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Computational Study of The Dynamics of Rumor Propagation in a Network

a dissertation submitted in partial fulfillment of the

requirement for the M. Tech. (Computer Science)

degree of the Indian Statistical Institute

By

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under the supervision of

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Dedicated to my Parents & my little brother,

– they are therefore I am.

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Submitted to the Department of Computer Science
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at the

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Abstract

In this thesis, I have computationally studied the propagation of rumors in a social scenario that is existent on a 2D-plane on a regular Lattice. The computational aspect of the study makes use of the Monte-Carlo Simulation technique to define local interaction criteria among the interacting subjects and the global spatio-temporal behavior of the Enclosed system is observed. The discrete time dynamical systems based on epidemiological models with adequate delays are used for the study. The results of the discrete time simulations and the characteristics of the model are analyzed and reported. Further work of focusing on formalizing the underlying stochasticity is proposed by means of a new realistic model of which only simulation based results are reported.

Thesis Supervisor: Subhash C. Nandy
Title: Professor

Acknowledgments

I am in a dubious swing in deciding whether a dissertation is akin to a novel or a short-story. Well that I am writing out chapters, it should probably be like a novel, but as Rabindranath Tagore had observed, a novel is like lighting up a whole room, but here I am at-most lighting up a minuscule portion of a Table top. So by his definitions it is not even a short story. A long winding essay with chapters it is.Two years at ISI kolkata has been many things, lots of ups & downs. I think whatever has been, has been for the better, for the life ahead from here is an adventurous journey. And I have met some amazing people as my batch-mates, it was a pleasure knowing each one of them. I think each one of them in their own way be it 'fight' or 'fun' or 'suggestions' or 'world bashing', have contributed towards moulding my mind into what it is now, and 'that collective contribution' is typing this out. Same goes to all my professors, I am indebted to ISI for each of it's moments.

I am highly indebted to my mentor, SCN sir as I address him, for putting up with me for the last 7 or more months. Furnishing the dissertation been quite a hacking through shrubbery of ideas. I am extremely grateful to sir for allowing me to proceed with the idea that I ultimately present here as these are the kind of things I am going to pursue from here onwards. In-spite of all these the care he has shown for me, I am just overwhelmed for real. Unbeknowest to him, he has been with me right from the beginning of my journey here, he was the first person to ask the first question to me in the grueling ISI interview.

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Chapter 1

Introduction

A Rumor is defined by the Google dictionary as an "information or story or report, that is being circulated but the truth of which is unverified or doubtful". It is interesting to observe that in a social scenario alongside a proper information a rumor story spreads equally like an 'wild fire'. With the advent of social media especially social networking sites the spread and impact of rumors can be seen vividly. Earlier what would be a simple gossip of a closed locality, can now easily cause a flash flood into the global community. Rumors arrives in various forms, and their span of habitability is also intriguing. While on one hand we see the quick and sudden childish rumors 'like the death of a celebrity', on the other hand we see 'political rumors' from misinterpretation of some forum discussion on an obscure community. While these may cater to people with distinctive and specific interests from very simple to complex tastes and may not live long, but there are rumors which never die and their out break in a social time line is seen over and over again. One good example of this later type are the Conspiracy theories. There are still millions who believe 'Moon landing was fake', 'Osama bin laden is not dead', 'Yeti lives up in high Himalayas' or even 'UFO s are a reality'. The lighter aspect of the situation is, shown enough dubious conspiracy logic, even die hard non-believers become believers even though they revert soon enough when they come back in touch with their rationalistic sides.

1.1 Motivation

Often one would say, rumor is like a disease. A look into the various epidemiological models of disease propagation can lead to a intuitive correlation of spread of a rumor sweeping through a section of a social scenario. Two treatments of a spreading scenario has been studied in this report based on epidemiological models viz. SIR and SIRS, where S represents *Susceptible*, I represents *Infected* and R represents *Removed or Recovered's.* Discrete time differential equations have been used, with time delay to model a more realistic propagation tendency, as one will observe. The models considers a homogeneous population resident on a planar lattice. Each site can have only one subject or a 'node' as we may be referring to them as. If we consider the node placement as a graph, then we have one node at each lattice point on the rectangular grid, each lattice point is a vertex and each vertex has a degree of *four*, i.e connected to the immediate North-South-East-West nodes only. Total initial and final population is constant. A rumor is introduced at one or more points in the the population and the spatial as well as global asymptotic tendency are analyzed both theoretically and by use of simulations using Monte Carlo technique. A better more realistic randomized simulation of this problem is also introduced in the end however formalizing it kept as further work.

1.2 Preliminaries

Few relevant terms are being introduced in this section, some of which have been already mentioned in the preceding text, and few of which will soon be used as we move to our next chapter on the mathematical model of our problem and the subsequent Simulation study of the problem.

1.2.1 Dynamical System

A dynamical system is a concept in mathematics where a fixed rule describes the time dependence of a point in a geometrical space. Examples include the mathematical models that describe the swinging of a clock pendulum (Simple harmonic pendulum), the flow of water in a pipe (Fluid motion), and the number of fish each springtime in a lake (Ecology). At any given time a dynamical system has a state given by a set of real numbers (a vector) that can be represented by a point in an appropriate state space (a geometrical manifold). Small changes in the state of the system create small changes in the numbers. The evolution rule of the dynamical system is a fixed rule that describes what future states follow from the current state. The rule is deterministic– for a given time interval only one future state follows from the current state.

The concept of a dynamical system has its origins in Newtonian mechanics. There, as in other natural sciences and engineering disciplines, the evolution rule of dynamical systems is given implicitly by a relation that gives the state of the system only a short time into the future. (The relation is either a differential equation, difference equation or other time scale.) To determine the state for all future times requires iterating the relation many times—each advancing time a small step. The iteration procedure is referred to as solving the system or integrating the system. Once the system can be solved, given an initial point it is possible to determine all its future positions, a collection of points known as a trajectory or orbit.

For simple dynamical systems, knowing the trajectory is often sufficient, but most dynamical systems are too complicated to be understood in terms of individual trajectories. The difficulties arise because,

1. The systems studied may only be known approximately—the parameters of the system may not be known precisely or terms may be missing from the equations. The approximations used bring into question the validity or relevance of numerical solutions. To address these questions several notions of stability have been introduced in the study of dynamical systems, such as Lyapunov stability or structural stability. The stability of the dynamical system implies that there is a class of models or initial conditions for which the trajectories would be equivalent. The operation for comparing orbits to establish their equivalence changes with the different notions of stability.

2. The type of trajectory may be more important than one particular trajectory. Some trajectories may be periodic, whereas others may wander through many different states of the system. Applications often require enumerating these classes or maintaining the system within one class. Classifying all possible trajectories has led to the qualitative study of dynamical sys-

tems, that is, properties that do not change under coordinate changes. Linear dynamical systems and systems that have two numbers describing a state are examples of dynamical systems where the possible classes of orbits are understood.

3. The behavior of trajectories as a function of a parameter may be what is needed for an application. As a parameter is varied, the dynamical systems may have bifurcation points where the qualitative behavior of the dynamical system changes. For example, it may go from having only periodic motions to apparently erratic behavior, as in the transition to turbulence of a fluid.

4. The trajectories of the system may appear erratic, as if random. In these cases it may be necessary to compute averages using one very long trajectory or many different trajectories. The averages are well defined for ergodic systems and a more detailed understanding has been worked out for hyperbolic systems. Understanding the probabilistic aspects of dynamical systems has helped establish the foundations of statistical mechanics and of chaos.

1.2.2 Discrete time dynamical equation

Change can be modeled using the formula,

$$change \ = \ future\ value \ - \ present\ value$$

If values that we monitor changes during discrete periods (for example, in discrete time intervals),the formula above leads to a difference equation or a dynamical system. In this case, we are dealing with a function that depends on discrete integer values – a sequence. A sequence of real numbers $\{a_n\}$can be represented by a recursive equation,

$$a_{n+1} \ = \ f(a_n)$$

with some initial value a_0. This relationship between terms of a sequence is called a dynamical system. A dynamical system allows us to describe the change from one state of the system to the next. At n-th stage, the change is described by

Change at stage n = future $(n+1)^{th}$ state $-$ present n^{th} state $= a_{n+1} - a_n$.

The difference, $a_{n+1} - a_n$ is frequently denoted by Δa_n and is called a change or n-th first difference. A difference equation is an equation of the form,

$$\Delta a_n = g(a_n)$$

A solution of a difference equation is a sequence a_n The solution can be given analytically (i.e. by the formula of a_n in terms of n), graphically, or numerically (i.e. as a table of a_n values for various values of n).

1.2.3 Delay differential equation

Delay differential equations (DDEs) are a type of differential equation in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times. A general form of the time-delay differential equation for $x(t) \in \mathbb{R}^n$ is,

$$\frac{d}{dt}x(t) = f(t, x(t), x_t),$$

where $x_t = \{x(\tau) : \tau \leq t\}$ represents trajectory of the solution in the past. In this equation, f is a functional operator from $\mathbb{R} \times \mathbb{R}^n \times C^1$ to $\mathbb{R}^n$.

1.2.4 Monte Carlo Simulation

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; i.e., by running simulations many times over in order to calculate those same probabilities heuristically just like actually playing and recording your results in a real casino situation: hence the name. They are often used in physical and mathematical problems and are most suited to be applied when it is impossible to obtain a closed-form expression or infeasible to apply a deterministic algorithm. Monte Carlo methods are mainly used in three distinct problems: optimization, numerical integration and generation of samples from a probability distribution.

Monte Carlo methods are especially useful for simulating systems with many coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures (e.g: cellular Potts model). They are used to model phenomena with significant uncertainty in inputs, such as the calculation of risk in business. They are widely used in mathematics, for example to evaluate multidimensional definite integrals with complicated boundary conditions. When Monte Carlo simulations have been applied in space exploration and oil exploration, their predictions of failures, cost overruns and schedule overruns are routinely better than human intuition or alternative "soft" methods.

The modern version of the Monte Carlo method was invented in the late 1940s by Stanislaw Ulam, while he was working on nuclear weapon projects at the Los Alamos National Laboratory. It was named, by Nicholas Metropolis, after the Monte Carlo Casino, where Ulam's uncle often gambled.Immediately after Ulam's breakthrough, John von Neumann understood its importance and programmed the ENIAC computer to carry out Monte Carlo calculations. In our experiments MC comes in handy as the tool for, randomly propagating the infection.

1.2.5 Epidemiology Models

Mathematical models of epidemiology are well studied. Mathematical models can project how infectious diseases progress to show the likely outcome of an epidemic. However here we will be talking about two models whose modified forms form the basis of our study. As mentioned earlier **S** stands for the currently 'Susceptible' section of the population, **I** stands for the currently 'Infected' section of the population in our focus & **R** denotes the section of the population who are currently 'Removed' from the infectibility. The standard continuous time dynamical system of the models of interest are being briefly explained here. The reference material on them are abundant though, so an interested reader can stumble upon lot of work in this regard. Since these models partition or compartmentalize the entire population into distinct sections, these models are also called compartmental models.

Basic SIR model

In 1927, W. O. Kermack and A. G. McKendrick created a model in which they considered a fixed population with only three compartments. susceptible, infected, and recovered. The compartments used for this model consist of three classes.

(a) $S(t)$: is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease.

(b) $I(t)$: denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category.

(c) $R(t)$: is the compartment used for those individuals who have been infected and then recovered from the disease. Those in this category are not able to be infected again or to transmit the infection to others.

The basic flow of this model is as follows

$$\mathrm{S} \rightarrow \mathrm{I} \rightarrow \mathrm{R}$$

Kermack and McKendrick assumed a Fixed population i.e $N = S(t) + I(t) + R(t)$ and derived the following system.

$$\begin{cases} \dot{S} = -\beta SI \\ \dot{I} = \beta SI - \gamma I \\ \dot{R} = \gamma I \end{cases} \tag{1.2.1}$$

Assumptions involved here are:

(a) An individual in the population must be considered as having an equal probability as every other individual of contracting the disease with a rate of β, which is considered the contact or infection rate of the disease.

(b) The population leaving the susceptible class as equal to the number entering the infected class.

(c) A number equal to the fraction (γ which represents the mean recovery rate, or $\frac{1}{\gamma}$ the mean infective period) of invectives are leaving this class per unit time to enter the removed class.

(d) These processes which occur simultaneously are referred to as the Law of Mass Action (which is an widely accepted idea).

(e) The rate of infection and recovery is much faster than the time scale of births and deaths and therefore, these factors are ignored in this model.

The SIRS model

This model is an extension of the SIR model, owing to the fact that removed population seeps back into the susceptible compartment as the system evolves over the time-line. Total population is also considered constant here supposing that no births and deaths take place during the time the time evolution of the system is observed. The basic flow of this system:

$$\mathrm{S} \rightarrow \mathrm{I} \rightarrow \mathrm{R} \rightarrow \mathrm{S}$$

The dynamical system in this case would be,

$$\begin{cases} \dot{S} &= -\beta SI + \sigma R \\ \dot{I} &= \beta SI - \gamma I \\ \dot{R} &= \gamma I - \sigma R \end{cases} \tag{1.2.2}$$

We will be formulating our experiment on these two models in discrete form and then introduce the changes accommodate our assumptions. Some analysis of the system will be performed and simulation results will be furnished.

Chapter 2

Dynamics of Rumor propagation

In this chapter we will be discussing some previous works in the related domain of rumor propagation and epidemiology and then move to modeling our problem.

2.1 A Brief Literature Survey

Social and biological interactions always have been a matter of interest to the scientists. Information diffusion, Prey-Predator, Host-Pathogen, Invasion- Extinction are some of the well known fields of application. Recently there has been a increased zeal of studying the social interaction both due to availability of Computational power and observable datasets from our virtual 'on-line social networks'. The spread, mutation and transformation of information is being studied. One of the earliest known works in rumor propagation was done by Daley and Kendall that studied stochastic behavior of rumors (1965). Studies of virality of a piece of new information (Topic of diffusion and emergence of virality, Rajyalaxmi, Bagchi, Das 2012), can give use characterization of the type of promotion that may lead to the condition which may be a boon for the marketing scenario.

It is well known in physics to study the global magnetic behavior of a system through Ising model that places the magnetic spins in a packed d-dimensional lattice. Essentially specifying the local behavior and observing the global behavior that results from homogeneous as well as inhomogeneous systems asymptotically has always been a matter of interest. Ostelli, Yoneki and

Leung in their Technical report of University of Cambridge (2010), studied rumor spreading in interacting communities with the help of Ising model. Similar studies have also been done to study complex interactions using a random complex Ising model by Son, Jeong and Noh.

Tilman and Kareiva proposed that spatial dynamics of an evolving population may lead to a different inter-specific interaction (1997).Invasion and extinction in Mean field approximation for spatial host-pathogen model by guiar, Rauch, Brar-yam (2003) studies the prey predator interaction in a spatial model. Recently, Uwe C. Tauber studied the stochastic oscillations in a population in a spatial prey-predator model (2010).

2.2 Proposed Mathematical Model and Justification

Influenced with the idea of studying and visualizing the rumor propagation in a spatial scenario, existing simple SIR and SIRS models were modified to our use with the constraint of constant total population. Here, we will study discrete time dynamical systems, which uses delay to model the process of rumor spreading by individuals in a social scenario. What is quite impossible to observe in a real life scenario, could be understood easily from observing and dwelling on the social networks for sometime. Since one of our major daily activities involve on-line social networks, which for the most of the time is labeled as waste of time, this could be termed as at least an useful job that has been done while logging hours of on-line activity.

2.2.1 Basic Rumor Propagation model

This model assumes an infectivity probability τ for a susceptible individual to accept a gossip from a gossiping individual with whom it is interacting. We assume a fixed delay of T_A during which a gossiping individual is interested in spreading a rumor and he is termed as a Believer (B), after which he loses interest and shows non-activity with regards to this specific gossip – he turns into a non-believer(N). This is a justified assumption because most of the individuals involved in a social network scenario tend to have a short attention span, and once the craze is over, the reigning rumor is simply discarded. A very well known citable example in this regard is the 'Why this Kolaveri Di' music phenomenon, that went viral and plagued India specific facebook users for months in mid 2012 but by end 2012 it was nowhere to be seen anymore. Another assumption we impose on the model is that the population is homogeneous on the grid, which allows us to ignore the local spatial interaction among the entities and only the time average of the population compartments is what matters (Uwe C. Tauber, stochastic oscillations in a population in a spatial prey-predator model (2010). A modification of the (1.2.1) is proposed as the 'Basic rumor propagation' model (BRP model here onwards) as follows.

$$\begin{cases} S_{t+1} = S_t - \tau S_t B_t \\ B_{t+1} = B_t + \tau S_t B_t - B_{t-T_A} \\ N_{t+1} = N_t + B_{t-T_A} \end{cases} \tag{2.2.1}$$

2.2.2 Analysis of the BRP model

System (2.2.1) without the time delay looks like:

$$\begin{cases} S_{t+1} = S_t - \tau S_t B_t \\ B_{t+1} = B_t + \tau S_t B_t - B_t \\ N_{t+1} = N_t + B_t \end{cases} \tag{2.2.2}$$

Equilibrium Points of the system are: $E_A : (0\ ,\ 0,\ n)$ and $E_B : (s,\ 0,\ (n-s))$ where n is the total population, and $0<s \leq n$. Since the total population is constant then we can replace the occurrence of N_t in the first two difference in terms of B_t and S_t, as at any given t, $n = S_t + B_t + N_t$. Hence, we can carry the analysis on the two dimensional system of $(S_t,\ B_t)$. So reduced system will have equilibrium points $E_A^* : (0\ ,\ 0)$ and $E_B^* : (s,\ 0)$.

2.2.3 Stability of E_A and E_B

Linearizing the system in the neighborhood of E_A^*, the system reduces to

$$\begin{pmatrix} S_{t+1} \\ B_{t+1} \end{pmatrix} = J \begin{pmatrix} S_t \\ B_t \end{pmatrix} \tag{2.2.3}$$

where

$$J = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

is the Jacobian matrix for the system of equations.The Eigen Values of the Jacobian matrix are 1 and 0 respectively. Since one eigenvalue of J is 1, the equilibrium point E_A^* is non-hyperbolic i.e this method is inadequate for studying the stability.Similarly, the Jacobian matrix for the system for the E_B^* will have 1 as one of the eigenvalue hence it is also inadequate for studying by this method.

However, although we simulate the system with delay, we do not present any analysis of it for the BRP model, as the situation is quite trivial, and also there is no change of the stability situation from the system with no delays.

2.2.4 Conspiracy Theory model

This model studies the kind of rumors that have a high tendency of reappearing in a social scenario again and again. Most apt example of this situation are the conspiracy theories. This is the reason of why we talked about 'moon landing hoax', 'Osama's death', or 'Yeti's Himalayan abode' in our very introduction of the thesis. Although the basic definition of susceptible, believers and non-believers remain the same but here non-believers after a resting period of T_B rejoin the pool of susceptible candidates. This can be attributed to the fact that in a social network both pro and anti of the hoax make very convincing counter points which at the moment of exposure can easily convince a person that what he is reading is the 'gospel', however it does not take long for the person to take a complete opposite stand in a short while, this in my opinion is the beauty of a social network. The concept from (1.2.2) and the modification of (2.2.1) will give us our conspiracy theory model (denoted as CT model here onwards):

$$\begin{cases} S_{t+1} = S_t - \tau S_t B_t + N_{t-T_B} \\ B_{t+1} = B_t + \tau S_t B_t - B_{t-T_A} \\ N_{t+1} = N_t + B_{t-T_A} - N_{t-t_B} \end{cases} \tag{2.2.4}$$

2.2.5 Analysis of the CT model

The system (2.2.4) without the delay parameters will be:

$$\begin{cases} S_{t+1} = S_t - \tau S_t B_t + \sigma N_t \\ B_{t+1} = B_t + \tau S_t B_t - \gamma B_t \\ N_{t+1} = N_t + \gamma B_t - \sigma N_t \end{cases} \tag{2.2.5}$$

where $\sigma = 1$ and $\gamma = 1$. Equilibrium Points of the system are: $E_A : (n,\ 0,\ 0)$ and $E_B : (\frac{\gamma}{\tau},\ \frac{\sigma(n-(\gamma/\tau))}{\gamma+\sigma},\ \frac{\gamma(n-(\gamma/\tau))}{\gamma+\sigma})$ where n is the total population, and $0 < s \leq n$.

Invoking the constant population condition we can again reduce the dimension by one. So the Equilibrium points of the reduced system, are $E_A^* : (n,\ 0)$ and $E_B^* : (\frac{1}{\tau},\ \frac{(n-(1/\tau))}{2})$. Now linearizing about the point E_A^* we have,

$$\begin{pmatrix} S_{t+1} \\ B_{t+1} \end{pmatrix} = J \begin{pmatrix} S_t \\ B_t \end{pmatrix} \tag{2.2.6}$$

where

$$J = \begin{pmatrix} 2 & -\tau n + 1 \\ \tau n & \tau n \end{pmatrix}$$

this implies eigenvalues are,

$$\lambda = \frac{2 + \tau n \pm \sqrt{(4 - 3\tau^2 n^2)}}{2}$$

so for the condition $|\lambda| < 1$, further simplification gives us, $6\tau^2 n^2 + 4\tau n + 4 < 0$ which is impossible. Hence $|\lambda| > 1$ for both eigen values, implying the system is unstable at E_A^* hence also unstable at E_A.

Now linearizing about the point E_B^* we have,

$$\begin{pmatrix} S_{t+1} \\ B_{t+1} \end{pmatrix} = J \begin{pmatrix} S_t \\ B_t \end{pmatrix} \tag{2.2.7}$$

where

$$J = \begin{pmatrix} (5-\tau n)/2 & -1 \\ 1 & 1 \end{pmatrix}$$

So the characteristic equation of J is

$$\lambda^2 - \frac{7-\tau n}{2}\lambda + \frac{7-\tau n}{2} = 0$$

. The two roots of this equation are,

$$\lambda_+ = \frac{-b+\sqrt{b^2+4b}}{2}$$

and,

$$\lambda_- = \frac{-b-\sqrt{b^2+4b}}{2}$$

where, $b = \frac{\tau n - 7}{2}$. For stability of this point we should have $0<\lambda_+<1$, however using λ_+ in this relation yields $0<b<b+1$, which is always true. Hence, $|\lambda_+| < 1$.

Also, for the stability of we should have $-1<\lambda_-<0$. Using the the λ_- in this relation we get $b<1/2$ which is not true for sufficiently large values of n as $0<\tau<1$. Hence, $|\lambda_-|>1$. So the system has one eigenvalue less than one and another greater than one about E_B^* so it is a saddle point, hence E_B is a saddle point in the system.

2.2.6 Delay analysis of the CT model

The system (2.2.4) after, dimension reduction is Linearized about an arbitrary point: $E_G : (s,\ b)$. The linearized form is,

$$\begin{pmatrix} S_{t+1} \\ B_{t+1} \end{pmatrix} = J \begin{pmatrix} S_t \\ B_t \end{pmatrix} + K \begin{pmatrix} S_{t-T_A} \\ B_{t-T_A} \end{pmatrix} + L \begin{pmatrix} S_{t-T_B} \\ B_{t-T_B} \end{pmatrix}$$

where J= $\begin{pmatrix} 1-\tau b & -\tau s \\ \tau b & 1+\tau s \end{pmatrix}$, K = $\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$, L = $\begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}$

Let $\begin{pmatrix} S_t \\ B_t \end{pmatrix} = \begin{pmatrix} S_0 \\ B_0 \end{pmatrix} \lambda^t = A_0\lambda^t$, using this in the previous equation we get,

$$A_0\lambda^t = JA_0\lambda^t + KA_0\lambda^{t-T_A} + LA_0\lambda^{t-T_B}$$

or,

$$[\lambda I_2 = J - K\lambda^{-T_A} + L\lambda^{-T_B}]A_0 = 0$$

where I_2 is the identity matrix of order two and the equation $\lambda I_2 = J - K\lambda^{-T_A} + L\lambda^{-T_B} = 0$ is called the characteristic equation.

At the point $E_G = E_A*$, this characteristic equation yields, $\lambda^{T_B}(1 - \lambda) = 1$ and, $\lambda^{T_A}(1 - \lambda + \tau n) = 1$.

From these two equations we observe a few interesting aspects of the system, for odd T_A and T_B,

1. We cannot conclude about the nature of the system for $\tau n = 1$
2. System becomes a saddle point for $\tau n > 1$
3. System becomes stable for $\tau n < 1$

This can give us a clue, that for sufficiently large population it is quite impossible that the rumor would be controlled and die out easily since it is quite hard to satisfy the constraint $\tau n < 1$, however since generally $\tau n > 1$ leads to a saddle point, so for certain initial conditions the system may become controlled.

The situation $E_G = E_B*$, leads to a similarly complex situation but the calculations become very complex so they are not pursued further.

Chapter 3

The Computational Study

In this, chapter we elaborate on the simulation based investigation of the mathematical model we have proposed. The 'Monte Carlo simulation technique' is our key to randomizing our model so that it behaves apparently realistically as if it has a mind of it's own. In the subsequent sections we explain our basic scheme of attacking the two models, and various nitty-grittys of the process. Experimental snapshots of the time evolving Lattice will be produced and also the global asymptotic characteristics of the system will be continuously observed from the evolving lattice.

3.1 Simulation Scheme for Basic Rumor Propagation model

We assume a uniformly distributed population on an $M \times N$ 2-D lattice with each individual on one lattice location. This is modeled by a $M \times N$ matrix Mat , with Mat_{ij} position on the matrix denoting the state of the individual at the lattice point (i, j). Total population in question is MN. $Mat_{ij} = 0$ implies that the candidate is susceptible to rumor, $Mat_{ij} = -1$ means the individual no longer believes in the rumor and goes dormant with respect to this piece of information i.e they do not try to influence the susceptibles with a negative rumor. $1 \leq Mat_{ij} \leq K$ implies the candidate in question is currently spreading the rumor. Here K is the number of days, the candidate stays active before turning into a non-believer or $Mat_{ij} = -1$. A rumor infectivity parameter $0 \leq \tau < 1$ denotes the probability of acceptance of the rumor by a susceptible upon coming in contact with a currently believer individual. However, a believer has only four degrees of freedom in terms of contacting a susceptible viz. West - East - South - North. At the start of the simulation, a random location on the lattice is made infected and we observe the plane as time evolves. The Scheme is as follows:

Algorithm 1 Scheme for Basic Rumor Propagation

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set K, τ, $MaxSteps$, $step \leftarrow 1$
while $step \leq MaxSteps$ **do**
  for $i = 0$ to M **do**
    for $j = 0$ to N **do**
      if $Mat_{ij} \ == \ -1$ **then**
        $Mat_{ij} \leftarrow -1$
      else if $Mat_{ij} > 1$ **then**
        $Mat_{ij} \leftarrow Mat_{ij} - 1$
      else if $Mat_{ij} \ == \ 1$ **then**
        $Mat_{ij} \leftarrow -1$
      else
        if $1 \leq Mat_{i,j-1} \leq K$ **and** r $\in [\ 0,\ 1\ ] < \tau$ **then**
          $Mat_{i,j-1} \leftarrow K$
        end if
        if $1 \leq Mat_{i,j+1} \leq K$ **and** r $\in [\ 0,\ 1\ ] < \tau$ **then**
          $Mat_{i,j+1} \leftarrow K$
        end if
        if $1 \leq Mat_{i-1,j} \leq K$ **and** r $\in [\ 0,\ 1\ ] < \tau$ **then**
          $Mat_{i-1,j} \leftarrow K$
        end if
        if $1 \leq Mat_{i+1,j} \leq K$ **and** r $\in [\ 0,\ 1\ ] < \tau$ **then**
          $Mat_{i+1,j} \leftarrow K$
        end if
      end if
    end for
  end for
  $step \leftarrow step\ +\ 1$
end while

3.2 Simulation Scheme for Conspiracy Theory model

We assume a uniformly distributed population on an $M \times N$ 2-D lattice with each individual on one lattice location. This is modeled by a $M \times N$ matrix Mat , with Mat_{ij} position on the matrix denoting the state of the individual at the lattice point (i, j). Total population in question is MN. $Mat_{ij} = 0$ implies th candidate is susceptible to Rumor, $Mat_{ij} = -1$ means the individual no longer believes in the rumor and goes dormant with respect to this piece of information i.e they do not try to influence the susceptible s with a negative rumor. $1 \leq Mat_{ij} \leq K$ implies the candidate in question is currently spreading the rumor. Here K is the number of days, the candidate stays active before turning into a non-believer or $Mat_{ij} = -1$. A rumor infectivity parameter $0 \leq \tau < 1$ denotes the probability of acceptance of the rumor by a susceptible upon coming in contact with a currently believer individual. However, a believer has only four degrees of freedom in terms of contacting a susceptible viz. West - East - South - North. However once an individual moves from infected to Removed state, we allow a fixed amount of time S after which the candidate switches back to Susceptible state again. At the start of the simulation, a random location on the lattice is made infected and we observe the plane as time evolves. The Scheme is as follows: Here, $r \in [0, 1]$ is performed by calling a pseudo-random number generator which follows a uniform distribution.

Algorithm 2 Scheme for Conspiracy Theory model

```
set K, τ, MaxSteps, step ← 1
while step ≤ MaxSteps do
  for i = 0 to M do
    for j = 0 to N do
      if Mat_ij == −1 then
        if counter_ij > 0 then
          counter_ij ← counter_ij − 1
        else
          Mat_ij ← 0
        end if
      else if Mat_ij > 1 then
        Mat_ij ← Mat_ij − 1
      else if Mat_ij == 1 then
        Mat_ij ← −1
        counter_ij ← S
      else
        if 1 ≤ Mat_{i,j−1} ≤ K and r ∈ [ 0, 1 ] < τ then
          Mat_{i,j−1} ← K
        end if
        if 1 ≤ Mat_{i,j+1} ≤ K and r ∈ [ 0, 1 ] < τ then
          Mat_{i,j+1} ← K
        end if
        if 1 ≤ Mat_{i−1,j} ≤ K and r ∈ [ 0, 1 ] < τ then
          Mat_{i−1,j} ← K
        end if
        if 1 ≤ Mat_{i+1,j} ≤ K and r ∈ [ 0, 1 ] < τ then
          Mat_{i+1,j} ← K
        end if
      end if
    end for
  end for
  step ← step + 1
end while
```

3.3 Simulation Results of Basic Rumor Propagation model

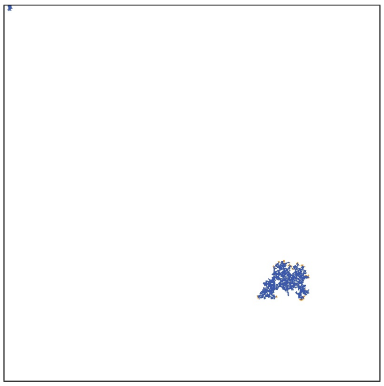

Figure 3-1: BRP model K=3, S=30 $\tau = 0.3$, step=155

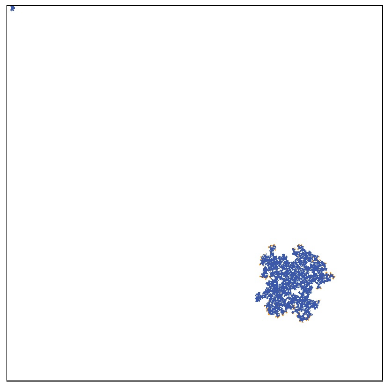

Figure 3-2: BRP model K=3, S=30 $\tau = 0.3$, step=250

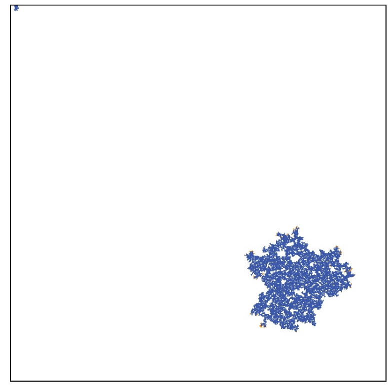

Figure 3-3: BRP model K=3, S=30 $\tau = 0.3$, step=339

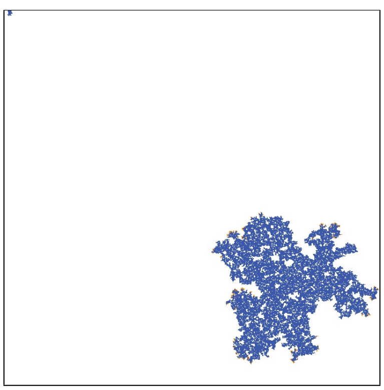

Figure 3-4: BRP model K=3, S=30 $\tau = 0.3$, step=500

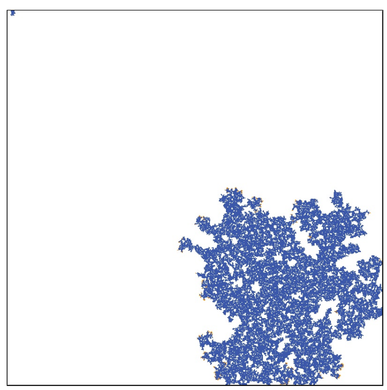

Figure 3-5: BRP model K=3, S=30 $\tau = 0.3$, step=700

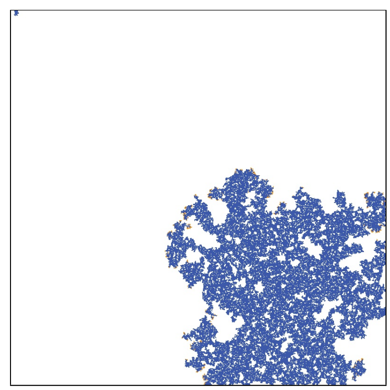

Figure 3-6: BRP model K=3, S=30 $\tau = 0.3$, step=800

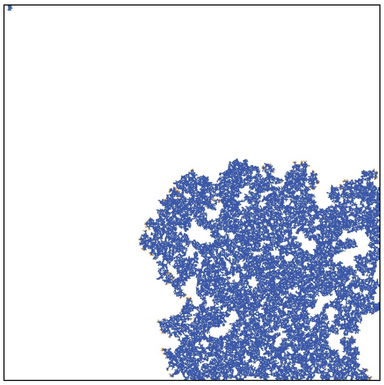

Figure 3-7: BRP model K=3, S=30 $\tau = 0.3$, step=900

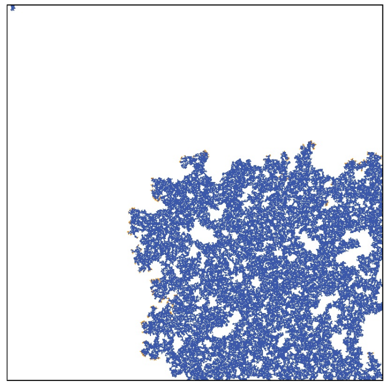

Figure 3-8: BRP model K=3, S=30 $\tau = 0.3$, step=1000

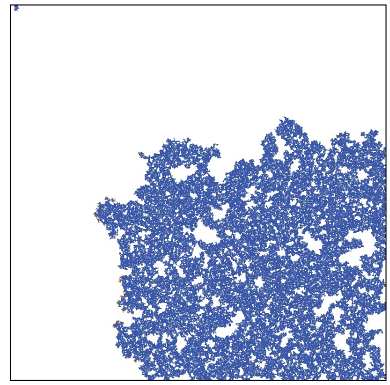

Figure 3-9: BRP model K=3, S=30 $\tau = 0.3$, step=1522

3.4 Simulation Results of The Conspiracy Theory model

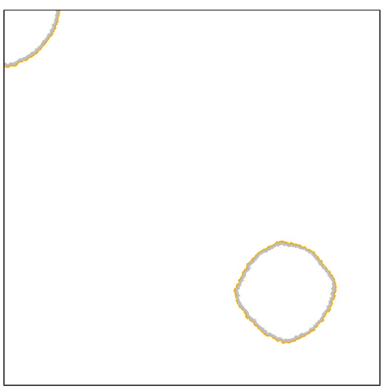

Figure 3-10: CT model K=10, S=4 $\tau = 0.3$, step=155

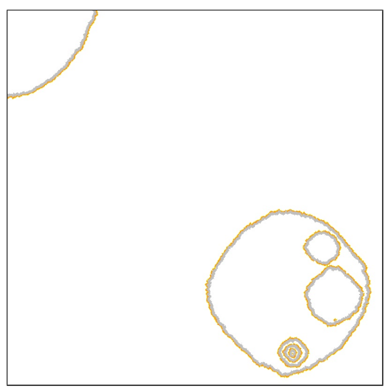

Figure 3-11: CT model K=10, S=4 $\tau = 0.3$, step=250

Figure 3-12: CT model K=10, S=4 $\tau = 0.3$, step=339

Figure 3-13: CT model K=10, S=4 $\tau = 0.3$, step=500

Figure 3-14: CT model K=10, S=4 $\tau = 0.3$, step=700

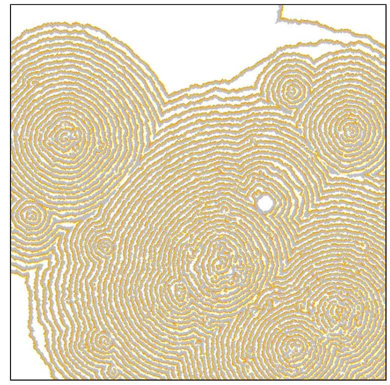

Figure 3-15: CT model K=10, S=4 $\tau = 0.3$, step=800

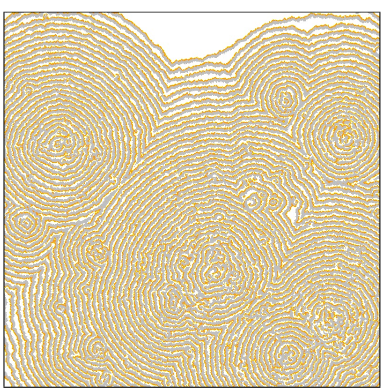

Figure 3-16: CT model K=10, S=4 $\tau = 0.3$, step=900

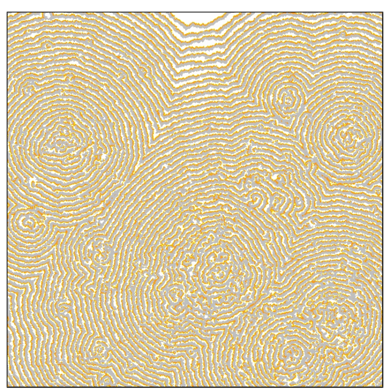

Figure 3-17: CT model K=10, S=4 $\tau = 0.3$, step=1000

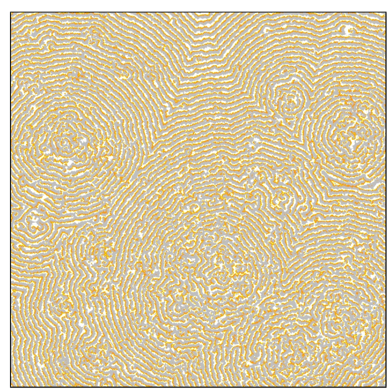

Figure 3-18: CT model K=10, S=4 $\tau = 0.3$, step=1200

Chapter 4

A Realistic model & Further work

Here we will be introducing a completely different scheme of simulation which takes the Basic SIR model, and introduces the concept of sparse population on the regular lattice, with each individual having 4-degrees of freedom of movement on the 2 dimensional plane. Effectively each individual, whether *Susceptible*, or, *Infected*, or *Removed* is allowed to perform a 2D random walk on the 2D lattice, and their coexistence and interaction leads to the evolution of the global model against time. The mathematical formalization of this model would not be provided in this document but the basic algorithm will be given. An outline of the further work which may be undertaken from here will be listed.

4.1 Sparse population - Random walk model

In both previous models the population was assumed homogeneous and immobile a person or node at location $(i\ , j\ )$ could only influence another person or node at $(k\ , h\ )$ only if a chain of influence arising from the ij^{th} node reaches kh^{th} node through any path via the intermediate nodes. However, in a realistic scenario, social interaction is not uniform and interactions are not restricted to just neighboring individuals. A node can easily interact with seemingly very distant nodes with respect to it's global scope and get influenced by it. Generally random graphs like Watts-Strogads graph is used, but the interest here was to model it on the 2D regular lattice. In this regard U.C Tauber's

2010 work on spatial prey predator model was an eye opener. So here we propose a modification for our BRP model that has the following distinct characteristics:

1. Total population in a $M \times N$ grid is $<<MN$, but remains constant during the observation.
2. The entire population is doing a 2-d random walk on the $M \times N$ grid with 4 degrees of freedom.
3. If a Believer individual meets a susceptible individual, he may turn him with a probability of τ.
4. A Believer turns into a non believer on his own with a probability σ.
5. Only one individual can stay at a lattice point at any point, i.e, stacking is not allowed.

The direct fall out of this constraints is we have a swarming lot of points, where individuals are randomly showing Brownian motion like activity on the observed grid. With the spatial immobility constraint removed points from far horizons can now easily interact given a sufficiently long iteration time. However in this report only the simulation scheme and some interesting simulation snapshots for our Sparse population- Random walk model (or SPRW model as is being mentioned) is presented. The detailed mathematical analysis is not undertaken as it seemed out of scope. As further work, stochastic formulation and subsequent mean field approximation theories can be applied to this model, more over delays can be implemented and also the this scheme could be extended to include the CT model. There is also a scope of introducing stacking effect, i.e allowing more than one entity to reside at a specific lattice grid point.

4.1.1 Simulation scheme for SPRW model

The meaning of the symbols are the usual from the BRP and CT model only σ, $InitSusSeed$ and $InitInfSeed$ are newly introduced. Here each node can be in 3 of the states. σ is the probability of moving from ij^{th} location to any of the four N-S-E-W locations.$InitSusSeed$ are total initial susceptibles and $InitInfSeed$ are initial Believers.

State 1: A Un-suspecting susceptible individual on a clandestine 2d walk on the $M \times N$ grid.

State 2: A Believer on a clandestine 2d walk on the $M \times N$ grid.

State 3: A Non-Believer on a clandestine 2d walk on the $M \times N$ grid.

Algorithm 3 Scheme for Sparse Population Random walk model

```
set τ, σ, MaxSteps, step
set InitSusSeed, InitInfSeed
Randomly seed M with InitSusSeed susceptibles and InitInfSeed believers
Set step ← 1
while step ≤ MaxSteps do
  {Infectivity cycle Update}
  {Movement cycle Update}
end while
```

Algorithm 4 Infectivity Cycle Update

```
τ, Mat
for i = 0 to M do
  for j = 0 to N do
    if Mat_{i,j} == 1 then
      if Mat_{i,j−1} == 2 and r ∈ [ 0, 1 ] < τ then
        Mat_{i,j} ← 2
      end if
      if Mat_{i,j+1} == 2 and r ∈ [ 0, 1 ] < τ then
        Mat_{i,j} ← 2
      end if
      if Mat_{i−1,j} == 2 and r ∈ [ 0, 1 ] < τ then
        Mat_{i,j} ← 2
      end if
      if Mat_{i+1,j} == 2 and r ∈ [ 0, 1 ] < τ then
        Mat_{i,j} ← 2
      end if
    end if
  end for
end for
```

Algorithm 5 Movement Cycle update

for $i = 0$ to M **do**
  for $j = 0$ to N **do**
    if $Mat_{i,j}\ ==\ 0$ **then**
      Do Nothing
    else
      if $Mat_{i,j+1}$ *is empty* **and** r $\in$ [0, 1] $< \sigma$ **then**
        $Mat_{i,j+1} \leftarrow Mat_{i,j}$
        $Mat_{i,j} \leftarrow 0$
        Continue to next iteration
      end if
      if $Mat_{i,j-1}$ *is empty* **and** r $\in$ [0, 1] $< \sigma$ **then**
        $Mat_{i,j-1} \leftarrow Mat_{i,j}$
        $Mat_{i,j} \leftarrow 0$
        Continue to next iteration
      end if
      if $Mat_{i-1,j}$ *is empty* **and** r $\in$ [0, 1] $< \sigma$ **then**
        $Mat_{i-1,j} \leftarrow Mat_{i,j}$
        $Mat_{i,j} \leftarrow 0$
        Continue to next iteration
      end if
      if $Mat_{i+1,j}$ *is empty* **and** r $\in$ [0, 1] $< \sigma$ **then**
        $Mat_{i+1,j} \leftarrow Mat_{i,j}$
        $Mat_{i,j} \leftarrow 0$
        Continue to next iteration
      end if
    end if
  end for
end for

4.2 Simulation Results of Basic Rumor Propagation model

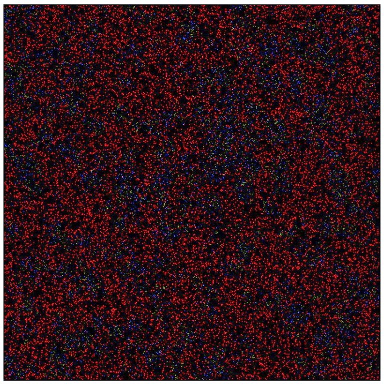

Figure 4-1: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=155

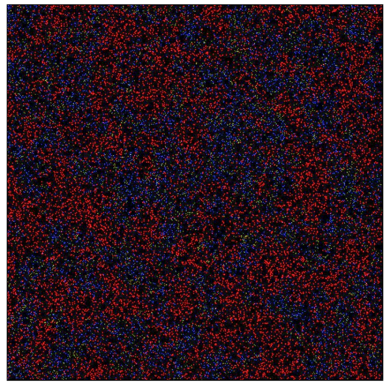

Figure 4-2: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=250

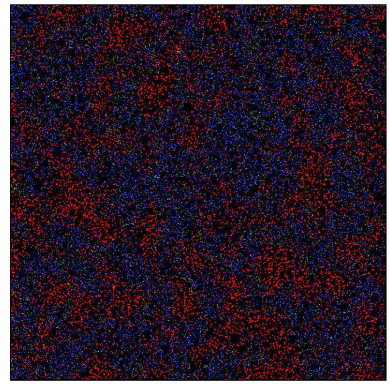

Figure 4-3: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=339

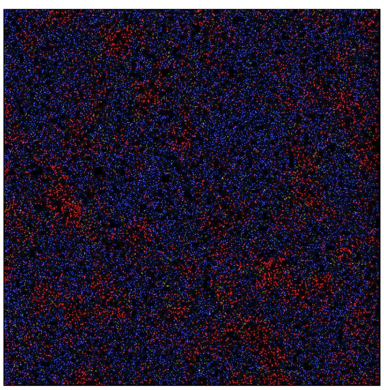

Figure 4-4: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=500

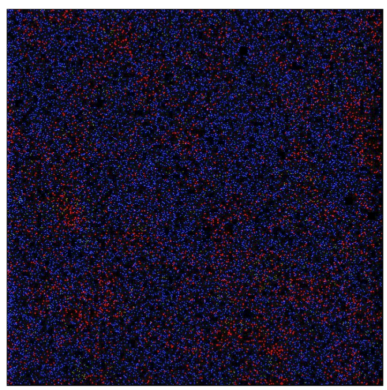

Figure 4-5: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=700

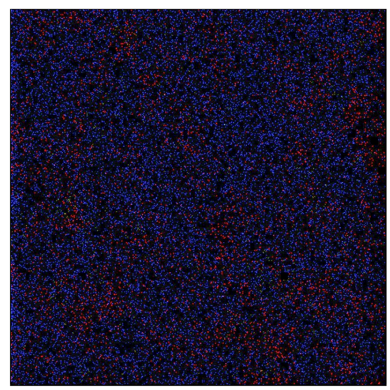

Figure 4-6: SPRW model $\tau = 0.6, \mu = 0.3, \sigma = 0.1$, step=800

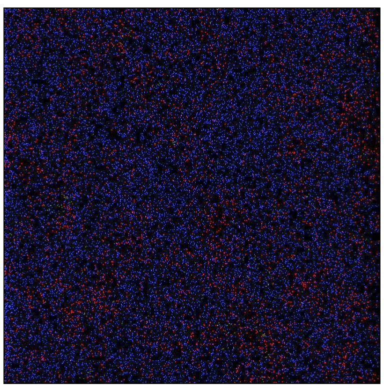

Figure 4-7: SPRW model $\tau = 0.6$,$\mu = 0.3$,$\sigma = 0.1$, step=900

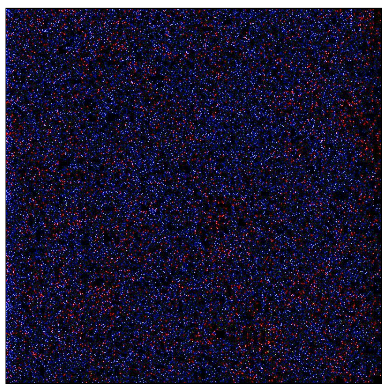

Figure 4-8: SPRW model $\tau = 0.6$,$\mu = 0.3$,$\sigma = 0.1$, step=1000

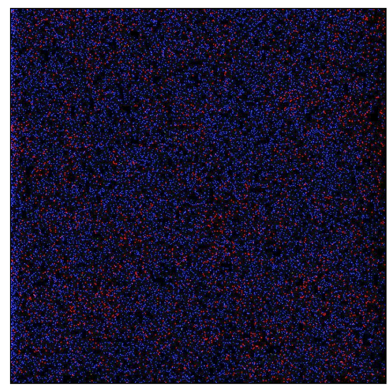

Figure 4-9: SPRW model $\tau = 0.6$,$\mu = 0.3$,$\sigma = 0.1$, step=1177

The References

The Preliminaries section about Dynamical Systems, Monte Carlo Simulation and Epidemiological models have been taken from the common web resource: Wikipedia.org. Hearty acknowledgements for the same. I am also thankful to the numerous Lecture notes that were available in the related topics, which aided in the process of my dissertation work.

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