THE NEW ERA AND WHAT IT MEANS

I - Introductory Remarks

The fact that we are here is mute evidence of our interest in knowing what statistical method has in store for each of us. We want to know how it can help us to do our job better. Ours is a utilitarian interest. All of us as engineers have heard much about statistical methods and their application in education, sociology, economics, etc. However, we have been inclined to stand aloof and say: "Well, these methods may be all right for the fellow who deals with such an inexact science as education or economics, but, thank goodness, we do not have to depend upon them because we are dealing with the application of exact sciences, such as physics and chemistry".

Then we read in one of the world's leading engineering journals - Engineering, 1927:

> "To-day the mathematical physicist seems more and more inclined to the opinion that each of the so-called laws of nature is essentially statistical, and that all our equations and theories can do, is to provide us with a series of orbits of varying probabilities."

Hearkening, we hear distinct rumblings of a revolution in the camp of the "exact" sciences. The concept of exact is overthrown for the moment at least and in its place statistical concepts hold sway.

Thus, writing in 1928 in his book, "Quantum Theory and Modern Physics", Professor Haas says:

> "A question which continually claims the center of philosophical interest is that of whether there is in nature any other kind of regularity than the purely statistical which on account of its generality might be common to physics and to other branches of knowledge, such as national economy, for example."

The year before, Professor Tolman wrote in his book, "Statistical Mechanics with Application to Physics and Chemistry":

> "The future of theoretical chemistry is dependent on its (statistical theory) application and there will be a mutual and advantageous interplay in the development of these two sciences."

But what of it? We as engineers have grown accustomed within recent years to cataclysmic upheavals in physical theory. Few of these have reached the height of general interest attained by the theory of relativity and yet, after everything has been said and done, how many of us today have changed our engineering practices because of relativity? True it is, relativity has its place but as yet it is not a useful tool for most of us. May it not be, therefore, that all of this new interest in statistical theory is but a tempest in a teapot so far as it touches us who are concerned primarily with those things which necessitate significant changes in our utilitarian mode of thinking?

Some of you, having felt the urge to learn something about statistical theory, may have acted as several friends of mine have under similar circumstances. You pick up a book on the subject to see just what it is all about. If the book is any good, it will contain a few Greek letters and all of us know what bugbears such things are. If such a reader goes further and dips into one of the important magazines of today, such as Biometrika, Metron, or the Scandanavian Actuarial Journal, where the new models of statistical machinery are being displayed, he will possibly come away feeling a little like he would after having looked at one of the recent books on the new Physics.

In other words, such a reader comes back from his brief excursion into the supposedly glorious land of modern statistics possibly with a firm conviction that, at least when looking through utilitarian glasses, the beauties of the generalities of the statistical method do not appear. Nevertheless, if we look about to see what others say who have made a more careful survey than we of the usefulness of statistical theory, we find comments such as this from Professor D. R. Buckingham of Harvard:[1]

> "In short, if it were not for the development of statistics, much of modern research would be impossible."

Can it be that these Greek characters at which we shudder really contain some truth worth having?

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1. "The Philosophy and Organization of Research", School and Society, June 15, 1929, pp. 755-764.

The well known and authoritative journal, Nature,[1] answers this question for us thus:

> "A large amount of work has been done in developing statistical methods on the scientific side, and it is natural for any one interested in science to hope that all this work may be utilized in commerce and industry. There are signs that such a movement has started, and it would be unfortunate indeed if those responsible in practical affairs fail to take advantage of the improved statistical machinery now available."

We might be tempted to discount these statements as coming from more or less academic sources, but, if we do, we are still faced by eulogies coming from practitioners of the method. Thus, three well known Germans, Becker, Plaut and Runge, writing in their book[2] on the use of mathematical statistics in problems of mass production say:

> "It is therefore important to every technician who is dealing with problems of manufacturing control, to know the laws of statistics and to be able to apply them correctly to his problems."

Similarly, Dr. Daeves who has been associated with the application of statistical theory in the Krupp Steel Works has this to say:[3]

> "Statistical research is the logical method for the control of operations, for the research engineer, the plant superintendent, and the production executive."

Naturally, therefore, most engineers find themselves in an uncertain state of mind in respect to the importance of statistical theory. On the one side they have heard its praises sung by theorists and practical men alike. On the other side they have been warned of the terrible and often cited sequence "Liars, Damned Liars, and Statisticians". I believe most of these men will agree, however, that the greater weight of evidence favors <u>more</u> instead of less interest in the new[4] statistical machinery.

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1. January, 1926.
2. Anwendungen der mathematischen Statistik auf Probleme der Massenfabrication.
3. "The Utilization of Statistics", Testing, March, 1924.
4. New at least to most engineers.

Of course those of us here today feel that way and, therefore, all of us are grateful to the two engineering organizations who have made this meeting possible so that we may lay definite plans for extending the usefulness of the "available statistical machinery". Particularly are we indebted to the two representatives, Mr. Hess and Mr. LePage, who have given so much of their personal time to make this meeting a success. As the one responsible for the technical part of the program, I wish to take this opportunity of expressing my hearty appreciation for the whole hearted cooperation of everyone taking part. I think that we are very fortunate indeed to have with us today as our chairman one of the very first men in this country to see and appreciate the advantages to be derived through the application of statistical theory and one who led in making such applications. The success of this meeting, however, depends largely upon how thoroughly all of you enter into the informal discussion coming after the somewhat formal remarks of those asked to take part in opening the discussion.

Personally, I shall try to do two things. First, I shall call attention briefly to four of the fundamental concepts which characterize this new era and make necessary a revision in many of the previously adopted methods of presenting and interpreting any and all kinds of data. Second, I shall indicate briefly and in a somewhat dogmatic way some of the changes which should be made in the presentation of practically all engineering and scientific data, if we are to make the best use of the rapidly mounting volume of such data.

II - Modern Scientific Concepts

A. Statistical Nature of Physical Properties

What engineer is there who is not interested in the physical properties of materials? What engineer does not have about him a table of the so-called physical and chemical constants? Yet how many of these so-called constants are really constant? This idea of constancy holds over from the older order of things.

It is interesting to contrast the old with the new concept of physical properties of materials. Take, for example, tensile strength, density, resistance, and so on. If we turn to one of the books on our table, we see that the tensile strength for a given kind of steel is so many pounds per square inch. Possibly it will add that the probable error of this measurement is so many pounds per square inch. Let us look at this information critically. Today we believe that there are but few physical quantities which have a single true fixed value (possibly one of these is the charge on an electron). In most cases, we believe that such quantities exist as a distribution function. Fig. 1 contrasts the old with the new conception.

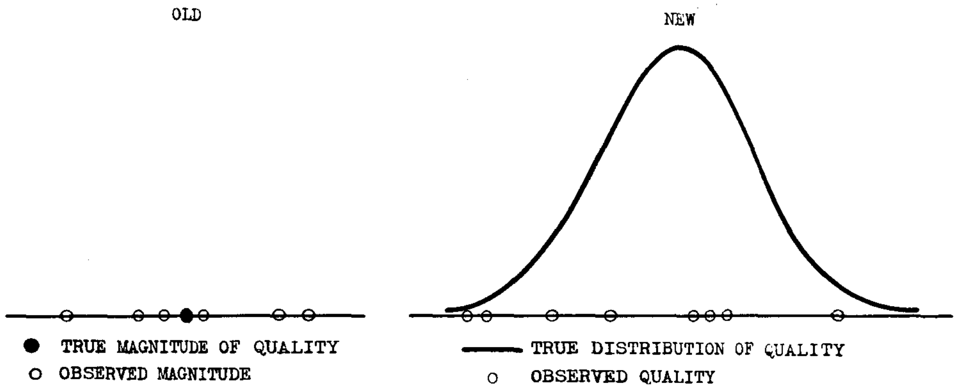


FIG. 1

Let us consider a little further the tensile strength of steel. Samples of this material which we cannot differentiate one from another do not show in general the same tensile strength. Instead they show a comparatively wide divergence in this property when measured in terms of the average tensile strength. What then can we mean by the tensile strength of steel? Obviously, it is not one fixed value. It must be some distribution. We cannot say, therefore, that the tensile strength or any other physical property of material is a certain given value. Instead, all that we can ever hope to say is that a certain proportion of a given kind of material, essentially the same so far as we can determine, will have a tensile strength or other physical property lying within any specified range.

This new concept of distribution is of vital interest to every engineer who makes a measurement and to every engineer who makes use of a physical property. Since there are but few engineers outside these categories, the applications are of general interest.

<u>Measurement</u>

How many measurements shall we take? Standard theory based upon the concept of a <u>true</u> value leads us to believe that we can increase the precision of the average of our measurements at will by increasing the number of measurements. It would seem to follow therefore that all we need to do is to decide upon the precision required and then take the correct number of measurements to attain this desired precision. This may lead to foolish conclusions as we shall now see. Suppose, for example, that we are measuring the length of the line AB.

FIG. 2

Standard theory assumes that the line has a <u>true</u> length, let us say, $\bar{X}'$, and that the method of measurement which we use has some true standard deviation, let us say, σ'. The theory then states that the standard deviation of the average of n observed values of the length of this line is $\frac{\sigma'}{\sqrt{n}}$. By making n sufficiently large, that is, by taking a sufficiently large number of observed values, this theory leads us to believe that we can measure the length of this line so that the chance of an error of, let us say, 10^{-20} centimeters is as small as we want to make it. But such a statement is ridiculous, as Prof. E. B. Wilson pointed out quite recently in Science. The length of the line has no true value in this sense. The molecules on the end of the line are jumping around in random fashion with mean free paths greater than the supposed precision of the measurement!

It follows that we must bring our concepts up to date if we are to answer correctly the important question: "How many observations?".

Standards for Properties of Materials

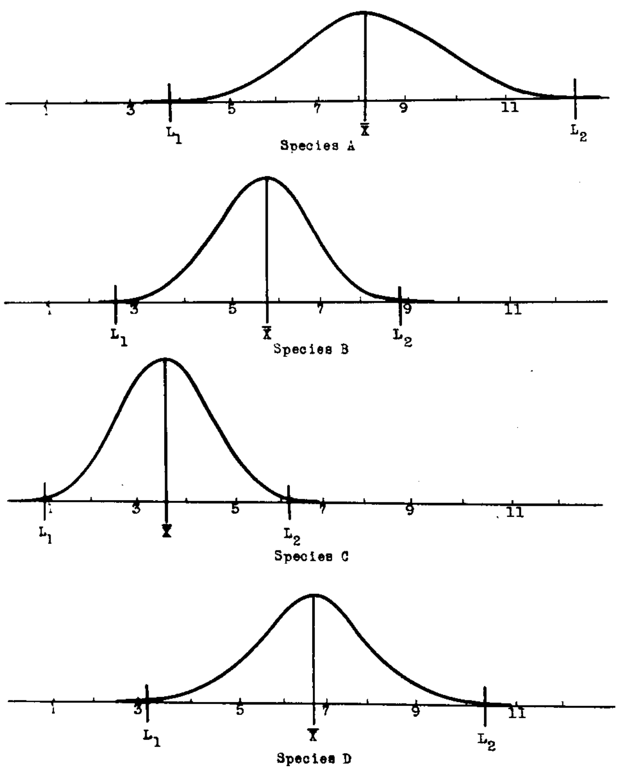
Since physical properties are distributions, standards for physical properties are distributions. Obviously this fact is of interest to all A.S.T.M. engineers because one of their problems is to establish and specify standards for the properties of raw materials. The fact is of just as much interest, however, to every engineer who makes use of this information as we shall now see by means of an illustration. Suppose that an engineer wishes to make use of the modulus of rupture of wood. He turns to an engineering handbook or some other source and often finds a single figure recorded for each species of wood. For example, on page 44 of the first edition of "Timber - Its Strength, Seasoning and Grading" by Harold S. Betts, the following values for the average modulus of rupture in pounds per square inch are given for several different kinds of woods:

	Modulus of Rupture
Cyprus	7110
Douglas Fir	8280
Eastern Hemlock	6685
Loblolly Pine	7870
Long Leaf Pine	8380
Norway Pine	5173
Red Spruce	5900
Red Wood	6980

TABLE 1

Is the engineer to draw the conclusion from such a table that every small specimen of long leaf pine, for example, has a greater modulus of rupture than every small specimen of all other species cited in Table 1? Obviously not. He realizes that there are differences in the moduli of rupture of pieces coming from the same tree and still larger differences in the moduli of rupture of pieces coming from different trees of the same species. However, so far as the table is concerned, no information is given to indicate the extent of this variability.

Figure 3 shows why the average does not tell the whole story. This figure indicates approximate standard distribution functions for modulus of rupture of round timbers from four species. In the first place it is apparent that the variability is large compared with the mean modulus of rupture. In the second place, it is evident that some pieces from each species will have the same modulus of rupture. What the engineer who makes use of such information really needs is an accurate distribution function which will tell him the proportion of pieces of material of a given species that may be expected to have a modulus of rupture within any two fixed limits.



Estimated Distribution of Modulus of Rupture for Different Species of Poles (99.7% included between L_1 and L_2)

WHY THE AVERAGE DOES NOT TELL THE WHOLE STORY

FIG. 3

B. Statistical Nature of Physical Laws

Previously cited quotations have called attention to the change in our concept of physical law. No longer do we believe that the relationships between physical quantities are functional in the strict mathematical sense of the term. Instead, we think of them as being statistical. Take as an illustration the relationship between tensile strength and hardness for some material such as steel. The older concept assumed the existence of a functional relationship between hardness and tensile strength represented graphically by a curve showing a one to one correspondence between these two properties. Observed deviations from this hypothetical curve were attributed to errors of measurement. In fact, many calibration curves of tensile strength in terms of hardness are based upon such a concept.

Today, however, we look at this situation in a different light. No longer do we believe that there is a one to one correspondence between such properties. Instead, we believe that there is a statistical distribution of pairs of values of two such quality characteristics corresponding to all possible samples of what we assume to be essentially the same material. This relationship between the old and the new is illustrated in Fig. 4. No longer then are we free to treat the deviations of an observed set of points from any curve of best fit as errors of measurement as, for example, is done in the application of the method of least squares.

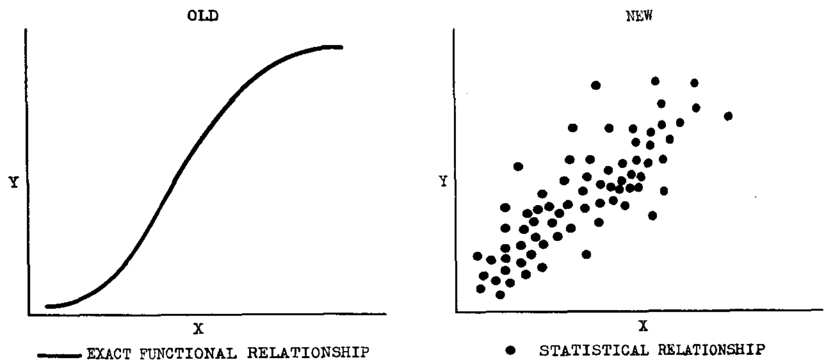


FIG. 4

We may illustrate the significance of this result by considering the problem of determining the relationship between tensile strength and hardness for, let us say, a given kind of aluminum die-casting, the data for which are given in Table 2. Let us make the assumption that the relationship is linear. By the customary method of least squares, we obtain the line of best fit by minimizing perhaps the deviations in ordinates (C, Fig. 5). However, upon the new assumption of statistical relationship, such a line of best fit does not have the usual significance, because the y deviations are no more errors than the x deviations, and in fact neither of these are errors in the customary sense of the term. Now, if we use the method of least squares and minimize the x deviations, we get a line (B, Fig. 5) which is distinctly different from that obtained by minimizing the y deviations. In a similar way we get a still different line,

Specimen	X Tensile Str. in psi	Y Hardness in Rockwells "E"	Specimen	X Tensile Str. in psi	Y Hardness in Rockwells "E"
1	29314	53.0	31	29250	71.3
2	34860	70.2	32	27992	52.7
3	36818	84.3	33	31852	76.5
4	30120	55.3	34	27646	63.7
5	34020	78.5	35	31698	69.2
6	30824	63.5	36	30844	69.2
7	35396	71.4	37	31988	61.4
8	31260	53.4	38	36640	83.7
9	32184	82.5	39	41578	94.7
10	33424	67.3	40	30496	70.2
11	37694	69.5	41	29668	80.4
12	34876	73.0	42	32622	76.7
13	24660	55.7	43	32822	82.9
14	34760	85.8	44	30380	55.0
15	38020	95.4	45	38580	83.2
16	25680	51.1	46	28202	62.6
17	25810	74.4	47	29190	78.0
18	26460	54.1	48	35636	84.6
19	28070	77.8	49	34332	64.0
20	24640	52.4	50	34750	75.3
21	25770	69.1	51	40578	84.8
22	23690	53.5	52	28900	49.4
23	28650	64.3	53	34648	74.2
24	32380	82.7	54	31244	59.8
25	28210	55.7	55	33802	75.2
26	34002	70.5	56	34850	57.7
27	34470	87.5	57	36690	79.3
28	29248	50.7	58	32344	67.6
29	28710	72.3	59	34440	77.0
30	29830	59.5	60	34650	74.8

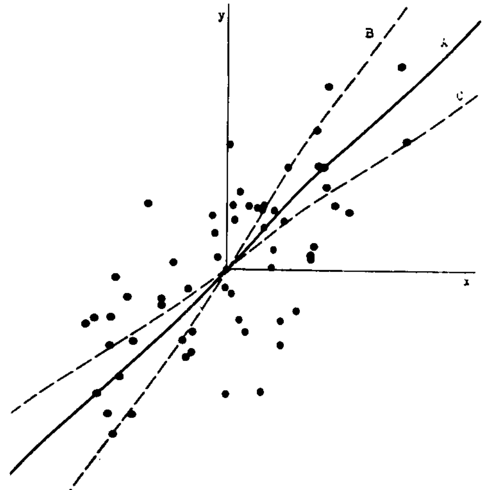
TABLE 2

if we minimize the squares of the perpendicular distance of the points from the fitted line. (A, Fig. 5).

Hence, whenever we have an engineering problem involving the relationship between two or more characteristics statistically related, we need to consider, further than is done customarily, the significance of the data. Obviously, these three lines of best fit are significantly different. Which one shall we choose? Customary theory leaves us in a quandary. Modern statistical methods lead us to think not so much of the line of best fit as of the frequency distribution in terms of the two properties. We shall return to this point in our consideration of ways and means for presenting data.

C. Law of Large Numbers

One of the fundamental objects of any applied science is to make use of previous data and experience in forecasting the future. Throughout the history of science, we have often been told that a complete knowledge of the physical laws governing the universe and a specification of the universe at any one instant in terms of these laws would make it possible for us to predict the state of the universe at any time thereafter with the same kind of precision that we can predict the occurrence of an eclipse. With Pope, it was believed that "Chance is but direction thou canst not see". Today, however, we are inclined to believe that the practical goal in this direction is merely prediction within limits. Specifically, all we can hope to say is that the probability of something happening within certain limits is so much. All of this is but descriptive of perhaps the best established law of nature, namely, the Law of Large Numbers. We can express the significance of this law in the following way. It assumes the existence of systems of chance or unknown causes which produce effects falling within a given range with a given probability. It goes further and states that the observed probability of the event happening within certain limits approaches as a statistical limit the probability of the event falling within these limits as the number of observations increases indefinitely.



RELATIONSHIP BETWEEN TENSILE STRENGTH AND HARDNESS - ALUMINUM DIE CASTINGS

FIG. 5

For our present purpose, this law simply means that, if the system of causes controlling, let us say, some physical property, such as the tensile strength of a given material, is subject to the Law of Large Numbers, then the observed distribution will approach the true hypothetical distribution as the number of observations increases indefinitely. It says something else of interest in this same connection, namely, that a future succession of observed values of some quantity such as tensile strength should not fall outside any given set of limits more than a given fraction of the time, provided the system of causes remains constant in the sense that observed deviations must be left to chance. Once again we touch upon something of interest to engineers and scientists alike. Deviation in a set of observed values is the rule and not the exception. The question is always present: "Should we leave such variations to chance?" The available statistical machinery gives us a practical criterion upon which to base an answer. It tells us how to establish limits within which the deviations may be expected to lie provided they should be left to chance. Naturally, the rule is not infallible. No scientific principle is. The most we can say is that it works. Let us consider a typical illustration.

In the production of a certain kind of equipment, considerable cost was involved in securing the necessary electrical insulation by means of materials previously used for that purpose. A research program was started to secure a cheaper material. After a long series of preliminary experiments, a tentative substitute was chosen and an extensive series of tests for insulation resistance were made, care being taken to eliminate known causes of variability. Table 3 gives the results of 204 observations of resistance in megohms taken on as many samples of the proposed substitute material. Reading from top to bottom beginning at the left column and continuing throughout the table gives the order in which the observations were made. The question is: "Should such variations be left to chance?"

No a priori reason existed for believing that the measurements forming one portion of this series should be different from those in any other portion. In other words, there was no rational basis for dividing the total set of data into groups of a given number of observations except that it was reasonable to believe that the system of causes might have changed from day to day as a result of changes in such things as atmospheric conditions, observers and material. In general, if such changes take place, we

5045	4635	4700	4650	4640	3940	4570	4560	4450	4500	5075	4500
4350	5100	4600	4170	4335	3700	4570	3075	4450	4770	4925	4850
4350	5450	4110	4255	5000	3650	4855	2965	4850	5150	5075	4930
3975	4635	4410	4170	4615	4445	4160	4080	4450	4850	4925	4700
4290	4720	4180	4375	4215	4000	4325	4080	3635	4700	5250	4890
4430	4810	4790	4175	4275	4845	4125	4425	3635	5000	4915	4625
4485	4565	4790	4550	4275	5000	4100	4300	3635	5000	5600	4425
4285	4410	4340	4450	5000	4560	4340	4430	3900	5000	5075	4135
3980	4065	4895	2855	4615	4700	4575	4840	4340	4700	4450	4190
3925	4565	5750	2920	4735	4310	3875	4840	4340	4500	4215	4080
3645	4190	4740	4375	4215	4310	4050	4310	3665	4840	4325	3690
3760	4725	5000	4375	4700	5000	4050	4185	3775	5075	4665	5050
3300	4640	4895	4355	4700	4575	4685	4570	5000	5000	4615	4625
3685	4640	4255	4090	4700	4700	4685	4700	4850	4770	4615	5150
3463	4895	4170	5000	4700	4430	4430	4440	4775	4570	4500	5250
5200	4790	3850	4335	4095	4850	4300	4850	4500	4925	4765	5000
5100	4845	4445	5000	4095	4850	4690	4125	4770	4775	4500	5000

ELECTRICAL RESISTANCE OF INSULATION IN MEGOHMS.

SHOULD SUCH VARIATIONS BE LEFT TO CHANCE?

TABLE 3

may readily detect their effect, if we divide the total number of observations into comparatively small sub-groups. If there is no reason for choosing a particular size of sub-group or sample in such conditions, it is taken as four. The block dots in Fig. 6-a show the successive averages of four. The dotted lines are the

SHOULD THESE VARIATIONS BE LEFT TO CHANCE?

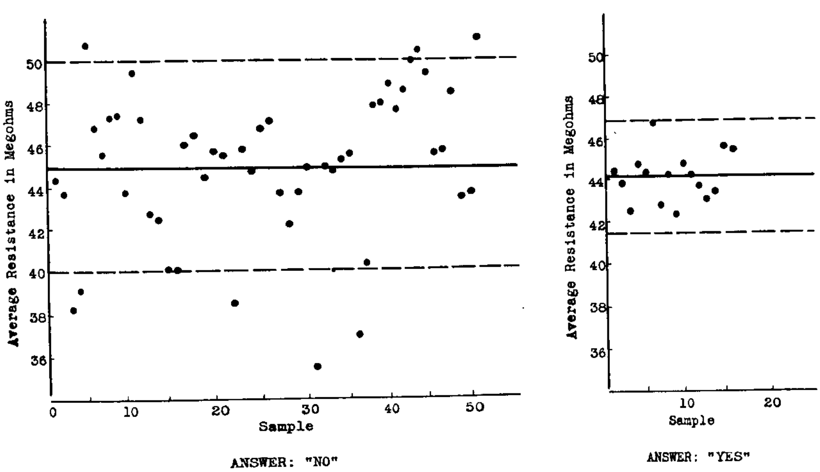


FIG. 6

limits within which these observations should fall provided the variability must be left to chance. Some of the observed values are seen to lie outside the limits. This was taken as an indication of the existence of causes of variability which could be found and eliminated. Further research revealed some of these causes of variability and after these had been eliminated another series of observed values was taken. The averages for the subgroups of this series are shown in Fig. 6-b. Here we see that all of the points fall within the limits. We assume, therefore, that it is not feasible for research to go much further in this case.

It must be recalled that this illustration is given not to prove that the method always works, but merely to indicate the rule instead of the exception as borne out by our experience. It may be of interest therefore to consider a case where there is every reason to believe that research has gone to a practical limit in removing the causes of variability. The outstanding series of observations of this type is perhaps that made by Millikan on the charge on an electron. Treating his data in a manner similar to that indicated above, we get the results shown in Fig. 7. All of the points are within the dotted limits. The indication of the test is consistent with the accepted conclusion that those factors which need not be left to chance had been eliminated before this particular set of data were taken.

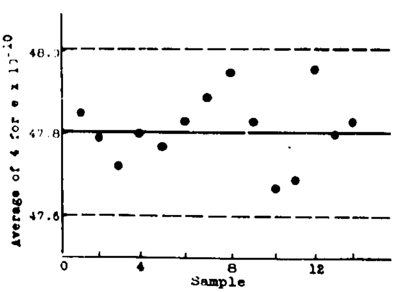


FIG. 7

Perhaps one of the most far reaching applications of modern statistical machinery is that just illustrated. Its applications appear in every field of science and engineering. The possible value of such a tool in any research or engineering work should not be overlooked.

D. <u>Every Set of Data a Sample</u>

Every set of data taken under supposedly the same essential conditions constitutes a sample of what may be obtained under these conditions. For example, if the data are a series of observed values of tensile strength of material, they constitute but a sample of what the unknown chance causes of variability can produce. Statistically, we often speak of a distribution in one or more dimensions as a universe and of a sample as being a sample from a given universe. In most of our engineering work, however, it is more meaningful to think of the universe as being the distribution of possible effects of a complex unknown chance cause system. Schematically, this may be pictured as in Fig. 8.

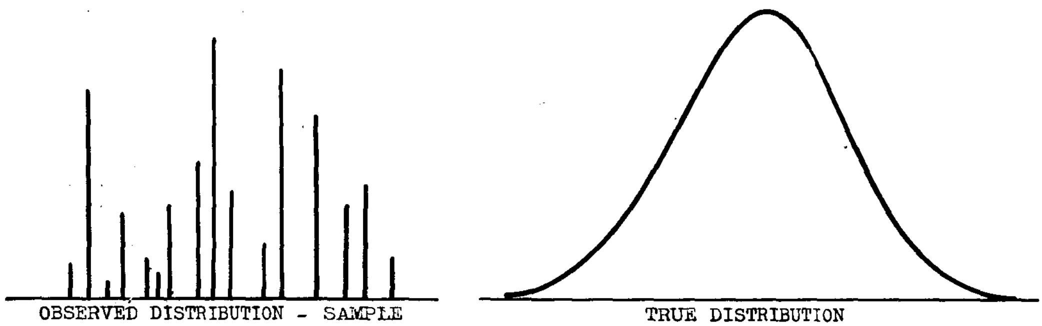


FIG. 8

What is the practical significance of this situation? It is simply this. We can never observe the true distribution of any quality such as tensile strength because to do so would require an infinite number of observations. Neither can we observe a distribution of relationship between tensile strength and any other property or in general between any two physical properties statistically related. What we must do, therefore, is to set up ways and means of trying to estimate the true distributions and true statistical relationships in terms of the observed data constituting a sample.

The importance of this situation can be emphasized by calling attention to the fact that the probable error of the mean as used by engineers is only one of a possible infinite number of probable errors depending upon the method we adopt for getting from the sample to the distribution. In general, when the samples are small, probable errors as customarily calculated are much smaller than they should be. Sometimes the probable errors given in engineering and scientific literature are roughly only three-fourths of the size that most generally accepted modern theory would indicate they should be. Thus, if our use of probable errors in engineering is to have real significance, we must take into account these modern developments.

Thus briefly we have caught a glimpse of the new order of things making it desirable for use to back off and consider anew the whole subject of analyzing, interpreting and presenting data. Today when we have large engineering projects under way demanding the accumulation of thousands of observed data, it will be a sheme if we do not make use of the new tools available for getting the most out of these data. It is this kind of feeling, I believe, that prompts every one here to be interested in one of the objects of this round table conference, namely, the formulation of some method of disseminating information in respect to the new and improved statistical machinery. Before closing my discussion, however, I wish to touch briefly and in a little more detail upon the problem of presentation of data, trying to show in so doing that several very definite changes should be adopted. It is not feasible at the present time to enter into the discussion of the reasons why. All that we can do is to present in a dogmatic fashion some of the results which recent investigations reveal. As in my previous illustrations, I have chosen to consider the presentation of data primarily because it is a subject in which every scientist and engineer is profoundly interested. The way results are given today often amounts to throwing away several per cent of the information contained in the original data. Furthermore, the way results are presented today often makes it impossible to use modern statistical machinery.

III - Presentation of Data

Let us consider the problem of presenting the essential information contained in a table of data such as shown in Table 4, recording the observed depths of sapwood and depths of penetration of preservative in 1370 telephone poles. Obviously it is not feasible to publish data in detail such as this. Neither is it desirable in most instances because data so published cannot readily be interpreted. In fact, the process of interpretation always necessitates the analysis of the data or the breakdown of the data as it were into a few simple functions. Suppose you were charged with the problem of presenting the results in this

X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
4.60	1.90	3.00	1.20	3.20	1.50	3.70	2.00	2.20	2.15	3.00	1.80	3.25	2.20	2.60	1.50	2.20	1.10	1.35	0.70	4.40	1.60	4.50	2.40	3.90	1.40	3.10	2.15
2.15	1.30	2.90	1.80	2.30	1.75	2.70	0.80	1.90	1.50	1.60	1.00	3.20	1.85	2.10	1.20	2.20	1.20	3.30	1.05	5.40	2.40	2.80	1.30	2.70	1.50	4.10	3.40
3.70	1.60	3.55	1.80	4.10	2.00	4.40	1.90	4.10	2.90	4.20	1.05	2.75	2.20	3.50	2.15	2.70	1.10	3.10	1.40	2.05	1.60	2.10	1.70	2.05	1.00	1.30	0.80
2.70	2.20	3.30	2.00	3.50	1.45	2.50	1.20	2.80	2.45	2.90	2.30	3.50	1.60	3.60	1.70	3.15	0.80	3.80	1.20	2.90	1.10	1.15	0.90	3.30	1.00	4.00	1.80
3.20	1.50	3.40	1.85	1.60	0.90	3.60	2.05	2.15	1.90	3.00	0.95	2.35	1.10	3.60	1.45	2.25	0.90	2.70	0.80	3.30	2.20	3.30	1.60	2.80	1.60	2.40	1.80
2.40	1.70	3.80	2.40	3.70	1.90	3.70	1.80	1.90	1.30	2.85	0.85	2.50	1.20	3.20	0.85	2.40	1.10	1.55	1.00	2.80	1.20	1.75	0.95	1.15	0.80	2.60	1.35
2.15	1.15	1.70	1.00	1.75	1.10	1.50	0.85	1.80	1.15	1.95	0.80	2.70	2.50	2.90	2.10	1.60	1.45	2.30	1.00	3.60	2.30	2.10	1.50	1.85	1.40	2.60	1.10
1.90	1.50	3.10	1.80	2.50	1.50	2.60	1.60	3.60	2.20	2.90	1.40	2.00	1.05	2.35	1.00	3.05	1.40	2.60	1.90	3.40	1.70	3.20	2.00	2.80	1.50	2.55	1.40
3.50	1.50	2.50	1.90	2.30	1.55	2.00	1.20	1.75	0.90	2.60	1.10	2.45	1.35	3.30	1.15	3.65	2.00	3.35	1.85	1.65	1.15	3.70	1.00	2.00	1.35	2.10	1.00
3.50	2.70	2.95	1.35	3.40	1.50	3.60	2.15	3.90	2.55	1.90	1.50	3.05	1.25	3.20	2.65	1.95	1.80	2.65	1.20	2.60	2.05	2.25	1.65	1.95	1.15	3.55	2.05
2.10	1.05	3.00	2.00	2.85	1.25	1.90	1.20	1.90	1.60	2.85	1.25	3.00	2.30	2.35	1.80	4.40	0.90	2.65	1.10	2.45	1.15	3.20	1.45	3.70	1.80	3.40	1.35
2.45	1.60	2.75	1.20	2.60	1.50	3.65	2.00	2.25	1.60	2.80	1.40	2.05	1.50	3.70	1.50	2.40	1.40	3.05	0.70	3.55	1.65	4.00	3.60	1.45	1.10	3.50	1.50
3.30	1.45	2.65	1.65	3.50	1.40	3.00	1.10	2.30	0.90	2.20	0.90	2.60	1.00	2.60	1.15	2.50	0.75	1.50	0.80	1.40	0.75	1.90	1.30	2.70	2.40	3.40	1.10
2.90	1.65	2.25	1.35	3.70	1.35	3.05	1.50	2.10	0.95	3.10	1.50	1.90	0.90	1.60	1.05	3.40	1.20	1.60	1.45	3.05	1.20	3.60	1.65	3.70	1.80	3.00	2.20
3.10	2.10	3.90	3.30	3.90	1.30	4.00	2.85	2.30	1.75	2.60	0.80	4.20	2.10	3.85	1.85	3.75	1.95	2.10	1.15	3.80	2.75	3.00	1.05	4.00	2.65	3.30	2.50
3.50	1.60	1.70	1.55	[illegible]	1.60	2.70	2.00	3.35	2.85	3.70	1.10	3.25	1.95	1.95	1.50	2.65	1.00	3.25	2.75	1.85	1.05	1.75	1.10	3.40	1.60	2.90	1.40
1.50	0.85	2.70	1.25	3.15	1.75	3.10	1.50	3.25	2.30	2.90	0.90	3.25	1.25	2.45	1.55	2.20	1.40	2.85	1.60	1.55	0.85	2.30	1.70	2.70	1.40	2.45	1.25
2.95	1.55	3.10	1.35	3.35	1.90	3.20	1.75	3.30	1.65	2.80	2.55	2.80	0.90	2.45	1.00	2.90	1.45	3.20	2.00	3.50	2.40	2.75	1.30	3.25	2.80	2.35	1.35
2.90	0.90	4.10	1.10	4.05	1.30	2.40	1.50	2.15	1.20	3.05	1.90	2.60	0.90	2.25	1.60	2.70	1.00	2.70	2.40	4.30	1.30	4.20	1.05	3.30	1.70	2.50	1.50
1.70	0.85	2.55	1.40	2.10	1.40	2.80	1.40	2.30	1.50	2.40	1.45	3.40	0.95	1.95	1.35	2.20	1.10	3.40	1.90	2.85	1.90	2.80	1.90	2.70	1.35	2.85	2.00
1.10	1.00	2.40	1.30	3.05	1.05	3.40	2.10	1.65	0.95	3.20	1.70	3.00	2.50	2.50	1.80	2.50	1.00	2.90	1.35	2.70	1.20	2.60	1.70	2.50	0.75	2.70	1.60
2.70	1.20	2.80	1.60	2.20	0.90	1.65	0.75	3.05	1.30	1.85	1.60	2.50	1.80	2.90	1.40	1.95	1.20	3.00	2.00	3.70	2.10	2.10	1.20	3.00	0.65	4.40	1.55
3.10	0.85	4.70	1.50	2.40	0.90	4.50	1.90	3.90	0.70	5.00	2.60	4.90	1.70	4.00	2.50	3.10	0.85	3.40	2.90	2.70	2.10	3.05	1.95	3.15	1.60	3.10	1.50
3.30	1.80	4.15	3.00	4.25	2.65	3.40	1.40	3.60	1.95	2.70	1.55	3.05	1.35	4.20	3.10	2.25	1.10	3.00	0.70	2.40	0.40	4.00	1.60	4.30	2.55	3.50	1.30
2.80	1.15	2.75	0.80	3.20	2.10	1.30	0.60	2.85	0.90	4.00	2.40	2.85	1.30	3.60	1.20	3.20	2.20	2.30	1.65	2.25	0.65	3.50	2.30	2.55	1.55	3.00	1.15
3.20	2.55	2.80	1.90	2.65	1.15	2.10	1.10	2.05	1.50	2.15	1.65	2.65	1.20	3.65	2.40	2.05	1.25	2.90	2.60	2.70	1.80	3.10	0.80	1.90	1.25	5.20	4.70
3.50	[illegible]	3.70	1.70	4.80	2.70	4.80	2.90	4.00	1.80	3.70	1.70	2.70	2.05	3.20	2.05	2.60	0.95	2.70	1.75	1.15	0.55	1.75	1.60	1.40	1.05	3.60	2.65
1.70	1.00	1.90	1.50	1.95	1.00	3.75	3.35	2.85	2.30	3.20	1.25	3.00	1.60	2.70	2.00	1.90	1.00	3.00	2.85	2.05	1.30	2.40	0.90	2.70	2.60	2.55	1.40
2.50	0.80	3.45	2.90	2.15	1.50	2.05	1.10	2.20	1.40	2.40	1.40	3.45	2.20	1.20	0.80	2.00	1.05	3.45	0.85	1.60	0.80	3.10	2.15	4.00	2.70	4.40	1.50
4.10	2.60	3.10	1.50	3.30	0.80	3.05	2.00	4.55	3.15	4.05	2.50	3.10	1.00	3.75	2.50	3.30	0.80	2.35	1.05	2.00	1.60	1.25	0.75	2.40	1.10	4.15	1.75
2.70	1.05	3.90	1.25	4.00	1.25	3.90	3.20	3.70	1.80	3.40	2.30	3.40	2.40	2.50	1.10	2.30	1.40	2.90	1.70	1.30	1.00	2.00	1.70	2.50	1.10	3.90	3.30
2.70	1.20	1.60	0.90	3.60	0.90	2.50	1.00	1.60	0.90	2.70	1.50	3.10	1.20	1.90	0.70	3.30	1.40	2.00	1.20	1.80	1.00	2.30	1.30	3.30	1.10	1.70	1.40
2.60	1.90	4.70	1.20	3.50	1.70	1.60	0.90	2.80	2.10	2.10	0.70	2.10	0.90	2.80	1.00	1.90	0.80	4.00	1.20	4.30	2.10	4.20	1.50	2.60	1.40	3.90	1.80
3.40	1.80	2.60	1.50	4.10	2.70	3.10	1.50	2.40	0.70	1.95	0.70	1.90	1.25	2.20	1.25	2.50	1.30	2.35	1.25	2.00	0.75	3.20	1.80	2.15	1.10	4.15	2.80
4.50	3.55	5.00	4.15	3.35	1.05	2.20	1.25	2.50	0.60	2.55	1.00	1.60	0.60	2.40	1.20	1.70	1.15	3.15	1.40	2.80	1.10	4.10	2.60	3.80	1.00	3.85	2.60
2.95	1.20	2.95	2.40	2.40	1.50	3.40	1.40	3.30	1.55	3.80	1.90	3.80	2.30	3.40	0.90	4.00	2.65	3.40	1.45	3.80	1.05	4.20	3.50	2.05	1.35	2.50	2.35
3.15	1.25	4.20	2.10	4.20	2.20	3.40	1.25	2.15	0.70	2.95	2.10	3.00	2.10	3.50	1.80	3.25	1.15	4.30	2.00	2.30	0.95	3.60	2.10	3.30	1.20	4.60	2.75
2.30	0.80	2.30	1.80	3.50	1.55	3.30	1.25	2.30	1.20	1.60	1.05	3.50	1.80	2.90	1.90	2.85	1.55	3.60	2.60	1.65	1.30	1.25	0.90	2.95	1.40	3.10	1.50
1.80	1.20	2.35	1.35	1.65	0.75	1.40	0.80	1.55	0.75	3.15	2.90	3.25	1.35	2.70	1.10	1.65	1.05	3.25	1.10	2.85	1.50	2.35	1.15	2.15	1.25	2.65	0.90
1.80	0.95	3.00	1.80	1.70	0.70	3.25	2.30	2.80	1.95	3.45	1.60	2.00	1.00	2.75	2.20	1.75	1.25	2.85	1.00	2.25	1.00	2.15	1.30	2.25	1.60	3.00	1.10
1.70	1.05	2.80	2.00	3.50	1.70	2.90	1.50	3.70	2.00	3.60	1.00	3.60	1.60	2.45	1.50	3.60	2.20	3.95	2.15	2.90	1.50	1.95	1.60	3.25	1.35	2.90	1.35
1.55	1.40	3.50	1.85	2.50	1.45	4.10	2.15	2.80	1.65	3.50	1.30	3.20	1.30	2.85	2.10	2.75	2.25	3.10	1.20	4.00	2.40	3.60	1.10	3.80	1.60	3.70	2.60
2.05	0.80	3.40	1.60	2.55	0.90	2.55	1.60	2.90	1.85	3.80	2.80	1.75	1.35	2.10	1.00	2.35	1.05	3.65	2.25	3.95	0.85	2.05	1.80	2.50	1.25	2.40	1.50
2.80	1.15	2.35	1.70	3.10	1.40	4.00	2.10	2.10	1.30	2.20	1.30	4.10	2.30	2.00	0.90	4.40	1.20	3.20	1.60	3.20	0.80	3.75	3.40	2.40	1.95	4.10	1.50
4.20	2.65	3.50	2.90	3.50	1.50	2.25	0.90	3.30	1.65	2.50	1.70	3.60	1.75	2.55	2.20	3.80	1.30	3.40	2.80	3.15	2.95	2.70	1.30	2.30	1.70	2.60	1.70
2.05	0.85	2.50	2.05	2.90	1.10	3.05	1.55	3.20	1.05	3.20	1.50	2.05	1.30	1.75	1.10	2.85	1.40	2.00	0.90	1.80	1.05	2.65	2.35	3.70	1.50	2.50	0.80
3.20	1.20	1.70	0.60	1.70	0.50	3.10	1.60	2.10	0.80	3.70	1.65	3.10	2.60	3.60	2.00	2.60	1.05	2.00	1.15	3.30	2.30	3.70	2.10	2.00	0.75	2.65	1.90
2.90	1.50	2.50	1.10	3.70	2.05	3.30	1.10	3.00	1.60	3.00	1.20	4.00	1.20	3.90	1.50	3.50	2.20	2.35	0.70	3.80	1.40	3.90	2.40	4.60	1.50	3.90	1.60
2.30	1.25	3.85	1.60	3.90	1.60	2.80	1.00	3.30	1.50	3.10	1.80	3.00	1.30	2.85	1.20	2.90	1.40	2.90	1.60	2.70	1.50	1.80	1.20	1.65	1.00	2.20	1.70
4.50	1.50	2.30	1.50	1.90	1.05	2.30	1.10	3.80	1.00	3.60	3.00	2.80	1.10	2.90	1.60	2.70	1.55	4.00	0.80	2.30	1.40	2.60	1.30	2.00	1.30	2.05	1.80
3.05	2.30	2.25	1.45	3.30	2.20	2.70	1.60	2.00	0.85	1.95	0.90	2.90	0.75	1.20	0.30	2.55	1.20	4.80	1.50	4.70	1.40	2.45	1.15	1.95	1.00	1.90	1.00
2.15	1.30	3.10	1.15	2.80	1.65	2.90	1.25	1.85	1.05	2.20	0.80	2.35	1.15	2.25	1.20	2.10	0.70	2.45	1.20	2.70	0.90	1.55	1.10	2.05	0.70	2.90	1.30
2.50	1.75	2.80	1.85	3.00	0.80	2.70	1.30	1.70	1.20	3.60	1.60	3.80	1.40	3.90	2.90	3.70	2.20	4.60	2.90	3.80	2.20	2.50	0.70	2.50	1.10	2.45	1.50
2.05	1.65	2.65	0.95	2.60	1.75	3.00	1.20	2.45	0.90	4.50	2.15	2.55	2.45	3.30	1.90	2.60	0.95	2.85	1.50	3.10	2.00	4.00	2.10	3.30	1.90	1.40	1.15
2.20	1.45	3.00	1.70	1.50	0.85	1.60	1.35	2.75	1.85	2.50	1.30	3.70	2.10	2.30	1.90	2.35	1.20	1.90	1.35	1.95	0.95	2.00	1.10	3.50	3.20	5.00	1.70
3.50	1.10	1.80	1.05	3.00	1.60	3.70	1.70	3.50	1.40	3.40	1.75	2.00	1.30	2.00	0.60	3.10	1.50	1.80	1.50	2.60	1.20	2.65	1.70	1.60	1.10	1.25	0.80
2.50	1.25	1.80	1.20	3.30	1.30	2.30	1.70	2.95	1.40	2.70	2.40	3.10	1.30	1.65	0.75	1.20	0.95	4.35	2.50	2.50	1.00	3.35	1.75	3.65	1.05	1.80	1.00
1.85	1.05	1.50	0.80	4.35	3.00	2.00	1.15	2.35	1.00	3.80	1.40	3.60	2.80	2.85	2.65	2.90	2.70	3.30	1.30	3.70	1.30	3.60	1.20	1.45	1.00	2.55	1.35
2.15	1.00	2.40	1.10	2.80	1.00	3.50	1.00	5.00	3.70	2.80	1.80	2.30	1.10	2.40	2.20	3.20	2.00	3.40	2.10	3.10	1.00	2.10	1.30	3.70	2.10	2.60	1.30
2.00	0.80	4.00	1.15	2.30	1.70	2.10	1.50	3.90	2.30	2.70	1.30	2.55	1.10	2.60	1.70	1.55	0.95	5.20	4.00	2.65	1.00	1.75	0.95	5.10	3.00	3.60	2.45
2.00	1.25	2.75	1.80	3.20	1.55	2.20	0.80	3.95	3.25	2.60	2.05	2.95	0.70	2.85	1.45	3.60	3.10	3.05	1.50	2.85	2.00	3.65	2.30	3.80	2.55	2.95	1.70
2.75	0.80	3.15	2.15	3.15	1.90	2.55	1.70	3.20	1.90	2.90	1.00	2.35	1.25	2.00	0.95	1.95	0.90	2.85	1.40	2.60	0.90	2.25	1.60	2.45	1.10	2.80	1.15
3.40	1.00	3.60	1.20	2.90	1.05	3.70	1.55	4.10	2.00	3.70	2.20	3.40	3.25	2.50	0.80	3.80	1.65	2.90	0.95	1.50	0.85	3.70	2.00	3.10	1.65	3.00	1.80
4.65	3.90	3.90	2.85	4.40	2.60	3.60	2.50	4.60	4.00	4.25	3.30	4.20	3.00	3.10	1.10	3.40	1.80	2.50	1.05	2.05	1.20	4.40	2.00	1.60	0.50	2.40	1.00
4.80	3.80	2.00	1.30	3.20	2.10	3.15	1.70	4.40	2.40	2.50	1.80	3.05	1.20	3.20	1.50	2.70	2.60	2.80	0.80	2.35	1.10	2.60	1.35	2.60	1.20	3.85	2.20
3.20	1.80	3.50	2.25	4.40	3.50	3.70	3.10	3.75	3.50	1.90	1.60	3.70	2.30	2.40	1.20	3.15	1.60	2.25	1.80	4.80	2.40	2.65	1.50	4.30	2.10	3.95	1.50
2.55	1.05	4.15	1.60	2.85	1.20	2.80	1.10	3.20	2.10	2.40	1.15	3.70	2.20	2.10	1.00	4.10	1.15	3.15	0.90	2.15	0.90	3.00	1.40	2.20	1.30	1.90	0.80
2.30	0.90	1.45	0.90	2.35	1.20	1.40	0.80	4.00	1.50	2.00	1.05	1.95	1.25	2.55	1.20	4.10	2.10	4.30	3.60	3.50	2.20	3.00	1.50	3.00	1.30	4.00	2.70
2.50	1.90	1.90	1.80	2.50	1.05	2.70	1.30	1.70	0.90	2.50	1.60	3.35	1.25	2.15	1.10	1.80	1.60	2.30	2.20	2.65	1.10	3.15	1.00	3.50	1.80	3.60	1.85
3.90	2.10	3.70	2.05	3.65	1.80	3.00	0.95	3.30	1.55	2.80	1.50	3.40	2.35	2.80	1.60	3.00	1.45	3.10	1.20	3.50	1.60	3.05	1.80	2.85	1.10	[illegible]	1.45
3.05	1.80	3.60	1.40	1.85	1.15	2.50	1.30	1.50	1.30	3.25	2.45	2.10	0.50	1.90	1.30	1.85	0.90	3.60	3.25	3.20	1.20	2.50	1.60	3.30	0.90	2.50	1.25
4.40	1.90	2.50	1.20	3.30	1.30	2.70	0.95	1.95	1.35	1.95	1.30	2.50	1.00	2.90	1.00	4.30	1.10	2.00	1.00	3.50	2.80	2.55	1.30	3.15	1.40	2.75	1.45
2.75	1.35	4.50	1.90	3.60	1.75	3.20	1.10	3.05	0.95	4.80	1.90	2.65	1.15	2.35	1.55	3.95	2.30	3.60	1.60	2.00	1.20	3.50	2.55	1.45	1.10	3.90	2.90
1.35	0.60	2.20	1.70	2.30	1.35	1.95	0.80	3.95	2.20	3.00	2.10	2.85	1.05	3.50	1.40	3.30	1.75	2.90	1.50	2.95	2.70	3.20	1.90	3.30	1.40	2.50	2.20
3.30	1.40	4.00	2.35	2.05	0.85	2.20	1.30	3.60	1.85	2.30	1.30	2.00	0.85	1.70	0.90	2.50	1.15	2.70	1.80	3.05	0.65	2.10	1.15	2.50	0.80	1.30	1.10
2.10	0.85	2.95	2.30	2.65	1.60	2.25	1.50	1.70	0.85	2.65	0.90	3.20	2.10	1.40	0.75	2.55	1.10	2.75	1.00	3.35	1.50	4.20	1.95	2.10	1.10	3.75	1.20
2.60	0.80	2.20	0.95	1.90	1.10	3.00	2.00	2.00	1.00	2.90	2.30	2.00	0.75	3.95	2.95	2.50	0.90	3.30	2.30	2.70	1.00	1.35	0.85	2.55	2.00	2.80	1.00
2.30	1.20	2.60	1.00	2.30	1.40	2.45	0.90	3.10	1.00	1.95	1.00	2.40	1.20	2.90	2.60	3.50	3.10	3.65	1.50	4.10	1.70	3.65	2.80	3.00	1.50	3.00	0.70
4.05	2.15	4.70	0.80	2.55	1.80	2.35	1.30	4.20	2.10	3.50	2.10	3.70	0.85	3.70	2.40	4.70	3.00	4.90	3.15	2.60	1.25	4.40	3.00	4.50	3.25	4.40	2.40
4.10	1.70	4.40	2.00	4.00	1.35	4.50	2.65	2.55	0.85	1.65	1.20	4.35	1.20	4.00	2.00	3.50	2.45	3.30	2.20	2.00	0.55	4.60	1.15	4.60	2.30	2.40	1.30
4.30	1.40	4.50	3.00	2.40	0.80	2.80	1.70	3.05	1.90	3.60	2.30	2.60	2.15	2.75	1.50	2.40	2.00	2.80	1.30	2.70	1.40	3.50	1.70	3.45	1.40	4.05	2.80
2.05	1.10	2.40	1.40	2.15	1.20	2.15	1.10	3.40	2.50	3.90	2.50	2.50	1.30	3.00	1.40	3.40	1.90	2.65	1.35	2.50	1.20	3.00	1.90	1.25	0.90	3.30	1.40
2.20	0.65	3.10	0.85	1.80	1.15	1.55	1.25	3.15	1.70	2.00	1.25	3.00	2.00	3.15	2.20	3.15	2.10	4.55	1.65	1.85	1.35	1.75	1.00	4.15	1.70	3.15	1.25
3.40	1.70	2.20	1.50	1.60	1.00	2.55	0.70	3.10	1.60	2.55	1.55	3.80	1.20	3.45	2.45	3.35	1.05	3.15	2.15	3.20	1.90	2.15	0.95	2.70	0.90	2.65	1.70
2.70	1.30	4.35	2.80	3.00	1.40	2.15	1.40	2.30	0.70	3.60	1.85	4.25	2.50	4.00	3.30	1.20	1.00	3.20	2.50	1.65	1.50	4.50	2.20	1.70	1.45	3.10	1.50
4.10	2.70	2.10	2.00	2.70	1.60	3.00	1.50	4.30	2.35	1.85	1.40	2.40	1.70	2.30	1.40	1.15	0.80	2.50	1.50	2.50	1.55	3.90	2.25	3.80	2.45	4.00	2.25
2.80	2.20	3.80	2.15	4.50	3.50	2.70	1.40	2.90	1.30	1.20	0.95	2.45	1.85	2.55	1.95	2.25	1.35	3.30	2.80	3.50	2.90	3.15	2.80	2.60	1.30	2.00	2.10
3.50	3.25	2.70	2.00	3.80	3.00	2.60	1.10	2.60	0.90	2.70	1.55	2.15	1.50	3.00	0.95	2.80	1.45	2.95	2.05	2.65	1.20	3.90	2.20	3.20	2.10	2.85	1.50
3.40	2.00	2.95	1.60	2.05	1.70	2.25	1.45	2.90	1.65	2.80	2.55	3.30	2.20	3.05	1.90	2.40	1.35	2.75	1.30	3.35	2.10	3.05	2.00	3.00	1.95	2.35	1.10
5.10	3.80	3.35	1.60	2.05	1.60	2.45	1.30	3.70	1.90	3.60	2.00	3.40	2.75	0.90	0.65	2.65	1.20	3.30	1.25	3.60	0.80	3.90	2.90	4.00	2.15	2.30	1.00
1.95	0.90	3.40	2.45	3.50	2.10	3.00	2.40	1.40	1.00	2.35	1.00	2.15	1.10	3.00	1.50	3.10	1.15	3.15	1.90	3.80	2.40	5.20	1.00	3.00	1.60	3.40	2.05
2.45	2.00	2.05	1.60	3.80	1.70	3.30	1.30	2.10	1.40	5.00	0.85	4.65	1.45	4.35	1.60	1.65	0.80	2.80	1.20	2.70	1.35	3.80	2.70	1.65	1.05	2.80	2.20
4.00	2.65	1.70	1.00	2.40	1.70	3.45	2.20	2.20	1.50	2.45	1.50	2.45	1.30	3.80	2.65	3.50	0.70	2.05	1.20	2.30	1.35	3.60	1.75	3.00	2.25	2.90	1.00
3.00	1.05	2.05	0.85	2.95	1.80	3.60	1.95	2.70	1.65	3.00	1.30	2.25	1.10	1.35	0.90	2.50	1.75	2.25	1.30	3.90	2.60	3.15	1.15	1.75	0.75	2.20	1.80
3.05	1.20	3.00	2.00	3.05	1.10	2.50	1.40	2.50	1.40	4.00	1.50	3.40	2.10	3.10	1.60	3.80	2.10	2.70	1.20	2.85	1.05	2.05	[illegible]	[illegible]	1.30	1.80	1.10
1.30	0.75	2.50	0.90	2.50	0.85	2.40	1.10	3.05	2.70	2.25	1.90	1.70	1.35	3.00	2.20	3.00	2.65	2.20	1.60	2.30	1.70	2.90	1.50	2.95	1.30	3.50	2.85
3.70	2.30	2.35	1.40	3.40	2.00	2.70	2.30	2.35	1.50	3.00	1.50	3.00	1.50	2.30	1.70	2.65	1.50	3.50	2.30	2.75	2.10	3.70	2.30	3.50	2.40	2.15	1.50
2.00	1.30	2.20	1.15	2.55	1.90	3.50	2.10	2.80	1.50	2.70	1.70	3.60	2.10	2.10	1.40	2.30	1.80	2.90	1.30	2.90	2.30	[illegible]	2.20				

TABLE 4

table in the briefest possible form and in a way that posterity would be given practically all of the information contained in the original set of data. How would you do it? The proposed answer is the tabulation of six quantities, namely, the average and standard deviation for each of the two properties, the correlation coefficient between these two properties, and the number of observations. Fig. 9-b shows this set of six statistics as we shall call them. These we believe contain the essential information of the original data. We shall understand that the essential information, to the best of our knowledge in the light of available methods of analysis, answers the questions for which the data were taken so that further analysis will not change our conclusions to a practical extent.

It is beyond the scope of the present paper to justify this contention. We can, however, see what this tabulation really gives us in this particular instance.

For example, the observed distributions of depth of sapwood and depth of penetration, respectively, are shown graphically in Fig. 9-c and 9-e. In a similar way, Fig. 9-g shows the scatter diagram. With the aid of the six statistics, we can obtain the smooth distributions shown in Fig. 9-d and 9-f closely approximating the observed distributions. In a similar way we can derive the ellipses shown in Fig. 9-h. For example, the two ellipses shown there were constructed to contain 50% and 99% of the observations respectively, and we see how closely they do this. True enough, distributions may be found where the closeness of check between the theoretical and observed distributions would not be as good as here illustrated, but even in such a case these six statistics should be included.[1]

Now let us return to consider what we could do by means of these six statistics calculated for the data in Table 2. It may readily be shown that the slopes of the three curves given in Fig. 5 may be calculated from the first five statistics. In other words, considered from the viewpoint of line of best fit, a tabulation of the five statistics enables us to obtain any one of the

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1. In other words non-linear regression would be involved.

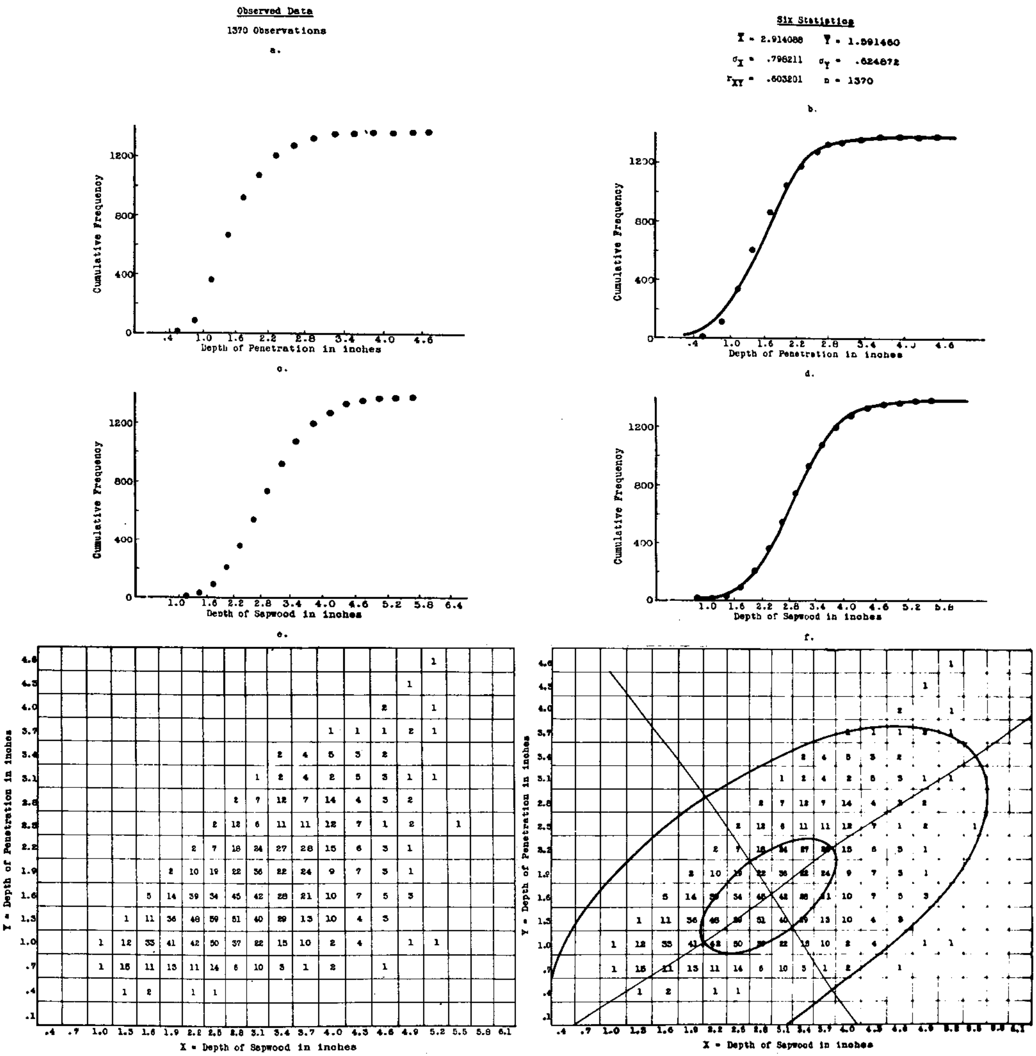

FIG. 9

three lines previously given without going through the methoa of least squares. It does more as is shown by Fig. 10 which is more or less self-explanatory in terms of what has already been said.

Before leaving this subject of presentation, we want to consider a little more carefully the significance of the average and standard deviation in respect to the essential information. The total information is obviously contained in the observed frequency distribution. Through the use of one of the simplest

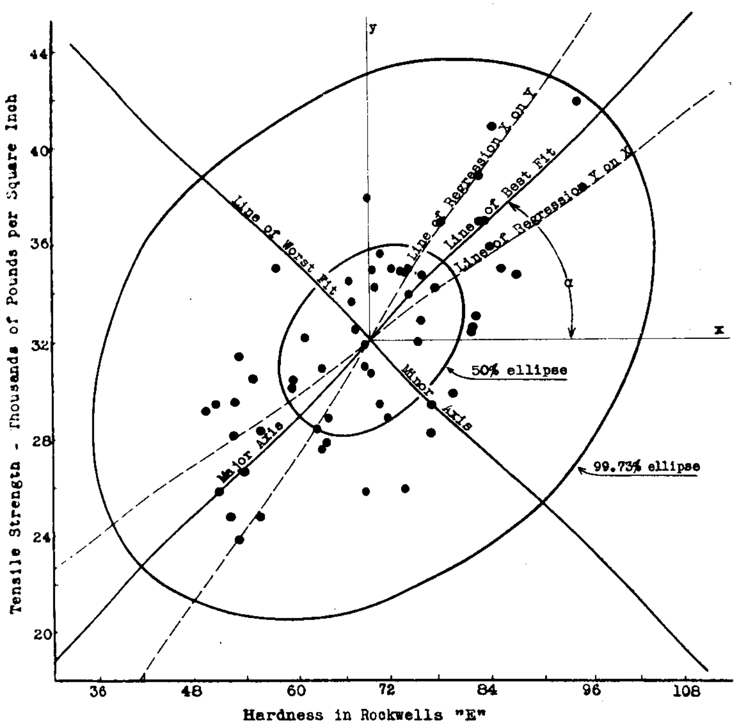

FIG. 10

theorems of mathematical statistics,[1] it is easy to show that the average and standard deviation make possible a remarkably close reproduction of the observed frequency distribution considering the fact that it is absolutely independent of the form of distribution of the observed data. For example, the curve in Fig. 11 is read in this way. The ordinate of this curve says that not less than the indicated per cent of the original observations lies within the average $\pm t\sigma$, where σ is the observed standard deviation.

Let us see what this means. Suppose we take any set of less than ten numbers, and calculate the average and standard deviation. This figure shows that all of the numbers lie within the range of the average ± 3 times the standard deviation. Carrying this on and making use of the figure, we see how much is really known from the average and standard deviation alone.

This is one of the justifications, of course, for believing that the average and standard deviation contain more essential information than any other pair of statistics. We must, however, consider one or two other illustrative reasons for such a belief. Why, for example, do we not use the mean error as is done so often in engineering and scientific work? The answer is that in the majority of cases to do so would amount to throwing away approximately 20% of the information given by the sample in respect to the standard deviation of the universe. Observations are

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1. Tchebycheff's Theorem.

generally too costly to permit of such wasteful practices.

Another reason why we should use the root mean square error is that for small samples we must make certain corrections which are in most instances large, and which are given only in terms of the root mean square error.[1]

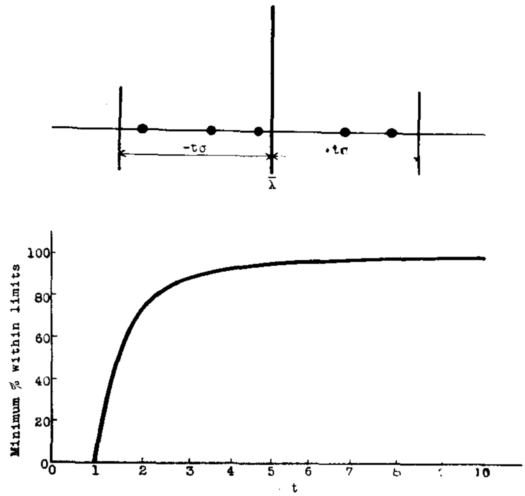

FIG. 11

Fig. 12 shows schematically the magnitude of the correction factor that must be applied to the observed standard deviation to obtain an estimate of the true standard deviation of the universe. This correction is clearly too large to be ignored. Thus in samples of five, such as are frequently used in practice, the estimate of true standard deviation which should be used is 29% larger than the standard deviation calculated from the sample which is customarily used.

IV - What Shall We Do About It?

Possibly this brief survey outlining some of the significant changes introduced in the modern physical concepts will be helpful in indicating how we should go about trying to give broader dissemination to statistical theory and its applications. From this viewpoint, it would appear undesirable for several engineering organizations to undertake the development of statistical theory as it

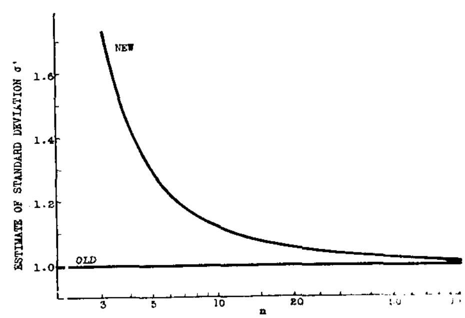

FIG. 12

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1. Furthermore many of the questions of interpretation involve the the use of this particular measure of dispersion and cannot be answered in terms of mean error or any other measure.

pertains solely to their work. What we really need, it would seem is a centralized organization taking into account the needs of the engineering profession at large. Enough has been said to show the generality of the methods.

Possibly sufficient interest can be aroused to make possible the publication of a supplement to the American Statistical Journal to present expository articles from authorities in the field exhibiting in a comprehensive and up-to-date way the improved statistical machinery referred to by "Nature".

This suggestion is thrown out at this point simply that you may keep it in mind while listening to the discussions which follow. I am sure that these will emphasize the need for statistical theory and hence will emphasize the need for some common medium of dissemination.