TABLES FOR STUDENTIZATION

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Let $(X_1, \ldots, X_k)$ be a random sample from a normal population $N(m, \sigma^2)$. Let y be a statistic, namely a function of $(x_1, \ldots, x_k)$. Suppose we know the distribution $F(X)$ of y/σ and we have an unbiased estimate s^2 of σ^2 which is based on n degrees of freedom and independent of y. The problem of finding out the distribution $F_n(y)$ (say) of y/s is called the problem of studentization. An approximate formulae for $F_n(X)$ was suggested by Hartley (1943) and it has been applied in various problems. *e.g.*, Nair (1947, 1948, 1952) and Pearson and Hartley (1943).

Moriguchi (1953) gave an elegant proof for the Hartley's formulae by making use of the following expansion :

$$F_n(X) = \sum_{\nu=0}^{N} b_\nu(n) x^\nu F^{(\nu)}(X) + R_N$$

where $F^{(\nu)}(X)$ is the ν-th derivative of $F(X)$ and

$$b_0(n) = 1$$

$$b_\nu(n) = \sum_{r=0}^{\nu} \frac{(-1)^{\nu-r}}{r!(\nu-r)!} \left(\frac{2}{n}\right)^{\frac{r}{2}} \Gamma\left(\frac{n+r}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)$$

$$|R_N| \leqslant |b_{N+1}(n)| \ |x^{N+1}| \max_{-\infty<u<+\infty} |F^{(N+1)}(u)|.$$

The tables give the values of $b_\nu(n)$. They were prepared in the following way. At first we tabulated $C(n) = \left(\frac{2}{n}\right)^{\frac{1}{2}} \Gamma\left(\frac{n+1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)$ then $b_\nu(n)$ were tabulated from the expressions :

$$b_1(n) = -b_2(n) = C(n) - 1$$

$$b_3(n) = -\frac{1}{6n}(4n - (4n+1)C(n))$$

$$b_4(n) = \frac{1}{12n}((4n+1) - (4n+2)C(n))$$

$$b_5(n) = -\frac{1}{120n^2}((16n^2+10n) - (16n^2+14n+3)C(n))$$

$$b_6(n) = \frac{1}{360n^2}((16n^2+18n+4) - (16n^2+22n+9)C(n))$$

$$b_7(n) = \frac{1}{5040n^3}((64n^3+112n^2+56n) - (64n^3+128n^2+86n+15)\,C(n))$$

$$b_8(n) = \frac{1}{10080n^3}((32n^3+80n^2+67n+12) - (32n^3+88n^2+88n+30)\,C(n))$$

and the rest were tabulated directly by making use of the relation

$$\Gamma(x+1) = x\Gamma(x).$$

For tabulation of $C(n)$, we made use of the tables (National Bureau of Standards (1951)), and an expansion

$$C(n) = 1 - \frac{1}{4n} + \frac{1}{32n^2} + \frac{5}{128n^3} - \frac{21}{2048n^4}.$$

As $b_0(n) = 1$ and $b_1(n) = -b_2(n)$, they are not listed in the tables.

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VALUES OF THE COEFFICIENTS IN THE MORIGUCHI EXPANSION OF THE DISTRIBUTION FUNCTION OF A STUDENTIZED STATISTIC

n	b_2	b_3	b_4	b_5
1	.2021155	−.0017620	.0177244	.0027516
2	.1137731	−.0019065	.0057388	.0003901
3	.0780822	−.0012705	.0028209	.0001195
4	.0600144	−.0008436	.0016721	.0000497
5	.0484672	−.0005937	.0011046	.0000250
6	.0406312	−.0004383	.0007835	.0000142
7	.0349696	−.0003361	.0005844	.0000088
8	.0306894	−.0002656	.0004525	.0000058
9	.0273408	−.0002140	.0003606	.0000041
10	.0246500	−.0001774	.0002941	.0000020
11	.0224408	−.0001480	.0002445	.0000022
12	.0205944	−.0001267	.0002004	.0000017
13	.0190287	−.0001092	.0001700	.0000013
14	.0176830	−.0000950	.0001528	.0000010
15	.0165165	−.0000834	.0001334	.0000008
16	.0154937	−.0000738	.0001176	.0000007
17	.0145001	−.0000658	.0001044	.0000006
18	.0137860	−.0000590	.0000933	.0000005
19	.0130058	−.0000531	.0000839	.0000004
20	.0124171	−.0000482	.0000758	.0000004
25	.0099476	−.0000314	.0000488	.0000002
30	.0082972	−.0000220	.0000340	.0000001
35	.0071164	−.0000163	.0000251	.0000001
40	.0062209	−.0000125	.0000192	
45	.0055307	−.0000099	.0000152	
50	.0049872	−.0000081	.0000124	
55	.0045349	−.0000067	.0000102	
60	.0041578	−.0000056	.0000086	
65	.0038386	−.0000048	.0000073	
70	.0035649	−.0000042	.0000063	
75	.0033277	−.0000036	.0000055	
80	.0031200	−.0000032	.0000048	
85	.0029368	−.0000028	.0000043	
90	.0027739	−.0000025	.0000038	
95	.0026281	−.0000023	.0000034	
100	.0024968	−.0000021	.0000031	

n	b_6	b_7	b_8	b_9
1	−.0013873	.0003532	.0001095	.0000293
2	−.0002163	.0000303	.0000076	.0000013
3	−.0000714	.0000067	.0000016	.0000002
4	−.0000321	.0000023	.0000005	.0000001
5	−.0000172	.0000010	.0000002	
6	−.0000102	.0000005	.0000001	
7	−.0000066	.0000003	.0000001	
8	−.0000045	.0000002		
9	−.0000032	.0000001		
10	−.0000024	.0000001		
11	−.0000018			
12	−.0000014			
13	−.0000011			
14	−.0000009			
15	−.0000007			
16	−.0000006			
17	−.0000005			
18	−.0000004			
19	−.0000004			
20	−.0000003			
25	−.0000002			
30	−.0000001			
35	−.0000001			

n	b_{10}	b_{11}	b_{12}	b_{13}
1	−.0000078	.0000020	.0000005	.0000001
2	−.0000002			
3				
4				
5				

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