INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2017-2018)
MS (Q.E.) II Year
Macroeconomics II

Date: 04.09.17 | Maximum Marks 40 | Duration 3 hours

Use separate booklets for group A & B

Group – A

Answer any one of the two

1. Is there any relationship between stability of equilibrium and hyperinflation? Explain your answer in the light of Cagan's model.

(20)

2. (a) Consider the following two rational expectation difference equations.

   (i) $y_t = \theta y_{t-1} + b E(y_{t+1}) + x_t$ with $\theta = 1 - b$

   (ii) $y_t = a E(y_{t+1}) + b E(x_{t+1}) + x_t$

   Here x_t satisfies a first order autoregressive process given by

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}$$

   for all t where $|\rho| < 1$ and ε_t is a white noise disturbance term

   (b) Distinguish between (i) Rational expectation and Adaptive expectation (ii) Active policy rule and Passive policy rule.

$(6 \times 2 + 4 \times 2 = 20)$

Group - B

Answer all

1) a) Show that the initial period consumption in the Ramsey model is a function of the initial asset and the present discounted value of lifetime wage earnings.

   b) Using that expression, work out the dynamics of per capita assets for a small open economy, facing a constant rate of interest in the world capital market.

P.T.O

In this context discuss the problems associated with either a very low or a very high rate of interest. [3+7] = [10]

2) a) Show that all paths other that the convergent one in the Ramsey model would either violate the transversality condition or one of the other necessary conditions for an optimum.

   Show that the No Ponzi condition together with optimality delivers what is required by the transversality condition

b) Assuming a Cobb-Douglas production function, work out the transitional dynamics of the savings rate in the Ramsey model. [5+5] = [10]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2017-2018)

MS(QE) II

Individual and collective choice

Date: 05.09.2017 Maximum Marks: 100 Duration: 3 hrs.

(1) Suppose the data for the problem are preferences of the n ($< \infty$) agents of a society over two alternatives x and y. Answer the following questions.

(a) Define a social welfare function. **(2)**

(b) Define the following properties of a social welfare function: symmetry among agents, neutrality between alternatives, positive responsiveness and non-triviality. **(4)**

(c) Define the majority voting social welfare function and the dictatorial social welfare function. **(4)**

(d) Show that a social welfare function satisfies symmetry among agents, neutrality between alternatives and positive responsiveness if and only if it is a majority voting social welfare function. **(20)**

(2) State and prove Arrow's impossibility theorem by giving all the relevant definitions. **(30)**

(3) Define quasi-ordering and ordering of a binary relation on the finite set of alternatives A (where $|A| \geq 3$). Also define oligarchy. Show that any oligarchy is a quasi-ordering. **(4+2+4=10)**

(4) Define scoring rules. Show that scoring rules always generate an ordering on the finite set of alternatives A. **(4+8=12)**

(5) Define single-peaked preferences on a finite set of alternatives A. Show that if preferences on A is strict, then the cardinality of the set of all possible single-peaked preferences (for a given linear order) is $2^{|A|-1}$. **(4+14)**

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2017-2018
MS (Q.E.) II Year
Econometric Methods II

Date: 6th September, 2017 Maximum Marks 60 Duration 2 hours

All notations are self-explanatory. This question paper carries 70 marks. You can answer any part of any question. However, the maximum that you can score is 60. Marks allotted to each question are given within parentheses.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic but endogenous. Assume that all other CLRM assumptions hold. Let Z be a matrix of instruments for X with rank as that of X.
   a. What essential assumptions must Z satisfy for the IV estimator to be consistent for β?
   b. Show that the OLS estimator of β is inconsistent.
   c. Show that the IV estimator is biased in general.
   d. Consider the transformed model as $Z'Y = Z'X\beta + Z'\epsilon$. Show that IV estimator is same as that of GLS estimator obtained from this transformed model.
   e. If the rank of Z is more than the rank of X, derive the form of IV estimator.
   f. How will you test for endogeneity of X? Derive the test.
   g. How will you test for heteroscedasticity using IV estimator?
   h. Show that OLS estimator is more efficient than the IV estimator when X is exogenous. [5+6+4+5+5+7+4+4=40]

2. Consider the following panel data model:
$$y_{it} = \alpha_t + x_{it}'\beta + \varepsilon_{it},$$
Where the x_{it} (k× 1) are time-individual varying regressors. Let $x_t = (x_{1t}, x_{2t}, \dots, x_{Nt})'$. Assume that $E[\varepsilon_{it}|x_t, \alpha_t] = 0$, and $E[\alpha_t|x_t] \neq 0$. $\sigma_\varepsilon^2 = Var(\varepsilon_{it})$.

   a. Provide a consistent estimator of β . Prove the consistency.
   b. How will you estimate α_t consistently? Prove the consistency.
   c. Discuss how will you test for the assumption $E[\alpha_t|x_t] = 0$.
   [10+10+10=30]

INDIAN STATISTICAL INSTITUTE
Mid Semester Examination: (2017- 2018)
MS (Q.E.) II Year
International Economics I

Date: 07·09·17 Maximum Marks 40 Duration 3 hours

Use separate booklets for group A & B

Group A

Answer all

Indicate whether the following statements are true or false, stating clearly the reasons for your answer.

1. In a pure exchange model with two countries and many goods, market equilibrium is stable if demand is identical and homothetic across countries and individuals.
2. In a Ricardian world with two countries and five goods, there must be one good which is produced by both countries in trade equilibrium.
3. In a Ricardian model, international trade expands the world production (and consumption) possibility frontier.
4. In a specific factors model with given international prices, a rise in the endowment of one of the specific factors leads to reductions in the return to both specific factors.

[5x4=20]

Group B

Answer all

1. Show that in a two agent setting, Walras stability guarantees that the recipient of a transfer necessarily gains. [10]

2. Consider a two country, two commodity trading world and show that a boom experienced by a country in its export sector can immiserize the country.

[10]

Indian Statistical Institute
Mid-Semester Examination: 2017-2018
MS(QE) II: 2017-2018
Industrial Organization

Date: 8/09/2017 Maximum Marks: 40 Duration: $2\frac{1}{2}$ Hours

You any TWO questions

1. Consider n firm Cournot oligopoly producing a homogenous good with market demand function in inverted form given by $P = a - X$ where X is the industry output, P is the price of the product and a is a demand parameter. The government imposes a tax t_i per unit output of firm i and wants to maximize its total tax revenue. Answer the following.

(a) Suppose $n = 2$ and $MC_i = c_i$. Find the optimal tax rates t_1 and t_2.

(b) Suppose $n \geq 3$ and $MC_i = c\ \forall i = 1, 2, \ldots\ldots n$. Find the optimal values of t_i, $i = 1, 2, \ldots\ldots n$.

[8+12 = 20]

2. (a) Suppose the market demand function is of the form $P = a - bQ; a, b > 0$. Firm 1 uses an old production technology which has marginal cost of Rs 15. Firm 2 uses a modern technology with marginal cost of Rs 10. In the current equilibrium the product price and aggregate output are respectively Rs 16.66 and 8.33. Then how much would firm 1 be willing to pay for the modern technology? Derive the effect on welfare when both firms use modern technology.

[6+4 = 10]

(b) Consider a duopoly producing a homogeneous product. Firm 1 produces one unit of output with one unit of labor and one unit of raw material. Firm 2 produces one unit of output with two units of labor and one unit of raw material. The unit costs of labor and raw material are w and r. The demand function is $p = 1 - x_1 - x_2$, where x_i denotes the supply of firm i; the firms compete in quantities.

(i) Compute the Cournot equilibrium.

(ii) How is firm 1's profit affected if the price of labour goes up? Give economic intuition of the result.

[5+5 = 10]

3. A monopolist produces two products, X_1 and X_2, at zero cost. There is a mass 1 of consumers. A fraction, γ, of consumers are heterogeneous described by their type θ which is distributed uniformly over the unit interval. The θ-consumer has willingness to pay $r_1 = \theta$ and $r_2 = 1 - \theta$ for each unit of X_1 and X_2, respectively. A fraction $\frac{(1-\gamma)}{2}$ of consumers has willingness to pay $r_1 = \frac{2}{3}$ and $r_2 = 0$, and the remaining fraction $\frac{(1-\gamma)}{2}$ of consumers has willingness to pay $r_1 = 0$ and $r_2 = \frac{2}{3}$. Let P_1 and P_2 be the prices under independent pricing strategy and P_b be the bundle price (a bundle consisting one unit of each product) under pure bundle pricing strategy. Answer the following.

(a) Suppose that $\gamma = 4/5$. Determine the prices under each of independent selling and bundle selling strategy. Which strategy is optimal for the monopolist?

(b) What will be the corresponding prices (i) when $\gamma = 0$ and (ii) when $\gamma = 1$?.

[12+(4+4) = 20]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2017-2018
M.S. (Q.E.), 2nd Year
Econometric Applications I

Date: 8 September 2017 Maximum marks: 100 Duration: 3 hours

[Answer question no. 1 and any **three** from the rest of the questions.]

1. (a) Using the following data on the Per-capita Expenditure (PCE) compute (i) the Head Count Ratio (H), (ii) the Income Gap Ratio (I) and (iii) the Sen's Index of Poverty (P). Assume that the poverty line is Rs. 400 per 30 days.

Table 1: The size distribution of population by PCE

PCE (Rs./30 days)	Percentage of population	Average PCE (Rs./30 days)
0 – 150	0.8	~~25.0~~ 150
150 – 200	2.5	~~36.1~~ 175
200 – 250	5.0	~~46.4~~ 225
250 – 300	8.0	~~55.8~~ 275
300 – 350	10.5	~~65.5~~ 325
350 – 425	16.3	~~78.2~~ 400
...	...	...

[25]

OR

1. (b) Suppose the urban population is just one-third of the rural population in a country. The average monthly income of the bottom 10% population of rural and urban sectors are Rs. 35 and Rs. 45, respectively. The respective Lorenz Ratios are 0.28 and 0.40. Assume that income follows lognormal distribution separately for rural and urban sectors. Calculate the percentage of people with income more than Rs.80/- for the country as a whole. [25]

2. State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? Derive Lorenz Curve and Lorenz Ratio of Pareto distribution. [2+6+5+12=25]

3. Define lognormal distribution. State and prove its properties including moments, quantiles, skewness, kurtosis, and moment distribution. [25]

4. Describe the problems in estimating poverty line. Also give a brief account of the different poverty measures that are used to find the poverty situation in a country. [15+10=25]

5. Write short notes on any two of the following:
   (a) Kapteyn's LPE model and its modification by Kalecki.
   (b) Normative measures of Inequality.
   (c) Three parameter lognormal distribution.
   (d) Measures of concentration in business and industry. [12½+12½=25]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2017-18

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 11/09/2017
Maximum marks: 40
Duration: 2 hours

Answer all questions rigorously.

Q1. In the principal agent problem with hidden information where the agent is of two possible types and has a linear cost function, fully derive all optimal contracts and then show that the principal will never offer a pooling contract. [5 + 5]

Q2. Consider the basic principal agent problem with hidden information where $S(q) = A\log_{10}(q-1), A > 0$.

a) Given values of $\bar{\theta}$, $\underline{\theta}$ and α, for what values of A is shutdown or partial employment optimal for the principal? Explain your findings. You do not have to derive optimal contracts for this answer. [10 + 5]

b) Suppose that $\bar{\theta} = 3$, $\underline{\theta} = 2$, $\alpha = 0.75$ and $A = 60$. Determine whether shutdown or partial employment is optimal for the principal. [5]

Q3. Suppose in the basic principal agent problem the principal's gross benefit function is $S(q, \theta)$, i.e., the agent's type directly affects the principal's benefit, and $S_{q\theta} > 1$.

a) Characterize the full information optimal contracts and show that $\underline{q}^* < \overline{q}^*$. Explain this result. [3 + 2]

b) Characterize the full employment optimal contract menu in the presence of hidden information. [5]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2017-2018)

MS(QE) II

Individual and Collective Choice

Date: 22.11.2017 Maximum Marks: 100 Duration: 3 hrs.

Note: Answer all questions.

(1) Define the plurality rule, the anti-plurality rule and the Borda count. Show that each of these three scoring rules generate a well-defined Arrovian social welfare function. **(10)**

(2) State and prove the Gibbard-Satterthwaite theorem when the individual preferences are rational and strict. Give all the relevant definitions. **(30)**

(3) Define single-peaked preferences (first define all the technical terms you will use in the definition). Assuming a given linear order, consider the social choice function on the domain of all single-peaked preferences that selects the rightmost peak under all profiles. Show that this social choice function is strategyproof. **(5+10=15)**

(4) Consider the pure public goods problem. Show that an efficient mechanism is dominant strategy incentive compatible if and only if it is from the class of VCG mechanisms. Give all the relevant definitions. **(25)**

(5) Define the pivotal mechanism for the pure public goods problem. Show that when the objective is to implement outcome efficiency, the pivotal mechanism satisfies dominant strategy incentive compatibility and feasibility. **(3+12+5=20)**

Indian Statistical Institute
First Semester Examination: 2017-18
M.S.(Q.E) – II Year

Econometric Applications I

Date: 24.11.2017 **Maximum Marks:** 100 **Duration:** 3 hours

Answer **any four** questions.

1. a) Define Engel elasticity. What is the classification of consumer goods based on Engel elasticity? Illustrate.

   b) Show that (i) the weighted sum of Engel elasticities for all items is 1.

   (ii) in a consumer's basket all commodities cannot be 'inferior goods'.

   c) Explain the methods of Engel and Rothbarth for measuring equivalence scales for children.

   d) Define 'Specific Concentration Curve' (SCC). Show that for 'inferior goods' the SCC lies above the egalitarian line.

   [5+ (3 + 3) + 8 + 6 = 25]

2. a) What are the different approaches to specifying a demand system?

   b) What is the Gorman-Polar form of cost function? What is the form of the Engel curve implied by this form of cost function?

   c) Show that for the Linear Expenditure System (LES) the non-compensated own price elasticities $[\mu_{ii} = -1 + (1 - b_i)\frac{c_i}{q_i}]$ are approximately proportional to the corresponding expenditure elasticities$[\eta_i = \frac{b_i}{w_i}]$.

   d) In the LES, an amount z of good 1 must be bought. Show that for $i = 2,3\ldots,n$.

   $$p_i q_i = p_i c_i + \frac{b_i}{\sum_{i=2}^{n} b_i}(x - p_1 z - \sum_{k=2}^{n} p_k c_k)$$

   where the symbols have their usual meanings.

   [6 + 4 + 7 + 8 = 25]

P.T.O.

3. a) Explain the difference between a 'censored distribution' and a 'truncated distribution?

   b) Write down the Tobit model to incorporate zero consumption with full specification of the distribution of the error term. Why are the assumptions underlying the standard linear regression model not tenable in such a case?

   c) Given the dynamic model for 'clothing'

   $$q(t) = \alpha + \beta s(t) + \gamma x(t)$$

   where $q(t)$: rate of demand at time t

   $x(t)$: income during the same time

   $s(t)$: inventory of 'clothing' at time t.

   and assuming that the stock is used up at a constant depreciation rate δ, find the short and long term derivatives of consumption with respect to income.

[3 + 12 + 10 = 25]

4. a) Define homothetic preferences. Show that under homothetic preferences the cost function is of the form

   $$C(u, p) = \alpha(p)u^*,$$

   where $u^* = f(u)$ and $\alpha(p)$ is linearly homogeneous in p, the vector of prices.

   b) Show that any demand system derived from maximization of a utility function subject to a budget constraint satisfies negativity of the Slutsky matrix.

   c) Suppose you are given the cost function

   $$C(u,p) = a(p)exp(\frac{b(p)}{1/lnu - \lambda(p)}),$$

   where $a(p)$ is homogeneous of degree one in prices, $b(p)$ and $\lambda(p)$ are homogeneous of degree zero in prices and u is the level of utility, with

   $$ln\, a(p) = \alpha_0^* + \sum_{i=1}^{n} \alpha_i^* \, ln\, p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{ij} \, ln\, p_i \, ln\, p_j \,,$$

   $$b(p) = \prod_{i=1}^{n} p_i^{\beta_i} \quad \text{and} \quad \lambda(p) = \sum_{i=1}^{n} \lambda_i \, ln\, p_i \,.$$

   (i) Derive the budget share function for item i.

   (ii) Obtain the appropriate restrictions on the parameters for the homogeneity condition to be satisfied.

[6 + 8+ (5+6) = 25]

P.T.O.

5. a) Describe the Ramsey-Samuelson-Diamond-Mirlees approach to the determination of optimal commodity taxation.

   b) How does one estimate the tax rates empirically?

   c) Assuming that consumer's demand behaviour is captured through the Linear Expenditure System, examine the redistributive impact and the effect of taxation on the worst-off household assuming that taxation is purely a redistributive mechanism amongst households.

[12 + 9 + 4 = 25]

Indian Statistical Institute
First Semestral Examination: 2017-2018
MS(QE) II: 2017-2018
Industrial Organization

Date:24/11/2017 Maximum Marks: 40 Duration: 3 Hours

Answer any THREE questions. Your total score cannot exceed 40.

1. Consider (an otherwise) Hotelling linear city model with the length of the city to be one. There are three firms; call them firm 1, firm 2 and firm 3. Price of the product is unity. The firms compete for market shares. Suppose that firm 1 is already located at the extreme left of the city. Now firm 2 and firm 3 decide their locations sequentially (firm 2 first, then firm 3). Find the optimal locations of firm 2 and firm 3. [15]

2. (a) There are two firms producing homogenous goods. Firm i has marginal cost of production c_i; $i = 1,2$; $c_1 < c_2$. Firms compete in prices and choose their prices sequentially. Find optimal prices if (i) firm 1 moves first, (ii) firm 2 moves first. [4]

   (b) Consider a duopoly in which homogeneous consumers of mass 1 have unit demand. The valuation for good i (supplied by firm i), $i = 1,2$, is $v(\{i\}) = v_i$, with $v_1 > v_2$. Marginal cost of production is assumed to be zero. Firms compete in prices.

   (i) Suppose that consumers make a discrete choice between the two goods. Find Nash equilibrium prices. [4]

   (ii) Suppose that consumers can decide to buy either of the goods or both goods. If they buy both goods, they are assumed to have a valuation $(\{1,2\}) = v_{12}$, with $v_1 + v_2 > v_{12} > v_1$. Firms compete in prices of its products (there is no additional price for the bundle). Characterize the Nash equilibrium. [7]

3. Consider asymmetric duopoly with linear market demand and constant marginal costs of production. Consider technology transfer from the low cost firm to the high cost firm. In the post-transfer situation both firms compete in quantities a la Cournot. Show that the quantity-based royalty licensing strictly dominates fee licensing. [15]

4. Consider an oligopoly market with $n \geq 2$ firms. Suppose k number of firms ($1 \leq k \leq n$) form a cartel and the cartel behaves as a Stackelberg leader; the remaining firms behave as Stackelberg followers. Will such cartel structure be stable in the sense that no insider (i.e., cartel member) will have any incentive to leave the cartel and no outsider (i.e., non-cartel member) will have any incentive to enter the cartel? [15]

5. Consider symmetric n firm oligopoly ($n \geq 2$) with linear market demand and constant marginal cost of production. One of these firms (say, firm 1) decides to multiple itself by opening independent subsidiaries. Then all firms compete non-cooperatively and firm 1's profit is the sum of profits of its subsidiaries. Suppose there is no cost of opening a subsidiary. If such a policy is optimal for firm 1, find the optimal number of subsidiaries to be opened. If there is a cost of $F > 0$ to open a subsidiary will your result undergo a change? [15]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2017-2018)

MS (Q.E.) II Year

International Economics I

Date: 27.11.17 Maximum Marks 60 Duration 3 hours

Group-A

Answer both questions.

1. Show that a country must gain from free trade if there is no distortion in the economy. Find a sufficient condition for positive gains from trade when there are tariffs, taxes and subsidies on commodities.

[3 + 12]

2. In a general equilibrium framework with production uncertainties, show that a country may gain or lose from trade depending on its risk aversion parameter.

[15]

Group-B

Answer both questions.

1. Show that in a three-agent setting, a transfer paradox might occur even when the equilibrium is Walras stable. In this context discuss the role of substitution effects in ensuring normal results.

[15]

2. Consider a 2 country, 2 commodity trading world, with perfectly competitive markets and show that imposing an ad-valorem export tax is the same as imposing an ad-valorem import tariff when the government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax.

[Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.] [15]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2017-18

MSQE II 2017-18

Incentives and Organisations

Date: 27 November 2017 Maximum marks: 50 Duration: 3 hours

Answer all questions

Q1. Consider the principal agent model with hidden action where there are two possible output levels, the agent is risk-averse and has two possible effort levels, low and high.

(a) What is the optimal contract if the principal wishes to implement low effort? [5]

(b) What is the optimal contract if the principal wishes to implement high effort? [10]

(c) Under what conditions will the principal choose to implement high effort? [5]

(d) Introduce the idea that the principal faces a cost of processing output, with the unit cost of processing equalling $\alpha > 0$ (in the basic model $\alpha = 0$). How will your answers to parts (a), (b) and (c) above change? [5]

Q2. Consider the principal agent model with hidden action where output can take any realisation in $[\underline{q}, \bar{q}]$, the agent is risk-averse and can take any effort in $[0, \bar{e}]$. Show that the first-order or local approach to deriving the optimal contract is valid if two conditions, the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) hold. [15]

Q3. Consider a principal agent model with hidden action where there are two possible output levels $\underline{q}$ and $\bar{q}$, the agent is risk-neutral and can take any effort in $[0, 1]$. Let $c(e)$ be the cost function for effort, with $c(.)$ thrice differentiable, and $c(0) = c'(0) = 0$, $c'(e) > 0$ for $e > 0$, $c'(1) = \infty$, $c''(e) \geq 0$, $c'''(e) \geq 0$. Also let $q(e) = Pr(q = \bar{q}|e) = e$.

(a) What effort does the principal implement in the first-best world (in the absence of hidden action)? [5]

(b) What effort does the principal implement in the presence of hidden action? [3]

(c) Show that the first best implemented effort is no smaller than the effort implemented in the presence of hidden action. [2]

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2017-2018
M.S. (Q.E.) II
Econometric Methods II

Date: 01.12.17 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 110 marks. You can answer any part of any question. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.

1. Consider the linear regression model: $y_i = x_i\beta + \varepsilon_i$. Assume that $E(\varepsilon_i|x_i) = 0$. Also assume that x_i and β are 1×1.
   a. Show that $E(x_i\varepsilon_i) = 0$, and $E(x_i^2\varepsilon_i) = 0$. Is $z_i = (x_i, x_i^2)'$ a valid instrumental variable for estimation of β?
   b. Define the 2SLS estimator of β using z_i as an instrument for x_i. How does this differ from OLS estimator?
   c. Find the efficient GMM estimator of β based on the moment condition $E(z_i(y_i - x_i\beta) = 0$. Does this differ from 2SLS and/or OLS estimators?

[(2+3)+(2+3)+(2+3)=15]

2. Consider the non-linear regression model: $y_i = \beta_1 \exp(x_i\beta_2) + \beta_3 \exp(w_i\beta_4) + \varepsilon_i$, where $E(\varepsilon_i|x_i, w_i) = 0$, $i = 1, \ldots n$.
   a. Propose moment functions that would make it possible to estimate $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)'$. Justify your answer.
   b. Derive the asymptotic distribution of your proposed estimator as obtained in (a).
   c. Suppose now that $E(\varepsilon_i|x_i) \neq 0$, and $E(\varepsilon_i|w_i) = 0$. Suppose there exists a $K \times 1$ vector $(K > 2)$, such that $E(\varepsilon_i|z_i) = 0$ and $E(z_ix_i) \neq 0$. Propose a GMM estimator of β. [5+10+5=20]

3. Let $y_t = \mu + \varepsilon_t$, $\varepsilon_t = u_t \times (\alpha_0 + \alpha_1\varepsilon_{t-1}^2)^{0.5}$, $u_t \sim i.i.d.N(0,1)$, $\alpha_0 > 0, \alpha_1 \geq 0$.
   a. Compute the first and second moments for y_t, (i) conditional on y_{t-1}; and (ii) unconditional.
   b. Write down the likelihood function based on T number of observations on y_t.
   c. Do you feel that the ML estimator of μ will be more efficient than that of the OLS estimator? Give the intuitive reason.
   d. Describe how you would test for H_0: $\alpha_1 = 0$.
   e. How will you estimate all the model parameters by GMM? Write down the moment conditions appropriately.

[(2+6)+7+5+7+8=35]

P.T.O.

4. Let $y_t = \mu + \lambda\, h_t + \varepsilon_t,\ \varepsilon_t = u_t \times (\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)^{0.5},\ u_t \sim i.i.d.\, N(0,1),\ \alpha_0 > 0, \alpha_1 \geq 0.$
   a. Derive the unconditional mean of y_t.
   b. Show that y_t is auto correlated. Derive the autocorrelation structure.

[6+9=15]

5. Consider the dynamic panel data model:
$y_{it} = \alpha_i + \rho\, y_{i\,t-1} + \beta x_{it} + \varepsilon_{it\ ,t=1,2,\ldots T;\ i=1,2,\ldots N}$. ε_{it} is i.i.d. with all ideal conditions. x_{it} is purely exogenous. $\alpha_i's$ are *i.i.d.* random variables with mean α and variance σ_α^2.
   a. Show that the OLS estimator of ρ is inconsistent for finite number of time series observations.
   b. Propose a GMM estimator of ρ. Write the moment conditions appropriately. Show that the proposed GMM estimator is consistent even when T is fixed.

[9+(4+12)=25]

Backpaper

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2017-2018

M.S. (Q.E.) II

Econometric Methods II

Date: 26 / 2 / 17 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. Marks allotted to each question are given within parentheses.

1. Consider the multiple linear regression model as $Y = X\beta + \epsilon$, where X is stochastic. Assume that data are independent across observations. Suppose $E(\epsilon_i|X_i) \neq 0$ but there are available instruments Z with $E(\epsilon_i|Z_i) = 0$ and $V(\epsilon_i|Z_i) = \sigma_i^2$, where dim(Z)> $dim(X)$. We consider the GMM estimator $\hat{\beta}$ that minimizes
$$G_N(\beta) = [\tfrac{1}{N}\sum Z_i(Y_i - X_i^{/}\beta)]^{\prime} W_N [\tfrac{1}{N}\sum Z_i(Y_i - X_i^{/}\beta)].$$
   a. Derive the limit distribution of $\sqrt{N}(\hat{\beta} - \beta)$.
   b. State how to obtain a consistent estimate of the asymptotic variance of $\hat{\beta}$.
   c. If errors are homoscedastic what choice $of\ W_N$ would you use?
   d. If errors are heteroscedastic what choice $of\ W_N$ would you use?

   [12 +8+7+8=35]

2. Consider the following panel data model:
$$y_{it} = \alpha_i + x_{it}^{/}\beta + z_i^{/}\delta + \varepsilon_{it},$$
Where the x_{it} $(k \times 1)$ are both time and individual varying regressors, the $z_i(m \times 1)$ are time invariant regressors. Let $x_i = (x_{i1}, x_{i2}, \dots, x_{iT})'$. Assume that $E[\varepsilon_{it}|x_i, z_i, \alpha_i] = 0$, and $E[\alpha_i|x_i, z_i] = 0$. Let $\sigma_\alpha^2 = Var(\alpha_i)$, and $\sigma_\varepsilon^2 = Var(\varepsilon_{it})$.

a. Let $c_i = \alpha_i + z_i^{/}\delta$. Find $Var(c_i)$ and compare it with σ_α^2.
b. Explain why estimation of the model by fixed effects transformation will lead to a larger estimated variance of the unobserved effects than if we estimate the model by random effects.
c. Show that both the fixed effect estimator and the random effect estimator of β are consistent.
d. Discuss how you will test for the assumption $E[\alpha_i|x_i, z_i] = 0$. [(4+4)+6+8+8=30]

3. Let $y_t = \mu + \varepsilon_t,\ \varepsilon_t = u_t \times (\alpha_0 + \alpha_1\varepsilon_{t-1}^2)^{0.5},\ u_t \sim i.i.d.\,N(0,1),\ \alpha_0 > 0, \alpha_1 \geq 0.$
   a. Compute the first and second moments for y_t, (i) conditional on y_{t-1}; and (ii) unconditional.
   b. Write down the likelihood function based on T number of observations on y_t.

P.T.O

c. Do you feel that the ML estimator of μ will be more efficient than that of the OLS estimator? Give the intuitive reason.
d. Describe how you would test for $H_0: \alpha_1 = 0$.
e. How will you estimate all the model parameters by GMM? Write down the moment conditions appropriately.

[(2+6)+7+5+7+8=35]

INDIAN STATISTICAL INSTITUTE
MID-SEMESTRAL EXAMINATION, 2017-2018
M.S. (Q.E.) II
Theory of Finance I

Date: 19.02.2018 Maximum Marks: 60 Time: 2 hours

Note: Clearly explain the symbols you use and state all the assumptions you need for any derivation. .

1. (a)Define certainty equivalent associated with an uncertain prospect. (2)

1. (b) Show that for a continuous, increasing utility function the associated relative cost of risk measure is continuous. Demonstrate also that this risk measure is positive only if the utility function is strictly concave. (8+5)

2. Show that in a two period simple binomial model in which the current stock price can assume only one of two values in the next period, in a probabilistic framework, non-arbitrage opportunities fail to exist if and only if there is at least one risk-neutral probability measure. (12)

3. Derive analytically the payoff function of a bullish price-spread strategy by giving necessary preliminaries. Explain its usefulness as a hedging instrument. (10)

4. Assume that a person's optimal portfolio consists of positive fractions of a risk-free prospect and a risky prospect. If his absolute risk aversion measure is decreasing in initial wealth, how does the amount invested in the risky prospect change? Prove or disprove your claim using rigorous arguments (10)

5. (a)Suppose of two stocks with uncertain returns an investor regards the former at least as good as the latter for all increasing utility functions. How can you express this using a graphical device? Demonstrate your assertion analytically. (10)

5. (b)'Second order stochastic dominance is necessary and sufficient for first order dominance.' Prove or disprove this statement using rigorous arguments. (3)

INDIAN STATISTICAL INSTITUTE

Mid-term Examination: 2017-18

MSQE II 2017-18

Auction Theory

Date: 20th February 2018 Maximum marks: 40 Duration: 2 hours

Answer all questions

Suppose there are N bidders participating in a first-price auction where the seller has 0 value for the object being sold. It is common knowledge that bidder valuations are drawn from [0, 1] according to the uniform distribution.

(I) Rigorously characterize symmetric, increasing, differentiable equilibrium bid functions, assuming one exists. [10]

(II) Argue that there is a unique such equilbrium bid function, assuming one exists. [5]

(III) Is the candidate bid function you have identified in parts (I) and (II) above an equilibrium? Argue rigorously. [10]

(IV) a) Suppose instead the cumulative distribution function is $F(x) = x^2$. What is the impact on the equilibrium bid function from (I), (II) and (III) above? [5]

b) What if instead the cumulative distribution function is $F(x) = x^{\frac{1}{2}}$? [5]

c) Provide a common explanation for your results from parts IV (a) and (b) above. [5]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2017-18

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 21-2-18 Maximum Marks: 40 Duration: 3 hours

Problem 1. Define single-peaked domain and min-max rule. Show that a social choice function is unanimous and strategy-proof on a single-peaked domain if and only if it is a min-max rule.

(10)

Problem 2. Justify your answer by a proof or a counterexample.

1. Strategy-proofness implies group strategy-proofness on any single-crossing domain.
2. Unanimity implies Pareto optimality under strategy-proofness on any single-dipped domain.

(15×2)

Economic Development

Mid-term Examination

MSQE I & II

Date: 22.2.2018

Time: 2 hours

Maximum Marks: 40

Question 1 is compulsory. Answer any one from the remaining two questions.

1. Consider a *static* economy consisting of two sectors: A and B. Labour is the only factor of production and the wage rates in the two sectors are given by

$$w_A = A_0 + aL_A$$
$$w_B = B_0 + bL_B$$

where w_A, w_B are wages and L_A, L_B are labour allocations in the two sectors. A_0, B_0, a and b are positive constants and $L_A + L_B = L$, the total labour endowment.

(a) Find all equilibrium allocations of labour and indicate which equilibria are stable.
(b) Now suppose each labour has to incur a cost c to move from one sector to another. Find all equilibrium allocations of labour and indicate which equilibria are stable in the changed situation.

[10 + 10]

2. Show that an economy with imperfect credit markets, lumpy investment cost of education, bequests and inheritance would converge to a bi-modal income distribution in the long run. How would the long run equilibrium change if credit markets were perfect?

[15+5]

3. In an economy where agents can choose to be either entrepreneurs or workers, show that there is trickle down growth if the return to investment is sufficiently high and if bequests are large enough.

[20]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2017-18
M.S.(QE) II YEAR
Mathematical Programming

Date: 23 February 2018 Maximum Marks: 60 Duration: $2\frac{1}{2}$ hours

---

Notation have usual meaning.

This paper carries 70 marks. However, maximum you can score is 60.

---

1 An auctioneer contemplates selling m items. Let $M = \{1, 2, \ldots, m\}$ be the set of items. From among the prospective buyers, n bids are received. A bid is represented by $B_j = (S_j, p_j)$ for $j = 1, 2, \ldots, n$, where S_j is the specified set of items the bidder wishes to acquire by paying the price p_j, $S_j \neq \phi$ and $S_j \subseteq M$. For example, $B_1 = (\{1,3,5\}, 13)$, $B_2 = (\{1,2,5\}, 20)$, and so on. The auctioneer intends to select the bids in order to maximize his/her revenue. Formulate this as an optimization problem.

[10]

2 Suppose that $\mathbf{d}$ is a direction of the polyhedral set $\mathrm{P} = \{\mathbf{x} : \mathbf{Ax} \leq \mathbf{b},\ \mathbf{x} \geq 0\}$. Then show that $\mathbf{d}$ is an extreme direction of P if and only if $\mathbf{d}$ is an extreme point of $\mathrm{D} = \{\mathbf{d} : \mathbf{Ad} \leq \mathbf{0},\ \mathbf{e}^{\mathrm{T}}\mathbf{d} = 1,\ \mathbf{d} \geq 0\}$.

[10]

3 Consider $S = \{\mathbf{x} : \mathbf{x}^{\mathrm{T}} = (x_1, x_2),\ -x_1 + 2x_2 \leq 4, x_1 - 3x_2 \leq 3,\ x_1 \geq 0, x_2 \geq 0\}$. Identify the extreme points and extreme directions of S. Express $(4,1)^{\mathrm{T}}$ with the help of Representation Theorem.

[3+5+7 = 15]

4 The following is the current simplex tableau (second iteration) of a given linear programming problem in canonical form with the objective to maximize $2x_1 - 3x_2$. The two constraints of the problem are $\leq$ type with non-negative right-hand-sides. In the tableau, x_3 and x_4 are slack variables.

	x_1	x_2	x_3	x_4	RHS
z	b	1	f	g	6
x_3	c	0	1	$\frac{1}{5}$	4
x_1	d	e	0	2	a

Find the unknowns a through g above.

[15]

[P.T.O.]

5 Suppose that at some iteration of simplex method with $\bar{\mathbf{x}}$ as current basic feasible solution, it is observed that $z_j - c_j < 0$ for each non-basic variable x_j. Prove that $\bar{\mathbf{x}}$ is unique optimal solution.

[10]

6 For the linear program:

$$
\begin{aligned}
\max \quad & x_1 - 2x_3 \\
\text{subject to} \quad & \\
& x_1 - x_2 \leq 1 \\
& 2x_2 - x_3 \leq 1 \\
& x_1, x_2, x_3 \geq 0,
\end{aligned}
$$

prove that $(3/2, 1/2, 0)$ is an optimal solution by using complementary slackness theorem.

[10]

——*** xXx ***——

Indian Statistical Institute
Mid Semestral Examination: (2017 – 2018)

M.S. (QE) – II year

Econometric Applications II

Date: 26.02.2018 | *Maximum Marks – 50* | *Duration: 2 hours*

Answer any <u>two</u> questions

1. (a) Suppose you are required to find the distribution of income from a given set of income data on n (large) individuals, given by $x = \{x_1, x_2, x_3 \ldots. x_n\}$. Let $p(a)$ denote the true density evaluated at point 'a' and $p_{h_n}(a)$ denote the estimate of $p(a)$ based on bandwidth h_n. State the underlying conditions for $p_{h_n}(a)$ to converge to $p(a)$.

   (b) Explain how the Kernel density estimation procedure can be viewed as an extension of the concept of Histogram.

   (c) Show that $lim_{h_n \to 0} p_{h_n}(a) = p(a)$.

   (d) When will you use a "nearest neighbour approach" to estimate density? Describe the steps involved in estimating density by k-nearest neighbour (k-nn) method.

[6 + 6 + 8 + 5 = 25]

2. (a) Explain the concept of kernel regression.

   (b) Derive the Nadaraya–Watson kernel regression estimator for the following budget share function of the *h-th* household.

$$w_h = m(\log x_h) + \varepsilon_h, \quad h = 1,2,\mathrm{K}\ ,H$$

   (c) Describe the k-nearest neighbour (k-nn) regression estimator.

   (d) Given the following data find the k-nn estimate for $x=4.5$ and $k=3$.

x	2.5	8.5	4	3.5	6	1	9.5	5	7
y	5	12	1	7	5	6	7	2	8

[4 + 8 + 4 + 9 = 25]

P.T.O

3. (a) Discuss the idea of Propensity Score Matching (PSM) and its applicability. Describe the different matching procedures.

(b) What do you mean by a semiparametric model? Describe the single index model and its estimation procedure by Ichimura's method.

(c) Show that the Nadaraya-Watson estimator is the weighted least squares estimator of the intercept of the regression equation $y_i = \alpha + \varepsilon_i$.

[10+10+5=25]

Indian Statistical Institute

Mid-semester Examination 2018, MSQE I and MSQE II

Course name: **Political Economy**

Subject name: **Economics**

Date: **27 February 2018**

Maximum marks: **75**

Duration: **3 hours**

1. This question pertains to direct aggregation of individual preferences $\{\succsim_i\}_i$ over alternatives in some set X to obtain a social preference by applying the majority rule, $\succsim_{MR}$. For simplicity, you may assume $X \subseteqq \Re$, discrete and finite. Either prove or give a counterexample for **any two** of the following four statements, clearly indicating whether you are proving or giving a counterexample: **(5 x 2 = 10 points)**

(i) If $\succsim_{MR}$ is intransitive over any $x, y, z \subseteqq X$, then exactly one of the following must hold:

(a) $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$

(b) $y \succsim_{MR} x \succsim_{MR} z \succsim_{MR} y$.

(ii) If $\succsim_i$ is complete $\forall i$, then $\succsim_{MR}$ is complete.

(iii) If individual preferences are *single-peaked* (SP), then $\succsim_{MR}$ is transitive. That is, SP $\implies \succsim_{MR}$ transitive.

(iv) Let X^* be the set of Condorcet winners. Then transitivity of $\succsim_{MR}$ is necessary for X^* to be non-empty. That is, $X^* \neq \phi \implies \succsim_{MR}$ transitive.

2. This question pertains to indirect aggregation of individual preferences in the context of a model of representative democracy as discussed in class. The standard assumptions of such a model remain the same, namely, there are n individuals in the economy each having well-behaved preferences over policy denoted by $\{\succsim_1, \succsim_2, \cdots, \succsim_n\}$. Moreover these preferences satisfy *Extremal Restriction* (ER). There are two candidates/parties/representatives, A and B, who try to maximize objective functions u_A and u_B. Let the objective function candidate A be as follows:

$$u_A(x_A, x_B) = \#\{i : x_A \succ_i x_B\} + \frac{1}{2}\#\{i : x_A \sim_i x_B\}.$$

We can define $u_B(x_A, x_B)$ likewise. Let $G := (\{A, B\}, \{X, X\}, \{u_A, u_B\})$. Consider the mixed extension of G and denote it by G'. Then show that

(i) G' has a Nash equilibrium. **(5 points)**

(ii) Any Nash equilibrium (p_A, p_B) of G' has support$(p_k) \subsetneqq X^*$, $k = A, B$, where X^* is the set of Condorcet winners in X. **(15 points)**

3. Suppose preferences of individuals for a service consists of two dimensions, p_1 and p_2. Suppose there are three individuals, a 'low' consumer, a 'middle' consumer and a 'high' consumer with bliss points $(1,1)$, $(2,3)$ and $(4,4)$ respectively. Suppose individuals can directly vote on alternative pairs. Prove or argue otherwise whether a 'median-voter' analogue may still hold good. That is, is it true that the $(2,3)$ alternative will beat all other alternatives in pairwise majority voting? Argue briefly (you may use diagrams). **(5 points)**

OR

This question pertains to part of the proof of the Sen and Pattanaik theorem (1969) as discussed in class.

Assume that individual preferences satisfy *Extremal Restriction* (ER). Suppose $\exists$ an individual i with strict preference between three alternatives x, y, z, such that $x \succ_i y \succ_i z$. Moreover assume that social ordering according to the majority rule satisfies the "forward cycle", that is, $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$. Under these circumstances, which kinds of individual preferences are feasible? **(5 points)**

(Note: The assumptions mentioned above are inconsistent and as we know, will lead to a contradiction. However, this question *does not* ask you to complete the proof and reach the contradiction.)

4. This question deals with lobbying under three possible and equally likely states of the world, namely θ_L, θ_M and θ_H. $\theta_L < \theta_M < \theta_H$, as discussed by Grossman and Helpman. Consider a single policy variable and a single lobby to answer the following questions:

(i) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. Recall $\delta > 0$ to be the size of divergence between the ideal policies of the policy maker and the lobby. What is the range of δ under which a full-revelation/fully-separating equilibrium is sustainable? **(9 points)**

(ii) Let $\theta_L = 1, \theta_M = 7$, and $\theta_H = 11$. What is the range of δ under which a partial-revelation/semi-separating equilibrium is sustainable? **(9 points)**

(iii) What happens to the range of δ that can be sustained in equilibrium when you move from an equilibrium in (i) to an equilibrium in (ii)? **(2 points)**

5. This question deals with lobbying as discussed by Grossman and Helpman. They conclude that "both the policymaker and the interest group may benefit from having lobbying not be free." Consider a single interest group, a single policy variable and two possible states of the world, to elucidate the above statement. **(20 points)**

OR

This question dwells on order-restricted preferences in the context of collective choice of tax-transfer schemes. There are n individuals, with preferences defined over two goods, a consumption good and leisure; let $c \in \Re_+$ denote units of the former and $l \in \Re_+$ those of the latter. Individual i's preferences over (c, l) are represented by a utility function of the Cobb-Douglas form

$$u_i(c, l) = c^{\alpha_i} l^{1-\alpha_i}$$

where $\alpha_i \in (0, 1)$. Suppose each individual has an endowment of 1 unit of time that can be allocated to leisure and work $(h = 1 - l)$ at a wage rate $w > 0$, with the price of the consumption good normalized to 1. Assume further that the collective decision to be made is over a set of proportional tax/transfer schemes on earned income. Specifically, the set of of possible tax/transfer schemes is $X \subseteq [0, 1] \times \Re$ with typical element (t, T) where $t \in [0, 1]$ is a proportional tax on labor income and $T \in \Re$ is a lumpsum transfer payment. Prove or argue otherwise that individual preferences over (t, T) schemes satisfy *Extremal Restriction*. (You may assume interior solutions throughout.) **(20 points)**

INDIAN STATISTICAL INSTITUTE
Second Semester Examination : 2017-18
M.S.(QE) II YEAR
Mathematical Programming

Date: 23 April 2018 Maximum Marks: 75 Duration: 3 hours

This paper carries 85 marks. Answer as much as you can.
However, maximum you can score is 75. Notations have usual meaning.

---

1 An engineering plant can produce five type of products: $P_1, P_2, \ldots, P_5$ by using two process - grinding and drilling. Each unit of any given product requires the following number of hours for each process, and contributes the following (in hundred \$) to the total net profit.

Production Planning Data

Product	P_1	P_2	P_3	P_4	P_5
Grinding time	12	20	0	25	15
Drilling time	10	8	16	0	0
Profit	55	60	35	40	20

The following points are also known.

(*i*) Each unit of every product takes 20 man-hours for final assembly.

(*ii*) The factory has 3 grinding machines and 2 drilling machines.

(*iii*) The factory works on 6 days a week with two shifts of 8 hours/day. Eight workers are employed in assembly, each working one shift a day.

(*iv*) If we manufacture P_1 or P_2 (or both), then at least one of P_3, P_4 and P_5 must also be manufactured.

Formulate the production planning problem as an ILP in order to maximize the net total profit.

[15]

2 Consider the transportation problem with m origins and n destinations. Let $\bar{A}$ be the coefficient matrix after one of the constraints (say, the last one) is dropped, i.e., $\bar{A}$ is an $(m+n-1) \times mn$ matrix. Show that every basis of $\bar{A}$ is upper triangular (possibly after appropriate permutation of its columns and rows).

[10]

[P.T.O.]

3 Consider the Hungarian method for solving assignment problem. Prove that at any iteration, the reduced cost matrix has the properties: (*i*) every element is non-negative, and (*ii*) every row and column has a zero element.

[6 + 9 = 15]

4 Consider the following profit matrix corresponding to different assignments of jobs to machines, where machines are in rows and jobs in columns. Obtain a profit maximizing (optimal) assignment under the usual assumptions.

	1	2	3	4	5
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

[13]

5 Consider the following linear programming problem:

$$\begin{aligned} \min\ z &= c^{\mathrm{T}}x \\ \text{subject to} & \\ Ax &\geq b \\ x &\geq 0. \end{aligned}$$

Let $\bar{x}$ be an optimal solution of this problem, and $\bar{z}$ be its associated objective value. If $\bar{y}$ is an optimal solution of its dual problem, show that $\bar{z} = \bar{y}^{\mathrm{T}}A\bar{x}$.

[10]

6 State whether the following statements, in connection with linear programming, are *True* or *False* using brief argument and/or counter-example.

(a) A non-basic feasible solution can never be optimal.

(b) Dual-feasibility of a basis implies its primal-optimality.

[4 + 6 = 10]

7 State convex programming problem. Show that every local optimal solution of convex programming problem is global optimal.

[2 + 10 = 12]

——*** xXx ***——

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2017-2018
M.S. (Q.E.) II
Theory of Finance I

Date: 24.04.18 Maximum Marks: 100 Time: 3 hours

Note: Clearly explain the symbols you use and state all the assumptions you need for any derivation. The paper carries 106 marks. You may attempt any part of any question. The maximum you can score is 100.

1. State the First Fundamental Theorem of Mathematical Finance. Demonstrate the theorem rigorously by proving all necessary preliminaries. (14)

2. A weakly risk-averse person who does not prefer less to more decides to invest in only one of the two uncertain prospects that are currently available. Develop an equivalent graphical device of this notion of dominance. Demonstrate your claim rigorously. (10)

3. When do you say that a future liability can be perfectly immunized by a bond with respect to interest rate risk? Develop a necessary and sufficient condition for this to hold. (10)

4. Give an analytical deduction of the Black-Scholes-Merton partial differential equation by giving necessary preliminaries, including derivation of the distribution of the stock price under geometric Brownian motion. (14)

5. Assume that a person's optimal portfolio consists of positive fractions of a risk-free prospect and a risky prospect. If his relative risk aversion measure is increasing in initial wealth, what can you say about the wealth elasticity of demand for the risky prospect? Prove or disprove your statement using rigorous arguments. (10)

6. Define a bottom straddle and determine its payoff function. Give a graphical exposition of the payoff function. When do you think a bottom straddle is appropriate as a hedging instrument? (2+5+3+2)

P.T.O

7(a). Does certainty equivalent remain invariant under affine transformations of the associated utility function? Justify your claim rigorously. (8)

7(b). Construct a utility function that exhibits risk loving behavior for small values of wealth and risk-averse behavior for high wealth levels. Identify the range of wealth for each situation.. (10)

8. Show how you can employ the riskless hedging principle to derive the price of a European call option in a two-period discrete time framework. (10)

9. Consider a sequence of net benefit schemes associated with a firm's investment in a project over a finite time horizon. Identify a set of sufficient conditions under which the corresponding positive internal rate of return is unique. (8)

Indian Statistical Institute
Second Semestral Examination: 2017 –2018
M.S. (Q.E) – II Year
Econometric Applications II

Date: **25.04.2018** Maximum Marks: **100** Duration: 3 hrs.

(Answer any <u>four</u> questions)

1. (a) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?
   (b) How can one estimate the sampling error of Laspeyres' price index using a regression framework?
   (c) What is 'Purchasing Power Parity (PPP)'? Why is it more appropriate than official exchange rate while making international comparison of level of living?
   (d) State the desirable properties that a PPP should satisfy. Describe the Geary-Khamis method of estimating PPP.

[4+5+5+ (3+8) =25]

2. (a) Explain the idea of resampling. What are the possible situations when resampling may be needed?
   (b) Describe the "Jackknife" and "Bootstrap" methods of resampling.
   (c) Show that under certain condition the bias in the Jackknife estimator is zero. State the condition clearly.

[5+10+10=25]

3. (a) Distinguish between the 'Unitary approach' and the 'Collective approach' for specification of household preferences. What are the different types of collective approach? Describe.
   (b) Starting with a Pareto efficient collective model, described as maximization of a Pareto weighted sum of household members' utility subject to an income constraint, demonstrate why Slutsky symmetry fails. Hence, specify a test for validity of the unitary framework.

(c) State the 'Symmetry plus Rank 1' (SR1) property in the above context. Show that to test for SR1 property, at least 5 goods are needed.

[8+8+ (2+7) = 25]

4. (a) Consider a general collective model specified by the following household utility function:

$$W(C^1, C^2, C^3, .., C^J, G; z, d) = \sum_j \mu_j (p, x, d) u^j (C^1, C^2, C^3, .., C^J, G; z),$$

where C^j is the consumption vector of the j-th member, G denotes consumption of public goods, z is the vector of demographic characteristics, x denotes household income, d is a vector of distribution factors, disjoint from z and $\mu_j(.)$'s are the Pareto weights.

Enumerate the different special cases by imposing restrictions on $\mu_j(.)$. For each case specify the type of model and state whether Slutsky symmetry is satisfied or not.

(b) What do you mean by 'Income pooling'? Does a collective model necessarily fail the 'income pooling' hypothesis?

(c) Show that if preferences are egoistic, then the household allocation problem using the utility function of the form as in (a) reduces to a two stage budgeting process. [Use a two-person household model].

[10+3+12 = 25]

5. (a) Enumerate the comparative features of parametric and nonparametric regression.

(b) Describe the procedure of bandwidth selection by minimizing the Integrated Squared Error (ISE) in the context of density estimation.

(c) In the nonparametric regression: $y_i = m(x_i) + \varepsilon_i$, describe the alternative methods of finding the derivative of $m(x_i)$.

(d) What could be a possible use of such derivatives in the context of demand analysis?

[5+8+10+2 = 25]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2017-18

MSQE II 2017-18

26 th

Auction Theory

Date: ~~26~~th April 2018 Maximum marks: 50 Duration: 3 hours

Answer all questions

1. Consider the independent private values model. Directly derive symmetric, increasing, differentiable equilibrium bid functions for the first-price all pay auction, assuming one exists (without appealing to the revenue equivalence principle). [10]

2. Consider the common values model, and assume a second-price auction.

a) Characterize symmetric, increasing, differentiable equilibrium bid functions, assuming one exists, and show there is a unique such function. [10]

b) Show that equilibrium exists. [5]

3. In the common values model show that expected seller revenue from the second-price auction is no less than that from the first-price auction. [10]

4. Suppose there are two independent random variables X_1 and X_2, both uniformly distributed over $[0, \omega]$. Suppose also there are two bidders, 1 and 2, with bidder i observing the realization of X_i (assume the distributions are common knowledge). The common value of an object being sold is $v = \frac{x_1+x_2}{2}$, where x_i is the realization of X_i. What is the symmetric equilibrium bidding strategy, assuming a second-price auction? [15]

Indian Statistical Institute

Economic Development I

MSQE I & II

Semestral Examination

Date: 27 April, 2018

Time: 3 hours

Maximum Marks: 60

Answer Question 1 and any two from the remaining.

1. Consider a scenario where there are two political parties: an incumbent in power and an opposition. Also, there is a large number of voters. Total number of voters is normalized to 1. There are two types of voters, type N comprising of a fraction γ of the total voters and type S comprising of the remaining . Each voter gets a noisy signal s which is uniformly distributed around the realized state of the economy θ. Type N voters vote for the incumbent if $E[\theta|s] \geq \bar{\theta}$, an exogenously specified level of performance of the economy. The S type is in a patron-client relationship with the political parties and votes on the basis of expected personal benefits. Finally, $\theta = e + \omega$ where e is the effort level of the incumbent and ω is a random shock. The incumbent maximizes its probability of winning net of effort costs by choosing the level of effort.
   (a) By suitably developing a model based on the above information show that an increase in the proportion of S voters reduces the effort level of the incumbent.
   (b) Now suppose that $\theta = \tau e + \omega$, $0 < \tau < 1$. Here τ measures the effectiveness of effort or alternatively the efficiency of the incumbent. Show that an increase in τ increases the probability of the incumbent to win the next election.

[12+8]

2. Show how lending with joint liability can reduce the inefficiencies due to (a) moral hazard; and (b) costly state verification.

[10 +10]

3. Developing a suitable model of repeated game, characterize an implicit insurance contract between two economic agents based on reciprocity. Show that in situations of extreme inequality the first best cannot be achieved.

[15+5]

P.T.O

4. Consider an agricultural market where a single commodity is sold over an interval [0,T] to satisfy demand which is linear in price at each point in time. There are n oligopolistic sellers in the market each with a storage cost c per unit of stock per unit of time. Show that

   (a) sellers with smaller initial stocks will leave the market early by exhausting their stocks.

   (b) If the possibility of international trade opens up and the sellers have the option of selling to the international market at a fixed price, only large sellers will sell to both domestic and international markets and small sellers will sell to the domestic market alone.

[12 + 8]

INDIAN STATISTICAL INSTITUTE

Final-Semestral Examination : 2017-18

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 27.4.18 Maximum Marks: 50 Duration: 3 hours

1. Consider a social choice problem with the set of players $N = \{1, 2\}$ and the set of alternatives $A = \{a_1, a_2, \ldots, a_5\}$. Let $\mathcal{D}$ be the (ordinal) single-peaked domain over A. Suppose the utility of a player $i \in N$ of an alternative $a \in A$ at a preference $P_i \in \mathcal{D}$ is defined as $u_i(a, P_i) = 6 - r(a, P_i)$, where $r(a, P_i)$ is the rank a at P_i. Consider a prior distribution μ over $\mathcal{D}^2$. Define the ex-ante utility of a social choice function $f : \mathcal{D}^2 \to A$ as
$$U(f) = \sum_{(P_1,P_2)\in\mathcal{D}^2} \mu(P_1, P_2) \sum_{i\in N} u_i\left(f(P_1, P_2), P_i\right).$$
Answer the following questions.
   (a) Find the social choice function(s) on $\mathcal{D}^2$ that maximize(s) the ex-ante utility with uniform prior distribution.
   (b) Find the strategy-proof social choice function(s) on $\mathcal{D}^2$ that maximize(s) the ex-ante utility with uniform prior distribution.

(10)

2. Show that if an allocation function on the unrestricted domain satisfies 2 cycle-monotonicity, then it satisfies 4 cycle-monotonicity.

(10)

3. Suppose $N = \{1, \ldots, n\}$ is the set of players, A is the set of outcomes with $|A| = m$, and for all $i \in N$, $T_i \subseteq \mathbb{R}^m$ is the type-space of player i. Consider the following affine-maximizer allocation function: for all $t_N \in \prod_{i\in N} T_i$,
$$f(t_N) \in \arg\max_{a\in A} \sum_{i\in N} \lambda_i t_i(a) + \kappa(a),$$

where, for all $i \in N$, $\lambda_i > 0$ is the weight parameter of player i, and for all $a \in A$, $\kappa(a)$ is the reservation price of the outcome a.

(a) Show that f satisfies k cycle-monotonicity for all $k \geq 2$.

(b) Find all payment functions $(p_i)_{i \in N}$ such that $(f, (p_i)_{i \in N})$ is incentive compatible.

(c) Is there a domain $\hat{T} = \prod_i \hat{T}_i$ and payment functions $p_i : \hat{T} \to \mathbb{R}$ for all $i \in N$ such that $(f, (p_i)_{i \in N})$ on $\hat{T}$ is incentive compatible, individually rational?

(10+5+5)

4. Prove or disprove the following: There are incentive compatible allocation functions other than affine maximizers and min-max rules on a (cardinal) single-peaked domain.

(10)

Indian Statistical Institute

Final Examination 2018, MSQE I and II

Course name: **Political Economy**

Subject name: **Economics**

Date: **4 May 2018**

Maximum marks: **100**

Duration: **3 hours**

1. In the context of voter turnout, Grossman and Helpman conclude, "The paradox in voting is ... the choice by a reasonably high percentage of eligible voters to bear the cost of voting." Elucidate the paradox. **(5 points)**

2. Within the Palfrey-Rosenthal framework of strategic voting as discussed in class, and using simple numerical figures for team sizes, demonstrate the forces of "competition" and "free-riding" that individuals face when deciding whether or not to vote. (You may assume the coin-toss rule for breaking ties and all individuals facing an identical voting cost given by c.) **(10 points)**

(Hint: Consider using simple 2x2 games and their Nash equilibria to make your point.)

3. Consider participation games as discussed by Palfrey and Rosenthal in the context of strategic voting. According to them, "The conclusion is that pure strategy equilibria fail to exist except for a few very special cases." Assume c, the identical cost of voting of all citizens, to be less than $1/2$, and coin-toss rule for breaking ties, to substantiate their conclusion. **(15 points)**

4. This question pertains to the characterization of "mixed-pure" equilibria as discussed by Palfrey and Rosenthal in the context of strategic voting. Suppose $M, N > 1$. (Recall M and N are the number of members in teams 1 and 2 respectively.) Let $k \in \mathbb{N}$ be such that $k \leq \min\{M-1, N\}$. Recall that c is the exogenously given cost of voting for each player. We know that if $c \leq \binom{M-1}{k}\left(\frac{k}{M}\right)^k\left(1-\frac{k}{M}\right)^{M-1-k}$, then there is an equilibrium of G_2 (status-quo rule) in which exactly k members of team 2 vote and all team 1 members vote with probability q. Show that q satisfies $c = \binom{M-1}{k}q^k(1-q)^{M-1-k}$. **(5 points)**

(Hint: This question only asks you to demonstrate the best response of a team 1 member, given the strategy of the other team - it *does not* ask you for the proof of the proposition.)

P.T.O

5. What is the expected turnout in a "$q-k$" or "mixed-pure" equilibria as discussed by Palfrey and Rosenthal? What are its drawbacks when positive voting costs of individuals and large electorates are considered? **(5 points)**

6. This question pertains to a parameterized version of Feddersen and Sandroni's ethical voting model. Let the fraction of ethical agents in groups 1 and 2, $\tilde{q}_1$ and $\tilde{q}_2$ respectively, be independently and identically distributed as $U[0,1]$. Let the fraction of the population in group 1 be deterministic and be given by $k \in (0, 1/2]$. Let cost of voting for each individual be random and be drawn from $U[0, \bar{c}]$. Let the payoff from 'doing one's part', D, be $> \bar{c}$. Let the social cost function be linear, that is $v(x) = x$. Recall w to be the parameter capturing the 'importance of election'.

(i) Let parameters $\bar{c}, w$ and k satisfy $\frac{\bar{c}}{w} > \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(20 points)**

(ii) Let $\frac{1}{(1-k)^2} \leq \frac{\bar{c}}{w} \leq \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(8 points)**

(iii) Let $\frac{\bar{c}}{w} \leq \frac{k}{(1-k)^2}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(8 points)**

(iv) Consider your answer in part (i). What can you conclude about the participation rates of the minority and the majority? **(6 points)**

(v) From (i), what can you conclude about the chances of winning of the minority versus that of the majority? **(6 points)**

(vi) What is total expected turnout? How does it vary with the 'level of disagreement' in the economy? How does it vary with average voting costs? How does it vary with the 'importance of the election'? **(4+4+2+2=12 points)**

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION, 2017-2018

M.S.(Q.E.) II year

Theory of Finance I

Back Paper

Date: 04/06/2018 Maximum Marks: 100 Time: 3 hours

Note: Clearly explain the symbols you use and state all the assumptions you need for any derivation.

1.Show that of two persons if the former is more risk-averse than the latter in the Arrow-Pratt absolute sense, then to avoid investment in an even prospect the maximum amount that the former would be willing to pay is higher than that of the latter. Define all preliminary necessities. (14)

2. Define certainty equivalent and explain it graphically. Demonstrate its continuity rigorously. (1+1+10)

3. Show that in a one-step Binomial model existence of a risk neutral probability is equivalent to non-arbitrage. (15)

4. Define and graphically explain first order stochastic dominance. Clearly demonstrate its expected utility equivalence. (1+1+10)

5. Derive Ito's Lemma rigorously and explain its usefulness by giving one example. (12+3)

6. (a) Determine the payoff function of butterfly-price spread strategy and plot the graph of this function. (8)

(b) Repeat the above steps for a bottom straddle. Make a systematic comparison between the two hedging policies. (8+4)

7 Assume that a person's optimal portfolio consists of positive fractions of a risk-free prospect and a risky prospect. If his absolute risk aversion measure is decreasing in initial wealth, how

P.T.O

does the amount invested in the risky prospect change? Prove or disprove your claim using rigorous arguments (12)