INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: (2016-2017)

MS (Q.E.) II Year

Macroeconomics II

Date:05.09.2016 Maximum Marks: 40 Duration: 3 hours

Group – A
Answer any two

1. What do you mean by a negative productivity shock? Analyse how it explains stagflation.

(2 + 8 = 10)

2. Analyse the validity of the policy impotence result under rational expectation hypothesis using an Aggregate Demand-Aggregate supply model.

(10)

3. What is adaptive expectation hypothesis? Analyse how it explains hyperinflation in the model of Cagan.

(10)

Group – B
Answer all questions

1. In an infinite horizon model, work out the dynamics of per capita asset accumulation for a small open economy, facing a constant rate of interest in the world capital market. In this context discuss the problems associated with either a very low or a very high rate of interest.

[10]

2. Assuming a Cobb Douglas production function, analyse how the savings rate behaves in an infinite horizon Ramsey model while in transit towards the steady state.

[10]

We can define $u_B(x_A, x_B)$ likewise. Let $G_{PL} := (\{A, B\}, \{X, X\}, \{u_A, u_B\})$. Consider the mixed extension of G_{PL} and denote it by G'_{PL}. Then show that

(i) G'_{PL} has a Nash equilibrium. **(5 points)**

(ii) Any Nash equilibrium (p_A, p_B) of G'_{PL} has support$(p_k) \subseteq X^*$, $k = A, B$, where X^* is the set of Condorcet winners in X. **(10 points)**

3. Suppose preferences of individuals for a service consists of two dimensions, p_1 and p_2. Suppose there are three individuals, a 'low' consumer, a 'middle' consumer and a 'high' consumer with bliss points $(1, 1)$, $(2, 3)$ and $(4, 4)$ respectively. Suppose individuals can directly vote on alternative pairs. Prove or argue otherwise whether a 'median-voter' analogue may still hold good. That is, is it true that the $(2, 3)$ alternative will beat all other alternatives in pairwise majority voting? Argue briefly (you may use diagrams). **(5 points)**

OR

This question pertains to part of the proof of the Sen and Pattanaik theorem (1969) as discussed in class.

Assume that individual preferences satisfy *Extremal Restriction* (ER). Suppose $\exists$ an individual i with strict preference between three alternatives x, y, z, such that $x \succ_i y \succ_i z$. Moreover assume that social ordering according to the majority rule satisfies the "forward cycle", that is, $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$. Under these circumstances, which kinds of individual preferences are feasible? **(5 points)**

(Note: The assumptions mentioned above are inconsistent and as we know, will lead to a contradiction. However, this question *does not* ask you to complete the proof and reach the contradiction.)

4. This question deals with lobbying as discussed by Grossman and Helpman. They conclude that "both the policymaker and the interest group may benefit from having lobbying not be free." Consider a single interest group, a single policy variable and two possible states of the world, to elucidate the above statement. **(20 points)**

OR

This question, again, deals with lobbying as discussed by Grossman and Helpman. They conclude that "As the number of possible states grows, full revelation becomes ever more difficult to achieve." Consider a single interest group, a single policy variable and let the possible states of the world increase from two to three, to elucidate the above statement. **(20 points)**

OR

This question dwells on order-restricted preferences in the context of collective choice of tax-transfer schemes. There are n individuals, with preferences defined over two goods, a consumption good and leisure; let $c \in \Re_+$ denote units of the former and $l \in \Re_+$ those of the latter. Individual i's preferences over (c, l) are represented by a utility function of the Cobb-Douglas form

$$u_i(c, l) = c^{\alpha_i} l^{1-\alpha_i}$$

where $\alpha_i \in (0, 1)$. Suppose each individual has an endowment of 1 unit of time that can be allocated to leisure and work ($h = 1 - l$) at a wage rate $w > 0$, with the price of the consumption good normalized to 1. Assume further that the collective decision to be made is over a set of proportional tax/transfer schemes on earned income. Specifically, the set of of possible tax/transfer schemes is $X \subseteq [0, 1] \times \Re$ with typical element (t, T) where $t \in [0, 1]$ is a proportional tax on labor income and $T \in \Re$ is a lump-sum transfer payment. Prove or argue otherwise that individual preferences over (t, T) schemes satisfy *Extremal Restriction*. (You may assume interior solutions throughout.) **(20 points)**

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
MID-SEMESTRAL EXAMINATION: 2016-17
M.S. (Q.E.) II Year
Econometric Methods II

Date: 09.09.2016 Maximum Marks: 100 Time: 3 hours

*[This question paper carries a total of **110** marks. You can answer any part of any question; but the maximum that you can score is 100. Marks allotted to each question are given within parentheses.]*

1. Consider estimating the standard probit model where $\text{prob}(y_i = 1) = \Phi(\beta' x_i)$ and y is a binary variable that takes on the value 0 or 1.

   (a) How would the estimated coefficients compare if one ran a probit model on the same data, except that y has been recoded to take on a value of 0 or 10?

   (b) Repeat the preceding question for the case when the model is a logit.

   (c) Do the same for a linear probability model and discuss how the coefficients should be interpreted in this case.

   [10+6+6= 22]

2. (a) Show that the log-likelihood function of the logit model is globally concave. Also interpret the marginal effects corresponding to the logit model in terms of 'odds ratio'.

   (b) Describe Berkon's minimum chi-square estimator for logit / probit model for grouped data.

   [11+11= 22]

3. (a) Explain what is meant by volatility in the context of financial time series. Also obtain the unconditional variance of a GARCH (p,q) process where $p>q$.

   (b) Show that an ARCH (1) process entails a symmetric distribution with tails heavier than the normal distribution.

   [11+11 = 22]

P.T.O

4. (a) Explain what is meant by QMLE, and then discuss briefly how robust inference about the parameters of an ARCH regression can be achieved based on QMLE under assumed distribution of normality .

   (b) What is 'leverage effect' in the context of stock markets? Is it relevant for other financial variables as well? Explain. Also discuss how 'leverage effect' is captured by the GJR model. .

[12+10 = 22]

5. (a) Explain why ARCH-M model is important in describing the risk-return relationship in a financial market. Discuss, with the help of a simple ARCH-M model, if taking more risk is always gainful to a potential investor in stock markets.

   (b) Consider a simple AR (1) model and ARCH (1) model for the conditional mean and conditional variance, respectively, for returns data. Obtain the h-step ahead point forecast as well as forecast interval for returns.

[12+ 10 = 22]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2016-2017
M.S. (Q.E.), 2nd Year
Econometric Applications I

Date: 11 September 2016 Maximum marks: 100 Duration: 3 hours

[Answer question no. 1 and any **two** from the rest of the questions.]

1. Suppose the population is divided into two groups. Group 1 consists of persons having income less than the overall mean income and Group 2 consists of the remaining persons. If the mean incomes of the two groups are Rs. 4000 and 12000 per month respectively, then find the share of income of the bottom 10% people in the population assuming that the income follows Pareto distribution. Also find the mean and median of the income of the bottom 10% people. [30+10=40]

2. (i) Suppose X is a size distribution. Define Lorenz Curve (LC) and hence Lorenz Ratio (LR) of X. Show that it can be expressed as

$$LR = 1 - 2\int_0^1 F_1 dF .$$

(ii) Prove that LR can also be found from

$$E|Y_1 - Y_2|/(2M),$$

where Y_1 and Y_2 are i.i.d. having the same distribution function F(.) and M is their common mean.
(iii) State and prove the moment distribution property of lognormal distribution. Hence derive the formula of Lorenz Curve of Lognormal distribution. State and prove its properties.

(iv) Show that if $X \sim LN(\mu, \sigma^2)$ then $LR = 2\Phi(\sigma/\sqrt{2}) - 1$. [7+6+12+5=30]

[Assume that the symbols have their usual meaning.]

3. (a) State and explain the (i) Scale Independence, (ii) Pigou-Dalton Principle of Transfer and (iii) Principle of Transfer properties of an inequality measure.
(b) Hence, examine the incomes of the following three groups of persons and arrange the groups in increasing order of their inequalities, if possible, giving sufficient reasons without taking any specific inequality measure.

Group-I: (10, 20, 30, 40), Group-II: (30, 30, 70, 70) & Group-III: (12, 12, 20, 36)

(c) Write the important steps in deriving the Atkinson's measure of inequality. How will you interpret the unknown parameter in the measure? [9+9+12=30]

4. Define three-parameter lognormal distribution and derive its pdf. State and prove its properties including Mean, Median, Mode, quantiles, dispersion, skewness, kurtosis, Lorenz Ratio, etc. How can one test whether a set of data is coming from a three parameter lognormal distribution? Also discuss some estimation procedures for this distribution. [5+12+5+8=30]

5. Write short notes on any two of the following:
   (a) Pareto law.
   (b) Kapteyn's LPE model and its modification by Kalecki.
   (c) Positive measures of Inequality. [15+15=30]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2016-2017)

MS(QE) II

Individual and collective choice

Date: 13.09.2016 Maximum Marks: 100 Duration: 3 hrs.

(1) Consider the social aggregation problem, where $A = \{x, y\}$ is the set of alternatives, $|A| = 2$, N is the finite set of agents and $|N| \geq 2$.
   (a) Suppose that the social welfare function satisfies symmetry and neutrality and the number of agents preferring x over y is the same as the number of agents preferring y over x. Then show that the social welfare function necessarily prescribes indifference between x and y.
   (b) Define non-triviality and positive responsiveness of a social welfare function.
   (c) Can you find a social welfare function that satisfies non-triviality and positive responsiveness but fails to satisfy Pareto? Justify your answer.
   (d) Find a *non-dictatorial* SWF F that satisfies Pareto, neutrality and non-triviality but fails to satisfy symmetry and positive responsiveness. **(10+4+8+8=30)**

(2) State Arrow's impossibility theorem by providing all the relevant definitions. Prove Arrow's impossibility theorem using the notion of an 'extremely pivotal' agent. **(10+20=30)**

(3) Define quasi-transitivity and acyclicity of a binary relation R on the finite set of alternatives A. State the field expansion lemma and the veto field expansion lemma. Using the veto field expansion lemma and the field expansion lemma show that any quasi-transitive social ordering satisfying unrestricted domain, weak Pareto and independence of irrelevant alternatives must be oligarchic. **(4+4+12=20)**

(4) State and prove the median voter theorem. Provide all the relevant definitions. **(4+10+6=20)**

P.T.O

Midterm syllabus-Individual and collective choice-2016

(1) Social welfare function with two alternatives
    (a) May's theorem
(2) Arrow's impossibility theorem
    (a) Two proofs of Arrow's theorem
        (i) Arrow's proof
        (ii) Proof by Geanakoplos
(3) Relaxing Arrow's axioms
    (a) Relaxing weak Pareto: Wilson's theorem
    (b) Quasi-transitivity, acyclicity, oligarchy and veto power
    (c) Domain restriction: Single peaked preferences
(4) Social Choice Function
    (a) The Gibbard-Satterthwaite Theorem

Indian Statistical Institute
Mid-Semester Examination: 2016-2017
MS(QE) II: 2016-2017
Industrial Organization

Date: 13/09/2015 Maximum Marks: 40 Duration: $2\frac{1}{2}$ Hours

You may answer ALL the questions. But your score will not exceed 40.

1. A monopolist produces two products, X_1 and X_2, at zero cost. There is a mass 1 of consumers. A γ fraction of consumers are heterogeneous described by their type θ which is distributed uniformly over the unit interval. The θ-consumer has willingness to pay $r_1 = \theta$ and $r_2 = 1 - \theta$ for each unit of X_1 and X_2, respectively. A fraction $\frac{(1-\gamma)}{2}$ of consumers has willingness to pay $r_1 = \frac{2}{3}$ and $r_2 = 0$, and the remaining fraction $\frac{(1-\gamma)}{2}$ of consumers has willingness to pay $r_1 = 0$ and $r_2 = \frac{2}{3}$. Let P_1 and P_2 be the prices under independent pricing strategy and P_b be the bundle price (a bundle consisting one unit of each product) under pure bundle pricing strategy. Answer the following.

(a) Suppose that $\gamma = 1$. Determine the prices under each of independent selling and bundle selling strategy. Which strategy is optimal for the monopolist?

(b) Suppose that $0 < \gamma < 1$. Then characterize the solution under independent selling for all γ.

[10+10=20]

P.T.O

2. (a) A monopolist produces a good at constant marginal cost $c < 1$. The market demand for the product is $D(p) = 1 - p$.

(i) Suppose that the monopolist cannot discriminate in pricing, hence charges a uniform price P_u per unit. Find the profit maximizing price and the corresponding profit of the monopolist.

(ii) Suppose that the monopolist can charge a two part tariff (F, p) where $F \geq 0$ is the fixed fee and p is the price per unit. Find the optimal values of F and p that maximize the monopoly profit.

(b) Suppose now that there are two types of consumers. The consumers of type 1 have demand $D_1(p) = 1 - p$ and the consumers of type 2 have demand $D_2(p) = 1 - \frac{p}{2}$. The population is of size 1, and there are equally many consumers of the two types. Finally assume in this question that $c = \frac{1}{2}$.

Calculate the two-part tariff that maximizes the profits of the monopolist. [Note that in equilibrium the monopolist may serve only one group of consumers or both groups.]

[(3+7) + 10=20]

3. Consider the monopoly problem of menu pricing when there are two types of consumers: low demand consumers and high demand consumers. Formulate the problem and write the relevant participation constraints and incentive constraints. Identify which of these constraints will be binding.

[5]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination: (2016-2017)

MS (Q.E.) II Year

International Economics I

Date: 15.09.2016 Maximum Marks 40 Duration 3 hours

Group A

Answer all questions

1. Consider a small open economy producing two goods, X and Y. Good X is produced with labour (L_X) and capital (K_X) through a smooth production function$F(L_X, K_X)$ exhibiting diminishing returns and constant returns to scale. Good Y is produced by labour alone and the amount of labour required to produce one unit of Y is a constant a_{LY}. The economy is endowed with some labour, which is mobile across sectors and some capital which is used only in sector X. Both factors are fully employed.
   (i) Find the effect of an exogenous rise in the relative price of good X on factor prices.
   (ii) Find the effect of an exogenous increase in labour endowment on the levels of output.

   [5 + 5 = 10]

2. Use a specific factors model to demonstrate that though in the short run the interests of the two sector specific capitals are opposed to each other, in the medium run as capital starts moving from the low return sector to the high return sector, the interest of capital as a whole is in conflict with that of labour.

   [10]

Group B

Answer all questions

1. Show that in a three-agent setting, a transfer paradox might occur even when the equilibrium is Walras stable. In this context discuss the role of substitution effects in ensuring normal results. [10]

P.T.O

2. Consider a 2 country, 2 commodity trading world, with perfectly competitive markets and show that imposing an ad- valorem export tax is the same as imposing an ad-valorem import tariff when the government redistributes all tax and tariff revenues lump sum. Also derive the optimal export tax.

   [Note: An export tax on good i means the following: $p_i(1+\tau) = p_i^*$, where p_i is the domestic price and p_i^* is the international price of good i and τ is the ad valorem export tax rate.] [10]

INDIAN STATISTICAL INSTITUTE
Supplementary Examination: 2016-2017
M.S. (Q.E.), 2nd Year, Semester I
Econometric Applications I

Maximum marks: 60 Duration: 2 hours Date: 26 October 2016

Note: Answer **Question no. 1** and any **two** questions from the rest.

1. Calculate the (i) Coefficient of Variation (CV), (ii) Relative Mean Deviation (RMD) and (iii) Lorenz Ratio of incomes of N individuals with incomes 1, 2, 3, ..., N units, respectively. What will happen if N goes to infinity? Explain. [20]

2. State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? State and prove some properties of Pareto distribution. [20]

3. Describe how one can construct Lorenz Curve and hence Lorenz Ratio graphically and numerically given grouped data. What happens if raw data are given instead of grouped data? [20]

4. Write short notes on any TWO of the following:
   (a) Law of Proportionate Effect (LPE) due to Kapteyn and its modification by Kalecki.
   (b) Lorenz Curve and Lorenz Ratio of two-parameter lognormal distribution.
   (c) Properties of three-parameter lognormal distribution. [10+10=20]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
FIRST SEMESTER SUPPLEMENTARY EXAMINATION: 2016-17
M.S.(Q.E.) II Year
Econometric Methods II

Date: 28.10.2016 Maximum Marks: 100 Time: 2 hours

Answer **ALL** questions.
Marks allotted to each question are given within parentheses.

1. a) Show that an ARCH (1) process entails a symmetric distribution with tails heavier than that of the normal distribution.

   b) Show that for a GARCH (1,2) process $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \beta_1 h_{t-1}$, a set of sufficient conditions for $h_t > 0$ is given by $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $\alpha_1 \beta_1 + \alpha_2 \geq 0$.

   (c) Discuss why EGARCH formulation is better than the usual GARCH model. Also obtain the standard properties of an EGARCH process.

   [12+7+11= 30]

2. a) Show that a GARCH (p, q) process for a time series $\{X_t\}$can be represented as an ARMA process of suitable orders for $\{X_t^2\}$.

   b) Show that the information matrix is block diagonal between the mean and the variance parameters of an ARCH (p) model in regression framework. Do you think that the same hold for an ARCH-M model? Justify your answer.

   [8+17 = 25]

3. a) Show that the log-likelihood function of the logit model is globally concave. Also interpret the marginal effects corresponding to the logit model in terms of 'odds ratio'.

   b) Describe Berkon's minimum chi-square estimator for logit/probit model in case of grouped data.

   c) Distinguish between truncated and censored models.

   [15+13+7= 35]

4. Consider a simple AR(1) model and ARCH (1) model for the conditional mean and conditional variance, respectively, for return data. Obtain the h-step ahead forecast interval for returns.

[10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2016-2017)

MS (Q.E.) II Year

Macroeconomics II

Date: 15/11/2016 Maximum Marks 60 Duration 3 hours

Group – A (Answer any two)

Q. 1. (a) How do you derive the equation of New-Keynesian Philips curve (NKPC) from the behavior of the firm? How does the NKPC differ from a traditional Philips curve?

(b) What are the properties of steady-state equilibrium in the New-Keynesian Business Cycle model. Analyse how a cost-push shock affects the steady-state equilibrium. ((3+2) + (2+8) = 15)

Q. 2. (a) How does a Real Business Cycle model differ from a neoclassical growth model?

(b) How does a Real Business Cycle model differ from a New-Keynesian Business cycle model?

(c) Using a two period model of household behavior under certainty, derive the equation of consumption smoothing as well as the equation of intertemporal substitution of labour. How are these two important to explain features of business cycle. (3+2+(8+2) =15)

Q. 3. Structural inflation is the result of supply-demand imbalance in the food market as well as of wage rigidity in the industrial sector – Explain the validity of the statement in the light of the contribution made by Cardoso (JDE, 1981) (15)

Group – B (Answer all questions)

1. In the Blanchard-Yaari model with cohort *dependent* wage, what would be the effect of a sharper decline in wage with respect to age on the steady state capital accumulation?

   Consider the same model, but now with zero population growth, cohort *independent* wages and open to international asset market with a constant rate of interest. Can you show that the aggregate savings in this model is negatively related to the level of assets?
   [Savings = Total income – Total Consumption, where Total income = wage income + interest income on assets. Also assume that the steady state exists in the model.]

   [10+5=15]

2. Show that with investments having convex adjustment cost, the capital stock exhibits smooth transitional dynamics even when the country is small in the international capital market facing a constant rate of interest.
   Show that if the production function and the investment adjustment cost function are both linear homogeneous then Tobin's marginal q would be equal to the average q.

   [10 +5=15]

Indian Statistical Institute
First Semestral Examination: 2016-2017
MS(QE) II: 2016-2017
Industrial Organization

Date: 18/11/2016 Maximum Marks: 40 Duration: 3 Hours

Note: Answer any THREE questions. Your total score cannot exceed 40.

1. Consider a Stackelberg leader-follower structure with n leaders and m followers: first, n leaders choose the quantities simultaneously, and then observing the total quantities of the leaders, m followers choose their quantities simultaneously; finally price of the product is set to clear the market. Let the market demand for the product be given by: $p(X) = 1 - X$; unit cost of production of each firm is assumed to be zero. Answer the following:
   a) Derive the payoff expression of each of the leaders and followers. [8]
   b) If one leader and one follower merge together and the merged firm behaves as a leader, will such a bilateral merger be profitable to the merging firms? 4]
   c) What would happen if the merged firm would behave as a follower? [3]

2. (a) Make clear the concept of parallel rationing rule and proportional rationing rule. [3]
   (b) Suppose that two identical firms in a homogeneous product market compete in prices. The capacity of each firm is 3. The firms have constant marginal cost equal to zero up to the capacity constraint. The market demand is given by the function $X(P) = 9 - P$ where P is the product price. If the firms set the same price, they split the demand equally. If the firms set a different price, the demand of each one of the firms is calculated according to the parallel rationing rule. (i) Show that $P_1 = P_2 = 3$ can be sustained as an equilibrium price. (ii) If parallel rationing rule is replaced by proportional rationing rule, will $P_1 = P_2 = 3$ remain to be a pure strategy equilibrium? Explain. [6+6=12]

3. Consider a duopoly market with market demand for the product given by $p = 2 - x_1 - x_2$, where is x_i is output of firm i. Firm 1's MC is 1 and this is common knowledge. However firm 2's MC is determined by Nature. It is $5/4$ with probability $1/2$ and $3/4$ with probability $1/2$. Firms choose quantities simultaneously and non-cooperatively. Find the expected payoff of each firm prior to nature's move in the following cases;

   (i) At the stage of production neither firm knows firm 2's cost. [6]

   (ii) At the stage of production firm 2's cost is private information. [6]

   (iii) At the stage of production both firms know their costs and this is common knowledge. [3]

4. Let there be **three** firms competing for a product innovation. An R&D activity involves setting up a research lab. It costs an amount $I = 1$ and has a success probability $\alpha = 1/2$. Further assume that if r firms compete in the product market, then each such competing firm derives a gross payoff V/r. Answer the following questions.

(a) Calculate the minimal value of V that ensures that each firm will invest in constructing a lab. [4]

(b) Suppose now that firm 3 went out of business, and that a foreign firm purchased the two remaining firms. Calculate the minimal value of V that would induce the foreign owner of the two firms to run two separate research labs instead of operating only one lab. [5]

(c) Given that the foreign firm owns two firms (1 and 2), if firm 3 (domestic firm) decides to remain in business, under what conditions will the foreign firm run its two research labs instead of only one lab whereas firm 3 will invest in its single lab? [6]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2016-2017)

MS(QE) II

Individual and Collective Choice

Date: 18.11.2016 Maximum Marks: 100 Duration: 3 hrs.

Note: Answer all questions.

(1) Assuming unrestricted domain, define the social choice function when there are two agents and the individual preferences are rational and strict. Define the full-range property, the unanimity property and strategyproofness of a social choice function. Show that if a social choice function satisfies unanimity, then it also satisfies the full-range property. Also show that if a social choice function satisfies the full-range property and strategyproofness, then it also satisfies the unanimity property. **(2+(3+3+4)+5+8=25)**

(2) Define single-peaked preferences (first define all the technical terms you will use in the definition). Assuming a given linear order, consider the social choice function on the domain of all single-peaked preferences that selects the leftmost peak under all profiles. Show that this social choice function is strategyproof. **(5+15=20)**

(3) State the Cubical Array Lemma for the pure public goods problem by giving all the relevant definitions. Using this lemma show that there is no balanced VCG mechanism for the pure public goods problem . **(8+15=23)**

(4) Define the pivotal mechanism for the pure public goods problem. Show that this mechanism is dominant strategy incentive compatible and feasible. **(2+10+5=17)**

(5) Define a simple lottery and a compound lottery. Give examples of two compound lotteries that reduce to the same simple lottery. Let $U : \mathcal{L} \to \Re$ be the utility function defined on the space of all simple lotteries $\mathcal{L}$. Show that the utility function has the expected utility form if and only if it is linear. **(2+3+10=15)**

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
M.S. (Q.E.) II Year (2016 – 17)
Semester I Examination
Econometric Methods II

Date: 21-11-2016 Maximum Marks: 100 Time: 3 hours

*This question paper carries a total of **120** marks. But the maximum that you can score is 100. Marks allotted to each question are given within parentheses.*

1. (a) Show that if a time series $\{x_t\}$ follows a GARCH (p, q) process, then the process can be represented as an ARMA process of suitable orders for the time series $\{x_t^2\}$.
   (b) Consider the following GARCH-M model for return y_t

$$y_t = \delta h_t^2 + \in_t$$

   where $\in_t | \varphi_{t-1} \sim N(0, h_t), h_t = \alpha_0 + \alpha_1 \in_{t-1}^2 + \beta_1 h_{t-1}, \varphi_{t-1}$ is the information set at t -1.
   (i) Obtain the unconditional second and fourth order central moments of the GARCH (1, 1) process.
   (ii) Find an expression for the expected return involving the parameters, and then comment if risk is worth taking.

(6+$\overline{8+6}$ = 20)

2. Consider the linear regression model: $Y = X\beta + \varepsilon, \varepsilon \sim N(0, \sigma^2 1_n)$, β is a $p \times 1$ vector of unknown parameters. Although the unconditional mean of ε is zero i.e., $E(\varepsilon) = 0$, you are not sure whether the conditional mean i.e., $E(\varepsilon | X) = 0$ or not. Propose a consistent estimator $\hat{\beta}$ for β under such a situation. Discuss how you would test the assumption $E(\varepsilon | X) = 0$. Also show that $\text{Cov}.(\hat{\beta} - \hat{\beta}_{ols}, \hat{\beta}_{ols}) = 0$ where $\hat{\beta}_{ols}$ is the OLS estimator of β.

(5+5+10 = 20)

3. (a) Suppose you have data on medical expenditures on a sample of individuals. Some of them, who did not have any ailments, or did not bother to go to the doctor even if they had ailments, had no expenditures. You are required to estimate a model where medical expenditure is regressed, *inter alia*, on income (of the individuals). You carry out this exercise by dropping the individuals with zero expenditures, and then estimating the model by OLS method. How would the estimated coefficient (associated with income) thus obtained compare with the one obtained by using an appropriate estimation method based on all observations? Explain your answer with derivations wherever required.

   (b) Suppose certain monetary/fiscal policies are set in place in an attempt to reduce unemployment rate across the country, and after some time an analysis is made of the experience of the various states. Due to historical developments, the states are likely to

P.T.O

have variable mixtures of industrial, commercial, private and public structure. Do you think a fixed effect modelling framework for the analysis of the underlying panel data would be appropriate? Explain your answer clearly.

(10 + 10 = 20)

4. (a) Consider the following model for stationary $\{y_t\}$

$$y_t = \Phi y_{t-1} + u_t, |\Phi| < 1, t = 1, 2, \ldots, n$$
$$u_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \text{ where } \varepsilon_t \sim WN(0, \sigma_\varepsilon^2).$$

Show that the OLS is not an appropriate method of estimation of the parameters of this model.

Could y_{t-2} and y_{t-3} be used as instruments? Justify your answer with derivations wherever necessary.

(b) Discuss how you can test the different hypotheses of interest in the context of panel data analysis.

$(\overline{6+6}+8 = 20)$

5. (a) Discuss how vector error correction model is related to cointegration. Also discuss why cointegration is called an exercise in constrained VAR model.

(b) Discuss how you would test the presence of cointegration in a system of equations with K (>2) variables.

(c) Discuss the relationship between Granger causality and cointegration.

$(\overline{6+4}+6+4 = 20)$

6. (a) Consider a multiple linear regression model where some regressors have been omitted. Discuss, with derivations wherever necessary, the standard properties of the OLS residuals of this regression.

(b) Describe White's IM test. Does this test reduce to the well-known Jarque-Bera test of normality? Give justifications for your answer.

(10 + 10 = 20)

Indian Statistical Institute

MSQE II First Semester Final Examination 2016 - 17

Course name: **Political Economy**

Date: **November 23 2016**

Maximum marks: **80**

Duration: **3 hours**

Instructions: **Answer all questions.**

1. In the context of voter turnout, Grossman and Helpman conclude, "The paradox in voting is ... the choice by a reasonably high percentage of eligible voters to bear the cost of voting." Elucidate the paradox. **(5 points)**

2. Consider participation games as discussed by Palfrey and Rosenthal in the context of strategic voting. According to them, "The conclusion is that pure strategy equilibria fail to exist except for a few very special cases." Assume c, the identical cost of voting of all citizens, to be less than $1/2$, and coin-toss rule for breaking ties, to substantiate their conclusion. **(10 points)**

3. Consider the expected turnout in a "$q - k$" or "mixed-pure" equilibrium as discussed by Palfrey and Rosenthal. They conclude that "turnout is quite strongly correlated with the relative sizes of the minority and the majority." Elucidate their conclusion. **(5 points)**

4. Consider usual participation games as discussed by Palfrey and Rosenthal but now introduce some uncertainty. Specifically, assume that there is uncertainty about the number of members in each of the teams. That is, assume that the total number of voters, N, is known, but the number of voters supporting alternative 1 is not known. Each voter knows which alternative he supports, but not the preferences of the others. Let p be the probability that any individual voter is a supporter of team 1. Assume c to be the constant voting cost of every individual. Assume N is even.

(i) Find the parametric restrictions under which a pure-strategy symmetric Nash equilibrium with positive turnout exists for this Bayesian version of the participation game. **(12 points)**

(ii) Suppose $p < 1/2$. What can you conclude about the participation rates of the majority team versus that of the minority? **(3 points)**

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(iii) Does introduction of uncertainty resolve the conundrum between positive voting costs and positive turnouts in large electorates? **(5 points)**

5. This question pertains to a parameterized version of Feddersen and Sandroni's ethical voting model. Let the fraction of ethical agents in groups 1 and 2, $\tilde{q}_1$ and $\tilde{q}_2$ respectively, be independently and identically distributed as $U[0,1]$. Let the fraction of the population in group 1 be deterministic and be given by $k \in (0, 1/2]$. Let cost of voting for each individual be random and be drawn from $U[0, \bar{c}]$. Let the payoff from 'doing one's part', D, be $> \bar{c}$. Let the social cost function be linear, that is $v(x) = x$. Recall w to be the parameter capturing the 'importance of election'.

(i) Let parameters $\bar{c}, w$ and k satisfy $\frac{\bar{c}}{w} > \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(14 points)**

(ii) Let $\frac{1}{(1-k)^2} \leq \frac{\bar{c}}{w} \leq \frac{1}{\sqrt{k(1-k)}}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(6 points)**

(iii) Let $\frac{\bar{c}}{w} \leq \frac{k}{(1-k)^2}$. Find the equilibrium fraction of ethical agents who vote in each group. Can you provide a brief intuitive explanation? **(6 points)**

(iv) Consider your answer in part (i). What can you conclude about the participation rates of the minority and the majority? **(4 points)**

(v) From (i), what can you conclude about the chances of winning of the minority versus that of the majority? **(4 points)**

(vi) What is total expected turnout? How does it vary with the 'level of disagreement' in the economy? How does it vary with the 'importance of the election'? **(6 points)**

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2016-17
M.S. (Q.E.), 2nd Year, Semester I
Econometric Applications I

Date: 25 November, 2016 Maximum marks: 100 Duration: 3 hours

Note: Answer Question 1 and any **three** from the rest of the questions

1. The following data show the percentage distribution of different races in Malaysia in a certain year among all households and non-poor households.

Races	Percentage distribution among	
	All households	Non-poor households
Malay	60	57.25
Chinese	30	35.72
Other	10	7.03
Total	100.0	100.0

If the overall Head Count Ratio (HCR) of Malaysia is 30.3%, then compute the Head Count ratio of each of the three groups in Malaysia. Later, it was found that the overall HCR is actually 20% instead of 30.3%, then what should be the percentage change in the HCR in each racial group? [12+13=25]

2. Define and compare Head Count Ratio, Income Gap Ratio and Sen's Index of Poverty. What are the limitations of Sen's poverty index? Describe some poverty measures, which overcome these problems? [18+2+5=25]

3. Discuss the properties of Cobb-Douglas production function. Also discuss the problems of estimation of its parameters. [12½+12½=25]

4. Describe some methods to account for household size in Engel Curve Analysis. [25]

5. Write down the criteria for a good measure of concentration in business and industry. Show how these criteria are satisfied by (i) the Herfindahl – Hirschman index and (ii) the Hall and Tideman index. Also discuss why Lorenz Ratio cannot be taken as a measure of concentration in business and industry. [22+3=25]

6. (a) Is it true that the demand for a commodity will always increase if the mean income increases and the inequality of income decreases for a given group of people? Explain your answer assuming a suitable form of the Engel curve and a specific income distribution.

   (b) Describe how you will estimate Engel elasticity using Specific Concentration Curve. (*You should show the derivations of the associated results.*) [12+13=25]

7. Write short notes on any two of the following:

   (a) Roy's Identity and Shephard's Lemma.
   (b) Pareto Law.
   (c) Linear Expenditure System. [12½+12½=25]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: (2016-2017)

MS (Q.E.) II Year

International Economics I

Date: 28·11·2016 Maximum Marks 60 Duration 3 hours

Answer all questions

Group-A

1. In a general equilibrium set up show that trade between two ex ante identical but ex post dissimilar countries is Pareto inferior or superior depending on whether the constant absolute risk aversion parameter is greater than or less than unity. Interpret the result.

[12 +3 = 15]

2. Consider a two-factor-two-good economy (say, the Home country) under autarky. Fixed input-output coefficients are given by $a_{L1} = 30$, $a_{L2} = 20$, $a_{K1} = a_{K2} = 1$, where a_{ij} is the amount of the ith factor required to produce one unit of the jth commodity. Endowment of the two factors labour and capital are given by $L = 500, K = 20$. All individuals have an identical utility function given by $U = X_1^{\alpha} X_2^{1-\alpha}, \alpha = 4/7$, where X_1, X_2 are the consumption (and production) of the two commodities under autarky.
   (a) Find equilibrium levels of output, factor prices and the relative commodity price under autarky.
   (b) Now consider another country (the Foreign country) with identical technology and demand but different endowments. Endowments in the Foreign country are given by $L^* = 700, K^* = 25$. Find equilibrium levels of output, factor prices and the relative commodity price under autarky in the Foreign country.
   [Hint: Note that both factors cannot be fully employed in autarky in the Foreign country]
   (c) Suppose both countries open up to free trade and both countries are price takers (i.e. small) in the world market. Suppose further that in the world market the price of good 1 relative to the price of good 2 is 5/4. Which good will the Home country export? Which good will the Foreign country export?

[5+6+4=15]

Group-B

3. Consider an economy producing a final good Y with the following production function: $Y = X^{\alpha} L_Y^{1-\alpha}$, where $X = \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}}$ and $0 < \alpha, \rho < 1$. L_Y is the amount of

P.T.O

labour and x_i is the amount of the ith intermediate input (they come in n varieties) used in the production of Y. To produce these intermediate goods one has to spend one unit of capital to begin with (this constitutes the fixed cost), and thereafter, each successive unit of the good is produced by employing one unit of labour alone. The total amount of labour and capital available to the economy is $\overline{L}$ and $\overline{K}$ respectively. The producers of the intermediate inputs are monopolistically competitive in the usual Dixit-Stiglitz sense. Labour is perfectly mobile between the Y sector and the intermediate goods sector and full employment of both the factors, labour and capital, prevails. Furthermore the Y producers can only earn zero profits in equilibrium (i.e. price is equal to average cost in Y sector). This constitutes the description of the economy.

Suppose now that this economy is allowed to trade (at the level of both the final good Y and the intermediate goods x_i s) with an economy which is in all respect similar to this country, except that it may differ in its total labour and capital endowments.

(a) Show that the wage rate would be equalized across countries.

(b) Show that the rental rate on capital would be equalized across countries.

(6+9=15)

4. Show how in Krugman (1981) distributional conflict is related both to difference in factor endowment ratios and economies of scale.

[15]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2016-17

Course name: MSQE II
Subject name: Selected Topics (Auction Theory)
Date: 21-02-2017
Maximum marks: 40
Duration: 2 hours

Suppose there are two bidders participating in a second-price auction where the seller has 0 value for the object being sold. It is common knowledge that bidder valuations are drawn from [0, 100] according to the uniform distribution.

(I) Rigorously characterize symmetric, increasing, differentiable equilibrium bid functions, assuming one exists. [10]

(II) Argue that there is a unique such equilbrium bid function, assuming one exists. [5]

(III) Is the candidate bid function you have identified in parts (I) and (II) above an equilibrium? Argue rigorously. [10]

(IV) Identify an asymmetric equilibrium profile of bid functions. [5]

(V) From the perspective of maximizing ex ante expected revenue, does the seller have a preference between whether the bidders bid according to the symmetric profile you have identified in parts (I) through (III) above or the asymmetric profile you have identified in part (IV) above? Provide a rigorous calculation. [10]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination : 2016-17
M.STAT. I YEAR, M.S.(QE) I and II YEAR
Optimization Techniques

Date: 22 February 2017 Maximum Marks: 75 Duration: $2\frac{1}{2}$ hours

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Notation have usual meaning.
This paper carries 85 marks. However, maximum you can score is 75.

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1 Two cities generate waste and their waste are sent to incinerators (furnaces) for burning. Daily waste production and distances among cities and incinerators are as below:

	Waste produced (ton/day)	Distance to incinerator (in km.)	
		A	B
City 1	500	30	20
City 2	400	36	42

Incineration reduces each ton of waste to 0.2 tons of debris, which must be dumped at one of the two landfills. It costs \$3 per kilometer to transport a ton of material (either waste or debris). Distances (in km) among the incinerators and landfills are given below.

	Capacity (ton/day)	Incineration Cost (\$/ton)	Distance to landfill (in km.)	
			Northern	Southern
Incinerator A	500	40	35	38
Incinerator B	600	30	51	48

Formulate a linear program that can be used to minimize the total cost of disposing waste of the cities. **[15]**

2 Let $\bar{\mathbf{x}}$ be an extreme point of a convex set $S(\subseteq R^n)$. Then $\bar{\mathbf{x}}$ lies on the boundary of S. **[8]**

3 Find all the extreme points and extreme directions of the polyhedral set given by:

$$\begin{aligned} x_1 + 2x_2 + x_3 &\leq 10 \\ -x_1 + 3x_2 &= 6 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

[12]

[P.T.O.]

4 Consider a maximization linear programming problem with extreme points $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ and $\mathbf{x}_4$, and extreme directions $\mathbf{d}_1$, $\mathbf{d}_2$ and $\mathbf{d}_3$, and with an objective function having cost vector $\mathbf{c}$ such that $\mathbf{c}^{\mathrm{T}}\mathbf{x}_1 = 5$, $\mathbf{c}^{\mathrm{T}}\mathbf{x}_2 = 7$, $\mathbf{c}^{\mathrm{T}}\mathbf{x}_3 = 4$, $\mathbf{c}^{\mathrm{T}}\mathbf{x}_4 = 7$, $\mathbf{c}^{\mathrm{T}}\mathbf{d}_1 = 0$, $\mathbf{c}^{\mathrm{T}}\mathbf{d}_2 = -3$ and $\mathbf{c}^{\mathrm{T}}\mathbf{d}_3 = 0$. Is this problem unbounded or has an optimal solution? Hence, depending upon your answer, give either the set of all rays or the set of all optimal solutions. **[8]**

5 The following is the current simplex tableau (second iteration) of a given linear programming problem in canonical form with the objective to maximize $2x_1 - 3x_2$. The two constraints are of $\leq$ type with non-negative right-hand-sides. In the tableau, x_3 and x_4 are slack variables.

	x_1	x_2	x_3	x_4	RHS
z	b	1	f	g	6
x_3	c	0	1	$\frac{1}{5}$	4
x_1	d	e	0	2	a

Find the unknowns a through g above. **[12]**

6 Suppose that simplex algorithm terminates declaring Big-M problem as unbounded with all the artificial variables are at zero level in the current solution. Show that the original problem of interest is unbounded. **[10]**

7 State and prove the complementary slackness theorem. **[10]**

8 Consider the linear program (P): min $\mathbf{c}^{\mathrm{T}}\mathbf{x}$ subject to $\mathbf{A}\mathbf{x} = \mathbf{b}_1$, $\mathbf{x} \geq 0$. Suppose that $\mathbf{u}_1$ is an optimal solution of P and $\mathbf{v}_1$ is optimal for its dual. Further, let $\mathbf{u}_2$ is an optimal solution of P when $\mathbf{b}_1$ is changed to $\mathbf{b}_2$. Show that

$$\mathbf{v}_1^{\mathrm{T}}(\mathbf{b}_2 - \mathbf{b}_1) \leq \mathbf{c}^{\mathrm{T}}(\mathbf{u}_2 - \mathbf{u}_1).$$

[10]

———*** xXx ***———

Indian Statistical Institute
Economic Development
Mid-Sem Examination
MSQE I & II

Date: 23.2.17

Time 2 hours

Answer both questions. Each question carries 20 marks.

1. Consider the model of occupational choice of Aghion and Bolton (1997) where agents can choose either to be self-employed with a low but certain income or to take up a risky project requiring a minimum amount of capital. In the latter project probability of success increases with privately observable efforts.
   (a) Find the wealth levels for which agents are credit rationed.
   (b) Show that if no one is credit rationed, all risky projects are funded in the long run, provided the rate of return on the project and the rate of savings (bequests) are high enough.
2. Show that if a small dose of noise is introduced into the History versus Expectations model of Krugman (1991), the problem of multiple equilibrium can be avoided.

Indian Statistical Institute,

MS(QE) Second Year, Mid Semester Exam: 2016-17

Topic: Bayesian Econometrics

Date: 24.02.17 Maximum Marks: 40, Duration: 2 hours.

Answer all questions. Show your works to get full credit. Marks will be deducted for untidiness and bad handwriting.

1. (a) Let $X_1, X_2, \ldots, X_n$ be iid observations from Poisson distribution with mean parameters λ, where λ follows a Gamma density with shape parameter=α and scale parameter=β with the following density:
$f(\lambda) = Ce^{-\frac{\lambda}{\beta}}\lambda^{\alpha-1}$, with C as the normalizing constant. Find the posterior estimate of λ under the squared error loss function.
(b) Show that the posterior estimate in (a) can be expressed as the weighted average of the maximum likelihood estimate ($\bar{X}$) and the prior mean ($\alpha\beta$). [6+4]

2. Financial regulations require banks to report their daily risk measures called Value at Risk (VAR). Let Y be the financial return of the bank and then for a given $p(0 < p < 1)$, the VAR is the value y^* satisfying $P(Y \leq y^*) = p$. The financial return depends on exchange rate (x). Based on a sample of n data points (Y_i, x_i), suggest a suitable statistical approach for estimating VAR at p=0.8. Specify the appropriate statistical model and the estimation method explicitly. [10]

3. Let $X_1, X_2, \ldots, X_n$ be iid observations from Bernoulli distribution with success probability=θ, where θ follows a Uniform(0,1) density. Show that under this setting, $X_1, X_2, \ldots, X_n$ are NOT marginally independent, but they are exchangeable. [10]

4. Suppose annualized data are collected from 6 different airlines from the United States for 2005-2014 (10 years) on several variables including output (revenue passenger miles),total cost (thousands of dollars), fuel price, and load factor (average capacity used). Suppose the goal is to predict the output (annualized) based on the other three variables. Suggest a suitable Bayesian approach for such prediction. Write down the data model, prior structure and the full posterior density explicitly. Use the following notations:
Y_{jt}= output (annualized) for the j-th airline at the t-th year, $j = 1, \ldots, 6$; and $t = 1, \ldots, 10$.
x_{1jt}= total cost (annualized) for the j-th airline at the t-th year.
x_{2jt}= fuel price (annualized) for the j-th airline at the t-th year.
x_{3jt}= load factor (annualized) for the j-th airline at the t-th year. [10]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2016-17

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 24-2-17 Maximum Marks: 40 Duration: 3 hours

Problem 1. Consider a Bayesian social choice problem with two players. Let A be the set of alternatives with cardinality at least 3. Let the prior belief μ_i of player $i \in \{1,2\}$ at a preference $P \in \mathbb{L}(A)$ be given by $\mu_i(P) = U(P, P')$ where U is the uniform probability distribution and P' is such that $r_1(P') = r_2(P)$, $r_2(P') = r_1(P)$, and $r_k(P') = r_k(P)$ for all $k \geq 3$. Is there a social choice function that is unanimous and Bayesian strategy-proof for (i) risk-lover (ii) risk-averse players?

(10)

Problem 2. Justify your answer by a proof or a counterexample.

1. Consider a finite set of alternatives A and a domain $\mathcal{D}$ of preferences over A such that for all $a, b \in A$, there exists $P \in \mathcal{D}$ with $r_1(P) = a, r_2(P) = b$. Then, $\mathcal{D}^n; n \geq 2$ is a dictatorial domain.

2. Let A be a set of finite alternatives, $n \geq 2$ and $\mathcal{D} \subseteq \bar{\mathcal{D}} \subseteq \mathbb{L}(A)$. Suppose $\mathcal{D}^n$ is dictatorial. Then, $\bar{\mathcal{D}}^n$ is dictatorial.

3. Let $\mathcal{D}$ be a single-dipped domain and $f : \mathcal{D}^2 \to A$ be unanimous and strategy-proof. Then, $f(P_1, P_2) \in \{r_1(P_1), r_1(P_2)\}$ for all $(P_1, P_2) \in \mathcal{D}^2$. Furthermore, there is an anonymous, unanimous and strategy-proof social choice function $f : \mathcal{D}^2 \to A$.

(10 × 3)

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2016-2017
M.S. (Q.E.) I, II Years and M. Math. I Year
Game Theory II

Date: 24.4.17 Maximum Marks: 100 Time: 3 hours

Note 1: Answer Parts (A) and (B) in separate answer scripts. Clearly explain the symbols you use and state all the assumptions you need for any derivation. Marks will be deducted substantially for any mistake you make in definitions and statements of assumptions, whenever you need them.
Note 2: The paper carries 110 marks. You may attempt any part of any question. The maximum you can score is 100.

A

1. Let $h = \{h_1, h_2, h_3, h_4\}$ be a set of four house owners whose preferences on the set of houses $H = \{H_1, H_2, H_3, H_4\}$ are given in the following table:

Table I : House Matching

h_1	h_2	h_3	h_4
H_3	H_1	H_1	H_2
H_2	H_2	H_4	H_3
H_1	H_4	H_2	H_1
H_4	H_3	H_3	H_4

Clearly establish how the house owners can be made better off through exchange of houses. (8)

2. Is the core of a bankruptcy game non-empty? Demonstrate your claim rigorously. (10)

3. Show that the Shapley value of a game is invariant under strategic equivalence. (5)

P.T.O

4. Define a voting game. Establish a necessary and sufficient condition for non-emptiness of the core of such a game in terms of a blocker.

(6)

5. Consider the problem of allocating costs for providing some service to a set of individuals. Assume that the following conditions hold: (a) all non-users of the service do not pay for it but all users should be charged equally; (b) the total cost of using the service is the sum of capital and operating costs, and (c) the service provider will recover the entire cost from the customers. Clearly demonstrate that there is a unique solution to this cost recovery game.

(10)

6. An firm I owns a factory and each member of a set L of laborers owns only his own labor skills. Laborers can produce nothing on their own and members of any non-empty subset S of $s = |S|$ laborers can produce exactly s units of output if they work in the factory. Formulate a coalition form game that models this situation. Justify your formulation.

(6)

7. State and prove the Bondareva-Shapley theorem by defining all necessary concepts.

(12)

8. Let $N = \{A_1, A_2, A_3\}$ be a set of 3 firms producing a homogenous output whose price function is $10 - x_{A_1} - x_{A_2} - x_{A_3}$, x_{A_i} being the output of firm A_i. The maximum output a firm can produce is 3. The cost function of firm A_i is $(1 + x_{A_i})$. The worth of any non-empty coalition $S \in 2^N$ is defined as

$$v(S) = \max_{x_{A_i}, A_i \in S} \min_{x_{A_j}, A_j \notin S} \sum_{A_i \in S} x_{A_i} \left(10 - x_{A_1} - x_{A_2} - x_{A_3}\right) - \left(\sum_{A_i \in S} \left(1 + x_{A_i}\right) \right).$$

Determine the numerical value of worth of each non-empty coalition. Also identify the set of core elements and determine its Shapley value.

(26)

P.T.O

9. Show that a solution to the two-person bargaining problem satisfying the four Nash axioms is the Nash product.

(7)

B

1. Suppose a weighted majority game has 5 players with weights 7, 3, 8, 2 and 1, given that the quota is 14. Using an efficient method determine the number of winning coalitions in the game and the number of swings for the player having weight 2.

(4+4 = 8)

2. Describe the "men propose" variant of the Gale-Shapley algorithm. In this algorithm, is it possible that before termination there arises a round where a man does not have any woman to propose to?

(8 + 4 = 12)

INDIAN STATISTICAL INSTITUTE

Final Examination: 2016-17

Course name: MSQE II
Subject name: Selected Topics (Auction Theory)
Date: 26·4·17
Maximum marks: 50
Duration: 3 hours

Answer all questions, rigorously wherever possible.

Q1. Consider an auction with two bidders. Valuations x are uniformly and independently distributed over $[0, 1]$ for both bidders. Suppose it is common knowledge that bidder 2 always follows the bidding strategy $\beta_2(x) = x^2$.

a) What is the best reply for bidder 1 in a first-price auction? [3]

b) What is the best reply for bidder 1 in a second-price auction? [3]

c) Are the expected payments of bidder 1, either at the interim or the ex-ante stage, the same across the two auction formats? [5]

d) In light of your answer to part (c), argue whether the revenue equivalence principle applies in this environment or not. [4]

Q2. Suppose there are two bidders with independent valuations x distributed over $[0, 1]$ according to distribution function $F(x) = \sqrt{x}$.

a) Derive the symmetric equilibrium bidding strategies in

i) the first-price auction, [5]

ii) the second-price auction, and [5]

iii) the all-pay auction. [5]

b) Suppose the seller can set an entry fee, with bidders finding out valuations only if they decide to enter. Can the seller increase revenue in any of three auction formats above by doing so? If so, what is the optimal entry fee in each relevant case? [2+3]

Q3. Suppose there are two bidders, participating in a first-price, sealed-bid common value auction. Each bidder's information (x_i for bidder i) regarding the value of the object follows the uniform distribution over [1,2]. These information signals are distributed independently.

a) Derive the symmetric equilibrium bid function if the common value of the object is $V = x_1 x_2$. [8]

b) Derive the symmetric equilibrium bid function if the common value of the object is $V = x_1 + x_2$. [7]

Indian Statistical Institute
Mid Semestral Examination: (2016 – 2017)

M.S. (QE) – II year

Econometric Applications II

Date: 27.02.2017 *Maximum Marks – 50* *Duration: 2 hours*

Answer any <u>two</u> questions

1. (a) Suppose you are required to find the distribution of income from a given set of income data on n (large) individuals, given by $x = \{x_1, x_2, x_3 \ldots . x_n\}$. Let $p(a)$ denote the true density evaluated at point 'a' and $p_{h_n}(a)$ denote the estimate of $p(a)$ based on bandwidth h_n. State the underlying conditions for $p_{h_n}(a)$ to converge to $p(a)$.

   (b) Explain how the Kernel density estimation procedure can be viewed as an extension of the concept of Histogram.

   (c) Show that $lim_{h_n \to 0} p_{h_n}(a) = p(a)$.

   (d) When will you use a "nearest neighbour approach" to estimate density? Describe the steps involved in estimating density by k-nearest neighbour (k-nn) method.

   [6 + 6 + 8 + 5 = 25]

2. (a) Explain the concept of kernel regression.

   (b) Derive the Nadaraya–Watson kernel regression estimator for the following budget share function of the h-th household.

$$w_h = m(\log x_h) + \varepsilon_h, \quad h = 1,2,\ldots,H$$

   (c) Describe the k-nearest neighbour (k-nn) regression estimator.

   (d) Given the following data find the k-nn estimate for x=4.5 and k=3.

x	2.5	8.5	4	3.5	6	1	9.5	5	7
y	5	12	1	7	5	6	7	2	8

   [4 + 8 + 4 + 9 = 25]

3. (a) What is 'selection bias'? What are the sources of such bias?

(b) Give an example of a bivariate sample selection model with a participation equation and an outcome equation. How would you estimate the parameters using Heckman's procedure? State the underlying assumptions clearly.

(c) Given a partial linear model of the form $y = z\beta + f(x) + \epsilon$, where z and x are scalars, how would you estimate β and $f(.)$?

[6 + 12 + 7 =25]

Indian Statistical Institute
Second Semestral Examination: 2016 –2017
M.S. (Q.E) – II Year
Econometric Applications II

Date: 28/04/17 Maximum Marks: **100** Duration: 3 hrs.

(Answer any four questions)

1. (a) Consider the following logarithmic form of the cost function $C(u, p)$:
$log\, C(u, p) = a_0 + \sum_{j=1}^{n} a_j\, log p_j + \frac{1}{2}\sum_j \sum_k c_{jk}^{*} log p_j log p_k + u b_0 \prod_j p_j^{b_j}$,
where notations have their usual meanings.

   (i) Derive the demand system in budget share form from the above cost function. What is this system called?

   (ii) Derive the conditions under which the demand system satisfies adding–up, homogeneity and symmetry properties.

   (b) What are 'exact' and 'consistent' aggregations in the context of consumer demand analysis? For each type of aggregation, give an example of a demand system conforming to the respective type.

[(6+12) +7 = 25]

2. (a) Show that any demand system derived from maximization of a utility function subject to a budget constraint satisfies negativity of the Slutsky matrix.

   (b) Describe the Ramsey-Samuelson-Diamond-Mirlees approach to the determination of optimal commodity tax rates.

   (b) Define True Cost of Living Index (TCLI). What is the difference between a TCLI and a standard price index number?

[8+12+5 = 25]

3. (a) Discuss the idea of Propensity Score Matching (PSM) and its applicability. Describe the different matching procedures.

   (b) What do you mean by a semiparametric model? Describe the single index model and its estimation procedure by Ichimura's method.

   (c) Show that the Nadaraya-Watson estimator is the weighted least squares estimator of the intercept of the regression equation $y_i = \alpha + \varepsilon_i$.

[10+10+5=25]

4. (a) What are the different approaches to specifying a demand system?

(b) What is the Gorman-Polar form of cost function? What is the form of the Engel curve implied by this form of cost function?

(c) Show that for the Linear Expenditure System (LES), the non-compensated own price elasticities are approximately proportional to the corresponding expenditure elasticities.

(d) In the LES, an amount z of good 1 must be bought. Show that for $i=2,3,\ldots,n$

$$p_i q_i = p_i c_i + \frac{b_i}{\sum_{i=2}^{n} b_i}(x - p_1 z - \sum_{k=2}^{n} p_k c_k),$$

where the symbols have their usual meanings.

[6+4+7+8 = 25]

5. (a) Distinguish between the 'Unitary approach' and the 'Collective approach' for specification of household preferences.

(b) What do you mean by 'Income pooling'? Does a collective model necessarily fail the 'income pooling' hypothesis?

(c) Starting with a Pareto efficient collective model, described as maximization of a Pareto weighted sum of household members' utility subject to an income constraint, demonstrate why Slutsky symmetry fails. Hence, specify a test for validity of the unitary framework.

(d) Consider a general collective model specified by the following household utility function:

$$W(C^1, C^2, C^3, .., C^J, G; z, d) = \sum_j \mu_j (p, x, d) u^j (C^1, C^2, C^3, .., C^J, G; z),$$

where C^j is the consumption vector of the j-th member, G denotes consumption of public goods, z is the vector of demographic characteristics, x denotes household income, d is a vector of distribution factors, disjoint from z and $\mu_j(.)$'s are the Pareto weights.

Enumerate the different special cases by imposing restrictions on $\mu_j(.)$. For each case specify the type of model and state whether Slutsky symmetry is satisfied or not.

[4+3+10+8 = 25]

Indian Statistical Institute

Semestral Examination

Second and Fourth Semesters, 2017

MSQE I & II

Economic Development

03.05.2017.

Maximum Marks 60

Time 3 hours

Answer question 1 and any two from the rest.

1. Consider a political economy scenario where there are two parties: an incumbent and an opposition. The state of the economy is $\theta = e + \delta$ where e is the effort put in by the incumbent and δ is a random shock which is normally distributed with mean zero and variance σ^2. Each voter gets a signal s about θ, where s is uniformly distributed over the interval $[\theta - \varepsilon, \theta + \epsilon]$ and $\varepsilon > 0$ is a noise. A voter votes for the incumbent if the conditional expectation of θ given s is at least as large as some exogenously given standard $\bar{\theta}$. The incumbent chooses e to maximize $W = p(e, \bar{\theta}) - C(e)$ where $p(e, \bar{\theta})$ is the endogenously determined probability of winning the election and $C(e)$ is the cost of effort put in by the incumbent. Assume that $\frac{dC(e)}{de} = \bar{c}$, a constant.
   (i) Determine the equilibrium effort choice and show that it is unique.
   (ii) How does the equilibrium effort choice and the probability of winning in equilibrium change if there is an increase in $\bar{\theta}$?
   (iii) How does the equilibrium effort choice and the probability of winning in equilibrium change if there is an increase in the marginal cost of effort $\bar{c}$?

[10+5+5]

P.T.O

2. In a model of endogenous growth and electoral democracy where only capital is taxed, show that an increase in the inequality in capital holding leads to a fall in the rate of growth.

[20]

3. Can group lending with joint liability mitigate the problem of moral hazard? Give your answer in terms of a suitable model.

[20]

4. Consider an economy where agents live for two periods. In the first period they can acquire t units of education where t is a choice variable and $t\epsilon[0,1]$. The total cost of acquiring t units of education is $C(t)$ with $C'(t) > 0, C''(t) > 0$. An agent acquiring t units of education in the first period earns $(1-t)w_u$ in the first period and $w_u + t(w_n - w_u)$ in the second period. Assume that $w_n > 2w_u$. Consumption and bequests take place at the end of the second period. An agent chooses his bequest (b) and consumption (c) to maximize his utility given by $U = c^{\alpha}b^{1-\alpha}$. At the beginning of his life an agent is endowed with an inheritance x.

   (a) First assume that there is a perfect credit market where an agent can borrow or lend as much as he wants to and the borrowing rate is equal to the lending rate. Find the level of education t acquired by an agent as a function of his inheritance. Also derive the long term distribution of wealth in the economy, for any given distribution of initial wealth.

   (b) Now assume that the credit market is imperfect, that is, the borrowing rate is greater than the lending rate. Find acquired education t as a function of inheritance and the long term distribution of wealth.

[10 + 10]

INDIAN STATISTICAL INSTITUTE

Final-Semestral Examination : 2016-17

Course Name: M.S. (Q.E.) II YEAR
Subject Name: The Theory of Mechanism Design
Date: 05.5.17 Maximum Marks: 50 Duration: 3 hours

1. Consider a social choice problem with the set of players $N = \{1, 2\}$ and the set of alternatives $A = \{a_1, a_2, \ldots, a_5\}$. Let $\mathcal{D}$ be the (ordinal) single-peaked domain over A. Suppose the utility of a player $i \in N$ of an alternative $a \in A$ at a preference $P_i \in \mathcal{D}$ is defined as $u_i(a, P_i) = 6 - r(a, P_i)$, where $r(a, P_i)$ is the rank a at P_i. Consider a prior distribution μ over $\mathcal{D}^2$. Define the ex-ante utility of a social choice function $f : \mathcal{D}^2 \to A$ as

$$U(f) = \sum_{(P_1, P_2) \in \mathcal{D}^2} \mu(P_1, P_2) \sum_{i \in N} u_i \left(f(P_1, P_2), P_i\right).$$

Then answer the following questions.

   (a) Find the social choice function(s) on $\mathcal{D}^2$ that maximize(s) the ex-ante utility with uniform prior distribution.

   (b) Find the strategy-proof social choice function(s) on $\mathcal{D}^2$ that maximize(s) the ex-ante utility with uniform prior distribution.

(10)

2. Show that, if an allocation function on the unrestricted domain satisfies 2 cycle-monotonicity, then it satisfies 4 cycle-monotonicity.

(10)

3. Suppose $N = \{1, \ldots, n\}$ is the set of players, A is the set of outcomes with $|A| = m$, and for all $i \in N$, $T_i \subseteq \mathbb{R}^m$ is the type-space of player i. Consider the following affine-maximizer allocation function: for all $t_N \in \prod_{i \in N} T_i$,

$$f(t_N) \in \arg\max_{a \in A} \sum_{i \in N} \lambda_i t_i(a) + \kappa(a),$$

where, for all $i \in N$, $\lambda_i > 0$ is the weight parameter of player i, and for all $a \in A$, $\kappa(a)$ is the reservation price of the outcome a.

(a) Show that f satisfies k cycle-monotonicity for all $k \geq 2$.

(b) Find all payment functions $(p_i)_{i \in N}$ such that $(f, (p_i)_{i \in N})$ is incentive compatible.

(c) Is there a domain $\hat{T} = \prod_i \hat{T}_i$ and payment functions $p_i : \hat{T} \to \mathbb{R}$ for all $i \in N$ such that $(f, (p_i)_{i \in N})$ on $\hat{T}$ is incentive compatible, individually rational, and budget balanced?

(10+5+5)

4. Show that there are incentive compatible allocation functions on a (cardinal) single-peaked domain that are not affine-maximizers.

(10)

INDIAN STATISTICAL INSTITUTE
Semestral Examination: 2016-2017
MS(QE), II Year
Bayesian Econometrics
Date:5th May, 2017 Maximum Marks 100
Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 105 marks. Marks allotted to each question are given within parentheses.

1. (a) Let $X_1, \ldots, X_n$ be iid sample from Bernoulli (p), and consider a uniform (0,1) prior on p. Show that the Bayes estimate of p is approximately equal to the corresponding maximum likelihood estimate as $n \to \infty$.
   (b) Define Bayes Factor and explain the usefulness of it in the context of Bayesian Hypothesis Testing.
   (c) What do we mean by highest posterior density (HPD) credible region for a parameter of interest?

   [10+5+5=20]

2. The pattern of sunny and rainy days on the planet Rainbow is a homogeneous Markov chain with two states. Every sunny day is followed by another sunny day with probability 0.8. Every rainy day is followed by another rainy day with probability 0.6.

   a) Today is sunny on Rainbow. What is the chance of rain the day after tomorrow?
   b) Compute the probability that April 1 in 2030 is rainy on Rainbow.
   [10+10=20]

3. (a) State the Accept-Reject algorithm and explain why it works.
   (b) For estimating a parameter of interest, say θ, in the Bayesian context, typically we generate from the posterior distribution $p(\theta|data)$ and estimate θ by its posterior sample mean. Suppose the posterior distribution is too complex to generate samples and hence we decide to use accept-reject algorithm for sample generation. However, after the sample generation (by the accept-reject algorithm) the statistician changes the prior distribution resulting in a completely differ-

P.T.O

ent posterior for θ. Suggest a suitable method of estimating θ using the sample originally generated from the older posterior density. **[10+10=20]**

4. Consider the following transition matrix

$$\begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix}.$$

(a) Assume that $0 < \alpha, \beta < 1$. Find the invariant distribution. Is the process aperiodic? Is it irreducible?
(b) Suppose $\alpha = \beta = 0$. Is there a unique invariant distribution? Is the process aperiodic?
(c) Suppose $\alpha = \beta = 1$. Is there a unique invariant distribution? Is the process aperiodic? Is it irreducible?
[15+15+15=45]

INDIAN STATISTICAL INSTITUTE

Back paper Examination: 2016-17

Course name: MSQE II
Subject name: Selected Topics (Auction Theory)
Date: 10·07·2017
Maximum marks: 100
Duration: 3 hours

Answer all questions

Q1. Derive rigorously the symmetric equilibrium bid function in the second-price (sealed-bid, winner-pay) auction, given a common value environment. [15]

Q2. Derive rigorously the symmetric equilibrium bid function in the third-price (sealed-bid, winner-pay) auction, given an independent private value environment. [15]

Q3. There are two bidders with common valuation for an object. Bidder i's private signal is X_i, where $X_i = S_i + T$, and S_1, S_2 and T are independent and uniformly distributed on $[0, 1]$. The value of the object is $V = X_1 + X_2$.

Consider sealed-bid, winner pay auctions.

a) Find the symmetric equilibrium bid function and expected seller revenue if a first-price auction is used. [15]

b) Find the symmetric equilibrium bid function and expected seller revenue if a second-price auction is used. [15]

c) Which auction format yields the seller higher expected revenue? [10]

Q4. There are three bidders, with common valuation for an object V. It is known that V is uniformly distributed over $[0, 1]$. Additionally, given $V = v$, for all bidders, signals X_i are independently distributed uniformly over $[0, 2v]$. What is the expected seller revenue from a sealed-bid, winner pay, second price auction in this environment? [30]

INDIAN STATISTICAL INSTITUTE
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SECOND SEMESTRAL BACK PAPER EXAMINATION 2016 - 17
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: 12.07.17 Maximum Marks: 100 Time: 3 hours

Answer any ALL questions. Marks allotted to each question are given within parentheses.

1. (a) Find the mean and ACF of the following ARMA (1, 1) process.

$$x_t = 2 + 1.1x_{t-1} - 0.3x_{t-2} + a_t - a_{t-1},\ a_t \sim WN(0, \sigma^2).$$

(b) Let $\{Y_t\}$ be an ARMA plus while noise process given by $Y_t = X_t + W_t$, where $\{W_t\} \sim WN(0, \sigma_w^2)$, $\{X_t\}$ is an ARMA (p, q) process satisfying $\phi(B)X_t = \theta(B)Z_t, \{Z_t\} \sim WN(0, \sigma_z^2)$, and $E(W_s, Z_t) = 0$ for all s and t. Show that $\{Y_t\}$ is stationary. Also find the autocovariances of $\{Y_t\}$ in terms of σ_w^2 and the ACVF of $\{X_t\}$.

(c) Find the 3-step ahead minimum MSE forecast at origin n of the following series.

$$x_t = 2x_{t-1} - x_{t-2} + a_t - 0.4a_{t-1} + 0.3a_{t-2},\quad a_t \sim WN(0, \sigma^2).$$

[6+8+6 = 20]

2. (a) Describe the ADF test for unit roots in a time series and comment on its power.

(b) Describe the Quandt-Andrews test for detecting the presence of a structural break in a time series.

[10+10 = 20]

P.T.O

3. (a) Suggest an appropriate procedure for obtaining seasonal indices in a monthly time series from which trend and cyclical components have been removed.

(b) Describe the HEGY test for testing the presence of seasonal and nonseasonal unit roots in a quarterly time series.

[8 + 12 = 20]

4. (a) Suppose the form of the transfer function of a transfer function noise model (TFNM) is given by

$$\upsilon(B) = (\omega_0 - \omega_1 B - \omega_2 B^2)/(1 - \delta B), \quad |\delta| < 1.$$

Find the impulse response weights of the underlying TFNM implied by this transfer function.

(b) Specify a simple intervention model along with all assumptions, and then discuss how different types of dynamic effects can be simulated using 'step' intervention.

[10 + 10 = 20]

5. (a) Define the spectral density function, $f(\lambda)$, of a stationary process and then show that it is nonnegative for all $\lambda \in [-\pi, \pi]$. Also find $f(\lambda)$ for an MA (1) process.

(b) State and prove the theorem on finding the spectral density function of a linear combination of stationary time series.

[10 + 10 = 20]