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A NOTE ON CONSTRUCTION OF SYMMETRIC FACTORIALS RETAINING FULL INFORMATION ON MAIN EFFECTS

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Kempthorne (1952), and Raghava Rao (1971), point out that it is impossible to construct a $(3^4, 3^3)$ design without confounding any main effct pencil. This assertion does not, however, seem to be true. For example, starting with 3 independent pencils (1, 0, 1, 1), (1, 1, 0, 1), (1, 1, 1, 0) and using standard methods (see, Raghava Rao (1971) one can construct a $(3^4. 3^3)$ design without confounding any main effect pencil.

In fact, as will be shown in this note, for every positive integers $s(\geqslant 2$ and not necessarily a prime power), $m(\geqslant 2)$, $k(1 \leqslant k \leqslant m-1)$, it is possible to construct an (s^m, s^k) design retaining full information on main effect contrasts. Following Mukerjee (1980), for such a construction it is necessary and sufficient that in each block the levels of each factor occur with equal frequency.

First take the case $k=m-1$. Then block size is s. Take an initial block given by the level combinations[(0,...,0), (1,...,1),...,(s-1,.. ,s-1)]. Then develop $s^{m-1}-1$ additional blocks each time taking some level combination not included in the previously constructed blocks and adding that level combination (coordinatewise and reduced mod(s)) to each level combination in the initial block. When s is a prime/prime power, this is equivalent to starting with (m-1) independent pencils(1,0,1,...,1,1) (1,1,0,...,1,1),..., (1,1,1,...,0,1), (1,1,1,..., 1,0) and applying the standard method as in Raghava Rao (1971).

For $k=1,2,..,m-2$, first generate an (s^m, s^{m-1}) *design by the procedure described in the preceding paragraph and call it D. Then* divide the s^{m-1} blocks in D into s^k mutually exclusive and exhaustive sets, each set with s^{m-k-1} blocks. Combine the blocks in each set to *form* a single block with s^{m-k} level combinations.

In the (s^m, s^k) designs $(k=1,\ldots, m-1)$ so generated, in each block the levels of each factor occur equally frequently and hence full information is retained on all main effect contrasts:

In fact, for such a construction it is not even essential that block size should be a power of s. It is enough to take block size a multiple of s. Thus to construct an s^m factorial in blocks of size u s (obviously then s^{m-1} is an integral multiple of u) one should first construct the design D and combine the blocks in D in mutually exclusive and exhaustive sets of cardinality u.

Example : With $s = 6$, $m=2$, $k = 1$, the design D *retaining full information* on all main effects will have blocks given by the rows of the following

(i)	00	11	22	33	44	55
(ii)	01	12	23	34	45	50
(iii)	02	13	24	35	40	51
(iv)	03	14	25	30	41	52
(v)	04	15	20	31	42	53
(vi)	05	10	21	32	43	54

Combining blocks (i) and (ii), (iii) and (iv), (v) and (vi) one gets a 6^2 design in 12-plot blocks retaining full information on main effects.

For s prime/prime power, the construction is also possible following Das (1964). In fact, the solution is not unique and in generating the design D one may take the level combinations in the initial block as $[(f_1(x),\ldots, f_m(x)), x = 0, 1, \ldots, s-1]$, where for $j=1, 2,\ldots, m$, $f_j(x)$ is any automorphic mapping (under addition (mod s)) over $\{0, 1, \ldots, s-1\}$. Then interpreting the set of s^m level combinations $\{(i_1, \ldots, i_m), 0\leqslant i_j\leqslant s-1; j=1, \ldots,m\}$ as a group under addition (componentwise and reduced (mod s)) the level combinations in the initial block form a subgroup and the other blocks are the cosets of the initial block.

It is not even essential to take the initial block as a subgroup of the s^m level combinations. In that case, however, some special approach should be adopted in generating the other blocks to avoid repetition of the same level combination. For example, in obtaining D, one may take the initial block as $[(x_{i_1}, x_{i_2}, \ldots, x_{im}), i = 0, 1, \ldots, s-1]$, where for each $j(1 \leqslant j \leqslant m)$, $(x_{0j}, x_{1j}, \ldots, x_{s-1,j})$ is a permutation of $(0, 1, \ldots, s-1)$ and then develop the remaining $s^{m-1}-1$ blocks as $[x_{i1}, x_{i2}+t_2, \ldots, x_{im}+t_m)$, $i=0, 1, \ldots, s-1]$, $0 \leqslant t_2, \ldots, t_m \leqslant s-1$, $(t_2, \ldots, t_m) \neq (0, \ldots, 0)$, addition being reduced (mod s).

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