EXPECTED VALUES OF MEAN SQUARES IN THE ANALYSIS OF INCOMPLETE BLOCK EXPERIMENTS AND SOME COMMENTS BASED ON THEM

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SUMMARY. Reference has been made to the logical status of Fisher's null hypothesis that all varieties under test have the same yield on any experimental plot. The types of departures from the null hypothesis, which the ratio of mean squares of varieties to error can detect with a reasonable chance, have been examined in the case of general incomplete block designs. It appears that the test ignores differences in varieties which are, in some sense, attributable to interaction between blocks and treatments. A study of the consequences of non-random allocation of subsets of varieties to blocks leads to a special property of the BIBD and some PBIBD designs. The effect of random indexing of varieties, i.e., of associating the given varieties with the symbols in which a design is represented, is also considered.

1. Introduction

In earlier papers (Rao, 1947, 1956) on general methods of analysis for incomplete block designs, the author has shown how combined intra and inter-block estimates and the expressions for their variances and covariances can be obtained from the theory of least squares under the hypothesis that *treatment* and *plot* effects are additive. The present paper is intended to clarify some of the points not fully elaborated in the earlier papers. Further, accepting Fisher's null hypothesis that an observed yield of a variety on a particular plot is purely a plot effect independent of the variety,[1] the types of departures from the null hypothesis which the analysis of variance test can detect have been examined. The latter is done by comparing the expected values of the mean squares for varieties and error in the analysis of variance under a general hypothesis that on each plot the varieties have possibly different yields, and plot × treatment and block × treatment interactions exist. The first attempt in this direction was due to Neyman (1935), who obtained the expectations for Randomized block and Latin square designs. Recently Wilk (1955), Wilk and Kempthorne (1957), and others have considered the two cases treated by Neyman under a more general set up.

The null hypothesis is sometimes stated as the equality of varieties with respect to the total yields over all plots of the experimental area *although* plot × treatment and block × treatment interactions may exist (Neyman, 1935). Some concern is expressed when it is found that under these conditions the expected mean square for varieties is smaller than that for error implying that the analysis of variance ratio test is not unbiased and probably insensitive.

It may be noted that when the total yields of some varieties over a given area are all equal, there must exist portions of the area over which they must differ if interactions exist. The validity of the null hypothesis that the total yields are the

[1] In this paper no distinction is made between variety and the more general term treatment. They have been used synonymously.

same over an experimental area then depends largely on what particular area has been chosen or was available for the experiment. Such a null hypothesis has, therefore, not the same logical status as Fisher's null hypothesis. We may now raise the question as to what type of departures from Fisher's null hypothesis are detectable by the variance ratio test of mean square for varieties to that for error. It turns out, in the cases examined before such as Randomized block and Latin square designs as well as for other designs considered in the present paper, the variance ratio test has only a small chance of detecting overall differences equal to or smaller in magnitude than the block×treatment interactions. Differences comparatively larger than the interactions have, however, a reasonable chance of detection. It is, perhaps, a desirable property of the test that it should ignore overall differences of the order attributable, in some sense, to the presence of interactions. The object of the experiment may not be to examine whether differences exist over the experimental area used, but to look for evidence whether the results of the experiment would justify an investigation on a large scale, over a wider area. The variance ratio test seems best suited for this purpose. Indeed, if the experimental area itself is chosen at random from a wider area the ratio of variances provides an unbiased test for examining the differences in varieties over the wider area.

Section 2 of this paper is devoted to a brief restatement of some of the results of the earlier paper (Rao, 1947) to clarify some of the statements made earlier and to explain the new notations used in the present communication.

2. RANDOMIZATION ANALYSIS OF INCOMPLETE BLOCK DESIGNS UNDER AN ADDITIVE MODEL

Let us consider an incomplete block design involving v varieties arranged in b subsets of k varieties each, such that every variety is used r times and any pair of varieties g and h occurs in λ_{gh} subsets. The actual layout of the experiment in b blocks of k plots is determined by the following randomization procedures.

R_1 : The subsets of varieties are assigned to the blocks at random.

R_2 : Within each block, the varieties of a subset are assigned to the plots at random.

The null hypothesis specifies that all varieties give the same yield on each plot of the experimental area. We may, however, write the yield of the g-th variety on the j-th plot of the i-th block as

$$\tau_g + x_{ij}, \qquad g = 1, \ldots, v \qquad \ldots \quad (2.1)$$

where the parameter τ_g is specific for the g-th variety and x_{ij} is independent of the treatment and may be considered as a plot effect. With the specification (2.1) the null hypothesis under test is

$$H_0: \ \tau_1 = \tau_2 = \ldots = \tau_v.$$

Let us define, with the usual notations for averages,

$$\sigma_p^2 = \frac{\Sigma\Sigma(x_{ij}-\bar{x}_{i.})^2}{b(k-1)}, \quad \sigma_b^2 = \frac{\Sigma(\bar{x}_{i.}-\bar{x}_{..})^2}{b-1}$$

so that σ_p^2 is the inherent average variation between plots within blocks and σ_b^2 is the variation between blocks. Consider a particular subset of varieties, say $\tau_1, \ldots, \tau_k$ without loss of generality, and represent the corresponding observed yields by $x^1, \ldots, x^k$. Then it is easy to see that under the randomization procedures R_1 and R_2,

$$E(x^i) = \tau_i + \bar{x}_{..}$$

$$V(x^i) = \frac{k-1}{k}\sigma_p^2 + \frac{b-1}{b}\sigma_b^2$$

$$\text{cov}\,(x^i x^j) = -\frac{1}{k}\sigma_p^2 + \frac{b-1}{b}\sigma_b^2$$

and there exists an orthogonal transformation

$$B/\sqrt{k} = (x^1 + \ldots + x^k)/\sqrt{k} \qquad \ldots \quad (2.2)$$

$$y_i = b_{i1}x^1 + \ldots + b_{ik}x^k, \qquad \ldots \quad (2.3)$$
$$i = 1, \ldots, k-1$$

such that the new variables are all uncorrelated and

$$V(B/\sqrt{k}) = k(b-1)\sigma_b^2/b$$
$$V(y_i) = \sigma_p^2, \quad i = 1, \ldots, k-1$$
$$E(y_i) = b_{i1}\tau_1 + \ldots + b_{ik}\tau_k.$$

An experiment with b blocks provides $b(k-1)$ observations of the type (2.3), which are all uncorrelated, have the same variance σ_p^2 and have as their expectations linear functions of the unknown parameters τ. Hence the theory of least squares can be used for obtaining the best linear estimates of treatment differences $\tau_i - \tau_j$, expressions for variances of estimates and the analysis of variance for testing any set of linear hypotheses. This supplies the theory of intra-block analysis.

If, in addition, we make an orthogonal transformation of the block totals (divided by $\sqrt{k}$)

$$z_0 = (B_1 + \ldots + B_b)/\sqrt{kb}$$

$$z_i = (c_{i1}B_1 + \ldots + c_{ib}B_b)/\sqrt{k} \qquad \ldots \quad (2.4)$$
$$i = 1, \ldots, b-1$$

we find that all z are uncorrelated, $V(z_i) = k\sigma_b^2$, $i = 1, \ldots, b-1$, and $E(z_i)$ is a linear function of the varietal effects. The $(b-1)$ observations (2.4), each with variance

$k\sigma_b^2$ together with the $b(k-1)$ observations (2.3), each with variance σ_p^2 yield combined intra and inter-block estimates by the use of weighted least square theory. The expressions for variances etc., involving the reciprocal of the variances

$$w = 1/\sigma_p^2 \quad \text{and} \quad w' = 1/k\sigma_b^2 \qquad \ldots \quad (2.5)$$

are given in the author's earlier papers (Rao, 1947, 1956) where a different interpretation was given to σ_b^2.

One useful result of the earlier papers was the derivation of the formulae for varietal differences, and variances and covariances in such a way that the same expressions can be used both for intra-block analysis and combined intra and inter block analysis by substituting appropriate values for the parameters.

To obtain estimates of σ_p^2 and σ_b^2, we use the expectations of mean squares in the analysis of variance of total sum of squares into blocks, varieties and error. Table 1 contains the relevant expectations.

TABLE 1. EXPECTATIONS OF MEAN SQUARES ASSUMING THE SPECIFICATION (2.1)

due to	d.f.	s.s.	expected mean square
blocks (ignoring varieties)	$b-1$	S_b	$k\sigma_b^2 + \frac{1}{b-1}\sum_{i<j}\left(\frac{r}{v}-\frac{\lambda_{ij}}{k}\right)(\tau_i-\tau_j)^2$
varieties (eliminating blocks)	$v-1-c$	$S_{v.b}$	$\sigma_p^2 + \frac{1}{k(v-1-c)}\sum_{i<j}\lambda_{ij}(\tau_i-\tau_j)^2$
error	g	S_p	σ_p^2
varieties (ignoring blocks)	$v-1$	S_v	$\frac{v(k-1)}{v-1}\sigma_p^2 + \frac{v(r-1)}{v-1}\sigma_b^2 + \frac{r}{v(v-1)}\sum_{i<j}(\tau_i-\tau_j)^2$
blocks (eliminating varieties)	$b-1$	$S_{b.v}$	$\frac{r-k(c+1)}{k(b-1)}\sigma_p^2 + \frac{v(r-1)}{k(b-1)}\sigma_b^2$

Note : $g = bk-b-v+1+c$, c = degrees of freedom confounded, which is $(v-1)$ minus the number of independent varietal contrasts estimable from intra-block analysis. The value of c is zero when all varietal differences are estimable as in any connected design for varietal trials. The relationship $S_b+S_{v.b} = S_v+S_{b.v}$ could be utilized in computing the sum of squares $S_{v.b}$ (or $S_{b.v}$) after obtaining directly $S_{b.v}$ (or $S_{v.b}$).

The estimates of σ_p^2 and σ_b^2 are obtained using the mean squares whose expectations are free from varietal differences.

$$\hat{\sigma}_p^2 = S_p \div g$$
$$\hat{\sigma}_b^2 = [kS_{b.v}-(v-kc-k)S_p/g]\div v(r-1).$$

For some designs, known as resolvable designs, it is possible to arrange the subsets into groups forming complete replications. The subsets forming a complete replication are assigned to contiguous blocks so that the variation between replications could be removed from the block differences. Defining σ_r^2 as the variance between replications the expectations given in Table 2 are obtained.

TABLE 2. EXPECTATIONS OF MEAN SQUARES IN THE FURTHER ANALYSIS OF BLOCK VARIATION, ASSUMING THE SPECIFICATION (2.1)

due to	d.f.	s.s.	expected mean square
replications (ignoring blocks)	$r-1$	S_r	$v\sigma_r^2$
blocks (within replications)	$b-r$	$S_{b\cdot r}$	$\frac{v-k(c+1)}{k(b-r)}\sigma_p^2+\frac{(v-k)(r-1)}{b-r}\sigma_b^2$
blocks (eliminating varieties)	$b-1$	$S_{b\cdot v}$	—

In such a case, the estimate of σ_b^2 is

$$\hat{\sigma}_b^2 = [S_{b\cdot r}-(v-kc-k)S_p/gk] \div (v-k)(r-1).$$

For further details the reader is referred to Rao (1947).

3. Expectations of mean squares when R_1 is not followed

We shall consider the situation when the subsets of varieties are not randomly assigned to the blocks but only the varieties within a block are randomized, i.e., only the procedure R_2 of Section 2 is followed. In such a case only intra-block estimation is possible. Considering the set up (2.1) let

$$\sigma_{pi}^2 = \sum_j (x_{ij}-\bar{x}_{i.})^2 \div (k-1), \qquad i = 1, \ldots, b \qquad \ldots \quad (3.1)$$

be the variation between plots within the i-th block. The transformed variables $y_1, \ldots, y_{k-1}$, considered in (2.3) arising out of the subset assigned to the i-th block have the variance σ_{pi}^2. Since σ_{pi}^2 is unknown and cannot be estimated unless some varieties are repeated more than once in each block, it is not possible to adopt the procedure of weighted least squares (of weighting the variables of each block by the reciprocal of the corresponding intra-block variance). Let us, therefore, examine the consequence of ignoring the differences in σ_{pi}^2 in estimation and tests of significance of varietal differences.

Let $v_{ij}\,\sigma_p^2$ denote the variance of $(\hat{\tau}_i-\hat{\tau}_j)$, the intra-block estimate of the difference between the i-th and j-th varieties, assuming a common variance σ_p^2 for all the blocks. The expressions v_{ij} for various types of standard designs discussed in literature are known. Let δ_s be the sum of v_{ij} for all possible pairs i and j of varieties included in the s-th block. Thus, if the s-th block has three varieties designated by 1, 4, 5 then $\delta_s = v_{14}+v_{15}+v_{45}$. The expectations of mean squares for varieties and error in the intra-block analysis of variance are given in Table 3.

TABLE 3. EXPECTATIONS OF MEAN SQUARES ASSUMING THE SPECIFICATION (2.1) WHEN R_1 IS NOT SATISFIED

due to	d.f.	s.s.	expected mean square
blocks (ignoring varieties)	$b-1$	S_b	—
varieties (eliminating blocks)	$v-1$	$S_{v\cdot b}$	$\frac{1}{k(v-1)}(\delta_1\ \sigma^2_{p1}+\ldots+\delta^b\ \sigma^2_{pb})+\frac{1}{k(v-1)}\sum_{j<i}\lambda_{ij}(\tau_i-\tau_j)^2$
error	g	S_p	$\frac{b(k-1)}{g}\sigma^2_p-\frac{1}{kg}(\delta_1\ \sigma^2_{p1}+\ldots+\delta_b\ \sigma^2_{pb})$

$$\sigma^2_p = (\sigma^2_{p1}+\ldots+\sigma^2_{pb}) \div b$$

From Table 3, we find that when $\tau_1 = \cdots = \tau_v$, the expected mean squares for varieties and error are not, in general, the same. By equating the coefficients of σ^2_{pi} in the two expressions, we obtain

$$\frac{\delta_i}{k(v-1)} = \frac{k-1}{g} - \frac{\delta_i}{kg} \text{ or } \delta_i = \frac{k(v-1)}{b}$$

so that the necessary and sufficient condition for the equality of expectations for error and varieties is that δ_i are all equal, i.e., the sum of variances of all comparisons within any subset of varieties assigned to a block should be the same, the variances being computed, in the usual way, under the assumption of no difference in intra-block variances.

It is easily seen that this condition is satisfied for the BIBD and PBIBD of the two-associate type with the special values $\lambda_1 = 1, \lambda_2 = 0$ such as the quasi-factorial. It may be of some interest to characterize an experimental design by the presence or absence of this property. Even if this condition is satisfied, there is the additional difficulty of estimating the exact variance of the estimated difference between two given varieties. The exact variance in such a case is a linear compound of the intra-block variances, which, in general, is not a constant multiple of the expected mean square for error except in the case of complete randomized blocks.

4. Expected mean squares under a non-additive model for a BIBD

In the general case of a non-additive model the following notations and definitions are used.

(i) x^a_{ij} = yield of the a-th variety in the j-th plot of the i-th block

(ii) $x^a_{i.}$, $x^a_{..}$, $\bar{x}^a_{i.}$, $\bar{x}^a_{..}$ represent sums and averages over the suffixes replaced by dots

(iii) Variance between plots within blocks

$$b(k-1)\sigma^2_p(a) = \Sigma\Sigma\,(x^a_{ij}-\bar{x}^a_{i.})^2, \quad a = 1, \ldots, v$$

(iv) Variance between block-means

$$(b-1)\sigma^2_b(a) = \Sigma\,(\bar{x}^a_{i.}-\bar{x}^a_{..})^2, \quad a = 1, \ldots, v$$

(v) Interaction variances, plot × treatment, within blocks

$$b(k-1)\, i_p^2(a, c) = \tfrac{1}{k} \Sigma\, \Sigma \{x_{ij}^a - x_{ij}^c - (\bar{x}_{i.}^a - \bar{x}_{i.}^c)\}^2 \quad a, c = 1, \ldots, v$$

(vi) Interaction variances, treatment × block, based on mean values of blocks

$$(b-1)\, i_b^2(a, c) = \tfrac{1}{k} \Sigma \{\bar{x}_{i.}^a - \bar{x}_{i.}^c - (\bar{x}_{..}^a - \bar{x}_{..}^c)\}^2 \quad a, c = 1, \ldots, v$$

(vii) Average variances summed over the varieties

$$\sigma_p^2 = \sum_a \sigma_p^2(a) \div v, \; \sigma_b^2 = \sum_a \sigma_b^2(a, a) \div v$$

$$i_p^2 = \sum_a \sum_c i_p^2(a, c) \div v(v-1), \; i_b^2 = \sum_a \sum_c i_b^2(a, c) \div v(v-1)$$

The expected mean squares in the case of BIBD are given in Table 4.

TABLE 4. EXPECTATIONS OF MEAN SQUARES IN THE ANALYSIS OF A BIBD (NON-ADDITIVE MODEL)

due to	d.f.	s.s.	expected mean square
blocks (eliminating varieties)	$b-1$	$S_{b.v}$	$\frac{v-k}{k(b-1)}\sigma_p^2 + \frac{g}{k(b-1)} i_p^2 - \frac{g}{b-1} i_b^2 + \frac{v(r-1)}{b-1}\sigma_b^2$
varieties (eliminating blocks)	$v-1$	$S_{v.b}$	$\sigma_p^2 - \frac{1}{k} i_p^2 + \frac{v-k}{v-1} i_b^2 + \frac{\lambda v}{k(v-1)} \Sigma(\tau_a - \bar{\tau})^2$
error	g	S_p	$\sigma_p^2 - \frac{1}{k} i_p^2 + i_b^2$

The expectations in Table 4 under the general set up enable us to examine the nature of the differences in the varieties which the analysis of variance test can detect. Under the null hypothesis, that all varieties have the same yield on each plot of the experimental area, the expectations of the mean square for error and varieties are the same as shown in Table 1. If this null hypothesis is not true then the expected mean square for varieties exceeds that for error only when

$$\frac{v-k}{v-1} i_b^2 + \frac{\lambda v}{k(v-1)} \Sigma(\tau_a - \bar{\tau})^2 > i_b^2$$

or

$$\sigma_\tau^2 - \frac{1}{b} i_b^2 > 0 \qquad \ldots \quad (4.1)$$

where $\sigma_\tau^2 = \Sigma(\tau_a - \bar{\tau})^2/(v-1)$, the variance between varietal effects. The relationship (4.1) shows that the analysis of variance test has a reasonable chance of detecting departures from the null hypothesis only when the overall differences in yields exceed a certain magnitude depending on the block × treatment interaction.

For examining the hypothesis $\sigma_\tau^2 = 0$, although i_p^2 and i_b^2 may not be zero, the analysis of variance test is somewhat conservative as in the case of Randomized block and Latin square designs (Neyman, 1935); the position is, however, slightly better for a BIBD. The difference between the expected values of mean squares for varieties and error in this case is $\{(k-1)/(v-1)\}i_b^2$ which is small when k is small compared to v. The corresponding difference for randomized blocks is i_b^2 apart from some difference in the magnitude of i_b^2 itself due to increased block size.

5. Expected mean squares in the case of a general incomplete block design

For a BIBD it is seen that under the hypothesis $\sigma_r = 0$, the expectations of mean squares for varieties and error agree upto terms containing intra-block variances and plot×treatment interactions. But this may not be true for a general incomplete block design. Let us consider a block containing variety a with $(k-1)$ others denoted by $1, 2, ..., k-1$ without loss of generality, and define $\sigma_p^2 V_a$ as the variance of the least square intra-block estimate of the contrast

$$(k-1)\tau_a-\tau_1-\cdots-\tau_{k-1}$$

under the additive model (2.1) where σ_p^2 is the intra-block error. For any standard design for which intra-block estimates are provided, the value of V_a can be obtained directly by first estimating the contrast and computing its variance. Or, if $\sigma_p^2 v_{ij}$ stands for the variance of the intra-block estimate of $(\tau_i-\tau_j)$, then

$$V_a = \Sigma\ k\, v_{ai}-\Sigma\, \Sigma v_{ij} \quad \ldots \quad (5.1)$$

where in the summations i and j vary over $a, 1, ..., k-1$.

Let $\Sigma_i V_a$ = sum of V_a for blocks in which the varieties i and a occur and

$$\xi_a = \frac{1}{k^2}\Sigma_a V_a$$

$$2\xi_{ac} = \frac{1}{k^3}\,(\Sigma_a V_c+\Sigma_c V_a-k^2 v_{ac}\lambda_{ac}). \quad \ldots \quad (5.2)$$

The expected value of the sum of squares due to varieties eliminating blocks is

$$\Sigma\, \xi_a\sigma_p^2(a)+\Sigma\, \Sigma\, \xi_{ac}i_p^2(a,c)-\Sigma\Sigma\left[\frac{\lambda_{ac}}{bk}+k\xi_{ac}\right]i_b^2(a,c)+\frac{1}{k}\sum_{i<j}\lambda_{ac}(\tau_a-\tau_c)^2 \quad \ldots \quad (5.3)$$

and that due to error is

$$\Sigma\left[\frac{r(k-1)}{k}-\xi_a\right]\sigma_p^2(a)-\Sigma\Sigma\left[\xi_{ac}+\frac{\lambda_{ac}}{k^2}\right]i_p^2(a,c)+\Sigma\Sigma\left[\frac{\lambda_{ac}}{k}+k\xi_{ac}\right]i_b^2\,(a,c) \quad \ldots \quad (5.4)$$

The condition for terms involving $\sigma_p(a)$ to be equal in the two expectations (5.3) and (5.4) is

$$\xi_a = \frac{v-1}{v}\ \text{(independent of } a\text{)}. \quad \ldots \quad (5.5)$$

This is true for BIBD, PBIBD of the quasi-factorial type, and linked block designs (LBD), though not for a general incomplete block design. Similarly for the terms involving plot×treatment variances to be equal

$$\xi_{se} = -\frac{(v-1)\lambda_{se}}{b(k-1)k^2} \qquad \dots \quad (5.6)$$

which is true for BIBD and PBIBD of the quasi-factorial type and not LBD. The condition for terms involving block × treatment variances to be equal is

$$\xi_{se} = -\lambda_{se}\frac{(v-1)(vr-v+1)}{kb^2(k-1)}. \qquad \dots \quad (5.7)$$

It may be examined whether this relation can ever be true and if true, what should be the nature of the design.

The actual expressions for LBD are computed to show the disagreement in the plot×treatment interactions terms. The expected sum of squares for varieties is, apart from the term involving varietal differences,

$$\frac{v-1}{v}\Sigma\sigma_p^2(a) - \frac{1}{bk}\Sigma\Sigma\lambda_{se}\left(\frac{1}{k}+\frac{r-\lambda_{se}}{r\mu}\right)i_p^2(a,c) + \frac{1}{br\mu}\Sigma\Sigma\lambda_{se}(r-\lambda_{se})i_b^2(a,c) \quad \dots \quad (5.8)$$

and that for error is

$$\frac{g}{v}\Sigma\sigma_p^2(a) - \frac{1}{bk}\Sigma\Sigma\lambda_{se}\left(\frac{b-1}{k}-\frac{r-\lambda_{se}}{r\mu}\right)i_p^2(a,c) + \frac{1}{b}\Sigma\Sigma\lambda_{se}\left(\frac{b-1}{k}-\frac{r-\lambda_{se}}{r\mu}\right)i_b^2(a,c) \dots \quad (5.9)$$

where μ is the number of varieties common to any two blocks in a linked block design.

For the coefficients of $i_p^2(a, c)$ in (5.8) and (5.9) to be the same, λ_{se} should be constant, in which case the design is a BIBD. How far the disagreement in the interaction terms can be considered as a drawback of the LB design remains to be examined.

As observed earlier there exist designs for which even the terms involving $\sigma_p^2(a)$ do not agree and it may be of some interest to obtain a classification of the designs with respect to the terms in which the expectations of mean squares for varieties and error agree.

6. Random indexing of varieties

In complete randomized block, Latin square and BIBD designs, the association scheme for the actual varieties is independent of the correspondence set up between the varieties and the symbols in which a design is represented. Thus, in a randomly chosen Latin square of order 4 using the symbols A, B, C, D it does not matter which of the four varieties is made to correspond with A, which with B and so on. But in a design like the quasi-factorial obtained by choosing, as blocks, the rows and columns of a square with s^2 symbols written in the s^2 cells there is the further problem of assigning the varieties to symbols. In this design differences of varieties chosen to correspond with symbols occurring in the same row or column are estimable with a higher precision than those not in the same row or column. So, if certain comparisons are deemed to be more important than others it may be possible to determine a correspondence which allows the estimation of these comparisons with a higher precision. If no such distinction could be made among the various possible comparisons then we may follow the procedure R_3 stated below.

R_3 : Obtain the correspondence between the varieties and the symbols in which a design is represented by randomly permuting the symbols over the varieties.

It is easy to verify that whatever may be the design used, with respect to the reference set generated by the randomization procedures R_1, R_2 of Section 2 and R_3 stated above, the following are true.

(i) The variance of the estimated difference between any two varieties is a constant independent of the varieties chosen.

(ii) The expected mean squares for varieties and error are same as those obtained for a BIBD (Table 4).

However, it is not suggested that the procedure R_3 justifies attaching the same precision to all estimated differences from the results of an experiment when the design is not balanced.

Note: If each observation in an experiment is subject to an additional independent random error (known as technical error) with variance σ_e^2, the expectations of mean squares in all the Tables of this paper will have the additional term σ_e^2 with coefficient unity. If σ_e^2 is large, the estimation of variance of block totals presents some difficulty. Some aspects of this will be considered in a subsequent publication.

References

Neyman, J., Iwaskiewicz, K. (1935): Statistical problems in agricultural experimentation. *Suppl. J. Roy. Stat. Soc.*, 2, 107.

Rao, C. R. (1947): General methods of analysis for incomplete block designs. *J. Amer. Stat. Ass.*, 42, 541.

——— (1956): On the recovery of inter-block information in varietal trials. *Sankhyā*, 17, 105.

Wilk, M. B. (1955): The randomization analysis of a generalised randomized block design. *Biometrika*, 42, 70.

Wilk, M. B. and Kempthorne, O. (1957): Non-additivities in a Latin square design. *J. Amer. Stat. Ass.*, 52, 218.

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