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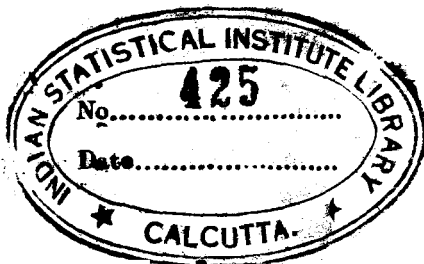
W. A. SHEWHART'S COLLECTION

CONTRIBUTION OF STATISTICS
TO THE
SCIENCE OF ENGINEERING

by

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Notes for lecture to be given at
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STATISTICAL CONTROL IN WAR EFFORT

Why

1. Reduce quantity of defective material both pieceparts and finished product. Hence save material and effort.
2. Reduce amount of necessary inspection. Hence save time.
3. Reduce tolerance range where desirable to save material or attain otherwise nonattainable result.
4. Attain maximum assurance that quality of product that cannot be inspected 100% because of destructive nature of test, will meet specified standards.
5. Efficient and accurate measure of wantability in both quantity and quality - operational research.

How

1. Provide sampling plans that give satisfactory consumer and producer risks. (1st and 2nd kind of errors).
2. Provide satisfactory statistical control procedures in production.
3. Provide
 - a) Effective sampling plans to determine what is wanted (spare parts, for example) in OPERATIONAL RESEARCH.

- b) Efficient research techniques for fingerprinting assignable causes in research and development, and
- c) Techniques for setting efficient tolerance limits under statistically controlled conditions.

How (continued)

- 1. Three correlated (circular) steps in mass production (any act of control).
 - 1.1 Private Industry Slide #20472
 - 1.2 War effort
- 2. Central position of statistics.
- 3. Two requisite kinds of statistical training.
 - 3.1 Training of men in production and inspection
 - 3.1.1 Sampling plans and control procedures.
 - 3.2 ~~3.1.2~~ Statistical consultant.

Exhibit: B-1274 ; Z1.1, 1.2, 1.3.

Object of Present Discussion

- 1. Consider some of the fundamental problems in control requiring the contribution of the statistical control consultant.
- 2. Indicate the kind of training in science and mathematics needed by the statistical consultant.

SOME FUNDAMENTAL CONCEPTS

Repetitive Operation

Any experimental procedure or rule of action described in such a way that it may be carried out again and again.

Examples:

- 1. Measurement of a physical constant.
- 2. Measurement of any quality.
- 3. Production process for each of the things we eat, wear, or use in any way.

Maximum Controlled Repetitive Operation

In terms of what we do

Any repetitive operation carried out under what the scientist or engineer would call "the same essential conditions" or, more strictly, under conditions that he can't do anything to control the outcome.

Ideal Example: Drawing from a bowl.

In terms of what we get

Slide 2 - Showing table of 144 values of thickness of relay inlays.
21612

Slide 3 - Plot of the results in Slide 2.
21617

- a) As they were taken.
- b) When drawn from a bowl.

Two

- a) Observed distribution.
- b) Order (runs) in sequence.

Characteristics

In terms of theoretical model

1. Concept of system of elemental causes as characterized by a given type of distribution function.
2. Concept of random order.

OBJECT OF APPLIED SCIENCE

Set up rules of action such that he may make valid prediction of the results to be expected if such rules of action are carried out.

Fundamental Experimental Fact

If a repetitive operation is carried out again and again under presumably the same essential conditions, the results $R_1, R_2, \dots, R_1, \dots$ will not in general be the same.

Fundamental Problem of Valid Prediction

1. Limitations of deterministic laws.
2. Limitations of probabilistic laws.
3. Necessity for discovering assignable causes before valid prediction is possible.

III

INSPECTION - TEST OF HYPOTHESIS

Use of statistical theory as a curative means of screening product even though assignable causes are present and controlling

1. Errors of 1st kind.
2. Errors of 2nd kind.

Given a repetitive process

$$X_1, X_2, \dots, X_1, \dots X_n, \dots$$

break up product into lots of N and from each choose a sample of n . Accept or reject on basis of number c of defects found.

Choice

- a) Lot size N
- b) Single or multiple sampling plan
- c) Sample sizes n_1, n_2, \dots
and
rejection numbers c_1, c_2, \dots
- d) Probabilities of errors of 1st and 2nd kind.

Slide 4

Practical rules or plans

Plan of single, double, or multiple sampling developed for each specific case to be used by man on job.

Literally hundreds of plans in operation in a single company. Many technical difficulties and practical compromises necessary.

Limitations of such sampling

Will reject more bad lots than good ones, but if quality of only one level of statistically controlled process average is offered, then accepted (uninspected portions) is no better than rejected.

One of greatest dangers is that one approaching applications from this angle will be like one of the blind men and the elephant.

He will fail to note that assurance of quality is not improved by sampling if quality is not in state of control.

Hence sampling plans should be set up so that total inspection will decrease as approach state of statistical control.

Theoretical Background Needed by Consultant

General theory of testing statistical hypotheses against errors of 1st and 2nd kinds.

<u>True Situation</u>	<u>Statement Advanced</u>	
	<u>$H = H_0$</u>	<u>$H \neq H_0$</u>
$H = H_0$	Correct	1st kind of error
$H \neq H_0$	2nd kind of error	Correct

Critical region - probability of 1st kind of error is size of the critical region.

Equivalent tests - those having same size critical region.

Power of test - probability of rejecting H_0 when the true hypothesis is alternative simple hypothesis H_1 .

Best critical region - that of greatest power.

Slide 5

#19000

DETECTING PRESENCE OF AND FINDING

ASSIGNABLE CAUSES

Show slide 3 again

#21617

Three Fundamental Questions

1. Are there any assignable causes present?
2. What are the assignable causes?
3. What will be resultant variation when assignable causes are removed?

CONTROL CHART Procedure for Production

Problem Establish practical fool-proof procedure for detecting presence of assignable causes.

Slide 6

#21616

Prerequisites

- a) Protect against errors of 2 kinds.
- b) Indicate which trouble enters.
- c) Rigorously correct in limit as assignable causes are removed.

Importance of breaking up into small subsamples.

Slide 7

#22369

Average Sigma / σ^2

<u>Size of Subsample</u>	<u>Observed Sequence</u>	<u>Drawn from Bowl</u>
4	12.6	16.5
12	14.7	17.1
24		
36	16.4	
72	17.3	
144	17.3	

Importance of Sampling Theory

One illustration:

Slide 8

#8889

Distribution of averages approach
normality.

Tolerance range theory

Question 2

RUN Chart and Other Procedures for Research

Problem - Establish methods for fingerprinting assignable causes.

Here no single rule is likely.

1. Serial Correlation

Thickness in arbitrary units of inlay on 144 relay springs - data of Slide 2.

$X_1, X_2, \dots, \dots, X_{143}, X_{144}$
 $X, X, \dots, \dots, X_{143}, X_{144}$
55, 72, 93, 80, 88
55, 72, 93, 54, 88

from Anderson's formula

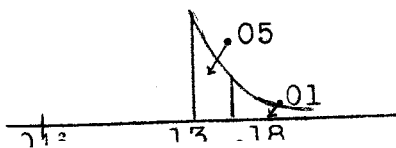
$$L R_N = \frac{LCN}{\sqrt{N}} = \frac{(X_1 X_{L+1} + X_2 X_{L+2} \dots X_N X_L) - (\sum X_i)^2}{\sum X_i^2 - (\sum X_i)^2 / N}$$

= serial correlation coefficient for lag L and N observations.

Taking L = 1 and N = 144

$$1 R_{144} = \frac{274119 - 252590}{295693 - 252590} = \frac{21529}{43103} = .4995$$

Test of significance: If no correlation exists, the probability is .01 that R will not exceed .182; .05 that it will not exceed .130.



$$x_2, \dots, x_{n_1+1} = 55, 72, \dots, 33, 20 \quad (7)$$

$$x_1', x_2', \dots, x_{n_2+1}' = 43, 26, \dots, 80, 88$$

∴ R.A. Fisher's formula

$$\bar{x} = \frac{1}{n_1+1} S(x), \quad \bar{x}' = \frac{1}{n_2+1} S(x')$$

$$= \frac{1}{n_1+n_2} \{S(x-\bar{x})^2 + S(x'-\bar{x}')^2\}$$

$$= \frac{\bar{x} - \bar{x}'}{2} \sqrt{\frac{(n_1+1)(n_2+1)}{n_1+n_2+2}}$$

∴ as shown above, first 72 values vs last values -

$$\bar{x} = 40.74$$

$$\bar{x}' = 43.03$$

ice

sid

30

16

$$= \frac{1}{71+71} \{16512 + 26402\}$$

$$= 302.2$$

$$= \frac{40.74 - 43.03}{17.38} \times 6 = -.7910$$

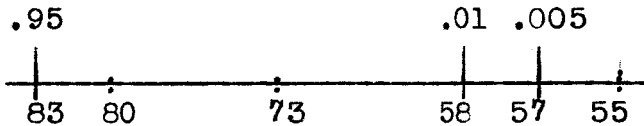
test of significance :- If the two samples are drawn from same population the probability of getting a t as large as $-.791$ is between .5 & .4

Statistical Methods for Research Workers - R.A. Fisher -

Criterion A

Number of Runs

<u>Observed Sequence</u>	<u>Theory</u>	<u>Sequence from bowl</u>
55	73	80



Criterion B

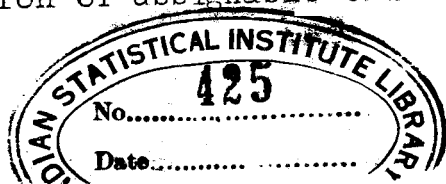
Probability of getting sequence with run of i or more

<u>Length i</u>	<u>One side</u>	<u>Either side</u>	<u>Both sides</u>
7	.36	.61	.20
8	.21	.36	.06
13	.006		

Advantages of run chart

1. Tells where to look.
2. Suggests nature of cause.
3. No computation necessary.

Note: Runs in seven sequence of 20 give no indication of assignable causes.



Question 3

What reduction in variance if assignable causes are removed?



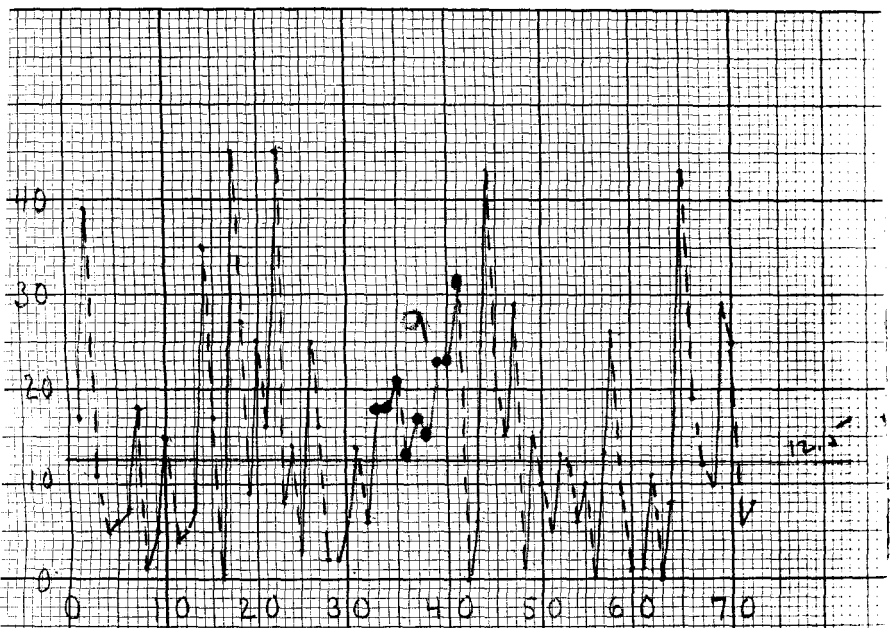
2. Average $\overline{\sigma}$ for small samples of n

$$n = 4 \quad \overline{\sigma} = 12.6$$

$$n = 144 \quad \overline{\sigma} = 17.5$$

What other methods?

3-4-43
DTH



144 Numbers. THICKNESS OF INLAY

odd-even

RUNS -

above or below

Chance of sequence
with run of 9 in. off

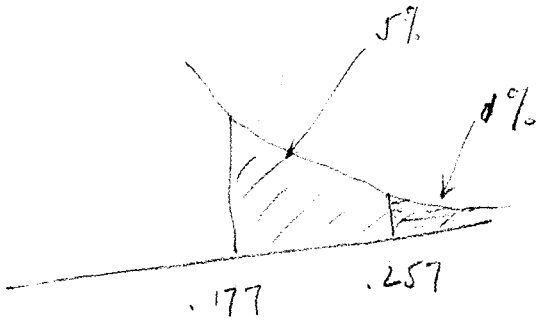
1	12
2	12
3	3
4	3
5	
6	1
7	
8	
9	1

32

$112 = 326$
 $100 = 29$
 $12 = 400$

for 72 values of μ $\sigma = 1$

$$L=1, R_{N=1000} = .6145$$



WHAT STATISTICAL CONSULTANT NEEDS TO KNOW

1. Run theory.
2. Other tests (characteristics of random sequence).
 - 2.1 serial correlation
 - 2.2 Wald-Wolfowitz test
 - 2.3 Analysis of variance, etc.
3. Effects of different kinds of causes on types of runs.
 - 3.1 Erratic effects.
 - 3.2 Running average effects.
 - 3.3 etc.

Note

None of this available in book form. Many tables unpublished. Much more needs to be done.

SETTING TOLERANCE LIMITSProblem

1. First attain state of statistical control.
2. Then set economic tolerance limits.

Fundamental problem in all applied science.

Slide 10

#16920

Involves choice:

n \bar{P}' Range
 e P'_e

Valid Tolerance Range Prediction

1. Given sample of n from controlled state.
2. Set limits that will cut off on the average \bar{P}' of universe.

2.1 For max - min limits

$$\frac{(1-\bar{P}')(n+1)}{2} = 1 \quad (1)$$

2.2 If normal

$$\bar{X} \pm t_{\bar{P}'} \sqrt{(n+1)/n} \sqrt{n/(n-1)} \text{ sigma} \quad (2)$$

Example 2.1 $n = 100$ $\bar{P}' = 99/101 = .980$

Observed in 40 samples of 100:

$P_1 = .984$

$P_2 = .988$

$P_3 = .980$

$\bar{P} = .9799$

$\dots \dots \dots$
 $P_{40} = .993$

Example 2.2

$$n = 100.$$

Choose $\bar{P}' = .900$ in Fisher's tables, then $t_{\bar{P}'} = 1.662$.

For forty samples of 100, percent P included between $\bar{X} \pm 1.668$ sigma, was as follows:

$$P_1 = .8804$$

$$P_2 = .9170$$

$$P_3 = .8997$$

.....

$$P_{40} = .8998$$

$$\bar{P} = .9001$$

Theoretically $40 \times .68 = 27.2$ should fall within the range $\bar{P}' \pm 1 \text{ sigma}_{P'}$.

Of the 40 samples of 100, 29 observed P's fell within these limits, where

$$\text{sigma}_{\bar{P}'} = \frac{t_{\bar{P}'}^2 e^{-t_{\bar{P}'}}}{P_i n}$$

$$= .0236 \quad \text{when } n = 40$$

3. Compute for Max-min range probability that neither tail will cut off more than fraction e

For $n = 40$ and $\bar{P}' = .9802$, $P_e = \underline{.99}$

Example:

Expected number of ranges cutting $e = .05$ is $.99 \times 40 = 39.6$.

Observed number = 39.

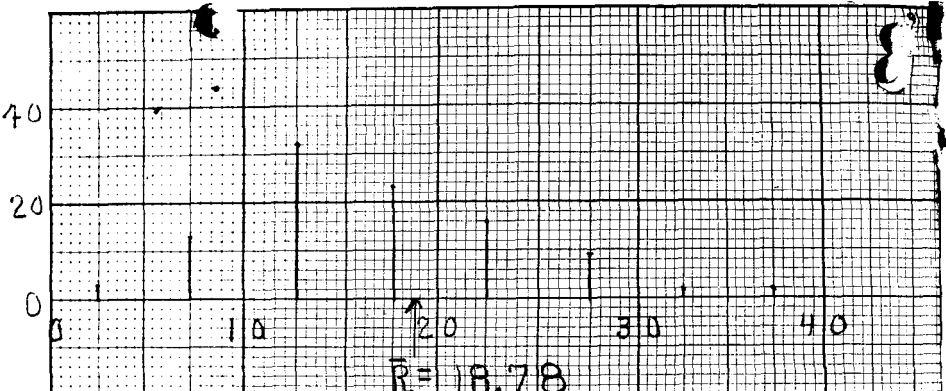
Saving Material

Even though, for a given n , both \bar{P}' and P_e' can be fixed satisfactorily, nevertheless we may want to close up tolerances to save material.

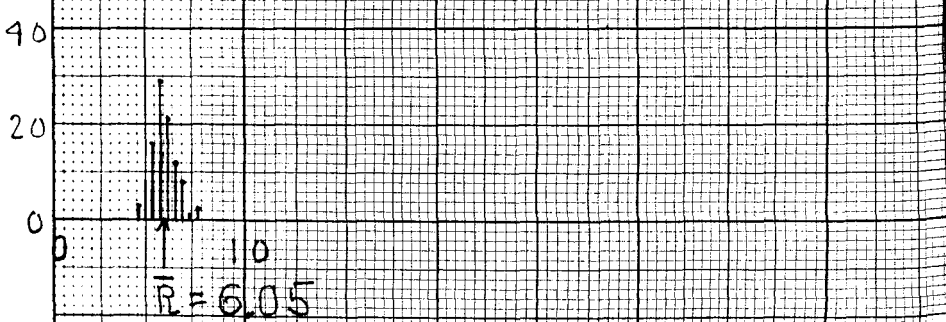
Discuss attached figure.

Difference Between Valid Prediction for tolerance range and Confidence Range

Slide 16921



Ranges equal $\bar{X} \pm 11.618\sigma$ for 100 Samples of 4



Ranges equal $\bar{X} \pm 3.0758\sigma$
for 100 Samples of 40

Frequency Distribution

of Ranges of $\bar{X} \pm t_{P,n} \sqrt{\frac{n}{n-1}} \frac{nt+1}{n} \sigma$
for $P = 99.72$

B. Analysis of variance W. A. SHEWHART'S COLLECTION

<u>Columns of size</u>	<u>Significant at</u>	
	<u>.05 Level</u>	<u>.01 Level</u>
4		✓
12		✓
36		✓

4. Some Disadvantages of such Tests

1. Long Computations.
2. Even though statistical significance is indicated, there is no indication of kind of trouble and where it comes in.

5. Run Chart

1. Runs up and down.
2. Runs above and below some percentile value.

Slide 9

#21617

Observe runs of 8, 13, 777, if median is used instead of average.

