INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2017-18
Course: Masters in Quantitative Economics Year I
Subject: Microeconomics I

Date: 04.09.17 **Maximum Marks**: 40 **Duration:** 3 hours

Answer all questions

1. Prove that, for a tight deterministic demand function, the Weak Axiom of Revealed Preference is equivalent to Samuelson's Inequality. State and establish the corresponding demand theorem from this result. (10 marks)

2. (a) Construct a tight (non-degenerate) stochastic demand function which satisfies (stochastic) non-positivity of the own-price substitution effect but violates stochastic substitutability. (5 marks)

   (b) Consider two competitive firms facing identical price vectors. Construct a (deterministic) supply function for each firm such that each firm individually violates the (deterministic) law of supply, but the aggregate stochastic representation satisfies the law of supply in its stochastic version. (5 marks)

3. (a) Suppose the universal set of alternatives is $X = \{a, b, c, d, e\}$. Construct a (non-degenerate) stochastic choice function F defined over $Z = \{\{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c, d\}, \{a, b, c, e\}\}$ which satisfies contraction consistency but violates the Weak Axiom of Stochastic Revealed Preference. (5 marks)

   (b) Prove that, if a binary preference relation is an ordering, then a choice function generated by it must satisfy Sen's α and β conditions. Construct an example of a choice function that satisfies the first but not the second. (5 marks)

4. Prove, for a deterministic supply function, that the Consistent Firm Choice and Non-reversibility conditions are, together, equivalent to the Weak Axiom of Profit Maximization. Prove also that Consistent Firm Choice and Non-reversibility are independent conditions. (10 marks)

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Mid-Semestral Examination : 2017-18

Course Name: M.S. (Q.E.) I YEAR
Subject Name: Game Theory I
Date: 5-09-2017 Maximum Marks: 50 Duration: 3 hours

Problem 1. Consider a Bayesian game Γ where the set of types for each agent is finite. Suppose that for all type profile $(t_1, \ldots, t_n)$, the complete information game induced by $(t_1, \ldots, t_n)$ has a pure strategy NE. Then, justify each of the following statements with a proof or a counter example.

(a) There exists a prior belief for each $i \in N$ such that Γ has a BNE.

(b) For all prior beliefs of each $i \in N$, Γ has a BNE.

(10)

Problem 2. Justify your answer with a proof or a counter example.
Every correlated equilibrium of a zero-sum game is an MNE. (5)

Problem 3. Let $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$. Consider the following utility functions of a player $i \in N$ defined on mixed strategies:

$U_i^{\max}(m) = \max\{u_i(s) \mid m(s) > 0\}$ for all $m \in \Delta S$,
$U_i^{\min}(m) = \min\{u_i(s) \mid m(s) > 0\}$ for all $m \in \Delta S$, and
$U_i^{avg}(m) = \frac{U_i^{\min}(m) + U_i^{\max}(m)}{2}$ for all $m \in \Delta S$.

Define the mixed strategy extension of G as $\hat{G} = \langle N, (M_i)_{i \in N}, (U_i)_{i \in N} \rangle$, where for all $i \in N$, M_i is the set of mixed strategies of i and U_i is the utility function of player i. Then, justify each of the following statements with a proof or a counter example.

(a) There exists an MNE of $\hat{G}$ when $U_i = U_i^{max}$ for all $i \in N$,

(b) There exists an MNE of $\hat{G}$ when $U_i = U_i^{min}$ for all $i \in N$, and

(c) There exists an MNE of $\hat{G}$ when $U_i = U_i^{avg}$ for all $i \in N$.

(15)

Problem 4. Justify each of the following statements with a proof or a counter example.

1. Every symmetric bidding strategy in a BNE of a first price auction is monotone.
2. Every monotone bidding strategy in a BNE of a first price auction is symmetric.
3. There exists a non-monotone bidding strategy in a BNE of a first price auction.

(10)

Problem 5. Consider a first price auction with $N = \{1, 2\}$, where the set of possible valuations (types) of each player is $\{1, 2, 3\}$ and the set of admissible bids (actions) for each player is $\{0, 1, 2\}$. Suppose that the beliefs of the players are independently and identically distributed according to uniform distribution. Then, justify each of the following statements with a proof or a counter example.

1. There exists a monotone and symmetric BNE of this auction.
2. Every equilibrium of this auction is monotone.

(10)

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M.S.Q.E. 1st Year : 2017-2018
Mid-Semester Examination
Subject: Mathematical Methods

Date: 06/09/2013 Time: 3 hours Marks : 100

Answer Group-A and Group-B on separate answer scripts.

Group-A

1. Show that the determinant of the *Vandermonde matrix*

$$A_n = \begin{bmatrix} a_1^{n-1} & a_1^{n-2} & \cdots & a_1 & 1 \\ a_2^{n-1} & a_2^{n-2} & \cdots & a_2 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_n^{n-1} & a_n^{n-2} & \cdots & a_n & 1 \end{bmatrix}$$

is $\prod_{1 \le i < j \le n}(a_i - a_j)$. [12]

2. Let A, B, C are arbitrary matrices for which matrix multiplication is defined. Then show that $A \cdot (B \cdot C) = (A \cdot B) \cdot C$. [5]

3. If an $n \times n$ square matrix A has a left inverse B and a right inverse C, then $B = C$. [5]

4. Construct two matrices A and B, such that $AB \neq BA$. [5]

5. Construct a singular matrix A, such that $A^2 = A$. [5]

6. Prove or disprove : If A is not the zero matrix and $AB = AC$, then $B = C$. [6]

7. Let A be an $n \times n$ invertible matrix with $n > 1$. Show that :

   (a) $\det(\text{adj } A) = (\det A)^{n-1}$.

   (b) $\text{adj}(\text{adj } A) = (\det A)^{n-2} A$. [4 + 8]

Group-B

1. Find cardinality of the following sets, with justification :

   (a) Set of all prime numbers.

   (b) Set of all real numbers in the closed interval $[-1.5, 11.2]$.

   (c) Set of real roots of the family of quadratic equations $ax^2 + bx + c$, where a, b, c are rational numbers. [10 + 5 + 15]

2. Describe the limiting behaviour of the following sequences as $n \to \infty$:

   (a) $\frac{\log n}{n}$.

   (b) $\frac{2n^2+1}{5n^2+4}$.

   (c) $\frac{n}{\sqrt{(n+1)}}$. [8 + 6 + 6]

INDIAN STATISTICAL INSTITUTE
MS (QE) I Year: 2017-2018

Mid-semester Examination (First Semester)

Statistics

Date: 07/09/17 Full Marks: 50 Duration: $2\frac{1}{2}$ hours.

Note: Answer all the questions.

1. (a) Suppose $P(A \cup B) = 0.6$ and $P(A \cup B^c) = 0.8$. Find $P(A)$.
   (b) If the events A and B are independent then show that A^c and B are also independent.

   [4+3 = 7]

2. An insurance company believes that people can be divided into two classes - those who are prone to have accidents and those who are not. The data indicate that an accident-prone person will have an accident in a 1-year period with probability 0.1 and the probability that a non-accident-prone person will have an accident in a 1-year period is 0.05. Suppose that the probability is 0.2 that a new policyholder is accident-prone. (a) What is the probability that a new policyholder will have an accident in the first year? (b) If a new policyholder has an accident in the first year, what is the probability that he or she is accident-prone?

   [4+3 =7]

3. Suppose X has a Poisson distribution and

$$P(X = 2) = 2P(X = 1).$$

   (a) Find the mean of the distribution.
   (b) Find $E\left[\frac{1}{X+1}\right]$.

   [2+5 = 7]

4. A random variable X has probability density function proportional to x^{-5}, for $x > 1$. Find $P[2 < X < 3]$.

   [5]

5. Suppose $U \sim U[0, 1]$. Find the median of $X = -\theta \ln(1 - U)$, $\theta > 0$.

   [5]

P.T.O.

6. Suppose X follows binomial distribution with parameters n and p. Derive the moment generating function of $Y = n - X$ and hence identify the distribution of Y.

[4+2 = 6]

7. Suppose $X \sim N(\mu, \sigma^2)$. Then

   (a) show with the help of momemnt generating function that all odd order central moments of X are zero [Moment generating function of X is $\exp(\mu t + \frac{t^2\sigma^2}{2})$].

   (b) find the probability density function of $Y = X^2$.

   (c) if $\mu = 200$ and $\sigma^2 = 25$, find a constant $c > 0$ such that $P(|X - 200| \leq c) = 0.95$.

[3 + 6 + 4 = 13]

Indian Statistical Institute
Mid-Semester Examination 2017
Course Name: MSQE First Year
Subject: Basic Economics

Date: 8.9.2017 | **Maximum Marks 40** | **Duration 2 Hours**

1. This is a problem in national income accounting. The following information is given about a hypothetical economy in a given period. The economy produced machinery, equipment and construction worth Rs.60,000 crore and the firms could sell all the consumption goods they produced. All the raw materials produced in the economy was fully used up as intermediate inputs in the same period. All imports consisted of machinery and equipment worth Rs.10000 crore, depreciation in the economy was Rs.8000 crore. The households' expenditure on produced goods and services was Rs.90000 crore of which Rs.10000 crore was spent in buying houses from the construction companies. Government's budget deficit was Rs.500 crore. The government collected Rs.2000 crore as direct and indirect taxes. The government did not pay any subsidy. Public sector enterprises' profit was zero. The government made transfer payments of Rs.600 crore and interest payment of Rs.100 crore. Trade deficit in the economy was given to be, Rs 2000 crore. Derive the values of C, I, G, X and M of the economy and, therefrom, compute the values of GDP and NDP of the economy in the given period. [23]

2. a) Explain the concepts of current account balance, capital account balance and official reserve settlement balance.

b) Suppose that in an economy in a given year the stock of foreign exchange of the central bank declined by Rs.15,000 crore to accommodate the balance of payments deficit at the prevailing exchange rate. The net inflow of foreign loan to the domestic sector was Rs.25000 crore. In addition foreigners purchased shares of domestic companies worth Rs.10000 crore. Domestic economic agents did not buy any foreign financial asset. Net investment in the domestic sector was Rs.80000 crore. Business transfers and net foreign transfers were zero, G = Rs.40000 crore, C=Rs.90000 crore, net indirect tax = Rs.20000 crore. Find out current account balance, capital account balance, official settlement balance and national income. [23]

Indian Statistical Institute
Mid-Semester Examination
Course name: MSQE-1st Year (2017)
Subject: Computer Programming and Applications
Date 08.09.2017 Full Marks 50 Time: 2 Hrs

(Answer all questions)

1. What is a Flow chart? Draw a Flow chart to find the sum of the following series for given values of x and k. [2+10]

$$1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\dots\dots\dots+\frac{x^k}{k!}$$

2. Draw a Flow chart to print the first N terms of the Fibonacci sequence. [10]

3. Write the algorithm of the Bubble sort technique. [10]

4. (a) What are the differences between while loop and do while loop in C programming language. [4]
   (b) (i) Convert $(1010111101)_2$ to Hexadecimal [2+2]
   (ii) Convert $(60D)_{16}$ to Octal

5. Which of the followings are invalid variable names in C and why? [4]
   ∑ float; character; Rs_vs_US*; int_type; msqe-2017; Double;

6. Write the output of the following program when executed. [6]

```
#include <stdio.h>
int x=0;
void main(){
        int x =0, k;
        for(k=0; k <=5; k++){
                x++;
                function1();
                function2();
        }
        printf("Value of x = %d\n", x);
}
void function1(){
        static int x=0;
        printf ("Value of x in function1 is =%d\n", x);
        x++;
}

void function2(){
        printf ("Value of x in function2 is =%d\n", x);
        x--;
}
```

Indian Statistical Institute
First Semester Examination 2017 - 18
Course Name: MSQE First Year
Subject Name: Basic Economics

Date: 20.11.2017 | Maximum Marks: 60 | Duration: 2.5 Hours

Answer all the following questions

1. Suppose the government cuts down subsidy and uses the resulting increase in saving to buy fighter planes from the USA. Examine the impact of this policy on the GDP of the domestic economy in the framework of the Simple Keynesian Model. [22]

2. Suppose the domestic economy is in recession. The gap between the current (equilibrium) GDP and the one that would have obtained in the absence of the recession is 1600 units. The RBI plans to tackle this recession by means of a monetary policy of reducing the repo rate. Sensitivity of investment with respect to the lending rate to the investors charged by the banks is – 40 units. By how much should the lending rate be changed from its existing level of 6 percent to eliminate the recession? In the light of this result, comment on the efficacy of the monetary policy in tackling the recession. [22]

3. Suppose the CRR and the currency-deposit ratio in an economy are 0.25 and 0.5 respectively. Start with an initial equilibrium situation, where the banks are fully loaned up. The economy has a fixed exchange rate regime. Now suppose the central bank buys 100 units of foreign exchange from the market. In the light of the information given above, answer the following questions:

(a) Show the resulting changes in the stocks of high-powered money and money supply, when the amount of excess demand for bank credit existing in the initial equilibrium situation is adequate for the money multiplier process to work out fully.

(b) How will your answer to (a) change, if the excess demand for credit in the initial equilibrium situation were 40 units?

(c) How will your answer to (b) change, if the change in money supply were due to a commercial bank having taken a loan of 60 units from the central bank? [7+7+8]

Note on Q2 :- Assume that the marginal propensity to spend on domestically produced goods = $\frac{1}{2}$

Indian Statistical Institute
Semester Examination
Course name: MSQE-1st Year (2017)
Subject: Computer Programming and Applications
Date 20.11.2017 Full Marks 100 Time: 3 Hrs

(Answer all questions)

1. Draw a Flow Chart to find the sum of the following series for given values of x and k

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\ldots\ldots\ldots\ldots(-1)^k+\frac{x^{2k}}{(2k)!}$$

[10]

2. Given two one-dimensional arrays A and B which are sorted in ascending order. Write a C program to merge these two arrays into a single sorted array C (without using sorting algorithm) that contains every item from array A and B in ascending order.

[10]

3. Write a C program to compute the frequencies of different English lower case characters in an input file INPUT (the input text file INPUT contains upper case letters, lower case letters and punctuation symbols).

[10]

4. Find the output of the following program and justify your answer.

```
#include<stdio.h>
int x=20, k;
int func1();
int func2();
void func3();

int main() {
k= func1();
printf("Here value of funcl is = %d\n",func1());
func2();
printf("Value of x after func2 = %d\n", func2());
func3(&k);
printf("After func3 value of x = %d and value of k= %d\n",x,k);
}

int func1(){
static int x =0;
x=x+2;
return x;
}

int func2(){
int x =0;
```

P.T.O

```
x=x+2;
return x;
}

void func3(int *p) {
x=x+2;
*p = *p +1;
}
```
[8]

5. A is an M X N matrix and B is an N x R matrix. Write a C program to compute the product AB. [Read M, N, and R as well as the values of the matrix as input from keyboard] [10]

6. Write the differences between (a) malloc and calloc functions (b) call by value and call by address in C. [8]

7. (a) Write the postfix and prefix notations of the following infix expression
   A+B^C*(D+(E-F)/G)-H
   (b) Convert $(1010111101001)_2$ to Hexadecimal. [3+3+2]

8. (a) Write the algorithm to delete the first node of a Singly linked-list. [6]
   (b) Write the algorithms for PUSH and POP operations of STACK using array. [4+4]

9. Write the algorithm for Binary search technique. [6]

10. Write a program (using bubble sort technique) to sort a one dimensional array of N elements. [8]

11. Write short notes (any 2)
    (a) Switch statement (b) Recursion function (c) Continue statement [4+4]

INDIAN STATISTICAL INSTITUTE

End-Semestral Examination: 2017-18

Course: Master's in Quantitative Economics Year I

Subject: Microeconomics I

Date: 22nd November, 2017 **Maximum Marks:** 60 **Duration:** 3 hours

Answer all questions. Students may consult their notes or study material, but not each other.

1. Consider a competitive firm producing according to a production function:

$$A = F(K, L),$$

where A represents the amount of output produced, K represents the amount of fixed input (capital), and L represents the amount of variable input (labour). The quantity of capital cannot be changed in the short run, but the (non-storable) output and variable input can be readily bought and sold at prices p and w, respectively. Future p can be either $\overline{p}$ or $\underline{p}$ with equal probability, $\overline{p} > \underline{p} > 0$. However, w is known with certainty. The firm chooses L to maximize expected utility, on the basis of a VNM utility function $u = \pi^{\alpha}$, $\alpha > 0$. How do expected profit, output and employment change with an increase in α? Interpret your findings. What do your results imply for the direction of capital flow? **(20 marks)**

2. Consider a household endowed with $\bar{l}$ amount of labour, which it has to allocate between urban employment and rural employment, so as to maximize expected utility. The household's indirect utility function is given by: $Eu(m) = \overline{m} - \frac{kVar_m}{2}; k > 0$; where $\overline{m}$ is mean household income and Var_m is the variance of household income. The rural wage rate is given by a function $w_r = \frac{z}{l_r}$, $z > 0$, where l_r is the amount of household labour allocated to rural occupations. The urban wage rate can be either W or 0 with equal probability. Show how the extent of urban migration changes with changes in (i) $\bar{l}$, (ii) W (iii) z and (iv) k. Explain your results. **(20 marks)**

3. Suppose that an individual's initial wealth is Rs 100. Consider a lottery which provides either Rs 21 with probability 1/3 or Rs 44 with probability 2/3. Find the certainty equivalent and the risk premium associated with this lottery, when the individual's utility function over monetary amounts is (a) $u = x^{1/2}$, (b) $u = x^2$ and (c) $u = x$. **(20 marks)**

Indian Statistical Institute
First Semester Examination : 2017–2018

M.S.Q.E. 1st Year
Subject: Mathematical Methods
Date: 24 /11/2017 Marks: 100 Time: 3 hours

Notations used are as explained in the class.
Answer Group-A and Group-B on separate answer scripts.

Group-A

1. For any two matrices A and B for which AB can be defined, show that

   (a) $\mathcal{N}(AB) \supseteq \mathcal{N}(B)$,
   (b) $\mathcal{N}((AB)^T) \supseteq \mathcal{N}(A^T)$,
   (c) $\mathcal{C}(AB) \subseteq \mathcal{C}(A)$,
   (d) $\mathcal{R}(AB) \subseteq \mathcal{R}(B)$. [10]

2. Prove that a set of vectors $\{x_1, x_2, \ldots, x_m\}$ in a vector space V is linearly dependent if and only if at least one of the vectors in the set can be written as a linear combination of the others. [5]

3. Let $T : V \to W$ be a linear transformation from a vector space V to a vector space W. Then prove that the kernel $Ker(T)$ and the image $Im(T)$ are subspaces of V and W, respectively. [5]

4. Let $T : V \to W$ be a linear transformation from a vector space V to a vector space W. Prove that, T is one-to-one if and only if $Ker(T) = \{0\}$. [5]

5. If x and y are vectors in an inner product space V, then prove that $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$. [5]

6. Let A be an $n \times n$ matrix. Then prove that

   (a) the determinant of A is the product of the n eigenvalues, and
   (b) the trace of A is the sum of the n eigenvalues.

   [9]

7. Find two matrices A and B such that $det(A) = det(B)$, $tr(A) = tr(B)$, but A is not similar to B. [6]

8. Show that a real symmetric $n \times n$ matrix A is positive definite if and only if all the eigenvalues of A are positive. [5]

Group-B

Answer any five questions, each question carries 10 marks.

1. Find the cardinality of the set of real roots of the family of quadratic equations $ax^2 + bx + c$ where a, b, c are rational numbers. [10]

2. Given x, an irrational number, prove that for every $\epsilon > 0$, the interval $(x-\epsilon, x+\epsilon)$ contains a rational number. [10]

3. Let $a_0 = 1$, $a_1 = 1$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$. Discuss convergence of sequence $\frac{a_{n+1}}{a_n}$. [10]

4. Using ϵ-δ method, find $\lim_{x \to 2}(x^2 + x - 4)$. [10]

5. Discuss the continuity of the following function.

$$\begin{aligned} f(x) &= +1 \quad \text{if } x > 0 \\ &= 0 \quad \text{if } x = 0 \\ &= -1 \quad \text{if } x < 0. \end{aligned}$$

[10]

6. let $g : \mathbb{R}^+ \to \mathbb{R}^+$ be defined as $g(x) = \sqrt{x}$. From first principles, find derivative of g at $x = 2$. [10]

7. A student wishes to allocate her available time of 60 hours per week between 2 subjects in such a way as to maximize her grade average. Let g_i functions give the expected grade as a function of study time $g_1 = 20 + 20\sqrt{t_1}$, $g_2 = -80 + 3t_2$, where t_i is the time spent studying subject $i (= 1, 2)$. Find out how she would allocate t_1 and t_2. [10]

1. Find the cardinality of the set of real roots of the family of quadratic equations $ax^2 + bx + c$ where a, b, c are rational numbers. [10]

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INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

MS (QE) I Year: 2017-2018

Statistics

Date: 27/11/17 Maximum Marks: 100 Duration: 3 hours.

Note: This paper carries 110 marks. However, maximum you can score is 100.

1. (a) State and prove the theorem of total probability. [2+6 =8]

   (b) The probability that a machine produces a defective item is 0.01. Each item is checked as it is produced. Assume that these are independent trials. Find the expected number of items to be checked to get the first defective item. [7]

   (c) Suppose the waiting time of a customer in a queue has an exponential distribution with mean waiting time 25 minutes. What is the probability of waiting for another 10 minutes given that the customer has already waited for 20 minutes? [5]

2. (a) Let X_1 and X_2 be two independent random variables with repective means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. Find the variance of $Y = X_1X_2$. [5]

   (b) Suppose X_1 and X_2 are two independent Poisson random variables with respective parameters λ_1 and λ_2. Find the conditional distribution of X_1 given $X_1 + X_2 = t$. [6]

   (c) Suppose X and Y are two random variables with $\text{Var}(X) = 2$, $\text{Var}(Y) = 4$ and $\text{Cov}(X, Y) = -1$. Find the correlation coefficient between $3X + 1$ and $2Y - 8$. [4]

3. The joint pmf of X and Y is

$$P(X = x, Y = y) = \frac{x+1}{12}, \quad x = 0, 1 \text{ and } y = 0, 1, 2, 3.$$

   (a) Find the marginal distributions of X and Y. [6]

   (b) Find $P(X + Y \leq 2)$. [4]

4. Suppose Z_1 and Z_2 are independent standard normal variables. Let $X = Z_1$ and $Y = \rho Z_1 + \sqrt{1-\rho^2}Z_2$, where $-1 \leq \rho \leq 1$. Find the joint density function of X and Y. Identify the distribution you have derived. [8+2 =10]

5. Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with the probability density function

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \ \lambda > 0.$$

   (a) Derive the maximum likelihood estimator (MLE) of λ. [3]

   (b) Find the Fisher information for λ. [3]

   (c) Find the MLE of mean (μ) of X. Show that the MLE of μ is an unbiased estimator of μ. [2+4 =6]

   (d) Find a sufficient statistic for λ. [3]

P.T.O.

6. Suppose X denotes the weight in grams of a 50-gram snack pack of peanuts. Assume that the distribution of X is $N(\mu, \sigma^2)$. A random sample of 15 peanut packs yielded the folllowing data:

$54.69, 50.90, 50.30, 49.92, 50.22, 51.03, 52.09, 49.12, 48.25, 49.17, 51.22, 52.34, 50.87, 53.25, 50.65.$

   (a) Find a 95% confidence interval for μ. [6]

   (b) What is the probability that an individual snack pack selected at random is filled with less than 50 grams of peanut? [4]

7. Given $f(x, y) = 2$, $0 \leq x \leq 1$, $0 \leq y \leq x$, find the equation of the regression curve of y on x. [5]

8. Suppose you have paired data (y_1, x_1),...,(y_n, x_n), where $y_i = \beta_0 + \beta_1/x_i + \epsilon_i$, $i = 1, \ldots, n$ and the model errors $\epsilon_1, \ldots, \epsilon_n$ are uncorrelated and have mean 0 and variance σ^2.

   (a) Give explicit expressions for the least squares (LS) estimators of β_0 and β_1. [4]

   (b) Prove that the two LS estimators are unbiased. [3]

   (c) Given the set of data (2,1), (3,1/2), (3,1/3), (4,1/4), (5,1/4), find the LS estimators. (Note that the first number in the pair is the value of y) [5]

   (d) Assume that the model errors have normal distribution. Describe, with justification, a test of $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 \neq 1$ with level 0.10, expressed numerically in terms of the appropriate quantile of a suitable distribution. (Clearly specify which quantile and which distribution you would use.) [6]

9. Suppose you are testing $H_0 : p = 1/2$ against $H_1 : p = 2/3$ for a binomial variable X with $n = 3$. What values of X would you assign to the critical region if you wish to have level 1/8 and you wish to maximize the power of the test? [7]

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2017-18

M.S. (Q.E.) I YEAR

Game Theory I

Date: 01.12.2017 Maximum Marks: 50 Duration: 3 hours

1. Justify your answer by a proof or a counterexample.

   (i) (a) Every 3×3 symmetric game has a pure strategy Nash equilibrium.

   (b) If a 3×3 symmetric game G does not have any pure strategy Nash equilibrium, then every Nash equilibrium of G is evolutionarily stable.

   (ii) There exists a positive discount factor δ such that the infinitely repeated Prisoner's Dilemma game with discount factor δ has an SPNE.

   [2×10]

2. Consider an auction where the bidders simultaneously submit sealed bids. The highest bid wins the object and every bidder pays the seller the amount of his/her bid. The valuation of the agents are identically and independently distributed in the interval $[0, 1]$ according to the distribution function F with density f (assume all the properties of the distribution function discussed in class).

   (i) Find a (strictly increasing) symmetric bidding function. Is it unique?

   (ii) Does this auction have a weakly dominant bidding strategy?

   [10]

3. Show that any convex combination of a finite collection of correlated equilibrium payoff profiles of a normal form game G is a correlated equilibrium payoff profile of G.

[10]

4. Consider a two-stage (two times repeated) Prisoner's Dilemma game with the pay-off matrix in the base stage as shown in the table below

1 \ 2	D	C
D	4,4	0,5
C	5,0	2,2

Find all p such that playing C in the first stage and playing $(p(C), (1-p)(D))$ in the second stage is an equilibrium of this (repeated) game.

[10]

INDIAN STATISTICAL INSTITUTE

First Semester Backpaper Examination: 2017-18

MS (QE) I Year: 2017-2018

Statistics

Date: 26.12.17 Full Marks: 100 Duration: 3 hours.

Note: Answer all questions.

1. Box A contains 10 items of which 4 are defective, and box B contains 6 items of which 2 are defective. An item is drwan at random from each box.
   (a) What is the probability that one item is defective and one not. [4]
   (b) If one item is defective and one is not, what is the probability that the defective item came from box A? [6]

2. Suppose X, Y and Z are independent Bernoulli random variables with respective parameters 1/2, 1/3 and 1/4. Find $E[e^{X+Y+Z}]$. [5]

3. Suppose $X_1 \sim \text{Binom}(n_1, p)$ and $X_2 \sim \text{Binom}(n_2, p)$, and X_1 and X_2 are indepndently distributed. Find the distribution of $X_1 + X_2$. [5]

4. Customers arrive at a travel agency at a mean rate of 5 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, find the probability that more than 4 customers arive in a given hour. [4]

5. Suppose $X \sim N(\mu, \sigma^2)$. Find the moment generating function of $Y = \left(\frac{X-\mu}{\sigma}\right)^2$. Hence identify the distribution of Y. [5+2 = 7]

6. Suppose Y have a uniform distribution $U(0,1)$. Find the distribution of $X = -\theta \ln U$, $\theta > 0$. [4]

7. Let X and Y have joint density

$$f(x, y) = 4e^{-2x}, \quad \text{for } 0 < y < x < \infty.$$

   (a) Find the conditional density of Y igiven $X = x$. [5]
   (b) Find $P(1 < Y < 2 | X = 3)$. [3]
   (c) Find the correlation coefficient between X and Y. [7]

8. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, 1)$.
   (a) Show that $\bar{X}^2 - 1/n$ is an unbiased estimator of μ^2. [4]
   (b) Find a 95% confidence interval for μ. [4]

P.T.O.

9. Let $X_1, \ldots, X_n$ be a random sample from the distribution with pdf

$$f(x) = \frac{x}{\theta^2} e^{-x^2/2\theta^2}, \quad x > 0, \ \theta > 0.$$

(a) Find a sufficient statistic for θ. [3]

(b) Show that maximum likelihood estimator of θ is a function the sufficient statistic. [5]

(c) Find the Fisher information for θ and hence asymptotic variance of the MLE of θ. [2+2 =4]

10. Prove that the function of a random variable X that is the best predictor of another random variable Y, in the sense of minimum mean squared error of prediction, is $E(Y|X)$. [5]

11. Given $\bar{X} = 28$ for a sample of 50 of a normal variable for which $\sigma = 5$, test $H_0 : \mu = 30$ against $H_1 : \mu = 29$ at level 0.05. [5]

12. Thirty samples of soil were analyzed for carbon content by two different methods. One method is quite accurate but expensive, whereas the other method is cheap but not very reliable. Calculations with the thirty pairs of x and y (cheap measurement and expensive measurement, respectively) sample values gave $\sum x_i = 60$, $\sum y_i = 90$, $\sum x_i^2 = 300$, $\sum y_i^2 = 750$, $\sum x_i y_i = 420$.

(a) Find the equation of the least squares line of y on x. [5]

(b) Assuming that the regression line should pass through the origin, find the equation of the least squares line of y on x. [5]

(c) Calculate the residual sum of squares in each of the above cases. [Hint: It is the difference between $\sum y_i^2$ and sum of squares of fitted values.] [5]

(d) Test the hypothesis that the regression line passes through the origin, against the hypothesis that it does not pass through the origin, at level $\alpha = 0.05$. [5]

INDIAN STATISTICAL INSTITUTE

Backpaper: 2017-18

Course: Master's in Quantitative Economics Year I

Subject: Microeconomics I

Date: 27.12.17 **Maximum Marks:** 100 **Duration:** 3 hours

Answer all questions.

1. Prove that a choice function satisfies Houthakker's WARP, if and only if it also satisfies Sen's α and β conditions. (20 marks)

2. Prove that Samuelson's Weak Axiom of Revealed Preference implies the Law of Demand, but that the converse is not true. (20 marks)

3. Prove that the Consistent Firm Choice and Non-reversibility conditions are, together, equivalent to the Weak Axiom of Profit Maximization. (20 marks)

4. Explain how the relative measure of (local) risk aversion may be interpreted in terms of an agent's willingness to accept gambles. (20 marks)

5. (i) Set up a simple model to show how a risk-averse agent's choice of optimal insurance depends on the insurance premium. (10 marks)

   (ii) Explain whether the following statement is true, false or uncertain: if an individual exhibits a diminishing coefficient of absolute risk aversion, she must exhibit a diminishing coefficient of relative risk aversion as well. (10 marks)

Indian Statistical Institute

Semester Examination (Back paper)

Course name: MSQE-1st Year (2017)

Subject: Computer Programming and Applications

Date 28.12.2017 Full Marks 100 Time: 3 Hrs

1. Write a C program which gets the number N as input and prints the number N, N-times, the number N-1, N-1-times, and so on. (For example if N is 5 then it prints the number 5 five times, the number 4 four times, the number 3 three times, the number 2 two times and the number 1 one time.) [10]

2. Write a C Program to find the sum of the series for given values of x and k

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots\dots\dots + \frac{x^k}{k!}$$

[10]

3. Write a C program which writes the odd digits of an input number N (>=10) in reverse order.

[10]

4. Draw a Flow chart to generate first N terms of the Fibonacci sequence.

[10]

5. Write a C program which will produce binary representation of a given integer number N. [10]

6. Write a C program to compute the H.C.F of two positive integers L and K.

[10]

7. What are the differences between (a) call by value and call by address (b) local and global variables. [4+4]

8. Convert $(11010111101)_2$ to Octal, and $(60D)_{16}$ to binary. [3+3]

9. Which of the followings are invalid variable names in C and why?

   &a; auto; char; float_type; Double; [5x2=10]

10. Write the algorithm of the Bubble sort technique. [10]

11. Write short note (i) fseek function (ii) static variable [3+3]

INDIAN STATISTICAL INSTITUTE
First Semester Examination (Back Paper): 2017-18

M.S. (Q.E.) I YEAR

Game Theory I

Date: 29.12.17 Maximum Marks: 50 Duration: 3 hours

1. Compute all correlated equilibria in the game

1/2	c	s
c	0,0	-2,1
s	1,-2	0,0

by using the definition of correlated equilibrium.

(25)

2. Justify your answer by a proof or a counter example.
Suppose that a stage game G does not have any pure strategy Nash equilibrium. Then, for every discount factor $\delta \in (0,1)$, the infinitely repeated game G^{∞} with discount factor δ does not have a pure strategy Nash equilibrium.

[25]

3. Justify your answer by a proof or a counter example.
   (i) Every evolutionarily stable strategy is a Nash equilibrium.
   (ii) Every Nash equilibrium in a symmetric game is evolutionarily stable.

[15+10]

4. (i) Find all SPNE of the infinite horizon Rubinstein bargaining problem (for arbitrary $\delta \in (0,1)$).

(ii) Find all NE and all SPNE of the infinite horizon Rubinstein bargaining problem when $\delta = 0$ and $\delta = 1$.

[15+10]

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination (2017-2018)
MS(QE) I
Microeconomics II

Date: 19.02.2018 Maximum Marks: 100 Duration: 3 hours

(1) Define rationality, continuity and monotonicity of a preference relation R on the commodity space $X \subseteq \Re^L_+$. Show that if a preference relation R on $X = \Re^L_+$ is rational, continuous and monotonic, then there exists a utility function $u : \Re^L_+ \to \Re$ that represents it. **(6+14=20)**

(2) Let R be a preference relation defined on X and let $u(.)$ be a utility function representing it. Show that R on X is strictly convex if and only if any utility function $u(.)$ representing it is strictly quasi-concave. **(10)**

(3) Suppose that $f(.)$ is the production function associated with a single-output technology, and let $Y \subset \Re^L$ be the production set of this technology. Show that Y is convex if and only if the production function $f(z)$ is concave. **(15)**

(4) Suppose that the production set $Y \subset \Re^L$ is convex. Then every efficient production $y \in Y$ is a profit maximizing production for some non-zero price vector $p \geq 0$. **(10)**

(5) Consider an economy consisting of I consumers (indexed $i = 1, \ldots, I$), J firms (indexed $j = 1, \ldots, J$) and L commodities (indexed $l = 1, \ldots, L$). Each consumer i is characterized by a consumption set $X_i \subset \Re^L$ and a *rational* preference relation R_i defined on X_i. Each firm j has the production possibilities summarized by the production set $Y_j \subset \Re^L$. We assume that Y_j is non-empty and closed. The initial resources of commodities in the economy, that is, the economy's endowments are given by a vector $\hat{\omega} = (\hat{\omega}_1, \ldots, \hat{\omega}_L) \in \Re^L$. Thus, the basic data on preferences, technology, and resources for this economy are summarized by $(\{(X_i; R_i)\}_{i \in I}, \{Y_j\}_{j \in J}, \hat{\omega})$. Answer the following questions.

(a) Define Pareto optimal and weak Pareto-optimal allocations. By imposing appropriate restrictions on the preferences, state and prove an equivalence result between Pareto-optimality and weak Pareto-optimality. **(4+16=20)**

(b) State and prove the first fundamental theorem of welfare economics by giving all the relevant definitions. **(25)**

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2017-2018)
MS (Q.E.) I Year
Macroeconomics I

Date: 20·02·2018 Maximum Marks 40 Duration 3 hours

Group A

Answer the following.

1. Consider the following one-period competitive equilibrium representative agent model. There is a representative consumer, who has a utility function $u(c,l)$, where c is consumption and l is leisure. The market real wage is w, and the rental rate on capital is r. The consumer is endowed with one unit of time and k_0 units of capital. The representative agent has a production technology given by

$$y = \alpha(k + g) + n,$$

Where y is output of consumption goods, k is the capital input for the firm, g is the quantity of public goods provided by the government, n is the labour input, and $0<\alpha<1$. The firm chooses its capital input and labour input treating g as given. The government levies a lump-sum tax τ on the consumer in order to finance public goods expenditure, with $g = \tau$. Public goods are like roads and bridges, i.e. they are public capital that generates private output.

(a) Assume that g is exogenous. Define a competitive equilibrium and derive a set of equations that solve for output, consumption, leisure, labour supply, the real wage rate and the rental rate on capital in a competitive equilibrium.

(b) Again, with g exogenous, determine the effects of an increase in g on output, consumption, leisure, labour supply, the real wage rate, and the rental rate on capital. Carefully explain your results, and what they depend on. In particular, how does the value of α matter, and why?

(c) Now, suppose that the government sets g in order to maximize the welfare of the representative consumer in a competitive equilibrium. Determine the optimal value of g and explain your results.

P.T.O

(d) Construct the Social Planner's problem for this economy. How does it differ from the competitive solution framework? Does the first welfare law hold here?

(7+5+4+4=20)

Group B

Answer all questions

1. In an appropriate new Keynesian model derive the long run multiplier of a balanced budget increase in government expenditure and show that with variety effect absent, such multiplier is smaller than what would obtain in the short run.

   (10)

2. Show that in the flexible price equilibrium that obtains in the Blanchard and Kiyotaki model, money is completely neutral. (10)

[Note: You need not derive the Dixit –Stiglitz demand functions, just use them directly.]

Economic Development

Mid-term Examination

MSQE I & II

Date: 22.2.2018

Time: 2 hours

Maximum Marks: 40

Question 1 is compulsory. Answer any one from the remaining two questions.

1. Consider a *static* economy consisting of two sectors: A and B. Labour is the only factor of production and the wage rates in the two sectors are given by

$$w_A = A_0 + aL_A$$
$$w_B = B_0 + bL_B$$

where w_A, w_B are wages and L_A, L_B are labour allocations in the two sectors. A_0, B_0, a and b are positive constants and $L_A + L_B = L$, the total labour endowment.

(a) Find all equilibrium allocations of labour and indicate which equilibria are stable.

(b) Now suppose each labour has to incur a cost c to move from one sector to another. Find all equilibrium allocations of labour and indicate which equilibria are stable in the changed situation.

[10 + 10]

2. Show that an economy with imperfect credit markets, lumpy investment cost of education, bequests and inheritance would converge to a bi-modal income distribution in the long run. How would the long run equilibrium change if credit markets were perfect?

[15+5]

3. In an economy where agents can choose to be either entrepreneurs or workers, show that there is trickle down growth if the return to investment is sufficiently high and if bequests are large enough.

[20]

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MID-SEMESTRAL EXAMINATION 2017- 18
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 23.02.2018 Maximum Marks: 50 Time: 2 hours

*This question paper carries a total of **60** marks, but the maximum that you can score is **50**. You can answer any part of any question. Marks allotted to each question are given within parentheses.*

1. Examine whether the following statements are **true** or **false** or **uncertain**. Give brief explanations in support of your answers.

   (a) If a time series is non-stationary, then the two methods *viz.*, method of removal of deterministic components and the method of differencing, lead to the same transformed series.

   (b) Weak stationarity always implies ergodicity.

   (c) Any invertible time series is necessarily stationary.

   (d) The moving average method is not an appropriate method of obtaining deterministic trend if the underlying trend in the time series is non-linear.

   [$4\times 4 = 16$]

2. Suppose that a monthly time series contains trend, seasonality and noise components. Discuss how you would obtain the trend and seasonality of the series.

   [8]

3. (a) Derive the conditions (in terms of the parameters involved) for stationarity of an AR (2) process.

   (b) Find the ACF of an MA (2) process.

   [$6 + 4 = 10$]

P.T.O

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2017-2018)

MS (Q.E.) I Year

Econometric Methods I

Date: 26.02.2018 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. You can answer any part of any question.

Question 1

In a linear regression model $Y = X\beta + \varepsilon$, β is a kx1 vector of parameters, and the disturbance term satisfies the Gauss Markov assumptions. $Var(\varepsilon) = \sigma^2 I$

(i) If you wish to carry out a joint significance test for $\beta_1, \beta_2, \beta_3$ based on F test, what should be the constraint matrix?

(ii) Subsequently, compute $E(RSS_H - RSS)$ in terms of $\beta's$. (10 + 10 =20)

Question 2

Anirjit is the postmaster at the Nischindipur post office. Three parcels have arrived at the post office today morning, which he has to weigh before they are delivered. Unfortunately, the weighing machine isn't perfectly accurate. Therefore, he adopts the following strategy. Three parcels are weighed singly, in pairs, and all together giving the weight-readings Y_{ijk} where i,j,k $\in \{0,1\}$. Clearly suffix '1' denotes the presence of the particular parcel and '0' denotes its absence. E.g Y_{101} denotes the weight reading corresponding to when the first and the third parcels are weighed together. Find the least square estimate of the individual weights of the parcels. (20)

Question 3

$Y_1 = \beta_1 + \varepsilon_1$

$Y_2 = 2\beta_1 - \beta_2 + \varepsilon_2$

$Y_3 = \beta_1 + 2\beta_2 + \varepsilon_3$

$(\varepsilon_1, \varepsilon_2, \varepsilon_3)^T \sim N(0, \sigma^2 I_3)$

Derive the F statistic for testing the hypothesis H: $\beta_1 = \beta_2$ (15)

P.T.O

Question 4

If the Gauss Markov assumptions are satisfied, Prove Cov (ε^* ,β^*) = 0, where β^* is the estimated parameter and ε^* is the fitted residual.

(15)

Question 5

Alpesh decides to throw a stone from the roof of the library building. The projectile will follow a parabolic path. Snita, standing on the ground takes snapshots of the projectile at different points of time. Is the linear regression technique suitable to estimate the path of the projectile? Is there a way of estimating the magnitude of the initial velocity with which Alpesh threw the stone and the angle of projection? Give detailed reasons.

(15)

Question 6

Consider the simple linear regression model Y = Xβ + ε, X is nxp of rank p. Var(ε) = $V \neq I$

(i) Find the projection matrix P^*_{GLS} corresponding to the GLS estimation. (such that $Y^* = X\beta^*_{GLS} = P^*_{GLS}\, Y$)

(ii) Show that P^*_{GLS} is idempotent but not necessarily symmetric.

(iii) Prove $(Y - X\beta^*_{GLS})^T\, V^{-1}\, (Y - X\beta^*_{GLS})$ follows chi square distribution with degree of freedom *(n-p)*.

(10 + 10 + 15 = 35)

Indian Statistical Institute

Mid-semester Examination 2018, MSQE I and MSQE II

Course name: **Political Economy**

Subject name: **Economics**

Date: **27 February 2018**

Maximum marks: **75**

Duration: **3 hours**

1. This question pertains to direct aggregation of individual preferences $\{\succsim_i\}_i$ over alternatives in some set X to obtain a social preference by applying the majority rule, $\succsim_{MR}$. For simplicity, you may assume $X \subseteq \Re$, discrete and finite. Either prove or give a counterexample for **any two** of the following four statements, clearly indicating whether you are proving or giving a counterexample: **(5 x 2 = 10 points)**

(i) If $\succsim_{MR}$ is intransitive over any $x, y, z \subseteq X$, then exactly one of the following must hold:
(a) $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$
(b) $y \succsim_{MR} x \succsim_{MR} z \succsim_{MR} y$.

(ii) If $\succsim_i$ is complete $\forall i$, then $\succsim_{MR}$ is complete.

(iii) If individual preferences are *single-peaked* (SP), then $\succsim_{MR}$ is transitive. That is, SP $\Longrightarrow \succsim_{MR}$ transitive.

(iv) Let X^* be the set of Condorcet winners. Then transitivity of $\succsim_{MR}$ is necessary for X^* to be non-empty. That is, $X^* \neq \phi \implies \succsim_{MR}$ transitive.

2. This question pertains to indirect aggregation of individual preferences in the context of a model of representative democracy as discussed in class. The standard assumptions of such a model remain the same, namely, there are n individuals in the economy each having well-behaved preferences over policy denoted by $\{\succsim_1, \succsim_2, \cdots, \succsim_n\}$. Moreover these preferences satisfy *Extremal Restriction* (ER). There are two candidates/parties/representatives, A and B, who try to maximize objective functions u_A and u_B. Let the objective function candidate A be as follows:

$$u_A(x_A, x_B) = \#\{i : x_A \succ_i x_B\} + \frac{1}{2}\#\{i : x_A \sim_i x_B\}.$$

P.T.O

We can define $u_B(x_A, x_B)$ likewise. Let $G := (\{A, B\}, \{X, X\}, \{u_A, u_B\})$. Consider the mixed extension of G and denote it by G'. Then show that

(i) G' has a Nash equilibrium. **(5 points)**

(ii) Any Nash equilibrium (p_A, p_B) of G' has support$(p_k) \subseteqq X^*$, $k = A, B$, where X^* is the set of Condorcet winners in X. **(15 points)**

3. Suppose preferences of individuals for a service consists of two dimensions, p_1 and p_2. Suppose there are three individuals, a 'low' consumer, a 'middle' consumer and a 'high' consumer with bliss points $(1, 1)$, $(2, 3)$ and $(4, 4)$ respectively. Suppose individuals can directly vote on alternative pairs. Prove or argue otherwise whether a 'median-voter' analogue may still hold good. That is, is it true that the $(2, 3)$ alternative will beat all other alternatives in pairwise majority voting? Argue briefly (you may use diagrams). **(5 points)**

OR

This question pertains to part of the proof of the Sen and Pattanaik theorem (1969) as discussed in class.

Assume that individual preferences satisfy *Extremal Restriction* (ER). Suppose $\exists$ an individual i with strict preference between three alternatives x, y, z, such that $x \succ_i y \succ_i z$. Moreover assume that social ordering according to the majority rule satisfies the "forward cycle", that is, $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$. Under these circumstances, which kinds of individual preferences are feasible? **(5 points)**

(Note: The assumptions mentioned above are inconsistent and as we know, will lead to a contradiction. However, this question *does not* ask you to complete the proof and reach the contradiction.)

4. This question deals with lobbying under three possible and equally likely states of the world, namely θ_L, θ_M and θ_H, $\theta_L < \theta_M < \theta_H$, as discussed by Grossman and Helpman. Consider a single policy variable and a single lobby to answer the following questions:

(i) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. Recall $\delta > 0$ to be the size of divergence between the ideal policies of the policy maker and the lobby. What is the range of δ under which a full-revelation/fully-separating equilibrium is sustainable? **(9 points)**

(ii) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. What is the range of δ under which a partial-revelation/semi-separating equilibrium is sustainable? **(9 points)**

(iii) What happens to the range of δ that can be sustained in equilibrium when you move from an equilibrium in (i) to an equilibrium in (ii)? **(2 points)**

5. This question deals with lobbying as discussed by Grossman and Helpman. They conclude that "both the policymaker and the interest group may benefit from having lobbying not be free." Consider a single interest group, a single policy variable and two possible states of the world, to elucidate the above statement. **(20 points)**

OR

This question dwells on order-restricted preferences in the context of collective choice of tax-transfer schemes. There are n individuals, with preferences defined over two goods, a consumption good and leisure; let $c \in \Re_+$ denote units of the former and $l \in \Re_+$ those of the latter. Individual i's preferences over (c, l) are represented by a utility function of the Cobb-Douglas form

$$u_i(c, l) = c^{\alpha_i} l^{1-\alpha_i}$$

where $\alpha_i \in (0, 1)$. Suppose each individual has an endowment of 1 unit of time that can be allocated to leisure and work $(h = 1 - l)$ at a wage rate $w > 0$, with the price of the consumption good normalized to 1. Assume further that the collective decision to be made is over a set of proportional tax/transfer schemes on earned income. Specifically, the set of of possible tax/transfer schemes is $X \subseteq [0,1] \times \Re$ with typical element (t, T) where $t \in [0,1]$ is a proportional tax on labor income and $T \in \Re$ is a lumpsum transfer payment. Prove or argue otherwise that individual preferences over (t, T) schemes satisfy *Extremal Restriction.* (You may assume interior solutions throughout.) **(20 points)**

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SECOND SEMESTRAL EXAMINATION (2017-18)

M.S.(Q.E.) 1st Year

Time Series Analysis & Forecasting

Date: 23.04.2018 Maximum Marks: 100 Time: 3 hours

Answer any *FIVE* questions. Marks allotted to each question are given within parentheses.

1. (i) Bring out the distinctions between trend and seasonality in a time series.

(ii) Examine, with derivations, if the following time series $\{x_t\}$ is stationary and invertible:

$$x_t + 1.9x_{t-1} + 0.88x_{t-2} = 2 + a_t + 0.6a_{t-1}$$

where $\{a_t\} \sim WN(0, 4)$. Find its mean, variance and autocovariances if these exist.

(iii) Show that an AR (p) process given as $x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + a_t$, $a_t \sim WN(0, \sigma_a^2)$, can be equivalently represented as $\Delta x_t = \varphi x_{t-1} + \sum_{j=1}^{p-1} \alpha_j^* \Delta x_{t-j} + a_t$, where $\varphi = (\alpha_1 + \alpha_2 + \cdots + \alpha_p) - 1$ and $\alpha_j^* = -(\alpha_{j+1} + \alpha_{j+2} + \cdots + \alpha_p)$.

[4+8+8 = 20]

2. (i) Let $\{Z_t\}$ be i.i.d. normal (0, 1) noise. Define $\{X_t\}$ as

$$X_t = \begin{cases} Z_t \text{ if } t \text{ is even} \\ \dfrac{Z_t^2 - 1}{\sqrt{2}} \text{ if } t \text{ is odd.} \end{cases}$$

Show that $\{X_t\}$ is $WN(0, 1)$, but not i.i.d.(0, 1) noise.

(ii) Find the range of values of c for which the time series $\{X_t\}$ defined as

$$X_t = X_{t-1} + cX_{t-2} + Z_t, \quad Z_t \sim WN(0, \sigma^2)$$

is stationary.

Also show that the special AR (3) process

$X_t = X_{t-1} + cX_{t-2} - cX_{t-3} + Z_t$, is non-stationary for all real values of c.

[10+10 = 20]

P.T.O

3. (i) Describe how the ADF test is used to conclude about the nature of trend in trended data.

(ii) Describe the Quandt-Andrews test for detecting the presence of a structural break in a time series. Do you think this test has any limitation(s)? Provide justification in support your answer.

[8+12 = 20]

4. (i) Describe the HEGY test for testing the presence of seasonal and non-seasonal unit roots in a quarterly time series. Discuss briefly the power problem, if any, of this test.

(ii) Describe the method of estimation of the parameters of an AR (1) model.

[12+8 = 20]

5. (i) Obtain the variance of the forecast error of h-step ahead minimum MSE forecast, $f_{n,h}$, at origin n.

(ii) Find the 2-step ahead minimum MSE forecast at origin n of the following time series $\{x_t\}$:

$$(1 - 0.8B)(1 - B)x_t = (1 + 0.4B)a_t, \qquad a_t \sim WN(0, \sigma_a^2).$$

(iii) Distinguish between (a) recursive and rolling methods of obtaining out-of-sample forecasts, and (b) in-sample and out-of-sample forecasts.

[4+8+8 = 20]

6. (i) Define spectral density function, $f(\lambda)$, of a stationary process, and then show that it is nonnegative for all $\lambda \in [-\pi, \pi]$. Also, find $f(\lambda)$ for an AR(1) process.

(ii) State and prove the theorem on finding the spectral density function of a linear combination of stationary time series.

[10+10 = 20]

INDIAN STATISTICAL INSTITUTE

End-Semestral Examination: (2017-2018)

MS(QE) I

Microeconomics II

Date: 25. 04. 2018 Maximum Marks: 100 Duration: 3 hrs

Note: Answer all questions

(1) State and prove the second fundamental theorem of welfare economics by giving all the relevant definitions. **(35)**

(2) Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{a_1, a_2\}$, $0 < a_1 < a_2 < \infty$ and $Pr(\theta = a_2) = \frac{1}{2}$. The cost of education is $c(e, \theta) = \frac{e^2}{2\theta}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type θ who chooses education level e and receives wage w. Assume that both worker types earn zero by staying home, that is $r(a_1) = r(a_2) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of e^* for which we can have a separating equilibrium. Justify your answer. **(7)**

(b) Consider the belief function $\mu^b(e) = \frac{e - \max\left\{0, e - \sqrt{2a_1(a_2 - a_1)}\right\}}{\sqrt{2a_1(a_2 - a_1)}}$ for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer. **(8)**

(3) Show that in any sub-game perfect Nash equilibrium of the screening game with unknown worker types, the low ability worker accepts $(\theta_L, 0)$ and the high ability worker accepts $(\theta_H, t^{(1)})$, where $t^{(1)}$, the task level assigned to the high type, satisfies $\theta_H - c(t^{(1)}, \theta_L) = \theta_L - c(0, \theta_L)$. Here the marginal (or average) productivity of a worker is $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H < \infty$, the probability that a worker is of high type is $\gamma \in (0, 1)$, and the opportunity cost of accepting employment to each type of worker is zero. **(35)**

(4) Consider the labour market model where the effort level of the tenant is neither observable nor verifiable. Show that in equilibrium, the incentive constraint is binding. **(15)**

Indian Statistical Institute

Economic Development I

MSQE I & II

Semestral Examination

Date: 27 April, 2018

Time: 3 hours

Maximum Marks: 60

Answer Question 1 and any two from the remaining.

1. Consider a scenario where there are two political parties: an incumbent in power and an opposition. Also, there is a large number of voters. Total number of voters is normalized to 1. There are two types of voters, type N comprising of a fraction γ of the total voters and type S comprising of the remaining . Each voter gets a noisy signal s which is uniformly distributed around the realized state of the economy θ. Type N voters vote for the incumbent if $E[\theta|s] \geq \bar{\theta}$, an exogenously specified level of performance of the economy. The S type is in a patron-client relationship with the political parties and votes on the basis of expected personal benefits. Finally, $\theta = e + \omega$ where e is the effort level of the incumbent and ω is a random shock. The incumbent maximizes its probability of winning net of effort costs by choosing the level of effort.
   (a) By suitably developing a model based on the above information show that an increase in the proportion of S voters reduces the effort level of the incumbent.
   (b) Now suppose that $\theta = \tau e + \omega$, $0 < \tau < 1$. Here τ measures the effectiveness of effort or alternatively the efficiency of the incumbent. Show that an increase in τ increases the probability of the incumbent to win the next election.

[12+8]

2. Show how lending with joint liability can reduce the inefficiencies due to (a) moral hazard; and (b) costly state verification.

[10 +10]

3. Developing a suitable model of repeated game, characterize an implicit insurance contract between two economic agents based on reciprocity. Show that in situations of extreme inequality the first best cannot be achieved.

[15+5]

P.T.O

4. Consider an agricultural market where a single commodity is sold over an interval [0,T] to satisfy demand which is linear in price at each point in time. There are n oligopolistic sellers in the market each with a storage cost c per unit of stock per unit of time. Show that
   (a) sellers with smaller initial stocks will leave the market early by exhausting their stocks.
   (b) If the possibility of international trade opens up and the sellers have the option of selling to the international market at a fixed price, only large sellers will sell to both domestic and international markets and small sellers will sell to the domestic market alone.

[12 + 8]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2017-2018)
MS (Q.E.) I Year
Macroeconomics I

Date: 02.05. 2018 Maximum Marks 60 Duration 3 hours

Group A

Answer the following questions.

1. Consider a two-period simple exchange economy. Here the consumer maximizes the utility function

$$u(c_1) + \frac{1}{1+\rho}u(c_2)$$

Subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

Derive the consumption-Euler equation for this model. Discuss the effects of changes in the interest rate (r) on the consumption-saving decision of the consumer depending on the cases where the consumer might be a net lender or net borrower or neither.

(3+7 = 10)

2. Consider the stochastic optimal growth model (Brock and Mirman, 1972). The preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

The production technology is given by

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}$$

The law of motion for capital stock is

$$k_{t+1} = i_t + (1-\delta)k_t$$

The resource constrain for the economy is

$$c_t + i_t = y_t$$

Construct the social planner's problem for this economy. Find the optimal decision rules for capital and consumption in this economy.

(3+7 = 10)

3. The representative agent's utility function is

$$\sum_{t=0}^{\infty} \beta^t \, u(c_t, l_t)$$

The agent's budget constraint is

$$c_t = w_t(1-l_t) - \tau_t - s_{t+1} + (1+r)s_t$$

For $t = 0,1,2, \ldots$

The government budget constraint is

P.T.O

$$g_t + (1+r)b_t = \tau_t + b_{t+1}$$

For $t = 0,1,2,\ldots.$
Define the no Ponzi game condition for this model. Interpret the condition intuitively. Find the aggregate life-time budget constraints for the government and for the consumer. Discuss the Ricardian Equivalence Theorem in this respect.

(1+2+2+2+3=10)

Group B
Answer all questions

1. Show that in an OLG model, introducing a 'Pay as you go' pension scheme will reduce the steady state per capita capital stock.
   What would be the effect of such a pension scheme on the steady state welfare?

   (15)

2. (a) In the Blanchard-Kiyotaki model; show that a coordinated reduction in all prices and wages, beginning from a situation of monopolistically competitive equilibrium, will raise real profits and also the utility.

   (b) Consider an economy with the representative agent having the utility function:

   $U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}$, $0 < \alpha, \gamma < 1$

   Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0 < \rho < 1$ and c_i is the consumption of the i^{th} variety. L is the labour supply, P is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. M is the money balances (and suppose M_0 is the initial endowment of money). The household budget constraint is given by:

   $PC + w(1-L) + M = M_0 + w + \pi - T$ where w is the money wage rate and π is the economy wide profits and T the taxes. Production of varieties is given by:

   $$Y_i = 0 \; if \; L_i \leq F$$
   $$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

   Y_i is the output of i^{th} variety and L_i is the labour employed in the production of the i^{th} variety.

   Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed n).

P.T.O

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where G takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and g_i is the government consumption of the i^{th} variety.

(ii) What would be the effect of such increase in government expenditure on P ?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ($M = M_0$).]

(10+5=15)

Indian Statistical Institute

Final Examination 2018, MSQE I and II

Course name: **Political Economy**

Subject name: **Economics**

Date: **4 May 2018**

Maximum marks: **100**

Duration: **3 hours**

1. In the context of voter turnout, Grossman and Helpman conclude, "The paradox in voting is ... the choice by a reasonably high percentage of eligible voters to bear the cost of voting." Elucidate the paradox. **(5 points)**

2. Within the Palfrey-Rosenthal framework of strategic voting as discussed in class, and using simple numerical figures for team sizes, demonstrate the forces of "competition" and "free-riding" that individuals face when deciding whether or not to vote. (You may assume the coin-toss rule for breaking ties and all individuals facing an identical voting cost given by c.) **(10 points)**

(Hint: Consider using simple 2x2 games and their Nash equilibria to make your point.)

3. Consider participation games as discussed by Palfrey and Rosenthal in the context of strategic voting. According to them, "The conclusion is that pure strategy equilibria fail to exist except for a few very special cases." Assume c, the identical cost of voting of all citizens, to be less than $1/2$, and coin-toss rule for breaking ties, to substantiate their conclusion. **(15 points)**

4. This question pertains to the characterization of "mixed-pure" equilibria as discussed by Palfrey and Rosenthal in the context of strategic voting. Suppose $M, N > 1$. (Recall M and N are the number of members in teams 1 and 2 respectively.) Let $k \in \mathbb{N}$ be such that $k \leq \min\{M-1, N\}$. Recall that c is the exogenously given cost of voting for each player. We know that if $c \leq \binom{M-1}{k}\left(\frac{k}{M}\right)^k\left(1-\frac{k}{M}\right)^{M-1-k}$, then there is an equilibrium of G_2 (status-quo rule) in which exactly k members of team 2 vote and all team 1 members vote with probability q. Show that q satisfies $c = \binom{M-1}{k} q^k (1-q)^{M-1-k}$. **(5 points)**

(Hint: This question only asks you to demonstrate the best response of a team 1 member, given the strategy of the other team - it *does not* ask you for the proof of the proposition.)

P.T.O

5. What is the expected turnout in a "$q - k$" or "mixed-pure" equilibria as discussed by Palfrey and Rosenthal? What are its drawbacks when positive voting costs of individuals and large electorates are considered? **(5 points)**

6. This question pertains to a parameterized version of Feddersen and Sandroni's ethical voting model. Let the fraction of ethical agents in groups 1 and 2, $\tilde{q}_1$ and $\tilde{q}_2$ respectively, be independently and identically distributed as $U[0,1]$. Let the fraction of the population in group 1 be deterministic and be given by $k \in (0, 1/2]$. Let cost of voting for each individual be random and be drawn from $U[0, \bar{c}]$. Let the payoff from 'doing one's part', D, be $> \bar{c}$. Let the social cost function be linear, that is $v(x) = x$. Recall w to be the parameter capturing the 'importance of election'.

(i) Let parameters $\bar{c}, w$ and k satisfy $\frac{\bar{c}}{w} > \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(20 points)**

(ii) Let $\frac{1}{(1-k)^2} \leq \frac{\bar{c}}{w} \leq \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(8 points)**

(iii) Let $\frac{\bar{c}}{w} \leq \frac{k}{(1-k)^2}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(8 points)**

(iv) Consider your answer in part (i). What can you conclude about the participation rates of the minority and the majority? **(6 points)**

(v) From (i), what can you conclude about the chances of winning of the minority versus that of the majority? **(6 points)**

(vi) What is total expected turnout? How does it vary with the 'level of disagreement' in the economy? How does it vary with average voting costs? How does it vary with the 'importance of the election'? **(4+4+2+2=12 points)**