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Basic Process Capability Indices: An Expository Review

Mohammed Z. Anis

SQC and OR Unit, Indian Statistical Institute, Kolkata, India
E-mails: zafar@isical.ac.in; mzanis@yahoo.com

Summary

A review of the four basic process capability indices has been made. The interrelationship among these indices has been highlighted. Attention has been drawn to their drawbacks. The relation of these indices to the proportion nonconforming has been dwelt upon and the requirement of the adequate sample size has been emphasized. Cautionary remarks on the use of these indices in the case of nonnormal distributions, skewed distributions, and autocorrelated data are also presented. The effect of measurement error on process capability indices has been dealt with in great detail.

Key words: Autocorrelation; measurement error; nonnormality; sample size; skewed distributions.

1 Introduction

With the world becoming borderless, at least as far as business is concerned, there is intense national and international competition amongst business organisations. This competition is compelling business organisations to manufacture defect-free products. To achieve this objective, companies have started adopting different strategies like Total Quality Management (TQM) and Six Sigma throughout their organisations. A part of the philosophy of these strategies requires the monitoring of the performance of the individual processes. These results are then compared with those of industry leaders through competitive benchmarking. One metric popularly used is the Process Capability Index (PCI) (Spiring, 1995). Essentially a PCI measures the variability of a process relative to its specification limits. Being unitless, these indices permit comparisons amongst hundreds of processes emanating from a whole range of production processes, industries, and even countries. Many (large) companies have instituted programmes that inherently make use of these indices to promote and drive quality improvement programmes throughout their organisations (Barnett, 1990; Gill, 1990; McCoy, 1991).

Moreover, the incorporation of capability analysis into a company's Six Sigma programme makes it a particularly important topic for management reporting. Briefly, Six Sigma is a quality and business improvement methodology that makes heavy use of statistical methods. It began in Motorola in the 1980s. While originating in manufacturing, it has expanded to financial services, health care, and even nonprofit organisations. The rapid spread of Six Sigma is due to the fact that it has delivered significant bottomline results. It is perhaps the largest and most important statistically based initiative in history. The Six Sigma methodology includes five steps, namely, definition of the measuring unit critical to quality (CTQ), measurement of the current process performance, analysis of the root cause and identification of the solutions, the improvements of the process quality, and control of the process quality. Recently, a session (Invited Paper Meet 74) was devoted to Six Sigma at the 56th session of the International Statistical Institute, held in Lisbon during August 22–29, 2007.

Historically, Feigenbaum (1951) and Juran (1951) first proposed 6σ as a measure of process capability. This represented process capability as a measure of the inherent variability of a process, but is divorced from customer specifications. Juran (1962) overcame this lacuna by comparing 6σ to the tolerance width as a method of determining the need for process improvement activities. Nevertheless, capability itself was still interpreted separately from specifications. Finally, Juran & Gryna (1980) proposed the first metric that directly compared process variability to customer specifications. They proposed a capability ratio:

$$\text{Capability ratio} = \frac{6\sigma\,\text{variation}}{\text{Total tolerance}}. \tag{1}$$

Like the capability ratio, all PCIs explicitly link process variability to customer specifications. Thus, they emphasize the suppliers' responsibility to satisfy those specifications. However, capability indices also have advantages over the capability ratio. They (generally) increase in value as the process performance improves. Furthermore, they indicate the relative benefits of improvement in both process location and variability.

However, it is also important to understand their limitations, especially because organisations are placing a greater emphasis on quality-related measures. One essential prerequisite for the process of improving quality via capability indices is that the process be in a state of statistical control. Further, these measures can themselves be distorted and may not accurately indicate the extent and type of improvements needed. A lack of understanding of process variability has caused a significant amount of controversy over the use of these indices (Kotz & Johnson, 1993, pp. 1–2). At times, the underlying correlation in a process when coupled with outliers can mask out-of-control points, and thereby make it appear to be in a state of statistical control. Ignoring these interactions may make the process appear better or worse than it actually is. This, in turn, may lead a manager to divert resources to improving processes where the returns are minimal but ignore areas that can lead to large reductions in variability and significant quality improvements. In the long run, this lack of understanding will frustrate the managers and affect the overall profitability of the organisation.

There is a large body of literature dealing with PCIs. Mention may be made of the books by Kotz & Johnson (1993), Kotz & Lovelace (1998), Wheeler (1999), Bothe (2001), and Pearn & Kotz (2006). Papers relating to PCI have appeared in journals of statistics, management science, industrial engineering, quality and TQM. Spiring *et al.* (2003) give a bibliography, whereas Kotz & Johnson (2002) provide a largely theoretical overview. However, most of these works have a theoretical flavour directed towards researchers. There is a paucity of literature, which unmasks the intricacies of these indices. This paper endeavours to fill this gap. In this paper, an attempt has been made to study the effectiveness of the existing PCIs in relation to the decision-making process of the users and advocate caution. It is hoped that this expository paper will be useful to the practicing engineer, the management personnel, and serious students of theoretical statistics.

The rest of the paper is organised as follows. Section 2 briefly dwells on the need for indices. Section 3 introduces the indices and their motivation; develops the interrelationships among them and discusses their estimators. Section 4 reviews the basics regarding index interpretation and process improvements and attempts to highlight the drawbacks. Section 5 discusses the indices in relation to the number of nonconforming (NC) products produced and the sample size required for any scientifically meaningful study. Section 6 considers the effect of nonnormality, correlation, and asymmetry in the calculation of PCIs. Section 7 deals with the effect of measurement error. Section 8 concludes the paper with a discussion.

2 The Need for an Index

The behaviour of a process is often described by a probability distribution. In order to assess its adequacy, the hypothesised distribution has to be compared with the corresponding specifications. A PCI attempts to summarise the process performance and hence is a function of the process distribution and the corresponding specification.

Shewhart (1931), Juran (1951), and Gryna (1988) discussed the use of process capability information to determine specification limits. They recommended that the tolerance width should not be tighter than 6σ. Feigenbaum (1951) and Juran (1951) referred to the use of process capability information to assign jobs to machines. Kane (1986) described six application areas for capability indices, which are as follows. The indices help in the prevention of NC products by establishing a benchmark capability. Being dimensionless, they facilitate communication between engineering and manufacturing departments and between manufacturers and suppliers. They aid in establishing the priority areas for process improvement and continuous improvement. The indices also provide information on the location and variability of a process and hence suggest the road map for process improvement. Finally, the indices can be used in audits to help establish the problem areas. Crain (1993), Chou (1994), and Schneider & Pruett (1995), among others, discussed the use of these indices in customer–supplier settings. Bulba & Ho (2006) used methods to obtain approximate confidence intervals for the variance of the output characteristic to make inference on capability indices when the variable of interest is not an observable one but is a function of a set of independent input variables. Bothe (2006) described a method for assessing the capability of a process to locate hole centres within a circular tolerance zone. These give a partial listing of the applications of a PCI.

The important objectives of a PCI have already been discussed by Tsui (1997). Suffice it to say that a PCI should be informative enough to guide the users in their decision problems *adequately* and *unambiguously*. Another desirable feature of a PCI is that its numerical value should increase when the variability decreases.

3 Process Capability Indices

It is assumed that there is only one quality characteristic (say X) of interest. Let U and L be the upper and lower specification limits, and let T be the "target value" and define $M = (U + L)/2$, and $D = (U - L)/2$. Let the underlying process mean and standard deviation be denoted by μ and σ, respectively. Unless otherwise stated, we shall assume that the quality characteristic is normally distributed.

Depending upon the situation, the specification for X can be one of the following types:

(a) Unilateral (one-sided, with target not specified)

- (i) Only U
- (ii) Only L

(b) Bilateral (two-sided, with target specified)

- (i) Centred target, that is, $T = M$
- (ii) Off-centred target, that is, $T \neq M$

3.1 Definition of the Indices and Their Motivations

Assuming the quality characteristic to be normally distributed with mean μ and variance σ^2, the process yield is, in general, given by

$$\%\,\text{Yield} = 100\left[\Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{L-\mu}{\sigma}\right)\right],$$

where Φ denotes the standard normal cumulative distribution function. The index C_p measures only the distribution spread (process consistency/precision), which only reflects the consistency of the product quality characteristic. The yield-based index C_{pk} provides lower bounds on process yield by taking the process location into consideration, which offsets some of the weaknesses in C_p, but can fail to distinguish between on-target and off-target processes. The index C_{pm} takes the proximity of process mean from the target value into account, which is more sensitive to process departure than C_{pk}. The index C_{pmk} provides a greater level of quality assurance with respect to process yield and process loss to the customers than the C_{pk} and C_{pm} indices.

3.1.1 The most basic indices

We begin with the most basic index.

(a) Unilateral with only U:

$$C_{pu} = \frac{U-\mu}{3\sigma}, \tag{2}$$

provided that $\mu \le U$

(b) Unilateral with only L:

$$C_{pl} = \frac{\mu-L}{3\sigma}, \tag{3}$$

provided that $L \le \mu$

(c) Bilateral with $T = M$:

$$C_p = \frac{U-L}{6\sigma}. \tag{4}$$

(d) Bilateral with $T \neq M$:

$$C_p^* = \min\left\{\frac{U-T}{3\sigma}, \frac{T-L}{3\sigma}\right\}. \tag{5}$$

These indices are from Kane (1986). C_p is the most basic capability index and is said to be a first-generation index. The observant reader will immediately notice that C_p is the reciprocal of Juran & Gryna's capability ratio (defined in Section 1).

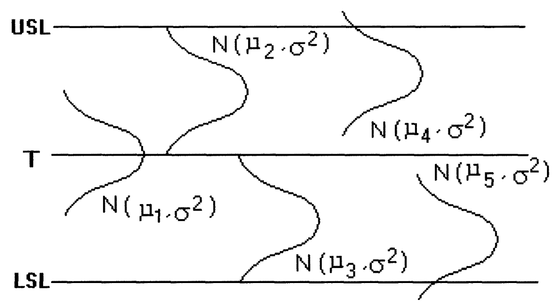

Figure 1. *Distributions associated with five populations.*

Conceptually, C_p compares the allowable process spread to the actual process spread, and can be thought of as indicating the potential of the process to produce conforming material. Consider a process that has both an on-target mean and a high C_p value (C_p^* value). By sufficiently shifting the process mean in the direction of any one of the specification limits, we can still obtain any proportion of items outside the specification limits and still maintain a high C_p value (C_p^* value). For example, in Figure 1, samples from any of the five normal distributions ($N(\mu_i, \sigma^2), i = 1, \ldots, 5$) will produce estimates of the $C_p(C_p^*)$ index that are almost the same. This is due to the fact that these five distributions have the same variance. As the actual process spreads are smaller than the allowable process spread, the process capability index (C_p^*) will be greater than 1, suggesting that the processes are capable. Only processes from distribution 1 are on target. It may be argued that processes from distributions 2 and 3 are still within the specification limits and hence should be judged capable, even though they are not centred at the target. However, the possibility of necessary adjustments could be costly. Finally, processes from distributions 4 and 5 are incapable of meeting the specifications required as both have substantial proportions of production beyond the specification limits. This example shows that the C_p and C_p^* indices simply relate the process spread to the specification limits and do not take into account the possible shifts of the process mean away from the target value.

Next we shall consider indices that take into consideration both the process mean and the process dispersion.

3.1.2 The C_{pk} index

To deal with violations of the centring assumptions, the following pair of indices was developed for the case $T = M$:

$$C_{pk} = \min\left\{\frac{U-\mu}{3\sigma}, \frac{\mu-L}{3\sigma}\right\}, \tag{6}$$

$$k = \frac{\mu - M}{(U-L)/2}. \tag{7}$$

The index k represents a measure of the distance that the process lies *off-centre*, and C_{pk} shows the reduction in process capability caused by the lack of centring. C_{pk} is said to be a second-generation index. Using the algebraic identity $\min\{a, b\} = [(a+b) - |a-b|]/2$, the definition of the C_{pk} can alternatively be written as:

$$C_{pk} = \frac{D - |\mu - M|}{3\sigma},$$

where $D = (U-L)/2$.

It should be noted that both Sullivan (1984) and Kane (1986) describe k as an absolute value; however, Palmer & Tsui (1999) feel it is useful for k to retain its sign.

When the target value T for the process mean is not necessarily equal to the mid-point M of the specification limits, the analogous indices are defined by

$$C^*_{pk} = \min(CPL^*, CPU^*), \tag{8}$$

where

$$CPL^* = \begin{cases} 0 & \text{if } |T - \mu| > T - L \\ \dfrac{T - L}{3\sigma}\left(1 - \dfrac{|T - \mu|}{T - L}\right), & \text{otherwise} \end{cases} \tag{9}$$

$$CPU^* = \begin{cases} 0 & \text{if } |T - \mu| > U - T \\ \dfrac{U - T}{3\sigma}\left(1 - \dfrac{|T - \mu|}{U - T}\right), & \text{otherwise} \end{cases} \tag{10}$$

$$k^* = \frac{\mu - T}{\min(T - L, U - T)}. \tag{11}$$

Interestingly Boyles (1991) shows that C_{pk} is essentially a measure of process yield only and can fail to distinguish between off-target and on-target process.

3.1.3 The C_{pm} index

The C_p and C_{pk} indices are appropriate measures of progress for quality improvement in which reduction of variability is the guiding principle and process yield is the primary measure of success. Taguchi (1986) has suggested a different approach to quality improvement in which reduction of variation from the target value is the guiding principle. In fact, Taguchi (1988) was the first author to propound the concept that there is a loss to society associated with missing the target. This concept of societal loss is difficult, if not impossible, to quantify. However, to be useful from a business perspective, a tool must be well defined, must be easy to use, and must have a quantifiable financial impact so that results can be attributed to the success of the business. Taguchi realised that just being within specification is not sufficient, so he developed the concept of the quadratic loss function to address the deficiency of the "goal post" approach to specification limits.

In this approach, any measured value x of a product characteristic X entails a monetary loss $L(x)$ to the customer as well as to the society in general. The loss function L is usually assumed to be well approximated by the symmetric squared error loss function,

$$L(x) = k(x - T)^2,$$

for some positive constant k, so that $L(T) = 0$; and any deviation from the ideal value T entails some positive loss to the consumer or to the society. The capability of the process is represented by the expected loss

$$E(L) = kE\{(x - T)^2\}.$$

This is a measure of process variation in terms of deviation of the characteristic X from the target T. The appeal of expected loss is that it expresses process capability in monetary units, and therefore enters naturally into management decision-making process.

Chan *et al.* (1988) introduced the so-called Taguchi capability index C_{pm} that is measurable and directly related to the quadratic loss of the measured feature. It is defined by

$$C_{pm} = \frac{U - L}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \tag{12}$$

assuming that the target value T is equal to the mid-point M of the specification limits. The C_{pm} is another second-generation index.

When $T \neq M$, the corresponding index is given by

$$C^*_{pm} = \min\left\{\frac{U - T}{3\sqrt{(\sigma^2 + (\mu - T)^2}}, \frac{T - L}{3\sqrt{(\sigma^2 + (\mu - T)^2}}\right\}. \tag{13}$$

Boyles (1991) presents the general statistical methodology for the capability index C_{pm} without the restrictive assumption $\mu = T$. Johnson (1992) exploits the relationship between the capability index C_{pm} and the expected squared error loss to provide an intuitive interpretation of C_{pm} in terms of the percentage loss. He shows that it is related to the expected relative loss L_e for the process, where the expected relative loss is defined as the ratio between the expected squared error loss and the value that the product is worth when the process mean μ is equal to its target T. In an interesting paper, Denniston (2006) provides the motivation for using C_{pm}. He shows that C_{pm} can indicate the probability of meeting the customer's product specification. It can be used to provide a better estimate of the cost of poor quality, and hence can be used to better manage product quality to the customer.

3.1.4 The C_{pmk} index

It is easy to observe that C_{pk} is derived from C_p by modifying the numerator, whereas C_{pm} is obtained by modifying the denominator. If the two modifications are combined, a new index C_{pmk}, first proposed by Pearn *et al.* (1992), is obtained. It is defined by

$$C_{pmk} = \min\left\{\frac{U - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - L}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}. \tag{14}$$

Observe that a manufacturing process satisfying the capability requirement "$C_{pk} \geq c$" may not satisfy the capability condition "$C_{pm} \geq c$". On the other hand, a process satisfying the capability requirement "$C_{pm} \geq c$" may not satisfy the capability requirement "$C_{pk} \geq c$". However, a manufacturing process does satisfy both capability requirements "$C_{pk} \geq c$" and "$C_{pm} \geq c$" if the process satisfies the capability requirement "$C_{pmk} \geq c$" as $C_{pmk} \leq C_{pk}$ and $C_{pmk} \leq C_{pm}$. Thus, the index C_{pmk} provides a greater level of quality assurance with respect to process yield and process loss to the customers than the other two indices. This is a desired property according to today's modern quality theory, as a reduction of process loss (variation from the target) is just as important as increasing process yield (meeting the specifications). Although C_{pk} remains the more popular and widely used index, C_{pmk} is arguably the most useful index to date for processes with two-sided specification limits. The index alerts the user if the process variance increases and/or the process mean deviates from its target and is designed to monitor normal and near-normal processes. For semiconductor and microelectronics manufacturing in particular, C_{pmk} is an appropriate index for capability measurement due to the high standard and stringent requirements on product quality and reliability.

The C_{pmk} index has a value of 0 when μ is at either specification limits (like C_{pk}); hence, it indicates closeness of μ to the specification limits. This is a desirable characteristic, which is not shared by C_{pm}. When the target value is not the specification mid-point, the maximum value for C_{pmk} is not the same as the maximum for C_{pk} and C_{pm} (recall that this maximum value is the C_p value). We can observe that this index explicitly takes into account that the process mean may not be midway between the specification limits and incorporates a penalty when μ deviates from the target T. The index is constructed so that the larger the index, the more capable the process. The C_{pmk} index is said to be a third-generation index.

In current practice, a process is called "inadequate" if $C_{pmk} < 1.00$, "marginally capable" if $1.00 \leq C_{pmk} < 1.33$, "satisfactory" if $1.33 \leq C_{pmk} < 1.50$, "excellent" if $1.50 \leq C_{pmk} < 2.00$, and "super" if $2.00 \leq C_{pmk}$ 2 .00 (Hsu *et al.,* 2007).

3.2 *Relationship among the Indices*

We shall now show the relationship among the different indices and get the bounds for their values.

It is easy to see that

$$C_{pk} = (1 - |k|)C_p, \tag{15}$$

and

$$C_{pm} = \frac{C_p}{\sqrt{1 + \frac{(\mu-T)^2}{\sigma^2}}}. \tag{16}$$

Clearly,

$$C_{pmk} \leq C_{pk} \leq C_p, \tag{17}$$

and

$$C_{pmk} \leq C_{pm} \leq C_p. \tag{18}$$

The relationship between C_{pk} and C_{pm} is less obvious. Using equations (15) and (16), we have $C_{pk} = (1 - |k|)C_{pm}\sqrt{1 + \frac{(\mu-T)^2}{\sigma^2}}$. In the special case when $T = M$, it can be shown that $C_{pk} < C_{pm}$, if $|\frac{\mu-M}{d}| < \frac{2}{9C_p^2}$, where

$$D = \frac{U - L}{2}. \tag{19}$$

Moreover, Parlar & Wesolowsky (1999) have observed that if $T = M$, then

$$C_{pk} = C_p - \frac{1}{3}\sqrt{\left(\frac{C_p}{C_{pm}}\right)^2 - 1}, \tag{20}$$

or equivalently

$$C_{pm} = \frac{C_p}{\sqrt{1 + 9(C_p - C_{pk})^2}}. \tag{21}$$

If $T = M$, we can also write

$$C_{pk} = -\frac{\beta}{3} + (\sqrt{1 + \beta^2})C_{pm}, \tag{22}$$

where

$$\beta = \frac{|\mu - T|}{\sigma}, \tag{23}$$

Boyles (1991) studied both C_{pk} and C_{pm} extensively, including the comparison of C_{pk} and C_{pm} as functions of μ and σ, without assuming $\mu = T$. He used a graphical technique to show that C_{pk} fails to address adequately the problem of process centring.

From the definition of C_{pmk}, it follows that

$$C_{pmk} = \frac{C_{pk}}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}}, \tag{24}$$

or equivalently

$$C_{pmk} = \frac{C_{pm} \times C_{pk}}{C_p} \text{ (using equation (16))}. \tag{25}$$

We note that

$$\begin{aligned} C_{pmk} &= \left(1 - \frac{|\mu - M|}{D}\right) C_{pm} \\ &= \frac{1}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}} C_{pk}. \\ &= \frac{\left(1 - \frac{|\mu - M|}{D}\right)}{\sqrt{\left\{1 + \left(\frac{\mu - T}{\sigma}\right)^2\right\}}} C_p \end{aligned} \tag{26}$$

If $\mu = M = T$, then clearly $C_p = C_{pk} = C_{pm} = C_{pmk}$. They differ in behaviour when $\mu \neq T$. By plotting the four indices as surfaces, we can get a feeling for the sensitivity with regard to departures of the process mean μ from the target value T, assuming that $T = M$. It is easy to see that, for fixed σ, when μ moves away from T, then C_p does not change; C_{pk} changes, but slowly; C_{pm} changes somewhat more rapidly than C_{pk}; but C_{pmk} is the one that changes most rapidly.

Proofs of some of these relationships, together with the conditions under which C_{pm} is greater than or less than C_{pk}, can be found in Kotz & Johnson (1999).

3.3 *Estimates of the Indices*

For using these indices meaningfully, they have to be estimated based on sample data. The estimate will depend upon how the statistics μ and σ are estimated. We shall assume that we have a random sample of size n, given as $\{X_1, X_2, \ldots, X_n\}$ from a stable process. On the basis of this assumption, we shall obtain a point estimate and confidence interval for these indices using the natural estimators for μ and σ.

However, for applications where routine-based data collection plans are in usage, the parameters μ and σ may be estimated using control charts. Again, from a practical perspective, manufacturing characteristics information about the process may be obtained by estimating the process capability using the past in-control data that includes multiple samples rather than a single sample. In such cases, the distribution of the estimated process capability index based on subsamples ought to be available. For brevity, we shall not discuss these two cases and refer the reader to Pearn & Kotz (2006) and the references therein.

Because of the sampling variation introduced by estimation, it is important and relevant to construct a confidence interval providing a range of values that includes the true index with a high probability.

3.3.1 Estimate of C_p Index

A natural estimator of C_p is given by

$$\hat{C}_p = \frac{U - L}{6S},$$

where $S = [\sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)]^{1/2}$ is the usual estimator of the process standard deviation σ, obtained on the basis of a random sample of size n from a stable process.

A $100(1-\alpha)\%$ confidence internal for C_p may be expressed as

$$\left[\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\hat{C}_p, \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\hat{C}_p\right],$$

where $\chi^2_{n-1,\alpha/2}$ and $\chi^2_{n-1,1-\alpha/2}$ are the upper $\alpha/2$ and $1-\alpha/2$ quantiles of a chi-squared distribution with $n-1$ degrees of freedom, respectively.

It should be noted that if the standard deviation σ is estimated based on control charts, then the appropriate sampling distribution should be used to get the confidence interval. For details, see Pearn & Kotz (2006).

3.3.2 Estimate of C_{pk} Index

The natural estimator $\hat{C}_{pk}$ is obtained by replacing the process mean μ and process standard deviation σ by their estimators $\bar{X}$ and S, respectively. Thus,

$$\hat{C}_{pk} = \frac{D - |\bar{X} - M|}{3S} = \left\{1 - \frac{|\bar{X} - M|}{D}\right\}\hat{C}_p.$$

The construction of the exact confidence intervals for C_{pk} is difficult because the distribution of $\hat{C}_{pk}$ involves the joint distribution of two noncentral t-distributed random variables.

Nagata & Nagahata (1992) proposed the following two-sided confidence interval for C_{pk}:

$$\hat{C}_{pk} \mp z_{\alpha/2}\sqrt{\frac{1}{9n} + \frac{\hat{C}^2_{pk}}{2(n-1)}}.$$

Nagata & Nagahata (1994) provide a thorough treatment of the construction of approximate confidence intervals for C_{pk}.

3.3.3 Estimate of C_{pm} Index

The index C_{pm} involves the unknown parameters μ and σ, which needs to be estimated from a sample. Boyles (1991) proposed the following estimator of C_{pm}:

$$\hat{C}_{pm} = \frac{D}{3\sqrt{S_n^2 + (\bar{X} - T)^2}},$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$; $S_n^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/n$. Note that $\bar{X}$ and S_n^2 are the maximum likelihood estimations (MLEs) of μ and σ^2, respectively. Hence, the estimator $\hat{C}_{pm}$ is also the MLE of C_{pm}.

Several authors have suggested approaches for constructing approximate lower confidence bounds for C_{pm}. Marcucci & Beazley (1988) propose using the ordinary chi-squared distribution to approximate the noncentral chi-squared distribution, resulting in the following approximate lower confidence bound:

$$\hat{C}_{pm}\sqrt{\frac{\chi^2_{n,1-\alpha}}{n}},\ 0 \le \alpha \le 1,$$

where $\chi^2_{n,1-\alpha}$ is the $(1-\alpha)$-th percentile of the ordinary central chi-squared variable with n degrees of freedom. When the process is on target, that is, $\mu = T$, this provides an exact bound; otherwise the bound is conservative.

3.3.4 Estimate of C_{pmk} Index

For a normally distributed process under statistical control, Pearn *et al.* (1992) suggested using the natural estimator of C_{pmk} given by

$$\hat{C}_{pmk} = \min\left\{\frac{U-\bar{X}}{3\sqrt{S_n^2+(\bar{X}-T)^2}}, \frac{\bar{X}-L}{3\sqrt{S_n^2+(\bar{X}-T)^2}}\right\},$$
$$= \frac{D-|\bar{X}-T|}{3\sqrt{S_n^2+(\bar{X}-T)^2}}$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S_n^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2/n$ are the MLEs of μ and σ^2, respectively. Chen & Hsu (1995) have shown that $\hat{C}_{pmk}$ is consistent and asymptotically unbiased. They have also proved that if $E(X^4) < \infty$, then $\hat{C}_{pmk}$ is asymptotically normal. They have derived an asymptotic $100(1-\alpha)\%$ confidence interval for C_{pmk} as

$$\hat{C}_{pmk} \mp z_{\alpha/2}\frac{\hat{\sigma}_{pmk}}{\sqrt{n}},$$

where

$$\hat{\sigma}^2_{pmk} = \left[\frac{1}{9(1+\delta^2)} + \frac{2\delta}{3(1+\delta^2)^{3/2}}\right]\hat{C}_{pmk} + \frac{72\delta^2 + D\left(\frac{m_4}{s_n^4}-1\right)}{72(1+\delta^2)^2}\hat{C}^2_{pmk},$$

where $\hat{\sigma}^2_{pmk}$ is the asymptotic estimator of $\mathrm{Var}(\hat{C}_{pmk})$; $z_{\alpha/2}$ is the upper $\alpha/2$ quantile of the standard normal distribution; and $m_4 = \sum_{i=1}^{n}(X_i-\bar{X})^4/n$, $\delta = (\bar{X}-T)/S_n$, and $S_n^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2/n$.

4 Index Interpretation and Drawbacks

It must be noted that a state of statistical control is required of the process for the capability index to have any long-term meaningful interpretation.

The index C_p only indicates the potential proportion conforming. A minimum value of $C_p = 1.33$ is generally used for an ongoing process (Montgomery, 2001, p. 361). If the C_p value is 1, and the process characteristic X is normally distributed and properly centred at the mid-point, that is, $T = M = \frac{U+L}{2}$, then the proportion (p) of NC items produced is rather small (0.27%). In spite of a high value (>1) of C_p, p can be more than 0.27% if the process is *not* properly centred.

This drawback of the C_p value index is taken care of by the C_{pk} index. Recall from equation (15) that $C_{pk} = (1-|k|)C_p$. This shows that C_{pk} is bounded above by C_p. The

C_{pk} index will achieve its maximum value (equal to the C_p value) when the process mean is at the specification mid-point. As the process mean drifts away from the specification mid-point, the C_{pk} value decreases linearly until it reaches a value of 0, when the process mean is equal to one of the specification limits.

The indices C_p, C_{pk}, and k can be used for process improvement. Adapting from Palmer & Tsui (1999), the steps for process improvement can take the following direction:

Step I: Obtain the estimates of C_p and C_{pk}.
Step II: If $C_{pk} < C_p$ and $C_p > 1$, then evaluate k.

(a) If $k > 0$, then adjust the process location to decrease the process mean until $C_{pk} = C_p$.
(b) If $k < 0$, then adjust the location to increase the process mean until $C_{pk} = C_p$.

Step III: If $C_p < 1.0$, then identify and remove sources of variability and go to Step II.

It is also interesting to note that the two indices C_p and C_{pk} can be associated with the stepwise loss function. Gunter (1989) has advocated caution in the use of the C_{pk} index.

The third index C_{pm} attains its maximum value when $\mu = T$, and decreases in value symmetrically, in a bell-shaped pattern, as the process mean shifts away from the target value. If the process mean is at the target value, that is, $\mu = T$, then from equation (16), we have $C_{pm} = C_p$. Unlike C_{pk}, the value of C_{pm} does not decrease to 0 as the process mean approaches the specification limits. The value of the C_{pm} index is independent of the closeness of process mean μ to the specification limits. Only the distance between μ and the target is considered. The entire curve for C_{pm} shifts with the target value, regardless of the actual locations of the specification limits. It should be noted that the target value is not necessarily the specification mid-point value.

As C_{pm} indicates the reduction in process capability due to shifts in the process mean away from the target, the pair of indices C_p and C_{pm} can be used to direct process improvement activities. Palmer & Tsui (1999) suggest one such road map. It is worth noting that C_{pm} was the first index to be developed that explicitly used a quadratic loss function. In view of the fact that C_{pm} is bounded above by C_p, it is clear that C_p index enjoys membership in both categories.

The term $(\mu - T)^2$ in the denominator of C_{pmk} in equation (14) may be viewed as an additional penalty to the process quality for the departure of the process mean from the target. This penalty ensures that C_{pmk} will be more sensitive to departure than C_{pk}, and therefore, is better able to distinguish between off-target and on-target processes.

It should be clearly understood that process capability cannot be adequately characterised by a single index. Bothe (2002) has shown that C_{pk} can be misleading, and is inappropriate for product features with asymmetric tolerances. He advises the reporting of C_p, C_{pk}, p_{LSL} (percentage nonconforming below the LSL), and p_{USL} (percentage nonconforming above the USL) to have a very good idea of what is happening regarding the process output and what actions are necessary to improve it.

Deleryd & Vännman (1999) and Vännman (2001, 2005) have introduced the concept of process capability plots as powerful tools to monitor and improve the capability of industrial processes. An advantage of process capability plots is that they instantly provide information about the location and spread of the characteristic under study. When the process is noncapable, the plots are helpful in trying to understand if it is the variability, the deviation from the target, or both that need to be reduced to improve the capability.

Daniels *et al.* (2005) address the problem of comparing two capability indices (they consider C_p, C_{pk}, and C_{pm}) for two different processes or the same process before and after an adjustment. They provide recommendations for selecting an appropriate method based on power and test size computations, whereas Melloy & Chandra (1992) demonstrate that the casual estimation of the proportion of NC items using the C_{pk} index can be very misleading when items are screened.

5 Indices, NCs, and Sample Size

In this section, we shall comment on the relationship of the indices to the number of the NC products and the sample size required for scientifically valid comparisons.

5.1 Indices and NC

Capability indices are popular because they provide single-number summaries to managers responsible for many processes running simultaneously. However, it should be noted that it is very difficult, if not altogether impossible, for a single index to capture the dynamics of a process.

If the process is normally distributed and centred at the mid-point, then a C_p value of 1 indicates that the proportion of NC products is 0.27%. It is easy to show that the probability of obtaining an NC value is

$$2\Phi(-3C_p). \tag{27}$$

where Φ (.) denotes the standard normal cumulative distribution function.

The index C_{pk} alone does not determine the proportion (p) of NCs; but provides an upper bound, given by

$$p \leq 2\Phi(-3C_{pk}). \tag{28}$$

However, C_p and C_{pk} together determine p by the equation

$$\Phi(-3(2C_p - C_{pk})) + \Phi(-3C_{pk}). \tag{29}$$

If the expected proportion NC is regarded as the most important criterion, then C_{pm} is unreliable, because the same value of C_{pm} can be associated with a wide range of values of the expected proportion NC. However, it should be noted that the motivation for C_{pm} does not arise from examining the number of NC products in a process but rather from requiring the ability of the process to be in the neighbourhood of the target. This motivation has little to do with the number of NC parts. However, the index provides an upper bound to the proportion of NC products, $p \leq 2\Phi(-3C_{pm})$. Moreover, it can be shown that $|\mu - T| < D/(3C_{pm})$. This inequality can be interpreted as a C_{pm}—value of 1 implies that the process mean μ lies within the middle-third of the specification range. As the design of C_{pm} is based on the average process loss relative to the manufacturing tolerance, the index C_{pm} provides an upper bound on the average process loss.

Similarly, the C_{pmk} index provides an upper bound to the proportion of NC products, $p \leq 2\Phi(-3C_{pmk})$. However, C_{pmk} is much more sensitive than other capability indices to movement in the process average relative to M. Further, C_{pmk} reveals the maximum information about the location of the process average. It can be shown that the distance between μ and M is less than $D/(1 + 3C_{pmk})$. This can be interpreted as a C_{pmk}—value of 1 implies that the process mean μ lies within the middle-fourth of the specification range. This is appreciably a very small interval.

Ranking the four common indices in an increasing order of sensitivity to departures of the process mean from the target value, we obtain (1) C_p (2) C_{pk} (3) C_{pm} (4) C_{pmk}. If the proportion of NC units (p) is of primary importance, then C_p and C_{pk} should be used as they are more closely concerned with the percentage of NC products.

5.2 Sample Size

Kane (1986) proposed the use of Operating Characteristic (OC) curve to analyse the sampling variation of C_p. He tests the hypothesis:

$$H_0: C_p \leq c_0 \text{ (process is not capable)}$$

versus

$$H_1: C_p > c_0 \text{ (process is capable)}$$

as a test for process capability ($C_p > c_0$). He has given a table for critical value determination for testing C_p for different sample sizes. Using this table, Kane (1986) points out that past automotive industry machine qualification practices that use sample size of $n = 30$ were, for the most part, inadequate. He points out that even though current automotive industry machine qualification use a C_p value of 1.33, accounting for nonsampling problems that make qualification runs different from production runs, these practices may not adequately account for sampling variability. Though both Charbonneau & Webster (1978, p. 112) and Montgomery (2001, p. 361) recommend that new equipment qualifications should use $C_p = 1.5$, neither of them recommends a sample size.

Chan *et al.* (1988) proposed an analogous approach to analyse the sampling variation of C_{pm}. They also present a table for determining sample size and critical values for testing C_{pm}. However, the same criticisms presented above are applicable to the OC curve approach for analysing the stochastic properties of $\hat{C}_{pm}$.

Chou *et al.* (1990) show that fairly large sample sizes are needed to determine C_{pk} precisely. For example, to be 95% certain that the true C_{pk} is not more than 10% below the measured sample estimated C_{pk} when the sample estimated C_{pk} is 1.33 (a fairly used target), one must have a sample size of about 350, assuming that the underlying quality characteristic has a normal distribution.

Shore (1997) has studied the effect of autocorrelation on process capability analysis. As correlated observations contain less information than noncorrelated ones, the sample size needed to arrive at an accurate estimate of the capability index is larger. In fact, Zhang (1998) has shown that for $n < 100$, the estimated values $\hat{C}_p$ and $\hat{C}_{pk}$ are quite variable, whereas the variations in $\hat{C}_p$ and $\hat{C}_{pk}$ decrease slowly when n is greater than 100. Hence, he recommends using a sample size $n > 100$ to avoid large variations in $\hat{C}_p$ and $\hat{C}_{pk}$.

6 Effect of Nonnormality, Asymmetry, and Autocorrelation

The theory of PCIs discussed so far is based on the assumption that the distribution of the underlying process characteristic is normally distributed and the observations are independent. However, there are situations when such assumptions are violated. The distribution of taper, ovality, concentricity, run-out, etc., are essentially nonnormal and skewed. The examples of nonnormality include impurity content in chemicals, measures of acidity/alkalinity, particle size powder, and measures of squareness, parallelism, and flatness in machined components. In Very Large Scale Integration (VLSI) processing technology, the distributions of boron-implanted atomic concentration in silicon are more and more negatively skewed from normality as implantation energy increases (Sze, 1988). Such a process displays natural nonnormal behaviour. Distributions of most measured critical features sizes of photomasks deviate significantly from normality. Nonnormality can also arise from truncation of data out of specification. As pointed out by Ryan (2000) and illustrated therein, normal distributions do not exist in practice. Gunter (1989) has expressed the view that in most real industrial processes, nonnormality is the norm. Similarly, in process industries, it is very common to expect and encounter autocorrelated data. We shall now consider how the PCIs are affected in such cases.

6.1 Nonnormal Data

Much work on these four popular indices has been done on the assumption that the measured quality characteristic is normally distributed (at least approximately). However, it is difficult to believe that a good industrial process must result in a normal distribution for every measured characteristic (Pyzdek, 1995). Munechika (1992) details several examples of machining processes that are inherently nonnormal. If the normality-based PCIs are used to deal with nonnormal processes, the results are generally incorrect, as expected. Somerville & Montgomery (1996–97) have observed that normality-based PCI cannot calculate the process fallout accurately when the underlying distribution is nonnormal. Gunter (1989) has shown three different distributions with identical values of C_p and C_{pk} but different proportions of NC parts. Kokcherlakota *et al.* (1992) give additional information on the effects of nonnormality on PCIs. A large body of literature has appeared on dealing with nonnormal data. The available methods include the empirical percentile method (Clements, 1989; McCormark *et al.,* 2000), and Monte Carlo simulation (English & Taylor, 1993). Chou *et al.* (1998) and Polansky *et al.* (1998) have suggested using Johnson's system of distribution curves to transform the nonnormal data into normality. Tang & Than (1999) review seven methods for performance comparison in their ability to handle nonnormal data. They also suggest applicable methods for each defined range of skewness and kurtosis under mild and severe departures from normality. In a recent paper, Wu & Swain (2001) proposed a method based on weighted variance to deal with nonnormal data. On the basis of simulation, they conclude that the weighted variance-based estimators perform best in both accuracy and efficiency and the recommended sample size for better estimating the nominal values would be at least 100. Ding (2004) presents a method to evaluate the PCI for a set of nonnormal data from its first four moments. Compared with some existing methods, his method gives a more accurate PCI estimation and shows less sensitivity to sample size. Pal (2005) suggests using the generalised lambda distribution (GLD) to evaluate nonnormal process capability indices as the GLD has an edge over the other family of distributions while modelling a process data.

6.2 Skewed Population

For many quality characteristics, such as circularity, cylindricity, straightness, and flatness, positive skewness in the inspection data is the norm and in fact is desirable. To deal with such data, Wright (1995) proposed an index C_s, which is basically an adaptation of the C_{pmk}, proposed by Pearn *et al.* (1992). Wright included a skewness term in the denomination to define

$$C_s = \min\left(\frac{U-\mu}{3\sqrt{\sigma^2+(\mu-T)^2+|\mu_3/\sigma|}}, \frac{\mu-L}{3\sqrt{\sigma^2+(\mu-T)^2+|\mu_3/\sigma|}}\right). \quad (30)$$

The absolute value of the skewness parameter, added to the denominator, has the effect of reducing the value of the index when asymmetry exists. This index extends the C_{pmk} index to handle situations where worsening capability is characterised not only by an increase in variance and/or deviation of the mean from the target but also an increase in skewness. He advocates the use of the C_s index for monitoring near-normal processes where loss of capability leads to asymmetry.

Nahar *et al.* (2001) modified Wright's index by simply omitting the absolute value sign in the denominator and subtracting the skeweness value. Thus, they propose

$$C_{sm} = \min\left(\frac{U-\mu}{3\sqrt{\sigma^2+(\mu-T)^2-(\mu_3/\sigma)}}, \frac{\mu-L}{3\sqrt{\sigma^2+(\mu-T)^2-(\mu_3/\sigma)}}\right). \quad (31)$$

Hence, a positive skewness value (in which "skewness is goodness" and a desirable characteristic) would cause the index to increase and a negative skewness value, an undesirable characteristic, would have the effect of decreasing the value of the index.

Chang *et al.* (2002) proposed a new method of constructing process capability index for skewed population based on a weighted standard deviation method, which decomposes the standard deviation of a quality characteristic into upper and lower deviations using different factors in computing the deviation above and below the process mean and adjusts the value of the index using decomposed deviations in accordance with skewness estimated from sample data. For symmetric populations, the proposed PCIs reduce to standard PCIs.

6.3 Autocorrelated Data

Traditionally, a process must be considered free from variation due to assignable causes before its capability can be determined. However, there are situations in which assignable causes are inherently present and it is very difficult, if not altogether impossible, to eliminate them; for example, the degradation of a cutting tool. This essentially means the presence of autocorrelation in the data. During the last decade, the determination of PCIs in the presence of autocorrelation has been discussed by a few authors. We shall attempt to provide an insight into their works.

In those situations where a variation due to an assignable cause occurs and is tolerated, traditional process capability measures cannot be meaningfully used. By allowing the process capability to be considered dynamic, Spiring (1991) proposed a procedure for assessing the process capability in such situations. That is, for process exhibiting variation due to a systematic assignable cause, the capability of the process is considered to be constantly changing as the process ages. The changing ability of the process can be monitored using a process capability index that considers both process variation and proximity to the target value. Though both C_{pk} and C_{pm} are appropriate indices, which take both the target and the spread into consideration, the author has developed the theory for the C_{pm} index (because the estimator of C_{pm} as suggested by Chan *et al.,* 1988 has a pdf that can be used to derive statistically based inferences from the sampling results). The total variation σ^2 is decomposed into two parts, namely, variation due to assignable causes (σ_a^2) and variation due to random cause (σ_r^2), that is, $\sigma^2 = \sigma_a^2 + \sigma_r^2$. In assessing process capability, σ_r^2 is only considered. It should be noted that assessing a constantly changing capability essentially requires that an instantaneous capability measure be made by examining the process capability over small time intervals. The advantage of the proposed capability measure is that it can be used to monitor and manage processes under the influence of systematic assignable cause. A similar exercise for C_{pmk} has been done by Pearn & Hsu (2007), whereas the present author is investigating for the C_{pk} index.

Jagadeesh & Babu (1994) have investigated the problem of process capability assessment in the presence of tool wear. They have used four different methods of estimating the process variability σ^2, and concluded that the results based on the different methods are not consistent.

In an influential paper, Shore (1997) dealt with the problem of process capability analysis when data are autocorrelated. He showed that autocorrelation affects the variance of the sample mean and hence the confidence interval associated with the sample mean. The author studied four approaches of estimating capability for autocorrelated data and concluded that when both performance and convenience in application are important, the model-free approach is superior.

Zhang *et al.* (1990) have shown that the sample variations in estimates of capability indices cannot be ignored and hence interval estimation should be considered. Assuming $\{X_t\}$ is a discrete Gaussian process, Zhang (1998) has shown that the variance of $\hat{C}_p(\hat{C}_{pk})$are functions of $C_p(C_{pk})$, the sample size n and the process autocorrelation ρ_i (from a lag of 1 to n).

Scholz & Vangel (1998) are concerned with the construction of tolerance bounds for C_{pk} when samples come in batches and the intrabatch correlation reduces the amount of independent information. They reduce the problem to the independent and identically distributed case by the simple device of effective sample size.

Noorossana (2002) has shown the variance estimate obtained from the original data is no longer an appropriate estimate to consider for process capability studies when observations are autocorrelated. He suggests using a combined procedure based on multiple regression and time-series modelling to remove the autocorrelation patterns that may be present in the data and also estimate model parameters effectively.

7 Measurement Error and PCIs

In spite of the large volume of work done on various aspects of process capability indices, the effects of measurement errors on these indices has received comparatively very little attention.

McNeese & Klein (1991–92) were perhaps the first one to point out that the variability inherent in the measurement systems and sampling techniques adds variability to the output from a process and hence affects the process capability. Hence it is necessary to have a capable measurement system, which they define as a system in statistical control with respect to the average and variation, whose average value is equal to the true value, and which is responsible for less than 10% of the total process variance. To decrease the total variability, it is necessary to determine where the greatest opportunity for improvement exists. This can be done by examining the components of variation. Usually, the components of variation are due to sampling, variation due to measurement system (gauge R & R), and error due to manufacturing process. Porter & Oakland (1991) also emphasize that process capability assessment is dependent upon the test or measurement method. They suggest that the capability of the test method should be at least two-and-a-half times the observed capability.

Persijn & Nuland (1996–97) also dwell upon the relationship between measurement system capability and process capability. They argue that process capability analysis is meaningful only if the measurement system is capable. For this purpose, they introduce the concept of measurement index (MI) and define it as the ratio of the process standard deviation (σ_p) to the standard deviation (σ_m) of the measurement method. Thus, $MI = \sigma_p/\sigma_m$. A capable measurement system has MI > 3, that is, the characteristics of the process can be measured and hence production can be assessed. In fact, the greater is the MI, the less important is the measurement error.

In a significant work, Mittag (1997) discusses the effects of measurement error on the performance of the four most basic process capability indices. When the error is a constant measurement error, the C_p index is unaffected, whereas the other three indices (C_{pk}, C_{pm}, and C_{pmk}) may be affected in either direction.

When the measurement error is random, instead of the true variable X, we observe the empirical variable X^e, where $X^e = X + V$, with V being the random error component. It is further assumed that the random variables X and V are stochastically independent and $E(V) = 0$. Clearly, the process variance is increased. Mittag (1997) has shown that stochastic measurement error always implies a decrease in the indices. For the indices C_p and C_{pk}, the extent of downward distortion only depends on the value of the error contamination degree $\tau = \sigma_v/\sigma$. In contrast, for the indices C_{pm} and C_{pmk}, the effect of a random measurement error is determined by τ as well as by $\delta = (\mu - M)/\sigma$. Whereas for C_p and C_{pk} the measurement error effects remains unchanged at $\mu \neq M$, the distortion of the C_{pm} and C_{pmk} weakens (up to being negligible) with increasing departure from the target. This implies that the index C_{pk} is inferior to the indices C_{pm} and C_{pmk} with respect to the robustness against normally distributed measurement errors.

Mittag's work is important as it shows that random and constant measurement errors can considerably falsify the results of process capability analyses. They emphasize that the accuracy of a capability analysis could be significantly influenced by the accuracy of the gauges. This fact underlines the importance of ensuring gauge capability before evaluating process capability and, consequently, measurement errors should receive greater attention.

However, although the analysis of Mittag (1997) is confined to considering the effects of measurement errors only on the behaviour of theoretical capability indices, such effects are not taken into account when PCIs are estimated for sample data. Bordignon & Scagliarini (2002) extend the analysis of Mittag (1997) to the inferential properties of the estimators of C_p and C_{pk} by considering the effects of measurement errors on the properties of capability indices estimated from sample data. They have shown that the C_p estimator obtained, from the sample data contaminated by random measurement errors, is biased, tending towards steady negative value as the sample size $n \to \infty$, and increases with the contamination degree τ. In contrast, the usual estimator of C_p in the measurement error-free case always has a positive bias going to zero as $n \to \infty$. They have also shown that $\text{Var}(\hat{C}_p^e)$ is never greater than $\text{Var}(\hat{C}_p)$. Therefore, when comparing the mean squared errors (MSEs) of the two estimators, the bias component plays a more important role. Similar results have been obtained for C_{pk} index. Scagliarini (2002) has analysed the properties of the estimator of C_p for autocorrelated data in the presence of measurement error. Later, Bordignon & Scagliarini (2006) study the behaviour of the estimator of C_{pm} in the presence of measurement error.

Pearn & Liao (2006) consider the estimation and testing of C_{pU} and C_{pL} in the presence of measurement error to obtain adjusted lower confidence bounds and critical values for true process capability. These can be used to determine whether the factory processes meet the capability requirement when the measurement errors are unavoidable.

Hsu *et al.* (2007) conduct a sensitivity study for the C_{pmk} index in the presence of gauge measurement errors. They consider the use of capability testing of C_{pmk} as a method for obtaining lower confidence bounds and critical values for true process capability when gauge measurement errors are unavoidable. Their research shows that using the estimator with sample data contaminated by measurement errors severely underestimates the true capability, resulting in an imperceptibly smaller test power.

8 Conclusion

This paper attempts to survey the four most popular process capability indices. In addition to introducing the indices, an attempt has been made to provide interpretation and point out the drawbacks of these indices. The popular misunderstanding with respect to the requisite sample size has been pointed out. The neglected aspects of nonnormal distribution, skewed distribution, and autocorrelated data have been studied. The impact of measurement error on the PCI has been highlighted. It is hoped that this paper will help in the correct and proper understanding and appreciation of the process capability indices and their correct applications. Incorrectly applied and/or interpreted, these indices can generate an abundance of misinformation that will confuse the shop floor personnel and the management alike, waste resources, and lead to incorrect decision making.

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References

Barnett, N.S. (1990). Process control and product quality. *Int. J. Qual. Reliab. Manage.*, **7**, 34–43.

Bordignon, S. & Scagliarini, M. (2002). Statistical analysis of process capability indices with measurement errors. *Qual. Reliab. Eng. Int.*, **18**, 321–332.

Bordignon, S. & Scagliarini, M. (2006). Estimation of C_{pm} when measurement error is present. *Qual. Reliab. Eng. Int.*, **22**, 787–801.

Bothe, D.R. (2001). *Measuring Process Capability*. Cedarburg, WI: Landmark Publishing.

Bothe, D.R. (2002). Discussion. *J. Qual. Technol.*, **34**, 32–37.

Bothe, D.R. (2006). Assessing capability for hole location. *Qual. Eng.*, **18**, 325–331.

Boyles, R.A. (1991). The Taguchi capability index. *J. Qual. Technol.*, **23**, 17–26.

Bulba, E.A. & Ho, L.L. (2006). Capability index for relational functions: inferential procedures. *Qual. Eng.*, **18**, 69–77.

Chan, L.K., Cheng, S.W. & Spiring, F.A. (1988). A new measure of process capability, C_{pm}. *J. Qual. Technol.*, **20**(3), 162–175.

Chang, Y.S., Choi, I.S. & Bai, D.S. (2002). Process capability for skewed populations. *Qual. Reliab. Eng. Int.*, **18**, 383–393.

Charbonneau, H.C. & Webster, G.L. (1978). *Industrial Quality Control*. Englewood Cliffs, NJ: Prentice Hall, 112.

Chen, S.M. & Hsu, N.F. (1995). The asymptotic distribution of the process capability index C_{pmk}. *Commun. Stat. - Theory. Methods.*, **24**(5), 1279–1291.

Chou, Y.M. (1994). Selecting a better supplier by testing process capability indices. *Qual. Eng.*, **6**, 427–438.

Chou, Y.M., Owen, D.B. & Borrego, A.S.A. (1990). Lower confidence limits on process capability indices. *J. Qual. Technol.*, **22**(3), 223–229.

Chou, Y.M., Polansky, A.K. & Mason, R.L. (1998). Transforming non-normal data to normality in statistical process control. *J. Qual. Technol.*, **30**(2), 133–141.

Clements, J.A. (1989). Process capability calculations for non-normal distributions. *Qual. Prog.*, **22**(9), 95–100.

Crain, C.R. (1993). Process capability indices and customer-supplier relations. *ASA Proc. Sect. Qual. Product.*, 156–158.

Daniels, L., Edgar, B., Burdick, R.K. & Hubele, N.F. (2005). Using confidence intervals to compare process capability indices. *Qual. Eng.*, **17**, 23–32.

Deleryd, M. & Vännman, K. (1999). Process capability plots—a quality improvement tool. *Qual. Reliab. Eng. Int.*, **15**, 213–227.

Denniston, B. (2006). Capability indices and conformance to specification: The motivation for using C_{pm}. *Qual. Eng.*, **18**, 79–88.

Ding, J. (2004). A method of estimating the process capability index from the first four moments of non-normal data. *Qual. Reliab. Eng. Int.*, **20**, 787–805.

English, J.R. & Taylor, G.D. (1993). Process capability analysis—a robustness study. *Int. J. Prod. Res.*, **31**, 1621–1635.

Feigenbaum, A.V. (1951). *Quality Control*. New York: McGraw Hill.

Gill, M.S. (1990). Stalking six sigma. *Business Month.*, January 1990, 42–46.

Gryna, F.M. (1988). *Quality Control Handbook*, 4th ed., Ed. J.M. Juran, pp. 14–16. New York: McGraw Hill.

Gunter, B.H. (1989). The use and abuse of C_{pk}. *Qual. Prog.*, **22**(3), 108–109.

Hsu, B.M., Shu, M.H. & Pearn, W.L. (2007). Measuring process capability based on C_{pmk} with gauge measurement errors. *Qual. Reliab. Eng. Int.*, **23**(5), 597–614.

Jagadeesh, R. & Babu, A.S. (1994). Process capability assessment with tool wear: an investigative study. *Int. J. Qual. Reliab. Manage.*, **11**, 51–62.

Johnson, T. (1992). The relationship of C_{pm} to squared error loss. *J. Qual. Technol.*, **24**, 211–215.
Juran, J.M. (1951). *Quality Control Handbook*, 1st ed., pp. 253–279; 404–409. New York: McGraw Hill.
Juran, J.M. (1962). *Quality Control Handbook*, 2nd ed., pp. 11–14; 11–27. New York: McGraw Hill.
Juran, J.M. & Gryna, F.M. (1980). *Quality Planning and Analysis*, 2nd ed., pp. 283–295. New York: McGraw Hill.
Kane, V.E. (1986). Process capability indices. *J. Qual. Technol.*, **18**(1), 41–52.
Kokcherlakota, S., Kocherlakota, K. & Kirmani, S.N.A.U. (1992). Process capability indices under non-normality. *Int. J. Math. Stat. Sci.*, **1**(2), 175–210.
Kotz, S. & Johnson, N.L. (1993). *Process Capability Indices*. New York: Chapman and Hall.
Kotz, S. & Johnson, N.L. (1999). Delicate relations among the basic Process capability indices C_p, C_{pk} and C_{pm} and their modifications. *Commun. Stat. - Comput. Simul.*, **28**(3), 849–866.
Kotz, S. & Johnson, N.L. (2002). Process capability indices—a review, 1992–2000 (with discussions). *J. Qual. Technol.*, **34**, 2–53.
Kotz, S. & Lovelace, C.R. (1998). *Process Capability Indices in Theory and Practice*. London: Arnold.
McCormark, D.W. Jr., Harris, I.R., Hurwitz, A.M. & Spagon, P.D. (2000). Capability indices for non-normal data. *Qual. Eng.*, **12**, 489–495.
McCoy, P.F. (1991). Using performance indexes to monitor production processes. *Qual. Prog.*, **24**(2), 49–55.
McNeese, W.H. & Klein, R.A. (1991–92). Measurement systems, sampling and process capability. *Qual. Eng.*, **4**(1), 21–39.
Marcucci, M.O. & Beazley, C.C. (1988). Capability indices: Process performance measures. *ASQC Qual. Congr. Trans.*, **42**, 516–523.
Melloy, B.J. & Chandra, M.J. (1992). Consequences of using the C_{pk} index to estimate the proportion of non-conforming items. *Int. J. Prod. Res.*, **30**, 2253–2263.
Mittag, H.J. (1997). Measurement error effects on the performance of process capability indices. *Frontiers in Statistical Quality Control*, Eds. H.J. Lenz & P.T. Wilrich, pp. 195–206. Heidelberg: Physica-Verlag.
Montgomery, D.C. (2001). *Introduction to Statistical Quality Control*, 4th ed. New York: John Wiley.
Munechika, M. (1992). Studies on process capability in machining processes. *Rep. Stat. Appl. Res. JUSE*, **39**, 14–29.
Nagata, Y. & Nagahata, H. (1992). Approximation formulas for the confidence intervals of process capability indices. *Rep. Stat. Appl. Res. JUSE*, **39**(3), 15–29.
Nagata, Y. & Nagahata, H. (1994). Appreciation formulas for the lower confidence limits of process capability indices. *Okayama Econ. Rev.*, **25**, 301–314.
Nahar, P.C., Hubele, N.F. & Zimmer, L.S. (2001). Assessment of a capability index sensitive to skewness. *Qual. Reliab. Eng. Int.*, **17**, 233–241.
Noorossana, R. (2002). Process capability analysis in the presence of autocorrelation. *Qual. Reliab. Eng. Int.*, **18**, 75–77.
Pal, S. (2005). Evaluation of non-normal process capability indices using generalized lambda distribution. *Qual. Eng.*, **17**, 77–85.
Palmer, K. & Tsui, K.L. (1999). A review and interpretations of process capability indices. *Ann. Oper. Res.*, **87**, 31–47.
Parlar, M. & Wesolowsky, G.O. (1999). Specification limits, capability indices and process centering in assembly manufacture. *J. Qual. Technol.*, **31**(3), 317–325.
Pearn, W.L. & Hsu, Y.C. (2007). Optimal tool replacement for processes with low fraction defective. *Eur. J. Oper. Res.*, **180**, 1116–1129.
Pearn, W.L. & Kotz, S. (2006). *Encyclopedia and Handbook of Process Capability Indices*. Singapore: World Scientific.
Pearn, W.L. & Liao, M.Y. (2006). One-sided process capability assessment in the presence of measurement errors. *Qual. Reliab. Eng. Int.*, **22**, 771–785.
Pearn, W.L., Kotz, S. & Johnson, N.L. (1992). Distributional and inferential properties of process control indices. *J. Qual. Technol.*, **24**(4), 216–231.
Persijn, M. & Nuland, Y.V. (1996–97). Relation between measurement system capability and process capability. *Qual. Eng.*, **9**(1), 95–98.
Polansky, A.K., Chou, Y.M. & Mason, R.L. (1998). Estimating process capability indices for a truncated distribution. *Qual. Eng.*, **11**(2), 257–265.
Porter, L.J. & Oakland, J.S. (1991). Process capability indices—an overview of theory and practice. *Qual. Reliab. Eng. Int.*, **7**, 437–448.
Pyzdek, T. (1995). Why normal distributions aren't [all that normal]. *Qual. Eng.*, **7**, 769–777.
Ryan, T. (2000). Discussion on "Controversies and contradictions in statistical process control". *J. Qual. Technol.*, **32**, 367–369.
Scagliarini, M. (2002). Estimation of C_p for auto-correlated data and measurement errors. *Commun. Stat. - Theory. Methods.*, **31**, 1647–1664.
Schneider, H. & Pruett, J. (1995). Use of process capability indices in the supplier certification process. *Qual. Eng.*, **8**, 225–235.

Scholz, F. & Vangel, M. (1998). Tolerance bounds and C_{pk} confidence bounds under batch effects. *Advances in Stochastic Models for Reliability, Quality and Safety*, Eds. W. Kahle, E. von Collani, J. Franz & U. Jensen, pp. 361–379. Boston: Birkhäuser.

Shewhart, W.A. (1931). *Economic Control of Quality of Manufactured Products*. New York: D. Van Nostrand.

Shore, H. (1997). Process capability analysis when data are autocorrelated. *Qual. Eng.*, **9**(4), 615–626.

Somerville, S.E. & Montgomery, D.C. (1996–97). Process capability indices and non-normal distributions. *Qual. Eng.*, **9**, 305–316.

Spiring, F.A. (1991). Assessing process capability in the presence of systematic assignable cause. *J. Qual. Technol.*, **23**(2), 125–134.

Spiring, F.A. (1995). Process capability: A total quality management tool. *Total Qual. Manage.*, **6**(1), 21–33.

Spiring, F., Leung, B., Cheng, S.W. & Yeung, A. (2003). A bibliography of process capability papers. *Qual. Reliab. Eng. Int.*, **19**, 445–460.

Sullivan, L.P. (1984). Reducing variability: A new approach to quality. *Qual. Prog.*, **17**(7), 15–21.

Sze, S.M. (1988). *VLSI Technology*, 2nd ed. New York: McGraw Hill.

Taguchi, G. (1986). *Introduction to Quality Engineering*. Tokyo: Asian Productivity Organization.

Taguchi, G. (1988). *Introduction to Quality Engineering, Designing Quality into Products and Processes*. Tokyo: Asian Productivity Organization.

Tang, L.C. & Than, S.E. (1999). Computing process capability indices for non-normal data: A review and comparative study. *Qual. Reliab. Eng. Int.*, **15**, 339–353.

Tsui, K.L. (1997). Interpretation of process capability indices and some alternatives. *Qual. Eng.*, **9**(4), 587–596.

Vännman, K. (2001). A graphical method to control process capability. *Frontiers in Statistical Quality Control*, Eds. H.J. Lenz & P.T.H. Wilrich, pp. 290–311. Physica: Heidelberg.

Vännman, K. (2005). The circular safety region: A useful graphical tool in capability analysis. *Qual. Reliab. Eng. Int.*, **21**, 529–538.

Wheeler, D.J. (1999). *Beyond Capability Confusion*. Knoxville: SPC Press.

Wright, P.A. (1995). A process capability index sensitive to skewness. *J. Stat. Comput. Simul.*, **52** , 195–203.

Wu, H.H. & Swain, J.J. (2001). A Monte Carlo comparison of capability indices when processes are non-normally distributed. *Qual. Reliab. Eng. Int.*, **17**, 219–231.

Zhang, N.F. (1998). Estimating process capability indexes for autocorrelated data. *J. Appl. Stat.*, **25**(4), 559–574.

Zhang, N.F., Stenback, G.A. & Wardrop, D.M. (1990). Interval estimation of process capability index C_{pk}. *Commun. Stat. - Theory. Methods.*, **19**, 4455–4470.

Résumé

Une revue des quatre index de capacité de procédé fondamentaux a été faite. L'inter-relation entre ces index a été soulignée. L'attention a été dessinée à leurs inconvénients. La relation de ces index au nonconformer de proportion a été demeurée sur et la condition de la taille d'échantillon suffisante a été soulignée. Les remarques d'avertissement sur l'usage de ces index dans le cas de distributions nonnormaux, les distributions déformés et les données d'auto-correspondu sont aussi présentés. L'effet d'erreur de mesure sur les index de capacité de procédé a été traité dans les moindres détails.

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