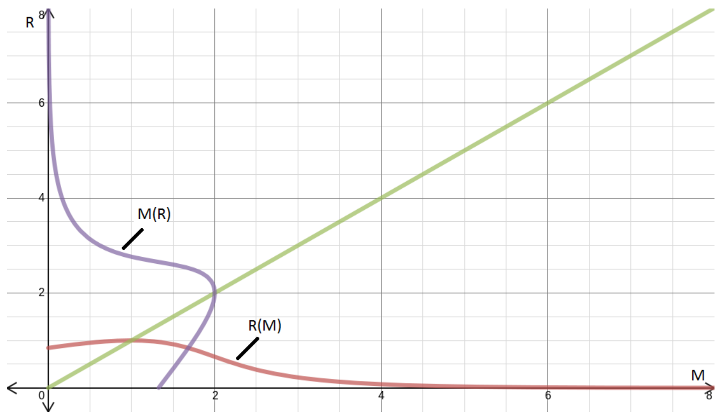


Figure 2.4: asymmetric, quadratic cost functions

intuitive, the player with cost advantages wins.

For the more general cost functions, we can have multiple pure strategy Nash equilibriums. We depict one particular case in the *figure 2.5*.

In this diagram, the points A and C are the two equilibrium points. At point B, the second order sufficiency condition for the maximization of the government's utility is not satisfied. Here no player, the government or the rebel, has a cost advantage throughout all levels of fighting.

We can summarize the main findings of this section in the following proposition:

Proposition 2 *Let Assumption 1 hold. Consider the linear symmetric cost functions ($C^R = kR, C^M = kM$ where $k > 0$) case. For $k \geq V/4$, we have a unique pure strategy Nash equilibrium ($R^* = 0, M^* = 0$). For $k < V/4$, there does not exist a pure strategy Nash equilibrium but there exists mixed strategy Nash equilibrium. For $k < V/4$, we can have pure*

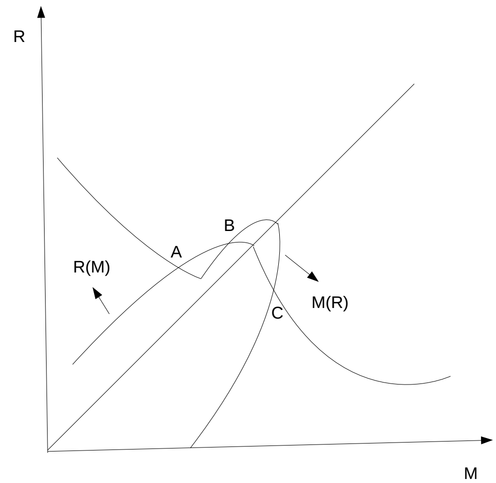


Figure 2.5: general cost functions

strategy Nash equilibrium if we have sufficient asymmetry in either cost functions or value functions. For symmetric quadratic cost functions ($C^R = R^2, C^M = M^2$), the sufficient condition for the existence of pure strategy Nash equilibrium is $V \leq 6\sqrt{3}$. In general, we can have multiple pure strategy Nash equilibriums.

Having established the existence and the nature of the equilibrium, we turn to the various comparative statics.

2.3.2 Comparative Statics

First, we consider comparative statics with regard to the level of D. Remember in this section, we are taking D as given, as exogenous. From the first order conditions, we derive $\frac{\partial R}{\partial D}$ and $\frac{\partial M}{\partial D}$. We perform comparative statics on the level of D so as to get a sense of the trade-off between M and D. Using the rebel's first-order-condition equation 2.4, we can workout the following:

$$\frac{\partial R}{\partial D} = \frac{C^R_{RD}}{P_{RR}V - C^R_{RR}} \tag{2.11}$$

The denominator, $P_{RR}V - C^R_{RR}$ is non-positive from the rebel's second order condition, 2.5. So, $\frac{\partial R}{\partial D}$ has the opposite sign as that of C^R_{RD}. If C^R_{RD} is positive, an increase in D increases the marginal cost of the rebellion activities. So, in response, the rebellion activities decrease.

The government's first-order-condition, equation 2.7, does not consist D directly so

$$\frac{\partial M}{\partial D} = 0 \tag{2.12}$$

This zero effect is due to the partial derivative which does not take into account the effect of D on R here. To get the net resultant effect we derive total derivative $\frac{dR}{dD}$ and $\frac{dM}{dD}$.

$$\frac{dR}{dD} = \frac{C^R_{RD}(P_{MM}V + C^M_{MM})}{(P_{RR}V - C^R_{RR})(P_{MM}V + C^M_{MM}) - (P_{RM}V)^2} \tag{2.13}$$

$$\frac{dM}{dD} = -\frac{P_{RM}C^R_{RD}V}{(P_{RR}V - C^R_{RR})(P_{MM}V + C^M_{MM}) - (P_{RM}V)^2} \tag{2.14}$$

So, $\frac{dR}{dD}$ has the opposite sign of C^R_{RD}. If an increase in D increases the marginal cost of the rebel, then an increase in D result in a decrease in the rebellion activities R. Unlike the partial differential case, now the $\frac{dM}{dD}$ is not zero and a change in D has an impact on M. The sign of this impact depends on, as expected, P_{RM} and C^R_{RD}. Consider the case when both are positive. Here $\frac{dM}{dD}$ is also positive. If P_{RM} is positive, an increase in M increases the marginal probability of success for the rebel. But here the cost effect dominates and the equilibrium M increases as a response to an increase in D.

We can summarize it in the following proposition:

Proposition 3 *Let Assumption 1 hold. Let $C^R_{RD} > 0$, i.e. an increase in the development activity increases the marginal cost of the rebellion activity. Then the following hold:*

1. *An increase in development activities, i.e. D decreases the equilibrium level of the rebellion activity.*

2. *An increase in development activities increases (decreases) the equilibrium level of the military activity if $P_{RM} > 0$ ($P_{RM} > 0$). If the rebel is cost efficient relative to the government, then an increase (decrease) in development activities increases (decreases) the equilibrium level of the military activity.*

Next, we analyze the effect of the rebel getting stronger. We capture this by replacing the rebel's cost function by $\zeta C^R(R, D)$ where, $\zeta > 0$. A decrease in ζ captures the increase in the rebel's strength. First, we analyze the effect on $\frac{\partial R}{\partial M}$. The rebel's utility changes to

$P(R, M)V - \zeta C^R(R, D)$.

The equation 2.4 changes to

$$P_R(R, M)V = \zeta C^R_R(R, D)$$

The modified $\frac{\partial R}{\partial M}$ will become $-\frac{P_{RM}V}{P_{RR}V - \zeta C^R_{RR}}$. First thing to notice is that the sign of the $\frac{\partial R}{\partial M}$ remains the same, as the denominator continues to be non-positive from the second-order sufficiency condition for the maximization of rebel's utility. Only the magnitude changes.

$$\frac{\frac{\partial R}{\partial M}}{\partial \zeta} = \frac{C^R_{RR} P_{RM} V}{(P_{RR}V - \zeta C^R_{RR})^2}$$

$\frac{\partial R}{\partial M}$ increases with an increase in ζ iff $P_{RM} \leq 0$ and decreases with an increase in ζ iff $P_{RM} \geq 0$. So, $\frac{\partial R}{\partial M}$ decreases with ζ if it is positive. In other words, cost function advantages reinforce the aggressive or passive responses of rebel.

The modified $\frac{\partial M}{\partial R}$ will remain same.

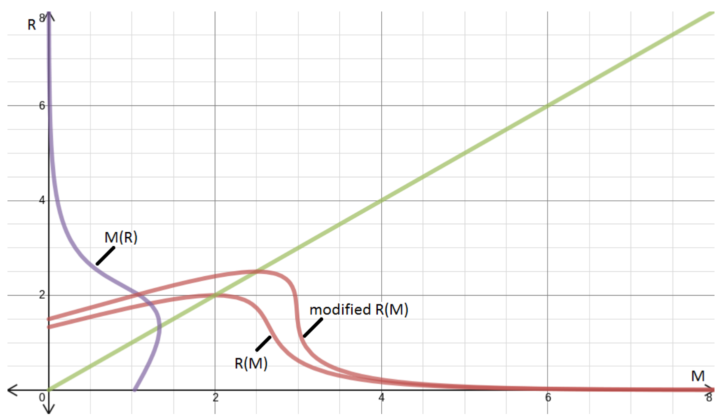


Figure 2.6: Comparative statics: rebel getting stronger

So, the effect of the rebel getting stronger, through cost advantages, depends on the rebel being a dominant player. If the rebel is the dominant player, it becomes more aggressive . If the rebel is not the dominant player, it becomes more accommodative. Cost efficiencies shift the reaction curve outward. If the stronger player becomes more cost efficient, the weaker plays an accommodative role. Suppose initially the rebel is stronger. Now if the rebel become more cost-efficient, the government plays accommodative role. As a result, R rises and M falls. This case is shown in *figure 2.6*. On the other hand, if the rebel is weaker initially, as in *figure 2.7*, the government responds with an increase in M. So, both R and

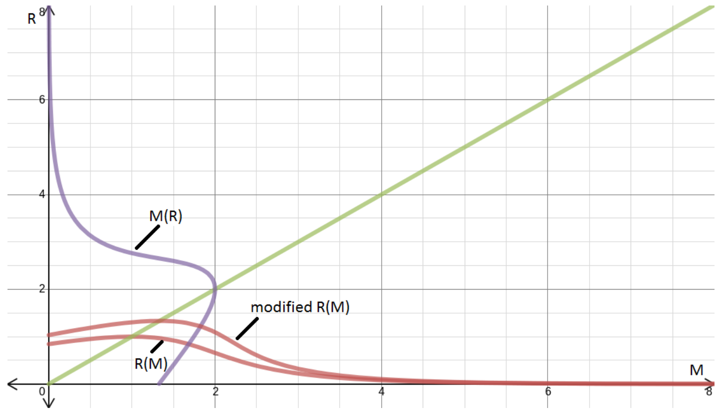


Figure 2.7: Comparative statics: rebel getting stronger

M rise.

This result is similar to the result from Esteban and Ray (2011). Esteban and Ray (2011) finds that an "increase in activism by one contending group will create escalation or deterrence depending on whether that group was weaker or stronger to start with". There if a moderate group become more aggressive, it escalates the conflict with both groups increasing the fighting efforts.

Now consider the effect on $\frac{dR}{dD}$. The equation 2.13 changes to

$$\frac{dR}{dD} = \frac{\zeta C^R_{RD}\{P_{MM}V + C^M_{MM}\}}{\{P_{RR}V - \zeta C^R_{RR}\}\{P_{MM}V + C^M_{MM}\} - (P_{RM}V)^2}.$$

Again, the sign remains the same only the magnitude changes.

$$\frac{\partial \frac{dR}{dD}}{\partial \zeta} = \frac{C^R_{RD}B\{BP_{RR}V - (P_{RM}V)^2\}}{\{AB - (P_{RM}V)^2\}^2}. \tag{2.15}$$

where $A = P_{RR}V - \zeta C^R_{RR}$ and $B = P_{MM}V + C^M_{MM}$.

For the difference-form contest success function, we have $P_{RR} = P_{MM} = -P_{RM}$. Using this, we get,

$$BP_{RR}V - (P_{RM}V)^2 = C^M_{MM}P_{RR}V.$$

Consider the case where $C^R_{RD} > 0$. Here we have, $\frac{\partial \frac{dR}{dD}}{\partial \zeta} < 0$ ($\frac{\partial \frac{dR}{dD}}{\partial \zeta} > 0$) if $P_{RR} < 0$ ($P_{RR} > 0$). Here, B is positive. We have $P_{RR} < 0$ in the region $R > M$ (upper side of the 45-degree line). Intuitively, as ζ goes down, the marginal cost of the rebellion activities goes down. The rebel needs to adjust the rebellion activities through its optimal responses (with regard to the development, in this case) to maintain the optimality. If the $P_{RR} < 0$, the rebel adjusts in the upper direction to maintain the optimality. As a result $\frac{dR}{dD}$ increases. If the $P_{RR} > 0$, the rebel needs to decrease $\frac{dR}{dD}$ to maintain the optimality.

Last, we consider the effect on $\frac{dM}{dD}$. The equation 2.14 changes to

$$\frac{dM}{dD} = -\frac{P_{RM}\zeta C^R_{RD} V}{(P_{RR}V - \zeta C^R_{RR})(P_{MM}V + C^M_{MM}) - (P_{RM}V)^2}. \tag{2.16}$$

$$\frac{\partial \frac{dM}{dD}}{\partial \zeta} = -\frac{P_{RM}VC^R_{RD}\{BP_{RR}V - (P_{RM}V)^2\}}{\{AB - (P_{RM}V)^2\}^2}. \tag{2.17}$$

where $A = P_{RR}V - \zeta C^R_{RR}$ and $B = P_{MM}V + C^M_{MM}$

As expected here the effect depend on the net sign of P_{RM} and C^R_{RD}.

We can summarize this case in the following proposition:

Proposition 4 *Let Assumption 1 hold. Suppose that the rebel gets stronger i.e ζ decreases. Then the following hold true:*

1. *The rebel become more aggressive (accommodative) with respect to M i.e $\frac{\partial R}{\partial M}$ increases (decreases) if $P_{RM} > 0$ $(P_{RM} < 0)$.*

2. *Suppose $C^R_{RD} > 0$. $\frac{dR}{dD}$ increases (decreases) if $P_{RR} < \frac{(P_{RM}V)^2}{BV}$ $(P_{RR} > \frac{(P_{RM}V)^2}{BV})$, where $B = P_{MM}V + C^M_{MM} > 0$.*

3. *$\frac{dM}{dD}$ decreases (increases) if $P_{RM}C^R_{RD}\{BP_{RR}V - (P_{RM}V)^2\} < 0$ $[P_{RM}C^R_{RD}\{BP_{RR}V - (P_{RM}V)^2\} < 0]$, where $B = P_{MM}V + C^M_{MM} > 0$.*

2.4 Endogenous D

In this subsection, we endogenize the development works D. The government first chooses D to maximize it's utility. The government and the rebel then observes this choices and pick M and R, respectively, to maximize their payoff. The utility functions of both the government and the rebel are same as earlier, given by equations 2.1 and 2.2 respectively. Reproducing the game, as given earlier:

Stage 1. The government moves first and chooses the D to maximize it's payoff.

Stage 2. The government and the rebel observe the government's choice of D and select M and R, respectively to maximize their payoff.

We solve for the SPNE of this game. It involves solution by the standard backward induction technique.

Stage 2. We first solve the second stage of the game, for a given D. We have already done it in the last section. *Proposition 1* characterizes the solution to this stage.

Stage 1. Now the government will incorporate the solution of the *Stage 2* and chooses D to maximize it's utility given by 2.1.

$$\{1 - P(R^*(D), M^*(D))\}V - C^M(M^*(D)) - C^D(D).$$

The interior solution will satisfy the following first-order-conditions:

$$D : -P_R \frac{dR}{dD} V - C^D_D = 0. \tag{2.18}$$

Assuming that the corresponding second-order condition is satisfied, $\frac{dR}{dD}$ must be negative for equation 2.18 to be satisfied. This implies that C^R_{RD} must be positive for the equilibrium development level to be positive. This is on intuitive line. If C^R_{RD} is positive, an increase in the development (D) increases the marginal cost of the rebellion activities (R) for the rebel. Here in our model, D does not have any (non-strategic) welfare utility for the government. If we introduce such welfare term in the government's utility (either by adding people utility as an increasing function of D or make V a function of D), we will have positive development level in equilibrium even if C^R_{RD} is negative. For the simplicity of exposition, we refrain from doing so here. Also notice, it is a restriction on second-order partial, we do not need to put any condition on, say, C^R_D.

Next, we perform the *comparative statics* with regard to the changes in the rebel's cost function, as in the earlier section. We replace the the rebel's total cost function C^R by ζC^R, where $\zeta > 0$. So a decrease in ζ shows that the rebel has become more stronger in cost efficiency sense. We would like to find out how the equilibrium level of the development D^* changes with respect to the ζ. Now, the government's utility is given by

$$\{1 - P\left(R^*(D,\zeta), M^*(D,\zeta)\right)\}V - C^M\left(M^*(D,\zeta)\right) - C^D(D)$$

Lets denote it by U^G. We maximize this with respect to D. Corresponding to the equation 2.18, we get the following first order condition:

$$U^G_D = -P_R\left(R^*(D,\zeta), M^*(D,\zeta)\right)\frac{dR}{dD}(D,\zeta)V - C^D_D = 0$$

From the first order necessary condition for the utility maximization of the rebel (from the second stage), 2.4, we know that $P_R(R,M)V = \zeta C^R_R(R,D)$. Using this in the government's utility function, we gets

$$U^G_D = -\zeta C^R_R\left(R^*(D,\zeta), D\right)\frac{dR}{dD}(D,\zeta)V - C^D_D = 0$$

We use this equation to get $\frac{dD}{d\zeta}$.

$$\frac{\partial U_D^G}{\partial D}dD + \frac{\partial U_D^G}{\partial \zeta}d\zeta = 0$$

$$\frac{dD}{d\zeta} = -\frac{\frac{\partial U_D^G}{\partial \zeta}}{\frac{\partial U_D^G}{\partial D}}$$

Now, from the second-order sufficiency condition (for the maximization of the government's utility w.r.t D), we have

$$\frac{\partial U_D^G}{\partial D} < 0$$

So, $\frac{dD}{d\zeta}$ has the same sign as that of $\frac{\partial U_D^G}{\partial \zeta}$.

$$\frac{\partial U_D^G}{\partial \zeta} = -\left\{C_R^R\frac{dR}{dD} + \zeta\left(C_{RR}^R\frac{\partial R}{\partial \zeta}\frac{dR}{dD} + C_R^R\frac{\partial\frac{dR}{dD}}{\partial \zeta}\right)\right\}$$

In the above expression, we know that $C_R^R > 0$, $\frac{dR}{dD} < 0$ (from 2.18), $\zeta > 0$, $C_{RR}^R > 0$, $\frac{\partial R}{\partial \zeta} > 0$ (we have established these in the comparative statics in the earlier section). So if $\frac{\partial\frac{dR}{dD}}{\partial \zeta} < 0$, we have $\frac{\partial U_D^G}{\partial \zeta} > 0$. This implies that $\frac{dD}{d\zeta} > 0$. From the last section's comparative statics exercise, we know that $\frac{\partial\frac{dR}{dD}}{\partial \zeta} < 0$ if $P_{RR} < 0$. $P_{RR} < 0$ in the region $R > M$. So in the region $R > M$, we have $\frac{dD}{d\zeta} > 0$. Suppose we start from a situation where the rebel has the cost advantages over the government. As a result we have in the equilibrium $R > M$ (as established in the last section). Now, suppose the rebel becomes even more stronger, i.e., the cost of rebellion activities goes down (ζ decreases). This comparative statics result tells us that the government's optimal response is to decrease the level of the development. If the rebels are dominant, the government plays an accommodative role. Consider the other case. Suppose the government has the cost advantages and the equilibrium is in the region $M > R$. Here we have $P_{RR} > 0$. Now the comparative statics result depends on the magnitude of the P_{RR} at the equilibrium point. If the P_{RR} is low enough then we still have $\frac{dD}{d\zeta} > 0$. But as P_{RR} increases, we can have the case where $\frac{dD}{d\zeta} < 0$. We can summarize this in the following proposition:

Proposition 5 *Let Assumption 1 hold. Suppose that the rebel gets stronger i.e ζ decreases. Then the followings hold true:*

1. *The government decreases equilibrium level of development if the rebel has the cost advantages (in the region $R > M$).*

2. *Suppose the government has the cost advantages (the region $R < M$). Here the result with regard to changes in equilibrium level of development, is ambiguous. So the equilibrium level of development might increase or, decrease.*

So, if the rebels become stronger, we might have lower development in the equilibrium. This is definitely the case when the rebels are dominant. Even when the government is dominant, we can have lower development in the equilibrium. If the government is dominant, we know that $P_{RR} > 0$. As explained earlier, we can still have $\frac{dD}{d\zeta} > 0$ if P_{RR} is sufficiently small.

2.5 General Value Function

Till now we have assumed the value attached with the state power/authority to be constant i.e V. However, it can very much be a function of the level of conflict taken place to achieve it, among other things. We all are aware of huge losses that conflict brings for the human life, society and the economy. Depending on the conflict technology, the effective level of conflict can be aptly described either by the aggregate level of conflict $(R + M)$ or the net level of conflict ($|R - M|$ or $(R - M)^2$), or a combination of both. In the traditional warfare, the aggregate level of conflict aptly captures the total causalities and damages. However, in the modern warfare, including cyber-war, the net level of conflict seems more suited. Here the causalities and damage depend more on the difference in the efforts/forces deployed. Consider the following form of the value function:

$$V - aR - bM - c(R - M)^2.$$

where, $a, b > 0$; and $c > 0$. Here if $c = 0$, we are in absolute traditional warfare whereas if $a, b = 0$ we are in absolute modern warfare.

Now, the modified utility function of the rebel is given by

$$P(R, M)\{V - aR - bM - c(R - M)^2\} - C^R(R, D). \tag{2.19}$$

and the utility function of the government is given by

$$(1 - P(R, M))\{V - aR - bM - c(R - M)^2\} - C^M(M) - C^D(D). \tag{2.20}$$

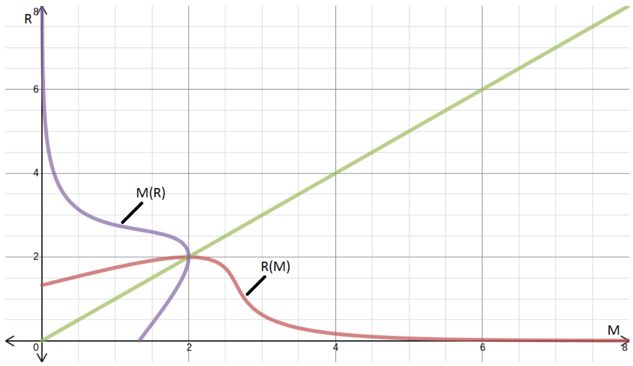
We derive the solution of the earlier game, with exogenously given D, with these modified utility functions. The first order condition equations 2.4 and 2.7 changes to

$$P_R\{V - aR - bM - c(R - M)^2\} - \{a + 2c(R - M)\}P - C^R_R = 0 \tag{2.21}$$

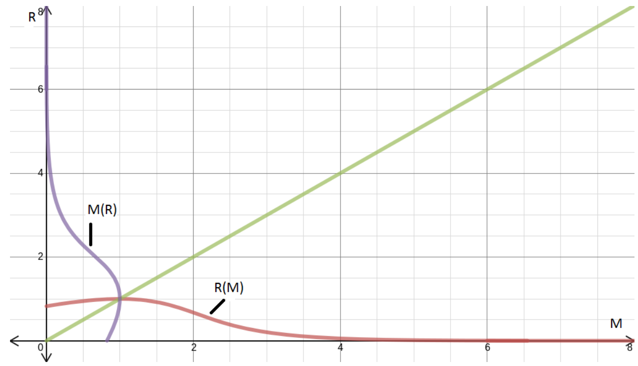
and

$$-P_M\{V - aR - bM - c(R - M)^2\} + \{b - 2c(R - M)\}(1 - P) - C^M_M = 0 \tag{2.22}$$

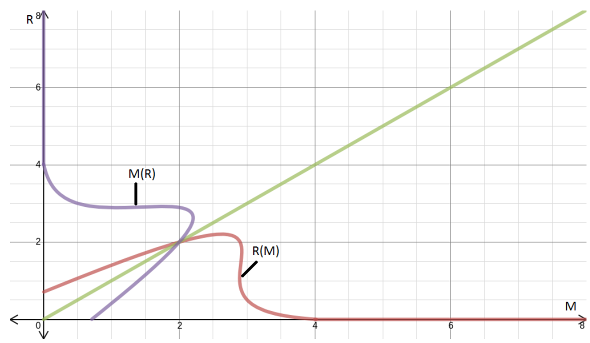
We also have corresponding second-order sufficiency conditions for maximization. Using these first order conditions we can derive $\frac{\partial R}{\partial M}$ and $\frac{\partial M}{\partial R}$ as earlier. Assuming these second order sufficiency conditions are satisfied, we can derive the best response functions. These equations are kind of analytical extension of the corresponding earlier equations, taking into account the new parameters a, b and c.



(a) constant value function: $a = 0, \quad c = 0$



(b) general value function: $a = 1, \quad c = 0$



(c) general value function: $a = 0, \quad c = 1$

Figure 2.8: general value function: traditional and modern warfare

The diagrams in *Figure 2.8* show the cases of a constant value function $(a, b, c = 0)$, general value function with traditional warfare $(a = b = 1, c = 0)$ and general value function with modern warfare $(a = b = 0, c = 1)$. We have taken $V = 8$ and the linear marginal cost function. These are for the particular parameters' value but still show the implications of the various forms. Compared to the constant value function, the traditional warfare general value function form admits the same shape but lesser level of conflicts. Here, for both the government as well as the rebel, the cost of fighting remain the same. However the benefit is now smaller. As a result the level of fighting goes down. The magnitude depends on the parameters a and b, here the symmetry comes from the assumption $a = b$. Now, when we move to the modern warfare, things are a bit different. The non-strategic benefit from the fighting goes down in this case also, although not by the similar magnitude. However, here matching opponent's fighting power not just increases the probability of winning but also increases the prize or the value itself. The magnitude of the net effect once again depend on the parameter c. Here this particular shape of the best response functions are due to the similarity of the quadratic forms of the cost function and the loss to the value.

So, all the analysis of the constant value case can be extended to the general value function case in a analytically equivalent way. However, compared to earlier version with constant value function, it is possible that a equilibrium might become stable from unstable (or otherwise also).

2.6 Alternative specifications of the contest success function

An alternative contest success function, often used in the literature, is the ratio form, where the probability of success is given by $\frac{R}{R+M}$. Hirshleifer (1989) provides a comparative study of ratio form and (logistic) difference form contest success functions. In a subsequent work, Jia, Skaperdas and Vaidya (2013) discusses different functional specifications of the ratio form (also referred to as the additive form) and (logistic as well as probit) difference form.

For the standard ratio form it is well known that $P_{RR} < 0$ and $P_{MM} > 0$, always. Here we also have $P_{RM} > 0$ if and only if $R > M$. Given that the difference-form also satisfies the property that $P_{RM} > 0$ if and only if $R > M$, and that, for the difference-form, most of our central results actually follow from this property, many of our results go through under the ratio form also. In particular, *Proposition 3(1)* goes through. Strikingly, *Proposition 5* becomes stronger under the ratio form, in that in case the rebel becomes stronger, the level of development necessarily decreases irrespective of who is the dominant player. This is because we always have $P_{RR} < 0$ in the case of ratio form.

There are some changes though. Suppose the rebel become stronger, i.e. ζ decreases. Then $\frac{dR^*}{dD}$ necessarily increases, whereas the effect on $\frac{dM^*}{dD}$ depends on who is the dominant player, the rebel, or the government. Recall that both these results are different from that under the difference-form (see *Proposition 4*).

Jia, Skaperdas and Vaidya (2013) have also considered a specification of ratio form, which allows for draws, i.e $P = \frac{R}{s+R+M}$, $s > 0$. Here $P^s = \frac{s}{s+R+M}$ is the probability of draw and $(1 - P - P^s)$ is the probability of winning for the government. For this specification as well, the signs of P_{RR}, P_{MM}, and P_{RM} remain the same. Hence the preceding analysis for the standard ratio form applies in this case also.

Similarly with other functional forms, as discussed in Jia, Skaperdas and Vaidya (2013), most of our results go through. This is because, qualitatively, our results depend on the signs of P_{RR}, P_{MM}, and P_{RM}. The general 'logit' specification mentioned in Jia, Skaperdas and Vaidya (2013), i.e $P = \frac{1}{1+e^{k(M-R)}}$, where $k > 0$, is same as the general difference form that we have considered in the *Appendix* of this chapter. For this general specification also, signs of P_{RR}, P_{MM}, and P_{RM} are same as that of the contest success function ($k = 1$) used in this chapter. They have also considered a 'probit' specification: $P = \Phi(M - R)$, where Φ is the cumulative distribution function of the standard normal distribution. Upon derivation, we see that here also the signs of P_{RR}, P_{MM}, and P_{RM} are same as that of the contest success function used in this chapter.[4] Hence for all these specification, most of the results and analysis go through.

Conceptually, we feel however that the difference form is more natural in this context. As pointed out by Hirshleifer (1989) also, the difference form has the property that both the rebel and the government has a positive probability of winning even if their respective conflict activities are zero. This seems to capture the notion that both the government, as well as the rebel has some measure of popular support, which cannot be totally over-turned by purely conflict activities.

2.7 Conclusion

In this chapter we have considered a conflict problem which in domestic (intra country) in nature. There is a incumbent government and a group of people are rebelling against the government. The rebels might have different motives. Suppose the rebels want to capture the state power. Such rebellion issues are present in many parts of the world. In particular, we want to study this setup, in the context of the Maoist problem in India. Now in this setup, a lot of people in popular media, as well as in academia believe that the lack of development is the root cause of such rebellion activities. Hence they advocate use of development instead of tough actions to counter such rebels. In this chapter we have studied conditions under which this kind of argument holds true.

We have used the difference-form contest success function for analysis. In the literature, there are issues related to the existence of pure strategy Nash equilibrium in the conflict game with this form of contest success function. So we begin by studying the existence of pure strategy Nash equilibrium. We have analyzed for both linear as well as non-linear cost functions. We have shown that for large and significant range of parameters, pure strategy Nash equilibrium exist. Non-existence of the pure strategy Nash equilibrium is mainly an

[4] $P_{RR} = P_{MM} = \frac{M-R}{\sqrt{2\pi}} e^{\frac{-(R-M)^2}{2}}$ and $P_{RM} = -P_{RR}$.

artifact of specific cost function (particularly linear) assumed. Even when we don't have pure strategy Nash equilibrium, we have mixed strategy Nash equilibrium.

Further we have analyzed the effect of changes in development level on the rebellion activities. Consider the case $C^R_{RD} > 0$ i.e. a increase in development increases the marginal cost of rebellion activities. Suppose there is an increase in development by some exogenous reason. For example, suppose the government build a road to increase connectivity. It decreases the equilibrium level of rebellion activity (R^*). However it might increase the equilibrium level of police and military deployment (M^*) by the government. It happens when rebels are stronger. Another interesting finding is that if the rebel gets stronger, the government might find it optimal to decrease the equilibrium level of development. This is surprising because with $C^R_{RD} > 0$, one might think that increasing D is best response. We also get the expected result that the equilibrium level of development is positive only if $C^R_{RD} > 0$.

So, basically even in a welfare state and when the government's development work increases the marginal cost of rebellion activities, there exists cases when strategically it is optimal for the government to *decrease* development. We have studied quite a few such scenarios in this chapter. This argument should be kept in mind while analyzing government's actions.

Here we have used a setup with just one rebel group. The analysis here can be expanded to the case of multiple rebel groups. Also it need not be a incumbent-rebel setup. Becsi and Lahiri (2007) and Lahiri and Vlad (2012) provide a setup for such cases. So this framework can be used in a wide range of strategic interaction contexts. Whenever a player has more than one instruments, particularly carrots and sticks options, insights from this chapter can be used. This setup can also be used in analyzing internal organization of a firm or industry federation where the policy of one party can be either accommodative or (hard) restrictive. Similar is true for internal organization of political parties.

2.8 Appendix

The ratio-form and the difference-form contest success functions are two most widely used forms in the literature: Ratio form- $P = \frac{R^k}{R^k+M^k}$, Difference form- $P = \frac{1}{1+e^{k(M-R)}}$.

We can have other forms as well, like winner-takes-all (as in the auction setup). But given the conflict setup in our context, we restrict our attention to these two types only.

Ratio-form probability functions:

$$P = \frac{R^k}{R^k + M^k}$$

$$\frac{\partial P}{\partial R} = \frac{kR^{k-1}M^k}{(R^k + M^k)^2}$$

$$\frac{\partial P}{\partial M} = -\frac{kR^kM^{k-1}}{(R^k + M^k)^2}$$

$$\frac{\partial^2 P}{\partial R \partial M} = \frac{kR^{k-1}M^{k-1}(R^k - M^k)}{(R^k + M^k)^3}$$

$$\frac{\partial^2 P}{\partial R \partial R} = \frac{kR^{k-2}M^k\{(k-1)M^k - (k+1)R^k\}}{(R^k + M^k)^3}$$

$$\frac{\partial^2 P}{\partial M \partial M} = \frac{kR^kM^{k-2}\{(k+1)M^k - (k-1)R^k\}}{(R^k + M^k)^3}$$

Here, we have $P_R \geq 0$ and $P_M \leq 0$. For $k \leq 1$, we have $P_{RR} < 0$ and $P_{MM} > 0$. So for $k \leq 1$, we do not have any inflection point. $k = 1$ is the most widely used version in the literature and this is one of the main criticism of this existing literature. For $k > 1$, they have an inflection point, but it occurs before the equality, like for P_{RR}, it happens at a point where $R < M$. Another main criticism of this form of the contest success function is that if one player is putting zero fighting effort, he looses everything (probability of winning become zero for him), irrespective of how small an effort the other player is putting. As a result it is not possible to sustain peace (even one-sided) in the equilibrium.

Difference-form probability functions:

$$P = \frac{1}{1 + e^{k(M-R)}}$$

$$\frac{\partial P}{\partial R} = \frac{ke^{k(M-R)}}{(1 + e^{k(M-R)})^2}$$

$$\frac{\partial P}{\partial M} = -\frac{ke^{k(M-R)}}{(1 + e^{k(M-R)})^2}$$

$$\frac{\partial^2 P}{\partial R \partial M} = \frac{k^2e^{k(M-R)}\{1 - e^{k(M-R)}\}}{(1 + e^{k(M-R)})^3}$$

$$\frac{\partial^2 P}{\partial R \partial R} = -\frac{k^2e^{k(M-R)}\{1 - e^{k(M-R)}\}}{(1 + e^{k(M-R)})^3}$$

$$\frac{\partial^2 P}{\partial M \partial M} = -\frac{k^2e^{k(M-R)}\{1 - e^{k(M-R)}\}}{(1 + e^{k(M-R)})^3}$$

Here also we have $P_R > 0$ and $P_M < 0$. But now we always have point of inflection at $R = M$. Also in this form, $P > 0$ even if $R = 0$ and $M > 0$. So unlike ratio form, here we can sustain peace as an equilibrium. It seems more suitable for our context of the civil conflicts. There has been issues with the existence of Nash equilibrium in this form but as we have already discussed, these issues are very case specific and can be worked with.

Chapter 3

Conflicts: Incomplete Information

3.1 Introduction

In this chapter, we complement the analysis done in the last chapter. Here we provide a foundation for the people's support to the government. It is often reported in the popular media, as well as in academia that a significant portion of the public support, or at-least have sympathy for, the rebels. The main reason behind this support or sympathy is said to be the lack of development in the concerned area. Due to this lack of development, the people form a perception that the government is not a benevolent government and it does not care for them. So, here the government can use development as a signaling device to convey it's benevolence.

In the literature on civil war and conflict, often inequality, polarization, and lack of development are associated with (and blamed as the causes for) such conflicts. While studying internal conflicts in south Asia, Iyer (2009) concludes- 'Analyzing the role of economic, geographic and demographic factors, I find that poorer areas have significantly higher levels of conflict intensity.' She supports it with extensive empirical analysis. Blattman and Miguel (2010) also express similar views- 'The most robust empirical finding in the existing literature is that economic conditions- both low income levels and slow growth rates- contribute to the outbreak of civil wars and conflicts in less developed countries.' In India, the Maoist problem is more acute in areas which are economically backward, particularly rural and hilly areas.

Collier and Hoeffler (2004) test alternative theories of grievances and greed as the causes of civil war. There are political and social issues such as high inequality, less political freedom, ethnic and religious divisions which are often related to rebellion problem. These issues are mainly related to grievances of the people. Whereas there are economic issues which are more related to opportunity and viability of rebellion activities. These can be classified as greed related issues. Capturing natural resources, outside finances and weak state institutions provide proxies for these issues. Collier and Hoeffler (2004) finds that economics variables have considerably more explanatory power for civil war. Bazzi and Blattman (2014) use export price shocks to test theories which relate income and conflict. Here authors differentiate between the factors influencing start/onset of a conflict and, continuation of a conflict. Our work, focusing on development, is more related to the conflict onset.

In India, we often see activists and columnists like, Arundhati Roy, Rahul Pandita and others, blaming lack of development and government's apathy for the rise of such conflict problems. According to a TOI-IMRB survey, published in the newspaper Times of India

(28th September, 2010), a significant portion of people seems to believe that the government has neglected them. They also do not agree with the hard and strict actions against the Maoists. This survey was conducted in some of the Maoist (severely) affected areas. So, in this context, the government can use development as a signaling device to convey it's benevolence.

We analyze this in a incomplete information framework. We use the standard signaling game model for the analysis here. We continue with the same context as in the last chapter. There is an incumbent government and a rebel group. The rebel group wants to overthrow the government and capture the state power through the rebellion activities (R). The government can use development (D) and/or brute force (M) to counter this rebel group. To fix the impact of D as a signaling instrument, we change the cost function of the rebel to $C^R(R)$. So here the rebel's cost depends on just the rebellion activities and not on the development level.[1] The government can be of two type: benevolent (B-type) and non-benevolent (NB-type). The people do not observe the government's original type, it is private information of the government.

The government first decide on a level of development. The people observe this decision and form a perception about the government. If they think that the government is a benevolent government and it cares for them, they support the government (in the ongoing conflict against the rebel). Once the public perception is formed, the government and the rebel engage in the conflict game, as earlier. Both types of the government cares for this support. It increases it's probability of winning the fight against the rebel. The benevolent type government cares for the people in a non-strategic way also, it's utility includes the people's utility from D. Let us denote the people's utility from the development work by $U^P(D)$, it increases in D. In this conflict, the people's support and sympathy; even the mass perception matters significantly. We were silent about this aspect in the last chapter. Here we analyze the formation of this support, sympathy and perception.

Here, we derive both, separating and pooling, perfect Bayesian equilibriums. We study the existence and nature of these equilibriums. Here it is a three players strategic interaction framework. Two players (government and rebel) are involved in a conflict game and they seek third player's (people's) support. Third player's support improves the player's position in the conflict game. We have used it to analyze domestic conflicts in a incumbent-rebel setup. However, it can be used in a wide range of contexts. This setup can be applied to

[1]The earlier form $C^R(R, D)$ also works fine and the all the analysis holds qualitatively but it make the algebra a lot messier.

the contexts of international trade and oligopolistic industries. We find that, in general, there will be over-signaling. In our context, equilibrium level of development will be too high. We apply intuitive criterion refinement to these equilibriums to get rid of multiplicity of equilibrium. The intuitive criteria works really sharply here. All pooling equilibriums get eliminated. All, but one, separating equilibrium get eliminated too. However, in the context of productive signals, there are problem with intuitive criteria. We discuss it in detail.

In next section, we specify the formal model.

3.2 The Model

The model comprises of two strategic players, *government* and *rebel*. We also have a non-strategic player, *people*. The people follow a mechanical decision rule, which we describe later in this section. First, people and government engage in a signaling game. The government chooses development level D, people observes D and decide whether to support the government or not. After this signaling game, government and rebel engage in a conflict game, similar to the one in last chapter. They fight to control the state power. Let us denote the worth of state power by V. The government chooses level of military resources (M) and the rebel chooses level of rebel activities (R), simultaneously. We will be using the solution concept of perfect Bayesian equilibrium (PBE). We will analyze the separating, as well as the pooling equilibrium cases.

In the last chapter, we have used the complete information framework. Here we are using the incomplete information framework. In this chapter, we are interested in analyzing the effect of government's ability to use development as a signaling device (of it's benevolence). In last chapter we have assumed that development (D) affect the cost of rebellion activities. Here, for simplicity, we have assumed that D does not enter directly in the rebel's cost function. However D affect rebel's cost of fighting, through people's support. So, the analysis here kind of explain the foundation behind the cost function of rebellion activities.

We now specify the modified contest success function:

$$P(R, M, x) = \frac{1}{1 + e^{M-R+x}}$$

Here x is the indicator for the people's support to the government. It takes the value α (a positive constant) if people support the government and it takes the value 0 if they do not support the government. Let ρ be the people's prior belief about a government being a benevolent type government. So given this information set, the people believe that any particular government is a benevolent type with probability ρ and non-benevolent type with probability $1-\rho$. We assume that $0<\rho<1$, we need this assumption for some of the analysis. Also, $\rho = 0 \quad \text{or} \quad 1$ are not the interesting cases to look at. We assume that people are non-strategic. In our model, the people are supposed to follow a mechanical rule. If the ex-post belief is that the government is benevolent with probability ρ, then they support the government with probability ρ and do not support the government with the probability $1-\rho$. For any given R and M, the people's support translate into higher probability of winning for the government.

We shall make the following assumptions (same as the last chapter) throughout this chapter.

Assumption 1 (A1).
(i) $P(R, M, x)$ is twice differentiable with $0 \leq P(R, M, x) \leq 1$. Moreover, $P_R(R, M, x) > 0$, $P_M(R, M, x) < 0$.
(ii) Let $C^R(R)$ be twice differentiable, with $C^R_R(R) > 0$ and $C^R_{RR} > 0$.
(iii) Let $C^M(M)$ and $C^D(D)$ be twice differentiable, with $C^M_M(M) > 0, C^M_{MM}(M) > 0$ and $C^D_D(D) > 0, C^D_{DD}(D) > 0$.

Let us also define the following property of the people's belief:

Definition **3.1** Monotonicity of People's Belief (MPB)*: The ex-post probability of the government being a benevolent type is non-decreasing in the level of development works done.*

We use this property particularly to restrict the class of off-the-equilibrium beliefs.

Now we proceed to specify the modified objective functions. The modified objective function for the government is-

$$(1 - P(R, M, x))V - C^D(D) - C^M(M) + \beta U^P(D)$$

$$\text{where} \quad \beta = \begin{cases} 1, & \text{if the government is benevolent;} \\ 0, & \text{otherwise.} \end{cases}$$

The rebel's objective function takes the form:

$$P(R, M, x)V - C^R(R) + \gamma U^P(D)$$

where, $0 < \gamma < 1$.

This range of γ provides the rationale for the mechanical rule followed by the people while deciding their support to the government. $\gamma < 1$ implies that the benevolent government cares more for the people relative to the rebel (for the benevolent government, $\beta = 1$). Moreover, $\gamma > 0$ implies that the rebels care more for the people relative to the non-benevolent type government. So, the people support the government if they think that the government is benevolent and do not support if they think that the government is non-benevolent. For other values of γ, the people either always support the rebel or always support the government. So, the values outside this range are not interesting from analysis point of view.

Having specified the objective functions, we proceed to specify the game-form:

Stage 1. Nature moves first and selects the government's type. The government observes this and chooses D to maximize it's payoff.

Stage 2. (Non-strategic stage) People observe the government's choice of D and decide whether to support the government, as per the mechanical rule outlined earlier.

Stage 3. Both the government as well as the rebel observe the D as well as the people decision and select M and R, respectively to maximize their payoffs.

The first two stages here form the signaling game between the government and the people. In this chapter, we are particularly interested in this signaling game. Next, we move to the analysis part.

3.3 Complete information case

Let us first solve the complete information counterpart of this game. This will serve as a benchmark case. Suppose there is no uncertainty about the government's type, the people know whether it is a benevolent type or not. They support the benevolent type and don't support the non-benevolent type. We solve for the sub-game perfect Nash equilibrium of this game. So first, for a given level of the development and the people's support, we solve the *Stage 3* of the above game. For a given (D, x), We solve the government's optimization problem (M^*) and the rebel's optimization problem (R^*), simultaneously.

$$\max_{M} (1 - P(R, M, x))V - C^D(D) - C^M(M) + \beta U^P(D)$$

$$\max_R P(R, M, x)V - C^R(R) + \gamma U^P(D)$$

Here R^* and M^* will depend on, apart from other things, whether the people support the government or not. Let us denote $R^*(x = \alpha)$ and $M^*(x = \alpha)$ as the solutions of the government-rebel conflict game when the people supports the government i.e the people think that the government is of benevolent type. Similarly, let us denote $R^*(x = 0)$ and $M^*(x = 0)$ as the solutions of the government-rebel interaction game when the people do not support the government i.e the people think that the government is of non-benevolent type. The probability of winning for the rebel, in the equilibrium, when the people support the government, is given by $\frac{1}{1+e^{M^*(x=\alpha)-R^*(x=\alpha)+\alpha}}$. Let us denote it by $P^*(x = \alpha)$. Similarly the probability of winning for the rebel, in the equilibrium, when the people do not support the government, is given by $\frac{1}{1+e^{M^*(x=0)-R^*(x=0)}}$. Let us denote it by $P^*(x = 0)$.

Now both types of the government accommodate these optimal functions in their utility function and maximize it by choosing the level of development. Since there is no uncertainty, the benevolent type knows that the people are supporting it and the non-benevolent type knows that people are not supporting it. Moreover they can not manipulate this belief. Therefore they accommodate this belief before maximizing their utilities with respect to the development level. So, in the equilibrium, the non-benevolent type government always chooses zero level of development. The benevolent type government solve the following maximization exercise:

$$\max_{D\geq 0}(1 - P^*(x = \alpha))V - C^D(D) - C^M(M^*(x = \alpha)) + U^P(D)$$

Let us denote the optimal level of development here as D^{**}. We can refer to this level of development as the non-strategic optimal level of the development.

3.4 Incomplete Information Case

Now we move to solve the incomplete information version. The people do not know the type of the government. To solve this game, we use the solution concept of the perfect Bayesian Nash equilibrium (PBE). We first solve the *Stage 3* of the game. After that, we use these solutions to analyze the signaling game between the government and the people. As true with the standard signaling game, we solve for the separating as well as the pooling equilibrium here.

Apart form the assumption *A1*, specified in last section, we also assume the following:

Assumption 2 (A2).
(i) $(1 - P^*(x = \alpha))V - C^M(M^*(x = \alpha)) > (1 - P^*(x = 0))V - C^M(M^*(x = 0))$
(ii) $U^P(D) - C^D(D)$ decreases monotonically on the either side of D^{**} where $D^{**} = \arg\max_D \{U^P(D) - C^D(D)\}$.

The first part says that the people's support increases the government utility, in a strategic sense. In the absence of this assumption, the people's support does not have any value as a strategic instrument. So the signaling game becomes unimportant. The second part insures that we have an *interval* that can be supported as the equilibrium development level.

Now, we solve for the separating equilibrium.

3.4.1 Separating Equilibrium Case

We follow the standard method to solve for the separating equilibrium. Let $D^*(B)$ and $D^*(NB)$ be the level of the development done by the benevolent type and the non-benevolent type government in the equilibrium.

Definition **3.2** *A* separating perfect Bayesian equilibrium *is an equilibrium in which both the benevolent type and the non-benevolent type choose different signals i.e different level of developments ($D^*(B) \neq D^*(NB)$).*

In the equilibrium, the people's belief must be consistent with these level of developments.

$$x^*(D^*(B)) = \alpha$$

$$x^*(D^*(NB)) = 0.$$

Lemma 1 *In any separating perfect Bayesian equilibrium, $D^*(NB) = 0$.*

Proof: Suppose in some separating perfect Bayesian equilibrium, $D^*(NB) > 0$. Notice it is still a separating equilibrium, so the people do not support the non-benevolent type government, i.e., $x^*(D^*(NB)) = 0$. Hence the non-benevolent type government can decrease the level of development, gain on development cost without loosing anything. As a result it's overall utility will increase. Hence the contradiction that $D^*(NB) > 0$ is the part of the equilibrium. ■

Having fixed the value of $D^*(NB)$, we need to fix the value of $D^*(B)$ which can be supported in the equilibrium. In what follows, we explore the following question- what values of $D^*(B) = \delta$ can be supported in the equilibrium? δ must be such that the benevolent type, as well as the non-benevolent type don't have any incentive to deviate.

First, consider the deviations by a non-benevolent type government. δ must be above a certain level such that the non-benevolent type finds it unprofitable to choose this level of the development and mimic as the benevolent type. Let U^G_{NB} denote utility of the non-benevolent type government. So, the following must be true-

$$U^G_{NB}(D = 0, x = 0) \;\geq\; U^G_{NB}(D = \delta, x = \alpha). \tag{3.1}$$

$$(1 - P^*(x = 0))V - C^M(M^*(x = 0)) \geq (1 - P^*(x = \alpha))V - C^M(M^*(x = \alpha)) - C^D(\delta)$$

or,

$$C^D(\delta) \geq V(P^*(x = 0) - P^*(x = \alpha)) + C^M(M^*(x = 0)) - C^M(M^*(x = \alpha)) \tag{3.2}$$

The right-hand-side of the inequality is positive from the assumption *A2*. Let us denote $\tilde{\delta}$ as the value of δ for which the above weak inequality is satisfied with equality. Notice that for all $\delta \geq \tilde{\delta}$, the above inequality is satisfied. So for any $\delta \geq \tilde{\delta}$, the non-benevolent type government does not have any incentive to deviate.

This condition is quite intuitive. The right-hand-side is the benefit from the mimicking as the benevolent type for the non-benevolent type. The left-hand-side is the cost of it. So, as long as the above condition holds, the non-benevolent type does not have any incentive to mimic as the benevolent type.

We also need to put similar restrictions from the benevolent type's behavior side. First notice that the δ must be above a certain level.

Lemma 2 *In any separating perfect Bayesian equilibrium with monotonicity of people's belief, $D^*(B) \geq D^{**}$.*

Proof: Suppose in some separating perfect Bayesian equilibrium, $D^*(B) < D^{**}$. Consider a deviation by the benevolent type government to the development level D^{**}. The people continue to support the government, due to monotonicity of people's belief property.

$U^P(D) - C^D(D)$ is maximized at D^{**}. Hence the utility of the benevolent type government is increased. So, in the equilibrium, we must have $D^*(B) \geq D^{**}$. ■

Now, δ must be below some level at which even the benevolent type prefer to let go of the people's support. Let U_B^G denote utility of the benevolent type government. So, the following must be true-

$$U_B^G(D = \delta, x = \alpha) \;\geq\; U_B^G(D = D^{**}, x = 0). \tag{3.3}$$

Notice that on the right-hand-side of the inequality, we don't have $D = 0$. For the benevolent type, the development level D enters into the objective function (with positive coefficient) through the utility of the people. So, the benevolent type government finds it apt to do some level of development, irrespective of whether it gains or not (in term of the probability of success through the people's support). This level of development, as defined earlier, is D^{**}.

$$\begin{aligned}(1 - P^*(x = \alpha))V - C^M(M^*(x = \alpha)) - C^D(\delta) + U^P(\delta) \geq (1 - P^*(x = 0))V \\ - C^M(M^*(x = 0)) - C^D(D^*) + U^P(D^*)\end{aligned}$$

or,

$$\begin{aligned}C^D(\delta) - U^P(\delta) \leq V(P^*(x = 0) - P^*(x = \alpha)) + (C^M(M^*(x = 0) \\ - C^M(M^*(x = \alpha))) + (C^D(D^*) - U^P(D^*))\end{aligned}$$

Let us denote $\hat{\delta}$ as the value of δ for which the above weak inequality is satisfied with equality. Observe that for all $\delta \in [D^{**}, \hat{\delta}]$, the above inequality is satisfied. For the existence of the separating equilibrium, we require $\hat{\delta} \geq \tilde{\delta}$.

Lemma 3 *In any separating perfect Baysian equilibrium with monotonicity of people's belief,* $\hat{\delta} \geq \tilde{\delta}$.

Proof: If $\tilde{\delta} \leq D^{**}$, this is true from *Lemma 2*. Consider the case when $\tilde{\delta} \geq D^{**}$. Observe that for $\delta = \tilde{\delta}$, the condition 3.3 reduces to

$$U^P(D^*)) - (C^D(D^*) - U^P(\delta = \tilde{\delta}) \leq 0.$$

We claim that this condition is satisfied for all $\tilde{\delta} \geq D^{**}$. At $\tilde{\delta} = D^{**}$, this is satisfied. Now if we increase $\tilde{\delta}$, the left-hand-side of the inequality goes down. Hence this inequality is satisfied. The $\hat{\delta}$ is defined as the value when this condition 3.3 is satisfied with equality. At $\delta = \tilde{\delta}$, this condition is satisfied with strict inequality. Hence $\hat{\delta} \geq \tilde{\delta}$. ■

Now, we have identified D^{**}, $\tilde{\delta}$ and $\hat{\delta}$. Using these three, we can characterize the possible equilibrium value of the development by the benevolent type government. We can summarize the above analysis in the following lemma:

Lemma 4 *The following level of development can be supported as equilibrium level of development by the benevolent-type government in the separating perfect Bayesian equilibrium:*

$$\text{If the } D^{**} < \tilde{\delta}, \text{ then } \delta \in [\tilde{\delta}, \hat{\delta}]$$
$$\text{If the } \tilde{\delta} \leq D^{**}, \text{ then } \delta \in [D^{**}, \hat{\delta}].$$

Proof: Consider the following off-the-equilibrium belief: observing any development level higher than δ, the people believe that it is coming from the benevolent type government. Also, upon observing any development level lower than δ, the public believe that it is coming from the non-benevolent type government.

Consider the first case $D^{**} < \tilde{\delta}$. Pick any $\delta \in [\tilde{\delta}, \hat{\delta}]$. The non-benevolent type government does not have any incentive to deviate to this point as $\delta \geq \tilde{\delta}$. From the condition 3.1, for any such δ, the non-benevolent type government does not have any incentive to mimic as the benevolent type. The benevolent type does not have any incentive to deviate either. Consider a deviation to the lower development level. Here the government looses the people's support. For no people's support, the benevolent type utility is maximized at D^{**}, by definition. From condition 3.3, we know that at D^{**} and no people's support, the benevolent type government utility is lower. So any deviation to the lower development level is not profitable. Now consider a deviation to the higher development level. It continues to get the people's support but now it is further away from the D^{**}. So it does not gain anything in the strategic (probability of winning) term whereas looses (or at-least does not gain) in the non-strategic term $(U^P(D) - C^D(D))$. So any such deviation is not profitable either.

Same arguments work for the other case, $\tilde{\delta} \leq D^{**}$, as well. ■

The following proposition summarizes the analysis for the separating equilibrium case:

Proposition 1 *Let assumptions* A1 *and* A2 *hold. There exists separating perfect Bayesian equilibrium where* $D^*(NB) = 0$ *and* $D^*(B)$ *is as characterized by* lemma 4*. The people's equilibrium beliefs are given by* $x^*(D^*(B)) = \alpha$ *and* $x^*(D^*(NB)) = 0$*. Further, no other separating perfect Bayesian equilibrium exists. We restrict off-the-equilibrium beliefs to the beliefs which satisfy monotonicity of people's belief (MPB) property. Here* $D^*(NB)$ *and* $D^*(B)$ *are equilibrium level of developments by the non-benevolent and benevolent type* *governments, respectively.* x^* *is the indicator of people's support to the government in the equilibrium and* α *is a positive constant.*

Proof: Again we work with the same off-the equilibrium belief as in *lemma 4*. Consider the following off-the-equilibrium belief: observing any development level higher than δ, the people believe that it is coming from a benevolent type government. Moreover, upon observing any development level lower than δ, the public believe that it is coming from a non-benevolent type government.

First, notice that for the given people's equilibrium beliefs, $D^*(NB) = 0$ and $D^*(B)$, as characterized by *lemma 4*, are optimal for the government. By *lemma 1* and *lemma 4*, any deviation by either the benevolent type government or the non-benevolent type government, is not profitable for them. The people's beliefs are also consistent with the government strategies. $R^*(x)$ and $M^*(x)$ also constitute the Nash equilibrium of the government-rebel interaction game, by construction. Further, no other separating equilibrium exists. Again, by construction, the development level can't be different from the ones that are characterized by lemma 1 and lemma 2 (for the reasonable off-the-equilibrium beliefs, satisfying monotonicity of people's belief). A consistent (updated using Bayes' rule, wherever possible) people's equilibrium belief also can't be different. ■

Here we have worked with a particular off-the-equilibrium belief. There can be many other such beliefs which are also consistent with the equilibrium and can support the given level of developments as the separating equilibrium outcomes. In fact, this very nature (none-uniqueness) of off-the-equilibrium beliefs that supports the multiplicity of equilibrium in the signaling game. We will be addressing this in the later section while performing the equilibrium refinement.

3.4.2 Pooling equilibrium case

Under the pooling equilibrium, both the benevolent type as well as the non-benevolent type government choose the same level of the development.

DEFINITION **3.3** *A* pooling perfect Bayesian equilibrium *is an equilibrium in which both the benevolent type and the non-benevolent type choose same signal i.e same level of development* $(D^*(B) = D^*(NB))$.

Let $D^*(B) = D^*(NB) = \delta$ be the optimal choice of the development level by both types of the government. Now the people seeing this level of development can no longer distinguish between the benevolent type and the non-benevolent type. Since this level of development is coming from the both types, for consistency of equilibrium belief, the people should stick to their prior belief. So after observing δ, the people must assign the probability ρ to the government being a benevolent type.

$$P(x = \alpha \mid D = \delta) = \rho$$

$$P(x = 0 \mid D = \delta) = 1 - \rho$$

Again to pin down the sustainable level of development in any pooling equilibrium, we impose incentive and rationality constraints for both types. First consider the non-benevolent type. For the pooling equilibrium to exist, the non-benevolent type government must find it profitable to put the development level of δ and get the people support with probability ρ rather than doing no development and not getting the people's support.

$$U^G_{NB}(D = \delta,\ Prob.(x = \alpha) = \rho)\ \geq\ U^G_{NB}(D = 0,\ Prob.(x = \alpha) = 0).$$

or,

$$\begin{aligned} C^D(\delta) \leq \rho V(P^*(x = 0) - P^*(x = \alpha)) + (C^M(M^*(x = 0) \\ - (\rho C^M(M^*(x = \alpha)) + (1 - \rho) C^M(M^*(x = 0))) \end{aligned}$$

The right-hand-side of the inequality is positive, from assumption *A2*. As earlier, we solve for the δ at which this weak inequality is satisfied with equality. Let us denote it by $\hat{\delta}$. Notice the above inequality is satisfied for all $\delta \leq \hat{\delta}$.

Another requirement for the existence of the pooling equilibrium comes from the willingness of the benevolent type to put the level of development δ. First observe that the *lemma 2* is true for the pooling equilibrium case as well. Moreover the following must be true: .

$$U^G_B(D = \delta,\ Prob.(x = \alpha) = \rho)\ \geq\ U^G_B(D = D^{**},\ Prob.(x = \alpha) = 0).$$

This inequality gives the minimum level of development that can be sustained under pooling equilibrium. For the pooling equilibrium to exist, both these inequality need to be satisfied simultaneously. First, we require $\hat{\delta} \geq D^{**}$. This will happen if the cost function C^D is not too high. Now, proceeding as similar to *lemma 3* and *lemma 4*, we can show that $[D^{**}, \hat{\delta}]$ can be supported as the level of the development in the pooling equilibrium.

Lemma 5 *The following level of development can be supported as the equilibrium level of development in the pooling perfect Bayesian equilibrium:*

$$D^*(NB) = D^*(B) = \delta \in [D^{**}, \hat{\delta}].$$

Proof: Consider the following off-the-equilibrium belief. Upon observing anything higher than δ, the public believes that it comes from the benevolent type government with probability ρ (stick to their prior belief) and upon observing anything lower than δ, the public believes that it comes from the non-benevolent type government.

Now, first consider the non-benevolent type government. They do not have incentive to deviate to the higher level of development. From any such deviation, they do not gain anything, in terms of people's support but lose in terms of cost of the higher development. They do not have any incentive to deviate to a lower level of the development either. By deviating to a lower level of development, they loose the people's support. In absence of people's support, their utility is maximized at $\delta = 0$. We have already shown that this is not a profitable deviation. For the benevolent type also, there is no profitable deviation. ■

We can summarize the pooling equilibrium in the following proposition:

Proposition 2 Let assumptions *A1* and *A2* hold. There exists pooling perfect Bayesian equilibrium where $D^*(NB) = D^*(B) = \delta \in [D^{**}, \hat{\delta}]$. The people's equilibrium beliefs are given by $Prob.(x = \alpha \mid D = \delta) = \rho$. Further, no other pooling perfect Bayesian equilibrium exists, for all beliefs, satisfying monotonicity of people's belief property.

Proof: We again use the same off-the-belief as in *lemma 5.* Consider the following off-the-equilibrium belief. Upon observing anything higher than δ, the public believes that it comes from the benevolent type government with probability ρ (stick to their prior belief) and upon observing anything lower than δ, the public believes that it comes from the non-benevolent type government.

Given the people's belief, the government does not have any incentive to deviate, from *lemma 5.* The people's belief is also consistent with the government choice of development in the equilibrium. By construction, R^* and M^* constitute the Nash equilibrium of the government-rebel conflict game. Moreover no other level of development can be support in any pooling equilibrium, for all possible beliefs. Any $\delta > \hat{\delta}$ can not be part of a pooling equilibrium, as the non-benevolent type will deviate to zero level of development. Any $\delta < D^{**}$ can not be part of a pooling equilibrium, as the benevolent type will deviate to D^{**}. ■

Once again, we have worked with a particular off-the-equilibrium belief. There can be many other such beliefs which are also consistent with the equilibrium and can support the given level of developments as the pooling equilibrium outcome.

3.4.3 Equilibrium Refinements

For the equilibrium refinement, we use the intuitive criterion. It is due to Cho and Kreps (1987). Consider a perfect Bayesian equilibrium with development levels $D^*(B)$ and $D^*(NB)$. It is said to survive the the intuitive criterion refinement if there does not exit a D such that the following two are true:

1. For one type, benevolent or non-benevolent, this D is not a profitable deviation for all possible beliefs of the people.

2. For the other type, it is a profitable deviation for some possible beliefs of the people.

Consider an off-the-equilibrium signal (say, $\bar{D}$). Suppose for some particular type of agents, even the best pay-off in case of $\bar{D}$ is no-greater than the equilibrium payoff. Now suppose such an off-the-equilibrium signal is observed. Then the people's belief should be such that it don't put any positive probability on the type for which even the best pay-off in case of $\bar{D}$ is no-greater than the equilibrium payoff. In many cases this refinement technique works really sharply. In the Spence schooling model, the only equilibrium that survives this refinement is the separating equilibrium in which the low type chooses zero education and the high type chooses minimum from the equilibrium range.

In our model, let us first consider the pooling equilibriums. Let $D^*(B) = D^*(NB) = \delta$ be the equilibrium level of the development by the both types of the government. In the pooling equilibrium, the utility of the non-benevolent type government is given by:

$$\rho\{(1-P^*(x=\alpha))V-C^M(M^*(x=\alpha))\}+(1-\rho)\{(1-P^*(x=0))V-C^M(M^*(x=0))\}-C^D(\delta)$$

We can rewrite this as,

$$\{\rho(1-P^*(x=\alpha)+(1-\rho)(1-P^*(x=0))\}V-\{\rho C^M(M^*(x=\alpha))+(1-\rho)C^M(M^*(x=0))\}-C^D(\delta)$$

For notational simplicity, let us denote

$$\{\rho(1-P^*(x=\alpha)+(1-\rho)(1-P^*(x=0))\} \quad \text{by} \quad A$$

and

$$\{\rho C^M(M^*(x=\alpha)) + (1-\rho)C^M(M^*(x=0))\} \quad \text{by} \quad B$$

.

Now we show the following:

Step 1: There exists $\delta' > \delta$ such the following is true:

$$AV - B - C^D(\delta) = (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta')$$

Step 2: There exists $\delta'' > \delta'$ such that the non-benevolent type does not have any incentive to deviate to δ'', irrespective of people's belief. Moreover, it is a profitable deviation for the benevolent type.

Step 1: Note that the following inequality holds:

$$AV - B - C^D(\delta) < (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta)$$

Here we have put the value of ρ equal to 1 in the RHS. This is true from the assumption *A2*. Now if we increase δ in the RHS of the inequality, the RHS will decrease. So we can find a $\delta' > \delta$ such that the above inequality is replaced by the equality.

$$AV - B - C^D(\delta) = (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta') \tag{3.4}$$

Step 2: For such δ', (for benevolent type government),

$$AV - B - C^D(\delta) + U^P(\delta) < (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta') + U^P(\delta') \tag{3.5}$$

This is true because $U^P(\delta') > U^P(\delta)$ for all $\delta' > \delta$.

Now we can choose δ'' which is slightly higher than δ' such that the equation 3.4 is replaced by inequality and the inequality 3.5 is maintained. So for this δ'', for the non-benevolent type,

$$AV - B - C^D(\delta) > (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta'') \tag{3.6}$$

and for the benevolent type,

$$AV - B - C^D(\delta) + U^P(\delta) < (1 - P^*(x=\alpha))V - C^M(M^*(x=\alpha)) - C^D(\delta'') + U^P(\delta'') \tag{3.7}$$

So, for the non-benevolent type, it is not profitable to move to δ'' from the earlier equilibrium δ even if it gets the full support from the people. However for the benevolent type, if it get the full support of the people, this is a profitable move.

Using the intuitive criterion argument, the benevolent type will deviate from δ to a such δ''. This is true for every level of development that can be supported in the pooling equilibrium. Intuitively, starting from any pooling equilibrium, the benevolent type can potentially deviate to a higher level of development (compared to the non-benevolent type) because it gets extra utility through the people's utility from the development. Here this utility advantage works as the efficiency advantage in the standard signaling model.

Hence all pooling equilibriums get eliminated through intuitive criterion refinement. Formally we summarize above findings in the following proposition:

Proposition 3 *Let assumptions* A1 *and* A2 *hold. No pooling equilibrium survives the intuitive criterion refinement.*

For the separating equilibriums, a similar argument rule out all cases but the one where the non-benevolent type government chooses zero level of development and the benevolent type government chooses either D^{**} or $\tilde{\delta}$, whichever is higher.

We can put formally this in the following proposition:

Proposition 4 *Let assumptions* A1 *and* A2 *hold. The intuitive criterion refinement eliminates all the separating perfect Bayesian equilibriums except the one where* $D^*(NB) = 0$ *and* $D^*(B) = \max\{\tilde{\delta}, D^{**}\}$.

Proof: *Proof.* We show it in two parts: first we show that all other equilibriums do not survive the intuitive criterion refinement. Second, we show that we can not rule out the $\Big(D^*(NB) = 0, D^*(B) = \max\{\tilde{\delta}, D^{**}\}\Big)$.

Consider a separating equilibrium where $D^*(NB) = 0$ and $D^*(B) > \max\{\tilde{\delta}, D^{**}\}$. Now consider a deviation to a development level D such that $\max\{\tilde{\delta}, D^{**}\} < D < D^*(B)$. This is not a profitable deviation for the non-benevolent type, even in the best case scenario. $D > \tilde{\delta}$ and for such D, the non-benevolent type's utility is lower from condition 3.1. However this is a profitable deviation for the benevolent type if the people continue to support it. Hence, as earlier, any such separating equilibrium is ruled out using the intuitive criteria.

Now, consider a separating equilibrium where $D^*(NB) = 0, D^*(B) = \max\{\tilde{\delta}, D^{**}\}$. Here, we can not find any such deviation. Consider the case where $\tilde{\delta} < D^{**}$. So we have $\max\{\tilde{\delta}, D^{**}\} = D^{**}$. Any deviation on the lower side as well as upper side is not profitable for the benevolent type, for all possible beliefs of the people. So using intuitive criteria, the people should think that any deviation here should come from the non-benevolent type only. Knowing this the non-benevolent type's utility is maximized at $D = 0$. Hence the intuitive can not rule out this case. Consider the other case where $\tilde{\delta} > D^{**}$. So we have $\max\{\tilde{\delta}, D^{**}\} = \tilde{\delta}$. Now a deviation to a D such that $D^{**} \leq D < \tilde{\delta}$ is a profitable deviation for the benevolent type if the people continue to support it. But this deviation is also profitable for the non-benevolent type if the people supports it. So we can not use any such D to apply the intuitive criteria. ■

The intuitive criterion imposes restrictions on off-equilibrium beliefs. It uses forward induction to restrict off-equilibrium beliefs to 'reasonable' ones. Let us assume that the people's behavior to be reasonable in the sense of forward induction. Here intuitive criterion implies that the non-benevolent type government will always be separated from the benevolent type, in the equilibrium. So, the non-benevolent type government will not be able to mimic the benevolent type government, if the people's behavior is 'reasonable' enough. These refinements make the comparative statics sharper. In this chapter we have used the intuitive criterion to pin down to a single separating perfect Bayesian equilibrium. Having pinned down the equilibrium level of development, we can now perform comparative statics.

The equilibrium level of development for the non-benevolent type government ($D^*(NB)$) is zero. This is independent of the parameters related to the cost functions of the government as well as the rebels, and the contest success function. The equilibrium level of development of the benevolent type ($D^*(B)$) is more interesting. Applying the intuitive criterion, we have $D^*(B) = \max\{\tilde{\delta}, D^{**}\}$. Here, D^{**} is related to the non-strategic and welfare benefits of development, whereas $\tilde{\delta}$ is related to the strategic benefit of development, for the government. If the non-strategic effect dominates the strategic effect, we have $D^{**} > \tilde{\delta}$. Here again the equilibrium level of development of the benevolent type ($D^*(B)$) is independent of the parameters related to the cost functions of government as well as rebels, and contest success functions. In the context of standard signaling model, it is equivalent to the case where type of the high-type agent is too high. Here, $D^*(B)$ depends on just $U^P(D)$ and $C^D(D)$. If cost of development works rises, $D^*(B)$ falls.

If strategic effect dominates non-strategic effect, we have $\tilde{\delta} > D^{**}$. Consider the effect of rebel getting cost efficient (C^R decreases). We can use the insights from the last chapter. There we have found that the government plays an accommodative role if, to start with, the rebel is stronger. Notice that here x serves the same purpose as M. So, if the rebel is stronger to start with, while the effect of such a change on $\tilde{\delta}$ is not obvious, the government's incentive to have a positive x will be lower. Hence, $\tilde{\delta}$ should decrease.

However, to know the precise effect, we need to specify the cost functions and utility function.

The intuitive criteria uses forward induction argument. In our model, the higher development level increases the utility of the public. Hence the public might strategically try to push development higher. Like knowing the above range of development level, which is sustainable in the equilibrium, the public might opt for such off-the-equilibrium beliefs which eliminates all development level except $\hat{\delta}$. It does not show up here as we have assumed non-strategic behavior on the people's part. However, on a intuitive level, it seems apt to apply the forward induction logic from both sides, the government as well from the people. When the signaling itself does not have any utility, we do not have to consider such possibilities. This observation is applicable to all the signaling cases where the signal itself has some value. So if we assume that the education has some value (which do not sound absolutely implausible!), in the traditional school-signaling model also, similar arguments make sense.

3.5 Conclusion

In this chapter, we have used an incomplete information framework to extend the analysis of *chapter 2*. Here we study a signaling game involving the government, the rebel and the people, where the people behave in a deterministic manner. The government can be of two types: benevolent and non-benevolent. The people support a benevolent government and do not support a non-benevolent one. This analysis provides a foundation of the cost function of rebel that we have used in last chapter.

We have explored both, separating and pooling perfect Bayesian equilibriums. We have studied the existence and the nature of these two equilibriums. Here it is a three players strategic interaction framework. Two players are involved in a conflict game and they seek third player's support. Third player's support improves the player's position in the conflict game. We have used it to analyze domestic (intra country) conflicts in a incumbent-rebel setup. However, it can be used in a wide range of contexts. This setup can be applied in context of international trade and oligopolistic industries. We find that, in general, there will be over-signaling. In our context, equilibrium level of development is be too high. We apply intuitive criterion refinement to these equilibriums. All pooling equilibriums get eliminated. All, but one, separating equilibriums also get eliminated. Intuitive criterion refinement uses forward induction technique to restrict off-equilibrium beliefs to 'reasonable' ones. These refinements make comparative statics sharper. However, in the context of productive signals, there are problem with intuitive criteria. When the signal has utility for the other player, the forward induction technique of intuitive criterion refinement seems less convincing.

Chapter 4

Centralization vs. Delegation: A Principal-Agent Analysis

4.1 Introduction

In a principal-agent setup, centralization refers to a contractual relationship where the principal contracts with all the agents directly. Delegation or decentralization refers to a contractual relationship where the principal contracts with some agents (a proper subset, to be precise) and asks them (or better say, give them the right) to contract with others. Centralization versus decentralization is a really old debate in social science and it is still wide open. In the context of organizational structures or contracts, this is a widely studied question. The literature tries to find the optimal organizational design. This literature tries to understand the internal organization of the firm, kind of trying to explain the *black box*. There are several works in mechanism design-contract theory literature which which study centralized and delegated contracts (see Mookherjee and Tsumagari (2004), Che and Kim (2006) and, Baron and Besanko (1999)).

Both centralized, as well as delegated contracts are quite common. For centralized contracts, consider the case of bundled goods. Often different brands come together and sell their products as a bundle. Here the customer is principal and different brands or companies are agents. So, the consumer can be thought of as contracting with all the agents, i.e., brands or companies. So different brands of the bundle are reliable separately. There is no main contractor or subcontractor in this case.

For delegated contracts, consider the personal computer industry. When someone buys a laptop, notebook or all-in-one PC, often it comes with several loaded softwares (Windows 7, MS-Office, e.t.c), which the computer manufacturer procure from different companies/suppliers. Even for hardware, often different companies supply sub-parts. So, if someone is buying a Lenovo notebook, it is not the case that Lenovo Corp. is manufacturing all major parts itself. It procures processor, motherboard, audio devices, e.t.c from different companies like Intel, Asus, AMD, SRS, e.t.c. In this case the customer can be thought of as the principal, the Lenovo Corp. as the main contractor and others like Intel, Asus, Gigabyte, Seagate, Microsoft, e.t.c as sub-contractors. The customer signs the explicit contract only with the Lenovo Corp. So, if there is any problem with, say, the audio of the laptop, one goes to the Lenovo service centre not the audio device producer, maybe SRS. Similar kind of contracts are found in auto-mobile industry. Company like Tata Motors, Ford Motor Company, Volkswagen Group and others also follow similar organizational setup for production.

There are many other examples of centralized, as well as delegated contracts. However, in a standard theoretical framework, it is hard to justify the presence of delegated contracts.

In a principal-agent framework, with no-collusion among agents, the revelation principle insures that centralized contracts can always achieve whatever delegated contracts can. This is a standard result in the literature. Here the centralized contracts can be thought of as direct contracts and the delegated contracts can be thought of as indirect contracts.

Recently many authors have tried to look at the possibility of collusion among agents as a justification for the existence of delegation (see Laffont and Martimort (1998), Baliga and Sjostrom (1998) and, Mookherjee and Tsumagari (2004)). The literature tries to identify conditions under which delegation outperforms centralization. Here, we first explore the centralized contracts, with and without collusion among agents. Then we proceed to study the delegated contracts. We provide a sufficiency condition under which delegation outperforms centralization. As is true with most of the literature related to information economics, this one is also mainly divided along two lines: adverse selection and moral hazard models. A general result is yet to emerge.

One crucial aspect of this literature is the way to capture the collusion among agents. Most of the literature seems to capture it through some enforceable side-contract among agents. However the literature is more or less silent on the issue of enforceability as well as the details of these side-contracts. Most of the papers deal in somewhat restrictive structure, like- either adverse selection or moral hazard, discrete costs, two agents, special information structure, e.t.c. This chapter is aimed to contribute to the existing literature which deals with the comparative analysis of centralized contracts and delegated contracts in the presence of collusion. It is an one-principal two-agents model. But the methodology allows for the any number of agents. It deals in a framework where adverse selection and moral hazard both exist. However it is a particular type of adverse selection-moral hazard setup. In this setup, adverse selection and moral hazard are deterministically related to each other. So possible set of manipulation on one dimension (say, AS) is restricted by the other (MH). We describe this formally in next section. In future works, we would like to extend it to a more general setup. Both, the types and efforts, of agents, are continuous. We start with analysing centralization with perfect collusion among agents.

Our motivation for starting with the perfect collusion, is a case study of the Boeing Corporation (Avery, 2008). It is related to the Boeing 787 Dreamliner project. Boeing 787 is a mid-size aircraft which was supposed to be a game-changer in the aircraft industry. Commercially it is a huge success. It has been in news for many wrong reasons like technical glitch, slow delivery, etc. To some extent, these problems are due to the changes made in supply-chain management for this project. However, we are interested in these changes for a different reason. To our interest, the Boeing Co. has changed the role of suppliers dramatically for this project. It has decreased the number of suppliers directly contracting (with the Boeing Corporation). These suppliers are now called global partners and they share the responsibilities to manage and extend the supply chain. They contract with other small suppliers and supply subsystems instead of the parts. Another interesting point is, the Boeing Co. is promoting regular meetings and collaborations among suppliers. It is like they are promoting *collusion* among agents.

There are others cases also where the procurer seems to encouraging collaborations among suppliers. These cases suggest that the procurer might also benefit from efficient collusion among suppliers. We have shown that this indeed is the case. We capture collusion among agents by their ability to manipulate types' representations and efforts in a coordinated way. Moreover they can manipulate their cost-accounts' books. This notion of collusion allows for cost-synergies in production of inputs which might benefit the principal as well.

To better understand these synergies, consider a simple example. Suppose an academic institute wants to build a computer lab and for that it wants to procure computer systems, hardware as well as software. It has a budget of 100 and it values one computer system at 38. These are common knowledge. There are two suppliers, A_1 and A_2. A_1 supplies the hardware and A_2 supplies the software. The production costs of both hardware as well as software (for one computer system) are 10. To simplify things further, assume that the academic institute doesn't have any information about the production costs of A_1 and A_2. Now consider a situation where both A_1 and A_2 quote a price of 19. They will be both supplying 2 units and their profits will be 18 each. The institute payoff will be zero. This is a Nash equilibrium in non-collusion scenario. Now allow for collusion between A_1 and A_2. If they can bring down the aggregate price to 33 instead of 38, they can earn a total profit of 39 instead of 36. The institute's payoff also increases to 15. Under Collusion, this is possible. Moreover, under collusion this is the unique Nash equilibrium. *The institute (the principal, in our model) also gains from the collusion among suppliers (the agents).* Notice here we don't need perfect collusion or the agents to have perfect information about each-other. This is one kind of synergy. As explained above, similarly there can be other synergies also. In the context of consumer goods, it can be thought of as the phenomenon of bundled goods. Many times, brands come together to offer discounts and all. Here the customer is principal and these brands suppliers. It can be argued that there exist situations where it is a win-win situation for both.

Most of the literature focus on restricting the collusion among agents. The implicit assumption behind this approach is that collusion among agents is bad for the principal. Here we would like to stress on the point that while forming coalition, the agents try to maximize there own utilities. The agents' and principal's interest need not always be in conflict. We provide an appropriate way to model collusion among the agents and we derive a sufficiency condition under which the principal prefers collusion among agent.

We also differ from the existing literature on the information structure. We assume that people at the same level of hierarchy know better about each-other as compared to people across hierarchy. So in this framework, the agents know more about each-other as compared to what the principal knows about agents. This seams a reasonable assumption in any institutional structure. Notice that in both the examples of delegation that we have given above, it can be argued safely that contractors/suppliers/agents know about each-others more than the customer/principal knows about them. In any society, people from same income-level/profession/background interact more with each-other, in general and hence know more about each-other as compared to someone from different income-level/profession/background. Similar information structures are used in evolutionary literature, literature on lobbying and tournaments, etc. Dubey and Sahi (2012) analyses the optimal prize allocation technique. They use an information structure where the principal doesn't have any information about the agents' skill however the agents knows about each-other skills.

We derive first-order-conditions for the optimality for both centralization with, and without collusion. This approach is useful in many other contexts as well. It provides a convenient way to derive Nash equilibrium outcome in different cases.

4.1.1 Related Literature

Recently there has been a lot of work in related areas, in particular mechanism design theory, contract theory and organization theory. For the sake of clarity, we are dividing this section into two subsections: mechanism design-contract theory literature and organization theory literature.

4.1.1.1 Mechanism Design and Contract Theory Literature

In mechanism design and contract theory literature, there are several papers along the lines of adverse selection and moral hazard frameworks. Laffont and Martimort (1998) is closely related to our work here in terms of modeling the collusion among the agents. This paper deals in an adverse selection framework where agents supply perfectly complementary goods and their costs take just two possible values. Here the collusion among agents is organized by a third party, who cares for both agents symmetrically. Authors find that in this setup, collusion does not have any bite on the organizational efficiency. This result crucially depends on just two possible realization of costs. They find that both centralization and delegation perform equally well. This result depends on their assumption of perfect complementarity and two cost types. They also consider limited communication case. In the presence of both limited communication and collusion possibilities, delegation strictly dominates centralization, if presence of limited communication restrict the centralization to treat both agents symmetrically. Baron and Besanko (1999) is also somewhat similar in formulation.

Another closely related paper is Mookherjee and Tsumagari (2004). It deals in a one-principal two-agents framework with adverse selection and collusion among agents. The principal and any particular agent share a common belief about the other agent's type. In the given setup, it is shown that delegating to one agent the right to subcontract with other agent always earns lower profit for the principal compared to the centralized setup. Here the collusion among agents consists of coordinating cost reports, reallocation of production assignments and payments received by the principal. This is done through an enforceable side-contract among agents which is not observable to the principal. The principal cannot observe production reallocation but can verify aggregate output. For side-contract, all the bargaining power rest with one particular agent. That particular agent make a take-it-or-leave-it offer to the other agent. So while comparing with delegation to that particular agent, centralization can achieve this outcome by offering a null contract to the other agent.

Che and Kim (2006) nicely sums up the issue of collusion having a detrimental effect on the principal's utility, in adverse selection framework. The principal can attain the second best outcome if the following three conditions are satisfied: 1. correlation of the colluding agent's type satisfy a sort of full rank condition, 2. transferable utility, and 3. agents take participation decision before collusion decision.

Baliga and Sjostrom (1998), Itoh (1993) and others address similar issues in a moral hazard framework.

4.1.1.2 Organization Theory Literature

There are several related works on collusion in industrial organization literature. In a oligopolistic framework, Salvo and Vasconcelos (2012) have shown that collusion among the producers can increase the consumer welfare for a significant range of parameters. In their competition regulation related writing, Farrell and Shapiro (2010) have argued that a merger among two firms need not always increase the price. A merger definitely decreases competition but it also bring synergies. So, the net effect depends on which of these two effect dominates.

In this chapter, we try to combine the insights from these two literature (mechanism design-contract theory and organization theory). The organization theory works, including works on mergers and acquisitions, provide important insights about collusion among players. These insights provide a natural and intuitive way to model collusion among agents in the mechanism design-contract theory literature.

4.2 The Model

Consider a procurement setup. There are three strategic players in this setup: one *principal* (P) and two *agents* (A_1 and A_2). Here the principal (P) wants to procure two goods q_1 and q_2 from two agents A_1 and A_2, respectively. First the principal announces a menu of contracts (explained later in the section). Then the agents choose the ones that maximize their respective utilities. The agents supply the goods accordingly. The principal then reimburses as per the contract.

Let us now introduce some notations. Let $V(q_1, q_2)$ denote the value that the principal gets out of this procurement process. Let the agent A_i's cost of producing q_i be denoted by $C^i(\beta_i, e_i, q_i)$. Here, β_i is the technological/productivity parameter of agent A_i. It can also be referred as agent's type. e_i is agent A_i's effort which results in cost reduction. Finally, the effort (e_i) entails disutility to the agent A_i, let us denote it by $\Psi_i(e_i)$. Let U^i denote the utility of agent A_i.

We shall carry the following assumption throughout this chapter.

Assumption 1.
(i) $V(q_1, q_2)$ is twice differentiable. Both inputs are essential i.e., $V(0, q_2) = V(q_1, 0) = 0$. Moreover $V(q_1, q_2)$ is increasing and concave in the inputs q_1 and q_2.
(ii) $C^i(\beta_i, e_i, q_i)$ is twice differentiable with $C^i_{\beta_i} > 0,\ C^i_{e_i} < 0,\ C^i_{q_i} > 0$.
(iii) $\Psi_i(e_i)$ is twice differentiable with $\Psi_i' > 0, \Psi_i'' > 0, \Psi_i''' \geq 0$.

We assume that the principal observes the total cost C_i for both the agents but cannot observe either an agent's type or effort. An agent knows her type before signing the contract. For principal, β_i is drawn randomly from some cumulative distribution function $F(\beta_i)$ with support $[\underline{\beta_i}, \overline{\beta_i}]$ and with density function $f(\beta_i)$. So, it is a restricted adverse selection-moral hazard (AS-MH) setup. In the literature, it has been shown that this setup is qualitatively equivalent to the adverse selection setup. We persist with this setup as it is more suitable and intuitive for the modeling of collusion among agents. Also later on we would like to extend the analysis to a general AS-MH setup.

We work with the following payment protocol for agent A_i, $\quad i = 1, 2$: $\quad$ the principal reimburses total cost C^i and in addition transfers some amount t^i. The transfer of t^i can be thought of as compensation towards the agent's effort as putting effort entails disutility or cost for the agent. We now write down the utility function of both, the principal and the agents, taken to be risk neutral. The utility function of the principal is given by:

$$V(q_1, q_2) - \Sigma_i(t^i + C^i) \tag{4.1}$$

Whereas the utility function of the agents are given by:

$$U^i = t^i - \Psi_i(e_i) \tag{4.2}$$

Now we define minimum effort function $E_i(\beta_i, C_i, q_i)$ from total cost function in the following manner [1]:

$$C^i = C^i(\beta_i, E_i(\beta_i, C_i, q_i), q_i)$$

Given the production technology, to produce a given output, a particular type agent also need to put some effort. The cost of production depends on the amount of effort put. Given our specifications, higher the effort, lower the cost; lower the effort, higher the cost. E_i is the minimum effort that a β_i type agent require to put in order to produce the output q_i at the cost C_i. Here we have, from *Assumption 1*,

$$E^i_{\beta_i} > 0, E^i_{C_i} < 0, E^i_{q_i} > 0.$$

Now we turn to first solve the complete information case. This will serve as a benchmark case for the rest of the analysis.

[1] We use *Assumption 1*, particularly $C^i_{e_i} < 0$, and implicit function theorem to derive function $E_i(\beta_i, C_i, q_i)$.

4.3 Perfect Information Case

Suppose the principal knows agents' types perfectly and can monitor their efforts perfectly too. Then the principal will simply maximize the following objective function with respect to quantities and efforts level and get it implemented by agents just subject to the individual rationality constraints (of agents).

$$\max V(q_1, q_2) - \Sigma_i(t^i + C^i) = V(q_1, q_2) - \Sigma_i(U^i + \Psi_i(e_i) + C^i)$$

subject to $\mathrm{U}^i \geq 0 \; for \; i = 1, 2.$

Since U_i appears with a negative sign in the objective function, following hold:

Lemma 1 *Let* Assumption 1 *hold. The principal's utility maximization implies* $U^i = 0$ *for* $i = 1, 2.$

Proof: Let $U^i > 0$ for at-least one i. So, $t^i > \psi_i(e_i)$ as $U^i = t^i - \psi_i(e_i) > 0$. Notice this is a complete information case, hence the principal can perfectly observe and monitor e_i. Let the principal decrease t^i to $t^i = \psi_i(e_i)$. This increases the principal's utility while the agent's IR constraint continues to hold. Hence in equilibrium, $U^i = 0$ for $i = 1, 2$.

First-order conditions for maxima will be given by following set of equations:

$$V_{q_i} = C^i_{q_i} \; for \; i = 1, 2. \tag{4.3}$$

$$\Psi'_i = -C^i_{e_i} \; for \; i = 1, 2. \tag{4.4}$$

These are standard marginal cost equals marginal benefit equations. We will have standard second-order sufficiency conditions. We now turn to the analysis of asymmetric information case.

4.4 Asymmetric Information

Now we consider the asymmetric information case described earlier in the introduction part. Here principal has several options regarding organisational design. We will be considering two particular (extreme) organisational setups:

Centralization:- The principal contracts with both agents directly.

Delegation:- The principal contracts with agent A_i and gives him the right to contract with agent A_j.

So one natural question to ask is, from principal's point of view which of the above setup is better? If we assume that agents behave in non-cooperative way or they don't collude in the process of their interaction with the principal, the answer to the above question is straightforward. Myerson (1982) ensures that centralization can always achieve the payoff (for the principal) whatever delegation can. This is an implication of generalized revelation principle. Here centralized contracts can be viewed as direct contracts whereas delegated contracts can be viewed as indirect contracts. Once we allow for collusion among agents, we can't claim the same.

4.4.1 Centralization with Collusion

In this subsection we analyze the centralized contracts, with collusion. In literature, one crucial and debated aspect is 'how to capture the collusion in such framework'. Here we will be trying to capture the notion of perfect collusion in one particular way which seems quite reasonable. Suppose there is one third party 'A' who knows both agents' type perfectly and can monitor their efforts perfectly too. This third party manages the coalition among agents. This third party can be thought of as trade union, labour union or industry federation. So from principal's point of view, it is like he/she is contracting with this third party A for the input vector (q_1, q_2). This is somewhat realistic in the context of labour union literature (bargaining models).

Let us denote $B = (\beta_1, \beta_2), e = e_1 + e_2$ and $Q = (q_1, q_2)$. Then if A can be characterized by total cost function $C(B, e, Q)$, this *'centralization with perfect collusion'* analysis can be seen as one principal-one agent problem which can be solved relatively easily and neatly.

We now turn to write down the game form for centralization with collusion case:

Stage 1. The principal offers a contract which is essentially a transfer schedule t(C, Q), where C is the cost observed by the principal and Q is the quantity of good produced.

Stage 2. Agents' types are realized and they take the participation decisions. If both agents decide to participate, the game continues to the next step, otherwise the game ends with principal and agents getting their outside options.

Stage 3. The agents cooperatively decide the optimal (Q, e) for the given t(C, Q). If they fail to reach the agreement they choose the quantities and efforts non-cooperatively.

Stage 4. The principal reimburses the realized costs and makes transfers. Payoffs for principal and agents are realized accordingly.

Stage 3 captures the collusion process here. Without collusion, the agents can misreport their types and efforts independently, subject to the principal observing the total costs. With collusion they can misreport in a coordinated way. Moreover the principal can no longer observe their total costs (C^1 and C^2) separately, the principal only observe $C^1 + C^2$. So the agents can misreport their types (β_i) and efforts (e_i) subject to the principal observing only $C^1 + C^2$. Hence here the agents or the coalition forming authority (A) can divert agent A_i's effort to the A_j's production activity. For simplicity, we assume here that A_i's effort and A_j's effort are homogeneous. So one unit of A_i's effort diverted to $q_j' s$ production activity works just like an extra unit of A_j's effort in cost-reduction in $q_j' s$ production.

Under this setup A (on behalf of the agents) will like to assign the efforts, e_1 and e_2, in the optimal way. For every level of total effort(e), e_1 and e_2 will solve the following problem (for a given transfer schedule):

$$\max_{(e_1,e_2)} t^1 + t^2 - \Psi_1 - \Psi_2$$

subject to $\mathrm{e}_1 + e_2 \equiv e$.

This will give an aggregate cost function $C(B, e, Q)$, as well as aggregate disutility function $\Psi(e)$. Now for some class/family of cost functions, we get aggregate cost function as $C(f(\beta_1, \beta_2), e, Q)$. In these cases, we can simply take $f(\beta_1, \beta_2) = \beta$ as the A's type. Separable cost functions are one such family of cost functions.

Suppose the aggregate cost function takes the form $C(\beta, e, Q)$ where $\beta = f(\beta_1, \beta_2)$. As earlier, define $E(\beta, C, Q)$ as

$$C \equiv C(\beta, E(\beta, C, Q), Q).$$

The principal will maximize his net expected payoff

$$\int_\beta [V(Q) - U - \Psi(e) - C] f(\beta) d\beta.$$

subject to individual rationality (IR) and incentive compatibility (IC) constraints. For simplicity we assume that outside options for both agents are zero. So IR constraints take the form

$$U(\beta) \geq 0 \text{ for all } \beta.$$

As is standard in the literature, the IC constraints come from the agents' optimization exercise. It is just like a standard Stackleberg case where principal incorporates agents' behaviour in his optimization. A derivation of this IC constraints is done in the *Appendix*. IC constraints take the form

$$\dot{U}(\beta) = -\Psi'(e)E_\beta(\beta, C, Q). \tag{4.5}$$

Intuitively, an agent of type $(\beta - \Delta\beta)$ can produce an output vector Q at the same cost as that of the agent with type β by decreasing her effort by $\Delta e = E_\beta \Delta\beta$. So above constraint implies that any agent don't have incentive to do it. Carroll (2012) shows that in our setup, the above local incentive compatibility implies full (global) incentive compatibility. He particularly uses the convexity of domains (types) and quasilinear preferences to show this.

Lemma 2 *Let* Assumption 1 *hold. Then the following is true:*

1. $\dot{U}(\beta) < 0$.

2. *The set of individual rationality (IR) constraints reduces to a single constraint:* $U(\overline{\beta}) = 0$.

Proof: Consider the agents' optimization problem. Suppose an agent of (true) type β announces his/her type α. The agents will announce his/her true type if it solves the following problem

$$\max_\alpha U(\alpha/\beta) = t(\alpha) - \Psi(E(\beta, C(\alpha), Q(\alpha)))$$

Evaluating the first-order conditions at $\alpha = \beta$, gives

$$\dot{U}(\beta) = -\Psi'(e)E_\beta < 0$$

Here, from *Assumption 1*, $\Psi'(e) > 0$ and $E_\beta > 0$. Hence $\dot{U}(\beta) < 0$.

Second part of the *lemma* is an implication of the first part. $U(\overline{\beta}) = 0$ and $\dot{U}(\beta) < 0$ ensures that the IR constraints for all the types are satisfied.

Now using *Lemma 2*, the principal's problem becomes

$$\max_{Q,e,U} \int_\beta [V(Q) - U - \Psi(e) - C] f(\beta) d\beta$$

subject to $\dot{U}(\beta) = -\Psi'(e)E_\beta(\beta, C, Q)$
and $U(\overline{\beta}) = 0$.

This can be thought as an optimal control problem with $Q(\beta)$ and $e(\beta)$ as control variables and $U(\beta)$ as state variable. We setup the *Hamiltonian* and use first-order necessary and second-order sufficiency conditions for maximization. We solve it in the the *Appendix*, here we are just stating the first-order conditions that govern the optimal choice of quantity and effort level.

The first-order conditions take the form

$$\Psi'(e) = -C_e - \frac{F(\beta)}{f(\beta)}[\Psi''(e)E_\beta + \Psi'(e)E_{\beta C}C_e] \tag{4.6}$$

$$V_{q_i} = C_{q_i} + \frac{F(\beta)}{f(\beta)}\Psi'(e)\frac{dE_\beta}{dq_i} \; for \; i = 1, 2. \tag{4.7}$$

These first-order conditions characterizes the efforts and quantities level in the equilibrium. These are kind of modified marginal cost equals marginal benefit equations. The second term on the right-hand side of the both equations captures the distortion due to the informational asymmetries.

We can summarize above findings in the following proposition:

Proposition 1 *Let Assumption 1 hold. Above equations 4.6 and 4.7 gives the Nash equilibrium outcome* (e^*, q_i^*)*, for the centralization with collusion case.*

4.4.2 Benevolent Principal

In the above analysis we have assumed that the principal does not care about the agents' welfare at all. In some cases this might not be an appropriate assumption. The government sector is a major example. Even in the private sector, employers don't ignore employees' welfare entirely. So let us consider the case where the principal does care about the agents' welfare. Allowing for such behaviour, the principal's objective function changes to

$$\max_{Q,e,U} \int_\beta [V(Q) - (1-\lambda)U - \Psi(e) - C] f(\beta) d\beta$$

where λ is kind of a motivation parameter (Besley and Ghatak (2005)).

It can also be referred as the degree of careness (by the principal) of agents' welfare. So the initial analysis is one special case where λ is equal to zero.

The first-order conditions become (derivation is shifted to the *Appendix*):

$$\Psi'(e) = -C_e - \frac{(1-\lambda)F(\beta)}{f(\beta)}[\Psi''(e)E_\beta + \Psi'(e)E_{\beta C}C_e]$$

$$V_{q_i} = C_{q_i} + \frac{(1-\lambda)F(\beta)}{f(\beta)}\Psi'(e)\frac{dE_\beta}{dq_i} \ for \ i = 1, 2.$$

So, as $\lambda \to 1$, i.e., principal assigns same weight to his/her payoff as that of agents' payoffs, effort and quantity level tend to the first best level showed earlier. The second term in the right hand side of both the equations are distortions due to asymmetric information. These distortions disappear as $\lambda \to 1$. The joint net payoff of principal and agents is maximized when the distortions are set to zero. In a sense here the principal internalizes the distortion. We can summarize this in the following proposition:

Proposition 2 *Let Assumption 1 hold. As $\lambda \to 1$, the centralized contract attains the outcome of the complete information case.*

This result hold for both, with and without collusion case.

4.4.3 Coalition Formation

In our setup of coalition design, the aggregate rationality of the coalition formation implies individual rationality for agents also. If as a group A_1 and A_2 can generate some net surplus from colluding, A can always distribute it in such a way that both agents will be better off. So the rationality constraint for the coalition formation takes the form

$$U_{collusion} \geq U^1_{noncollusion} + U^2_{noncollusion}$$

Further this rationality of coalition-formation will always be weakly satisfied because A always has an option to dictate agents to opt for the non-cooperative behaviour.

While maximizing principal's utility, the literature has generally focuses on the ways to restrict collusion among agents. The implicit assumption behind this approach is that collusion among agents hurts principal. Here we would like argue and demonstrate that this need not be the case always. We would like to stress the point that the agents' objective is to maximize their own utilities, not to hurt principal's utility. So there might be cases where the principal also prefers collusion to non-collusion. *This is possible because in case of collusion, there might be cost synergies* ($C \to min(C1 + C2)$) *and the principal might also get benefited from it.* This is applicable in other contexts as well, like consumer-producer case.

Formally, consider the following procurement setup. The principal P wants to procure two goods, q_1 and q_2, from two agents, A_1 and A_2, respectively. For simplicity, we modified the timings as the following: first the agents quote their prices, P_1 and P_2, and then the principal chooses the quantities, q_1 and q_2. The value that principal gets out of this procurement, is denoted by $V(q_1, q_2)$. V is increasing and concave in inputs, q_1 and q_2. Let us denote cost of production for agent A_1 by $C^1(q_1)$ and that of agent A_2 by $C^2(q_2)$. In this section, we have eliminated β and e from the cost functions. This is to simplify the analysis. Let us denote utilities of principal and agents by U^P and U^i, where $i = 1, 2$, respectively.

$$U^P = V(q_1, q_2) - P_1 q_1 - P_2 q_2.$$
$$U^i = P_i q_i - C^i(q_i) \quad \text{for} \quad i = 1, 2.$$

First, we solve the principal's optimization problem for given P_1 and P_2. Then we incorporate this in agents' utilities and solve the agents' optimization problem.

Principal's optimization:

$$\max_{q_1, q_2} V(q_1, q_2) - P_1 q_1 - P_2 q_2.$$

The interior solution must satisfy the following first-order necessary conditions:

$$V_{q_1} - P_1 = 0$$

$$V_{q_2} - P_2 = 0$$

We can solve these two first-order conditions to derive $q_i^*(P_1, P_2)$ for $i = 1, 2$. The second-order sufficiency condition is satisfied here, since we have assumed concavity of the value function V.

For agents' optimization, first we solve it for *without collusion* case. Both agents' maximize their utilities independently.

Agents' optimization:

$$\max_{P_i} P_i q_i^*(P_1, P_2) - C^i(q_i^*(P_1, P_2)) \quad \text{for} \quad i = 1, 2.$$

The interior solution must satisfy the following first-order necessary condition:

$$q_i^*(P_1, P_2) + P_i \frac{\partial q_i^*(P_1, P_2)}{\partial P_i} - \frac{\partial C^i}{\partial q_i} \frac{\partial q_i^*(P_1, P_2)}{\partial P_i} = 0 \tag{4.8}$$

Here if $q_i^*(P_1, P_2) > 0$ and $\frac{\partial q_i^*(P_1, P_2)}{\partial P_i} < 0$ (strictly positive demand and negative own price effect), we have $P_i - \frac{\partial C^i}{\partial q_i} > o$.

For maximization, the following second-order sufficiency condition also need to be satisfied:

$$\frac{\partial^2 U^i}{\partial {P_i}^2} < 0.$$

Let us denote the solution of this exercise as $\hat{P}_i$.

Now we solve the agents' optimization for *perfect collusion* case. Here agents' maximize their joint profit cooperatively.

$$\max_{P_1,P_2} \sum_i P_i q_i^*(P_1, P_2) - C^i(q_i^*(P_1, P_2))$$

The interior solution must satisfy the following first-order necessary conditions:

$$\frac{\partial U^1}{\partial P_1} + \frac{\partial U^2}{\partial P_1} = 0 \tag{4.9}$$

$$\frac{\partial U^2}{\partial P_2} + \frac{\partial U^1}{\partial P_2} = 0 \tag{4.10}$$

Again we will have corresponding second-order sufficiency conditions.

We analyze these two first-order conditions 4.9 and 4.10 at the price vector $(\hat{P}_1, \hat{P}_2)$. First, we do it for equation 4.9. Similar analysis hold for 4.10. At this price vector, the first part of the left-hand-side (LHS) of the equation is zero, from equation 4.8. If we assume strictly positive demand for both goods, negative own price effect and negative cross price effect ($\frac{\partial q_2^*}{\partial P_1} < 0$), the second part is negative. So the LHS is negative at the price vector $(\hat{P}_1, \hat{P}_2)$. Moreover, at this price vector,

$$\frac{\partial^2 (U^1 + U^2)}{\partial {P_1}^2} < 0 \quad \text{if} \quad \frac{\partial^2 q_2^*}{\partial {P_1}^2} < 0.$$

Let us denote the price vector, which satisfy equations 4.9 and 4.10, as $(\bar{P}_1, \bar{P}_2)$. So, if we have strictly positive demand for both goods, negative own price effect and negative cross price effect and $\frac{\partial^2 q_2^*}{\partial {P_1}^2} < 0$, these conditions imply $\bar{P}_i < \hat{P}_i$ for $i = 1, 2$. Lower prices imply that the principal‘s utility is higher when agents collude perfectly. All these conditions are satisfied for $V(q_1, q_2) = {q_1}^{\alpha_1} {q_2}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$.

We can summarize this in the following proposition:

Proposition 3 *Let Assumption 1 hold. Let* $V(q_1, q_2) = {q_1}^{\alpha_1} {q_2}^{\alpha_2}$ *, where* $\alpha_1 + \alpha_1 < 1$*. Then the principal prefers perfect collusion compared to no collusion (among agents). This* *is true for all $V(q_1, q_2)$, for which we have strictly positive demand for both goods, negative own price effect and negative decreasing cross price effect.*

Proposition 3 has an immediate welfare implication. Here social welfare is sum of the principal's and the agents' utilities. For $V(q_1, q_2) = {q_1}^{\alpha_1}{q_2}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$, social welfare is higher under perfect collusion among agents, compared to no collusion case. Here social welfare consists of principal's utility and agents' utilities. From *Proposition 3*, we know that principal's utility is higher in perfect collusion case. Moreover from the construction, the sum of agents' utilities is also higher. This follows from a revealed preference argument because the agents' utilities, in the case of perfect collusion, come from the maximization exercise where no-collusion was also possible. We can summarize this in the following proposition:

Proposition 4 *Let Assumption 1 hold. Let $V(q_1, q_2) = {q_1}^{\alpha_1}{q_2}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$. Then the social welfare is higher in the case of perfect collusion compared to no collusion (among agents). This is true for all $V(q_1, q_2)$, for which we have strictly positive demand for both goods, negative own price effect and negative and decreasing cross price effect.*

A somewhat similar argument can also be found in the literature on corruption. Bag (1997) explore ways to control corruption in hierarchies. However in it's conclusion, the author mentions about not modeling the cost and benefits of corruption. Bac and Bag (2006) revisits this issue. Here the authors do the cost-benefit analysis and identify the conditions under which collusion among the agent and the supervisor benefit the principal.

In a recent work Deltas, Salvo and Vasconcelos (2012) has shown that collusion among oligopolist producers can increase the consumer surplus. It is a different setting, but the arguments are on somewhat similar lines. These kind of arguments are also present in the literature on mergers and anti-competition policies. In Farrell and Shapiro (2010), the authors talk about cost synergies or efficiencies. They propose the idea of net upward pricing pressure. The merger generates two effects: lesser competition and cost savings (better coordination). The first one tends to increase the pricing whereas the second one acts in opposing direction. If the second factor dominates then net upward pricing pressure can be negative and in these cases merger can beneficial instead of being harmful.

4.4.4 Centralization without Collusion

Under centralization, we can have the outcome either with or without collusion. Even if beneficial, the without collusion outcome can occur (because of coordination failures, among other reasons). In the no-collusion case, the timings of the game changes to the following:

Stage 1. The principal offers a contract which is essentially a transfer schedule $t(C, Q)$, where C is the cost observed by the principal and Q is the quantity of goods produced.

Stage 2. Agents' types are realized and they take the participation decisions. If both agents decide to participate, the game continues to the next step, otherwise the game ends with principal and agents getting their outside options.

Stage 3. Both agents independently decide the their optimal (q_i, e_i) for the given t(C, Q). They produce and supply their optimal quantities.

Stage 4. The principal reimburses the realized costs and makes transfers. Payoffs for principal and agents are realized accordingly.

For *non-collusion* case, the optimization problem becomes

$$\max_{Q,e,U} \int_{\beta_1} \int_{\beta_2} [V(Q) - U^1 - U^2 - \Psi_1(e_1) - \Psi_2(e_2) - C^1 - C^2] f(\beta_1, \beta_2) d\beta_1 d\beta_2$$

subject to
$\dot{\mathrm{U}}^1(\beta_1) = -\Psi_1'(e_1) E^1_{\beta_1}$
$\dot{\mathrm{U}}^2(\beta_2) = -\Psi_2'(e_2) E^2_{\beta_2}$
$\mathrm{U}^{(}\overline{\beta}_1) = 0$
$\mathrm{U}^{(}\overline{\beta}_2) = 0$

This is a two dimensional optimal control problem. We use the Hamiltonian technique to solve this. We derive the first-order conditions for the optimality in the *Appendix*. For without collusion case, it is a well established result in the literature that the centralization perform (at-least weakly) better than any other forms of organisational structure. It is based on the application of generalized revelation principle. However the technique that we use here to derive equilibrium outcomes (e_i^*, q_i^*) can be quite useful in many contexts. It provides a convenient way to derive Nash equilibrium outcome.

4.5 Delegation

In our setup, delegation corresponds to the case where the principal contracts with one agent, giving him the right to contract with the other agent. Recall the examples of the personal computer industry and the automobile industry given in the introduction section. As assumed throughout the chapter, we work with the information structure where the agents at the same hierarchy knows more about each-other than across hierarchy.

Consider the setup that we have used in the *coalition formation* subsection. Suppose the principal delegate, say agent A_1, the right to contract with agent A_2. Now agent A_1 will choose the price vector which maximizes his profit. Here A_1 can set the other agent's (A_2's) utility to zero by just paying him the production cost of q_2, given the information structure that we have assumed. Given this, A_1 will choose the price vector $(\bar{P}_1, \bar{P}_2)$, as it maximizes the joint profit. So under delegation, principal's utility is given by:

$$U^P(\bar{P}_1, \bar{P}_2) = V(q_1^*(\bar{P}_1, \bar{P}_2), q_2^*(\bar{P}_1, \bar{P}_2)) - \bar{P}_1 q_1^*(\bar{P}_1, \bar{P}_2) - \bar{P}_2 q_2^*(\bar{P}_1, \bar{P}_2) \tag{4.11}$$

Under centralization, the principal's utility will be $U^P(\bar{P}_1, \bar{P}_2)$, if there is perfect collusion among agents. In absence of perfect collusion, the principal's utility will be different. Under centralization, we might not have perfect collusion among agents due to various coordination reasons. Under the conditions described in the subsection *coalition formation*, i.e. strictly positive demand for both goods, negative own price effect and negative cross price effect and $\frac{\partial^2 q_2^*}{\partial P_1{}^2} < 0$, we know that the principal's utility is maximized at the price vector $(\bar{P}_1, \bar{P}_2)$. In particular, this is true for $V(q_1, q_2) = q_1{}^{\alpha_1} q_2{}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$. So for $V(q_1, q_2) = q_1{}^{\alpha_1} q_2{}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$, delegation outperforms centralization, at-least weakly.

Under centralization, we can have the outcome either with collusion or without collusion. Whereas under delegation, we will always have the outcome with collusion. So for the cases where the collusion is beneficial for the principal, delegation outperforms the centralization.

Proposition 5 *Let Assumption 1 hold. Let $V(q_1, q_2) = q_1{}^{\alpha_1} q_2{}^{\alpha_2}$, where $\alpha_1 + \alpha_1 < 1$. Then delegation outperforms centralization, atleast weakly. This is true for all $V(q_1, q_2)$, for which we have strictly positive demand for both goods, negative own price effect, and negative and decreasing cross price effect.*

One variant of the above information structure can be that the people in the same hierarchy incur a lower cost to acquire the knowledge about each-other. This setup is similar to the one used by Fahad Khalil in his several works with his co-authors (Cremer, Khalil, and Rochet (1998), Cremer and Khalil (1992)). This variant is more suitable for exposition purpose. In present context, one agent's cost to get the information about the other agent is lower than the principal's cost to know about that agent. In this context, what delegation does is to initiate the process of one agent (main contractor) investing to acquire the information about the other agent (sub-contractor). This avoids the possibility of co-ordination problem (which may arise in centralization case). So, in the context of the Boeing example, delegation acts to improve the efficiency of the collusion. In the context of delegation, the coalition formation authority A acts as a captive institution for the main contractor. In a asymmetric setup, it is better to delegate to the agent whose marginal cost to acquire information about other agent is lower.

4.6 Conclusion

In this chapter, we have studied centralized and delegated contracts, in a procurement setup. Here we have allowed for the possibility of collusion among agents. We have explored centralized contracts with perfect collusion among agents. Collusion among agents is captured by their ability to misreport their types and efforts in a coordinated way. Using dynamic optimization technique, we have provided first-order conditions which characterize Nash equilibrium outcomes, for both the centralization with and without collusion cases. While comparing these two cases, we found that, in some cases, collusion among agents can benefit the principal as well. Hence, in general, we should be looking at the cost-benefit analysis of collusion rather than just ways to block collusion.

We have used this insight in the comparative analysis between centralization and delegation. Using this, we get a sufficiency condition under which delegation outperforms centralization. Delegation removes possibility of coordination failure among the agents.

4.7 Appendix

IC Constraints (for *lemma 2*)

Consider the agents' optimization problem. Suppose an agent of type β announces his/her type α. The agents will announces his/her true type if it solves the following problem

$$\max U(\alpha/\beta) = t(\alpha) - \Psi(E(\beta, C(\alpha), Q(\alpha)))$$

Evaluating the first-order conditions at $\alpha = \beta$ gives

$$\frac{dt(\beta)}{d\beta} - \Psi'[E_c \frac{dC}{d\beta} + \sum_i E_{q_i} \frac{dq_i}{d\beta}] = 0$$

Using $U(\beta) = t(\beta) + \Psi(E(.))$, we get ,

$$\dot{U}(\beta) = \frac{dt(\beta)}{d\beta} - \Psi'[E_c \frac{dC}{d\beta} + \sum_i E_{q_i} \frac{dq_i}{d\beta}] - \Psi'(e)E_\beta$$

$$\Rightarrow \dot{U}(\beta) = -\Psi'(e)E_\beta < 0$$

<u>Dynamic Optimization: finite horizon, continuous time</u>

Dynamic optimization techniques are very much used in areas like macroeconomics, economics of growth, etc. Here we are providing a basic outline of the technique suited for the concerned problem in the chapter. For the detailed analysis and proof, any standard textbook of dynamic optimization (Pontryagin et al. (1962), Chiang (1993)) can be consulted. The following borrows heavily from Lorenzoni (2009).

Suppose the instantaneous payoff is given by $f(t, x(t), y(t))$, where $x(t) \in X$ and $y(t) \in Y$. t denotes the time element. Here $x(t)$ is state variable and $y(t)$ is control variable. The agent chooses or controls $y(t)$ to maximize the payoff. The state variable depends on the agent's choice of control variable and represent the dynamics of the system. In a typical macroeconomics example, the consumption choices serve as the control variable and the capital serves as the state variable. The capital formation dynamics captures the state of the economy. In the current scenario, the effort (e) will be the choice variable of the agent and the utility (u) acts as the state variable. The state variable dynamics act as the constraint to the optimization problem:-

$$\dot{x}(t) = g(t, x(t), y(t))$$

We assume that both f and g are continuously differentiable functions. The problem is to maximize

$$\int_0^T f(t, x(t), y(t))\, dt$$

subject to the constraint $\dot{x}(t) = g(t, x(t), y(t))$ for all $t \in [0, T]$ and given the initial condition $x(0)$.

We use the Hamiltonian technique to setup the Hamiltonian as,

$$H(t, x(t), y(t), \lambda(t)) = f(t, x(t), y(t)) + \lambda(t)g(t, x(t), y(t))$$

Necessary condition for optimality:
If x^* and y^* are optimal, continuous and interior then there exists a continuously differentiable function $\lambda(t)$ such that

$$H_y(t, x^*(t), y^*(t), \lambda^*(t)) = 0$$

$$\dot{\lambda}(t) = -H_x(t, x^*(t), y^*(t), \lambda^*(t))$$

$$\Omega x^*(t) = -H_\lambda(t, x^*(t), y^*(t), \lambda^*(t))$$

and, $\lambda(T) = 0$.

Sufficiency condition for optimality:
Define

$$M(t, x(t), \lambda(t)) = \max_y H(t, x(t), y, \lambda(t))$$

If x^* and y^* are two the continuous functions that satisfy above necessary conditions for some continuous function $\lambda(t)$, X is a convex set and $M(t, x(t), \lambda(t))$ is concave in x for all $t \in [0, T]$, then x^* and y^* are optimal.

This technique can be extended to suite the contexts (multiple control variables, multi-dimensional optimization) applicable in our problems.

FOC for principal‘s problem (for *proposition 1*)

$$\max \int_\beta [V(Q) - U - \Psi(e) - C] f(\beta) d(\beta)$$

subject to

$$\dot{U}(\beta) = -\Psi'(e)E_\beta$$

$$U(\overline{\beta}) = 0$$

The Hamiltonian becomes

$$H = (V(Q) - U - \Psi(e) - C)f(\beta) - \mu(\beta)\Psi'(e)E_\beta$$

So the first-order conditions become,

$$\frac{\partial H}{\partial U} = -\dot{\mu}(\beta)$$

$$\Rightarrow f(\beta) = \dot{\mu}(\beta)$$

integrating both sides and using $\mu(\underline{\beta}) = 0$ (Since $U(\underline{\beta}) > 0$) gives

$$\mu(\beta) = F(\beta)$$

Now using $\mu(\beta) = F(\beta)$ in the H, the other first-order conditions $\frac{\partial H}{\partial e} = 0$ and $\frac{\partial H}{\partial q_i} = 0$ for $i = 1, 2$ straightway give the earlier stated first-order conditions.

$$\Psi'(e) = -C_e - \frac{F(\beta)}{f(\beta)}[\Psi''(e)E_\beta + \Psi'(e)E_{\beta C}C_e]$$

$$V_{q_i} = C_{q_i} + \frac{F(\beta)}{f(\beta)}\Psi'(e)\frac{dE_\beta}{dq_i} \; for \; i = 1, 2.$$

For the second-order sufficiency conditions to be met, we need to assume the followings: the domain of u is convex and the max functions (as defined in the above section) are concave in u.

Two dimensional optimal control problem
(for centralization without collusion, *subsection 1.4.4*)

We have to solve the following problem:

$$\max \int_{\beta_1}\int_{\beta_2} P(U, e_1, Q) f(\beta_1, \beta_2) d\beta_1 d\beta_2$$

subject to

$$\Delta\beta_2 \dot{U}_1 \beta_1 = \Delta\beta_2 g_1(\beta_1, e_1, q_1)$$

$$\Delta\beta_1 \dot{U}_2 \beta_2 = \Delta\beta_1 g_2(\beta_2, e_2, q_2)$$

where, $\Delta\beta_i$ is the range of β_i. The Lagrange method can be used to get the solution of above problem.

$$\ell = \int_{\beta_1}\int_{\beta_2} P(U, e_1, Q) f(\beta_1, \beta_2) d\beta_1 d\beta_2 + \int_{\beta_1} \mu_1(\beta_1)[g_1(\beta_1, e_1, q_1) - \dot{U}_1\beta_1]\Delta\beta_2 d\beta_1 + \int_{\beta_2} \mu_2(\beta_2)[g_2(\beta_2, e_2, q_2) - \dot{U}_2\beta_2]\Delta\beta_1 d\beta_2$$

$$\Rightarrow \ell = \int_{\beta_1}\int_{\beta_2} [P(U, e_1, Q) f(\beta_1, \beta_2) + \mu_1(\beta_1) g_1(\beta_1, e_1, q_1) + \mu_2(\beta_2) g_2(\beta_2, e_2, q_2)] d\beta_1 d\beta_2 - \int_{\beta_1} \mu_1(\beta_1)\dot{U}_1\beta_1\Delta\beta_2 d\beta_1 - \int_{\beta_2} \mu_2(\beta_2)\dot{U}_2\beta_2\Delta\beta_1 d\beta_2$$

Now,

$$\frac{d[U_1\mu_1]}{d\beta_1} = \dot{U}_1\mu_1 + \dot{\mu}_1 U_1$$

$$\Rightarrow \int_{\beta_1} \frac{d[U_1\mu_1]}{d\beta_1} d\beta_1 = \int_{\beta_1} \dot{U}_1\mu_1 d\beta_1 + \int_{\beta_1} \dot{\mu}_1 U_1 d\beta_1$$

$$\Rightarrow \int_{\beta_1} \dot{U}_1\mu_1 d\beta_1 = \textbf{constant} - \int_{\beta_1} \dot{\mu}_1 U_1 d\beta_1$$

Lets denote $P(U, e_1, Q) f(\beta_1, \beta_2) + \mu_1(\beta_1) g_1(\beta_1, e_1, q_1) + \mu_2(\beta_2) g_2(\beta_2, e_2, q_2)$ by $\mathbf{H}$. Then ℓ becomes

$$\ell = \textbf{constant} + \int_{\beta_1}\int_{\beta_2} [\mathbf{H} + \dot{\mu}_1 U_1 + \dot{\mu}_2 U_2] d\beta_1 d\beta_2$$

Now using the pointwise maximisation, we get the first-order conditions as

$$\frac{\partial H}{\partial e_i} = 0 \ for \ i = 1, 2.$$

$$\frac{\partial H}{\partial q_i} = 0 \ for \ i = 1, 2.$$

$$\frac{\partial H}{\partial U_i} + \mu_i = 0 \ for \ i = 1, 2.$$

Again we need to make similar assumptions as above for the second-order sufficiency conditions to be met.

Now we use this technique to derive the solution for the *cetralisation without collusion* case. First, we form the Hamiltonian

$$H = [V(Q) - U^1 - U^2 - \Psi_1(e_1) - \Psi_2(e_2) - C^1 - C^2] f(\beta_1, \beta_2) - \mu_1(\beta_1)\Psi_1'(e_1)E^1_{\beta_1} - \mu_2(\beta_2)\Psi_2'(e_2)E^2_{\beta_2}$$

The first-order codition for the otimality becomes:

$$\Psi_i'(e_i) = -C^i_{e_i} - \frac{F(\beta_i)}{f(\beta_i)}[\Psi_i''(e_i)E^i_{\beta_i} + \Psi_i'(e_i)E_{\beta_i C^i}C^i_{e_i}] \text{for} i = 1, 2.$$

$$V_{q_i} = C^i_{q_i} + \frac{F(\beta_i)}{f(\beta_i)}\Psi_i'(e_i)\frac{dE^i_{\beta_i}}{dq_i} \text{for} i = 1, 2.$$

Bibliography

Armstrong, M. and J.-C. Rochet (1999): "Multi-dimensional screening:: A user's guide," *European Economic Review*, 43, 959âĂŞ979.

Avery, S. (2008): "Boeing Executive Named Supply Chain Manager of the Year".

Azam, J.-P. (1995): "How to Pay for the Peace? A Theoretical Framework with References to African Countries," *Public Choice*, 83, 173–84.

Bac, M. and P. K. Bag (2006a): "Beneficial collusion in corruption control: The case of nonmonetary penalties," *Journal of Development Economics*, 81, 478–499.

——— (2006b): "Beneficial collusion in corruption control: The case ofnonmonetary penalties," *Journal of Development Economics*, 81, 478–499.

Bag, P. K. (1997): "Controlling Corruption in Hierarchies," *Journal of Comparative Economics*, 25, 322–344.

Baliga, S. and T. Sjostrom (1998): "Decentralization and Collusion," *Journal of Economic Theory*, 83, 196–232.

Baron, D. P. and R. B. Myerson (1982): "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50, 911–930.

Bazzi, S. and C. Blattman (2014): "Economic Shocks and Conflict: Evidence from Commodity Prices," *American Economic Journal: Macroeconomics*, 6, 1–38.

Becsi, Z. and S. Lahiri (2007): "Bilateral War in a Multilateral World: Carrots and Sticks for Conflict Resolution," *The Canadian Journal of Economics*, 40, 1168–1187.

Besley, T. and M. Ghatak (2005): "Competition and Incentives with Motivated Agents," *American Economic Review*, 95, 616–636.

Besley, T. and T. Persson (2010): "State Capacity, Conflict, and Development," *Econometrica*, 78, 1–34.

Bhattacharya, P. and M. Graham (2009): "On institutional ownership and firm performance : a disaggregated view," *Journal of Multinational Financial Managment*, 19, 370–394.

Blattman, C. and E. Miguel (2010): "Civil War," *Journal of Economic Literature*, 48, 3–57.

Burgess, M. (2003): "A Brief History of Terrorism".

Cai, H. (2003): "A Theory of Joint Asset Ownership," *The RAND Journal of Economics*, 34, 63–77.

Celik, G. (2009): "Mechanism design with collusive supervision," *Journal of Economic Theory*, 144, 69–95.

Chassang, S. and G. P. i Miquel (2008): "Mutual Fear and Civil War," *BREAD Working Paper No. 165.*

Che, Y.-K. and J. Kim (2006): "Robustly Collusion-Proof Implementation," *Econometrica*, 74, 1063–1107.

Chiang, A. C. (1992): *Elements of Dynamic Optimization*, McGraw-Hill.

Cho, I.-K. and D. M. Kreps (1987): "Signalling Games and Stable Equilibria," *Quarterly Journal of Economics*, 102, 179–221.

Collier, P. and A. Hoeffler (2004): "Greed and grievance in civil war," *Oxford Economic Papers*, 56, 563–595.

Cremer, J. and F. Khalil (1992): "Gathering Information before Signing a Contract," *The American Economic Review*, 82, 566–578.

Cremer, J., F. Khalil, and J.-C. Rochet (1998): "Strategic Information Gathering before a Contract Is Offered," *Journal of Economic Theory*, 81.

de Mesquita, E. B. and E. S. Dickson (2007): "The propaganda of the deed: Terrorism, counterterrorism, and mobilization," *American Journal of Political Science*, 51, 364–381.

Deltas, G., A. Salvo, and H. Vasconcelos (2012a): "Consumer-surplus-enhancing collusion and trade," *The RAND Journal of Economics*, 43, 315–328.

——— (2012b): "Consumer-surplus-enhancing collusion and trade," *The RAND Journal of Economics*, 43, 315–328.

den Steen, E. V. (2010): "Interpersonal Authority in a Theory of the Firm," *American Economic Review*, 100, 466–490.

Dixit, A. K. (2007): *Lawlessness and Economics - Alternative Modes of Governance*, Princeton University Press.

Dubey, P. and S. Sahi (2012): "The Allocation of a Prize (Expanded)".

Esteban, J. and D. Ray (2011): "A Model of Ethnic Conflict," *Journal of the European Economic Association*, 9, 496–521.

Farrell, J. and C. Shapiro (2010): "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition," *The B.E. Journal of Theoretical Economics*, 10, 1–41.

Fearon, J. D. (1995): "Rationalist Explanations for War," *International Organization*, 49, 379–414.

Fudenberg, D. and J. Tirole (1984): "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look," *American Economic Review*, 74, 361–66.

——— (1991): *Game Theory*, MIT Press.

Garfinkel, M. R. and S. Skaperdas (2007): *Handbook of Defense Economics*, Elsevier, chap. Economics of Conflict: An Overview.

Gottlieb, D. and H. Moreira (2013): "Simultaneous Adverse Selection and Moral Hazard," .

Grossman, H. I. (1991): "A General Equilibrium Model of Insurrections," *American Economic Review*, 81, 912–21.

——— (1994): "Production, Appropriation, and Land Reform," *American Economic Review*, 84, 705–12.

——— (2002): "Constitution or Conflict?" *NBER Working Paper No. 8733.*

Hirshleifer, J. (1989): "Conflict and rent-seeking success functions: Ratio vs. difference models of relative success," *Public Choice*, 63, 101–112.

——— (1995): *Handbook of Defense Economics*, Elsevier, chap. Theorizing about conflict, 165–189.

——— (2001): *The Dark Side of the Force: Economic Foundations of Conflict Theory*, Cambridge University Press.

Holmstrom, B. and P. Milgrom (1991): "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics, & Organization*, 7, 24–52.

IMRB-TOI (2010): "Times of India (28th September, 2010)".

Itoh, H. (1993): "Coalitions, incentives and risk-sharing," *Journal of Economic Theory*, 60, 410–418.

Iyer, L. (2009): "The Bloody Millennium: Internal Conflict in South Asia," HBS Working Paper Number: 09-086.

Jia, H., S. Skaperdas, and S. Vaidya (2013): "Contest functions: Theoretical foundations and issues in estimation," *International Journal of Industrial Organization*, 31, 211–222.

Kets, W., G. Iyengar, R. Sethi, and S. Bowles (2011): "Inequality and network structure," *Games and Economic Behavior*, 73, 215–226.

Khalil, F. and J. LawarrÃl'e (1995): "Collusive Auditors," *The American Economic Review*, 85, 442–446.

Kreps, D. M. (1999): *A Course in Microeconomic Theory*, Prentice Hall of India.

Kreps, D. M. and J. Sobel (1994): *Handbook of Game Theory*, Elsevier Science, vol. 2, chap. Signalling, 849–867.

Laffont, J.-J. and D. Martimort (1998): "Collusion and Delegation," *The RAND Journal of Economics*, 29, 280–305.

Lahiri, S. and V. Vlad (2012): "Peace Dividends in a Trade-theoretic Model of Conflict," *Economics Bulletin*, 32, 737–745.

Lorenzoni, G. (Fall 2009): *14.451 Lecture Notes 10.*

Macho-Stadler, I., J. Perez-Castrillo, and R. W. (Translator) (1995): *An Introduction to the Economics of Information: Incentives and Contracts*, Oxford University Press, second ed.

Melumad, N. D., D. Mookherjee, and S. Reichelstein (1995): "Hierarchical Decentralization of Incentive Contracts," *The RAND Journal of Economics*, 26, 654–672.

MOOKHERJEE, D. (1984): "Optimal Incentive Schemes with Many Agents," *The Review of Economic Studies*, 51, 433–446.

MOOKHERJEE, D. AND M. TSUMAGARI (2004): "The Organization of Supplier Networks: Effects of Delegation and Intermediation," *Econometrica*, 72, 1179–1219.

PAVLOV, G. (2008): "Auction design in the presence of collusion," *Theoretical Economics*, 3, 383–429.

PONTRYAGIN, L. S., V. G. BOLTYANSKII, R. V. GAMKRELIDZE, AND E. F. MISHECHENKO (1962): *The Mathematical Theory of Optimal Processes*, John Wiley & Sons.

POWELL, R. (2007): "Defending against Terrorist Attacks with Limited Resources," *The American Political Science Review*, 101, 527–541.

SIQUEIRA, K. AND T. SANDLER (2006): "Terrorists versus the Government: STRATEGIC INTERACTION, SUPPORT, AND SPONSORSHIP," *Published Articles & Papers*, Paper 57, cREATE Research Archive.

THEILEN, B. (2003): "Simultaneous moral hazard and adverse selection with risk averse agents," *Economics Letters*, 79, 283–289.

TIROLE, J. (1992): "Collusion and the theory of organizations," in *Advances in Economic Theory: Sixth World Congress*, ed. by J. Laffont, Cambridge University Press, vol. 2.

VIVES, X. (2005): "Complementarities and Games: New Developments," *Journal of Economic Literature*, 437–479.

——— (2007): "Supermodularity and supermodular games," Occasional Paper.

WILLIAMSON, O. E. (1967): "Hierarchical Control and Optimum Firm Size," *Journal of Political Economy*, 75, 123–138.