

INDIAN STATISTICAL INSTITUTE
 Mid-Semestral Examination: 2011-2012
 MS (Q.E.) and MStat II Year
Econometric Methods II

Date: 01/09/11

Maximum Marks 100

Duration 3 hours

All notations are self-explanatory. You can answer any part of any question.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic but endogenous. Assume that all other CLRM (ideal conditions) assumptions hold. Let Z be a matrix of instruments for X . Number of columns in Z may be more than that of X .
 - a. Find the GMM estimator of β . Show that the GMM estimator is consistent.
 - b. Find the GMM estimator of β when data is heteroscedastic.
 - c. Show that more number of instruments lead to better estimates of β than the case when one has exact number of instruments.

[(5+5)+10+10=30]

2. Consider a standard dynamic panel data regression model as $Y_{it} = \alpha_i + \rho Y_{i,t-1} + \epsilon_{it}$, with all CLRM assumptions.
 - a. Show that both fixed and random effect estimates of ρ are inconsistent.
 - b. Propose an IV estimator for ρ . Show that your proposed estimator is consistent.
 - c. Describe Arellano-Bond's GMM estimator for ρ . [(5+5)+10+10=30]

3. The following question concerns the determinants of low birthweight among newborn infants in the US. The dependent variable called "undewt" is a dummy for whether the infant weighs less than 100 ounces at birth. The covariates are mother's years of education, the infant's gender and the family's annual income in 000 dollars.

| Variable | Description |
|----------|--|
| undewgt | equals 1 if birthweight less than 100 ounces |
| motheduc | mother's yrs of educ |
| male | equals 1 if male child |
| faminc | 1988 family income, \$1000s |

The summary statistics are as follows:

| <u>Variable</u> | <u>Mean</u> | <u>Std. Dev.</u> | <u>Min</u> | <u>Max</u> |
|-----------------|-------------|------------------|------------|------------|
| undewgt | 0.149 | 0.356 | 0.000 | 1.000 |
| motheduc | 12.936 | 2.377 | 2.000 | 18.000 |
| male | 0.521 | 0.500 | 0.000 | 1.000 |
| faminc | 29.027 | 18.739 | 0.500 | 65.000 |

A probit regression of undewgt on the covariates produces the following output. The column marked "z" is simply the coefficient divided by the std error, the column marked P>z is the 2-sided p-value corresponding to this z-statistic.

| | | | |
|-----------------------------|---------------|---|--------|
| Probit regression | Number of obs | = | 1387 |
| | LR chi2(3) | = | 15.41 |
| | Prob > chi2 | = | 0.0015 |
| Log likelihood = -576.77002 | Pseudo R2 | = | 0.0132 |

| <u>Y=undewgt</u> | <u>Coef.</u> | <u>Std. Err.</u> | <u>z</u> | <u>P>z</u> |
|------------------|--------------|------------------|----------|---------------|
| motheduc | -0.011 | 0.020 | -0.560 | 0.578 |
| male | -0.238 | 0.083 | -2.860 | 0.004 |
| faminc | -0.006 | 0.003 | -2.200 | 0.028 |
| _cons | -0.625 | 0.237 | -2.640 | 0.008 |

- Write down the probit model that corresponds to this output. Please be clear about the assumptions implicit in the model.
- What can we conclude from the row corresponding to the regressor faminc?
- Calculate the predicted probability of being underweight for a male child born to a mother with 10 years of education and annual family income equal to 30 thousand dollars?
- How would the predicted probability change if all else equal, it was a female child?
- How would you test that the change in the previous question is statistically significant?

$$[(4+2)+5+5+5+4=25]$$

- The following question concerns the determinants of smoking among poor pregnant mothers in the UK. The dependent variable called "smoke" captures how many cigarettes a woman smoked on average while pregnant. It is censored above at 20. The covariates are the woman's years of education (motheduc), her husband's years of education (fatheduc) and the family's annual income (faminc) in 000 pounds. The summary statistics are as follows:

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|----------|----------|--------|-----------|-------|--------|
| smoke | 1388.000 | 1.924 | 5.156 | 0.000 | 20.000 |
| faminc | 1388.000 | 29.027 | 18.739 | 0.500 | 65.000 |
| fatheduc | 1192.000 | 13.186 | 2.746 | 1.000 | 18.000 |
| motheduc | 1387.000 | 12.936 | 2.377 | 2.000 | 18.000 |

The output of a tobit regression of smoke on the covariates is as follows.

Number of obs = 1191
 LR chi2(3) = 63.28

Prob > chi2 = 0.0000
 Log likelihood = -3505.7626
 Pseudo R2 = 0.0089

| smoke | Coef. | Std. Err. | t | p-value |
|----------|---------|-----------|-------|---------|
| faminc | -0.0204 | .0089 | -2.30 | 0.022 |
| fatheduc | -0.069 | .0688 | -1.01 | 0.311 |
| motheduc | -0.321 | .0770 | -4.16 | 0.000 |
| _cons | 7.503 | .8220 | 9.12 | 0.000 |
| sigma | 4.82 | .1027 | | |

- Write down the likelihood function corresponding to the above tobit model.
- What can we infer from the row corresponding to the covariate faminc in the tobit output table?
- What *policy* conclusions can we draw from the estimated effects of the covariates on the woman's smoking behavior – i.e., if you were advising the government about ways to reduce smoking for would-be mothers, what would be your recommendation?
- How can we test the joint significance of the two regressors fatheduc and motheduc in this model? (you do not have to perform the test, just write down what calculations you would do). Intuitively, what are we testing here?
- Calculate the predicted amount of smoking by a pregnant woman who went to school for 20 years, whose husband went to school for 20 years and the family income is 70 thousand pounds. Is this a reasonable number?

[5+5+5+5+5=25]

INDIAN STATISTICAL INSTITUTE
M.S. (Q.E.), 2nd Year
Mid-Semester Examination: 2011-2012
Econometric Applications I

Date: 03 September 2011

Duration: 3 hours

Maximum marks: 100

[Answer question no. 1 and any **three** from the rest of the questions]

1. Suppose the students of first year in a college either come from Bengali medium schools or from English medium schools. How will you compare the two groups of students on the basis of marks obtained in a certain examination using a regression model with dummy variable giving a value '0' for students of English medium school and '1' for the other group? Clearly state the assumptions you are making at each stage. What will happen if you take '1' and '3' instead of '0' and '1' respectively?
[20+5=25]
2. Calculate (i) Coefficient of Variation (CV), (ii) Relative Mean Deviation (RMD) and (iii) Lorenz Ratio of incomes of N individuals with incomes 1, 2, 3, ..., N units, respectively. What will happen if N goes to infinity? Explain. [25]
3. Write down the desirable properties of a measure of inequality. Examine variance of logarithms in the light of these properties. [25]
4. (i) Describe how one can construct Lorenz Curve and hence Lorenz Ratio graphically and numerically given grouped data.
(ii) Write down the important steps in deriving Atkinson's measures of inequality based on the Social welfare Function Approach. How can one interpret the unknown parameter in the measure? [12+15 = 25]
5. Write short notes on any two of the following:
 - (a) Kapteyn's Law of Proportionate Effect and it's modification due to Kalecki.
 - (b) Positive Measures of Inequality
 - (c) Estimation of parameters of (two-parameter) lognormal distribution from grouped data.
 - (d) Properties of Pareto distribution. [12½+12½=25]

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: (2011-2012)
MS (Q.E.) II Year
International Economics I

Date: 08.09.2011

Maximum Marks 40

Duration 3 hours

Group A

Answer any three questions

All questions carry equal marks

- (1) 'In the presence of factor-intensity reversal in the HOS model, Stolper-Samuelson theorem is not valid' – Examine the validity of this statement.
- (2) Consider a 3 X 2 specific factor model where labour endowment is infinitely large and wage rate is fixed. Show that the direct price-out response is positive but cross price output response is nil.
- (3) What is a production possibility locus? Show that this locus slopes negatively in the HOS model as well as in the specific factor model.
- (4) Introduce a third sector in the specific factor model that produces a non-traded final good with labour as the only input and with a linear production technology. Formulate the model and analyse how factor prices are determined.

Group B

Answer all questions

1. Show that in a two agent setting, with equilibrium being Walras' stable, a transferor is never a gainer. (9)
2. Considering a 2 country, 2 commodity trading world, show that imposing an ad- valorem export tax is the same as imposing an ad-valorem import tariff when the government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax. Report if you find anything interesting in the result. [Note: An export tax on good i means the following: $p_i(1 + \tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.]

(10)

Mid-Semester Examination
Economic Development I
MSQE I & II

Date: 9.9.11

Maximum Marks: 40

Time: 2 hours

Answer question 1 and either 2 or 3.

1. Consider two infinitely-lived agents with identically and independently distributed income streams every period. For each agent, income can take two possible values, 100 and 0, with equal probability. In any period, the utility function of each agent is given by $U(c) = c^{1/2}$, where c is consumption in that period. The agents are in an informal insurance arrangement in which if in a period income realizations are different, the high income agent makes a transfer to the low income agent. By assumption, the agents are unable to save or store. Also assume that the rate of discount $r = 25\%$.

(a) Find the first best contract and check whether it is implementable.

(b) Suppose the income is in terms of grains and the agents are able to store their grains only for the next period. After two periods, the grains are rotten. Can storage be a good alternative to informal insurance?

[15+5]

2. A risk neutral landlord gives his land to a risk-averse farmer for cultivation but cannot observe his work effort. The output, which is observable, depends on the effort and the realization of a random term. Assuming that the output can be either high (H) or low (L) and the effort level can be either 0 or 1, with high effort increasing the probability of high output, show that share tenancy is the best incentive contract. [20]

3. Show, in terms of a suitable model, how a less developed economy can get stuck in a bad equilibrium due to coordination failure and how a big push by the government can take the economy to a good equilibrium. [20]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2011-12

Course name: MSQE II
Subject name: Game Theory II

Date: 10.9.11.

Maximum marks: 60

Duration: 2 hours

Please answer parts A and B on separate answer books. Each part is worth 30 marks.

Part A

Q.1. (a) Define balancedness of a family of subsets of a player set. [1]

(b) State the Bondareva-Shapley Theorem and prove its sufficiency part. [1+14]

Q.2. Define the core and dominance core of a game. Show that the former is a subset of the latter. When do they coincide? Prove your claim. [2+3+4]

Q.3. Consider the dominance relation over imputations, i.e., the dominance relation between two imputations via a coalition. "This relation is not reflexive, not symmetric, but transitive". Do you agree or disagree with this statement? Justify your answer. [5]

Part B

Consider the sealed-bid, independent private value auction model done in class. There are N bidders. Values are drawn from the set $[0, \bar{x}]$ according to the continuously differentiable, increasing distribution function F .

(I) Suppose the seller wishes to sell using a first-price auction.

Show that the symmetric equilibrium bid function, assuming bidders with 0 value submit a 0 bid, is $\beta(x) = E(Y_1 | Y_1 < x)$, where Y_1 is the highest-order statistic of $N - 1$ independent draws from the distribution F , i.e., the highest of the values of $N - 1$ bidders. [10]

(II) Suppose additionally $\bar{x} = 1$, and F is uniform.

What is the expected revenue to the seller? What happens to expected seller revenue as the number of bidders becomes very large? [5+5]

(III) Suppose additionally the seller is contemplating setting a reserve price.

What is her optimal reserve price? What happens to optimal reserve price as the number of bidders becomes very large? [5+5]

12.09.11

Date: ~~29.08.11~~

Maximum Marks 40

Duration 3 hours

Answer Group A and Group B in separate scripts

Group A

Answer any two questions

1.(a) Suppose, rates of return of two risky assets 1 and 2 are **perfectly negatively** correlated and their standard deviations are σ_1 and σ_2 . Show that a portfolio P having w proportion of fund invested in asset 1 and the rest in asset 2 have a rate of return with standard deviation given by

$$\sigma_P = w\sigma_1 - (1-w)\sigma_2.$$

Suppose, $\sigma_1 = 0.2$ and $\sigma_2 = 0.8$, and mean rates of returns of the two assets are, $\mu_1 = 0.4$ and $\mu_2 = 0.9$. Can you find a particular portfolio of these two assets which behaves exactly like a *risk-free* asset yielding a particular rate of return with *certainty*? Explain.

(b) Consider CAPM with a risk-free asset (with return, r_f). Let V be the variance-covariance matrix. The weight vector and the expected return of the *optimal* portfolio M of risky assets (i.e. the tangency portfolio of the mean-variance frontier) are given by

$$w_M = \lambda V^{-1} (e - \mathbf{1} \cdot r_f) \quad [\mathbf{1} = \text{vector with each element 1; } e = \text{vector of assets' mean returns}]$$

$$E(r_M) = \lambda H + r_f \quad [\lambda = \text{Lagrange multiplier; } H \equiv (e - \mathbf{1} \cdot r_f)' V^{-1} (e - \mathbf{1} \cdot r_f)]$$

Using the above results, compute $\sigma^2(r_M)$ and show that $\lambda = \sigma^2(r_M) / \{E(r_M) - r_f\}$.

Derive the equation for the so-called *security market line*. Interpret this relation.

[4 + 6] = [10]

2. Write a note on **any one** of the following topics:

- (a) Arbitrage pricing theory with two factors;
- (b) Equity-premium puzzle.

[10]

3. Consider a two-period asset pricing model based on intertemporal expected utility maximization and consider an investor who wishes to invest in the current period in a risk-free asset as well as a number of risky assets.

Set up the investor's problem. From the optimality conditions show that the expected return of an asset equals the risk-free rate plus a risk-premium. Show that risk-premium here involves the covariance of marginal utility of consumption with the asset's return. Interpret this relation.

[10]

PTO

Group B

Answer all questions

1) In the Blanchard Yaari model, show that if wages are falling more sharply with age, higher will be the per capita capital stock in steady state. [10]

2) In an infinite horizon model, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital market.

In this context discuss the problems associated with either a very low or a very high rate of interest. [10]

INDIAN STATISTICAL INSTITUTE
 First Semestral Examination: (2011-2012)
 MS (Q.E.) II Year
Macroeconomics II

Date: 14.11.11

Maximum Marks 60

Duration 3 hours

Group A

Answer any two

1. A Robinson Crusoe economy experiences productivity shock, $\ln z_t$ (i.i.d. with zero mean). Crusoe lives forever, gives labour l_t , produces output y_t and consumes c_t in period t . Labour yields disutility. He maximizes expected present value of lifetime utility:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right] = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{\gamma \sigma}{\sigma+1} l_t^{\frac{\sigma+1}{\sigma}} \right\} \right] \quad (\gamma \text{ and } \sigma > 0; \beta < 1) \text{ subject to}$$

(i) $c_t + k_{t+1} = y_t = z_t k_t^\alpha l_t^{1-\alpha}$ ($0 < \alpha < 1$ and capital, k , depreciates fully after one period).

Let $v(k_t, z_t)$ denote the value function which satisfies the following *Bellman* equation

$$(ii) \quad v(k_t, z_t) = \max_{c_t, k_{t+1}, l_t} \left[\left\{ \ln c_t - \frac{\gamma \sigma}{\sigma+1} l_t^{\frac{\sigma+1}{\sigma}} \right\} + \beta E_t \{ v(k_{t+1}, z_{t+1}) \} \right]$$

An educated guess about the solution of (ii) is that $v(\cdot)$ takes the following form:

(iii) $v(k_t, z_t) = \theta_0 + \theta_1 \ln k_t + \theta_2 \ln z_t$ { $\theta_0, \theta_1, \theta_2$ are constants to be determined. }

(a) Using (i) and (iii), maximize (ii) with respect c_t (or k_{t+1}) and l_t . Show that neither the optimal value of k_{t+1}/y_t nor that of l_t depends on k_t and z_t .

(b) Find θ_1 and θ_2 as well as optimal values of k_{t+1} and l_t in terms of α and β .

[9 + 6] = [15]

2. An individual maximizes *two-period* utility (under *certainty*). He consumes c_i and works l_i at a *given* (and known) real wage rate w_i in period i ($i = 1, 2$). Let β (less than unity) be the discount factor and r be the given (and known) rate of interest on savings made in period 1. The individual (with total available labour in each period normalized to unity) *maximizes* the following *utility function*:

$$U = \ln c_1 + b \ln(1 - l_1) + \beta \{ \ln c_2 + b \ln(1 - l_2) \} \quad \text{subject to his budget constraint.}$$

- (a) Write down the *budget constraint* of this individual. Find *optimal* values of his labour supply l_1 , l_2 and of the Lagrange multiplier associated with this constraint in terms of w_1 , w_2 , r and the parameters b and β .
- (b) What would be the effect of a *fall* in r on his labour supply in two periods? Give an intuitive explanation also. On the basis of your result comment on the issue of *intertemporal substitution of leisure*.

[9 + 6 = 15]

3. Write a note on calibration of a real business cycle model, describing briefly the methodology used.

[15]

Group B

Answer all

1. Show that with investments having convex adjustment costs, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest. Find out the conditions on the production function and on the investment adjustment cost function under which Tobin's marginal q would be equal to the average q .

(10+7)

2. Consider a two period OLG model which can potentially accommodate money in equilibrium. Assume that agents live for two periods and are endowed with Y units of good in the first period when they are young and earn nothing in the second period of their life; when they are old. These agents can potentially carry over the first period endowment of goods in the second period, in which case they will get in the second period $\frac{1}{1+\delta}$ times what they carry over from the first period, where $-1 < \delta$.

Agents have a two period utility function given by $U = \ln c_t^y + \frac{1}{1+\rho} \ln c_{t+1}^o$, where c_t^y and c_{t+1}^o denote the consumption of a young agent in period t and the consumption of the agent in period $t+1$ (when she is old) respectively and $0 < \rho$. The old agent also gets a transfer from the government financed through seigniorage where the transfer to the old in period $t+1$ denoted by $T_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}}$, where M_{t+1} and P_{t+1} denote the money supply and price level in period $t+1$. The government follows a simple monetary rule given by $M_{t+1} = (1 + \alpha)M_t$, with $-1 < \alpha$.

Derive the condition on the rate of inflation for which agents would hold money in equilibrium.

Also show that, in such a case where money is held in equilibrium, the rate of inflation is equal to the growth rate of money. (13)

INDIAN STATISTICAL INSTITUTE
 First Semestral Examination: 2011-12
 M.S. (Q.E.), 2nd Year, Semester I
 Econometric Applications I

Date: 16.11.2011

Maximum marks: 100

Duration: 3 hours

[Note: Answer any four questions. Marks allotted to each question are given at the end of the question]

1. (a) Suppose X is a size variable having the distribution function $F(\cdot)$. Define Lorenz Ratio (LR) of X . Show that it can be expressed as

$$LR = 1 - 2 \int_0^1 F_1 dF,$$

where the symbols have their usual meanings. Prove that LR can also be obtained from

$$E|Y_1 - Y_2| / (2M),$$

where Y_1 and Y_2 are i.i.d. having the distribution function $F(\cdot)$ and M is their common mean.

(b) Show, in particular, that if $X \sim LN(\mu, \sigma^2)$ then $LR = 2\Phi(\sigma/\sqrt{2}) - 1$. [20+5=25]

2. (a) Consider the following production function $Q = f(K, L)$ with two factors of production – capital (K) and labour (L). Define elasticity of substitution (σ). Prove that the following expressions for σ are equivalent.

$$\sigma = \frac{R(LR + K)}{KL \left(R \frac{\partial R}{\partial K} - \frac{\partial R}{\partial L} \right)} = \frac{R(LR + K)}{KL \left(\frac{d^2 K}{dL^2} \right)} = \frac{Q_K Q_L (K Q_K + L Q_L)}{-KLT},$$

where $T = Q_K^2 Q_{LL} + Q_L^2 Q_{KK} - 2Q_K Q_L Q_{KL}$ and R is the MRTS. Other symbols have their usual meanings. Hence prove that σ can be simplified to $\sigma = Q_K Q_L / (Q Q_{KL})$, if the production function is homogeneous of degree 1.

(b) State the statistical problems involved in the estimation of Cobb-Douglas Production function. [20+5=25]

3. Define the three-parameter lognormal distribution. State and prove its properties. How can one test whether a set of data is coming from a three-parameter lognormal distribution? Also discuss an estimation procedure for this distribution. [2+14+3+6=25]

[Any result of two-parameter lognormal distribution used should be proved]

4. Suppose you have a sample of size n from a Pareto distribution. Describe different procedures of estimating the parameters of the distribution for (i) grouped and (ii) ungrouped cases of sampled data. [25]

5. (i) Write down the popular forms of Engel curves.
 (ii) How will you choose an Engel curve form amongst this list? Give justifications for your answer. [12½+12½=25]

6. Write short notes on any two of the following:

- (a) Treatment of household size in Engel curve analysis.
 (b) Measures of poverty.
 (c) Methods of estimation of Engel elasticities proposed by Iyengar. [12½+12½=25]

MSQE II - Theory of Finance I

Final Exam. / Semester I 2011-12

Time - 3 hours/ Maximum Score - 50

19/11/11

NOTE : SHOW ALL YOUR WORK. RESULTS USED MUST BE CLEARLY STATED.

1. Let $\{S_n : n = 0, 1, \dots\}$ be a sequence of random variable such that,

$$S_{n+1} = \begin{cases} S_n + 1 & \text{with probability } p \\ S_n & \text{with probability } r \\ S_n - 1 & \text{with probability } q, \end{cases}$$

with $S_0 = c$. a positive integer, $0 < p, q, r < 1$, $p \neq q$ and $p + q + r = 1$.

- (a) (7 marks) Let $X_n = S_n - n(p - q)$. Let $Z_n = (q/p)^{S_n}$. Define an appropriate filtration $\{\mathcal{F}_n\}$ and show that, $\{X_n\}$ and $\{Z_n\}$ both are martingales with respect to $\{\mathcal{F}_n\}$.
- (b) (7 marks) Let a and b be integers such that, $0 < a < c < b$. Define $T = \inf\{n : S_n = a \text{ or } b\}$. Assume further that $E(Z_T) = E(Z_1)$. Find
- $P(S_n \text{ hits } b \text{ before } a)$,
 - $E(S_T)$.

2. (a) (2 marks) Let $\{X_t\}$ be a continuous time stochastic process. Write down the exact conditions under which $\{X_t\}$ can be said to be a standard Brownian motion.
- (b) (4 marks) Let $\{B_t\}_{t \geq 0}$ be a standard Brownian motion. Define a process $\{X_t\}_{t \geq 0}$ as follows:

$$X_t = \begin{cases} tB_{\frac{1}{t}} & \text{for } t \neq 0 \\ 0 & \text{for } t = 0 \end{cases}$$

Show that $\{X_t\}$ is a standard Brownian motion.

- (c) (4 marks) Let $a > 0$. Define, $\Lambda = \sup\{t > 0 : B_t = at\}$. Here Λ is the last time, the (standard) Brownian motion $\{B_t\}$ touches/crosses a line at . Find the distribution of Λ .
- (d) (4 marks) Let $\sigma > 0$. For $t \geq 0$, define,

$$S_t = S_0 e^{(r - \frac{\sigma^2}{2})t + \sigma B_t}$$

Let S_t , be the price of a stock in Rupees (under risk-neutral model) Let $\sigma = .3$, and $r = 6\%$ be the volatility per annum for the process. Find the probability that the price of the stock would never be more than the current price, $S_0 = 2$, in terms of Rupees after 2 years from now.

3. Assume that S_t satisfies,

$$dS_t = \mu S_t dt + \sigma S_t dB_t,$$

where $\{B_t\}$ is a standard Brownian motion,

- (a) (6 marks) Let $X_t = \frac{c}{S_t}$. Use Itô's formula and derive the SDE for X_t . Hence or otherwise, find X_t , in terms of μ , σ and B_t .
- (b) (8 marks) Let S_t is the price of one unit of Euro in terms of Rupees. Suppose a contract that pays 2.5 Euro if the price of one unit of Euro goes Rs.75 or above, and pays 1.5 Euro if the price of one unit of Euro goes Rs.48 or below, at the end of the maturity. What should be the initial price of the contract (under risk-neutral valuation), if the $\sigma = 0.4$, $r_d = 8\%$, $r_f = 2\%$, $T = 6$ months, and initial cost of one unit of Euro is Rs.60.

4. (8 marks) Suppose that there is a riskless asset in strictly positive supply and investors prefer to hold efficient frontier portfolios. Assume that borrowing at the riskless rate r_f is prohibited. Show that for any feasible portfolio q

$$E[r_q] = E[r_{zc(m)}] + \beta_{qm}(E[r_m] - E[r_{zc(m)}]), \quad E[r_m] > E[r_{zc(m)}] \quad \text{and} \quad E[r_{zc(m)}] \geq r_f.$$

5. (6 marks) Let $X = \mathcal{R}^2$, equipped with lexicographic ordering. Show that any one of the following:

- (a) For any point $x \in X$ elements inferior to x and elements superior to x do not both form open sets.
- (b) There does not exist countable set $Y \subset X$, such that for every $x_1, x_2 \in X$ with $x_1 < x_2$ there exist $y \in Y$, $x_1 < y < x_2$.

All the best.

Indian Statistical Institute
Semester Examination: 2011-2012
MS(QE) I/ M.Stat.II: 2011-2012

Game Theory I

Date: 21/ 11/2011

Maximum Marks: 50

Duration: 3 Hours

Group A: Answer any FOUR questions. Your total score cannot exceed 50.

1. (a) What is the 'trigger strategy' in an infinitely repeated Prisoners' Dilemma game? Show that if the discount factor δ is close to one, the trigger strategy is a Nash equilibrium. Is this a subgame perfect Nash equilibrium also?
(b) Derive the relevant condition on δ for the following game, given that the cooperative outcome is sustained. There are two firms, 1 and 2, interacting repeatedly in a Cournot fashion. The market demand for the homogeneous good is $p = a - Q$, where Q is the quantity demanded at price p , and the firms are symmetric, with cost function $C_i = cQ_i$, $0 < c < a$, where Q_i is the quantity supplied by firm i ; $i = 1, 2$.

[6+7=13].
2. A seller throws his house for sale. It has a listed price of Rs. 1,50,000/- which is the buyer's maximum price, and its invoice price is Rs 1,20,000/- which is the seller's reservation price. Consider an alternating offer game in the Rubinstein framework. Assume that each player has the common discount factor, $\delta = 1/2$.
 - (i) If the buyer is the player to give the opening offer, what will be the price at which trade will occur if the game is played for three periods only?
 - (ii) If the game has an infinite time horizon, what will be the optimal buyer's offer?
 - (iii) If the seller, instead of the buyer, gives the opening offer, what will be the result?

[5+5+3=13]
3. (a) There are three one-rupee notes to be divided between two players. It's a two period game. In the first period player 1 proposes a division. If player 2 accepts it, the division is implemented, and if it is rejected the game goes to the second period in which case player 2 gives a counter offer. Again, if it is accepted, this is implemented, but if it is rejected, no player gets anything (say, the money goes to a charity fund). Each player has time preference given by the (common) discount factor, δ ; $0 < \delta < 1$. Find the subgame perfect Nash equilibrium of this game.

(b) Find all mixed strategy Nash equilibria of the following two player game:

| | L | M | R |
|---|--------|--------|--------|
| B | (2, 2) | (0, 3) | (1, 2) |
| S | (3, 1) | (1, 0) | (0, 2) |

[7+6=13]

4. Two people have Rs. 10/- to divide between themselves. They use the following procedure. Each person names a number of rupees (a non-negative integer), at most equal to 10. If the sum of the amounts the people name is at most 10, then each person receives the amount she named (and the remainder, if any, is destroyed). If the sum of the amounts that the people name exceeds 10, and the amounts named are different, then the person who named the smaller amount receives the amount and the other person receives the remaining amount. If the sum of the amounts that the people name exceeds 10 and the amounts named are the same, then each person receives Rs 5/-.

- (a) Construct the payoff matrix and solve the game by iterated elimination of weakly dominated strategies.
- (b) Write down the payoff function of each player, determine their best response functions, plot them in a diagram and finally find the Nash equilibria of the game.

[6+7=13]

5. A legislature has k members, where k is an odd positive integer. Two rival bills, X and Y , are being considered. The bill that attracts the votes of a majority of legislators will pass. Interest group X favors bill X , whereas interest group Y favors bill Y . Each group wishes to entice a majority of legislators to vote for its favorite bill. First, interest group X gives an amount of money to each legislator, then interest group Y does so. Each interest group wishes to spend as little as possible. Group X values the passing of bill X at Rs $V_X > 0$ and the passing of bill Y at zero, and group Y values the passing of bill Y at Rs $V_Y > 0$ and the passing of bill X at zero. Each legislator votes for the favored bill of the interest group that offers her the maximum money. But a legislator to whom both groups offer the same amount of money votes for bill Y .

How do you proceed to solve for the subgame perfect Nash equilibrium of the game?

[13]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2011-2012)
MS (Q.E.) II Year
International Economics I

Date: 21.11.11

Maximum Marks 60

Duration 3 hours

Group A

Answer any two

1. In equilibrium, the ratio of foreign capital to domestic capital in a less developed economy varies positively with its relative technological backwardness and inversely with the North-South capital endowment ratio. – Examine the validity of this statement in terms of a static North-South model with perfect capital mobility. (15)
2. In the Brander-Spencer model with Cournot competition between the home firm and the foreign firm in the home market, analyse the effect of an increase in the tariff rate (i) on the level of production of these two firms and (ii) on the consumer surplus. (8+7)
3. Analyse the validity of the following statements.
 - (a) Factor price equalization theorem is valid in the 3x2 specific factor model.
 - (b) Walras law is valid in the 2x2 HOS model.
 - (c) Production possibility locus slopes negatively in the HOS model but not in the specific factor model.

Group B

Answer all questions

1. Consider two Ricardian countries which can potentially produce two goods (1 and 2) with consumers having identical preference given by $u = C_1^\alpha C_2^{1-\alpha}$, $0 < \alpha < 1$.

The technological coefficients are

$$a_{l,1} = 2$$

$$a_{l,2} = 4$$

$$a_{l,1}^* = 10$$

$$a_{l,2}^* = 5$$

Total labour endowments are

$$L = 80$$

$$L^* = 400$$

- a) Find out the range of α for which both the countries will strictly gain by engaging in trade.
- b) Say L and L^* are not given and $\alpha = \frac{1}{4}$. Find the range of $\frac{L}{L^*}$ such that countries remain completely specialized under trade.

[All notations have their usual meaning] (15)

2. Show that in a three-agent setting, a transfer paradox may occur even when the equilibrium is Walras stable. In this context discuss the role of substitution effects towards ensuring normal results. (15)

or

Show how trade driven distributional conflict is related to factor endowment ratios and economies of scale. (15)

INDIAN STATISTICAL INSTITUTE
End-Semestral Examination: 2011-12

Course name: MSQE II

Subject name: Game Theory II

Date:

Duration: 3 hours

Answer all questions. Please answer parts A and B on separate answer books. Each part is worth 50 marks.

Part A

Q. 1: Define a Shapley Value and demonstrate its uniqueness. [3 + 10]

Q. 2: Clearly establish the relationship between the kernel and the bargaining set of a coalitional form game. [6]

Q.3: Define a market game and show that it has a non-empty core. [3+7]

Q.4: (a) Establish the interchangeability property of a strictly competitive game. Explain whether this property holds for a general game? [4+2]

(b) Person A has an old car to sell. The car is worth nothing to him unless he can sell it. Persons B and C want to buy the car. B values it at Rs. 5,000, while C values it at Rs. 6,000. The game consists of each of the prospective buyers bidding on the car and A either accepting one of the higher bids or rejecting both of them. Formulate the characteristic function of the game. [5]

(c) It is known that for a superadditive game an imputation is in the core if and only if it is not dominated by any imputation. Is this assertion true without superadditivity? Justify your answer. [10]

Part B

Q.1: Consider the sealed-bid, common value auction model done in class.

(a) Show that there is a unique symmetric equilibrium in the second-price auction. [15]

(b) Show also that it yields higher expected revenue than the first-price auction. You may use results derived in class regarding the first-price auction. [20]

Q. 2: Consider the independent private value model done in class. There are N bidders. Values are drawn from the set $[0, \bar{x}]$ according to the continuously differentiable, increasing distribution function F . Suppose the seller wishes to sell using a first-price auction and that $\bar{x} = 1$, $N = 2$ and $F(x) = x(2 - x)$.

(a) What is the symmetric equilibrium bid function? [5]

(b) What is the expected revenue to the seller? [5]

(c) What is her optimal reserve price? [5]

Indian Statistical Institute
First Semester Examination
MSQE I & II
Economic Development I

Date: 24.11.11
Maximum Marks: 60
Duration: 3 hours

Answer question number 1 and any two from the remaining three

1. Ramu is a cobbler. He collects hides from carcasses of dead animals, dries them up in the sun and makes shoes. On a sunny day he can dry up a skin and make one pair of shoes which earns him X . On a rainy day, skins cannot be dried and therefore Ramu earns nothing. A sunny day occurs with probability p and a rainy day occurs with probability $(1-p)$. In addition, Ramu has some money which he lends and earns R per day as interest. Ramu is risk neutral, has a daily discount factor $\beta < 1$ and an infinite life time.
- (a) Let V and W denote the discounted value of Ramu's future life time income on a sunny day and on a rainy day respectively. Express V and W in terms of X, β, p and R .

Now suppose that Ramu can buy a drying machine with the money he has. This allows him to dry the skin he gathers on a rainy day as well as on a sunny day. Accordingly, if he buys the machine, he earns an income X every day irrespective of whether it is rainy or sunny. But, of course, he has to forego his daily interest income. Moreover, the machine has a probability $(1-q)$ of getting stolen on any particular day in future. If the machine is stolen, Ramu comes back to his old technology and his money is gone forever.

- (b) On which day is he likely to buy the machine, a sunny day or a rainy day? Give reasons for your answer.
- (c) Let U be his life time income when he buys the machine. Express U in terms of X, β, p and q .
- (d) Prove the following proposition: If $X(1-p\beta) > R > X(1-\beta)$, then there exists $\bar{q} \in (0,1)$ such that for all $q \leq \bar{q}$, Ramu has no incentive to buy the machine. What does this tell you about property rights?

[9+3+9+9=30]

2. Demonstrate, in terms of a suitable model, that in a less developed region with a large informal sector, where property rights are imperfect and political patronage is essential for survival, democracy might reduce efficiency.

[15]

3. Characterize the optimal contract between a farmer who takes a production loan and a trader-cum-money lender who gives the loan and markets the product, when the product market and the credit market are interlinked. Show that the contract is Pareto efficient.

[15]

4. In a model of increasing returns, show how history and expectations can interplay to determine the long run equilibrium as well as the dynamic path to the long run equilibrium.

[15]

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2011-2012
MS (Q.E.) II and MStat II Year
Econometric Methods II/ Econometric Methods

Date: 28/11/11

Maximum Marks 100

Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 110 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\epsilon_i|X_i) \neq 0$ but there are available instruments Z with $E(\epsilon_i|Z_i) = 0$ and $V(\epsilon_i|Z_i) = \sigma_i^2$, where $\dim(Z) > \dim(X)$. We consider the GMM estimator $\hat{\beta}$ that minimizes

$$G_N(\beta) = \left[\frac{1}{N} \sum Z_i(Y_i - X_i'\beta) \right]' W_N \left[\frac{1}{N} \sum Z_i(Y_i - X_i'\beta) \right].$$

- Derive the limit distribution of $\sqrt{N}(\hat{\beta} - \beta)$.
- State how to obtain a consistent estimate of the asymptotic variance of $\hat{\beta}$.
- If errors are homoscedastic what choice W_N would you use?
- If errors are heteroscedastic what choice W_N would you use?

[12+8+7+8=35]

2. Consider a latent variable modeled by $y_i^* = x_i'\beta + \epsilon_i$, with $\epsilon_i \sim N(0, 1)$. Suppose we observe only $y_i = \begin{cases} 1, & \text{if } y_i^* < u_i \\ 0, & \text{if } y_i^* \geq u_i, \end{cases}$

where u_i is known constant for i^{th} individual.

- Find $\Pr[y_i = 1|x_i]$.
- How would you estimate β consistently? Discuss in details.
- Suppose the distribution of ϵ_i is unspecified with zero mean and variance 1. Assume that $u_i = u \forall i = 1, \dots, N$. How would you estimate $\Pr[y_i = 1|x_i]$? Discuss in details.

[7+6+12=25]

3. a. Consider a two-variable VAR (1) model. Discuss the different modelling situations that arise out of restrictions on the parameters of this model from consideration of stationarity or otherwise.
- b. Describe the ECM-based test for cointegration in a single-equation set-up, and then indicate why this test can be considered to be better than the residual-based test.
- c. Discuss how you would carry out causality test in situations of cointegration.

[10+8+7=25]

P.T.O.

4. Suppose that $y_i = \mu(x_i) + \varepsilon_i$ where $\varepsilon_1, \dots, \varepsilon_n \stackrel{iid}{\sim} N(0, \sigma^2)$, $x_i = \frac{i}{n+1}$ for $i = 1, \dots, n$, and $\mu(x) = \exp(x)$ consider the local average estimate of $\mu(1/2)$ based on average over the interval $[1/2 - h, 1/2 + h]$.
- Derive simplified (as much as possible) expressions for the bias and variance of the estimate.
 - Show that variance of the estimate tends to 0 as $nh \rightarrow \infty$.
 - Show that bias of the estimate tends to 0 as $h \rightarrow 0$.

[10+8+7=25]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2011-2012) (Back paper)

MS (Q.E.) II Year

International Economics I

Date *27.12.11*

Maximum Marks: 100

Duration: 3 hours

Group A

Answer any two

1. Introduce following modifications in an otherwise 3x2 specific factor model:
Labour endowment is infinitely large and the wage rate is institutionally fixed.
(a) Formulate the model and describe its working.
(b) Show that the ratio of output of the two sectors varies positively with the ratio of specific capital of these two sectors. (10+15)
2. Analyse how Brander-Spencer justify the existence of optimum tariff. (25)
3. State the assumptions of Findlay model of North-South trade. Explain the concept of long run equilibrium in this model. Analyse intuitively why trade is an engine of growth to South in this model. (12+5+8)

Group B

Answer all

1. Explain the mechanics through which capital inflow in Dei's model necessarily leads to welfare improvement. How is this result strikingly different from the Brecher, Diaz-Alejandro result where the import competing capital intensive sector is tariff protected and the country is small. (25)
2. Show how pattern of trade (intra/ inter industry) is related to factor endowment ratios and economies of scale. (25)

P.T.O

Syllabus: (International Economics I): Professor Manash RanjanGupta

(Group A)

(Up to mid semester)

1. Competitive general equilibrium modes: HOS, specific factor

(After mid semester)

2. North- South model : Findlay (AER, 1980), Wang (JIE, 1990)
3. Trade policy under imperfect competition: Spencer-Brander model- Optimum tariff, optimum export subsidy.

Syllabus: (International Economics I): Dr. Brati Sankar Chakraborty

(Group B)

1. The Basic Exchange Model.
2. The Ricardian Model.
3. Comparative Statics in a Basic Trade Model: growth, terms of trade and welfare (the immiserization results), transfers (related 'presumptions') and paradox thereof.
4. Tariffs, terms of trade, volume of trade, optimal tariff, domestic prices (Metzler Paradox)
5. Growth/capital inflow under tariff regime (Johnson, Brecher, Diaz-Alejandro, Dei).
6. Increasing Returns and trade.

INDIAN STATISTICAL INSTITUTE

Back paper Examination: 2011-2012

MS (Q.E.) II and MStat II Year

Econometric Methods II/ Econometric Methods

Date: 30.12.11

Maximum Marks 100

Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 100 marks. You can answer any part of any question. Marks allotted to each question are given within parentheses.

1. (a) Discuss briefly about the problems of 'normalisation' and 'exogeneity' in the context of cointegration in the single-equation approach.

(b) Describe the Johansen's procedure for estimating a cointegrated system of equations specified by a VAR (p) model.

[8+7=15]

2. Suppose that we have the transformed regression model $h(y) = \beta_0 + \beta_1 X_1 + \dots + \beta_d X_d + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$, ε is independent of the X 's and h is a smooth and increasing function. Let $\mu(X_1, \dots, X_d) = E(y | X_1, \dots, X_d)$.

(i) Show that μ is a smooth and increasing function of $\beta_1 X_1 + \dots + \beta_d X_d$.

(ii) Show that the vector of first partial derivatives of μ is a scalar multiple of the vector $(\beta_1, \dots, \beta_d)$.

(iii) Show that the matrix of second partial derivatives of μ is of rank one with $(\beta_1, \dots, \beta_d)$ as an even vector.

[5+5+5=15]

3. Consider the multiple linear regression model as $Y = X\beta + \varepsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\varepsilon_i | X_i) \neq 0$ but there are available instruments Z with $E(\varepsilon_i | Z_i) = 0$ and $V(\varepsilon_i | Z_i) = \sigma_i^2$, where $\dim(Z) > \dim(X)$. We consider the GMM estimator $\hat{\beta}$ that minimizes

$$G_N(\beta) = \left[\frac{1}{N} \sum Z_i (Y_i - X_i' \beta) \right]' W_N \left[\frac{1}{N} \sum Z_i (Y_i - X_i' \beta) \right].$$

a. Show that GMM estimator of β is same as the IV estimator of β for a particular choice of W_N .

b. Show that the IV estimator is consistent.

c. If errors are homoscedastic what choice W_N would you use?

d. If errors are heteroscedastic what choice W_N would you use?

[7+6+6+6=25]

4. Consider a latent variable modeled by $y_i^* = x_i' \beta + \varepsilon_i$, with $\varepsilon_i \sim N(0, 1)$. Suppose we observe only $y_i = \begin{cases} 1, & \text{if } y_i^* < u_i \\ 0, & \text{if } y_i^* \geq u_i \end{cases}$

where u_i is known constant for i^{th} individual.

- a. Find $\Pr[y_i = 1|x_i]$.
- b. How would you estimate β consistently? Discuss in details.
- c. Suppose the distribution of ε_i is unspecified with zero mean and variance 1. Assume that $u_i = u \forall i = 1, \dots, N$. How would you estimate $\Pr[y_i = 1|x_i]$? Discuss in details.

[7+6+12=25]

5. Consider a standard dynamic panel data regression model as $Y_{it} = \alpha_i + \rho Y_{i,t-1} + \varepsilon_{it}$, with all CLRM assumptions.

- a. Show that both fixed and random effect estimates of ρ are inconsistent.
- b. Propose an IV estimator for ρ . Show that your proposed estimator is consistent.

[10+10=20]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2011-2012)
MS (Q.E.) II Year
Advanced Topics in International Economics

Date: 20.02.2012

Maximum Marks 40

Duration 3 hours

Group A

Answer all

1. Refer to the Krugman (1991) model of Economic Geography. Begin with a situation where all mobile factors of production are located in one single location. Now, how would changes in transport cost destabilize (if at all) such an allocation of factors?
(10)
2. Derive the condition under which the Stolper- Samuelson theorem would remain valid in its magnification version when one of the sectors is subject to increasing returns to scale.
(10)

Group B

Answer all

1. Consider a model of monopolistic competition where the firms (indexed by ' i ') differ with respect to the level of productivity (i.e. the production function of a firm ' i ' assumes the form: $x(i) = a(i) l(i)$ where ' x ' denotes the level of output, ' a ' is the productivity parameter and ' l ' is the amount of labor employed by the firm). The firms have identical fixed costs (in units of labor) given by ' f '. Derive the least restrictive conditions which ensure factor price equalization if two countries having the economic structure as given above trade in the commodities in the presence of:
 - a) No fixed cost associated with export
 - b) A fixed cost given by ' g ' (in units of labor) associated with export

2. Consider a model of monopolistic competition (set up above) when there is a fixed cost (in unit of labor given by 'g') associated with export. Provide an explanation why an exposure to trade forces the least productive firms to exit or shut-down in any country and this is not explained by increased foreign competition brought about through trade.

[6]

3. Explain why, in a model of monopolistic competition (set up above) when there is firm heterogeneity, fixed cost associated with export and endogenous choice of technology (a traditional technology with high marginal cost with low fixed cost and a modern technology with low marginal cost with high fixed cost), market integration brought about by trade favors (if at all) the modern technology over the traditional one for the most efficient firms.

[8]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2008-2009)

MS(QE) II

Social Choice and Political Economy

Date: 22.02.2012 Maximum Marks: 40 Duration: 2 hrs.

Note: Answer Group A and Group B in separate answer scripts.

Group A

- (1) Define the utilitarian rule. State and prove a set of necessary and sufficient conditions for a social welfare ordering to be the utilitarian rule. (2+7=9)
- (2) (a) Define cardinally measurable, non-comparable utility information. Show how it leads to intra-personal utility difference comparability. Is the converse true? (1+2+3=6)
(b) Define strong neutrality. Show how independence of irrelevant alternatives and Pareto indifference drop out as implications of strong neutrality. (1+2+2=5)

Group B

- (1) Consider the social aggregation problem, where $A = \{x, y\}$ is the set of alternatives, $|A| = 2$, N is the finite set of agents and $|N| \geq 2$.
 - (a) Suppose that the social welfare function satisfies symmetry and neutrality and the number of agents preferring x over y is the same as the number of agents preferring y over x . Then show that the social welfare function necessarily prescribes indifference between x and y . (4)
 - (b) Find a social welfare function that satisfies Pareto, neutrality but fails to satisfy positive responsiveness and symmetry. (2)
- (2) State and prove the Field Expansion Lemma by giving all the relevant definitions. (14)

INDIAN STATISTICAL INSTITUTE
Mid Semester Examination: 2011-2012 (Second Semester)
MS(QE) II Year
Environmental Economics

Date: 24 February, 2012 **Maximum Marks:** 50 **Duration:** 1 Hour 30 Minutes

1. (a). To choose an appropriate regulatory measure out of emission fee and quantity regulation, how is the decision of a regulator affected when regulator is uncertain about marginal abatement cost of polluters.
- (b). It is known that impacts of emissions by polluting firms on ambient pollution concentrations differ by location of firms. In that situation how should an efficient emission fee for each firm be set by a regulator following equimarginal principle?
- (c). Two identical firms save money from polluting. Marginal savings from emitting an amount e_i by i^{th} firm are given by $10 - 2e_i$. Two firms differ in their impact on ambient pollution concentrations. Two units of emission from firm 1 result in one unit of ambient pollution. Firm 2 has twice the impact on the ambient environment from the same amount of emission.
- (i) Find the appropriate amount of emission at firm level and at ambient level.
- (ii) If total of 6 units of emission permits (on ambient) are distributed, where firm 1 is given 2 unit and firm 2 is given 4 units and they are allowed to trade, how many permits will each firm end up with and what will be the price?

[10+10+10=30]

2. (a) Explain the concept of total economic value of environmental services.
- (b). Suppose a consumer derives utility from consumption of a good x and personal hygiene H . H depends on air pollution level α and medical care M which is the defensive expenditure to reduce the effect of air pollution on health. Consumption of both x and M involves money and time. Suppose market prices of x and M are p_x and p_M , respectively. Consumer's total income consists of wage income W (for L hours, available for work, at wage rate w per hour) and asset income A . Assuming no change in prices explain how you can estimate empirically the marginal cost of change in air pollution level.

[8+12 = 20]

MSQE II - Theory of Finance II
Midsem. Exam. / Semester II 2011-12
Time - 2 hours/ Maximum Score - 30

27-02-2012

1. (4+4+4=12 marks)

Let stocks in a financial market follow the trinomial model (i.e., U, M, D , three states), and the interest is fixed. (a) With two stocks and a bond would the market be complete? Under what condition? Give the answer, justifying mathematically (if you can replicate any portfolio, etc.).

(b) Let the payoff for an option be $(\max(S_{1T}, S_{2T}) - K)^+$, T is the maturity time and asset prices at time t are denoted by S_{1t} and S_{2t} , respectively.

(i) Using one-step trinomial model with complete market condition, calculate the hedging necessary (i.e., Δ_1 and Δ_2) to keep the portfolio risk-neutral.

(ii) Also, under the same condition, find the corresponding risk-neutral measure.

2. (4+3+4=11 marks)

(a) Can GARCH(1,1) be a good model for stock returns? Write the model clearly. Then give your answer justifying it mathematically, whether (i) it exhibit clustering of volatility; (ii) the return is negatively correlated with the change in volatility.

(b) If the GARCH(1,1) is not a satisfactory one, do you suggest any alternative model? Justify.

3. (6+4=10 marks)

Give reasons whether the put-call inequalities for options on an asset hold.

(a) $C_A + Ke^{-rT} \leq P_A + S \leq C_A + K$;

(b) $P_E \leq P_A$.

All prices given are initial prices. The suffix A and E refer to American and European options, respectively.

All the best.

Indian Statistical Institute
MS(QE) II: Mid-Semester Examination: 2011-12
Industrial Organization

Date: ~~09/09~~ 2012

Maximum Marks: 40

Duration: 3 Hours

1. There are two periods, $t \in \{1, 2\}$. The inverse demand for light bulbs in each period is

$$p = \frac{1}{50}Q$$

where p is the rental rate for one bulb per period and Q is the total number of bulbs used per period. The monopolist has zero marginal cost to produce bulbs. Bulbs sold in period 1 last through period 2. Suppose the monopolist can only sell bulbs, not rent them. Furthermore, the monopolist cannot in period 1 commit to its behavior in period 2. Let the interest rate be 10% (or equivalently the discount factor be $1/(1+0.1) = 0.909$)

- (a) Solve for the equilibrium rental rates charged by the monopolist who chooses to rent the product.
- (b) Find the equilibrium purchase prices if the monopolist chooses to sell the product.
- (c) Does the monopolist gain if the bulb lasts for only one period? Explain.

[2+3+5=10]

2. A software firm wants to sell two different products: a word processor (Letter), and a spreadsheet program (Accounts). The marginal cost of producing an additional digital copy of either programme is zero. Consumers are heterogenous The following table shows you the willingness-to-pay (WTP) of each group of consumers for each product, as well as the number of consumers in each group:

| WTP for: | | | |
|------------------|-------------|--------|----------|
| User Type | No of users | Letter | Accounts |
| Writers | 40 | 50 | 0 |
| Number Crunchers | 40 | 0 | 50 |
| Generalists | 20 | 30 | 30 |

- (a) What is the profit-maximizing price for each product if the firm cannot observe the buyers type, and sells each program separately?
- (b) Suppose that the firm decides to offer the two products in a bundle, called "Office Suite", What is the maximum profit that the firm can make with its Office Suite?
- (c) If the firm considers selling both the Office Suite and the separate programs side by side, what are the profit-maximizing prices for the separate programs? Who will buy the bundle, who will buy the separate products? Compare profits of the three strategies.

[3+3+4=10]

3. A restaurant manager realizes that the demand for soft drinks is more elastic among students and is trying to determine the optimal pricing schedule for soft drinks. Specifically, he estimates the following average demands;

$$\text{Students: } q_s = 18 - 5p$$

$$\text{Non-students: } q = 10 - 2p$$

The two age groups visit the restaurant in equal numbers on average. Soft Drinks cost the Restaurant Rs 2 per bottle.

- (a) If the market cannot be segmented, what is the uniform monopoly price?

- (b) If the restaurant can charge a fixed fee per customer as well as a price per bottle, what fixed fees and prices would be set for each group by the perfectly discriminating monopolist?
- (c) If it is impossible to distinguish the students from non students., what is the optimal entry fee and price per bottle of drink that will attract both types of customers. ?

[2+3+5=10]

4. Suppose two firms in an industry face linear inverse demand curves $p_i = 7 - q_i - q_j$, $i = 1, 2; i \neq j$. Firms compete in a two-stage game; first they set capacity and then they set price or output. At the first stage firms set capacities, at this stage the marginal costs of capacity is 6. Suppose that firms have zero marginal costs of production up to the installed capacity and that production above capacity is not feasible. In case of rationing, rationing is assumed to be efficient (consumer valuations and unit prices are matched from the highest to lowest).

- (a) Suppose that firms have given capacities \bar{q}_1 and \bar{q}_2 , respectively. Show that the Nash equilibrium in prices in the stage game after capacities have been set satisfies $p_1 = p_2 = p^* = 7 - \bar{q}_1 - \bar{q}_2$
- (b) Determine the subgame perfect equilibrium of the two-stage game in which firms first set capacities and then prices.

[5+5=10]

5. There are two firms in the market. They produce perfect substitutes at cost $C(q) = \frac{q^2}{2}$. The demand is $p = 1 - (q_1 + q_2)$

- (a) Compute the Cournot Equilibrium.
- (b) Suppose now that firm 1 has the opportunity to sell the same output in another market as well. The quantity sold on this market is x_1 , so firm 1's cost is $(q_1 + x_1)^2/2$. The demand on the second market is $p = a - x_1$. Consider the Cournot game in which firm 1 chooses q_1 and x_1 and firm 2 chooses q_2 simultaneously. Show that $q_1 = (2 - a)/7$ and $q_2 = (5 + a)/21$ over the relevant range of a . Show that for $a = \frac{1}{4}$ a small increase in a hurts firm 1.

[4+6=10]

6. Consider a linear city of length 1. Consumers are uniformly distributed over the length of the city and each consumer buys exactly one unit of a product supplied by two firms, $i = 1, 2$. Each consumer is to pay a price for the product and a transport cost proportional to the square of the distance (i.e., td^2 , where t is the transportation cost per unit distance and d is the consumer's distance to the selected shop). Assume that there is no production cost and that firm 1 is already located at one edge of the city. Now firm 2 is to decide its location in the city. Finally, they compete in prices. Find the optimal location of firm 2 and the prices they will charge for their products

[10]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2011-12

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 10.9.12
Maximum marks: 30
Duration: 2 hours

Question 1

Consider the basic hidden information model done in class. Suppose the principal's benefit function is $S(q) = 12q$, and the agent cost function is $C(q, \theta) = \theta q^2$. The agent knows his own type, i.e., the value of θ , while the principal knows that θ is either 1 or 2 with equal probability. Output is observable and the principal can give a transfer to the agent. If the output is q , and the principal gives a transfer t , the net payoff of the principal is $12q - t$, while the net payoff of the agent is $t - \theta q^2$.

Assuming the agent as well as the principal have outside option 0, and that the principal is the residual claimant, has full bargaining power and offers contracts, what is the principal's net expected payoff from the interaction? [20]

Question 2

Consider an economy consisting of agent A and a unit measure of workers. If A appoints a worker i , she produces $2\alpha_i$ units of a consumption good from which A obtains payoff $2\alpha_i$. If A does not appoint a worker, his payoff is 0. A can appoint at most one worker.

If worker i is appointed, she produces $2\alpha_i$ units of the good at cost α_i^2 . If a worker is not appointed, her payoff is 0. A worker's type, i.e., the value of α , is known only to herself, while A knows that α is distributed uniformly in the population of workers over $[0, 1]$.

A can offer a wage contract w , such that any worker who accepts it and is appointed produces $2\alpha_i$ and gets wage w . If he gets multiple acceptances, he appoints one of the accepting workers randomly.

What wage does A offer to maximise net expected payoff? How does your answer change if the workers' productivity parameter is 4 instead of 2? [5 + 5]

Indian Statistical Institute
Mid Semestral Examination: (2011 – 2012)
M.S. (QE) – II year
Econometric Applications II

Date: 02.03.12

Maximum Marks – 60

Duration: 2 hours

(Answer any **three** questions)

1. (a) Suppose the income variate x follows a Gamma distribution with density function

$$f(x) = \frac{1}{\Gamma\alpha} e^{-x} x^{\alpha-1}, \quad x \geq 0$$

Find the Lorenz curve and Lorenz Ratio.

- (b) Explain the following in the context of inequality measurement.
- (i) Symmetry axiom of population
 - (ii) Population principle.
 - (iii) Pigou-Dalton transfers principle.

- (c) Given the individual utilities

$$u(x_i) = \begin{cases} C + D \frac{x_i^{1-\varepsilon}}{1-\varepsilon}, & \varepsilon \neq 1 \\ C + D \log x_i, & \varepsilon = 1 \end{cases}$$

write down the expression for Atkinson's measure of inequality.

[8 + 6 + 6 = 20]

2. a) Consider two income profiles $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ with $x_1 \leq x_2 \leq \dots \leq x_n$ and $y_1 \leq y_2 \leq \dots \leq y_n$.
- (i) Define Lorenz dominance of x with respect to y . State the assumptions clearly.
 - (ii) Why is the ordering of income profiles, generated by the LC comparison, a quasi-ordering? Illustrate with an example.
 - (iii) What is a generalized Lorenz curve (GLC)? What is the advantage of GLC over LC?

- b) Show that $L(p) = p - ap^{c_1}(1-p)^{c_2}$; $a > 0$, $0 < c_1 \leq 1$, $0 < c_2 \leq 1$, is a valid Lorenz curve.

[9+11=20]

3. a) Discuss the different techniques of fitting a specified model $f(x, \theta)$, θ being the vector of parameters, to a (grouped) income distribution data.
- b) How do you judge the goodness of fit of the fitted model?

[8+12=20]

4. a) Explain the 'Direct method' and 'Income method' of poverty measurement.
- b) In the context of measurement of poverty, state the axioms (i) Replication invariance, (ii) Scale invariance and (iii) Decomposability.

What would be the problem if a poverty index does not satisfy these axioms?

- c) (i) Consider the income profile $x_1 \leq x_2 \dots \leq x_n$ and the poverty measure

$$P = \left[\frac{1}{q} \sum_{i=1}^q (z - x_i)^\alpha \right]^{1/\alpha}, \text{ where } q: \text{ number of poor and } z: \text{ poverty line.}$$

Examine the index in light of the above axioms.

- d) What are "ethical" poverty measures?

[4 + 9 + 5 + 2 = 20]

INDIAN STATISTICAL INSTITUTE
SECOND SEMESTRAL EXAMINATION (2011-2012)
M.S.(QE) II
Advanced Topics in International Economics

Date: 23.04.2012 Maximum Marks: 60 Duration 3^{1/2} hours.

Group A
Answer all

1. Show that even when there is a static welfare loss due to temporary monopoly enjoyed by those who undertake innovations, the overall dynamic gains from innovations can be sufficient to justify such static loss. (15)

2. In the Dollar (1986) model, what would happen to the northern real wage if there is a growth of the southern labour force? In this context, what can you say about the dynamics of the terms of trade? (15)

Group B

Answer as much as you like. The maximum marks you can score is 30

1. Consider an economic setup of two countries, where a single **homogenous** good (say X) is produced. One unit of the good is produced using one unit of labour. In addition to labour, to set up a plant, a firm has to incur a fixed cost in the form of ' f ' units of capital. Export entails a further transport cost (of iceberg type) so that for every unit shipped, only τ fraction reaches the exported country. Firms compete as Cournot oligopolists. The countries differ only with respect to the amounts of their

endowments of labour and capital (say L, L^*, K and K^* respectively) and engage in trade amongst them. Set up economic structure of the above example mathematically (You don't need to solve analytically).

Now suppose the firms are able to relocate a part of their production process outside their country of origin to the other country (this outside country is termed as the host country. Also note that there are only **two** countries). To do so, the firms need a **fixed** additional 'v' amount of the **host country's** labour in addition to their unitary labour requirement per unit of output produced in the host country. But they don't require any additional amount of capital in the host country. **Given** the equilibrium amounts of the endogenous variables of the previous section (this is also a hint), obtain the condition/s which if satisfied ensures that the equilibrium for this extension is characterized by firms who relocate parts of their production process outside their country of origin.

[25]

2. Consider an economic setup of two countries (one starred and the other non – starred), where two goods (say X and Y) are produced with labour as the only factor of production. The preferences of all individuals in the countries are identical and the utility structure is given by: $U = (X_c^\rho + Y_c^\rho)^{1/\rho}$ ('c' in the subscript denotes consumption). There is a single firm **in any sector, in any country**. Firms compete as Bertrand oligopolists and their productivities (i.e. the amount of the output produced per unit of labour) takes values of $\theta_x, \theta_x^*, \theta_y, \theta_y^*$ with $\frac{\theta_x}{\theta_x^*} > \rho \frac{\theta_x}{\theta_x^*} > \frac{1}{\rho} \frac{\theta_y}{\theta_y^*} > \frac{\theta_y}{\theta_y^*}$. The countries differ only with respect to the amounts of their endowments of labour (say L and L^*) and engage in trade in both the goods. Set up economic structure of the above example mathematically and solve for the endogenous variables.

[25]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2011 - 20012)

MS(OE) II Year

Environmental Economics

Date : 25 April, 2012

Maximum Marks : 100

Duration : 3 hrs.

**The Question Paper carries 120 Marks.
You can answer any part of any question.
The maximum you can score is 100.**

1. (a) According to Coase theorem, there is no reason to regulate externalities as long as property rights are clearly defined; either the victim or the polluter may have the right. Explain the statement with example.
- (b). Suppose there is a paper mill by the side of a river discharging wastewater into the river. In the downstream, there is a fishery whose production is affected by the polluted river water. Suppose the cost function of producing the quantity of paper P is $C_p = P^2 + 8$ and that of producing fish product F is $C_f(F, P) = F^2 + FP + 4$. The prices of fish and paper are Rs. 12.0 and Rs. 14.0, respectively. What will be the production and profit of each firm if
- (i) two firms are independent,
 - (ii) two firms are merged,
 - (iii) paper mill has the right to pollute and
 - (iv) fishery has the right to get clean water.
- [12+10=22]
2. Consider a non-renewable resource; say oil whose stock is given at X_0 . Let p_t and q_t be respectively the price and quantity of extraction in period t . Assume constant marginal cost of extraction c and fixed rate of interest r for all periods.
- (a) State and derive the condition for optimal extraction policy taken by a firm in a perfectly competitive market and by a monopolist.
- (b) Assume a linear (inverse) demand function: $p_t = p^b - \beta q_t$, where β is a positive constant and p^b is the backstop price.
- (i) Starting from 0^{th} period what will be optimal time (T) when the firms will stop extraction?
 - (ii) Compare the results graphically.
- [15+8+5=28]

3. (i). Explain why the forest is considered to be a separate type of renewable resource.

(ii). Suppose the 'best next use' of land is rotational forestry for an infinite period of time and owner of the forest considers only the timber supply following even-aged management strategy.

(a) Find the condition for determining the optimal rotation period (years) of a forest assuming stumpage price P_t , rate of discount (rate of interest) r and establishment cost C are exogenous and given. Explain the economic interpretation of the condition.

(b) Explain the role of discount rate, stumpage price and establishment cost in determining the optimal rotation age and supply of timber in the market

[2+8+8=18]

4. What do you mean by resource scarcity? Explain 'Unit Cost' method of measurement of resource scarcity.

[2+12=14].

5. What do you mean by Sustainable Development? Explain the Hartwick's approach for optimal adjustments to the National Accounts to maintain the sustainable development process.

[2+10=12]

6. (a). Does conventional theory of international trade take into account the environmental factor?

(b). Do you think that less developed countries are more responsible for degrading the environment than their developed counterparts? Explain your views.

[2+12=14]

7. Do you agree with the view that the hedonic price theory can be applied for environmental valuation provided the underlying property market is in equilibrium? Justify your views.

[12]

Indian Statistical Institute
MS(QE) II (Second Semester): 2011-2012
Industrial Organization
Final Examination

Date: 27/04/12

Maximum Marks: 40

Duration: 3 Hours

1. (a) A monopolist (M) faces demand curve $p = 1 - q$ for a product in each of two periods, 1 and 2. Its unit cost of production is $0 < c < 1$ in period 1 and $c - \lambda q_1^*$ in period 2, where $\lambda > 0$ and q_1^* is the first period output of the monopolist (so the firm learns by doing). Assume the discount factor between the two periods to be 1. Derive q_1^* in equilibrium.

(b) Suppose now that M faces an entrant (E) in the second period; E has unit cost c and an entry cost F . If entry occurs, the firms play Cournot game. Find second period output and profits of the two firms.

(c) If M follows entry deterrence strategy (and accordingly decides q_1), what will be the corresponding first period output of M? Also derive its total profits under entry deterrence.

[3+3+4=10]

2. The market inverse demand function for flights between two cities is $q = 12 - p$ where p denotes the airfare, and q denotes the number of passengers in a given period. There are no production costs. In stage I, an entering airlines denoted by e , invests in capacity k (the number of passengers that can be flown) and sets an airfare p^e . None of these can be changed in stage II of the game. In stage II the incumbent airlines denoted by I , sets its airfare p^I . Therefore the demand facing each airlines in stage II is

$$q^I = \begin{cases} 12 - p^I & \text{if } p^I \leq p^e \\ 12 - k - p^I & \text{if } p^I > p^e \end{cases} \quad \text{and} \quad q^e = \begin{cases} k & \text{if } p^e < p^I \\ 0 & \text{if } p^e \geq p^I \end{cases}$$

Assuming that the entrant's choice of capacity and price $\{k, p^e\}$ is limited to two pairs only (2, Rs 3/-) and (4, Rs.1/-) compute the subgame-perfect equilibria of this game.

[10]

3. A monopoly manufacturer of an intermediate good sells a quantity q to a monopoly retailer. The retailer faces demand $q = 1 - p$ where p is the final price. The retail cost is 0 and the manufacturer's cost is $C(q) = \frac{q^2}{2}$. Before the manufacturer chooses the price or tariff the retailer can make a promotional/advertising investment a that increases the demand to $q = 2 - p$ and is observed by the manufacturer.

- (a) What is the level of investment under a linear price/tariff $T(q) = p_w q$ where p_w is the price charged by the manufacturer to the retailer?
- (b) What is the level of investment under a two-part tariff?
- (c) Does the manufacturer make more investment under a two-part tariff?

[3+4+3=10]

4. A buyer and a seller enter a contract for a product. The expected value of the product to the buyer is V . The cost of production depends on whether the seller makes an investment of I . This investment is non-contractible but the buyer can tell whether the seller has made it. If the seller makes this investment, the cost of production falls by $(1+r)I$. The cost of production is then $C - (1+r)I + I$. The *ex ante* gain from exchange is:

$$V - C + (1+r)I - I = V - C + rI$$

Both the buyer and the seller know that regardless of the price they agree on *ex ante*, they will be obliged to renegotiate the contract and split the gain from exchange equally *ex post* (Nash bargaining solution). The Nash bargaining price is p such that the buyer's surplus (*ex post*) is equal to the seller's surplus (*ex post*). If the seller makes the investment, the Nash bargaining price, p is such that: Buyer's surplus = $V - p$ = Seller's surplus = $p - C + (1+r)I - I = p - C + rI$

- (a) Find the Nash bargaining price if the seller **does not** make the cost-reducing investment.
- (b) Find the seller's *ex post* profit if he **does** make the investment.

[5 + 5=10]

5. A buyer B buys an input from a monopolistic seller S who produces it at a cost 0. At date 2, S has all the bargaining power and can make a take-it-or-leave-it price offer p . If B accepts p , trade takes place ($q = 1$). If the buyer rejects, there is no trade ($q = 0$). The buyer's valuation of the good is $v \in \{\bar{v}, \bar{v} + \Delta\}$, $\bar{v} > 0, \Delta > 0$ and is private information of B . At date 1, B can take an unobservable action $a \in \{a_L, a_H\}$, at his own cost $c(a_H) = c > 0 = c(a_L)$ which determines the probability that $v = \{\bar{v} + \Delta\}$. If he takes a_H then $\Pr\{v = \bar{v} + \Delta\} = \frac{3}{4}$. If he takes a_L then $\Pr\{v = \bar{v} + \Delta\} = \frac{1}{4}$. At date 0, q and a are both non-contractible, but it is possible to contract on the share $\alpha \in [0,1]$ of the B 's profits that S receives. Given α , the utility functions of the buyer and seller are:

$$U_B = (1 - \alpha)q(v - p) - c(a)$$

$$U_S = qp + \alpha q(v - p)$$

- (a) What are the first best levels of q and a ?
- (b) Show that under non-integration ($\alpha = 0$) there is an unique equilibrium under which S charges $p = \bar{v} + \Delta$ and B chooses $a = a_L$.

(c) Show that under full integration ($\alpha = 1$) S charges $p = \bar{v}$ but B still chooses $a = a_L$.

(d) Show that if $c \leq \frac{1}{6} \bar{v}$ there exists an $a \in [0,1]$ such that the first best can be achieved in equilibrium.

[2+2+2+4=10]

6. Consider 2 firms producing compatible network goods, A (Airtel) and B (BSNL). Each network, i , has an expected network size $n_i^e = m_i + q_i^e$ where $m_i \geq 0$ is the size of installed base -- who are locked in to their respective networks (i.e cannot switch) -- and q_i^e is the expected number of new customers. New customers of type θ , uniformly distributed on $[0,1]$, obtain a net surplus

$$U_i(\theta) = \theta + h_i - p_i$$

from adopting the good of firm i at price p_i where h_i is the expected network benefit of good i , $h_i = v[m_i + q_i^e] + s[m_j + q_j^e]$ and $s < 1$ is the degree of compatibility between the two networks. Firms choose capacities simultaneously and compete a la Cournot over new customers. Given these capacities, prices adjust at levels such that (i) consumers are indifferent between the goods offered by the two firms and (ii) demand is equal to supply. Consumers observe quantities before making actual consumption decisions. In this framework show that:

- (a) Enhanced compatibility leads to a market expansion effect leading to a higher consumers' surplus.
- (b) Enhanced compatibility is less attractive to a firm that is more efficient or enjoys larger installed base.

[5 + 5=10]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2011-2012)

MS(QE) II

Social Choice and Political Economy

Date: 30.04.2012

Maximum Marks: 100

Duration: 3 hrs.

Note: Answer Group A and Group B in separate answer scripts.

Group A

- (1) Consider the income distributions over population sizes m and n respectively, where $m \neq n$. Develop necessary and sufficient conditions for welfare ranking of these distributions. Explain empirical relevance of this result. (10+3=13)
- (2) (a) Define second order stochastic dominance and demonstrate its equivalence with expected utilitarian rule, where the utility function is increasing and strictly concave. (10)
(b) Show that the s -th order stochastic dominance implies all higher order stochastic dominance but the converse is not true, where $s \geq 1$ is an integer. (5)
- (3) Define Harsanyi's aggregation equation and state his theorem under Pareto indifference by giving necessary preliminaries. (2+2=4)
- (4) State a set of sufficient conditions for expressing an income distribution based welfare function as the product of the mean income (efficiency) and a decreasing function of inequality. Demonstrate your claim rigorously. (8)
- (5) Assume that individual utilities in an n -person society are cardinally measurable and non-comparable, where $n \geq 3$. Show how under welfarism this information along with the weak Pareto principle can order all utility 2-tuples in \mathfrak{R}^2 . (10)

Group B

- (1) Consider a social welfare function F defined over a finite set of alternatives A with $|A| \geq 3$ for a finite set of agents N . Show that if F satisfies unrestricted domain, independence of irrelevant alternatives and non-imposition then one of the following statements must be true.
- (a) F is null.
 - (b) There exists $x, y \in A$ such that for all profiles with the property that whenever $xP_i y$ for all $i \in N$, we have xPy .
 - (c) There exists $x, y \in A$ such that for all profiles with the property that whenever $xP_i y$ for all $i \in N$, we have yPx . **(12)**
- (2) Consider the preference relation \mathcal{R} defined on the finite set of alternatives A with $|A| \geq 3$. Answer the following questions.
- (a) Define reflexivity, completeness, quasi-transitivity and acyclicity of \mathcal{R} on A .
 - (b) Show that \mathcal{R} on A is reflexive, complete and quasi-transitive if and only if $\forall x, y, z \in A$ such that xPy and yRz we get xRz .
 - (c) Demonstrate via an example that acyclicity over triples is not sufficient for maximal elements to exist. **(8+10+5=23)**
- (3) Let A be a finite set of alternatives with $|A| = m$ and let \mathcal{P}_{\geq} denote the set of all strict single-peaked preferences given a linear order \geq on A . Show that the total number of such single peaked preferences is 2^{m-1} . **(15)**

MSQE II - Theory of Finance II
Final Exam. / Semester II 2011-12
Time - 3 hours/ Maximum Score - 50
Date: 2nd May, 2012

1. (4+4+4+4=16 marks)

Assume the following short rate model for r ,

$$dr = (b - ar)dt + \sigma dW_t$$

with $a(> 0)$, b , σ are constants and W is a standard Brownian motion independent of $r(0)$.

- (a) Show that, as $t \rightarrow \infty$, distribution of $r(t)$ converges to $N(b/a, \sigma^2/2a)$.
- (b) Note that the above r can take negative value, perhaps with low probability, depending on the value of the parameters a , b and σ . Take $z = r^2$ and find dz . This can give you a CIR model on short rate.
- (c) What should be the risk-neutral price of a zero coupon bond at time zero which matures at time T assuming that the market follows the above short rate model on r ?
- (d) Do the models on r or r^2 give an affine structure? Justify.

2. (4+6+4=14 marks)

Let the stocks in a financial market follow the trinomial model (i.e., U, M, D , three states), and the interest is fixed.

- (a) With two stocks and a money account would the market be complete? If so, under what condition? Give the answer, justifying mathematically (if you can replicate any portfolio, etc.).
- (b) Let the prices of two stocks, at time t , be denoted by S_{1t} and S_{2t} , respectively. Assume that a derivative pays $\max((S_{1T} + S_{2T}) - K)^+$, where T is the maturity time and K is a positive constant (that may be called the strike price).
 - (i) Using one-step trinomial model with complete market condition, calculate the hedging necessary (i.e., Δ_1 and Δ_2) to keep the portfolio risk-neutral.
 - (ii) Also, under the same condition, find the corresponding risk-neutral measure.

3. (4+3+4+4=15 marks)

Suppose firms have tax liability but do not pay taxes on the interest payment on loan which is charged at rate of $R_D = 5\%$. If T_c is the flat rate of tax (ignoring the progressive factors). If a firm needs Rs.1000cr for some investment with the expected return 80cr. of Rupees. Assume their initial wealth is Rs.10000cr and it is fully unlevered firm with outstanding share 4 billions.

- (a) Calculate, the yield per share if the funding is done
 - (i) Q percent from borrowing and the rest through IPOs (assuming $0 \leq Q \leq 100$);
 - (ii) from their retained earnings.

(b)(i) Would the yield in (a)(i) be same for every value of Q ? If so, justify, if not, find the value of Q that maximises the yield and find the maximum value.

(ii) Would the investment that gives the maximum yield in (a)(i) be preferable than the investment from the retained earnings? Justify your answer.

4. (5+5=10 marks)

Justify if the following holds or not.

(a) The Put price of a European option is always cheaper than that of the American options;

(b) Given a forward rate model one may not be able to find a short rate model that is consistent with the forward rate model;

All the best.

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2011-12

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 12/04/05/12
Maximum marks: 50
Duration: 3 hours

Note: Answer any one question in Part 2

Part 1 (20 marks)

A risk-neutral entrepreneur E has a project which requires a unit of capital to be initiated. The opportunity cost of capital is 0. The project produces 2 with probability p , and 0 otherwise. p is chosen by E and the cost of p is $c(p) = \frac{p^2}{2\alpha}$, where α is E 's type. The cost is borne by E .

(a) Suppose E can supply the capital required. Does he initiate the project, and what is the effort level if he does? (5)

(b) Now suppose E has no capital. A risk-neutral financier F can supply the capital required. E 's type is private information, while the probability chosen by him is observable, as is the output obtained. F can offer a take-it-or-leave-it contract to E , offering him the required capital in exchange for a return which can be conditioned on the output obtained as well as the probability chosen. E is liquidity constrained, so the return received by F cannot exceed project output. F knows that $\alpha \in \{\frac{1}{4}, \frac{1}{2}\}$ and $\Pr(\alpha = \frac{1}{4}) = \frac{1}{2}$ (E knows his own type). What is the optimal contract? (15)

Part 2 (30 marks)

Q1: Consider the basic binary moral hazard problem with a risk-averse agent. Assume the principal always wishes to implement high effort.

a) How would you define the notion of agent productivity in this setting? (10)

b) Does increased agent productivity necessarily increase the principal's net expected payoff? Argue rigorously. (20)

Q2: Consider a variant of the hidden action contracting model with continuous effort, limited liability, and risk-neutral agent. Output is contractible, and there are three output levels: y_0, y_1, y_2 , with $y_2 > y_1 > y_0 \geq 0$. A contract defines a transfer to the agent given output, so any contract is defined by (t_0, t_1, t_2) , with $t_i \geq 0$, for all i by limited liability.

The agent can take effort $e \in [0, \frac{1}{2}]$. The cost of effort is $c(e)$, which is thrice continuously differentiable, strictly increasing and strictly convex, with $c(0) = c'(0) = 0$, $c'''(e) \geq 0$.

Denote the probability that output is y_i , given e , as π_i . Assume $\pi_0 = 1 - e$, $\pi_1 = e(1 - e)$, $\pi_2 = e^2$.

a) Show that in any optimal contract, $t_0 = 0$. (10)

b) Using the earlier result, show that in any optimal contract, $t_1 = 0$. (10)

(c) Now derive the optimal contract. What effort level does the optimal contract induce? (5 + 5)

Indian Statistical Institute
Second Semestral Examination: 2011 –2012
M.S.(Q.E) – II Year/ M.Stat. – II Year
Econometric Applications II

Date: 8.5.2012

Maximum Marks: 100

Duration: 3 hrs.

(Answer any **four** questions)

1. (a) Define Lorenz Curve (LC) that incorporates both discrete and continuous type size distributions of income, given the distribution function

$$F : [0, T] \rightarrow [0, 1]$$

- (b) Find the LC for log-logistic distribution using the above definition.
(c) Describe the alternative procedures for estimation of Lorenz curve from grouped data.
(d) What are the 'relative' and 'absolute' measures of income inequality? Define 'normative' and 'positive' measures of inequality. Give examples.

[4+5+8+8=25]

2. (a) Define homothetic preferences. Show that under linear homogeneity of preferences, the cost function is of the form

$$C(u, p) = \alpha(p)u^*$$

where $u^* = f(u)$ and $\alpha(p)$ is linearly homogeneous in p , the vector of prices.

- (b) Suppose you are given the cost function

$$C(u, p) = \alpha(p) \exp\left(\frac{b(p)}{1/\ln u - \lambda(p)}\right),$$

where $\alpha(p)$ is homogeneous of degree one in prices, $b(p)$ and $\lambda(p)$ are homogeneous of degree zero in prices and u is the level of utility, with

$$\ln \alpha(p) = \alpha_0 + \sum_{i=1}^n \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \delta_{ij} \ln p_i \ln p_j,$$

$$b(p) = \prod_{i=1}^n p_i^{\beta_i} \quad \text{and} \quad \lambda(p) = \sum_{i=1}^n \lambda_i \ln p_i.$$

- (i) Obtain the appropriate restrictions on the parameters for the homogeneity conditions to be satisfied.
(ii) Derive the budget share function for item i .
- (c) What are 'exact' and 'consistent' aggregations in the context of consumer demand analysis? For each type of aggregation, give an example of a demand system conforming to the respective type.

[7 + (5+5) + 8 = 25]

3. (a) Describe the Ramsey-Samuelson-Diamond-Mirlees approach to the determination of optimal commodity taxation.
- (b) Assuming that consumer's demand behaviour is captured through the Linear Expenditure System, examine the redistributive impact and the effect of taxation on the worst-off household assuming that taxation is purely a redistributive mechanism amongst households.
- (c) Explain the 'double hurdle model' (with discrete random preference regimes) for modelling 'zero expenditure'.

[12 + 5 + 8 = 25]

4. (a) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?
- (b) What is "Purchasing Power Parity" (PPP)? How do you estimate PPP between two countries given their domestic price and quantity vectors?
- (c) What is 'sample selection bias'? Give an example of a bivariate sample selection model with a participation and an outcome equation. How would you estimate the parameters using Heckman's procedure?

[3+10+12=25]

5. (a) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans.
- (b) "D-F technical efficiency is necessary, but not sufficient for Koopmans technical efficiency"—explain this statement diagrammatically for both input and output oriented measures.
- (c) Show (diagrammatically) that the input and output oriented measures of technical efficiency are equal under constant returns to scale (use a single input single output set up).
- (d) Describe the Corrected Least Squares (COLS) method of estimating technical efficiency from cross section data using a deterministic production frontier.
- (e) Describe the Modified Least Squares (MOLS) method of estimating technical efficiency from cross section data using a deterministic production frontier and a Half Normal distribution of efficiency (u).

$$[u \sim \text{HalfNormal}(0, \sigma_u^2) \Rightarrow f(u) = \frac{2}{\sigma_u \sqrt{2\pi}} \exp(-u^2 / 2\sigma_u^2)]$$

[5 + 6 + 4 + 5 + 5 = 25]

Back Paper

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2011-2012)

MS(QE) II

Social Choice and Political Economy

Date: 25.6.12

Maximum Marks: 100

Duration: 3 hrs.

Note: Answer Group A and Group B in separate answer scripts.

Group A

- (1) Show that the Atkinson-Kölm-Sen index of inequality is a relative index if and only if the underlying welfare function is homothetic. Provide a policy interpretation of this index. (8+2=10)
- (2) Develop an axiomatic characterization of positional dictatorship in the welfarism framework. (15)
- (3) Show how first order stochastic dominance becomes equivalent to the expected utilitarian rule criterion, where the utility function is increasing. (10)
- (4) Define Harsanyi's aggregation equation. Give an example to illustrate this. (2+5=7)
- (5) Demonstrate the Dasgupta-Sen-Starrett Theorem for income distributions with variable population size. (8).

Group B

- (1) Define quasi-transitivity and acyclicity. Consider a reflexive and complete binary relation R defined on the set of alternatives A which is finite but has more than two elements. Using this information prove the following:
 - (a) The binary relation R is quasi-transitive if and only if for all $x, y, z \in A$, xPy and yRz implies xRz .
 - (b) The binary relation R is acyclic if and only if the maximal set $M(S, R)$ is non-empty for all non-empty $S \subseteq A$.(4+8+8=20)

- (2) State and prove the Median Voter Theorem, giving all the relevant definitions. **(10+5=15)**
- (3) Define strategyproofness, unanimity and the full range property of a social choice function when agents have strict preferences and the set of alternatives is A which is finite and has more than two elements. Show that if the social choice function satisfies strategyproofness and the full range property then it also satisfies unanimity. (Note that the set of agents may be more than two) **(4+2+2+7=15)**

Indian Statistical Institute

Second Semestral Examination: 2011 –2012
M.S.(Q.E) – II Year/ M.Stat. – II Year
Econometric Applications II

Back paper

Date: 27.06.12

Maximum Marks: 100

Duration: 3 hours

Answer **all** questions.

1. a) What are the different approaches to specifying a demand system?
- b) What is the Gorman-Polar form of cost function? What is the form of the Engel curve implied by this form of cost function?
- c) Show that for the Linear Expenditure System (LES), the non-compensated own price elasticities are approximately proportional to the corresponding expenditure elasticities.
- d) In the LES, an amount z of good 1 must be bought. Show that for $i = 2, 3, \dots, n$,

$$p_i q_i = p_i c_i + \frac{b_i}{\sum_{i=2}^n b_i} (x - p_1 z - \sum_{k=2}^n p_k c_k)$$

where the symbols have their usual meanings.

[6 + 4 + 7 + 8 = 25]

2. a) Explain the difference between a 'censored distribution' and a 'truncated distribution'?
- b) Write down the Tobit model to incorporate zero consumption with full specification of the distribution of the error term. Why are the assumptions underlying the standard linear regression model not tenable in such a case?
- c) Given the dynamic model for 'clothing'

$$q(t) = \alpha + \beta s(t) + \gamma x(t)$$

where $q(t)$: rate of demand at time t

$x(t)$: income during the same time

$s(t)$: inventory of 'clothing' at time t ,

and assuming that the stock is used up at a constant depreciation rate δ , find the short and long term derivatives of consumption with respect to income.

[3 + 12 + 10 = 25]

3. Write short notes on the following:

- a) Application of complete demand models to demand projection;
- b) The Almost Ideal Demand System.

[25]

4. a) Define the input and output oriented measures of 'technical efficiency' due to Debreu-Farrell (D-F) and Koopmans.

b) "D-F technical efficiency is necessary, but not sufficient for Koopmans technical efficiency"—explain this statement diagrammatically for both input and output oriented measures.

c) Show (diagrammatically) that the input and output oriented measures of technical efficiency are equal under constant returns to scale (use a single input single output set up).

d) Describe the Corrected Least Squares (COLS) method of estimating technical efficiency from cross section data using a deterministic production frontier.

e) Describe the Modified Least Squares (MOLS) method of estimating technical efficiency from cross section data using a deterministic production frontier and a Half Normal distribution of efficiency (u).

$$[u \sim \text{HalfNormal}(0, \sigma_u^2) \Rightarrow f(u) = \frac{2}{\sigma_u \sqrt{2\pi}} \exp(-u^2 / 2\sigma_u^2)]$$

[5 + 6 + 4 + 5 + 5 = 25]

INDIAN STATISTICAL INSTITUTE

Back Paper Examination (2011 – 2012)

MS(QE) II Year

Environmental Economics

Date : 28.06.2012

Maximum Marks : 100

Duration : 3 hrs.

1. In a single species competitive fishery the initial stock of fish is X_0 and production function is $q = \theta XE$, where q is the harvest rate, θ is a positive constant and E is effort involved in catching fish. The simple cost function is, $c(X, q) = c(X)q = wE$, where $c(X)$ is average cost and w is the cost per unit of effort. The biological growth function is given by $g(X) = \gamma X(k - X)$, where k is carrying capacity level and γ is a positive constant. The firm's objective is to maximize present value of net benefit over an infinite time horizon when discount rate is r . Competitive price is p which is constant.
- (a) Determine the 'Fundamental Rule' for renewable resources in the context of this model.
- (b) Determine the optimal harvest rate, optimal stock and optimal effort level in this dynamic model for a sole ownership fishery firm.

[15+10=25]

2. Consider a non-renewable resource industry, say oil, consisting of number of small firms and a few big firms. A cartel, formed by the big firms, acts as a price discriminator and the small firms, who behave like competitive firms, are price takers. Let q^f and q^{ca} be the quantity of extraction by competitive firms and cartel respectively in period t . Let p , X and T respectively be the output price, resource stock and terminal time. Assume marginal cost of extraction is constant and it is lower for the cartel relative to that for the competitive firms. The rate of interest remains fixed at r for all periods. Also assume that the demand function is an inverse demand function with a backstop price p^b . All the firms have the objective to maximize discounted future profits when (a) initial stock is given and (b) change in stock is determined by the extraction rate. State and derive the optimal extraction policy of each type of firm.

[15]

6

3. What do you mean by resource scarcity? Explain the Neo-classical model for measuring resource scarcity.

[3+12=15]

4 (a). Explain the difference between emissions-based and ambient-based tax systems for pollution control and how they are determined.

(b). Two chemical units having marginal abatement cost functions $5 - e_1$ and $8 - 2e_2$, respectively exist in a locality. A monitoring centre is located in a place nearby the units. Marginal damage function due to ambient air pollution is observed as $MD(e) = e$, where e is the sum of e_1 and e_2 . Find the optimal level of ambient pollution as well as the emission level of each unit. Also find out the optimal emission fee for controlling emission level.

[20+5=25]

4. Develop a methodology to measure the benefits from protecting the residence of Swami Vivekananda-a heritage building in Kolkata, the services of which are not sold in the market. Also outline the theoretical basis for developing the methodology.

[20]