A NOTE ON OPTIMAL POLICIES IN DUAL ECONOMIES*

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I. INTRODUCTION

In the August 1980 issue of this *Journal,* Kaushik C. Basu discusses optimal subsidy policies within a Harris-Todaro economy as described by Bhagwati and Srinivasan [1973, 1974].[1] The latter showed that a first best could be reached with a single policy, viz., the uniform subsidy. However, as correctly pointed out by Basu, "A serious problem with the Bhagwati-Srinivasan (BS) optimal subsidy is that a particular component of the subsidy formula is the marginal product of labor *in the optimal situation.*" Basu solves the informational problem by showing that any subsidy greater than or equal to the one described by BS gets us to the first best. In particular, a subsidy equal to the fixed minimum wage is sufficient for optimality. Moreover, the first best equilibrium is unique.

Basu assumed fixed relative prices. In this note we introduce an aggregate demand curve and, therefore, allow prices to vary. Under homotheticity of preferences we show that (i) if the propensities of labor and nonlabor are the same to consume, then price flexibility does not change the Basu analysis (i.e., there is a range of subsidies that gets us to first best and the equilibrium is unique); and (ii) if the propensities differ,[2] there is still a range of subsidies that gives us Pareto efficiency, but each subsidy in this range gets us to a different equilibrium.

II. THE MODEL

There are two sectors, modern (M) and rural (R). They produce outputs X_M and X_R, using labor L_M and L_R, respectively. The production functions are

$$(1) \qquad X_i = f_i(L_i), \qquad f'_i > 0, \qquad f''_i < 0, \qquad i = M,R,$$

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1. See also Srinivasan and Bhagwati [1975].
2. We shall assume the propensity to consume rural produce by laborers is greater than that of nonlabor. If we think of rural produce as food, this means that laborers spend a larger portion of their income on food.

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where

$$\sum_i L_i \leqslant 1, \tag{2}$$

i.e., labor availability is fixed and, by assumption, equal to unity.

Profit maximization in each sector entails

$$pf'_M = w_M \tag{3}$$

and

$$f'_R = w_R, \tag{4}$$

where p = price of modern output in terms of rural produce and w_i = wage in the ith sector in terms of rural output. In keeping with the Harris-Todaro [1970] model,

$$w_M \geqslant \overline{w} > w_e, \tag{5}$$

where w_e is the market-clearing wage. Migration equilibrium in the labor market is given by

$$w_R = (w_M L_M)/(1 - L_R). \tag{6}$$

Using (1), (2), (3), (5), and (6), Basu solves the system, assuming prices fixed. He then proves his results using the above model.

Fixity of prices was guaranteed by the assumption of a small open economy. To allow for price flexibility, we close the economy and introduce an aggregate demand relationship. Output is demanded by two groups in this economy, labor and nonlabor (e.g., the owners of the specific factors). Let I_L denote labor income and $(pX_M + X_R - I_L)$ nonlabor income. Then for equilibrium in the rural market

$$I_L b_L(p) + (pX_M + X_R - I_L) b_K(p) = X_R, \tag{7}$$

where $b_j(p)$ = fraction of jth group's income spent on rural produce ($j = L,K$), $b'_j > 0$, and $I_L = w_M L_M + w_R L_R$. Invoking Walras's Law, we can solve the above system of equations.

Two cases are possible: (I) $b_L(p) = b_K(p)$ for all $p > 0$ and (II) $b_L(p) \neq b_K(p)$. Within this framework we consider the uniform wage subsidy. We assume that all subsidies are financed by nondistortionary taxes. The implicit assumption is that the economy can somehow finance its own subsidies. In other words, if output does not change and the wage subsidy is increased, then there is a redistribution of income from nonlabor to labor. Our first best is defined as follows: (a) $pf'_M = f'_R$, that is, marginal productivities of labor are equated across the two sectors; (b) $L_M + L_R = 1$, that is, we are at full employment; and (c) producer's price of output equals consumer's price for output.

III. RESULTS

Case I: $b_L(p) = b_K(p) = b(p)$, *for all* p

Let S^* be the BS optimal uniform wage subsidy. Then $S^* = \overline{w} - f'_R(L_R^*)$, where the corresponding equilibrium labor allocations are given by L_R^* and L_M^* and the price by p^*.[3] It is our objective to show that there is a unique equilibrium that is also first best. First, we show that for any subsidy $S > S^*$, $L_R + L_M = 1$; i.e., we are at full employment. For, if not, then $L_R + L_M < 1$, which implies that $w_M = \overline{w}$, $w_R < \overline{w}$ [Basu, 1980]. There are two possibilities: $L_R \leqslant L_R^*$ and $L_R > L_R^*$. If $L_R \leqslant L_R^*$, then $f'_R(L_R) \geqslant f'_R(L_r^*) \Longrightarrow w_R - S \geqslant \overline{w} - S^* \Longrightarrow w_R > \overline{w}$, which is not possible. If $L_R > L_R^*$, then $L_M < L_M^* \Longrightarrow f'_M(L_M) > f'_M(L_M^*)$ and $w_M = \overline{w}$. Since $S > S^*$, $\overline{w} - S < \overline{w} - S^* \Longrightarrow pf'_M(L_M) < p^*f'_M(L_M^*) \Longrightarrow p < p^*$. From (7), $X_R/X_M = pb(p)/(1 - b(p))$. Since $L_R > L_R^*$ and $L_M < L_M^*$, we know that $X_R/X_M > X_R^*/X_M^*$. Hence, $pb(p)/(1 - b(p)) > p^*b(p^*)/(1 - b(p^*))$. Since $b'(p) > 0$, it follows that $p > p^*$, which is contrary to what we have just shown, completing the first part of the proof.

We now show that with $S > S^*, L_R = L_R^*, L_M = L_M^*$ and $p = p^*$, i.e., the equilibrium is unique. For $L_R = L_R^*$ and $L_M = L_M^*$, see Basu [p. 193]. It is easy to see that with $S > S^*$, if $L_R = L_R^*$ and $L_M = L_M^*$, we have (7) holding with $p = p^*$, which completes the proof.[4]

Thus, for all $S \varepsilon [S^*, \infty)$ we have a unique equilibrium that is also the first best; i.e., we get the Basu result. Recall that we assume all subsidies are domestically financed by nondistortionary taxes. If the subsidy is financed by gifts from abroad, then the Basu result breaks down because his assumption of fixed prices can no longer be sustained, and this leads to a different allocation of labor between the two sectors.

3. With a uniform wage subsidy S, the equation system (1)–(7) would have to be adjusted in an obvious way. For example, (3) now becomes $pf'_M = w_M - S$.

4. BS define S^* to be $\overline{w} - f'_M(L_M^*)$. Our specification can now be seen to be equivalent.

Case II: $b_L(p) \neq b_K(p)$

Here we prove the existence of a first best uniform wage subsidy S^*, as in BS, and then go on to show that all subsidies greater than S^* give us different first best equilibria.

To sketch a proof of existence, we first show that the following pair of equations have a unique solution with $p > 0$ and $L_M \leqslant 1$:[5]

$$\text{(A)} \quad pf_M'(L_M) - f_R'(1 - L_M) = 0$$

$$\text{(B)} \quad [pf_M(L_M) + f_R(L_R) - \overline{w}]\, b_K(p) + \overline{w} b_L(p) = f_R(L_R).$$

As L_M increases, $f_R'(1 - L_M)$ increases, and $f_M'(L_M)$ decreases. So for (A) to hold, p must increase. Also, if we assume well-behaved production functions, then $\lim_{L_M \to 0} f_M'(L_M) = \infty$ and $\lim_{L_M \to 0} f_M'(1 - L_M)$ is a finite number. Thus, for equation (A) to hold, p must be going toward zero as L_M goes toward zero. Diagrammatically, we can plot (A) as in Figure I. As for (B), if L_M increases, p must fall. To see this, rewrite (B) as $pX_M b_K(p) + \overline{w}(b_L(p) - b_K(p)) + X_R(b_K(p) - 1) = 0$. If $b_L(p) > b_K(p)$,[6] then $\overline{w}(b_L(p) - b_K(p)) > 0$. $X_R(b_K(p) - 1) < 0$ because $b_K(p) < 1$. Recall that $b_j'(p) > 0, j = L,K$. As L_M increases, X_M increases, and for (B) to hold, p must go down. Also, as L_M approaches zero, $pX_M b_K(p)$ approaches zero. If p approaches zero, then equation (B) cannot hold. Indeed, p approaches a positive constant given by $\overline{w}(b_L(p) - b_K(p)) + f_R(1)[b_K(p) - 1] = 0$. Also, as L_M approaches 1, for (A) to hold, p must approach infinity, for $\lim_{L_M \to 1} f_R'(1 - L_M) = \infty$. Thus, we have a solution with $p > 0$ and $L_M < 1$. Let us call this p^* and L_M^*.

Now define S^* as $\overline{w} - p^* f_M'(L_M^*)$, and use this as our uniform subsidy. Then we know that (7) holds[7] because (B) holds. Equation (3) holds because $p^* f_M'(L_M^*) = \overline{w} - S^*$. If $L_R^* = 1 - L_M^*$, then $f_R'(L_R^*) = p^* f_M'(L_M^*)$ and only at $L_R^* = 1 - L_M^*$ is, therefore, $f_R'(L_R^*) = \overline{w} - S^*$. Therefore, (4) holds. Also, since $L_R^* = 1 - L_M^*$, (6) holds. Thus, S^* is our first best uniform subsidy.

Now, let us introduce wage subsidy, $S > S^*$. We show that the equilibrium will change. For, if $L_M = L_M^*$, $X_M = X_M^*$, and $p = p^*$, then $w_M - S = pf_M'(L_M) = p^* f_M'(L_M^*) = \overline{w} - S^*$, or $w_M =$

5. We follow the same method of proof of existence as in Bhagwati and Srinivasan [1973].

6. See footnote 2.

7. If every laborer is receiving $\overline{w}$ and we are at full employment, then $I_L = w_M L_M + w_R L_R = \overline{w}$.

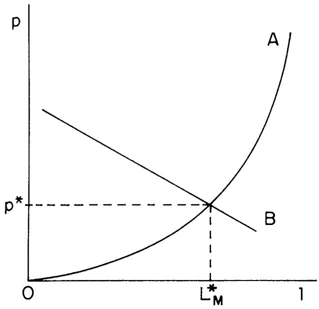

FIGURE I

$\overline{w} + (S - S^*) > \overline{w}$. If workers behave in the way postulated by Harris and Todaro [1970], then whenever wages are greater than $\overline{w}$, the economy is in full employment. Therefore, $L_R = L_R^*$. But then, $pX_Mb_K(p) + X_R(b_K(p) - 1) + I_L\ (b_L(p) - b_K(p)) = p^*X_M^*b_K(p^*))\ X_R^*(b_K(p^*)-1) + \overline{w}(b_L(p^*) - b_K(p^*))$. This implies that $\overline{w} + (S - S^*) = \overline{w} - S^*$, which is impossible. On the other hand, at $S > S^*$, we can easily show that $w_M = w_R = \overline{w} + S - S^*$ gives a first best equilibrium.

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