INDIAN STATISTICAL INSTITUTE

Mid-Semester examination: 2017-18

Course Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: Operations Research-I

Date of Exam: 04.09.17 **Total Marks: 100** **Duration: 3 hrs.**

Answer all the questions.

1. (a) An indigenous mobile manufacturer produces two brands of mobiles. Long-term projections indicate an expected demand of at least 100 brand-I and 80 brand-II mobiles each day. Because of limitations on production capacity, no more than 200 brand-I and 170 brand-II mobiles can be made daily. To satisfy a contract, a total of at least 200 mobiles much be despatched each day. Each brand-I mobile sold, results in a \$2 loss, but each brand-II mobile produces a \$5 profit. Formulate the optimization problem to maximize the net profit.

   (b) Show graphically how many of each brand should be made daily to maximize net profit? What is the value of expected net profit to be maximum?

   [*Mark all extreme points, feasible region, constraint lines and use line of the objective function to find out the optimal solution.*]

   [10+8=18]

2. Prove that every basic feasible solution (b.f.s) of a LPP is an extreme point of the convex set of all feasible solutions (f.s.).
   Show that the set of vectors $a^1 = (2,-1,0)$, $a^2 = (3,5,1)$ and $a^3 = (1,1,2)$ form a basis in E_3. **[6+4=10]**

3. Consider the following LP problem. Find the optimal solution using the Simplex method. What evidence indicates that an alternate optimal solution exists?

   Maximize Return on Investment = $2X_1 + 3X_2$

   Subject to $6X_1 + 9X_2 \leq 18$

   $9X_1 + 3X_2 \geq 9$

   $X_1, X_2 \geq 0$ **[8+2=10]**

4. What is the role of an artificial variable in Simplex algorithm? When and how an improved b.f.s is constructed from a current b.f.s? Under what conditions infeasibility and degeneracy are observed in simplex tableau?

[2+5+3=10]

5. If the set of all feasible solutions K of a LPP is a Convex Polyhedron, then at least one extreme point must be obtained which is optimal.

[12]

6. Write down the alternative conditions that hold for Primal-Dual problems. Prove that If x^* is any f.s. of primal and w^* is any f.s. of dual such that $c^t x^* = b^t w^*$, then x^* is optimal f.s. of primal and w^* is optimal f.s. of dual.

[4+4=8]

7. (a) What is the degeneracy of basic feasible solution (b.f.s) of a transportation problem (T.P.)? Describe the conditions for a non-degenerate b.f.s of a T.P.
   (b) Determine an initial b.f.s by Minimum Cost method and test for the optimal solution for the following T.P.

		Distrn. centres				
		D1	D2	D3	D4	Supply
Plants	P1	19	30	50	10	7
	P2	70	30	40	60	9
	P3	40	8	70	20	18
	Demand	5	8	7	14	**34**

[5+15 = 20]

8. Prove the following two results of duality:
   (a) If x^* is any feasible solution (f.s.) of primal and w^* is any f.s. of dual such that $c^t x^* = b^t w^*$, then x^* is optimal f.s. of primal and w^* is optimal f.s. of dual.
   (b) If the primal has unbounded solution, then dual is infeasible.

[6+6 = 12]

9. (a) Explain the basic differences between PERT and CPM techniques under project management.
   (b) Define the terms: Critical path, Direct cost, Indirect cost, Time-cost slope

[5+5 = 10]

Indian Statistical Institute

Mid-Semestral Examination : 2017-18

M-TECH(QR&OR) -- 1st YEAR (E - STREAM)

PROBABILITY

{Answer all the questions}

Date:05.09.17 **Full marks: 100** **Time: 3 hours**

[Symbols have their usual meaning]
Answer any FIVE questions

1. a) Derive the expression of mean and variance of a hypergeometric distribution.
   b) Find the mode of the above distribution.
   c) Prove that Hypergeometric distribution with parameters (N,n,p) will approach to Binomial distribution with parameter (n,p) as $N \to \infty$.

   [10+6+4=20]

2. a) Define monotonic sequence of events. Let $\{A_n\}$ be monotonic sequence of events, each belonging to sigma field of events $A \subseteq \Omega$, then prove that,
   $\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$.
   b) State and prove Bonferroni's inequality.

   [10+10=20]

3. a) Define rth order raw moment , rth order central moment and rth order factorial moment. Express rth order central moment in terms of rth order raw moment.
   b) Write down the probability density function of Gamma distribution with parameter α and p. Find out its moment generating function, mean, variance and skewness. Comment on the shape of the distribution.

   [6+14=20]

4. a) A machine normally makes items of which 10% are defective. An inspector selects a sample of size 15. If it contains no defective item then the lot is accepted. What is the probability that the lot will be rejected?
   b) In certain examination the score of a group of students is normally distributed with mean 58 and standard deviation 6.5. The pass mark is 45. Find the proportion of the students who fail in that examination.

   [10+10=20]

5. a) Suppose A, B, C are independent components of a system with survival probability 0.3, 0.9, 0.9 respectively. The system will survive if A and at least one of B and C survive. What is the survival probability of the system?

   b) In a college 4% of male-students and 1% of female-students are taller than 6 feet. 60% of college students are female. If a student is selected at random and found to be taller than 6 feet what is the probability that she is a female-student?

   [10+10=20]

6. a) 10 letters are placed in 10 envelopes at random. Find the probability that each letter will be placed in wrong envelope.

P.T.O

b) Fit a Poisson distribution to the following data.

# accidents(x)	Observed frequencies
0	56
1	156
2	132
3	92
4	37
5	22
6	4
7	0
8	1

[10+10=20]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2017-18

Course Name: **M. Tech. (QR&OR) - I Year**

Subject Name: **Quality Management & Systems**

Maximum Marks: 100 Duration: 2 hours

Date: 06 September 2017

Note: Answer all the questions.

1. State the different dimensions of quality proposed by David A. Garvin. Explain each of the dimensions briefly.

[8 + 12 = 20]

2. Define the customer satisfaction model proposed by Kano. Explain the model with a real life example.

[12 + 8 = 20]

3. Write short notes on any two of the following personalities, explaining their contribution to quality management.
   (a) Joseph M. Juran
   (b) W. Edwards Deming
   (c) Armand Vallin Feigenbaum

[10 + 10 = 20]

4. Explain EFQM Excellence Model in brief.

[10]

5. Assignments.

[30]

INDIAN STATISTICAL INSTITUTE
M.Tech (OR&OR) I Year: 2017-18
Mid-Semestral Examination
Programming Techniques & Data Structures
Date: 7th September, 2017 Duration: 2:00 Hours Marks: 50
Note: ***Answer all the questions.***

1. Choose the correct alternatives for the following: **[2x5=10]**

i) The output of the following program would be:

```
void main()
    {
       int i = -1, j = -1, k = 0, m = 2, n;
       n=i++&&j++&&k++||m++;
       printf("%d %d %d %d %d", i, j, k, m, n);
    }
```

(a) 0 0 1 3 1 (b) -1 0 1 3 0 (c) 0 -1 1 2 1 (d) -1 -1 0 2 1

ii) The output of the following program would be:

```
void main()
    {
       int i, j;
       for (i=1; i<=5; i++)
       {
         if (i%2)
        continue;
        printf("%d ",i);
       }
    }
```

(a) 2 4 (b) 1 3 5 (c) 1 2 3 4 5 (d) blank (i.e., no output)

iii) If a two dimensional array int $a[10]$ [20] is represented as an array of pointers, then the element $a[4][5]$ can be denoted by:
(a) $*(a + 4) + 5$ (b) $*a[4] + 5$ (c) $*(*(a + 4) + 5)$ (d) $a[4] + 5$

iv) A function in C must contain
(a) no return statement; (b) at most one return statement;
(c) exactly one return statement; (d) zero, one or more return statements.

v) The output of the following program would be:

```
void main()
    {
       int i;
      for (i = 0; i < 100; i = i + 3);
       printf("\ni = %d", i);
    }
```

(a) 33 (b) 34 (c) 99 (d) 102

[P.T.O.]

2. i) Explain the characteristics of the variables v1, v2, v3, v4, v5 and v6 as declared in the following program segment.: **[6]**

```
void main()
        {
        int v1=1;
        extern int v2;
        static int v3;
           printf("\n Test of variables");
           f();
        }
void f()
   {
      int v4, v5=2;
      static int v6;
      printf("\n Welcome to ISI, Kolkata");
   }
```

ii) Explain the meaning of the following declarations: **[2]**

```
float (*p) [25];
float (*p) ( );
```

3. Write a C program that deletes a substring from a given string without using string library functions. For example, let the given string be "Indian Statistical Institute" and the substring be "tatistical" then the resultant string will be "Indian S Institute". **[6]**

4. Square of 12 is 144. 21, which is the reverse of 12 has a square 441, which is same as the reverse of 144. Write a program to find out all such numbers/pairs in the range of 10 to 100. **[6]**

5. Write a program to multiply two matrices. NB: Check proper conditions for matrix Multiplication. **[6]**

6. Let the positive integer n be supplied as input. You have to determine positive integers k and j such that n is a product of exactly k prime factors of which exactly j are distinct. For example, when $n = 24$ (= 2*2*2*3), The output would be $k = 4$ and $j = 2$, and when $k = 64$ the output would be $k = 6$ and $j = 1$. **[7]**

7. Define a structure called '*employee*' to store information of an employee (*e_no, e_name, basic_pay, DA, HRA, gross_pay*).
   Write a program in C to input the *e_no, e_name* and *basic_pay* of several employees. The program will calculate the *DA* (=120% of *basic*), *HRA* (=15% of *basic*) and *gross_pay* (=*basic* + *DA* + *HRA*) of all employees and display the details of the employee having the highest salary. **[7]**

INDIAN STATISTICAL INSTITUTE

Mid-Sem Examination: 2017-18

M. Tech. (QR & OR), 1st Year, 1st Semester, E Stream

Subject: Statistical Methods I

Date: 08. 09. 2017 **Duration:** 3 hours

Note: This paper carries a total of 100 marks. Answer as many questions as you can.

1. Construct a histogram of the following observations. Draw your conclusions from the histogram. Compute the mean, standard deviation and skewness using the grouped data.

8.9	9.3	7.1	8.8	8.5	7.9	6.9	17.5	7.4	25.3
7.6	7.5	8.1	9.2	8.8	6.5	7.9	8.8	9.0	6.1
7.2	8.4	6.7	8.6	8.6	9.3	8.5	8.2	10.0	8.9
8.3	8.9	8.5	8.7	8.4	8.2	7.7	19.2	7.1	7.3
7.2	9.3	7.0	8.1	9.0	7.3	7.6	7.2	8.0	6.2
7.5	9.3	7.9	8.0	8.5	7.4	6.9	8.6	8.2	6.9
21.1	9.4	11.0	8.0	7.4	7.7	8.6	7.6	7.4	6.5
7.9	9.9	7.7	6.6	9.0	6.4	7.5	8.1	7.4	8.8
7.5	6.3	7.7	9.9	8.5	8.7	7.2	5.7	7.6	8.7
7.2	7.3	10.2	10.4	5.8	10.0	6.8	7.3	7.9	7.9

[12+4+(2+3+4)=25]

2. Classify and define the various types of data. Give two examples of each type of data.

[6+4=10]

3. Define simple and stratified random sampling. Compare the advantages and disadvantages of the two.

[4+4=8]

4. Derive the expression for the variance of sample mean in case of simple random sampling with replacement.

[15]

5. Consider simple random sampling without replacement. Prove that the sample variance is an unbiased estimate of population variance.

[12]

P. T. O

6. In a stratification with two strata, the values of the W_h and S_h are as follows:

Stratum	W_h	S_h
1	0.8	2
2	0.2	4

Compute the sample sizes n_1 and n_2 in the two strata needed to satisfy the following conditions: (i) The standard error of the estimated population mean y_{st}-bar is to be 0.1 and $n = n_1 + n_2$ is to be minimized. (ii) The standard error of the estimated mean of each stratum is to be 0.1. (Note: the notations have their usual meaning)

[15+15=30]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

Programme Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Course: Operations Research-I

Date of Exam: 20-11-2017 **Max Marks: 75** **Duration:** 3 hrs.

Assignment: 25 marks

Question no. 1 and 7 are compulsory. Use of scientific calculator/ RMMR table is allowed.

Answer as many questions as you can.

1. Explain the following terms:
   (a) Least cost project schedule
   (b) Safety stock and Stock out cost
   (c) Little's law

[5+5+5=15]

2. As a Project Manager, find the following
   a) Critical Path and expected project completion time;
   b) Least Cost schedule and the corresponding project cost;
   c) 36 days schedule and the corresponding project cost.

   Given: Indirect cost per day = Rs. 175/-

   The information on activities, their durations for completion and corresponding cost figures are tabulated in the next page.

Activity Node	Normal duration (days)	Crash duration (days)	Cost (Rs.)	Crash Cost (Rs.)
(1,2)	8	6	1800	2200
(2,3)	16	11	1500	2200
(1,4)	12	9	2400	3000
(3,5)	14	9	1800	2400
(4,5)	15	14	800	2000
(5,6)	10	8	2000	4000

Comment on the solutions obtained in b) and c).

[5+6+7=18]

3. (a) Derive the optimal order quantity for quantity discount model after stating the most important assumptions and underlying constraints.

   (b) A digital printing press has an annual demand of 1,400 portable hard disk. The cost of a hard disk is Rs. 400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is Rs. 25 per order. If the company orders in quantities of 300 or more, it can get a 5% discount on the cost of the hard disks. Should the company take the quantity discount? Assume the demand is constant.

   [5+7=12]

4. What are the assumptions of a Production Order Quantity (POQ) model and how it is different from EOQ model? Derive the conditions for which POQ model approaches to EOQ model. Explain the significance of POQ model parameters towards shortage/stock out situation.

   [5+3+2=10]

5. State all the assumptions of a (M/M/1/FCFS/∞/∞) model. Derive the expression for the expected number of customers under steady state conditions for a non-empty queue. Give some real life examples of encouraging and discouraging queues.

   [3+6+3=12]

6. Customers arrive at the first class ticket counter at a rate of 12/hr. There is one clerk serving the customer at a rate 30/hr.
   a) What is the probability that there is no customer in the counter?
   b) What is the probability that there are 2 more customers in the queue?
   c) What is the probability that there is no customer waiting to be served?
   d) What is the probability that probably a customer is being served and nobody is waiting?

[2+2+2+2=8]

7. SONY Appliances sells and services several brands of home appliances. Past sales for a particular model of LED television have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.4	0.15	0.2	0.05

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.15	0.45	0.40

Based on cost considerations as well as storage space, the company has decided to order 10 of these each time an order is placed. The carrying cost is Rs. 200 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 3,000 per stock out. The company has decided to place an order whenever there are only 2 television sets left at the end of the week. Simulate 10 weeks of operation for SONY with currently 5 units in inventory. What would be the weekly stock out cost and weekly carrying cost under this situation?

[12+3=15]

INDIAN STATISTICAL INSTITUTE
First-Semester Examination : 2017-18
M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY

Note : Answer any FIVE questions

[Symbols have their usual meaning]

Date:22.11.17 **Full marks:100** **Time: 3 hours**

1. a) State and prove Chebyschev's inequality. What does it provide?
   b) State and prove central limit theorem due to Lindeberg and Levy. Explain its application in the field of SQC.

[(6+2)+(10+2)=20]

2. a) Derive the probability density function of t distribution.
   b) Explain one application of t distribution in the field of statistical test of hypothesis.
   c)) Let χ_1^2 and χ_2^2 be two independent χ^2 variables with degree of freedom n_1 and n_2 respectively. Let Z = $\frac{\chi_1^2}{\chi_2^2}$. Derive the distribution of Z.

[10+3+7=20]

3. a) Let $X_i \sim$ iid N(0,1), i = 1,2. Let $Y = X_1/ X_2$. Find the distribution of Y. Find a measure of central tendency of Y.
   b) Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ and the correlation coefficient between X and Y be ρ. Derive the joint distribution of X and Y.

[(6+4)+10=20]

4. a)) Let $A_1, A_2, \ldots\ldots, A_r$ be r events not necessarily mutually exclusive. Find the probability of occurrence of exactly m events (m<r).
   b) There are three chests, each having 2 drawers. Each drawer of first chest contains one gold coin. Each drawer of second chest contains one silver coin. Third chest contains one gold coin in one drawer and one silver coin in another. A chest is open at random. A drawer is open and it is found to contain a gold coin. What is the probability that the coin in the other drawer is also a gold coin?

[12+8=20]

5. a) Suppose that the number of telephone calls received by an operator from 11 AM to 11-30 AM follows a Poisson distribution with λ=5. Find the probability that in the next 3 days he will receive a total of one call.
   b) A card from a pack of 52 cards is lost. From the remaining card, two cards are drawn and are found to be spades. Find the probability that the missing cards is also a spade.
   c) What is the probability that at least two students have the same birthday if there are r students in a class?

[8+6+6=20]

P.T.O

6. a) Let X and Y be two random variables with E(X) = 5, Var (X) = 2, E(Y) = 10, Var (Y) = 3, and correlation coefficient between X and Y be – 0.4.

Let Z_1 and Z_2 be two random variables defined as follows

$Z_1 = 3X + 5Y$, and $Z_2 = 2X - 3Y$.

Find $E(Z_1)$, Var (Z_1), $E(Z_2)$, Var (Z_2) and correlation coefficient between Z_1 and Z_2. (state the results you have used).

b) A person is paying Rs 50.00 for each participation of the following game: He is drawing 2 cards from a deck. He gets Rs 20.00 if he draws 2 red kings. He gets Rs 30.00 if he draws 2 queens. He gets Rs 30 if he draws 1 red jack and another black ace. Otherwise he gets nothing. What is his expected gain?

[10+10=20]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

Course Name: **M. Tech. (QR&OR) - I Year**

Subject Name: **Quality Management & Systems**

Date: 24 November 2017 **Maximum Marks: 100** **Duration: 2 hours**

Note: Answer all questions.

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1. Define quality function deployment. Describe the structure of House of Quality.

   Or, Define six sigma methodology and explain the various steps of implementation, in brief.

   [5 + 8 = 13]

2. Write short note on any five of the following terms.

   (a) Service Quality

   (b) Adequacy Audit and Compliance Audit

   (c) First Party Audit and Second Party Audit

   (d) Kano Model of Customer Satisfaction

   (e) Different Views of Quality

   (f) Different Scales of Measurement

   (g) Quality Management Principles

   [5 x 5 = 25]

3. Write short notes on any two of the following personalities, explaining their contribution to quality management.

   (a) Frederick Taylor (c) Henry Fayol

   (b) Max Weber (d) Abraham Maslow

   [2 x 6 = 12]

4. Following are the objective evidences of an internal audit conducted according to ISO 9001: 2015 QMS. For any four objective evidences, provide your explanation and justification on being classified as a non-conformance or not.

   (a) Waste records show that waste was removed only once per week on 4 occasions during last three months instead of twice per week as stated in the procedure PR-6.8.

   (b) The foreman in the Machine Shop was referring to revision 03 of the drawing GT1653/13, but was not sure whether that was the latest revision or not.

   (c) QMS Objectives are found to be measured routinely, but no improvement is observed.

   (d) Testing procedure specifies that the temperature to be controlled within 40 ± 2°C. A calibrated thermometer of increment 1°C is used for this purpose.

   (e) The company Quality Manual has not been revised since April 2015.

   (f) An Inspector was using a documented instruction detailing how the testing should be carried out, but the testing instruction did not contain any signature.

   [4 x 5 = 20]

5. Assignments.

   [30]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

M. Tech. (QR & OR), 1st Year, 1st Semester, E Stream

Subject: Statistical Methods I

Date: 27. 11. 2017 **Maximum Marks: 100** **Duration:** 3 hours

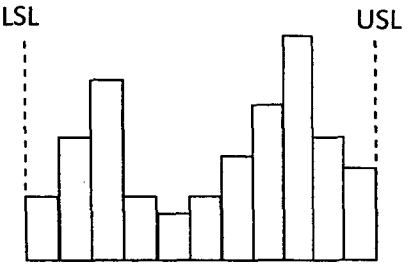
Note: Answer all the questions.

1. Compare the usefulness and limitations of Histogram and Probability plot. What, if any, do you gain by constructing a probability plot based on grouped data instead of the individual observations?

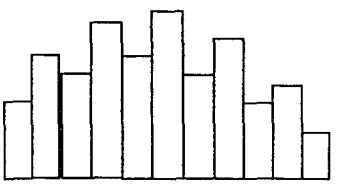
[6+4=10]

2. Describe the process that could have generated the following two types of histograms?

(i)



(ii)



Assume that the samples are taken from a lot received from a supplier in case (i) and that samples are taken from a lot representing a production of six hours.

[5+5=10]

3. Both the cases in question no. 2 indicate the presence of assignable causes in the process. However, in many cases the situation (ii) is likely to be less problematic. why?

[5]

4. Write the estimator of population mean under stratified random sampling. Derive the variance of the estimator in case of sampling without replacement.

[3+10]

5. The measurement of the diameter of a certain product yielded the following results (in mm):

4.5, 7.8, 6.0, 5.1, 6.8, 5.7, 3.6, 4.8, 5.9, 3.9, 3.7, 6.0, 3.7, 5.0, 4.1, 4.2, 4.7, 7.7, 6.4, 5.1, 4.9, 4.6, 6.5, 4.7, 5.7, 5.1, 5.9, 5.0, 2.9, 4.7

Assume that the diameter is Normally distributed and the specifications are 5±2 mm.

(i) Estimate the expected percentage of non-conforming products.

P.T.O

(ii) Estimate the expected percentage of non-conforming products by counting the number of non-conforming products in the sample.

(iii) Which of the above two estimates is better? Give qualitative justifications.

(iv) Give the outline of any possible method for quantitative evaluation of the two estimates.

[10+2+3+7=22]

6. Assume that the observations given in questions 5 is taken from the lot produced on day 1. Another sample of size 20 is taken from the lot produced on day 2 and the following results are obtained:

5.1, 5.2, 5.7, 9.0, 4.5, 8.2, 6.3, 7.3, 7.2, 4.5, 3.4, 4.3, 11.0, 3.9, 5.9, 7.0, 7.5, 6.5, 10.0, 6.4

Construct a Q-Q plot of the two samples and draw your conclusions from the plot.

[12]

7. Explain the method of maximum likelihood estimation and the method of moment estimation using an example in each case. Take two different examples other than the Bernoulli distribution.

[7+7=14]

8. Write an efficient algorithm for generating 100 random observations from the following discrete distribution of a random variable X:

Interval of X	Probability
2-4	.1
5-8	.4
9-13	.5

[14]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

M.Tech (OR&OR) I Year

Programming Techniques and Data Structures

Date: 30/11/2017 Duration: 3 hours

Note: This paper carries 125 marks. Answer as much as you can. Maximum marks you can get is 100

1. a) Write a C function to perform reversal (in terms of traversal order) of a singly linked list. **(7)**

   b) Write a C function to perform deletion of node from any position, as specified by the user, of a doubly linked list. **(8)**

2. a) Using stack, write a pseudocode to check if an expression has duplicate parenthesis. For example: ((7+3)/((12*5))) **(5)**

   b) Write a procedure in C to implement a queue using two stacks. One stack is used to enqueue elements, while the other is used to dequeue elements. That is, when an element is added to the queue, it is pushed on the IN-stack. When an element is removed from the queue, it is popped off the OUT-stack. If the OUT-stack is empty, the content of the IN-stack is transferred to the OUT-stack. Use an example to illustrate the enqueue and dequeue operations. **(10)**

queue made with two stacks

3. a) Define an AVL tree. Why is AVL tree an important data structure? **(1+2 = 3)**

   b) Insert the following keys in the order, as shown below, to construct an AVL search tree:
   64, 1, 44, 26, 13, 110, 98, 85

   Mention the height of each node after insertion. Also, illustrate the balancing steps whenever applicable. **(6)**

   c) A binary tree T has 9 nodes. The inorder and preorder traversals of T yield the following sequence of nodes:

Inorder:	E	A	C	K	F	H	D	B	G
Preorder:	F	A	E	K	C	D	H	G	B

   Draw the tree T. **(6)**

4. a) What are the advantages of using the binary search trees as a data structure? **(2)**

   b) Suppose the following list of letters is inserted in the order into an empty binary search tree:
   J, R, D, G, T, E, M, H, P, A, F, Q

   (i) Draw the final tree, and (ii) show the outcome of preorder, inorder and postorder traversals of the tree. **(4+3 = 7)**

[P.T.O.]

c) Explain with an example how a node N is deleted from a binary search tree, when
   i) N has no children,
   ii) N has exactly one child, and
   iii) N has two children. **(2x3 = 6)**

5. a) Consider the following 4-digit employee numbers: 9614, 7148, 3205, 2345 **(1x4 = 4)**

   Determine the 2-digit hash address of each number using
   i) the division method, with m = 97;
   ii) the mid-square method;
   iii) the folding method without reversing; and
   iv) the folding method with reversing.

   b) Suppose a table T has 11 memory locations, T[1], T[2], ..., T[11], and suppose a file F consists of 8 records Z, Y, X, E, D, C, B and A with the following hash addresses:

Record:	Z	Y	X	E	D	C	B	A
H(k):	1	5	11	4	11	2	8	4

   Show how the file F is represented in memory. Use linear probing for collision resolution. **(2)**

   c) Explain the term collision in the context of hashing. Write one advantage and one disadvantage of using linear probing for collision resolution. Which collision resolution procedure can be used to overcome the disadvantage of linear probing? **(1+2+1 = 4)**

6. a) Define a Max-Heap. **(2)**

   b) Show steps of building a heap from the following list of numbers: **(4)**
   44, 30, 50, 22, 60, 55, 77, 58

   c) Show step-by-step how the root node is deleted from the resultant heap. **(4)**

7. Write the algorithmic steps for converting an algebraic expression in infix form to postfix one. Demonstrate step-wise this conversion of the expression: ((A + B) * D) ^ (E - F) **(5+5=10)**

8. a) Write any sorting algorithm that takes $O(n*\log_2 n)$ time on average, for sorting n numbers. **(5)**

   b) Analyse the time complexity and write the disadvantages of radix sort. **(3+2 = 5)**

   c) Assuming that you are given a set of numbers in a sorted order, write an efficient C program to search for a query number. Mention the limitations of using such an algorithm, if any. **(4+1 = 5)**

9. Write short notes on the following: **(5x4 = 20)**

   a) 2-3 tree

   b) Fibonacci search algorithm

   c) Threaded binary tree

   d) Priority queue

INDIAN STATISTICAL INSTITUTE

Back Paper Examination: 2017-18

M.Tech (OR&OR) I Year

Programming Techniques and Data Structures

Full Marks: 100 Duration: 3 hours

Date : 27/12/2017

Group – A *(Multiple Choice Questions)*

1. Choose the correct alternative(s) for the following questions: **(2x10 = 20)**

i) Consider the following program:

```
main()
{
     int i = -3, j = 2, k = 0, m;
     m = ++i && --j && ++k;
     printf("\n %d %d %d %d", i, j, k, m);
}
```

Its output is:

a) -2 3 0 1 **b)** -2 1 1 1 **c)** -3 1 1 0 **d)** -3 2 0 1

ii) A variable of register storage class is declared:

a) for accessing it multiple times like loop counters **b)** to retain its value in between function calls **c)** for accessing it from outside the program **d)** for making it accessible from all the functions

iii) You cannot perform binary search among a sequence of numbers:

a) when the numbers are in a linked list **b)** when the numbers are not sorted **c)** both a and b **d)** none of these

iv) Too many recursive calls may run into stack overflow:

a) true **b)** false

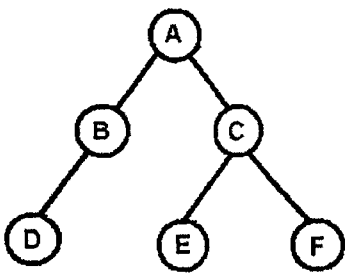
v) Consider the following program:

```
main()
{
     char str1[] = "Hello";
     char str2[] = "Hello";
     if(str1 == str2) printf("Equal");
     else printf("Unequal");
}
```

Its output is:

a) Equal **b)** Unequal **c)** error **d)** none of these

vi) Consider the following binary tree:



Its array representation is:

a) A, B, D, C, E, F **b)** A, B, C, D, '\0', E, F **c)** D, B, A, E, C, F **d)** none of these

[P.T.O.]

vii) The post-order traversal of the tree in Question 1(vi) is:
a) D, B, A, E, C, F **b)** A, B, C, D, E, F **c)** D, B, E, F, C, A **d)** A, B, D, C, E, F

viii) For a single threaded binary tree,
a. left null pointers are made to point to inorder predecessor **b.** right null pointers are made to point to inorder successor **c.** both a and b **d.** none of these

ix) Let MAX_SIZE be the size of the array used in the implementation of a circular queue. While inserting an element in the queue, REAR is manipulated as:
a) (REAR % 1) + MAX_SIZE **b)** REAR % (MAX_SIZE + 1)
c) (REAR + 1) % MAX_SIZE **d)** REAR + (1 % MAX_SIZE)

x) In hashing, the collision resolution technique that can be used to avoid clustering is:
a) quadratic probing **b)** double hashing **c)** both a and b **d)** none of these

Group – B *(C Programming)*

Answer any four from the following five questions in this section **(10x4 = 40)**

2. Bob, the builder, takes an umbrella to his work only if the weather is sunny or rainy. Write a C program in order to automate this recommendation for carrying an umbrella, based on the input about the weather. If you use *if-else* or *switch-case* condition statements in your C code, justify your preference for the condition statements. **(6+4 = 10)**

3. Write a C program using recursive function to print the Fibonacci series and the sum of the series up to *n* terms. **(7+3 = 10)**

4. Differentiate between: (*any two*) **(5x2 = 10)**
   i) *while* loop and *do-while* loop
   ii) function *call-by-value* and function *call-by-reference*
   iii) *#define SIZE 5* and *const int SIZE = 5;*

5. Write a C program to print the following pattern: **(10)**

```
    *
   * *
  * * *
 * * * *
```

6. Write the pseudo-code for bubble sort. How can you detect if the array is already sorted before all passes are complete? How will you modify the pseudo-code to avoid the extra unnecessary iterations of bubble sort after you find that your array has already been sorted? **(7+1+2 = 10)**

Group – C *(Data Structures)*

Answer any two from the following three questions in this section **(20x2 = 40)**

7. Write a C program to implement the stack data structure using arrays, i.e., your code should include push(), pop() and display() functions, and should be called with appropriate arguments from main(). **(20)**

8. Velma loves using array data structure. One day a super-genius computer geek, Fred, tells her that her computer memory is almost full, still it can store about 100 integers but not in contiguous memory locations. Can she continue using arrays? Why not? What kind of linear data structure will she now prefer to store her data (integers)? If you were to assist her in writing some C functions for this purpose, how will your function look like for deletion of elements from a user-entered location specific to this data structure? Remember in order to convince her to accept your function, you are required to display the remaining elements for the data structure in the appropriate order after deletion. **(1+2+2+10+5 = 20)**

9. In order to have speedy search, Bluto likes to store numbers in binary search trees. Popeye disagrees with him, and says AVL trees are better data structures. They sought for your expert opinion to resolve their conflict. Who do you think is right? Give examples to justify your answer. Explain all the cases, with examples, for adding a new element in an AVL tree. **(5+15 = 20)**

INDIAN STATISTICAL INSTITUTE

Back Paper Examination: 2017-18

M. Tech. (QR & OR), 1st Year, E Stream

Subject: Statistical Methods I

Date: 29.12.2017 **Duration: 3 hours**

Note: Answer all the questions. Full marks are 100.

1. (a) Classify the four types of data. Give two examples of each type. (b) Draw the flow chart of the data collection process.

[(3+4) +5=12]

2. In a study of the possible use of sampling to cut down the work in taking inventory in a stock room, a count is made of the value of the articles (y) on each of the 36 shelves in the room. The values to the nearest rupee are as follows.

29, 38, 42, 44, 45, 47, 51, 53, 53, 54, 56, 56, 56, 58, 58, 59, 60, 60

60, 60, 61, 61, 61, 62, 64, 65, 65, 67, 67, 68, 69, 71, 74, 77, 82, 85

It is required that the estimate of total value made from a sample be correct within Rs. 100.00, apart from a 1 in 20 chance. An advisor suggests that a simple random sample of 12 shelves will meet the requirements. Do you agree? (Given $\sum y = 2138$, $\sum y^2 = 131682$)

[12]

3. Explain the inverse transform method for generating random numbers from a specified distribution using two examples.

[14]

4. Define the terms 'statistic', 'sampling distribution' and 'standard error'. Why do we need to study sampling distributions? Describe the methods for constructing 95% confidence intervals for the Binomial proportion p, when the sample size (n) is small and also when n is large.

[6+3+10=19]

5. (a) Define point and interval estimates. Give examples. (b) Describe the three criteria normally used to judge the quality of an estimator. (c) Name three methods of estimation and describe any one of them with an example.

[4 + 6 + 6 = 16]

P.T.O

6. Assume the time taken (y) by the police to arrive at a bank after the alarm starts follows exponential distribution. The following data on y (minutes) are obtained from 11 trial runs.

3.4, 2.4, 4.2, 10.2, 7.8, 3.1, 5.2, 7.2, 3.9, 2.9, 9.5

What is the probability that a robber entering the bank will be caught if he takes two minutes to finish off his business?

[12]

7. Suppose the process of police arrival as mentioned in question 6 is modified in order to reduce the arrival time. Following data are obtained after the modification.

2.2, 3.1, 3.2, 6.3, 4.7, 2.8, 4.0

Construct a Q-Q plot to examine whether the process has really improved or not.

[15]

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination: 2017-2018 (Second Semester)

M. Tech(QR & OR) I Year (E & S Stream)

Reliability-I

Date: February 19 , 2018 Full Marks: 65 Duration: $2\frac{1}{2}$ hours.

Note: Answer all questions.

1. (a) Consider a system with three components. Let x_1, x_2 and x_3 be the binary state variables of the components and ϕ be the structure function of the system. The states of the components and system are given in the following Table. Check whether the system is coherent.

x_1	x_2	x_3	$\phi(\underset{\sim}{x})$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	1	0	1
1	0	1	1
1	1	1	1

(b) Show that if ϕ is a coherent structure, then its dual is also coherent.

[6 + 6= 12]

2. Show that the redundancy at component level is better than the redundancy at system level for any coherent system.

[6]

3. (a) Define min path sets of a coherent system. Write down the structure function of a coherent system in terms of its min path structures, with justification.

(b) Consider a coherent system with n components. Let Let T_i be the lifetime of the ith component. Then show that the lifetime of the system T can be expressed as

$$T = \max_{1 \le j \le p} \min_{i \in P_j} T_i,$$

where $P_1, \ldots, P_p$ are the min path sets.

[(2+3)+3 = 8]

4. Let T be the lifetime random variable of a unit with the hazard function

$$\lambda(t) = \begin{cases} \lambda_1, & \text{for } 0 \le t \le t_1 \\ \lambda_2, & \text{for } t_1 < t \le t_2 \\ \lambda_3, & \text{for } t > t_2. \end{cases}$$

Find the reliability function $R(t)$ and hence find $E(T)$.

[6 + 6 = 12]

P.T.O.

5. Consider a coherent system ϕ with n associated components. Let $\rho_1(\underset{\sim}{x}), \ldots, \rho_p(\underset{\sim}{x})$ be the min path structures and $\kappa_1(\underset{\sim}{x}), \ldots, \kappa_k(\underset{\sim}{x})$ be the min cut structures corresponding to ϕ. Show that

$$\coprod_{j=1}^{k} P[\kappa_j(\underset{\sim}{X}) = 1] \leq P[\phi(\underset{\sim}{X}) = 1] \leq \coprod_{j=1}^{p} P[\rho_j(\underset{\sim}{X}) = 1].$$

Hence show that if the components are independent with reliabilities $p_1, \ldots, p_n$, then

$$\coprod_{j=1}^{k} \coprod_{i \in K_j} p_i \leq h(\underset{\sim}{p}) \leq \coprod_{j=1}^{p} \prod_{i \in P_j} p_i$$

[To proof (b), you may use the results:
If $X_1, \ldots, X_n$ are associated binary random variables, then

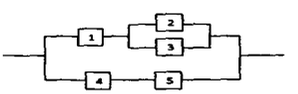
$$\text{(i) } P\left[\prod_{i=1}^{n} X_i = 1\right] \geq \prod_{i=1}^{n} P[X_i = 1] \quad \text{(ii) } P\left[\coprod_{i=1}^{n} X_i = 1\right] \leq \coprod_{i=1}^{n} P[X_i = 1].]$$

[4+4 =8]

6. Consider a series system with n components. The lifetimes of the components are i.i.d. with common cumulative distribution function $F(t) = 1 - \exp(-\lambda t^{3.7})$, $t > 0$, $\lambda > 0$. Show that the system has IFR lifetime distribution [5]

7. Consider a system with the reliability block diagram shown in the Figure. Find all the min path sets and min cut sets of the system. Compute the structural importance of components 1 and 4. Find the structure function of the system by using a modular decomposition with three modules.

[4 + 6 + 4 = 14]



INDIAN STATISTICAL INSTITUTE

MID-SEMESTRAL EXAAMINATION: 2017 – 18 (Second Semester)

M. Tech. (QR-OR); I year; E-Stream

Subject: Elements of Stochastic Process

Date: 20.02.2018 **Full Marks: 60** **Duration: 2 hours**

Note: Unless stated otherwise, a Markov Chain will mean a discrete time parameter Markov Chain with stationary transition probabilities.

(1) Define the following:
(a) Stochastic Process (b) State space and Parameter space of a Stochastic Process (c) Markov Chain (d) Transition Probabilities.

(4x5) = [20]

(2) Suppose $\{X_n | n=0,1,2...\}$ is a Markov Chain. Derive the joint distribution of the random variables $X_0, X_1, ...X_n$.

[15]

(3) State and prove the Chapman-Kolmogorov Equation for a Markov Chain (Prove all the results that you use). Interpret the equation.

[15]

(4) A meter measuring a sequence of electric impulses records the highest voltage that has passed through it up to any given time. Suppose that the impulse passing through the meter at the n^{th} moment is Y_n and its probability mass function is:

$P(Y_n=k) = 1/(M+1)$ for $k = 0,1,2,....M$ (M is a positive integer)

Denote by X_n the meter reading at time n.
Show that $\{Xn | n=0,1,2...\}$ is a Markov Chain and find the transition matrix of the chain.

[10]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2017-18
Course: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: SQC

Date of Exam: 21/02/2018 **Max. Marks: 100**

Duration: 3 hrs.

Answer All Questions.

1) The thickness of a printed circuit board is an important quality characteristic. Data on board thickness (in inches) are given in the following Table for 25 samples of three boards each.

Data on thickness of printed circuit board

Sample number	*X-bar*	*R*
1	0.06350	0.0011
2	0.06277	0.0009
3	0.06307	0.0005
4	0.06317	0.0004
5	0.06257	0.0011
6	0.06253	0.0021
7	0.06313	0.0014
8	0.06257	0.0006
9	0.06273	0.0010
10	0.06317	0.0002
11	0.06343	0.0008
12	0.06277	0.0007
13	0.06320	0.0005
14	0.06387	0.0014
15	0.06317	0.0025
16	0.06293	0.0004
17	0.06233	0.0015
18	0.06287	0.0004
19	0.06320	0.0007
20	0.06347	0.0011
21	0.06230	0.0012
22	0.06197	0.0010
23	0.06307	0.0002
24	0.06330	0.0006
25	0.06273	0.0007

a) Investigate whether the process was in control or not based on the trial control limits.

P.T.O

b) Draw the $\bar{X}$-R chart and prescribe the central lines and control limits for future control purposes.

c) Obtain the process capability index C_p of the process if the specification limits are 0.0631 ± 0.00047.

d) Considering an underlying normal distribution find the percentage of non-conforming thickness measurements for the specification limits given above.

[10+10+4+6 = 30]

2) Suppose that m preliminary samples are available each of size n, and let S_i be the standard deviation of the ith sample. Derive the central lines and 3-sigma control limits for the S-chart and $\bar{X}$-chart. Also establish the method of construction of an $\bar{X}$-S control chart when the sample size varies.

[10+5 = 15]

3) Considering the fact that the distribution of relative range is known, derive the central lines and 3-sigma control limits for the R-chart and $\bar{X}$-chart if m preliminary samples are available each of size n.

[15]

4) The mortgage loan processing unit of a bank monitors the costs of processing loan applications. The quantity tracked is the average weekly processing costs, obtained by dividing total weekly costs by the number of loans processed during the week. The processing costs for the most recent 20 weeks are shown in the following table. Set up appropriate control charts for these data.

Weeks	Cost x	Weeks	Cost x
1	310	11	294
2	288	12	299
3	297	13	297
4	298	14	299
5	307	15	314
6	303	16	295
7	294	17	293
8	297	18	306
9	308	19	301
10	306	20	304

[30]

5) Define the following:
   a) Rational Subgroup
   b) Statistical Control
   c) Type I Error
   d) Type II Error
   e) Juran's Feedback Loop

[5×2 = 10]

INDIAN STATISTICAL INSTITUTE

Second Mid-Semester Examination (2017 – 2018)

Course Name : **M.Tech (QR & OR)**

Subject : **Industrial Engineering and Management**

Date : 22-02-2018

Maximum Marks : **40**

Duration : **120 minutes**

Question Paper

Answer *all* questions

1. (a) With respect to the Table as given below the home builder wants to finish the following project with minimum investment. The indirect cost is Rs 10,000/- per week. Crash the activities in three stages only and suggest the optimal investment accordingly.

Table: Normal Activity and Crash Data

Activity	Predecessor	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost (in Rs 1000)	Crash Time (in Rs 1000)
A	----	2	1	180	280
B	A	4	2	320	420
C	B	10	7	620	860
D	C	6	4	260	340
E	C	4	3	410	570
F	E	5	3	180	260
G	D	7	4	900	1020
H	G	9	6	200	380
I	C	7	5	210	270
J	I	8	6	430	490
K	F, J	4	3	160	200
L	F, J	5	3	250	350
M	E, H	2	1	100	200
N	K,L	6	3	330	510

P.T.O

(b) What is the significance of controlling of a project? Explain with an example. **[15+5]**

2. (a) What do you mean by standardization? Explain its advantages and disadvantages.

   (b) Specify the layout of a Big Railway Station with the explanation. **[5+5]**

3. Write short notes on ***any two***: **[2X5=10]**

   (a) Significance of SIMO chart

   (b) Work sampling

   (c) Productivity analysis

   (d) Ineffective Time due to shortcomings of management

   (e) Function of a manager

INDIAN STATISTICAL INSTITUTE

Mid -Semester Examination: 2017 – 18

M. Tech (QROR), E-Stream, Semester II

Statistical Methods – II

Date: 23/02/2018 Maximum Marks: 50 Duration: $\mathbf{1\frac{1}{2}}$ Hrs.

Note: Answer Question 1 and any three (3) from the rest.

1. a) For an $r \times c$ contingency table with entries f_{ij}, $i = 1,2,\cdots,r$ $j = 1,2,\cdots,c$, find the expression for chi-squared test for independence stating the hypothesis to be tested.

b) In a survey sampling example randomly selected 610 respondents were cross classified based on two variables sex of the respondent (X) and social class of the respondent (Y). The cross classification table is given below:

Social Class (Y)	Sex (X)		TOTAL
	Male	Female	
Upper Middle	33	29	62
Middle	153	181	334
Working	103	81	184
Lower	16	14	30
TOTAL	305	305	610

Use the chi square test of independence to determine if variables X and Y are independent of each other. Use the 5% level of significance.

[13+7 = 20]

2. The sodium content of twenty 300-gram boxes of organic corn flakes was determined. The data (in mg) are as follows:

131.15	130.69	130.91	129.54	129.64	128.77	130.72	128.33	128.24	129.65
130.92	131.15	130.69	130.91	129.54	129.64	128.77	130.72	128.33	128.24

a) Can you support a claim that mean sodium content of this brand of cornflakes is 130 milligrams? Use $\alpha = 0.05$.

b) Compute the power of the test, if the true mean sodium content is 130.5 milligrams.

[6+4 = 10]

P. T. O

3. Two machines are used to fill plastic bottles with dishwashing detergent. The standard deviation of fill volume are known to be 0.10 and 0.15 fluid ounces for machine 1 and machine 2 respectively. Two random samples of 12 bottles and 10 bottles are selected from machine 1 and machine 2 respectively and the sample mean fill volumes are $\bar{x}_1 = 30.87$ fluid ounces and $\bar{x}_2 = 30.68$ fluid ounces.

a) Assuming normality, test the hypothesis that both machines fill to the same mean volume. Use $\alpha = 0.05$.
b) What is the *P*-value of the test in part (a)?
c) Construct a 95% confidence interval on the mean difference in fill volume

[6+2+2 = 10]

4. A designed experiment is done to assess how moisture content and sweetness of a pastry product affects a tester's rating of the product. In the experiment four moisture levels and two sweetness levels are studied. Two pastries are prepared at each combination and rated by the tester. Following table gives the data, after coding (original rating - 60):

Sweetness	**Moisture Level**			
	1	2	3	4
1	4, 1	12, 11	23, 26	28, 34
2	13, 16	20, 23	29, 33	35, 40

a) State the hypotheses of interest in this experiment.
b) Analyze the data using analysis of variance assuming both the factors to be fixed. Use $\alpha = 0.05$.

[3 + 7 = 10]

5. a) Write down the linear statistical model of a two factor mixed effect design with n replicates, where factor A, with a levels, is random and factor B, with b levels, is fixed. Explain the terms and assumptions made.

b) Obtain the expression for expected mean squares of the terms of the model in part (a).

[2+2+6 = 10]

Indian Statistical Institute
Second Semester Examination : 2017-18
M. Tech (QR & OR) I Year (E & S Stream)

Sub: Reliability-I

Date: 16/04/2018 Maximum Marks: 100 Time: 3 hours.

Note: The paper carries 110 marks. Answer any part of any question, but the maximum you can score is 100.

1. (a) Show that the min path sets of a coherent system are the min cut sets of its dual.
   (b) Show that the component with highest reliability is most important to a parallel system with indepndent components.
   (c) What is accelerated life testing?

   [7 + 5 + 3 =15]

2. Consider a coherent system with four independent components. The system functions if and only if components 1 and 2 functions and at least one of the remaining components functions. Let T_i be the lifetime of the ith component, for $i = 1, 2, 3, 4$. Suppose T_1 and T_2 have constant hazards λ_1 and λ_2, respectively, and T_3 and T_4 have the same constant hazard λ_3.
   (a) Check whether the system lifetime is IFR or DFR.
   (b) Find the mean residual life of the system at age t.
   (c) Construct fault tree for the failure of the system.

   [8 + 5 +3 =16]

3. Consider a series system with n associated components. Suppose $R_i(t)$ and $R_s(t)$ denotes the reliability of the i-th component and the system, respectively. Show that

$$R_s(t) \geq \prod_{i=1}^{n} R_i(t)$$

   [10]

4. Suppose an s-out-of-n system is functioning under a stress X with the c.d.f. $F_X(x) = 1 - e^{-\theta x}$, $\theta > 0$, $x > 0$. The strengths of the components are i.i.d. with the c.d.f. $F_Y(y) = 1 - e^{-\lambda y}$, $\lambda > 0, y > 0$. Find the reliability of the system.

   [10]

5. Consider a 2-unit standby redundant system with a switch. The standby unit carries no load while in standby mode. The lifetime of the primary active unit has constant failure rate λ_1. The lifetime of the standby unit has constant hazard rate λ_2. Let p_s be the probability that switch activates the standby unit when required. Find the reliability of the system. Hence find the expected lifetime of the system.

   [6+4=10]

P.T.O.

6. Suppose a unit can fail by any one of the k failure modes. Let T be the failure time and J be the failure mode. Assume that T is continuous. Suppose $\lambda_j(t)$ and $F_j(t)$ denotes the mode-specific hazarad rate and subdistribution function, respectively, for mode $j = 1, \ldots, k$.

   (a) Show that $F_j(t)$ can be expressed completey in terms of the mode-specific hazard rates.

   (b) Assume that $\lambda_j(t) = \lambda_j$, for $j = 1, \ldots, k$. Derive the maximum likelihood estimators of λ_j, for $j = 1, \ldots, k$, based on random right cenosred data.

   [5 + 5 = 10]

7. The service life (in hours) of a semiconductor has an exponential failure time distribution. Ten semiconductors are placed on life test until four fail. The failure times for these four semiconductors are 585, 972, 1460 and 2266. Identify the censoring scheme with justification. Derive the maximum likelihood estimator (MLE) of the median lifetime. Give an estimate of the asymptotic variance of the MLE of the median lifetime.

   [2+ 7 + 4 = 13]

8. Suppose the lifetime (in years) of a memory chip in a mainframe computer has a Weibull failure time distribution with hazard rate $\lambda(t) = \alpha\lambda t^{\alpha-1}$, $\alpha > 0$, $\lambda > 0$. Fifty chips were placed on test and the number of survivors was recorded at the end of each year, for a period of 8 years. Find the least squares estimates of α and λ. Write down the likelihood function to obtain the maximum likelihood estimates of α and λ [Do not derive the estimates].

Year	1	2	3	4	5	6	7	8
Number of survivors	47	39	29	18	11	5	3	1

   [12 + 3 = 15]

9. (a) Show that the Kaplan-Meier estimator becomes empirical survival function when there is no censoring.

   (b) The following censored data reflect the failure times in months of a new laser printer. Censored times result from removal of the printer due to upgrades. Give the Kaplan-Miere estimate of the reliability of this printer over its 2-year warranty period.

   $$8, 33, 15+, 27, 18, 24+, 13+$$

   [5+6 =11]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination 2017 - 18

M. Tech. (QR OR); I Year; E-Stream

ELEMENTS OF STOCHASTIC PROCESSES

Date : 18/04/2018 Maximum Marks : 100 Time :3 $^1/_2$ hours

Notes:

(i) The paper carries 120 marks. You can answer as much as you can. The maximum you can score is 100.

(ii) "Markov Chain" will mean a discrete time parameter, time homogeneous Markov Chain.

(iii) The symbols have their usual meanings.

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(1) Define the following:

(a) Irreducible Markov Chain.
(b) Recurrence and Transience.
(c) Hitting Time of a state.
(d) First passage probabilities. (4×5)=[20]

(2) Suppose we have two boxes, labeled 1 and 2, and d balls labeled 1,2,...d. Initially some of these balls are in box 1 and the remainder are in box 2. An integer is selected at random from 1,2,...d, and the ball labeled by that integer is removed from its box and placed in the other box. This procedure is repeated indefinitely with the selections being independent from trial to trial. Let X_n denote the number of balls in box 1 after the nth trial. Show that $\{X_n|n=0,1,2...\}$ is a Markov Chain. Find its state space and transition matrix. [20]

(3) State and prove the Basic Renewal Equation for a Markov Chain. [20]

(4) Show that $P_{ij}(s) = \delta_{ij} + F_{ij}(s)\, P_{jj}(s)$ for $i, j \in S$ and $|s| < 1$ [20]

(5) Show that if i is recurrent and i communicates with j, then j is also recurrent. [20]

(6) Suppose $\{X_n, n=0,1,2...\}$ is a M.C. with state space S, transition matrix $P = \left((p_{ij})\right)_{i,j\in S}$ and initial distribution $\pi = (\pi_i|i\epsilon S)$. Show that

(a) $P(X_0 = i_0, X_1 = i_1, \ldots, X_n = i_n) = \pi(i_0)p_{i_0 i_1}p_{i_1 i_2}\cdots p_{i_{n-1} i_n}$

(b) $P(X_0 = i_0|X_1 = i_1, \ldots, X_n = i_n) = P(X_0 = i_0|X_1 = i_1)$ whenever the L.H.S is defined. (10+10)=[20]

INDIAN STATISTICAL INSTITUTE

Second End-Semester Examination (2017 – 2018)

Course Name	:	**M.Tech (QR & OR)** *I year*
Subject	:	**Industrial Engineering and Management**
Date	:	**20/04/2018**
Maximum Marks	:	**60**
Duration	:	**180 Minutes**

Question Paper

Direction: **Question 1 is compulsory.** Attempt ***Any two*** questions from rest.

1. (a) Explain the role of a finance manager of an organization.

   (b) Prepare a typical income statement of an organization. What do you mean by current assets and non-current assets?

   (c) How does material handling equipment improve the performance of a manufacturing organization?

   (c) ABC Co. is considering a potential project with the following forecasts:

Description	**Now**	T_1	T_2	T_3
Initial Investment (Rs million)	(1000)			
Disposal proceeds (Rs million)				200
Demand (millions of units)		5	10	6

T_1, T_2, T_3 have their usual meanings.

The initial investment will be made on the first day of the new accounting period.

The selling price per unit is expected to be Rs 100 and the variable cost Rs 30 per unit. Both of these figures are given in today's term.

Tax is paid at 30% after one year of the concerned accounting period.

Working capital, that will be required, is equal to 10% of annual sales. This will need to be in place at the start of each year.

WDA's are available at 25% reducing balance.

The company has a real required rate of return of 6.8%.

P.T.O

General inflation is predicted to be 3% p.a. but the selling price is expected to inflate by 4% and the variable costs by 5% p.a.

Determine the NPV of the project. **[5+ (2 ½ +5) +2 ½ +15]**

2. (a) Discuss the 3 E's of the time value for money.

(b) Define 'Annuity". Prove the following relation:

$$AF = \frac{1 - (1+r)^{-n}}{r}$$

where, AF, n, and r are as annuity factor, number of years, and interest rate, respectively.

(c) How do you estimate IRR? Explain with an example. **[5 + (1+4) + (3+2)]**

3. (a) A company XYZ has a demand of 45,000 components p.a. The cost of each component is Rs 4.50. There is no lead time between order and delivery, and ordering cost is Rs 100 per order. The annual cost of holding one component in inventory is estimated to be Rs 0.65.

A 0.5% discount is available on orders of at least 3,000 components and a 0.75% discount is available if the order quantity is 6000 components or above.

Calculate the optimal order quantity.

(b) What do you mean by 'risk' and 'uncertainty' in financial management?

(c) Briefly describe the role of probability modeling in evaluating the financial risk of a project?

[10 + (1+1) + 3]

4. (a) A time study of an assembly line worker in a car plant produced the following results: Cycle time of 3.0 minutes; worker performance rating of 90 percent; average allowance for the activity 10 percent of the normal time.

Calculate the standard time.

(b) How are "Motion and Time Study" and "Wage Incentives" related?

(c) Write explanatory notes on 'conveyors'. **[5+5+5]**

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2017-18

Course Name: M. Tech. (QROR) I Year

Subject Name: Statistical Quality Control

Date: 23/04/2018 Maximum Marks: 100 Duration: 3 hours

[All Symbols Have Usual Meaning]

Answer All Questions.

1. a) What's the role of OC curve in $\bar{x}$ and R charts?

   b) Prove that for an $\bar{x}$-chart if the mean shifts from the in-control value μ_0 to another value $\mu_1 = \mu_0 + \delta\sigma$, then, $\beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n})$.

   c) Prove that ARL $= \frac{1}{1-\beta}$ for an out-of-control process.

   d) What are the assumptions for constructing a sloping $\bar{X} - R$ control chart?

   e) Prove that under the assumptions in d) above, the parameters for estimating the central trend line are $\alpha = \bar{\bar{x}}$ and $\beta = \frac{\sum_{i=1}^{m} h_i \bar{x}_i}{\sum_{i=1}^{m} h_i^2}$, where α is the intercept and β is the slope of the straight line equation and h, an odd number, is the revised subgroup numbers with $\sum_{i=1}^{m} h_i = 0$.

   [2+4+4+3+7=20]

2. A paper mill uses a control chart to monitor the imperfection in finished rolls of paper. Production output is inspected for twenty days, and the resulting data are shown below. Use these data to set up an appropriate control chart for nonconformities per roll of paper. Does the process appear to be in statistical control? What center line and control limits would you recommend for controlling current production?

P. T. O

Data on Imperfections in Rolls of Paper

Day	Number of Rolls Produced	Total Number of Imperfections	Day	Number of Rolls Produced	Total Number of Imperfections
1	18	12	11	18	18
2	18	14	12	18	14
3	24	20	13	18	9
4	22	18	14	20	10
5	22	15	15	20	14
6	22	12	16	20	13
7	20	11	17	24	16
8	20	15	18	24	18
9	20	12	19	22	20
10	20	10	20	21	17

[20]

3. a) Define repeatability, reproducibility and accuracy associated with measurement error.

   b) State the standard criterion for evaluating the measurement process with respect to its percentage error.

   c) Twenty units of a product are measured by an instrument by three operators. Each unit of the product has been measured twice by each operator. The USL and LSL of the product are 60 and 5 respectively. It has been found that $\bar{x}_1 = 22.30, \bar{R}_1 = 1.00$; $\bar{x}_2 = 22.28, \bar{R}_2 = 1.25$; $\bar{x}_3 = 22.60, \bar{R}_3 = 1.20$. Calculate *P/T* ratio and conclude about the measurement process.

[6+4+10 = 20]

4. a) A company has been asked by an important customer to demonstrate that its process capability ratio C_p exceeds 1.33. It has taken a sample of 50 parts and obtained the point estimate $\hat{C}_p = 1.52$. Assume that the quality characteristic follows a normal distribution. Can the company demonstrate that C_p exceeds 1.33 at the 95% level of confidence?

   b) The molecular weight of a particular polymer should fall between 2100 and 2350. Fifty samples of this material were analyzed with the results $\bar{x} = 2275$ and $s = 60$. Assume that molecular weight is normally distributed. Calculate a point estimate of C_{pk} and a 95% confidence interval on C_{pk}.

   c) In designing a fraction nonconforming control chart with center line at $p = 0.20$ and three-sigma limits, what is the sample size required to yield a positive lower control limit? What is the value of n necessary to give a probability of 0.50 of detecting a shift in the process to 0.26?

[6+ (3+5) + (3+3) = 20]

P. T. O

5. Prove that the economic centering of a process following normal distribution is $\mu = \frac{U+L}{2} + \frac{\sigma^2}{U-L} \ln \frac{C_1 + C}{C_2 + C}$, where U and L are upper and lower specification limits, C is the profit per unit for within specification production and C_1 and C_2 are losses per unit due to producing under specification and over specification respectively.

[20]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2017 – 2018
M.Tech (QROR), First Year (E-Stream)

Statistical Methods – II

Date: 27.04.2018 Maximum Marks: 100 Duration: 3 Hours

Note: ***Answer all the questions. Maximum you can score is 100.***
All calculations are to be rounded to 3 decimal places.
Notations and symbols used are as in the class.

1. a) Write down the linear statistical model for experimental design involving one factor where the factor has "a" levels chosen by the experimenter. Explain the model terms and the assumptions, if any.
   b) Write down the hypothesis to be tested in an one-way ANOVA for the model in (a) and show that under the null hypothesis mean square treatment (MS_T) is an unbiased estimate of σ^2.
   c) Show that the error mean square (MS_E) is the pooled estimate of the group variances.

   [(2+2) + (2+8)+6=20]

2. a) What is repeated measures deign of experiment and when it is used.?
   b) An experiment was carried out to test the effect of four memory drugs. 5 people were selected and everyone in the study tried all the four drugs and took a memory test after each one. The data, i.e. result of the memory test, are given below.

Subject	Drug 1	Drug 2	Drug 3	Drug 4
1	30	28	16	34
2	14	18	10	22
3	24	20	18	30
4	38	34	20	44
5	26	28	14	30

   Does memory drugs affect the performance in memory test? Use 5% level of significance.

   [(2+2) + 16 =20]

3. a) Consider a simple linear regression model

$$y_i = a + bx_i + \epsilon_i, \qquad i = 1,2,\cdots,n$$

with $E(\epsilon_i) = 0$, $Var(\epsilon_i) = \sigma^2$ and $Cov(\epsilon_i \epsilon_j) = 0$ for $i \neq j$.

   i) Find the least square estimators of the model parameters.
   ii) Further show that these estimators are minimizer of the sum of squared residuals.

b) Show that

i) $Var(\hat{b}) = \frac{\sigma^2}{S_{xx}}$ and *ii)* $E(SS_R) = \sigma^2 + b^2.S_{xx}$.

[(7+3) + (5+5)=20]

4. An engineer is studying the effect of cutting speed on the rate of metal removal in a machining operation. However, the rate of metal removal is also related to the hardness of the test specimen. Five observations are taken at each cutting speed. The amount of metal removed (y) and corresponding hardness of the specimen (x) are shown in the following table. Analyse the data suitably. Does cutting speed affect the average metal removal? Use $\alpha = 0.05$.

Cutting Speed					
1000		**1200**		**1400**	
y	x	y	x	y	x
68	120	112	165	118	175
90	140	94	140	82	132
98	150	65	120	73	124
77	125	75	125	92	141
88	136	85	133	80	130

[20]

5. a) Assuming that the least square estimator of the regression coefficient vector $\boldsymbol{b}$ for a multiple linear regression model involving p regressor variables $x_1, x_2, \cdots, x_p$ and $n\,(>p)$ observations, $(x_{i1}, x_{i2}, \cdots, x_{ip}, y_i)$, $i = 1,2,\cdots,n$, is given by $\hat{\boldsymbol{b}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$, where $\boldsymbol{y}$ is a column vector of order n, $\boldsymbol{X}$ is the design matrix of order $(n \times (p+1))$ and $\boldsymbol{b}$ is column vector of order $p+1$, show that

i) $E(\hat{\boldsymbol{b}}) = \boldsymbol{b}$, *ii)* $var(\hat{\boldsymbol{b}}) = \sigma^2(X^TX)^{-1}$

b) Nine experts rated four brand of Columbian Coffee. A rating in a 7-point scale (1 = extremely unpleasing, 7 = extremely pleasing) is given for each of the four characteristics: taste, aroma, richness and acidity. The table in the following page displays the summated ratings, i.e. total of the above four ratings.

Expert	Brand			
	A	B	C	D
1	24	26	25	22
2	27	27	26	24
3	19	22	20	16
4	24	27	25	23
5	22	25	23	21
6	26	27	24	24
7	27	26	22	23
8	25	27	24	21
9	22	23	20	19

At 0.05 level of significance, use appropriate non-parametric test to determine whether the four brands of Columbian Coffee have different median summated ratings?

[(4+6) + 10=20]

6. a) Given below two samples, of sizes 10 and 8 respectively, collected from two continuous distributions. Can it be assumed that both samples are drawn from the same distribution? Use Kolmogorov-Smirnov Test at 0.05 level of significance.

Sample 1	1.2	1.4	1.9	3.7	4.4	4.8	9.7	17.3	21.1	28.4
Sample 2	5.6	6.5	6.6	6.9	9.2	10.4	10.6	19.3		

b) Under the assumption that mean Life Expectancy (LE) at birth will depend upon the Gross National Product per capita (GNP), an indicator of average income level of a country, 10 countries are randomly selected from a list of countries and data on LE and GNP were collected. Following table gives the LE and GNP for those ten countries.

Country	GNP	LE
Algeria	2360	65
Nepal	340	59
Mongolia	780	62
El Salvador	940	64
Ecuador	1120	66
Malaysia	1940	74
Ireland	7750	74
Argentina	2520	71
France	16090	76
Sierra Leone	240	47

Use Spearman's Rank Correlation Test to determine whether correlation between GNP and LE is statistically significant at the 5% level of significance.

[10 + 10=20]