INDIAN STATISTICAL INSTITUTE
KOLKATA

M.TECH (COMPUTER SCIENCE) DISSERTATION

Approximation Algorithm for Base Station Placement on Vertices of a Convex Region

---

A dissertation submitted in partial fulfillment of the requirements
for the degree of Master of Technology
in
Computer Science

Author:
Gopinath MISHRA
Roll No:MTC1209

Supervisor:
Prof. Sandip DAS
Advanced Computing
and Microelectronics
Unit

Advanced Computing and Microelectronics Unit

Indian Statistical Institute, Kolkata

Certificate

This is to certify that the Project entitled ”*Approximation Algorithm for Base Station Placement on Vertices of a Convex Region*” has been submitted by Gopinath Mishra(MTC1209) under my supervision in partial fulfillment of the degree of Master of Technology in Computer Science of Indian Statistical Institute, Kolkata, during the academic year 2013-2014.

Roll No	Names of Student
MTC1209	Gopinath Mishra

Prof. Sandip Das
(Dissertation Supervisor)

Date:30 June 2014

Acknowledgment

At the end of the course, it is my pleasure to thank everyone who has helped me on this way.

First of all I want to express my sincere gratitude to my supervisor, Prof. Sandip Das for interducing me to this interesting problem and for his step by step guidance. I have learn a lot from him. For all his advice, encouragement and the way he helped me to think this problem in a boarder prospective, I will remain grateful.

I would also like to thank all professors at ISI who have made my educational life exciting and helped me to gain a better outlook in computer science. Specially I would like to thank Prof. Subash Chandra Nandy, and Dr. Arijit Bishnu for giving me a good outlook in different aspects of Algorithms during coursework. SCN Sir's use full suggestions during midterm evaluation helped me a lot to complete the thesis.

Special thanks to Aritra da, Ayan da, Soumen da for their suggestions in midterm evaluation and subha da for various discussion in course works and thesis works. I would also like to thank all my classmates who has made my life at ISI delightful, Special thanks to Badri and Ratan.

My most important acknowledgment goes to my family and friends who have filled my life with happiness. Most significantly to my parents who have always encouraged me to pursue my passions and instilled a love of knowledge in me.

Contents

List of Figures

Abstract

Facility Location problem has been an interesting research topic in the area of Wireless (Sensor) Networks, Operation Research, Computational Geometry and other related area of Computer Science. A lot of work have been done in this area where the objective is to place k base station of equal(minimum) range to cover entire interior region of a given polygon. Many variations have been studied in the literature by restricting the base station location on the boundary of the polygon, on a given edge etc.We focus on the problem in other way round i.e. given a Convex Region P and a real number R the objective is to find minimum number (say k) of base station position(s), if possible, where we can place our base station(s) such that each and every interior point of P is covered by atleast one of the k base station(s). Here the constraint is that the base stations can only be placed on vertices of the Convex Region. This problem of finding k base station position(s)to cover polygonal region is named as $MinRegionCover(P, R)$. The minimum value of given R for which $MinRegionCover$ can be found out, we name that value as $\mathcal{L}(P)$. In this work we find $\mathcal{L}(P)$ in $O(n)$ time and decide whether given R is sufficient to find $MinRegionCover$ of given P along with an approach for $MinRegionCover(P, \mathcal{L}(P))$. We also have proved that at most 5 base stations are sufficient to cover a regular region P when $R \geq \mathcal{L}(P)$ and that can be found in $O(n)$ time. In general none of the variation of Base Station Placement problems is known to be NP-hard or Polynomial time solvable though polynomial time algorithms for some special cases are known. Our main work is an optimal $O(n^2)$ algorithm to find $MinRegionCover$ of a Convex Polygon P such that line joining farthest pair is an edge of the P, which leads to an approach to find constant factor approximation algorithm for $MinRegionCover$ for general Convex Polygon in polynomial time.

Chapter 1

Introduction

1.1 Base Station Placement Problems

Due to recent growth in the demand of mobile communication services in several typical environments, the development of efficient systems for providing specialized services has become an important issue in mobile communication research. An important sub-problem in this area is the base station placement problem, where the objective is to identify the location for placing the base stations. Mobile terminals communicate with their respective nearest base station, and the base stations communicate with each other over scarce wireless channel in a multi-hop fashion by receiving and transmitting radio signals. Each base station station emits signal periodically and all the mobile terminals with in its range can identify it as its nearest base station after receiving such radio signal. Here the problem is to position the base stations such that the mobile terminal at any point in the entire area of interest can communicate with at least one base station, and the total power required for all the base station in the network is minimized. A different variation of the problem arises when some portions of the target region is forbidden for placing the base stations, but the communication inside those regions need to be provided.For example, we may consider a large water body or a stiff mountain. In such cases, we need some specialized algorithms for efficiently placing the base stations on the boundary of the forbidden zone to provide services inside that region. Some cases may also arise such that we are allowed to place our base station only at some given location say only on vertices.

For simplicity, we assume that the region $\mathcal{P}$ is convex, and all the k base stations are similar, in other words,their range /power requirement are same, and the power requirement(cost) for a base station of range r is proportional to r^2.

In this thesis some constrained versions of Base stations placement on vertices of the given Convex Region are studied.

1.2 Related Works

The k base stations placement problem can be formulated as classical k center problem in computational geometry. The simplest version of this problem is Euclidean 1-center problem by sylvester[24]. The first algorithmic result on this problem is by Elzinga and Hearn[10], which gives an $O(n^2)$. Later Shamos and Hoey[25] improved the time complexity to $O(nlogn)$. Lee[15] proposed the farthest point voronoi diagram which can also be used to solve 1-center in $O(nlogn)$. Finally Megiddo[19] gave an $O(n)$ prune and search algorithm.

Several constrained versions of Euclidean 1-center problems are also studied in literature. Megiddo[19] has also studied a problem of minimum enclosing circle where center lies on a given straight line.Hurtado, Sacristan and Toussaint [13] used linear programming to give an $O(n + m)$ time algorithm for finding minimum enclosing circle whose center satisfies m linear inequality constraints. The query version of the minimum enclosing circle problem is studied by Roy et al. [22], where the given points need to be preprocessed such that given an arbitrary query line, the minimum enclosing circle with center on the query line can be reported efficiently. The preprocessing time and space requirement of this algorithm are $O(nlogn)$ and $O(n)$ respectively, and the query time complexity is $O(log^2n)$.

For the 2-center problem, the first work is due to Sharir[23], where an $O(nlog^9n)$ time algorithm is presented. The best known algorithm for this problem was proposed by Chan[9]. It suggests two algorithms. The first one is a deterministic algorithm, and it runs in $O(nlog^2n(loglogn)^2)$ time; the second one is a randomized algorithm that runs in $O(nlog^2n)$ time with high probability.

Several other constrained variations of the $k - center$ problem may be of interest in the domain of mobile communication and sensor network. Alt et al.[5] considered the problem of computing the centers of k circles on a line to cover a given set of points in $2D$. The radius of the circles may not be the same. The objective is to minimize the sum of radii of all these k circles. They proposed an $O(n^2 logn)$ time algorithm for solving this problem.

A variation of this problem is the discrete $k-center$ problem, where the objective is to find two closed disks whose union can cover a given set P of n points, and whose centers are a pair of points in P. Bilo et al.[8] proved that the discrete $k-center$ problem in $1D$ can be solved in polynomial time, and is NP-hard in higher dimension. Agarwal et al.[7] first studied the case where $k = 2$, and proposed an $O(n^{4/3} logn)$ time algorithm. Recently, Kim et al.[14] proposed much efficient algorithms for both the standard and discrete versions of the 2-center problem where the points to be covered are vertices of a convex polygon. Their algorithms run in $O(nlog^3 nloglogn)$ and $O(nlog^2 n)$ respectively. The discrete $k-center$ problem is known to be NP-complete[18]. Hwang et al.[11] proposed an $O(n^{\sqrt{k}})$ time algorithm for the discrete $k-center$ problem. Therefore, it makes sense to search for efficient approximation algorithms and heuristics for the general version [12,21]. Lev-Tov and Peleg[16,17] proposed another variation of the discrete $k-center$ problem in the context of mobile communication. Here the positions of k base stations are given, and the objective is to find the radius of coverage of each base station such that each point in P is covered by at least one of the base stations and the sum of radii of coverage of all these base stations is minimized. A polynomial time approximation scheme for this problem is also proposed[16]. Detailed review on this topic can be found in [27].

Many similar problems can also be found in the mobile communication and sensor network literature. Sohn et al.[26] assumes that two sets of points B and R , called blue and red points, are given. The objective is to cover all the red points with circles of radius p (given apriori) centered at minimum number of blue points. Here the blue points indicate the possible positions of base stations, and red points indicate the target locations where the message need to be communicated. A heuristic algorithm using integer linear programming is presented along with experimental results. Azad et al.[6] studied a different variation where n base stations (of same range) are placed on the boundary of a square region, and m sensors are uniformly distributed inside that region. The sensors are also allowed for limited movement. The entire time span is divided into slots. At the beginning of each time slot, depending on the positions of the sensors,k base stations need to be activated. The proposed algorithm finds a feasible solution (if exists) in time$O(mn + nlogn)$ time.

Sansanka Roy et al.[1,2] have also studied some variations of $k - centre$ problem on boundary of convex region. They gave an $O(n)$ algorithm for $1 - centre$ problem where center lies on the boundary and an $1 + \epsilon$ factor approximation algorithm for $regioncover(k)$ problem that runs in $O((n + k)log(n + k) + nlog(1/\epsilon))$, where all k base stations lie on a specified boundary.

In this paper we focus on the facility location problem on vertices of a convex Region. We basically focus on the problem named $MinRegionCover$ to find minimum set of facilities that are required to cover entire region given a real number R as radius. We find minimum possible value of $R((\mathcal{P}))$ that is required for a given polygon in $O(n)$ time. We give an approach for $MinregionCover(\mathcal{P}, \mathcal{L}(\mathcal{P}))$ and $MinregionCover(\mathcal{P}, R)$ where R is any thing.

Chapter 2

Problem Definition and Some Results

2.1 Some Definitions

Below some definitions are given which are used in the following in this thesis.

Definition *(PRegion) Given a Convex Region $\mathcal{P}$ and a point p(p is a vertex of $\mathcal{P}$) and a real number R , PRegion($p, R, \mathcal{P}$) (written as PRegion(p, R) for simplicity) is defined as PRegion(p, R)=$CR(p, R) \cap \mathcal{P}$. Where $CR(p, R)$ is the circular region of radius R centered at p.*

Definition *(PolygonalVoronoiRegion) Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ in anticlockwise order $VoronoiRegion(p)$ is the intersection of $\mathcal{P}$ and the voronoi region of $p \in \{p_1, p_2, \ldots, p_n\}$ and denoted as $VR(\mathcal{P}, p)$ or simply $VR(p)$. Here the sites of the voronoi diagram are the vertices of the convex Region.*

Definition *(VoronoiDiameter) VoronoiDiameter(p)is the distance of the farthest point inside $VR(\mathcal{P}, p)$ i.e. farthest from p and is denoted as $VD(p)$. $VD(p) = max_{q \in VR(p)} d(p, q)$, where $d(., .)$ refers to euclidean distance.*

Definition *(LowerBoundCoveringRadius) Given a Convex Region P with its vertex set $\{p_1, p_2, \ldots, p_n\}$in anticlockwise order $LowerBoundCoveringRadius(\mathcal{P})$ is defined as $\mathcal{L}(\mathcal{P}) = max_{i \in [n]} VD(p_i)$. Refer figure 2.1., $d(p_i, c) = d(p_j, c) = d(p_k, c) = \mathcal{L}(\mathcal{P})$*

Definition *(CriticalPoint) By the definition of $\mathcal{L}(\mathcal{P})$ it is clear that there exists a point c inside the convex region and a vertex $p \in \{p_1, p_2, \ldots, p_n\}$ such that $d(p, c) = \mathcal{L}(\mathcal{P})$, Here c is called Critical Point of $\mathcal{P}$. Refer figure 2.1.*

Definition *($\epsilon - disk$) Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ in order $\epsilon - disk = CR(q, \epsilon)$, where q is the critical point and $\epsilon < max_i VD(p_i) - max_{i \neq j : VD(p_j) = max_l VD(p_l)} VD(p_i)$. Refer figure 2.1., circle of radius ϵ centered at c is $\epsilon - disk$.*

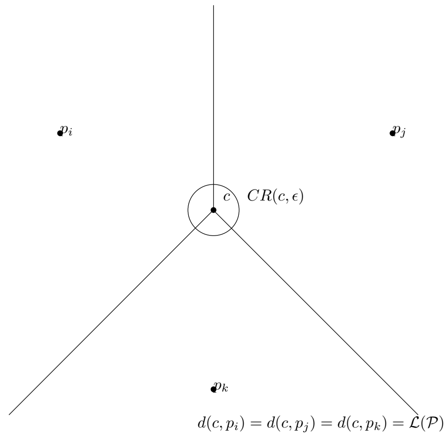


Figure 2.1: $CriticalPoint$ and $\epsilon - disk$

2.2 Problem Definition

Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ in anticlockwise order and a real number R as radius the objective is to find , if possible, $\{f_1, f_2, \ldots, f_k\} \subseteq \{p_1, p_2, \ldots, p_n\}$, $1 \leq k \leq n$, such that$\mathcal{P} \subseteq \cup_{i=1}^{k} CR(f_i, R)$ or $\mathcal{P} = \cup_{i=1}^{k} PRegion(f_i, R)$ and k is minimum.

Definition *(MinRegionCover) The problem posed above for finding $\{f_1, f_2, \ldots, f_k\}$ is referred to as MinRegionCover($\mathcal{P}, R$). For simplicity we say $MRC(\mathcal{P}, R) = \{f_1, f_2, \ldots, f_k\}$.*

2.3 Some Results

2.3.1 On Critical Point of a Convex Region

Lemma 2.3.1 *Critical Point of a Convex Region $\mathcal{P}$ is a voronoi vertex(inside the region $\mathcal{P}$) of voronoi diagram of $\{p_1, p_2, \ldots, p_n\}$ where $\mathcal{P}$ is a convex region and $\{p_1, p_2, \ldots, p_n\}$ is its vertex set.*

Proof *Let q be the critical point of $\mathcal{P}$. It is obvious that q is either a voronoi vertex or a point on the boundary through which a voronoi edge passes through it. Let's assume that q is not a voronoi vertex, then there must exist $p_i, p_j \in \{p_1, p_2, \ldots, p_n\}$ such that q lies on the bisector of p_i and p_j say e and there exists an edge of $\mathcal{P}$ on which q lies, let that edge be e_1. Here e is either perpendicular to e_1 or e forms an obtuse angle at q in exactly one of $VR(p_i)$ or $VR(p_j)$, w.l.o.g. say $VR(p_i)$, then there exists a point y on e such that $d(y, p_i) > d(x, p_j)$, which contradicts definition of q. Hence the claim holds.* □

2.3.2 On Lower Bound of Covering Radius

Lemma 2.3.2 *Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, n\}$ in anticlockwise order and a real number R as radius, we can find $MinRegion-Cover$ iff $R \geq \mathcal{L}(\mathcal{P})$.*

Proof *Let by contrary assume that $R < \mathcal{L}(\mathcal{P})$ and q be the critical point defined earlier, by definition of q, $d(q, p_i) \geq \mathcal{L}(\mathcal{P}), \forall i \in [n]$. If $R < \mathcal{L}(\mathcal{P})$ then $q \notin \cup_{i=1}^{n} CR(p_i, R)$. Hence the claim holds.* □

Remark *$\mathcal{L}(\mathcal{P})$ is the lower bound for R to find $MinRegionCover(\mathcal{P}, R)$.Here as well as In the following sections R refers to the given radius and $\mathcal{L}(\mathcal{P})$ as defined above.*

2.3.3 On Regular Convex Polygon

Lemma 2.3.3 *Given a Regular Convex Region $\mathcal{P}$ having n vertices $\{p_1, p_2, \ldots, p_n\}$, we can find $MRC(\mathcal{P}, R)$ =$\{f_1, f_2, \ldots, f_k\}$ such that $k \leq 5$ whenever $R \geq \mathcal{L}(\mathcal{P})$.*

Proof The claim is trivially true for $n \leq 5$ and we can also verify that $k = 3$ for $n = 6$.So,w.l.o.g let's assume that $n \geq 7$.

It is trivial that $\mathcal{L}(\mathcal{P})$ = Radius of circumcircle $\mathcal{C}$ of $\mathcal{P}$. Let's draw a hexagon $abcdefa$ of side length $\mathcal{L}(\mathcal{P})$ which is also circumscribed by $\mathcal{C}$ such that f_1 coincides with a. Refer figure 2.2. Assume origin of $\mathcal{C}$ as origin of the co-ordinate system and define x axis arbitrarily. Find angular position of the vertex set of $\mathcal{P}$. Assign each vertex $p \in \{p_1, p_2 \ldots, p_n\}$ to region $r \in \{ab, bc, cd, de, ef, fa\}$ iff p lies on arc r. Observe that each region contains atleast one vertex. Now assign $f_2 = p(\in \{p_1, p_2, \ldots, p_n\})$ such that p is in the region bc and nearest to c. Similarly $f_3 = p(\in \{p_1, p_2 \ldots, p_n\})$ such that p is in the region cd and nearest to d, $f_4 = p(\in \{p_1, p_2, p_n\})$ such that p is in the region de and nearest to e and $f_5 = p(\in \{p_1, p_2 \ldots, p_n\})$ such that p is in the region ef and nearest to f. Now it is easy to observe that $\mathcal{P} \subseteq \cup_{i=1}^{5} CR(f_i, R)$ or $\mathcal{P} = \cup_{i=1}^{5} PRegion(f_i, R)$and hence the claim holds. $\square$

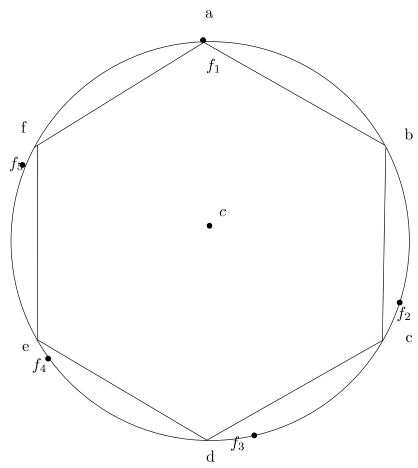


Figure 2.2: MRC of a Regular Convex Region

Observation *The bound given in Lemma 2.3.3 is tight. To be specific $k = 5$ for $n = 5$ iff $R = \mathcal{L}(\mathcal{P})$*

Chapter 3

Finding MinRegionCover with Lower Bound of Covering Radius

In the previous chapter we have defined $\mathcal{L}(\mathcal{P})$. An approach for finding $MRC(\mathcal{P}, \mathcal{L}(\mathcal{P}))$ is given in this chapter with some important observations. Though the algorithm is not completely solved but our approach gives a direction to solve the problem.

3.1 On Compulsory facilities when $R = \mathcal{L}(\mathcal{P})$

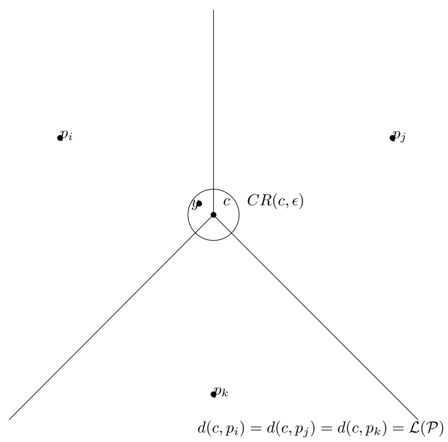


Figure 3.1: Compulsory facilities when $R = \mathcal{L}(\mathcal{P})$

Lemma 3.1.1 *Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ in anticlockwise order and a real number $R = \mathcal{L}(\mathcal{P})$, $MinRegionCover = \{f_1, f_2, \ldots, f_k\} \subseteq \{p_1, p_2, \ldots, p_n\}$ can be found out such that $k \geq 3$. We need atleast 3 facilities to cover $\epsilon - disk$ or say $\mathcal{P}$ if $R = \mathcal{L}(\mathcal{P})$.* □

Proof *Let q be the critical point,and by lemma 2.3.1, q is a voronoi vertex.So, there exists atleast three vertices such that q is inside their voronoi region. For simplicity assume that q is the intersection of voronoi regions of three vertex $p_i, p_j, p_k \in \{p_1, p_2, \ldots, p_n\}$i.e. $VR(p_i), VR(p_j), VR(p_k)$. Refer figure 3.1. Let's assume that $p_i \notin MRC(\mathcal{P}, R)$ w.l.o.g. there exists a point $y \in VR(p_i) \cap \epsilon - disk(\mathcal{P})$ such that $y \notin \cup_{f \in MRC} CR(f, R)$, Hence the claim holds.* □

Lemma 3.1.2 *Given a Convex Region $\mathcal{P}$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ in anticlockwise order and a real number $R = \mathcal{L}(\mathcal{P})$ and c be its critical point. If c is in the voronoi region of m points, we need at most 5 points out of those m points to cover $\epsilon - disk$ around c.*

Proof *The claim is trivially true for $m \leq 5$, Hence assume that $m > 5$. Now it is clear that we can find a circle of radius $R = \mathcal{L}(\mathcal{P})$ centered at c that has all m points on its circumference. Now the proof is a variation of lemma 2.3.3.* □

3.2 Approach for $MinRegionCover$ when $R = \mathcal{L}(\mathcal{P})$

Below an observation is given that leads to an approach for finding $MRC(\mathcal{P}, \mathcal{L}(\mathcal{P}))$

Observation *From the definition of $\epsilon - disk$ the minimum number of facilities required to cover it are necessary facility to cover a given convex region $\mathcal{P}$.*

Algorithm 1: *$MinRegionCover$ of a given $ConvexPolygonwithR = \mathcal{L}(\mathcal{P})$*

Input: A Convex Region $\mathcal{P}$.
Output: MRC of $\mathcal{P}$.

```
begin
    Find the critical point(s) and (3 to 5) necessary facilities.
    Divide P into smaller regions by fixing necessary facilities.
    Cover each smaller part separately.
end
```

Remark *It is obvious that $\mathcal{P}$ will be divided in to disjoint regions as facility points in one partition will be at distance more than $R(\geq \mathcal{L}(\mathcal{P}))$ from any points in other partition, but the difficulty is that each small part is not in normal polygonal structure. Refer figure-*3.2*. Here $UV-1$, $UV-2$ and $UV-3$ are three, at most, uncovered regions after fixing the three compulsory facilities p_i, p_j and p_k.*

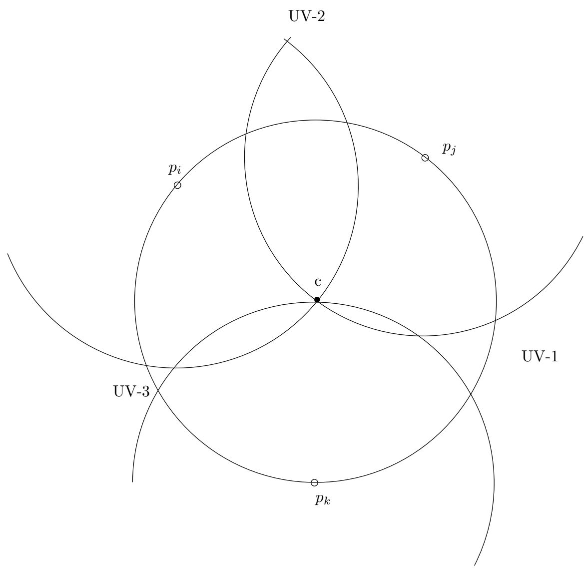


Figure 3.2: Uncovered Regions after fixing Critical Points

Chapter 4

Given Radius Finding MinRegionCover

In this chapter basically two approach for finding MRC of convex region $\mathcal{P}$ and a real number R if $R \geq \mathcal{L}(\mathcal{P})$,otherwise our procedure reports fail.

4.1 Naive Approach

Definition *(Optional facility)Given a Convex Region* $\mathcal{P}$ *with its vertex set* $V = \{p_1, p_2, \ldots, p_n\}$ *in anticlockwise order and a real number* $R \geq \mathcal{L}(\mathcal{P})$*, a point* $p \in V$ *is said to be optional iff* $VR(p, R) \subseteq \cup_{q \in V, q \neq p} CR(q, R)$*.*

Lemma 4.1.1 *A vertex of* p *of a given Convex Region* $\mathcal{P}$ *is optional facility point iff* $VR(p, R) \subseteq \cup_{q \in V \setminus \{p\}, VR(p) \cap VR(q) \neq \phi} CR(q, R)$ *(assume* $R \geq \mathcal{L}(\mathcal{P})$*).*

Proof It follows from the definition of Voronoi diagram.

Lemma 4.1.2 *Given a Convex Region* $\mathcal{P}$ *with vertex set* V *and a real number* $R \geq \mathcal{L}(\mathcal{P})$*, a point* $p \in V$ *is optional or not can be tested in* $O(n_1 log n_1)$ *(*n_1 *denotes number of edges in* $VR(p, R)$*). In fact each* $v \in V$ *can be tested in* $O(nlogn)$*.*

Following is an approach for finding $MinRegionCover$ given a Convex Polygon and a real number R.

Algorithm 2: *MinRegionCover of a given ConvexPolygon*

Input: A Convex Region $\mathcal{P}$ with vertex set V and a real number R .
Output: MRC of P, if possible.

1 **begin**
2   Find $\mathcal{L}(P)$
3   **if** $R < \mathcal{L}(P)$ **then**
4   Return FAIL;
5   Find set of *optional facilities* A as defined above;.
6     $B = V \setminus A$;
7   Initialize $MRC = B$;
8   Add some facility of A dynamically in such a way to minimize $|MRC|$
9 **end**

Remark *Step* 2 *can be performed in* $O(n)$ *time as we can draw voronoi diagram of a set of point in convex position in* $O(n)$ *time*[3]*. Except step 8 remaining steps are easy and can be performed in* $O(nlogn)$*. Some dynamic programming techniques may be used to solve step* 8*.*

4.2 Efficient Approach

Before explaining our approach-2 for any Convex Region we need the algorithm for finding MRC of a$HalfConvexPolygon$ as defined below.

Definition *(HalfConvexPolygon)A Convex Region* P *with vertex set* $\{p_1, p_2, \ldots, p_n\}$ *in clock wise order is said to be HalfConvexfPolygon if* (p_1, p_n) *is the farthest pair and* $p_1(x) < p_2(x) < p_n(x)$*, where line joining* p_1 *and* p_n *is* x *axis of our co-ordinate system and* $p_i(x)$ *denotes* x *co-ordinate of* p_i*,* $\forall i \in [n]$*. Refer figure-*4.1*.*

4.2.1 MinRegionCover of a HalfConvexRegion

Following contains the algorithm for finding $MinRegionCover$ of a given $HalfConvexRegion$.

Algorithm The basic idea of our algorithm is as follows. We start from the leftmost facility of(one point of the farthest pair) and find its $PRegion$ and add it to our MRC(F in algorithm). We then go to other vertices in clock

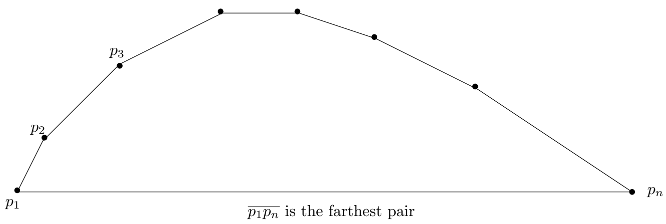


Figure 4.1: $HalfConvexPolygon$

wise order and find their $PRegion$. We maintain two variables $Last_{added}[1]$ and $Last_{added}[2]$ to store two recently added facilities. At each instance we check whether $PRegion$ of currently considered facility point is contained in $PRegion$ of $Last_{added}[1]$ or vice versa. We delete the facility point accordingly from F. If none of them is true then we check whether $PRegion$ of $Last_{added}[1]$ is contained in the union of $PRegion$ of currently considered facility point and $Last_{added}[1]$ then we delete $Last_{added}[1]$ from our solution F and we set $Last_{added}$ variable accordingly.

Following is a formalism to our idea. $Cover(P, R, p, q)$ in Algorithm-3 returns true iff $PRegion(q, R) \subseteq PRegion(p, R)$.

Algorithm 3: *MinRegionCover of a given HalfConvexPolygon*

Input: A *HalfConvexPolygon* P with its vertex set $\{p_1, p_2, \ldots, p_n\}$ and a real number R as radius.

Output: MRC of P iff possible.

begin
    Find $\mathcal{L}(P)$;
    if $R < \mathcal{L}(P)$ **then**
        Return FAIL;
    Initialize $F = \{p_1\}$;
    $Last_{added} = \{p_1, NULL\}$;
    foreach $i = 2(1)n$ **do**
        if $(Cover(P, R, p_i, Last_{added}[1]))$ **then**
            $F = F \setminus Last_{added}[1]$;
            $F = F \cup \{p_i\}$;
            $gotostep20$;
        else if $(!Cover(P, R, Last_{added}[1], p_i))$ **then**
            $F = F \cup \{p_i\}$;
            $Last_{added}[1] \longleftarrow \{p_i\}$;
        else
            $gotostep - 21$;
        if $(Last_{added}[2] \neq NULL)$ **then**
            if $(PRegion(Lastadded[1], R) \subseteq PRegion(p_i, R) \cup PRegion(Last_{added}[2], R))$ **then**
                $F = F \setminus \{Last_{added}[1]\}$;
            else
                $Last_{added}[2] \longleftarrow Last_{added}[1]$;
        $Last_{added}[1] = p_i$;
        $i \leftarrow i + 1$;
    $ReturnF$;
end

Lemma 4.2.1 *Let P be a HalfConvexPolygon and $\{p_1, p_2, \ldots, p_n\}$ be its vertex set in order. If $p_1 \notin CR(p_i, R)$ then $p_1 \notin CR(p_j, R)$, $\forall p_j(x) > p_i(x)$, where R is the given radius and x axis as considered above.*

Proof *Let by contradiction assume that there exists a p_j such that $j > i$ and $p_1 \in CR(p_j, R)$. Let's draw an circle of radius R centered at p_1, then* *by assumption $p_i \notin CR(p_1, R)$ and $p_j \in CR(p_1, R)$. Observe that $p_n \in CR(p_1, R)$, otherwise P wouldn't be convex. So, $d(p_1, p_n) \leq R$ and also As (p_1, p_n) is the farthest pair we can say $d(p_1, p_n) \geq d(p_1, p_i) > R$, which is not possible.* □

Theorem 4.2.2 *Algorithm 3 solves $MinRegionCover(P, R)$ optimally, where P is a $HalfConvexPolygon$.*

Proof *Let $\mathcal{A} = \{p_{i_1}, p_{i_2}, p_{i_3}, \ldots, p_{i_k}\}$ is returned by our algorithm and it is not optimal, then there exists $\mathcal{B} = \{p_{l_1}, p_{l_2}, p_{l_3}, \ldots, p_{l_{k'}}\}$* such that $k' < k$ and $P \subseteq \cup_{j=1}^{k'} CR(p_{l_j}, R)$. It is easy to observe that $i_1 \geq l_1$ and $i_k \leq l_{k'}$. In the view of Algorithm-1 we can say that $i_1 = l_1$ and $i_k = l'_k$. Let's divide vertex set of P in to regions as follows. $Region_r = \{p_{i_r+1}, \ldots, p_{i_{r+1}}\}$ $\forall r \in [k]$ and $p_{i_{k+1}} = p_n$. By pigeonhole principle there exists a region that does not have any point of $\mathcal{A}$, observe that Which is not possible. Hence the claim holds.□

Time Complexity Analysis

Observation $PRegion(v)$, v is a vertex of the given convex region, is convex.

Observation *Let P be given $HalfCovexRegion$ with its vertex set $\{p_1, p_2, \ldots, p_n\}$ then $CR(p_i, R)$ intersects with polygon boundary with at most 4 points, to be more specific at most two intersection point on $\overline{p_1 p_n}$ and at most two on the chain (one to the left of p_i and one to the right of p_i and one to the left of p_i) $\widehat{p_1 p_2 \ldots p_n}$.)*

Theorem 4.2.3 *Algorithm-3 runs in $O(nlogn)$ time.*

Proof *From above observation it is clear that we can find $PRegion$ of a particular vertex in $O(logn)$ by binary search. Intersection and union of two convex region can be computed in $O(logn)$[4]. So, each execution of loop can be done in $O(logn)$, hence the claim holds.* □

4.2.2 Approach for a Convex Polygon

Given a convex polygon $\mathcal{P}$ and a real number R we have to follow the following steps.

Step 1 : Find $\mathcal{L}(\mathcal{P})$ and proceed iff $R \geq \mathcal{L}(\mathcal{P})$.

Step 2 : Find the farthest pair say (p_1, p_l).

Step 3 : Construct two polygons namely $\mathcal{P}_u$ having vertex set $\{p_1, p_2, \ldots, p_l\}$ and $\mathcal{P}_l$ having vertex set $\{p_l, p_{l+1}, \ldots, p_1\}$ in order.

Step 4 : Find MRC of $\mathcal{P}_u$ and $\mathcal{P}_l$ using $Algorithm - 3$.

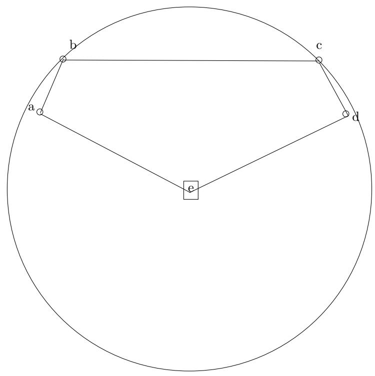
Step 5 : Merge $MRC(P_u)$ and $MRC(P_l)$ as suggested in next part.

Remark *$\mathcal{P}_u$ and $\mathcal{P}_l$ are $HalfConvexPolygon$ as defined above.*

4.2.3 On Merging MRC of $\mathcal{P}_u$ and $\mathcal{P}_l$

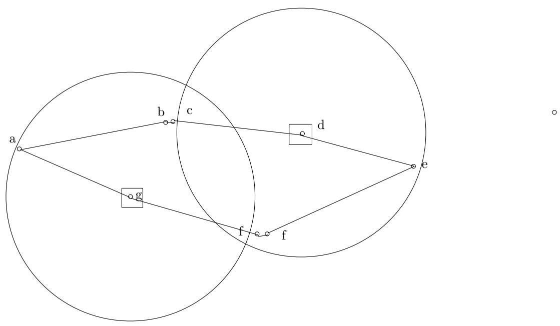
From the above procedure, we can do each step easily except step-5. Though we can find $MRC(\mathcal{P}_l, R)$ and $MRC(\mathcal{P}_u, R)$ optimally but merging the two optimal solution is not so straight forward. Some observations with figures are given below.

Observation *It is not necessary that* $MRC(\mathcal{P}, R) > MRC(\mathcal{P}_u, R)$ *(resp.* $MRC(\mathcal{P}_l, R)$*, Refer figure* 4.1*. In fact there exists a case as illustrated in figure* $MRC(\mathcal{P}, R) < MRC(\mathcal{P}_u, R)$, $MRC(\mathcal{P}_l, R)$*, Refer figure* 4.2*. In figure* 4.2 *and* 4.3 *, given radius* $R =$ *radii of the circles shown and optimal facilities are marked by squares.*



$MRC(\mathcal{P}, R) = 1$, $MRC(\mathcal{P}_u, R) = 2$, $MRC(\mathcal{P}_l, R) = 1$

Figure 4.2: A case where $MRC(\mathcal{P}_u, R) > MRC(\mathcal{P}, R)$



$MRC(\mathcal{P}, R) = 2$, $MRC(\mathcal{P}_u, R) = 3$, $MRC(\mathcal{P}_l, R) = 3$

Figure 4.3: A case where $MRC(\mathcal{P}_u, R), MRC(\mathcal{P}_u, R) > MRC(\mathcal{P}, R)$

Remark One may think that merging $MRC(\mathcal{P}_u, R)$ and $MRC(\mathcal{P}_u, R)$ will give 2-approximate solution, but from the above observation it is clear that we may not get a 2-approximate solution in some cases.

Observation *It may be possible that $\mathcal{L}(\mathcal{P}_u)(resp.\mathcal{L}(\mathcal{P}_l)) > R \geq \mathcal{L}(\mathcal{P})$, it is illustrated by figure-4.4. No facility point in upper polygon can cover the center but it is covered by a facility point in lower polygon as shown below.*

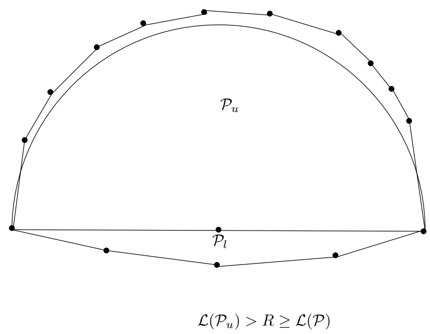


Figure 4.4: Example where $\mathcal{L}(\mathcal{P}_u) > R \geq \mathcal{L}(\mathcal{P})$

Remark To avoid some criticality we assume that if $R \geq \mathcal{L}(\mathcal{P})$ then $R \geq \mathcal{L}(\mathcal{P}_u)$ and $\mathcal{L}(\mathcal{P}_l)$.

Definition *Given a $HalfConvexPolygon$ P with vertex set V, $Interval(v)$ or $I(v), v \in V$ is defined as the intersection of $CR(v, R)$ and the line joining farthest pair of points, where R is the given radius.Let $\overline{v_1 v_n}$ be the line joining farthest pair, $I(v) = CR(v, R) \cap \overline{v_1 v_n}$ and $I_l(v)(resp.I_r(v)$ denotes left (resp. right) end point of $I(v)$. Refer figure 4.5*

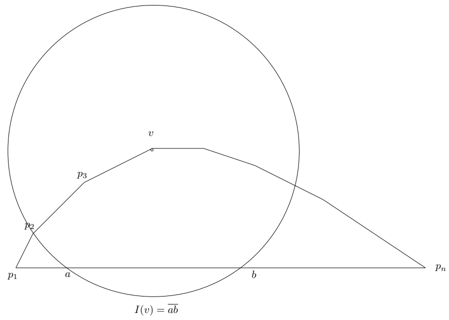


Figure 4.5: Interval on farthest pair

Below an algorithm is given to merge $MRC(\mathcal{P}_u, R)$ and $MRC(\mathcal{P}_l, R)$.

Algorithm 4: *Merging of $MRC(\mathcal{P}_l, R)$ and $MRC(\mathcal{P}_u, R)$*

Input: $MRC(\mathcal{P}_u, R) and MRC(\mathcal{P}_l, R)$.
Output: $Approximate solution to MRC(\mathcal{P})$.

begin
  Initialize $F = MRC(\mathcal{P}_l, R) \cup MRC(\mathcal{P}_u, R)$;
  Find $\mathcal{I}_u(resp.\mathcal{I}_l) = \{I(v) : v \in MRC(\mathcal{P}_u)(resp.MRC(\mathcal{P}_l)\}$.;
  Sort $\mathcal{I}_u(resp.\mathcal{I}_l)$ according to ascending order of left end point.//let $\mathcal{I}_u(resp.\mathcal{I}_l)$ has $n_u(resp.n_l)$ elements;
  $count_l = 0$; $count_u = 0$;
  $i = 1$; $j = 1$;
  while *($i \leq n_u$ and $j \leq n_l$)* **do**
    if ($\mathcal{I}_u[i] is\ a\ subset\ of\ \mathcal{I}_l[j]$) **then**
      $count_u \leftarrow count_u + 1$;
      $i \leftarrow i + 1$;
      if *($count_u == 3$)* **then**
        $F = F \setminus \mathcal{I}_u[i - 1]$;
        $count_u = 2$
    else if ($\mathcal{I}_l[j]\ is\ a\ subset\ of\ \mathcal{I}_l[i]$) **then**
      $count_l \leftarrow count_l + 1$;
      $j \leftarrow j + 1$;
      if *($count_l == 3$)* **then**
        $F = F \setminus \mathcal{I}_l[j - 1]$;
        $count_l = 2$;
    else
      $count_l = 0$; $count_u = 0$; **if** $(\mathcal{I}_u[i])_l < \mathcal{I}_l[j])_l$ **then**
        $i \leftarrow i + 1$;
      else
        $j \leftarrow j + 1$;
  Return F
end

Observation *Let p, q be two vertices of a given half convex region P such that* $p, q \in MRC(P, R)$ *then* $I(p) \cap I(q) \neq I(p)(resp.\ I(q))$.

Lemma 4.2.4 *Let* $\mathcal{P}$ *be a given convex region and* $\mathcal{P}_u$ *and* $\mathcal{P}_l$ *as defined earlier and* $p \in MRC(\mathcal{P}_u, R)$ *and* $q_1, q_2, q_3 \in MRC(\mathcal{P}_l, R)$ *such that* $I(p) \cap I(q_1)(resp.\ I(q_2), I(q_3)) = I(q_1)(resp.\ I(q_2), I(q_3))$ *and* $I_l(q_1) < I_l(q_2) < I_l(q_3)$ *then* $\cup_{v \in V} PRegion(v, R) = \cup_{v \in V \setminus q_2} PRegion(v, R)$ *, where* $V = MRC(\mathcal{P}_u, R) \cup MRC(\mathcal{P}_l, R)$.

Proof Let $p = \overline{xy},\ q_1 = \overline{a_1 a_2},\ q_2 = \overline{b_1 b_2}$ and $\overline{c_1 c_2}$ as in figure 4.6. It is obvious that $a_1 < b_1 < c_1$ and $a_2 < b_2 < c_2$. Let's divide the proof in to two cases.
Case-1($c_1 \leq a_2$)From the figure 4.6 it is clear that presence of q_2 in $MRC(\mathcal{P}, R)$ does not make any difference to it. Hence the claim holds.
Case-2($c_1 > a_2$) From the figure 4.6 it is clear that there is a small region r such that $r \notin \cup_{v \in V_l \setminus q_2} PRegion(v, R)$, where $V_l = MRC(\mathcal{P}_l, R)$, but $r \in CR(p, R)$, hence the claim holds.

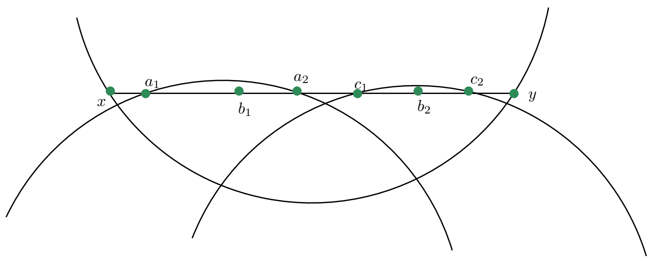


Figure 4.6: Merging $MRC(\mathcal{P}_u, R)$ and $MRC(\mathcal{P}_l, R)$

Theorem 4.2.5 *Given a Convex Polygon* $\mathcal{P}$ *and a real number R such that if* $R \geq \mathcal{L}(\mathcal{P})$ *then* $R \geq \mathcal{L}(\mathcal{P}_u)$ *,* $\mathcal{L}(\mathcal{P}_l)$*, our algorithm finds a 3-factor approximate solution.*

Proof *Due to Theorem* 4.2.2 *we get optimal solution for both* $MRC(\mathcal{P}_u)$ *and* $MRC(\mathcal{P}_l)$ *that are input to Algorithm-*4*. From the above observation no interval corresponding to a vertex of* $MRC(\mathcal{P}_u)(resp. MRC(\mathcal{P}_l))$ *is subset of another interval of* $MRC(\mathcal{P}_u)(resp. MRC(\mathcal{P}_l))$*. Step-*17*(resp. Step-*12*) deletes one vertex in* $MRC(\mathcal{P}_l)$ $(resp. MRC(\mathcal{P}_u))$ *when* 3 *intervals corresponding to* $\mathcal{P}_l(resp. \mathcal{P}_u)$ *are subset of a interval of* $\mathcal{P}_u$ $(resp. \mathcal{P}_l)$*. By lemma* 4.2.4 *it does not make any difference to a solution of* $MRC(\mathcal{P})$*. So at most* 3 *facilities will be left at end of one iteration one corresponding to* $\mathcal{P}_u$ $(resp.\mathcal{P}_l)$ *and two corresponding to* $\mathcal{P}_l(resp.\mathcal{P}_u)$. *Out of those three atleast one is necessary in our optimal solution, Hence the claim holds.* □

Theorem 4.2.6 *Algorithm-*4 *runs in* $O(nlogn)$ *time and* $O(n)$ *space.*

Proof *Sorting takes* $O(nlogn)$ *time and other steps can be performed in* $O(n)$*time, hence total of* $O(nlogn)$ *is required and* $O(n)$ *space required to store the intervals.* □

Theorem 4.2.7 *Running time of the procedure given in section* 4.2.2 *for finding* MRC *is* $O(nlogn)$.

Proof *Step-*1 *runs in* $O(n)$[3], *step-*2 *in* $O(nlogn)$ *and step-* 3 *in* $O(n)$. *By Theorem* 4.2.3 *step-*4 *runs in* $O(nlogn)$ *and step-*5 *in* $O(nlogn)$ *by Theorem* 4.2.6, *hence the claim holds.* □

Chapter 5

Conclusion and Future Works

A variation of base station placement problem is given in this thesis where the base stations can only be placed on vertices of a given convex region. We have focused the problem in other way around i.e. given R we have given an $O(nlogn)$ time $3-$factor approximation algorithm to find $MinRegionCover$ under a condition $if\ R \geq \mathcal{L}(\mathcal{P})\ then\ R \geq \mathcal{L}(\mathcal{P}_u),\ \mathcal{L}(\mathcal{P}_u)$. Our work can be extended to find MRC in general i.e. the given Polygon not necessarily satisfies the above said condition. Our first approach for MRC can further be extended.

We have given some results in chapter-2 that can be used by other variations of facility location problem in future. We have also studied another constrained version in chapter-3 to find $MRC(\mathcal{P}, R)$ when $R = \mathcal{L}(\mathcal{P})$. Future work also can be carried out to establish the detailed algorithm and to extend this idea to find $MRC(\mathcal{P}, R)$ for any R.

This work also can be extended to the case where the region is not necessarily convex and to the variations of the problem where power requirements by different base stations are not equal.

The base station placement problem on the vertices on convex polygon interduced in this thesis can also be studied in more general i.e. to be specific let the vertices of given convex polygon are colored by two colors and we are only allowed to place our only on the green vertices.

Bibliography

[1] Gautam K. Das, S. Roy, S. Das and S. C. Nandy, Variation of base station placement problem on the boundary of a convex region, International Journal of Foundations of Computer Science, Vol 19,No-2, pp. 405-427, 2008.

[2] S. Roy, D. Bardhan and S. Das, Efficient algorithm for placing base stations by avoiding forbidden zone, Proc. of the 2nd International Conference on Distributed Computing and Internet Technology, LNCS-3816, pp. 105-116, 2005.

[3] Alok Aggarwal, Leonidas J. Guibas, James Saxe, Peter W. Shor, A Linear-Time Algorithm for Computing the Voronoi Diagram of a Convex Polygon, Discrete Computational Geometry 4, pp. 591-604, 1989

[4] B. Chazelle, D. B. Dobkin, Intersection of Convex Objects in Two and Three Dimensions, Journal of the Association for Computing Machinery, Vol. 34, No. 1, January 1987, pp. 1-27.

[5] H. Alt, E. M. Arkin, H. Bronnimann, J. Erickson, S. P. Fekete, C. Knauer, J. Lenchner, J. S. B. Mitchell and K. Whittlesey, Minimum-cost coverage of point sets by disks, Proc. of the 22nd Annual ACM Symposium on Computational Geometry, pp. 449-458, 2006.

[6] A. P. Azad and A. Chockalingam, Mobile base stations placement and energy aware routing in wireless sensor networks, Proc. of the IEEE Wireless Communication and Networking Conference, vol. 1, pp. 264-269, 2006.

[7] P. K. Agarwal, M. Sharir and E. Welzl, The discrete 2-Center problem, Proc. of the 13th Annual ACM Symposium on Computational Geometry, pp. 147-155, 1997.

[8] V. Bilo, I. Caragiannis, C. Kaklamanis, and P. Kanellopoulos, Geometric clustering to minimize the sum of cluster sizes, Proc. of the 13th European Symposium on Algorithms, pp. 460-471, 2005.

[9] T. M. Chan, More planar two-center algorithms, Computational Geometry: Theory and Application, vol. 13, pp. 189-198, 1999.

[10] J. Elzinga and D. W. Hearn, Geometrical solutions to some minimal location problems, Transputer Sciences, vol. 6, pp. 379-394, 1972.

[11] R. Z. Hwang, R. C. Chang and R. C. T. Lee, The searching over separators strategy to solve some NP-hard problems in subexponential time, Algorithmica, vol. 9, pp. 398-423, 1993.

[12] D. S. Hochbaum and D. Shmoys, A best possible heuristic for the k-center problem, Mathematics of Operations Research, vol. 10, pp. 180-184, 1985.

[13] F. Hurtado, V. Sacristan and G. T. Toussaint, Some constrained minimax and maxmin location problems, Studies in Locational Analysis, vol. 15, pp. 17-35, 2000.

[14] S. K. Kim, and C. -S. Shin, Efficient algorithms for two-center problems for a convex polygon, Proc. of the 6th Annual International Conference on Computing and Combinatorics, LNCS-1858, pp. 299-309, 2000.

[15] D. T. Lee, Farthest neighbor Voronoi diagrams and applications, Report80-ll-FC-04, Dept. Elect. Engrg. Comput. Sci., Northwestern Univ., Evanston, IL, 1980.

[16] N. Lev-Tov, D. Peleg, Exact Algorithms and Approximation Schemes for Base Station Placement Problems, Proc of the 8th Scandinavian Workshop on Algorithm Theory, pp. 90-99, 2002.

[17] N. Lev-Tov and D. Peleg, Polynomial time approximation schemes for base station coverage with minimum total radii, Computer Networks, vol. 47, pp. 489-501, 2005.

[18] A. Marchetti-Spaccamela, The p-center problem in the plane is NP-complete, Proc. of t h e 19th Allerton Conference on Communication, Control and Computing, pp. 31-40, 1981.

[19] N. Megiddo, Linear-time algorithms for linear programming in R3 and related problems, SIAM Journal on Computing, vol 12, pp. 759-776, 1983.

[20] K. Mehlhorn and S. Naher, The LEDA Platform of Combinatorial and Geometric Computing, Cambridge University Press, 1999.

[21] J. Plesnik, A heuristic for the p-center problems in graphs, Discrete Applied Mathematics,vol. 17, pp. 263-268, 1987.

[22] S. Roy, A. Karmakar, S. Das and S. C. Nandy, Constrained minimum enclosing circle with center on a query line segment, Proc. of t h e 31st International Symposium on Mathematical Foundations of Computer Science, LNCS-4162, pp. 765-776, 2006.

[23] M. Sharir, A near-linear algorithm for the planar 2-center problem, Discrete Computational Geometry, vol. 18, pp. 125-134, 1997.

[24] J. J. Sylvester, A question in the geometry of situation, Quarterly Journal of Mathematics, vol. 1, p. 79, 1857.

[25] M. I. Shamos and D. Hoey, Closest-point problems, Proc. of the 16th Annual IEEE Symposium on Foundations of Computer Science, pp. 151-162, 1975.

[26] S. Sohn and G. Jo, Optimization of base stations positioning in mobile networks, Proc. of the 2006 International Conference on Computational Science and its Applications, LNCS-3981, pp. 779-787, 2006.

[27] C. -S. Shin, J. -H. Kim, S. K. Kim and K. Chwa, Two-center problems for a convex polygon, Proc. of the 6th Annual European Symposium on Algorithms, LNCS-1461, pp. 199-210, 1998.