INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part I: 1968-69

PERIODICAL EXAMINATIONS

General Science-1: Chemistry Theory

[13]

Date: 23.?.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any three questions.

1. Write out the structures of the following compounds.
   a) 2-methyl, but 1:3 diene.
   b) 2-Ethyl. 3 methyl but 1 - ene
   c) 3:4:4 Trimethyl pent-2-ene
   d) 5-propyl 2:3 diethyl octane
   e) 2:3 dichloro 2:3 dimethyl pentane.

2. i) Write out the structures, and also indicate the number of π and σ bounds in the following compounds.

   (a) C_2H_2, (b) C_2H_4, (c) C_6H_6.

   ii) Mention the values of the bond angles in each of the following compounds,

   (a) CH_4, (b) C_2H_4, (c) C_2H_2.

3. What is meant by the term unsaturated compounds? Give a few examples of such substanus, showing how their reaction differ from those of saturated compounds.

4. Write short notes on (any three) of the followings.
   a) Inductive effect, (b) π-electron
   c) Electro-valency, (d) Free-radicals.

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PERIODICAL EXAMINATIONS

MATHEMATICS-1: Group A

2A

Date: 14.10.68 Maximum Marks: 50 Suggested Time: $1\frac{1}{2}$ hours

QUESTION BOOKLET: PAGES 0 - 4

INSTRUCTIONS:

1. Questions 1 to 23 carry each 2 marks.
   Question 24 carries 4 marks.

2. Answer all questions.

3. Answers should be written on the ANSWER SHEET separately provided, by putting a tick (✓) mark in the appropriate place.

4. All rough work should be done on the question booklet itself.

For Questions (1), (2), (3),

S_1 = The set of all even integers.

S_2 = The set of all odd integers.

1. Let $x \in S_1$ and $y \in S_2$, then $x\,y$ belongs to
   (a) $S_1 \cap S_2$; (b) S_2 ; (c) S_1 ; (d) none of these.

2. Let $x \in S_1$ and $y \in S_2$, then $x + 7y$ is a member of
   (a) S_1 ; (b) $S_1 \cap S_2$; (c) S_2 ; (d) none of these.

3. Let $x \in S_1$ and $y \in S_2$ then $x + 4y$ is an element of
   (a) S_2 ; (b) $S_1 \cap S_2$; (c) S_1 ; (d) none of these.

4. If A = (4, 5, 6, 7)
   B = (6, 7, 8, 4), then $A \cup B$ is
   (a) (4, 5, 6, 8); (b) (6, 7, 8, 5); (c) (6, 7);
   (d) (4, 5, 6, 7, 8).

5. If X = (1, 2, 4, 5, 6), then the number of subsets of X is equal to
   (a) 64; (b) 31 ; (c) 2^6-1 ; (d) 32.

6. If A is a subset of B then $A \cap B$ is
   (a) A ; (b) B ; (c) empty set ; (d) none of these.

7. If A and B are disjoint then $A \triangle B$ is equal to
   (a) $A \cup B$; (b) Empty set ; (c) $A \cap B$;
   (d) none of these.

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8. If A = The set of all roots of the equation $x^2 - 2 = 0$,
and B = $(+\sqrt{2}, -\sqrt{2})$, then $A \triangle B$ is equal to
(a) Empty set ; (b) $(\sqrt{2}, -\sqrt{2})$; (c) $(-\sqrt{2})$;
(d) $(+\sqrt{2})$.

9. Let X = (1, 2, 3, 4, 6, 7)
A = (1, 2, 3, 4)
B = (3, 4, 6)
Then $A \triangle B^C$ is equal to
(a) (1, 2, 6); (b) (1, 2, 3, 4, 7); (c) (7)
(d) (3, 4, 6, 7).

10. Let A, B, D be three sets. Then $A \cap (B^C \cup D)$
is equal to
(a) $(A \cap B^C) \cap (A \cap D)$; (b) $(A \cap B^C) \cup (A \cap D)$.
(c) $(A \cup B^C) \cap (A \cup D)$; (d) none of these.

11. If A = The set of all prime numbers,
and B = The set of all even integers, then $A \cap B$ is equal t
(a) Empty set ; (b) odd integers ; (c) even integers
(d) (2).

12. If A = The set of all rectangles which are not parallelogram
and B = The set of all parallelograms which are not squares,
then $A \cap B$ is equal to
(a) all rectangles; (b) all squares; (c) all parallelograms;
(d) empty set.

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13. Let X be a universal set and A, a subset of X, then $A \triangle A^C$ is equal to

(a) A ; (b) A^C ; (c) Empty set ; (d) X.

14. $(A \cup B^C)^C$ is equal to

(a) $A^C \cap B^C$; (b) $A^C \cup B$; (c) $A^C \cup B^C$; (d) $A^C \cap B$.

15. $A \cup (B \cap D)$ is equal to

(a) $(A \cap D) \cup (B \cap A)$; (b) $(A \cup B) \cap D$;

(c) $(A \cup B) \cup (D \cap A)$; (d) none of these.

16. $(A - A \cap B) \cup B$ is equal to

(a) $A \cap B$; (b) $A - B$; (c) $B \cup A$; (d) none of these.

17. $A \triangle B$ is equal to

(a) $(A \cap B^C) \cup (A^C \cap B)$; (b) $(A-B) \cap (B-A)$;

(c) $(A \cup B) \cap (A-B)$; (d) $(B-A) \cap (A \cup B)$.

18. Let $A = (-3, -1, 0, 1, 3)$

$B = (-1, 0, 1)$

$D = (-3, 0, 3)$.

Then $(A \triangle B) \triangle D$ is equal to

(a) (0) ; (b) $(-1, 3)$; (c) $(1, -3)$; (d) $(-3, -1, 0, 3)$.

19. If α, β are the roots of the equation $x^2+2x-1 = 0$, then the equation whose roots are $1/\alpha$, $1/\beta$ is

(a) $x^2-2x+1 = 0$; (b) $x^2-2x-1 = 0$;

(c) $x^2+2x-1 = 0$; (d) $x^2+2x+1 = 0$.

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20. If 5 is a root of the cubic equation $x^3 - p.x + 10 = 0$, then p is equal to

(a) 135 ; (b) 27 ; (c) 5 ; (d) none of these.

21. Let x be a root of the equation $x^2 - 2 = 0$ then x is

(a) a rational number; (b) a positive integer;

(c) an irrational number; (d) a negative integer.

22. If $(1+x)^n = C_0 + C_1.x + C_2.x^2 + \dots + C_n.x^n$ then $C_1 + C_2 + \dots + C_n$ is equal to

(a) 2^{n-1} ; (b) 0 ; (c) 2^n ; (d) $2^n - 1$.

23. If $\alpha, \beta, \gamma, \delta$ are the roots of the biquadratic equation $x^4 - x^3 - 2x^2 + 1 = 0$ then $\alpha + \beta + \gamma + \delta + \alpha\beta\gamma\delta$ is equal to

(a) -2 ; (b) 2 ; (c) 0 ; (d) -1.

24. If α, β, γ are the roots of the equation $x^3 - x + 1 = 0$, then $\alpha^3 + \beta^3 + \gamma^3$ is equal to

(a) -3 ; (b) 3 ; (c) 1 ; (d) -1.

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2B

PERIODICAL EXAMINATIONS

Mathematics-1: Mathematics - Group B

Date: 14.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Find the points which are at a distance 5 from (-3, -4) and at a distance 13 from (5, 12).

b) Find the point of intersection of the lines given by

$$2x - 5y + 1 = 0 \quad \text{and} \quad x + y + 4 = 0.$$

c) What is the equation of the line through the point (2,1) and parallel to the line joining (2,3) and (3,-1)? [6+3+5]=[14]

2.a) Compute the acute angle between the lines

$$3x + y - 7 = 0 \quad \text{and} \quad x + 2y + 9 = 0.$$

b) What must be the value of 'k' in order that the three lines $4x + y - 3 = 0$, $kx + 2y - 3 = 0$, and $2x - y - 3 = 0$ should meet in a point? [6+6]=[12]

3. For what value of 'λ' does the equation given by $12x^2 + 36xy + \lambda y^2 + 6x + 6y + 3 = 0$ represent two straight lines? [8]

4. OA, OB are two fixed straight lines, A and B being fixed points. We take two points P, Q on these lines such that the ratio AP:BQ is constant. Show that the locus of the middle point of PQ is a straight line. [16]

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PERIODICAL EXAMINATIONS

Subject: English

Date: 21.10.68 Maximum Marks: 100 Time: 3 hours.

Note: Answer Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

GROUP A

Maximum Marks: 50; Suggested time: $1\frac{1}{2}$ hours.
Answer all questions.

1. Supply 'a', 'an' 'some', 'any', 'the' where necessary.
He was hungry after ___ swimming in ___ morning, but as there weren't ___ sweets in ___ shop, he bought ___ cake. [5]

2. Make these 2 sentences into 1 by adding a connecting relative pronoun.
What was the name of the film? You saw it last night. [1]

3. Show the difference in meaning between these sentences by explaining each sentence.

(i) My friend who works in London, has bought a new car.
My friend, who works in London, has bought a new car.

(ii) His house which is by the river is very large.
His house, which is by the river, is very large. [4]

4. Supply the correct tense (Past, Present or Present Perfect) to the verbs in brackets.

My elder brother (join) the army when he (be) seventeen. He (serve) in India when the first World War (break) out. He (continue) his training there for a time, and soon (become) an officer. Afterwards he (fight) in Iraq and Palestine. I (expect) you (hear) how he (win) a medal for bravery. [10]

5. Add the appropriate relative pronoun. If it can be omitted enclose it in brackets.

i) What is the name of the girl ___ lives next door?
ii) She is the tallest girl ___ I've ever seen.
iii) Bring me all the books ___ are on the table. [3]

6. Put the following sentences into the passive voice.

i) Has someone mended the chair yet?
ii) The painter will be painting the house from Tuesday to Saturday.
iii) People expect that the economic situation will improve very soon.
iv) Someone has found the boy the people wanted (2 passives). [4]

7. If the main sentence is true, only one of the following sentences is true. Make your choice.

A. At Camford University 414 candidates passed their final examinations this year and 59 failed, whereas last year 411 passed and 83 failed.

i) There were fewer candidates for the final examinations at Camford University this year than last year.
ii) The final year students at Camford University were of higher academic standard this year than last year.
iii) The examiners for Camford University were more lenient this year than last year.

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iv) The number of students at Camford University is decreasing. [1]

B. Although it is more expensive to go by train, Mr. Smith goes to his office every morning by train and not by bus because the train journey is 40 minutes shorter than the bus journey.

i) Mr. Smith has a 40 minute train journey between his home and his office.

ii) Mr. Smith dislikes travelling by bus.

iii) Mr. Smith would spend less money on fares if he travelled to work by bus instead of by train.

iv) It is always cheaper to travel by bus than by train. [1]

8. Nearly 30 years ago when Bernard Shaw visited New Zealand he suggested that the geothermal capacity of North Island, so abundantly evident from its geyser and lot springs, could be harnessed for industrial power. Now a geothermal power station at Wairakei has been delivering 6.500 Kw. of electricity to the national grid for some months. It is the first natural-steam-driven generating plant in the southern hemisphere and its development has proved so satisfactory that up to 27 bores to tap steam down to 3000 feet and to deliver 70,000 Kw. of electricity are scheduled.

A second phase of development at the site will increase the power output to 150,000 Kw. and it is estimated the natural steam potential in the area can support development of up to 260,000 Kw. Once testing and installation are complete - and this is the trickiest past of the operation - natural steam is the cheapest power source in the world. The cost of all these stages of New Zealand's Wairakei project is estimated at £ 21 million.

Wairakei is in fact only one small outcrop of the vast thermal region of New Zealand's North Island. The whole region extends across the centre of the island in a huge triangle, whose points are marked by the active volcano Ruapehu to the south, the spa town of Te Aroha in the west and an offshore island that rises blazing out of the Bay of Plenty on the eastern seaboard. The United Kingdom consultants who have been advising the New Zealand government on the development estimate that there are 7,390 million British Thermal Units looked up in this triangle waiting to be used.

It is possible that, when this new source of natural power is exploited, the steam may give out when released. Virtually nothing is known of the source of the underground heat, nor of its geophysical cause. At the present state of knowledge the chances are even for steam production from underground to continue unimpaired.

i) Make up sentences of your own to show the meanings of the following words and phases:

satisfactory; scheduled; potential; outcrop; exploit; harness for; evident from; lock up. [8]

ii) Write 4 short paragraphs on geothermal power in New Zealand under the following headings:

Potential, advantages, difficulties, prospects. [8]

iii) Write one sentence answer to the following questions on the passage.

a) What gave Bernard Shaw the idea that geothermal power could be developed in New Zealand?

b) Why was it decided to enlarge the Wairakei installation?

c) In what way is Wairakei unique?

d) What other possibilities are there of exploiting natural gas in New Zealand?

e) Why is it difficult to estimate the cost before production begins? [5]

GROUP B

Maximum Marks: 50; Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1. Write a precis of the following passage and add a suitable title:- [20+2]=[22]

'The universal tendency of the human mind is to shrink from the trouble of thinking out any of its so-called opinions. People become mentally indolent, too indolent to judge for themselves. Upon every conceivable subject they take their opinions ready-made. The memory thus becomes a store-house of conventional ready-made opinions, and these eventually harden into irrational convictions.

People are influenced in their opinion by the prevailing fashion. They fear singularity more than error; they accept numbers as the index of truth, and they follow the crowd. The dislike of labour, the fear of unpopularity, the danger even of setting up individual opinions against established convictions, strengthen this inclination. People take their opinions from their favourite newspaper, from the accepted beliefs of the society in which they move, or of the party or church to which they attach themselves, from tradition, from custom, from hereditary association, from any source except that of careful independent thought. If they are asked why they believe a particular thing, they will say, I have it on good authority, or I read it in a book, or it is matter of common knowledge, or everybody in the village believes it, or I learned it at school.

These replies mean that they have accepted information from others, without making any attempt to verify it, and without thinking the matter out for themselves. The causes of such beliefs are thus obvious, though such causes are clearly not reasons. But the causes may become reasons if we are able to recognize that our teachers, our family, and our neighbours are competent and truthful persons, and possess adequate information. Reasons of this kind are probably the principal ground on which, in mature life, we accept the great mass of our scientific, historical and other convictions. I believe, for instance, that the diameter of the sun is about 850,000 miles for no other reason than that I believe in the competence of the persons who have made the necessary observations and calculations.

2. Replace the underlined words, using the verb 'take' together with an adverbial or prepositional particle, and making any necessary changes in word order.

a) The teacher said I ought to start learning French.

b) He resembles his father in many ways.

c) The son assumed control of the business on the retirement of his father.

d) The secretary wrote the letter in shorthand.

e) He sopke English so well that people frequently assumed him to be an Englishman. [1(

3. Form verbs ending in - ate according to the definitions given, and write sentences illustrating their use:-

a) work together with someone

b) make complex

c) pacify or soothe

d) take part in

e) turn into vapour. [10]

4. Complete the table with related words belonging to the Specified Form classes:

Noun	verb	adjective	adverb
terror	------	-------	------
-------	Signify	-------	------
Destruction	---------	--------	-------
----------	---------	astonishing	-------

[9]

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PERIODICAL EXAMINATIONS

Economics-1

Date: 28.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A,B, and C in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A: Micro Economic (Theory)

Maximum Marks: 40 . Suggested time: 1 hour
Answer any two questions.

1. Given a consumer's total utility curve for a commodity, the price of the commodity per unit, and the utility of each rupee to the consumer, find by a geometrical construction the quantity of the commodity the consumer would like to buy. [20]

2.a) Define price-elasticity of demand at a point on a given demand curve.

b) If the demand curve is a negatively sloped straight line, show that price-elasticity of demand must be different at different points on it. [20]

3. How will the total consumer's expenditure on a commodity be affected by a small change in its price if the demand for the commodity is (a) elastic, (b) inelastic? Give reasons for your answer. [20]

GROUP B: Social Frame Work, National Income and Accounting.

Maximum Marks: 30 Suggested time: 1 hour

Answer Q.4 and any two of the rest.

1. Explain why 'Economic life may be looked at as a system of exchanges'. Add a short note on any other way of looking at it'. [12]

2. Should all kind of services be regarded as part of production? Explain your answer and state the problem as it was seen by Adam Smith. [12]

3. What are the problems connected with the assessment of the results of a year's production process? [12]

4. Write short notes on any one of the following:

a) single use goods and perishable goods. [6]

b) producer goods and consumer goods. [6]

c) relation between labour force and age and sex structure of the population. [6]

GROUP C: Indian Economic Structure

Maximum Marks: 30 . Suggested time: 1 hour
Note: Answer all questions.

1. What do you mean by 'economic structure'? Discuss the main indicators of economic development and also explain their limitations. [15]

2. Fully examine the trend of aggregate and per capita national income in India since 1948-49. [15]

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PERIODICAL EXAMINATIONS

Statistics-1: Statistics Theory and Practical

Date: 4.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. 100 children took three examinations. 40 passed the first, 39 passed the second and 48 passed the third. 10 passed all three, 21 failed all three, 9 passed the first two and failed the third, 19 failed the first two and passed the third.

Present the above information in a suitable tabular form with appropriate headings. Also find how many children passed at least two examinations. [9+6]=[15]

2. Explain briefly the uses of the bar diagram and its variants. [15]

3. Draw the less than type ogive of the following frequency distribution:

no. of rooms:	1	2	3	4	5	6	7	8	9
no. of houses:	17	32	49	28	12	5	4	0	1

[10]

4. State briefly why the histogram is preferred for continuous variates and the frequency polygon for discrete variates. [10]

5. Discuss the relative merits and demerits of the mean and the median as measure of location of a frequency distribution. [15]

6. The following shows the frequency distribution of weights of adult males born in a certain country:

weight (lb.)	no. of males	weight (lb.)	no. of males
90 - 100	2	180 - 190	304
100 - 110	26	190 - 200	174
110 - 120	133	200 - 210	75
120 - 130	338	210 - 220	62
130 - 140	694	220 - 230	33
140 - 150	1240	230 - 240	10
150 - 160	1075	240 - 250	9
160 - 170	881	250 - 260	3
170 - 180	492	260 - 270	1

a) Draw the histogram of this frequency distribution of weights.

b) Calculate mean, median, mode and the 9th decile of the distribution. [10+8+6+5+6]=[35]

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PERIODICAL EXAMINATIONS

General Science-1: Biology Theory

Date: 11.11.68. Maximum Marks: 100 Time: 3 hours

Note: Answer question number one and any three from the remaining. Illustrate your answers with suitable diagrams wherever necessary. Marks allotted for each question are given in brackets [ ].

1. Draw suitable labelled diagrams (only) of any five of the following:-

   (a) Euglena, (b) Trichonympha, (c) Section of a sponge,
   (d) Body wall of Hydra, (e) Segment of a tapeworm,
   (f) King crab, (g) Typical insect. [5 X 5]=[25]

2. Compare and contrast the distinguishing characteristics of the members of phylum Coelenterata and phylum Echinodermata, with notes on at least two members from each phylum. [15+10]=[25]

3. Give an account of the life-history of Schistosoma and discuss possible measures to eradicate the parasite. [20+5]=[25]

4. Discuss the economic importance of insects in general and outline the life-history of silk moth pointing out how silk is obtained, [15+10]=[25]

5. Write short notes on any five of the following:-

   (a) Entamoeba histolytica, (b) Aurelia, (c) Palolo worm, (d) Leech, (e) Octopus, (f) Economic importance of earthworms. [5 X 5]=[25]

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PERIODICAL EXAMINATIONS

General Science-1: Biology Practical

Date: 8.11.68 Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

<u>Note:</u> Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe the specimen (A) provided with suitable diagrams. [15+15]=[30]

2. Draw suitable diagrams and identify the specimen (B) provided. [14+6]=[20]

3. Identify and comment upon specimens C to L. [10X3]=[30]

4. Practical records. [20]

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(B)

MID-YEAR EXAMINATIONS

English

Date: 18.12.68   Maximum Marks: 100   Time: 3 hours

Note: Write Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50   Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

I. Insert 'a','an','the','some' or 'any' if necessary.

1) As ___ boy was going to ___ school by ___ tram, he wrote ___ essay about ___ houses he had seen. [5]

2) Put into the passive voice.

I've only used this pen once since the day I had it mended (2 passives). [2]

3) Make the following sentences into one sentence by adding the necessary relative pronoun.
This is an inflatable umbrella. I use it during the monsoon. [1]

4) Insert 'in', 'on' or 'at' in the appropriate spaces.

We arrived ___ Ranchi ___ exactly six O'clock and stayed ___ our aunt's home for one night. As we left ___ the morning, I remembered I had left my key ___ my coat which was ___ the chair. [6]

5) Add the necessary relative pronouns in the appropriate spaces.
The lecturer ___ came yesterday was late because the train ___ he caught had been derailed. [2]

6) Below the passage you will find several choices of words for each blank space in the passage. Fill in the blank spaces with the best choice.

After leaving school he (1) . French in Paris for two years then (2) to America where he now (3). He (4) to England once or twice ... (5) English quite well, but (6) yet the opportunity of visiting European countries. [6]

(1) A. has studied B. studied C. had studied
D. was studying.

(2) A. had moved B. will have moved C. has been moving
D. moved.

(3) A. is living B. has been living C. will have lived
D. lives.

(4) A. is visiting B. has visited C. had visited
D. visited.

(5) A. knows B. is knowing C. has known D. had known.

(6) A. will not have B. has not had C. had not had
D. is not having.

7) Re-word the following sentences using 'too' or 'enough'.

The student was very clever ___ could solve any mathematical problem.

The window was so dirty they couldn't see out of it. [2]

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8) Put the adverbs in the brackets into their correct order in the sentence. [

The soldier sang (last year, gaily, at midnight, in the square). [

II. Read the passage carefully then answer the questions below.

Never before has abstract mathematics been applied to so many problems. To meet the demands of industry, technology and other sciences, mathematics has been enlarged almost beyond recognition.

Whilst applied mathematicians have been involved with everyday problems, the pure mathematician seems to have lost touch with the world. Yet this abstractness can be useful, as it may help the mathematician see beyond the superficial details to the simple pattern beneath. For instance, celestial mechanics once used only by astronomers to calculate the positions of planets and comets, is now of practical value in calculating the orbits of the earth's satellites.

Even mathematical games can be useful. For example, mathematicians are still trying to find a general rule for calculating the number of ways a particle can travel from one corner of a rectangular net to another corner without crossing its own path. From the solution they will be able to tell chemists about the build-up of the long chain molecules of polymers.

Mathematicians have solved many new problems, they have developed new statistical methods for controlling quality in high-speed industrial mass production, they have developed Operations Research techniques used by businessmen for organising production & distribution, they have created an information theory for communications engineers to evaluate telephone, radio and television circuits, they have investigated human behaviour through game theory, used in both military and business strategy; the design of automatic control systems in such complex schemes as factory production lines and supersonic aircraft has been analysed - next they must turn to space flight.

Although mathematicians have only just become involved in biological and social sciences, these sciences are already becoming more mathematical. Biologists are applying information theory to inheritance, sociologists are using statistical methods to control their sampling. From these links biometrics, econometrics, psychometrics etc., have developed.

With the arrival of the computer, mathematicians can solve problems which before would have taken years of calculation, and in designing and programming computers they have had to develop new techniques. While computers have as yet contributed little to pure mathematical theory, they have been used to test certain relationships among numbers. It now seems possible that some day a computer will discover and prove a new mathematical theorem.

Give short answers, in your own words, to the following questions.

i) How have mathematicians been affected by technological progress?

ii) What is a general use of pure mathematics?

iii) How would mathematicians have dealt with problem now solved by computers? [6]

GO ON TO THE NEXT PAGE

2. Find phrases in the passage which mean the same as.
   i) no longer concerned with ordinary problems of life
   ii) methods of approach to problems, as used in commerce or in battles
   iii) the little things which on the surface seem important. [3]

3. Explain the meaning of the following words from the passage, by putting them into sentences of your own.

   Analyse, link, technique. [6]

4. Write, in your own words, a short account of the importance of higher mathematics in the advancement of science and technology. Give any examples you think will make your account clearer. [8]

5. Suggest a title for the passage. [1]

Group B

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

Write a precis of the following passage:-

In all civic unrest there are always a certain number of men who, either from excited passions, or fanatical conviction, or evil intentions, or just from a perverse taste for disorder, do all they can to push things as far as possible: they propose and support the wildest suggestions, and fan the flames whenever they begin to die down: they want the riot to burst all bounds and restraint. But, to balance these, there are always a certain number of others who are working with equal ardour and persistence to produce the opposite results; some moved by friendship or partiality for the people threatened, other with no other motive than a pious and instinctive horror of bloodshed and atrocity. May heaven bless them! In each of these rival parties unanimity of desire creates, even without any prior arrangements, an immediate uniformity of action. The mass or, one might almost say, the material for the tumult consists of a haphazard mixture of people who tend towards one or the other of these extremes; moved partly by hot-headedness, partly by knavery, partly by a desire to see justice done as they understand it, partly by the urge to see something exciting happen; ready for savagery or pity, for hatred or adoration, as the opportunity for wallowing in any of these feelings comes up; longing to hear and to believe astounding news every moment, needing someone to shout at, to applaud or to deride. 'Long live' and 'Death to' are the words they bring out most readily. And anyone who has succeeded in persuading them that such and such a person does not deserve to be hanged by the neck, need use no more words to persuade them that the same man is worthy of being carried in triumph. They are actors or spectators, instruments or obstacles, according as the wind is blowing; ready to keep quiet when they hear no more cries to repeat, to stop when there is no longer anyone to incite them, and to disperse when they hear many voices agreeing and no-one contradicting that it is time to go; and on getting back home they ask each other what it was all about. [20]

(Oxford G.C.E.)

2. Replace the word 'Said' in the sentences by one of the words at the head [5]

suggested   shouted   claimed
boasted   admitted

a) 'I can speak six languages fluently', he said.

b) 'Let's go to the cinema this evening', he said.

c) 'Stop that noise in the classroom', said the teacher.

d) 'That car you are driving is my property', said the man.

e) 'Yes, I broke the windows with my catapult', said the boy.

3. Complete the sentences with suitable clauses:-

You had better repeat your instructions carefully

(a) so that - - - - - - - - - - -

(b) in case - - - - - - - - - - -

(c) if - - - - - - - - - - -

(d) unless - - - - - - - - - - -

(e) before - - - - - - - - - - - [10]

4. Complete the sentences with participial phrases introduced by the verbs in brackets, using the form suggested and retaining the punctuation given:-

(a) The accused was led out of the court, still firmly - - - - - - - - - - - (maintain, present participle)

(b) I last saw him - - -(go, present participle)

(c) The game keeper caught a man - - - (shoot, present participle).

(d) - - - - - - - -, they decided not to spend their ~~holding~~ holid in Kashmere (tell, Passive perfect participle)

(e) The platform was crowded with people - - - - - - - - - - - (were, present participle) [10]

5. Form verbs ending in -fy according to the definitions given:-

(a) make pure

(b) fill with terror

(c) invest with glory

(d) report

(e) make clear. [5]

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MID-YEAR EXAMINATIONS

Mathematics-1: Set Theory, Algebra and Trigonometry

Date: 19.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) State Demorgan Laws. [4]

b) Simplify: (No proof is needed)

i) $A \cup (B \cap C)$

ii) $A \cap (B \Delta C)$ [2+2]=[4]

c) Find the number of subsets of a set consisting of n elements, where n is a positive integer. [6]

2.a) Explain clearly the terms: function, Domain and Range of a function. [3+3]=[6]

b) When do you say that a set is an enumerable infinite set. [5]

c) Show that the set of all integers is an enumerable infinite set. [10]

3. Prove or disprove the following statements:

a) If A, B, C are any three sets then

$(A \cup B) - C = A \cup (B - C)$. [5]

b) Let A be the set of all positive integers and B be the set of all rational numbers. Define $f(n) = n^2$ if n is a positive integer, then

i) f is a one-one function.

ii) Range of f = B. [2+3]=[5]

4.a) State Binomial theorem. [4]

b) If $(1+x)^n = C_0 + C_1 . x + C_2 . x^2 + \ldots + C_n . x^n$ prove that

$$C_0^2 + C_1^2 + \ldots + C_n^2 = \frac{(2n)!}{(n!)^2}.$$ [9]

c) State Remainder theorem. [4]

d) Solve $x^3 - 6x^2 + \frac{23}{4}x + \frac{9}{2} = 0$, given that the roots are in arithmetical progression, [10]

5.a) Define the 'modulus' and 'amplitude' of a complex number. [2+2]=[4]

b) Find the modulus and amplitude of

$$\frac{1+i}{2+3i}.$$ [6]

c) Define the multiplication of two complex numbers. [3]

d) State Demoivre's theorem. [5]

e) If P_1, P_2 represent the points corresponding to two complex numbers Z_1, Z_2 in the Argand diagram find the point corresponding to the complex number $Z_1 + Z_2$. Justify your answer. [10]

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MID-YEAR EXAMINATIONS

Mathematics-1: Coordinate Geometry and Calculus

Date: 20.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [.].

1. Answer any two sub-divisions:

a) Show that the middle point of the line joining (5,1) and (3,7) is also the middle point of the line joining (20,9) and (-12,-1). What geometrical conclusion can be drawn from this fact? [6]

b) The axes being rectangular the equation of a certain curve is

$$x^2 + y^2 - 4x + 6y = 14.$$

What will this become if the origin is transferred to (2, -3) without changing the directions of the axes? [6]

c) Find the equation of a line parallel to $3x-8y = 21$ and making an intercept of (-7) on the x-axis. [6]

2. Prove that the product of the lengths of the perpendiculars from (α, β) on the lines given by

$$ax^2 + 2hxy + by^2 = 0 \text{ is } \frac{a\alpha^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}}.$$ [14]

3. Show that the line $x+y = 2$ touches the circles $x^2+y^2 = 2$ and $x^2 + y^2 + 3x + 3y - 8 = 0$ at the same point. [8]

4. Find the equation of the circle whose diameter is the common chord of the circles:

$$x^2 + y^2 + 2x+ 3y+ 1 = 0; \quad x^2+ y^2+ 4x+ 3y+ 2 = 0.$$ [10]

5. Show that the length of the common chord of the two circles $x^2+y^2+ 2\lambda x+ c = 0$, $x^2+y^2+ 2my- c = 0$ is given by $2\sqrt{(\lambda^2- c)(m^2+ c)/(\lambda^2+ m^2)}$. [14]

6. Find the length of the latus rectum and the positions of the vertex, focus, and equation to directrix of the parabola $y^2 + 2x - 4y + 3 = 0$. [3+2+2+3]=[10]

7. PROVE: EITHER

If $(x_1,y_1)(x_2,y_2)(x_3,y_3)$ be three points on the parabola $y^2 = 4ax$ the normals at which meet in a point, then $y_1 + y_2 + y_3 = 0$. [12]

OR

If a chord of the parabola $y^2 = 4ax$ subtend a right angle at the vertex, the tangents at its extremities meet on the line $x+ 4a = 0$. [12]

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8. Find the following limits. (Answer any four sub-divisions)

a) $\lim\limits_{x \to -1} \frac{x^2+2x+1}{x+1}$. [3]

b) $\lim\limits_{x \to 5} \frac{2x}{x^2-25}$. [3]

c) $\lim\limits_{x \to 0} \frac{\sin 3x}{x}$. [3]

d) $\lim\limits_{x \to 0} \frac{5 + 3x + x^2}{1 - x}$. [3]

e) $\lim\limits_{x \to (-3)} \frac{x^3 + 27}{x^2 + 5x + 6}$. [3]

f) $\lim\limits_{t \to 0} \frac{7t + t^2 - t^3}{t(t + 2)}$. [3]

9. In what range is each of the following functions continuous? Discuss your answer in full. [Attempt any two sub-divisions].

i) $f(x) = \frac{x}{x^2 - 3x + 2}$. [4]

ii) $f(x) = \begin{cases} \frac{x^2 - x}{x - 1} & \text{for } x \neq 1 \\ 1 & \text{for } x = 1 \end{cases}$ [4]

iii) $f(x) = 2u + \frac{1}{u}$ $u = x - 3$. [4]

iv) $f(x) = \frac{x^2 - 9}{x^2 - 2x - 3}$. [4]

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MID-YEAR EXAMINATIONS

Economics-1

Date: 21.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A, Group B and Group C in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Micro Economic Theory

Maximum Marks: 40 Suggested time: 1 hour

Answer any two questions

1. Is it right to say that a two-commodity consumer's indifference map consists of a family of negatively sloped parallel straight lines? Give reasons for your answer. [20]

2. Explain why
   i) no two indifference curves have a common point,
   and ii) no part of an indifference curve is parallel to either axis. [20]

3. Define price effect, income effect and substitution effect and show that price effect is the algebraic sum of the other two effects. [20]

Group B

Social Frame Work, National Income and Accounting.

Maximum Marks: 30 Suggested time: 1 hour

Answer any two questions.

1. What are the considerations and what are the methods available to distribute the labour among different occupations in an economy? [15]

2. When unemployment comes, the constructional trades suffer particularly badly - why? [15]

3. Write short notes on any three of the following:
   a) Fixed capital and constant capital;
   b) Working capital and variable capital;
   c) Reserve Stocks of single use capital goods;
   d) Ordinary shares, preference shares and bonds. [15]

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Group C

Indian Economic Structure

Maximum Marks: 30   Suggested time: 1 hour

Answer any two questions.

1. Indicate the general structural features of underdeveloped economies. Critically examine if the economic problems of all the underdeveloped countries are similar. [10+5]=[15]

2. Analyse the sectoral origin of national income in India and its pattern of change during the period from 1948-49 to 1964-65. [15]

3. a) Indicate the age-structure of the population in India and explain its economic significance.

   b) Explain the concepts:

      i) Population at working age; and

      ii) labour force. [9+6]=[15]

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MID-YEAR EXAMINATIONS

Statistics-1: Statistics Theory

Date: 23.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Suppose you are preparing a frequency distribution for summarising a large number of observations. How would you choose the number and the positions of the class-intervals for this purpose? [16]

2. Let x be a variable assuming values $1,2,\ldots,k$, and let $F_1 = n, F_2, \ldots, F_k$ be the corresponding cumulative frequencies of the greater than type. Show that the arithmetic mean is given by

$$\bar{x} = \frac{1}{n} \sum_i F_i .$$ [10]

3. EITHER

If $x_1, x_2, \ldots, x_n$ are non-negative observations on a variate x, prove that their arithmetic mean cannot be less than their geometric mean. [16]

OR

Let $x_1, x_2, \ldots, x_n$ be n observations on a variate x. Find the value of x_0 for which the sum of absolute deviations $\sum_i |x_i - x_0|$ is the smallest. [16]

4. Why is the standard deviation widely used as a measure of dispersion? State its important properties (without proof). Show how it is related to the sum of squares of the differences between all possible pairs of observations. [8+8+6]=[22]

5. EITHER

Write a short note on the range, explaining its frequent use in statistical quality control. [12]

OR

Define the coefficient of variation and explain its uses. [12]

6. Describe fully, with computational layout, the calculation of mean, s.d., β_1 and β_2 from a frequency distribution table with equal intervals. Mention Sheppard's corrections and indicate when they should be applied. (You need not prove any relation.) [18+6]=[24]

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MID-YEAR EXAMINATIONS

Statistics-1: Statistics Practical

Date: 24.12.68. Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. The following information was obtained from the Year Book of the Commonwealth Bureau of Census and Statistics, Australia, No.39, 1953, p. 564:

Among the permanent new arrivals to Australia during 1950 there were 101949 males and 72591 females, and of these 21210 males and 19948 females belonged to the age group 'under 15 years'. The number of males and females in the age-group '15 and under 45' were 71801 and 43175 respectively; the corresponding numbers for the age-group '45 and under 65', were 7784 and 7824, and for the age-group '65 and over', 1154 and 1640, respectively.

Present the above information in a tabular form with appropriate headings, adding appropriate totals.

Express the numbers in different age-groups as percentages of the total number in all the age-groups, separately for each sex, and present these percentages also in the same table. [15]

2. Give a suitable diagrammatic representation of the following data:

Sector	Percentage of Working Force of a Certain Country	
	1870	1950
Farming	47	11
Proprietors, officials and professionals	8	21
Lower salaried	2	18
Industrial wage earners	26	43
Others	17	7
Total	100	100

[20]

3. EITHER

Below are given some data on wages of three groups of agricultural labourers:

Group	No. of labourers	Daily wage rate (Rs.)	
		mean	s.d.
men	270	3.50	0.62
women	130	2.25	0.46
children	40	1.50	0.28

Find mean and s.d. of the wage rates considering all the three groups of labourers together. [20]

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3. OR

The following shows the number of runs made by two batsmen in a number of innings:

Batsman A : 9, 0, 186, 72, 106, 34, 66, 135, 2, 27

Batsman B : 25, 51, 12, 67, 89, 47, 33, 59, 8, 78

Comment on the performances of the two batsmen after calculating means, standard deviations and coefficients of variation of their scores. [20]

4. EITHER

Find mean and variance of the following frequency distribution of percentage certain content obtained in 178 determinations on a certain mixed powder:

percentage carbon (class mark)	frequency
4.145	1
4.245	2
4.345	7
4.445	20
4.545	24
4.645	31
4.745	38
4.845	24
4.945	21
5.045	7
5.145	3

[35]

OR

For calculating the moments of a frequency distribution of annual message use (x) of 995 telephone subscribers, a computer introduced a transformed variable $u = (x - 525)/50$, and obtained the following sums:

$$\Sigma fu = -956, \quad \Sigma fu^2 = 9676, \quad \Sigma fu^3 = -22952,$$

$$\Sigma fu^4 = 283564,$$

where f denotes frequency. Find the mean, the s.d. and the β_1 and β_2 coefficients of the distribution, applying Sheppard's corrections for grouping. [35]

5. Practical Records. [10]

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MID-YEAR EXAMINATIONS

General Science-1: Physics Theory

Date: 25.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Define surface tension and angle of contact. How is surface tension connected with the surface energy? Derive an expression for the capillary rise in a tube dipped vertically into a liquid. What is Jurin's law? [3 X2+5+9]=[20]

2. Explain what is meant by the phenomenon of viscosity. What is Newton's law in this context? Hence define the coefficient of viscosity. What are its dimensions? [4+2+3+3]=[12]

3. Explain the following terms: Atomic number, isotope, nucleon, alpha particle. What is Pauli's exclusion principle? Distribute the electrons of L shell (n = 2) into the different sub-shells using the Pauli's principle. [4 X3+3+9]=[24]

4. Describe with a schematic diagram the experiment of Geiger and Marsden that established the existence of atomic nucleus. Describe the construction and the principle of action of a linear accelerator. [12+12]=[24]

5.a) Calculate the radius of the first Bohr orbit of hydrogen, with the following data:

$$h = 6.62\ 10^{-27} \text{ erg. sec.}, \quad m = 9.11\ 10^{-28} \text{ gm.},$$

$$e = 4.80\ 10^{-10} \text{ e.s.u.}$$ [12]

b) Calculate the excess pressure inside a spherical soap bubble of diameter 1 inch blown with a soap solution of surface tension 25 dynes per cm. [8]

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MID-YEAR EXAMINATIONS

General Science-1: Physics Practical

Date: 9.12.68 Maximum Marks: 100 Time: 3 hours

		Marks
1.	Perform the experiment as indicated in Card A.	60
2.	Practical Note Book	10
3.	Class work	20
4.	Oral test	10

Distribution of marks of Q.1.

Theory and working formula	:	10
Table	:	40
Calculation	:	5
Accuracy	:	5

N.B. Special credit would be given for intelligent multiplication of data.

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MID-YEAR EXAMINATIONS

General Science-1: Chemistry Theory

Date: 26.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for question are given in brackets [ ].

1. An organic comp. contains 85.72 per cent of carbon, 14.28 per cent of Hydrogen. Mol. wt. of the compound is 98. This comp when treated with Bromine water and alkaline solution of $KMnO_4$ separately the colour is atonce discharged. When treated with Ozone forms Ozomide, wh which On hydrolysis gave dimethyl ketone as one of the products. Write down the name and structural formula of the compound, also give equations of the reactions involved [25]

2. Give a brief account of geometrical and optical isomerism. [25]

3. Write notes on
   a) Hofmann reaction
   b) Aldol condensation
   c) Hell-volhand zelinsky Reaction
   d) Markownikoff's Rule. [25]

4. What happens when
   a) Primary, Secondary and Tertiary alcohols are seperately oxidised by alkaline $KMnO_4$ solution.
   b) Acid chloride, aldehydes, and Esters are seperately reduced.
   c) Sodium derivatives of alcohols are heated with alkyl halide. [25]

5. Starting from ethyl alcohol how can you prepare.
   i) Propoonic acid
   ii) Butanone
   iii) n-Butyric Acid. [25]

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PERIODICAL EXAMINATIONS

Economics-1

Date: 3.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A, Group B and Group C in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A: Micro-economic theory

Maximum Marks: 40 Suggested time: 1 hour

Answer any two questions.

1. Show that a two-commodity consumer's demand curve for a normal commodity must be negatively sloped. [20]

2. Define inferior commodity. When can a two-commodity consumer's demand curve for an inferior commodity have (a) a positive slope, (b) a negative slope? Justify your answer. [20]

3. Define a firm's isoquant map. Prove that no two isoquants can have a common point. [20]

4. What conditions should a firm satisfy in order to produce (i) the largest output at a given total cost and (ii) a given output at the least total cost? Justify your answer.

[N.B. For Q.3 and Q.4 assume that the firm produces a single commodity and uses only two inputs whose prices are given].

GROUP B: Social Frame Work, National Income and Accounting

Maximum Marks: 30 Suggested time: 1 hour

Note: Answer all questions.

1. Why are joint stock companies preferable to partnership Firms? [15]

2. In drawing a balance sheet for the nation, what are the problems connected with the evaluation of capital goods? [15]

GROUP C: Indian Economic Structure

Maximum Marks: 30 Suggested time: 1 hour

Note: Answer any two questions.

1. Examine the main features of underdevelopment in the Indian Economy. [15]

2. Describe the production pattern of the main agricultural crops of India indicating their regional distribution. [15]

3. Examine the main causes of low agricultural productivity in India indicating the per acre productivity of selected crops. [15]

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PERIODICAL EXAMINATIONS

General Science-1: Physics Theory

Date: 10.3.69 Maximum Marks: 50 Time: 2 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe, with a neat diagram, the construction and the principle of action of a cyclotron. [5+6+7]=[18]

2. Give briefly the history of the discovery of cosmic rays. [14]

3. Establish the Kinetic - theory equation and hence prove the Avogadro's hypothesis. [14+4]=[18]

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PERIODICAL EXAMINATIONS

[20]

General Science-1: Biology - Botany Theory

Date: 17.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Rewrite the following statements correctly without changing the portion within brackets:

   a) (The class Fungi imperfecti is so named because of) the development of imperfect sex organs in its members.

   b) (Ascomycetous fungi are characterised by the presence of) aseptate hyphae and asci containing exogenous spores known as aeciospores.

   c) (Tropic movement may be defined as) a special kind of movement of locomotion exhibited by tropical plants.

   d) (The following are the essential factors required for successful germination of a seed namely) low soil nutrient, optimum light and high atomospheric CO_2.

   e) (A phytohormone may be defined as a) stationary chemical substance produced in a specific plant organ where it exerts its specific effect at a very high concentration. [3X5]=[15]

2. Write a short account on different kinds of movements in plants. [20]

3.a) Define growth.

   b) Enumerate different internal and external factors that affect growth. [4+16]=[20]

4.a) What is meant by germination of a seed?

   b) What are the various conditions necessary for germination?

   c) What are the roles of different factors?

   d) Can you suggest an experiment to determine that a specific temperature level is required for germination? [2+5+8+10]=[25]

5. Write short notes on:

   a) Apical dominance
   b) Grand period of growth
   c) Classification of fungi
   d) Role of plant growth substances in agriculture and horticulture. [4 X5]=[20]

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PERIODICAL EXAMINATIONS

Statistics-1: Statistics Theory and Practical

Date: 24.3.69 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ].

GROUP A

Answer any four questions.

1. Define the correlation coefficient r_{xy} between two variates x and y, and show that if u and v are linear functions of x and y respectively, then r_{xy} and r_{uv} are numerically equal. [3+12]=[15]

2. Suppose you have fitted a linear regression $y = a + bx$, by least squares method, to a set of n observation-pairs $(x_1, y_1), \ldots, (x_n, y_n)$ on two variates x and y.

Prove the following results (all symbols have the usual meaning):

(i) $r_{\hat{y}e} = 0$ (ii) $r^2_{xy} = 1 - \dfrac{\Sigma e^2}{\Sigma (y - \bar{y})^2}$ [15]

3. Express the angle between the two lines of regression in a bivariate problem in terms of the two s.d.'s and the correlation coefficient r_{xy}. Discuss the cases $r_{xy} = -1$, 0 and +1, with appropriate diagrams. [10+5]=[15]

4. Explain the least squares method of fitting a parabolic regression $Y = a + bx + cx^2$ to a set of observations $(x_1, y_1), \ldots, (x_n, y_n)$. Give the normal equations and the computational layout. What short-cuts are available when the observations form a time series? [12+3]=[15]

5. Define the correlation ratio η_{yx} and prove that it cannot be smaller than the correlation coefficient r_{xy}. Discuss the following cases briefly with scatter diagrams: $\eta_{yx} = 0$, $\eta_{yx} = 1$ and $\eta_{yx} = |r_{xy}|$. [4+5+6]=[15]

6. Discuss the significance of the correlation coefficient, quoting necessary results. In what way the correlation ratio is an improvement over the correlation coefficient? (You need not prove any result.) [9+6]=[15]

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GROUP B

Answer all questions.

7. EITHER

Compute the correlation coefficient between Federal Reserve Bank discount rates and the discount rates of commercial banks from the correlation table given below. Apply usual checks on the calculations.

discount rates (%) of commercial banks (mid-points)	discount rates (%) of federal reserve banks (midpoints)												To[tal]
	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	
8.00										1	1	2	[illegible]
7.50										7	9	1	17
7.00								5	4	63	9	36	117
6.50							4	22	10	22	1	3	62
6.00					9	21	146	150	8	32			366
5.50				1	90	164	175	45	-	-	-	-	475
5.00			4	25	111	196	65	1	-	-	-	-	402
4.50			16	27	122	96	3	-	-	-	-	-	264
4.00	1	9	19	19	29	-	-	-	-	-	-	-	77
3.00	4	2	1	-	9	-	-	-	-	-	-	-	16
Total	5	11	40	72	370	477	393	223	22	125	20	42	180[illegible]

[40]

OR

Find the constants of the least squares regression line of y on x from the following observations on some cities in the US in 1950:

City No.	x: average family income (thousand dollars)	Y: average family consumer expenditure (thousand dollars)
1	3.60	3.40
2	5.10	4.52
3	3.78	3.90
4	3.04	3.09
5	5.04	4.58
6	3.15	3.22
7	3.55	3.47
8	4.00	3.55
9	2.93	2.85
10	5.33	5.05

Draw the scatter diagram and show the regression line passing through the scatter. [25+10+5]=[40]

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PERIODICAL EXAMINATIONS

ENGLISH

Date: 31.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1. Put the following sentences into the past tense.

   a) They must do it at once.
   b) You are not to choose an expensive one.
   c) He won't have to read the whole book, will he?
   d) You don't need to call him 'Sir'. [4]

2. Supply the correct tense of the verbs in brackets.

   a) I'll come to see you before I (leave) for England.
   b) I would have come sooner if I (know) you were there.
   c) When I (see) him, he (sit) asleep in a chair.
   d) He (discover) to his horror that he (eat) the maggot.
   e) If I (be) a ghost, I would frighten all the people I dislike.
   f) We (meet) you tomorrow after you (finish) your work.
   g) I will give you good marks, if you (be) good. [10]

3. Write a few paragraphs on ONE of the following:

   i) My ambition in Life.
   ii) My favourite Festival.
   iii) A narrow escape.
   iv) What the world will be like in 2,000A.D.
   v) Harvest time. [20]

4. Read the passage carefully and answer the following questions:

There are several general considerations which may profitably be borne in mind when approaching the subject of statistics.

First, it is both a science and an art. It is a science in that its methods are basically systematic and have general application; and an art in that their successful application depends to a considerable degree on the skill and special experience of the statistician, and on his knowledge of the field of application, e.g. economics. Statistical methods are not a kind of automatic machine into which numbers can be put and from which perfect results can be taken. Nevertheless, the subject is not a closed mystery, and I believe that it is not necessary to be a statistician to appreciate the general principles underlying it.

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As a science, the statistical method is a part of the general scientific method, and is based on the same fundamental ideas and processes. It teaches the scientific method in terms of things of everybody experience, and inculcates a habit of scientific approach to ordinary economic, social, and political problems. It will be seen, however, that statistical methods have their own special features. These arise from the fact that the data are not simple, like those that usually result from a well-designed and well-controlled scientific experiment, but are relatively complex, being the result of a number of causes all operating together without control. Statistics deals with figures that are subject to uncontrolled variation.

Another feature that statistics has in common with other scientific subjects is that it is not finished and complete; it is always developing. Despite its power and essential usefulness, it has limitations and imperfections; but future developments will undoubtedly reduce these.

The scope of the subjects included under statistics is wide; and few, if any, statisticians are expert in all branches. Some specialize in the development of the mathematical theory underlying statistical methods, and are essentially mathematicians. Others are interested in the methods themselves, both elementary and advanced, and in their general application to almost any field, although they often have also some special experience of one field. There are also statisticians who are able to use with confidence only elementary methods - perhaps fairly simple tables, diagrams, and averages - but who have a very wide and deep knowledge of some field of application.

a) Why is statistics both a science and an art? [5]

b) What are the features statistics has in common with other scientific subjects? [6]

c) What are the special features of statistics? [5]

d) Suggest a suitable title to the above passage. [1]

GROUP B

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1. Write out the sentences using the verbs in brackets in the gerund, participle, or infinitive form:-

(a) After (get)(known) him better, I regretted (judge) him unfairly.

(b) A job worth (do) is worth (do) well.

(c) I should prefer (go) to the Cinema rather than (sit) here (listen) to the radio.

(d) I would advise (you, wait) before (decide) (accept) his offer.

(e) If I catch (you, cheat) again, I shall make you (stay) in after School (do) some extra work. [10]

2. Complete the sentences with an adverbial clause as indicated:-

(a) He speaks English much better .... (comparison)

(b) We left the car .... (Place)

(c) Such was his anxiety ... ... (Result)

(d) ... ... , I shall expect to see you more often (Time)

(e) We arranged to hire a Coach ... ... (Purpose) [10]

3. Form verbs ending in - ise according to the definitions given, and write sentences illustrating their use:-

(a) Prepare for movement or action

(b) reduce to a minimum

(c) go through carefully and correct where necessary

(d) say that one is sorry for doing wrong

(e) happen at the same time. [10]

4. Fill in the blanks:

(a) I am tired ---- depending --- tram service --- getting --- town and back.

(b) I don't want to burden you --- my worries --- the future.

(c) The loudspeaker was blaring --- dance music --- the hour.

(d) The books are --- loan --- us --- a private library.

(e) It is no use your begging --- mercy --- me. [10]

5. Write in about 150 words on how to make a good impression at an interview. [10]

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PERIODICAL EXAMINATIONS

MATHEMATICS

Date: 7.4.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

GROUP A

Set theory, Algebra and Trigonometry.

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1.a) State clearly (i) Cauchy-Schwartz inequality
(ii) Arithmetic and Geometric inequality. [5+5]=[10]

b) If the sum of the sides of a triangle is a constant show that the area is greatest when the triangle is equilateral. [10]

c) Show that $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \dots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} \geq n$,

where each $a_i > 0$. [5]

2.a) Explain clearly the terms 'Field of sets', and 'Ring of sets'. [8]

b) Is it true that every ring is a field? Justify your answer. [7]

c) Let X be the set of all positive integers.
Define $\mathfrak{F} = \{A: A \subseteq X, \ A \text{ is finite or } A^c \text{ is finite}\}$
Show that $\mathfrak{F}$ is a field of sets. [10]

GROUP B

Coordinate Geometry and Calculus.

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1.a) In the curve $y = a e^{\frac{x}{a}}$, if ψ (psi) be the angle which the tangent at any point makes with the axis of x, prove that $y = a \tan \psi$.

b) Prove that if a particle move so that the space described is proportional to the square of the time of description, the velocity will be proportional to the time and the rate of increase of the velocity will be constant. [5+5]=[10]

2. If $f(x) = \frac{1}{1 - x}$ prove $fff(x) = x$

Also if $f(x) = a + bx$ show that

$f^4(x) = ffff(x) = a\left(\frac{b^4-1}{b-1}\right) + b^4 x$. [3+4]=[7]

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3. Find $\frac{dy}{dx}$ in the following cases [Attempt any three sub-divisions].

i) $y = \log \frac{x^2 + x + 1}{x^2 - x + 1}$, (ii) $y = (x+a)^m (x+b)^n$

iii) $y = \sqrt{\frac{1 - x}{1 + x}}$ (iv) $y = \sin^{-1} \frac{1}{\sqrt{1+x^2}}$.

[6+6+6]=[18]

4.a) Find the n^{th} derivative of $e^{ax} \sin (bx)$.

b) Find the maximum and minimum values of the function given by

$$y = \frac{x^2 - x + 1}{x^2 + x + 1}.$$

OR

b) A closed box whose base is a rectangle twice as long as it is wide, is to have a total surface area of 108 sq. inches. What is the greatest capacity the box can have?

[6+9]=[15]

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ANNUAL EXAMINATIONS

English

Date: 19.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A
Maximum Marks: 50
Answer all questions

1. Put the verbs in brackets into the correct tense.

a) When we (go) to see them last night, they (play) cards; they (say) they (play) since 6 O'clock.

b) I (go) if I had known about it.

c) In a few minutes time, when the clock (strike) six, I (wait) here for three-quarters of an hour.

d) If I (have) the courage, I would have answered him back. [8]

2. Reword the following sentences using 'too' or 'enough'.

a) Asis is so fat he can't tie up his own shoes.

b) I have very little ink; I won't be able to finish this letter. [2]

3. Insert the adverbs in brackets in their correct place in the sentence.

Saroj arrives (always, at ten O'clock, punctually, at the Institute). [2]

4. Form the following conversation into reported speech.

'Suppose we change the subject', the March Hare interrupted'. 'I vote the young lady tells us a story'.

'I'm afraid I don't know one', said Alice a little alarmed at the idea.

'Then the Dormouse shall', cried the March Hare and the Mad Hatter. 'Wake up, Dormouse!'

Dormouse feebly said, 'I wasn't asleep. I heard every word you were saying'.

'Tell us a story', said the March Hare.

'Yes, please do', pleaded Alice. [5]

5. Reword the following sentences using 'have' or 'get' with a past participle

a) Order someone to send the Cokes round to the hostel.

b) I asked a man to mend my shoes. [2]

6. Add 'a', 'the', 'a', 'an', 'some', 'any', or a relative pronoun where necessary. Put '0' where nothing is required.

— film - I saw yesterday, but - name I cannot remember, was - strange one. I could not understand - language, yet everyone was roaring with - laughter. [6]

7. Write a few paragraphs on ONE of the following:

a) Travel is the best education.
b) What makes my life worthwhile.
c) Leisure time is wasted time. [15]

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8. Comprehension.

a) The claim that sociology is a scientific discipline raises more interesting questions. The physical sciences have long held pride of place as examples of what sciences really should be. The verification of hypotheses by controlled experiment, the abstraction of such physical characteristics as are measurable and the discovery of functional dependence between one measurement and another, the establishment of broad unifying theories, in terms of which a teeming variety of phenomena are explained, together with that predictive accuracy which enables the scientist to apply his theories to the construction of bridges, steam-engines, aeroplanes, television sets and atomic bombs, all these have set a standard of 'scientific' research. If you cannot experiment, if you cannot measure, if you cannot be confident in your social engineering, you cannot be said to be engaged in scientific study at all. However, problems of microphysics in general, and the principle of indeterminism, in particular, have given rise to a great deal of uneasy reflexion. The changes in theory which have followed one another so swiftly have made us less certain that what 'science teaches' to-day will be what science will teach to-morrow. It is now realized that the abstract mathematical theories which apply to certain aspects of reality need not be of the same order as those which are appropriate for dealing with other aspects. The notion of what is meant by a science should not be taken to be the closest approximation to the procedures and formulations of the physical sciences; it is something far more general.

Answer all the following questions in your own words.

i) How is sociology different from the physical sciences.

ii) How can we now justify calling it a science. [6]

b) If the main sentence is true, only one of the following sentences is true. Make your choice.

A. Simuliid flies are common in many parts of the tropics; furthermore, throughout Central Africa and in parts of Central America they are carriers of a particularly unpleasant disease, known as Onchocerciasis, which may cause blindness.

i) Simuliid flies, by carrying the disease known as Onchocerciasis, are a major cause of blindness in the tropics.

ii) A disease which can result in blindness is carried by the Simuliid flies in certain tropical areas.

iii) Throughout Central America, Simuliid flies act as disease carriers.

iv) Onachcerceriasis, a disease which destroys the vision of those who catch it, is carried by the Simuliid fly throughout Central Africa and parts of Central America.

B. 'The Theory of Laminar Flow' by the late Professor J.B.Horrocks, was first published by the Fordham Press in 1946, and a second edition thoroughly revised by Prof. H.R.Smith, was published in 1957.

i) The author of 'The Theory of Laminar Flow' is Professor H.R.Smith.

ii) 'The Theory of Laminar Flow' was written between 1946 and 1957.

iii) The first edition of Horrocks' 'The Theory of Laminar Flow' was published by Fordham Press.

iv) Professor Smith revised 'The Theory of Laminar

Group B

Maximum Marks 50

Answer all questions.

1. Complete the sentences, observing a correct sequence of tenses:-
   (a) Should you need my help again, ...
   (b) If only you would read more carefully, ...
   (c) If you had taken my advice, ... [6]

2. Change the clauses underlined into participial phrases, making any necessary changes in word order:-
   (a) As he had witnessed the crime, he was expected to give evidence in court.
   (b) When I visit a strange city, I like to have a guide-book with me.
   (c) It strikes me that he is an intelligent man. [6]

3. Complete the sentences with adjectives ending in ___ able, __ ible, or __ uble, derived from the verbs given in brackets:-
   (a) Only a limited number of types of fungi are (eat)
   (b) His moods are very (change)
   (c) Men may die, but their words are (destroy, negative). [6]

4. Fill in the blank spaces:-
   (a) Write ... pencil.
   (b) She sat ... her aunt and uncle.
   (c) The streets are lit ... electricity.
   (d) He is ... the phone.
   (e) I've nothing to write ... [5]

5. Complete the table:-

Noun	Verb	Adjective
Stability	-	-
-	tolerate	-
-	-	equal

[3]

6. Put the verbs in brackets into the correct forms:-
   (a) I saw him (help) her (cook) the dinner.
   (b) Are you going (keep) me (wait) all day?
   (c) (See) is (believe)
   (d) He refused (allow) her (think) for herself.
   (e) Children are supposed (obey) their parents without (ask) why. [5]

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7. Make a precis of the passage and add a suitable title:-

Interest in the consumer as a factor in economic activity is of recent origin. Adam Smith and his followers assumed that the consumer would be the final arbiter of economic activity, for by his power to buy or withhold his patronage he could determine what was produced, how much was offered for sale, and the price at which goods were sold. No one really believed this; and virtual control over economic life gravitated into the hands of the producers. Through advertising and monopoly, producers controlled consumers' desires and prices. The consumer soon became the 'forgotten man' of economics. Resentment against this condition has found an outlet in two directions: through legislation and through consumer organizations. Through the former the consumer has secured protection against the most dangerous and fraudulent practices of producers; through the latter consumers have been using their power to bargain and demand fair treatment, on the threat of withholding patronage and establishing a consumer controlled system of production and distribution.

Only the most fanatical supporters of co-operation, however, look upon it as an eventual substitute for the present economic system. More conservative persons recognize that the great value of the co-operative movement lies in offering competition and a 'yard stick' for private business enterprise. Co-operative business activity will probably never be able to enter the field of large-scale industry where heavy overhead requires a type of financing not open to co-operative group s. Furthermore, the co-operative undertakings are subject to the fluctuations in business activity which undermine the stability of the economy as a whole. The consumer co-operative movement is not yet in a position to act as a stabilizing factor, albeit there are some who claim that the co-operatives were responsible for the ease with which the storm of depression was weathered in Scandinavian countries. In recent years the organized consumers speaking through their co-operative societies have been notably instrumental in forcing consideration for the consumers' problems into the forefront of government economic activity. [19

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ANNUAL EXAMINATIONS

Mathematics-1: Set Theory, Algebra and Trigonometry

Date: 20.5.69 Maximum Marks: 100 Time: 3 hours

Note:- Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Set Theory

This group carries a total of 65 marks. Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) Define the symmetric difference of two sets A and B. Illustrate by a Venn diagram. [1+2]=[3]

b) State De'Morgan Laws. Illustrate by Venn diagrams. [4]

c) When do you say that two sets are disjoint? Let A_i, $1 \leq i \leq n$, be sets. Express

$$\bigcup_{i=1}^{n} A_i$$

as a disjoint union of n sets. [2+5]=[7]

2.a) Find the number of subsets of a set consisting of n elements where n is a positive integer. [6]

b) When do you say that a set A is countable infinite? Show that the set of odd integers and also of rational numbers are countable infinite sets. [2+3+5]=[10]

3. Explain clearly the terms 'Field' and 'Ring' of subsets of X. Give one example of each. [3+3+2+2]=[10]

4. State which of the following relations are correct and which incorrect and justify your answer in each case.

a) For any three sets A, B, C

$$A \cup (B - C) = (A \cup B) - C$$ [5]

b) If R_1 and R_2 are rings of subsets of X then $R_1 \cap R_2$ is a ring of subsets of X. [5]

c) If F_1 and F_2 are fields of subsets of X then $F_1 \cup F_2$ is a field of subsets of X. [5]

d) Let X be the set of all positive integers. Let R be the set of all finite subsets of X.

If $A \in R$ define n(A) to be the number of elements in the set A then

$$n(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2)$$

where $A_i \in R$, $i = 1,2$. [5]

e) Every ring is a field. [5]

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Group B: Algebra and Trigonometry

This group carries a total of 61 marks. Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

5.a) State Cauchy-Schwartz inequality and prove it. [2+5]=[7]

b) Show that

$$\sum_{i=1}^{n} a_i \cdot \sum_{i=1}^{n} \frac{1}{a_i} \geq n^2$$

where each a_i is greater than zero. [4]

c) Show that

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \dots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} \geq n$$

where each a_i is a positive number. [6]

6.a) Define the product of two complex numbers. Show that this product is associative. [2+3]=[5]

b) State De-Moivre's theorem for a positive integral power and prove it. [2+4]=[6]

c) Define the terms, 'modulus' and 'principal amplitude' of a complex number. [2+2]=[4]

d) Find the 'modulus' and 'principal amplitude' of

$$\frac{(1+i)^2}{(1-i)^3} \quad \text{where } i^2 = -1. \qquad [3+4]=[7]$$

7.a) State Binomial theorem for a positive integral exponent and prove it. [2+6]=[8]

b) If $(1+x)^n = C_0 + C_1 \cdot x + C_2 \cdot x^2 + \dots + C_n \cdot x^n$

where n is an even positive integer show that

$$C_1 + C_3 + \dots + C_{n-1} = C_0 + C_2 + \dots + C_n. \qquad [5]$$

c) State remainder theorem and prove it. [2+6]=[8]

d) For what values of a,

$$4x^4 - (a-1)x^3 + ax^2 - 6x + 1$$

is divisible by $(x-1)$? [5]

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ANNUAL EXAMINATIONS

Mathematics-1: Co-ordinate Geometry and Calculus.

Date:21.5.69 Maximum Marks: 100 Time: 3 hours.

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) If p and p' denote the perpendiculars from the origin upon the lines $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then prove that

$$4p^2 + p'^2 = a^2.$$ [4]

b) Find the angle between the lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$. Find also the co-ordinates of their point of intersection and the equations of lines drawn perpendicular to them from the point $(3,-2)$. [3+3+3+3]=[12]

2. Write down the equation to the pair of straight lines joining the origin to the points of intersection of the straight line $y = mx + c$ and the curve

$$x^2 + y^2 = a^2.$$

Prove that they will be at right angles if $2c^2 = a^2(1+m^2)$

[4+5]=[9]

3.a) Given the three circles: $x^2+y^2-16x+60 = 0$;
$3x^2+3y^2-36x+81 = 0$; and $x^2+y^2-16x-12y+84 = 0$.
Find (i) the point from which the tangents to them are equal in length and (ii) this length. [5+3]=[8]

b) Find the equation to the circle which passes through the origin and cuts orthogonally each of the circles
$x^2 + y^2 - 6x + 12 = 0$ and $x^2 + y^2 - 2x - 2y + 7 = 0$. [8]

4. Given that $y = e^{\tan^{-1}x}$. Prove the following

i) $(1 + x^2)y_2 + (2x-1)y_1 = 0$ and

ii) $(1 + x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$

where the suffixes denote the derivatives of the corresponding order. [6+8]=[14]

5. Investigate the maximum and minimum values of the expression $3x^5 - 25x^3 + 60x$. [12]

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Group B

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) For what point on the parabola $y^2 = 4ax$ is the normal equal to twice the sub-tangent? [6]

b) The normal at the point $(at_1^2, 2at_1)$ meets the parabola $y^2 = 4ax$ again in the point $(at_2^2, 2at_2)$;

Show that $t_2 = -t_1 - \frac{2}{t_1}$. [6]

2. In the hyperbola $4x^2 - 9y^2 = 36$, find the axes, the co-ordinates of the foci, the eccentricity and the latus rectum. [3+4+2+4]=[13]

3. Find the equation to the tangent at the point (α, β) on the following curves [Attempt any two sub-divisions].

i) $x^2 + y^2 = c^2$ (ii) $y = \log x$

iii) $y = \tan^{-1} x$ (iv) $y = e^x$. [5+5]=[10]

4. Find $\frac{dy}{dx}$ in the following cases: [Attempt any three sub-divisions]

i) $y = \text{Sin}(\sqrt{x})$ (ii) $y = \text{Sin}^{-1} \frac{a + b\cos x}{b + a\cos x}$

iii) $y = (e^{\tan^{-1} x})[\log(\text{Sec}^2 x^3)]$ (iv) $y = e^{e^x}$.

[7+7+7]=[21]

5. EITHER

a) If $y = \sqrt{\text{Sin}\, x + \sqrt{y}}$ show that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$. [10]

OR

b) If S_n denotes the sum of a G.P. to n terms prove that $(R-1)\frac{dS_n}{dR} = (n-1)S_n - nS_{n-1}$ where R is the common ratio. [10]

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ANNUAL EXAMINATIONS

Economics-1: Micro-economic Theory

Date: 22.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any two questions.

1.a) Define the isoquant map of a single-output, two input firm. [11]

b) Assuming that the input prices are given, find the conditions which a firm should satisfy in order to

i) produce a given output at the least cost and
ii) produce the largest output at a given cost. [7+7]=[14]

2.a) In what respects does a firm's short-period cost differ from its long period cost? [5]

b) Explain why in the short period a firm's average variable cost curve and (overall) average cost curve are both U-shaped. [20]

3. Prove the following:

a) The output of a firm for which the overall average cost is the lowest is greater than the output for which the average variable cost is the lowest. [15]

b) If a firm's average variable cost decreases as its output increases, then its marginal cost is less than its average variable cost. [7]

c) When a firm's average cost is the least, it is equal to the marginal cost. [3]

Group B

Maximum Marks: 50

Answer any two questions.

1. Prove the following: If a two-commodity consumer thinks that he will be better off if he has more of one commodity and no less of the other, then in his indifference map

a) No two indifference curves will have a common point , [12]

b) no part of an indifference curve will have a positive slope; [6]

c) no part of an indifference curve will be parallel to either axis. [7]

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2. A consumer's indifference map in a two-commodity world consists of negatively sloped parallel straight lines.

a) Show that the consumer will either (i) buy only one of the two commodities or (ii) spend his income just anyhow. [15]

b) Prove that in which of the two ways the consumer will behave will depend on the ratio of the prices of the two commodities, but not on the consumer's spendable income. [10]

3. The demand curve for a commodity is a negatively sloped straight line.

a) What do the intercepts of the demand curve on price and quantity axes represent? [7]

b) At which point on the curve is the modulus of the price-elasticity of demand equal to 2? [7]

c) Can the price-elasticity of demand have the same value at two different points on the curve? Give reasons for your answer. [1

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ANNUAL EXAMINATIONS

Economics-1

Date: 23.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Social Frame Work, National Income and Accounting

Maximum Marks: 50

Answer any two questions.

1. Why and what qualifications are necessary to the fundamental equations of earning and spending of the social income due to the economic activities of the state? [25]

2. Explain the statement 'in the case of an open economy, we are not able to establish the identity between social income and net social product either on the earning side or on the spending side; but the two ways of reckoning must be consistent with one another'. [25]

3. Assuming a closed economy and no economic activity of the state, show how one can build up the estimates of social product and/or social income from the accounts of individual firms. [25]

Group B: Indian Economic Structure

Maximum Marks: 50

Answer question 1 and any two questionsfrom the rest.

1. EITHER

Describe the age-structure of the population in India and explain its economic significance. [10]

OR

Indicate the variations in the levels of per capita consumer expenditure of the Indian people in rural and urban sectors. [10]

2. Do you find any relationship between,

a) per capita national income and distribution of labour force in different sectors; and

b) per capita national income and sectoral origin of national income,

in different countries of the world? Quote statistical data in support of your argument. [20]

3. Examine the trends of agricultural production in India during the period of planning. To what extent have improvements in productivity and the extension in acreage contributed to the rise in agricultural production in this period? [20]

4. Indicate the distinctive features of the industrial structure of India before independence. Examine to what extent the Five Year Plans have made an attempt to correct the imbalances in the structure. [20]

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ANNUAL EXAMINATIONS

General Science-1: Chemistry Practical

Date: 24.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Determine the strength of Pottassium Permanganate solution against oxalic acid of given strength. [50]

2. Find out the organic elements present in the given organic sample. [25]

3. Laboratory Note Book. [10]

4. Viva Voce. [15]

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ANNUAL EXAMINATIONS
Statistics-1: Statistics Theory

Date: 26.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 40

Answer any two questions.

1. Explain the concept of the regression of y on x. Establish the relation

$$s_e^2 = s_y^2(1 - r^2)$$

for the variance of residuals e from the linear regression of y on x. (All symbols have the usual meaning.) What light does the relation throw on the correlation coefficient? [5+8+7]=[20]

2. Given n observation-pairs $(x_1,y_1),(x_2,y_2),\dots,(x_n,y_n)$ on two variates x and y, how would you fit a regression equation $y = ab^x$ by the method of least squares? Describe the method fully giving computational layouts. Mention any graphical test(s) you might carry out before choosing this type of equation. [14+6]=[20]

3. Write short notes on any two:

(i) The index of correlation, (ii) Limitations of the correlation coefficient, (iii) The origin of the term 'regression'. [2×10]=[20]

Group B: Maximum Marks: 60. Answer any three questions.

4. Bring out the advantages of the method of orthogonal polynomials in the fitting of polynomial regressions of y on x. [20]

5. Define Spearman's rank correlation coefficient and obtain its expression (in terms of the sum of squares of the differences between ranks) for the case where ties are present in both the rankings. [5+15]=[20]

6. Define the generalized correlation coefficient between two variates x and y, and show that the product-moment correlation coefficient and Kendall's rank correlation coefficient can be regarded as special cases of the generalized coefficient. [5+8+7]=[20]

7. Explain the concept of association between two attributes A and B, giving appropriate illustrations. Mention the common measures of association for a 2 × 2 contingency table and examine the circumstances in which they attain the values ± 1. [8+5+7]=[20]

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ANNUAL EXAMINATIONS

Statistics-1: Statistics Practical

Date: 27.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. The following shows the monthly expenditure on clothing (y) and on all consumer items (x) for ten families:

Family No.	1	2	3	4	5	6	7
Monthly expenditure in Rs. on:							
Clothings	16.6	17.1	22.6	24.7	28.2	27.6	33.9
All items	174.4	198.3	252.9	297.4	342.1	387.5	468.2

Family No.	8	9	10
Monthly expenditure in Rs. on:			
Clothings	47.7	57.2	71.1
All items	570.1	869.2	1253.3

Draw a scatter diagram and comment on the nature of the dependence of y on x. [10+5]=[15]

2. EITHER

The following bivariate frequency distribution is based on an enquiry on the production of jute in West Bengal:

weight in gm. of dry jute fibre (class-mark)	length in cm. of green plant (class-mark)						
	111.5	127.5	143.5	159.5	175.5	191.5	207.5
1.175	12	25	15	1			
2.775	1	4	33	59	29	3	
4.375	1		4	28	35	14	2
5.975				2	20	18	1
7.575				1	1	14	5
9.175					4	8	2
10.775						3	2
12.375							3

Find the correlation ratio of the weight of dry fibre on the length of green plant. [30]

OR

The following gives some information on the relationship between marks obtained by students at their first attempt at B.A. examination (denoted y) and their marks at Intermediate Arts examination (x), both marks being expressed in percentage form:

interval of x:	30-34	35-39	40-44	45-49	50-54
no. of students:	18	56	57	35	16
average of Y:	37.2	41.3	44.2	47.0	51.8

Given that the standard deviation of y is 8.76, find (i) the linear regression of y on x, and (ii) the correlation coefficient between x and y. [30]

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3. Below is indicated the variation in hardness along the length of a 18-inch long bar of steel:

distance from one end of bar (inch) (x):	0	3.6	7.2	10.8	14.4	18.0
hardness (y):	250	298	374	454	558	671

Fit a second degree polynomial regression of y on x and display the fit graphically. (Credit will be given for use of the technique of orthogonal polynomials.) [35]

4. Viva Voce [10]

5. Practical Record [10]

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ANNUAL EXAMINATIONS

General Science-1: Physics Theory

Date: 28.5.69 | Maximum Marks: 100 | Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 60

Answer any three questions.

1. Deduce the dimensions of the following quantities: Constant of gravitation, surface tension, coefficient of viscosity, Young's modulus. [12]

   Assuming that the period T of vibration of a tuning fork depends upon the length l of its prongs, on the density ρ and Young's modulus Y of the material, find by the method of dimensions a formula for the period of vibration. [8]

2. Deduce the equations relating the bulk modulus, modulus of rigidity, Young's modulus and Poisson's ratio of an elastic isotropic solid material. [7+7]=[14]

   Prove that Poisson's ratio is less than 0.5 but cannot be less than -1. [6]

3. What is surface tension? Derive an expression for the capillary rise in a tube. What is Jurin's law? [4+,9]=[13]

   Water can rise upto a height h in a certain capillary tube. Suppose the tube is immersed in water so that only a length h/2 is above the surface of water. Will the water flow out of the tube like a fountain? Explain your answer. [7]

4. What are the critical constants of a gas? Write down the Vander waal's equation for a gas and calculate the values of the critical constants in terms of the constants of this equation. Hence deduce the reduced equation of state. [6+2+7+5]=[20]

Group B

Maximum Marks: 40

Answer any two questions.

1.a) What is radioactivity? Deduce the radioactive decay law. Define 'half life' of a radio-element and correlate it to its decay constant. [3+4+6]=[13]

b) Explain what you mean by an isotope. Enumerate, with nuclear symbols, the different isotopes of hydrogen. [4+3]=[7]

2. Describe, with a neat diagram, the construction and principle of action of a Wilson cloud chamber. How would you distinguish the track of a β-particle from that due to an α-particle. [4+6+6+4]=[20]

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3. Write notes on any two of the following:

   a) Discovery of cosmic rays.

   b) Fusion

   c) Bubble chamber

   d) Cyclotron. [10 +10]=[20

4. Derive Bohr's expression for the frequency of radiation emitted due to electronic transitions in a hydrogen atom. What is the significance of the negative sign to the total energy of the electron? [17+3]=[20

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ANNUAL EXAMINATIONS

General Science-1: Physics Practical

Date: 29.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Perform the experiment as indicated in card A. [60]

2. Laboratory Note Book [10]

3. Class work [20]

4. Oral [10]

Distribution of Marks of Q. 1

Theory (working formula)	6
Accuracy	6
Tabular recording of data	40
Calculation	8

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ANNUAL EXAMINATIONS

General Science-1: Biology: Theory

Date: 30.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 60

Answer question 1 and any two other questions.

1. Rewrite the following statements correctly without changing the portion given within the brackets.

a) (When a hypanthodium inflorescence develops into a fruit) it is known as syconus.

b) (A legume is defined as) a dry indehiscent fruit having an inferior ovary of five united carpels.

c) (Flowers with imbricate corolla have their petals) twisting regularly either to the left or to the right.

d) (In a hypogynous flower, the ovary is placed) below the rest of the floral parts.

e) (The tuberous part of the potato plant is the modified) adventitious root.

f) (Geotropic movement may be defined as) a special kind of oscillatory movement found in tropical plants.

g) (Basidiomycetes fungi are characterised by) aseptate hyphae and basidia with endogenous spores known as basidiospores.

h) (The essential factors required for germination are) soil nutrients, optimum light and minimum CO_2.

i) (In hypogeal germination the cotyledons) penetrate the soil and function as roots.

j) (Leaf-spot of rice and stem rust of wheat are caused by the pathogenic) bacteria Bacillus subtilis and Psalliota campestris respectively. [10×2]=[20]

2. What is a fruit? Give the classification of fruits. Write an illustrated account of the fleshy fruits. [2+8+10]=[20]

3. Write an account on any two of the following

A. Homology and analogy,

B. Stipules,

C. Tendril climbers. [10+10]=[20]

What is a root? What are the functions of roots? Enumerate the aerial modifications of roots giving an example for each modification. [3+5+12]=[20]

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Group B

Maximum Marks: 40

Answer any two of the following questions.

1. Define growth. What is meant by grand period of growth? What are the different external and internal factors that affect growth? [2+6+12]=[20]

2. Write a brief account of the various kinds of movements in plants. [20]

3. What are the important characteristic features of the different classes of fungi? Write the life history of any economically important saprophytic fungus that you have studied. [8+12]=[20]

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ANNUAL EXAMINATIONS

General Science-1: Biology Practical

Date: 31.5.69. Maximum Marks: 100 Time: 3 hours

Note: The number of marks allotted to each question is given in brackets [ ]. Answer all questions.

1. Give a detailed botanical description of specimen A supplied with suitable labelled sketches of different parts. [10+10]=[20]

2. Perform the experiment as indicated in the card B. [15]

3. Comment on specimens/experiments C - G. [5 X5]=[25]

4. Identify specimens H - L. [5 X2]=[10]

5. Viva Voce [10]

6. Practical records. [20]

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PERIODICAL EXAMINATIONS

Mathematics-2: Calculus

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Date: 16.9.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. The number of marks allotted to each question is given in brackets [].

1. Do the following limits exist? Give a reason for your answer.

a) $\lim_{x \to 0} \left[\frac{x}{x}\right]$; b) $\lim_{x \to 0} x\sqrt{1+(1/x^2)}$. [10]

2. Evaluate the following limits

a) $\frac{x^3 - 3x + 1}{2x^4 - x^2 + 2}$ as $x \to \infty$

b) $\frac{1 - \sin x}{\cos x}$ as $x \to \pi/2$

c) x^x as $x \to 0+$

d) $(\cot x)^x$ as $x \to 0+$ [10]

3. Suppose that $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = c$, and $f(x)$, $g(x)$ $h(x)$ are defined and that $f(x) \leq g(x) \leq h(x)$ for all x with $0 < |x-a| < b$. Then $\lim_{x \to a} g(x) = c$. [10]

4. State and prove the Rolle's theorem. Deduce now the mean value theorem. [10]

5.a) Consider a closed right circular cylinder with a given surface area. What radius and altitude Provide maximum volume?

b) Find the dimensions of the rectangle of maximum area which can be des-cribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ [20]

6. A bridge is 30 ft. above a Canal. A motor boat going 10 ft./sec. passes under the center of the bridge at the same instant that a man walking 5 ft./sec. reaches that point. How rapidly will they be separating 3 sec. later? [10]

7. An arc light is 15 ft. above a side walk. A man 6 ft. tall walks away from the point under the light at the rate of 5 ft./sec. How fast is the shadow lengthening when he is 20 ft. away from the point under the light? [10]

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8. Draw the graph of function determining its points of inflection, relative maximum and relative minimum

$$f(x) = x^2 \sqrt{3 - x^2}$$ [10]

9. Find the Taylor's expansion for the following functions.

1) $f(x) = \sqrt{x}$ at $a = 4$

2) $f(x) = \dfrac{1}{1 + x^2}$ at $a = 0$

(upto and including the term $(x - a)^4$). [10]

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PERIODICAL EXAMINATIONS

Date: 23.9.68 Maximum Marks: 100 Time: 3 hours

Note: Answer the two groups in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Statistics-2: Probability

Max.marks: 50

Note: Answer all questions.

1.a) Describe the sample space and determine its cardinality in the experiment of distributing r balls into n cells in the following cases:

i) The balls and the cells are distinguishable.

ii) The balls are indistinguishable and the cells distinguishable.

b) In a certain family four girls take turns at washing dishes. Out of a total of four breakages, find the probability that at least 3 breakages were caused by the eldest girl. [7 +3]=[10]

2. Explain how Hypergeometric probability distribution can be used in estimating the total number of fishes in a pond. Justify the rationale behind the estimation technique involved in the solution. [10]

3. Construct a suitable probability model in the following two experiments.

i) The random placing of balls into n cells continues until for the first time a ball is placed into a cell already occupied.

ii) We fix a cell (say cell number 1) and continue the procedure of placing balls as long as this cell remains empty [5+5]=[10]

4.a) Define 'path from (0,0) to the lattice point (x,y), x > 0'.

b) Establish a necessary and sufficient condition for the existence of a path from (0,0) to (x,y), x > 0.

c) What does the 'reflection principle' say? [3+3+4]=[10]

5. Assignments. [10]

Group B: Statistics (Theory and Practical)

Max. Marks: 50

Note: Answer all questions.

1.a) If (x+ k) is the number of trials required to get k successes when the probability of success in a singletrial is p, evaluate the probability distribution of the random variable X. [10]

b) From random number tables generate a sample of 20 from a population having the distribution given by (a). Estimate p from the sample. [10]

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2. $(x_1, x_2, \ldots, x_n)$ is a sample of size n from a population whose first four central moments are μ_1, μ_2, μ_3 and μ_4 respectively.

a) Compute the first four moments of the sample mean $(\bar{x})$, and show that for large n, $\beta_1(\bar{x})$ is nearly zero and $\beta_2(\bar{x})$ is approximately equal to 3. [20]

b) Compute the expected value of the sample variance. [10]

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PERIODICAL EXAMINATIONS

Date: 14.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and B in separate answerscripts.

Group A

Economics-2: Economic Theory

Maximum Marks: 50 . Suggested Time: $1\frac{1}{2}$ hours

Answer any three questions. All questions carry equal marks.

1. Explain how different types of deposits are created by commercial banks. Can these bank deposits be regarded as money? Give reasons for your answer.

2. Discuss the quantity and interest effects of the central bank's open market policy.

3.a) What are the factors that determine the ex ante level of national income?

b) 'Given the savings function and a given amount of net investment, the equilibrium level of income is that at which households wish to save an amount equal to the given investment'. Elucidate the statement.

4. Briefly describe the multiplier mechanism.

5. Give in brief the views of Duesenberry and Friedman about the relationship of consumption to income.

Group B

Economics-2: Indian Economics

Maximum Marks: 50. Suggested time: $1\frac{1}{2}$ hours.

Answer any two questions. Marks allotted for each question are given in brackets [ ].

1. Discuss the changes that have taken place in the direction and composition of India's foreign trade since independence. [25]

2. Examine the trend of India's foreign trade during the period of planning. What are the main causes of the growing deficit in India's balance of payments? [25]

3. Indicate the distinctive features of the agricultural structure of India. [25]

4. Discuss in full, the consequences of land-tenure systems introduced in India during the period of British rule. [25]

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PERIODICAL EXAMINATIONS

General Science-C: Physics Theory

Date: 21.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Define the terms:

moment of inertia and radius of gyration.

What is the Physical Significance of moment of inertia?

Derive an expression for the moment of inertia of a solid sphere about a diameter. [3+3+4+6]=[16]

2. Deduce the differential equation for the simple harmonic motion of a particle. Solve the equation and show that the motion is isochronous.

Suppose a smooth straight tunnel is bored through the earth and a body is dropped into it. Show that the body would execute S.H.M. (Assume the earth to be a uniform sphere). [4+4+3+9]=[20]

3. Obtain an expression for the time period of a compound pendulum. Show that the centres of oscillation and suspension are interchangeable. [7+7]=[14]

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PERIODICAL EXAMINATIONS
General Science-2: Chemistry Theory

Date: 21.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Explain what is meant by 'isotonic solution'. What is osmotic pressure of a solution? [8]

b) State Raoult's law governing the vapour pressure of dilute solution and show how they can be employed to calculate the molecular wt of an organic substance. [12]

OR

At $20^{\circ}C$, 36 gm of mannitol dissolved in 1000 gm of water lowered the vapour pressure by 0.0614 mm. of mercury. The vapour pressure of water at $20^{\circ}C$ is 17.5391 m.m of mercury and its density 0.9982. Calculate the Mol. wt. of mannitol and osmotic pressure of the solution. [20]

2. Explain the significance and inter-relationship of the following:

a) Transport numbers.

b) Equivalent conductance at infinite dilution.

c) Ionic mobilities. [15]

3. State the law of mass action and apply it to any two of the followings equilibria:

a) $H_2 + O_2 = 2HO - 43,200$ cal

b) $Caco_3 = Cao + Co_2$

c) $Pcl_5 = Pcl_3 + cl_2$

Discuss fully the effect of temperature and pressure on these reactions. [15]

OR

Write notes on.

a) PH of a solution.

b) Equilibrium constant.

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PERIODICAL EXAMINATIONS

General Science-3: Geology

Date: 28.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer question 7 and any four from the rest. Marks allotted for each question are given in brackets [ ].

1. What are the major elements that constitute the earth's crust? What is a mineral? Name the important minerals which constitute: (i) granite, (ii) basalt. [10+5+5]=[20]

2. What is a sedimentary rock? Name the different environments in which a sedimentary rock can form. Briefly describe the mineralogical composition and physical properties of two important sedimentary rocks. [5+5+10]=[20]

3. How are the major mountain belts distributed on the surface of the earth? Do you think that the mountains existed in their present positions right from the beginning of the earth's history? Justify your answer. [5+15]=[20]

4. What is an unconformity? How is it generally detected? Mention some of the major unconformities in the geology of the Peninsular India. [5+10+5]=[20]

5. Name five minerals of economic importance. What are the rocks in which these minerals are generally found? Where are these deposits found in India? [5+5+10]=[20]

6. Comment on any five of the following:

   i) Cross-stratifications are found in river delta sediments.

   ii) Reef limestones bear skeletons of corals.

   iii) Fossils are not found in igneous rocks.

   iv) Human fossils are not found in the Mesozoic rocks whereas, Dinosaurs, found as fossils in these rocks, do not exist now.

   v) Rocks found in many of the high mountains contain fossils of marine organisms.

   vi) Coal beds of India are associated with sediments which were laid down by rivers.

   vii) Thickness of the earth's crust varies from the continents to the oceans. [4 × 5]=[20]

How do you like the subject of Geology? Do you think that a knowledge of the basic principles of geology can be of any use to a student of Statistics? Answer giving reasons. [5+15]=[20]

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PERIODICAL EXAMINATIONS

Statistics-2

Date: 4.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

Group A: Numerical Mathematics Theory and Practical

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1. Let $y_i = f(x_i)$, $x_i = x_0 + ih$ for $i = 0, 1, 2$ where

$$f(x) = a + bx + cx^2 .$$

Show that

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}(y_0 + 4y_1 + y_2).$$

Use this result to evaluate

$$\int_1^2 \frac{dx}{x}$$

by using nine ordinates. [25]

2. Write a FORTRAN program for Lagrange's Interpolation Formula using four ordinates. [25]

Group B: Statistics Theory and Practical.

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1.a) X_1 and X_2 are two independent normal variates with means μ_1, μ_2 and variances σ_1^2 and σ_2^2 respectively. Obtain the distribution of the random variables

$$U = X_1 + X_2.$$ [10]

b) If X_1 and X_2 are independent and are distributed as χ^2 with n_1 and n_2 degrees of freedom, show that $X_1 + X_2$ is also distributed as a χ^2. [10]

2. [illegible]

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2. The following tables gives the scores of 213 candidates in an admission test.

63	57	53	62	70	35	53	52	69	82
50	83	40	74	66	36	51	48	42	52
53	69	50	38	66	53	56	41	65	47
52	46	50	58	61	50	48	65	41	69
67	49	61	60	71	35	68	68	51	77
35	43	50	57	82	62	66	58	70	56
40	45	78	64	41	52	56	37	63	56
41	43	62	55	83	51	56	46	62	59
39	36	33	51	86	48	49	44	39	83
42	56	41	36	62	60	38	56	57	62
47	55	36	84	49	41	58	47	53	47
34	61	45	67	41	69	31	71	54	65
49	65	71	82	40	63	66	70	54	78
42	49	52	61	51	46	40	45	65	54
44	60	43	66	50	69	42	51	57	30
53	60	45	79	45	45	71	40	79	42
54	68	43	66	92	70	53	71	49	59
36	32	69	60	61	45	52	64	65	57
58	47	61	57	35	53	52	46	63	57
55	57	51	73	77	69	41	57	46	71
50	53	60	73	43	47	52	67	77	85
65	60	80							

a) Prepare a frequency distribution. [10]

b) Fit a normal distribution to the frequency distribution and compare it with expected frequencies. [20]

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PERIODICAL EXAMINATIONS

Mathematics-2: Matrix Algebra

Date: 11.11.68. Maximum Marks: 100 Time: 3 hours

Note: 1. Answer as many questions as you can. Each question carries 7 marks. The maximum marks you can score in this paper is 70; 30 marks are reserved for home assignments.

2. Read the questions carefully. Your answers should be brief and precise.

Instructions for questions 1-4

For any integer $m \geq 2$, let $\{Z_m = 0,1,2,\ldots, m-1\}$ and for any α, β in Z_m let $\alpha + \beta$ denote the remainder obtained by dividing the (ordinary) sum of α and β by m and $\alpha\beta$ denote the remainder obtained by dividing the (ordinary) product of α and β by m.

State whether the following assertions are true or false. Give reasons for your answer.

(1) Z_7 is a field.

(2) If m is not a prime number, then Z_m is not a field.

(3) In the field Z_{11}, $(-5) + 2(5^{-1}) = 5$.

(4) Let V be any 2-dimensional vector-space over the field Z_{13}. Then V has exactly 26 vectors.

Instructions for questions 5-9

Let R be the set of real numbers. Addition and scalar multiplication are defined in R^n by

$$(x_1,x_2,\ldots,x_n)+(y_1,y_2,\ldots,y_n) = (x_1+y_1,\ x_2+y_2,\ldots,\ x_n+y_n)$$

$$\alpha\ (x_1,x_2,\ldots,x_n) = (\alpha x_1,\alpha x_2,\ldots,\alpha x_n) \quad (\alpha \in R).$$

(5) In R^2, define $\lambda(x_1,x_2) = \sqrt{x_1^2 + x_2^2}$. Then is λ a linear functional?

(6) Let $A = \{(x_1,x_2) \in R^2 : x_1 = 3\}$. What is the span of A?

(7) In R^3, let $A = \{(x_1,\ x_2,\ x_3) : x_1 = 2x_2\}$ and

$B = \{(x_1,\ x_2,\ x_3) : x_2 = 2x_3\}$.

Then what is the span of $A \cup B$?

(8) Let $A = \{(x_1,x_2,x_3) \in R^3 : x_1+x_2 = 0\}$. Find a subspace of R^3 which is a complement of A.

(9) Let $S = \{(1,1,1),(1,2,2),(1,3,3)\}$. Does S form a basis for R^3?

(10) When is a finite set of vectors said to be a linearly independent set?

GO ON TO THE NEXT PAGE

(11) Let V_1, V_2 be two vector spaces over a field F. When are V_1 and V_2 said to be isomorphic?

(12) Let V be a vector space over a field F. Let $\{x,y,z\}$ be a set of vectors. Then show that $\{x+y,\ y+z,\ z+x\}$ is a linearly independent set if, and only if, $\{x,y,z\}$ is a linearly independent set.

(13) Let V be a finite dimensional vector space over a field F. Let S and T be subspaces of V such that $S \subset T$ and dimension of S = dimension of T. Then show that S = T.

Instructions for questions 14 and 15

Let P denote the set of all polynomials in a real variable t. Addition and scalar multiplication are defined as follows: For any $x, y \in P$ and $\alpha \in R$

$$(x+y)(t) = x(t) + y(t) \quad \text{for all } t \in R$$

$$(\alpha x)(t) = \alpha\, x(t).$$

(14) Define $\mu(x) = \int_0^1 t^2 x(t)\,dt$ for all $x \in P$.

Then is μ a linear functional on P ?

(15) Define $\eta(x) = \int_0^1 x(t^2)\,dt$ for all $x \in P$.

Is η a linear functional on P?

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MID-YEAR EXAMINATIONS

Mathematics-2: Calculus

QUESTION BOOKLET

Pages: 1 - 11 Roll No.

Date: 18.12.68 Maximum Marks: 100 Time: 3 hours

1. Answer all questions
2. Write your answers in the space provided under each question.
3. Marks allotted for each are given in brackets [ ].
4. Blank white papers are attached with this booklet for rough work.

INTEGRATION

1. Define a real step function on the real line. [2]

2. Show that the set of all step functions defined as above forms a vector space. [9]

GO ON TO THE NEXT PAGE

3. Define an integral for a step functions and give reason that it is well defined. [3]

GO ON TO THE NEXT PAGE

4. Show that the integral of a non-negative step function is non-negative. [3]

5. Suppose φ_n is a sequence of step functions, all vanishing outside the same interval $[a,b]$. Let M_n be the maximum value of φ_n and suppose that M_n tends to zero as n tends to infinity. Then prove that

$$\lim_{n \to \infty} \int_a^b \varphi_n \, dx = 0$$ [5]

GO ON TO THE NEXT PAGE

6. Give an example of a sequence of step functions g_n vanishing off (0,1], tending to zero for all x, and violating the conclusion in 5) above. [10]

7. Integrate log x between 0 and 1. [2]

GO ON TO THE NEXT PAGE

8. Integrate $e^{ax} \sin bx$. [3]

9. Define the term 'Field of subsets'. [2]

GO ON TO THE NEXT PAGE

10. Define the term 'measurable space'. [2]

11. Define the term 'measure'. [3]

12. When does a finitely additive measure become a countably additive measure. [5]

13. Justify your answer in Q.12. [10]

GO ON TO THE NEXT PAGE

DIFFERENTIAL EQUATIONS

1. Eliminate the arbitrary constants from $y = Ae^{x} + Be^{-x} + C$. [3]

2. Solve the equation $ydx - xdy = xydx$. [5]

GO ON TO THE NEXT PAGE

3. Solve the equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. [10]

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MID-YEAR EXAMINATIONS

Mathematics-2: Matrix Algebra

Date: 19.12.68 | Maximum Marks: 100 | Time: 3 hours

Note: Answer as many questions as you can out of questions 1 to 7. Marks allotted for each question are given in brackets [ ].

1. If A and B are two finite bases for a vector space V over a field F, show that A and B have same number of elements. [12]

2. Let X be a set. Let V denote the set of all real-valued functions defined on X. Addition and scalar multiplication in V are defined as follows:

$$(f+g)(x) = f(x) + g(x)$$
$$(\alpha f)(x) = \alpha f(x) \qquad \text{for all } x \in X,$$

where $f, g \in V$ and α is a real number.

a) Show that V is a real vector space. [5]

b) Let $f \in V$. f is said to be finitely non-zero if

$\{x \in X: f(x) \neq 0\}$ is a finite subset of X.

Let $S = \{f \in V: f \text{ is finitely non-zero}\}$.

Is S a subspace of V? [7]

3. If S and T are finite dimensional subspaces of a vector space V over a field F, show that

$$\dim(S) + \dim(T) = \dim(S+T) + \dim(S \cap T)$$

where $S+T$ denotes the span of $S \cup T$. [12]

4. If S, T and W are subspaces of a vector space V, show that $S \cap (T + (S \cap W)) = (S \cap T) + (S \cap W)$. [10]

5.a) Let A be a finite basis for a vector space V and let V^* denote the dual space of V. Define what we mean by the dual basis of A for V^*. [4]

b) Let V denote the real vector space R^3 with usual addition and scalar multiplication.

Let $x_1 = (3, -2, 3)$, $x_2 = (1, 2, 3)$, $x_3 = (2, 1, 4)$.

Then $A = \{x_1, x_2, x_3\}$ is a basis for V.

Let $A^* = \{x_1^*, x_2^*, x_3^*\}$ denote the dual basis of A for V^*. Let $x = (5, 4, 6)$.

Then find $x_1^*(x)$, $x_2^*(x)$ and $x_3^*(x)$. [10]

6.a) Define the annihilator of a subset of a vector space. [3]

b) If U is an m-dimensional subspace of an n-dimensional vector space V, show that its annihilator is an $(n-m)$ dimensional subspace of V^*. [12]

GO ON TO THE NEXT PAGE

6.c) For any two subspaces S and T of a vector space W, show that

$$(S+T)^0 = S^0 \cap T^0,$$

where A^0 denotes the annihilator of A for any subset A of W. [5]

7. Let V be a vector space over a field F. Let V^* denote the dual space of V and V^{**} the dual space of V^*.

a) Define the natural correspondence between V and V^{**}. [4]

b) When is V said to be reflexive? [4]

c) If V is finite dimensional, show that V is reflexive. [12]

8. Assignments. [20]

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MID-YEAR EXAMINATIONS

Economics-2: Economic Theory

Date: 20.12.68 Maximum Marks: 100 Time: 3 hours

Note: Attempt any five questions. Marks allotted for each question are given in brackets [ ].

1. 'Only in certain cases do changes in the quantity of money influence effective demand, and through it, under certain circumstances, prices'. Discuss this statement critically. [20]

2. Discuss whether the Central Bank's discount policy can, by itself, guarantee its control over the lending potential of commercial banks. [20]

3. Briefly explain the conditions required for equilibrium in the product and money markets. Graphically derive the IS and LM functions and hence show how the equilibrium levels of income and interest rate are simultaneously determined. [20]

4. In classical theory there is only one equilibrium level of real income and one equilibrium level of employment whereas the typical Keynesian would admit of several possible equilibrium levels of real income and employment. In what respects does the Keynesian differ from the classicist? Show that both the classical and Keynesian systems are self-consistent, given their initial assumptions. [20]

5. Distinguish between replacement investment and induced investment. Explain the acceleration principle and discuss the conditions under which it is valid. [20]

6. Give a dynamic analysis of the multiplier process when the change in investment is autonomous and given. How will the change in investment affect income when investment is a function of the rate of interest? Under what conditions will these two different situations have the same multiplier?

7. Write short notes on:

a) the interest effect of open market policy

b) the Pigou effect. [20]

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MID-YEAR EXAMINATIONS

Economics-2: Indian Economic Problems

Date: 21.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for each question are given in brackets [ ].

1. Discuss the changes that have taken place in the composition of India's foreign trade since independence. What are the main causes of the growing deficit in India's balance of payments? [25]

2. Examine the problems of agricultural labour in India and suggest remedial measures. [25]

3. Discuss the main aspects and objectives of land reform in India. [25]

4. Analyse the progress of land reforms in India in the light of the recent review by the Land Reforms Implementation Committee. [25]

5. 'Advantages of Co-operative farming arise mainly from its large size, joint management and individual proprietorship'; - Critically examine the statement.

   Do you agree with the view that the expansion of Co-operative forming in India would create more unemployment? Give reasons for your answer. [25]

6. Write short notes on the following:

   a) Permanent settlement.

   b) Report of the Kumarappa Committee.

   c) Concentration of land-ownership. [25]

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MID-YEAR EXAMINATIONS

Statistics-2: Probability

[52]

Roll No. [ ]

Date: 23.12.68. Maximum Marks: 100 Time: 3 hours

QUESTION BOOKLET

Pages: 1 - 8

Instructions to examinees:-

i) Write your answers in the space provided under each question.

ii) Marks allotted for each question are given in brakkets [ ].

iii) The whole paper carries 107 marks. Answer as much as you can.

iv) Do all the scratchwork in the question booklet itself.

1. A die is marked with the numbers 1,1,2,2,3,3, and another die is marked with the numbers -1,-1,0,0,1,1. These two dice are rolled once. Let X denote the score on the first die and Z the maximum of the scores. Find the joint probability distribution of X and Z. [10]

2. The following is the joint probability distribution of two random variables X and Y.

X \ Y	-2	1	4	7
-1	$\frac{1}{12}$	$\frac{1}{8}$	0	$\frac{1}{4}$
0	$\frac{1}{24}$	$\frac{3}{24}$	$\frac{1}{48}$	0
1	$\frac{3}{48}$	0	$\frac{1}{24}$	$\frac{1}{4}$

i) Write down the marginal probability distributions of X and Y:- [2]

ii) Are X and Y independent? [2]

iii) Find EX, EY, Var (X), Var (Y) and correlation coefficient between X and Y. [8]

GO ON TO THE NEXT PAGE

iv) Find the conditional probability distribution of $X/Y = -2$. [2]

v) Find $EX/Y = -2$ and $Var\ [X/Y = -2]$. [3]

GO ON TO THE NEXT PAGE

vi) Find the probability distribution of $X+Y$. [5]

vii) Find the probability distribution of X^3. [2]

3. Give a sufficient condition for the following to hold. [2]

$$EXY = EX.EY$$

4. If X and Y are independent, show that cor.coeff. correlation coefficient $(X, Y) = 0$. [2]

GO ON TO THE NEXT PAGE

5. Give an example testifying the fact 'Cor. coeff. (X,Y) = 0 need not imply X and Y are independent'. [5]

6. If two random variables X and Y assume only two values each, and if cov (X,Y) = 0, then show that X and Y are independent.

[Hint: Use the fact that correlation coefficient is unaltered under the change of origin and scale of the random variables]. [10]

7. Define independence of two events. Explain how will you generalise this notion to any finite number of events. [3]

GO ON TO THE NEXT PAGE

8. When do you say that two events are mutually exclusive. Generalise this notion to any finite number of events. [3]

9. Let Ω =(1,2,3,4,5,6) be a sample space associated with some experiment. Let A = (1,2,3,5) and B = (2,4,6) be two events. Let P be the uniform probability measure on Ω. Show that A and B are not independent under P. Construct a probability measure P' on Ω which gives positive probability to every sample point and under which A and B are independent. [10]

10. State carefully Chebychev's inequality. [3]

11. The random X has the following probability distribution

X :	0	1	2	3	.....
Pr. :	$\frac{1}{3}$	$\frac{1}{3} \cdot \frac{2}{3}$	$\frac{1}{3} \cdot (\frac{2}{3})^2$	$\frac{1}{3} \cdot (\frac{2}{3})^3$	.....

Find Pr. $[|X - 2| \geq 4]$ and compare this probability with the upperbound given by Chebychev's inequality. [5]

12. From a bag containing 3 red and 3 green marbles a person removes 3 marbles at random but does not inspect their colour. He then puts 3 blue marbles into the bag and again draws 3 at random. Find the probability that all the three marbles drawn being all of different colours. [10]

GO ON TO THE NEXT PAGE

13. Assignments. [20]

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MID-YEAR EXAMINATIONS

Statistics-2: Statistics Theory and Practical

Date: 24.12.68 Maximum Marks:100 Time: 3 hours

Note: Answer Q.1 and any three from the rest. Marks allotted for each question are given in brackets [ ].

1. In an experimental scheme, 342 samples were drawn from a certain population and a statistic T was calculated from each sample. The frequency distribution of T is given below. Fit an appropriate Pearsonian type distribution.

Frequency distribution of a statistic T

Class interval	Frequency	Class interval	Frequency
$0 \le T < 2$	4	$14 \le T < 16$	14
$2 \le T < 4$	23	$16 \le T < 18$	18
$4 \le T < 6$	51	$18 \le T < 20$	3
$6 \le T < 8$	69	$20 \le T < 22$	4
$8 \le T < 10$	59	$22 \le T < 24$	3
$10 \le T < 12$	57	$24 \le T < 26$	1
$12 \le T < 14$	35	$26 \le T < 28$	1
		Total	342

[40]

2. X and Y are two Gamma variates with parameters (α,p) and (α,q) respectively. Obtain the distribution of the random variable $Z = X/Y$. [20]

3. Show that as the sample size increase, the distribution of the sample mean approaches Normal distribution. [20]

4. $x_1, x_2, \ldots, x_n$ is a sample of size n from uniform distribution in the interval (0,1). The range R, of the sample is defined as the difference between the maximum observation and the minimum observation, i.e.

$$R = [\max_i (x_i) - \min_i (x_i)]$$

Compute the frequency density function of R. [20]

5. In a series of 12 tosses of a coin, it was observed that heads turned up 8 times. Is there any evidence to suspect that the coin is biased? [20]

6. The mean and standard deviation of the monthly income of workers employed in industries in a certain city in the two year 1945 and 1946 was estimated from sample surveys and are given below. Do you think that there was an improvement in the average income of the workers in the year 1946 as compared to 1945?

Mean and Standard Deviation of the Monthly Income of Workers

Year	Sample Size	Monthly income (rupees) Mean	Standard deviation
1945	230	82.4	18.6
1946	346	85.1	17.2

[20]

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MID-YEAR EXAMINATIONS

Statistics-2: Numerical Analysis Theory and Practical

Date: 25.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Write a program in FORTRAN to evaluate

$$f(x) = x^3 - x - 4$$

for $x = 1.0(0.0001)2.0$. [15]

2. A root of the equation

$$x^3 - x - 4 = 0$$

lies between 1 and 2. In order to evaluate this root to 4 places of decimals the following iterative procedure has been suggested

$$x_0 = 1.0$$

$$x_{i+1} = x_i^3 - 4 \qquad i = 1,2,\ldots$$

stop at the (n+1)-th step if $|x_{n+1} - x_n| < 0.00001$.

(a) Examine whether the above procedure will work. [15]

(b) Using any procedure you like, evaluate the root correct to 4 places of decimals. [15]

3. EITHER

The following table gives values of

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

for selected values of x: Find by interpolation

i) the value of $\Phi(x)$ for $x = 0.0234$ [12]

ii) the value of x for which $\Phi(x) = 0.55567$. [18]

x	$\Phi(x)$
0.00	0.50000
0.05	0.51994
0.10	0.53983
0.15	0.55962
0.20	0.57926

OR

If a table is available in which the above function $\Phi(x)$ is tabulated correct to seven places of decimals for values of x starting from 0.00 at intervals of 0.01 and we use linear interpolation there to obtain some value of $\Phi(x)$, what would be the accuracy of the result? [30]

4. EITHER

Solve the equations: $Ax = c$, where $x = (x_1, x_2, x_3)$

$$A = \begin{matrix} 1.000 & 0.313 & 0.280 \\ 0.313 & 1.000 & 0.652 \\ 0.280 & 0.652 & 1.000 \end{matrix} \quad \text{and } c = \begin{matrix} 0.182 \\ 0.554 \\ 0.747 \end{matrix}$$

[25]

OR

Write a program in FORTRAN for solving simultaneous linear equations. [25]

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MID-YEAR EXAMINATIONS

General Science-2: Physics Theory

Date: 26.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. State and explain Kirchhoff's laws of distribution of currents in a network of conductors. Find the current through the galvanometer in an unbalanced Wheatstone's bridge. Hence find the null condition. What is Specific resistance ? [7+9+3+2]=[21]

2. Explain what is meant by coefficient of self-induction. Deduce expressions for the growth and decay of current in a circuit containing an inductance L and a resistance R in series with a battery of e.m.f. E. Explain your results with the help of diagrams. What is the time constant of the circuit? [3+10+5+3]=[21]

3. Distinguish between the mean value and the root mean square value of an alternating current and find the relation between them. Derive an expression for the current in a circuit containing an inductance and a resistance when a sinusoidal alternating e.m.f. is applied to it.

[4+8+9]=[21]

4. What is a cantilever? Explain how you would determine experimentally the Young's modulus of the material of a rectangular bar by bending it in the form of a double cantilever. Deduce the relation you use. [3+7+11]=[21]

5. A telephone operates at a current of 120 milliamperes and has an inductance of 10 henries and resistance 100 ohms. If a 24-volt battery having negligible internal resistance is suddenly applied calculate the operating time. [16]

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MID-YEAR EXAMINATIONS
General Science-2: Chemistry Theory

[56]

Date: 27.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for each question are given in brackets [ ].

1.a) What is meant by the terms Internal Energy, and Enthalpy?

b) Establish the pressure volume relation in Reversible Adiabatic process. [25]

2. State second law of thermodynamics. Find an Expression for the efficiency of any reversible heat engine operating between the temps T_1 and T_2. [25]

3. Find the expressions for k_p for the following reactions.

(1) $N_2 + 3H_2 \rightleftharpoons 2NH_3$

(2) $SO_2 + \frac{1}{2}O_2 \rightleftharpoons SO_3$. [25]

4. i) 100.0 ml of 0.1 N Acetic Acid be mixed with 200.0 ml of 0.5N Na-Acetate. The PH of the resulting mixture at 25° is 5.757, calculate Ka, the dissociation constant of Acetic Acid.

ii) At 20°c the vapour pressure of ether is 442 mm of mercury. When 6.1 gms of a subs. are dissolved in 50 gm of ether, the vapour pressure falls to 410 mm. What is the molecular weight of the subs.? (Mol. wt. of ether = 74). [25]

5. i) State Hess's law. Calculate the heat of formation of the salt.

a) $Na_{(s)} \rightarrow Na_{(g)} - 26$ Kcal,

b) $Na_{(g)} \rightarrow Na^+ + \bar{e} - 119$ Kcal,

c) $\frac{1}{2} Cl_{2(g)} \rightarrow Cl_{(g)} - 29$ Kcal,

d) $Cl_{(g)} + \bar{e} \rightarrow Cl^- + 92$ Kcal

e) $Na^+ + Cl^- = NaCl_{(g)} + 128$ Kcal

f) $NaCl_{(g)} = NaCl_{(s)} + 52$ Kcal.

ii) Explain with illustration:- Heat of solution Heat of formation, Heat of dilution. [25]

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MID-YEAR EXAMINATIONS

General Science-3: Biology: Zoology Theory

Date: 28.12.68. Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. What are the characteristics of the chordates? Draw comparative diagrams of the fundamental plans of a non-chordate and a chordate. [20]

2. State the general characters of Hemichordata. Discuss the phylogenetic affinity of Balanoglossus. [20]

3. State the characters shown by Petromyzon, absent in Amphioxus and typical of Craniata. State also the specialised characters of Petromyzon. [20]

4. Describe a generalised vertebra. [20]

5. Rewrite any five of the following sentences making corrections if necessary but without changing the portion within brackets. [20]

a) [Trochophore larva of Balanoglossus resembles] tornaria larva of the Annelids.

b) [Balanoglossus shows resemblance to] Annelida by its circulatory system.

c) [Notochord is] not present in the adult Urochordates.

d) [Branchial sac of Herdmania is a] food collecting apparatus and at the same time it is respiratory in function.

e) [Hepatic portal system is] not present in Amphioxus.

f) [In India, Balanoglossus is found] in a island near Rameswaram.

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MID-YEAR EXAMINATIONS

General Science-3: Biology: Zoology Practical

Date: 30.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe specimen A supplied, giving detailed labelled sketches of the different external organs. Assign the specimen into the class to which it belongs. [16+4]=[20]

2. Identify specimens B - E. [4 X5]=[20]

3. Comment on specimens F - I. [4X3]=[12]

4. Draw dorsal and ventral views of specimen J and label its parts. [18]

5. Viva Voce. [10]

6. Practical records. [20]

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PERIODICAL EXAMINATIONS

Mathematics-2: Calculus

(60)

Date: 17.2.69 Maximum Marks: 100 Time: 2 hours

Note: All questions carry equal marks: Answer all questions.

1. Find the surface area of the solid of revolution for the following:

a) $f(x) = e^x \qquad 0 \leq x \leq 1$

b) $x = t^2/2, \qquad y = t^3/3 \qquad 1 \leq t \leq 3.$

2. Find the volume of the solid of revolution for the following:

a) $r = a(1 - \cos\theta) \qquad 0 \leq \theta < \pi/2$

b) $x = t^2/2, \qquad y = t^3/3 \qquad 1 \leq t \leq 3.$

3. If a continuous periodic function $f(x)$ is represented as

$$f(x) = \frac{a_0}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} (a_n \cos nx + b_n \sin nx) .$$

Find the fourier coefficients a_0, a_n and b_n.

4. Obtain the fourier series for the following functions.

a) $f(x) = e^x \qquad -\pi \leq x < \pi$

b) $f(x) = \cos^2 x \qquad -\pi \leq x < \pi .$

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PERIODICAL EXAMINATIONS

Mathematics-2: Matrix Algebra

Date: 2..2.69 Maximum Marks: 100 Time 3 hours

Note: Answer as many questions as you can. Twenty five marks are reserved for home assignments. Marks allotted for each question are given in brackets [ ].

1. Carefully define the following terms:

a) Linear functional
b) Isomorphism
c) Reflexive vector space
d) Annihilator
e) Direct sum of two subspaces
f) Linear transformation
g) Matrix of a linear transformation. [14]

2. If a vector space V is the direct sum of two subspaces M and N show that the dual space of V is the direct sum of the annihilator of M and the annihilator of N. [12]

3. If S and T are subspaces of a finite dimensional vector space W, show that

$$\dim (S+T) = \dim (S) + \dim (T) - \dim (S \cap T)$$ where $S+T$ denotes the span of $S \cup T$. [12]

4. Give an example of 3 subspaces S_1, S_2, S_3 of a vector space V such that

$$S_1 \cap (S_2+S_3) \neq (S_1 \cap S_2) + (S_1 \cap S_3) .$$ [10]

5. Let A be a linear transformation on the real vector space R^3 let $x_1 = (1,2,3)$ $x_2 = (2,3,1)$ $x_3 = (3,1,2)$. Suppose the matrix of A with respect to the basis $\{x_1, x_2, x_3\}$ is given to be

$$\begin{bmatrix} -1 & 2 & -4 \\ 7 & 3 & 9 \\ 8 & -4 & -2 \end{bmatrix}$$

Then find $A(x)$ where

$$x = (1,1,1).$$ [10]

6. Let P_3 denote the real vector space of polynomials (in a real variable) of degree at most 3.

Let $(Ax)(t) = x(t+1) - x(t)$ for every $t \in R$, $x \in P_3$.

Then show that A is a linear transformation on P_3.

If $x_1(t) = -2$, $x_2(t) = t$, $x_3(t) = 1 + t^3$ and $x_4(t) = t^2$, $(t \in R)$ then find the matrix of A with respect to $\{x_1, x_2, x_3, x_4\}$. [12]

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7.a) When is a linear transformation on a vector space said to be invertible?

b) Consider the linear transformation A defined in Q.6.

Is A invertible? Give reasons for your answer.

c) If a linear transformation T defined on a vector space is such that

$$T^3 - T^2 - T + 1 = 0,$$

then show that T is invertible. Also find T^{-1}. [13]

8. If A and B are linear transformations on a vector space V such that AB = 0. Does it necessarily follow that BA = 0? If so, give a proof. If not, give a counterexample. [10]

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PERIODICAL EXAMINATIONS

Economics - 2

Date: 3.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A: Economic Theory

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1.a) Explain the sacrifice principle of taxation. State the assumptions you may make in formulating such a principle. Represent graphically the three different sacrifice principles.

b) Suppose the utility of an income is given by the function

$$u(y) = c - \frac{1}{y}$$

where c is a constant. Discuss whether the tax system would be progressive, proportional or regressive when there is equal absolute sacrifice and when there is equal proportional sacrifice. [17+8]=[25]

2. Discuss the effect of a balanced budget on national income when taxation does not depend on income and when taxation is an increasing function of income. [25]

3. 'The way the budget affects the quantity of money and liquidity preference is of considerable influence in determining the total effect of a given combination of government expenditure and revenue on national income'. Discuss the statement critically. [25]

GROUP B: Indian Economic Problems

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1. Examine the various aspects of the problem of agricultural finance in India indicating the main findings of the All India Rural Credit Survey Committee. [25]

2. Critically examine the main features of the 'integrated scheme of rural credit' as recommended by the All India Rural Credit Survey Committee. [25]

3. Discuss the main problems of agricultural marketing in India and suggest remedial measures. [25]

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PERIODICAL EXAMINATIONS

Statistics-2: Probability

[63]

Date: 10.3.69 Maximum Marks: 100 Time: 3 hours

Note: The whole paper carries 131 marks. Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) Define 'convergence in Probability'. [5]

b) Examine whether the following sequence of random variables converges in probability or not.

$$X_n : \quad 0 \qquad n^2$$
$$\text{Pr.}: \quad \frac{n-1}{n} \qquad \frac{1}{n} \qquad n \geq 1$$ [8]

2.a) $\{X_n, n \geq 1\}$ is a sequence of random variables such that

$$EX_n \to \mu \quad \text{as } n \to \infty \quad \text{and}$$
$$\text{Var}(X_n) \to 0 \quad \text{as } n \to \infty.$$

Show that $\{X_n, n \geq 1\}$ converges in probability to μ. [10]

b) State and prove a version of the weak Law of Large Numbers. [10]

3.a) Define 'generating function of a sequence of real numbers $\{a_n; n \geq 0\}$' and 'probability genreraring function of a non-negative integer valued random variable X'. [4]

b) Let P denote the probability generating function of a non-negative integer valued random variable X. Derive the following formal identities

(i) $EX = P'(1)$

(ii) $\text{Var}(X) = P''(1) + P'(1) - [P'(1)]^2$. [7]

4.a) Define the convolution of two sequences of real numbers $\{a_n, n \geq 0\}$ and $\{b_n, n \geq 0\}$. [3]

b) Find the convolutions of the following pairs of sequences.

i) $a_k = 1$ for every $k \geq 0$
$b_k = 1$ for every $k \geq 0$

ii) $a_k = k$ for every $k \geq 0$
$b_k = 1$ for every $k \geq 0$

iii) $a_0 = a_1 = \frac{1}{2}$, $a_k = 0$ for every $k \geq 2$
$\{b_n, n \geq 0\}$ is any arbitrary sequence. [6]

c) Let $\{c_k, k \geq 0\}$ be the convolution of $\{a_k, k \geq 0\}$ and $\{b_k, k \geq 0\}$. Find the relationship between their generating functions. [5]

d) If X and Y are non-negative integer valued mutually independent random variables with generating functions $A(\cdot)$ and $B(\cdot)$ respectively, show that the generating function of X+Y is $A(\cdot)\,B(\cdot)$. [5]

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5.a) Find the probability generating function of a negative Binomial probability distribution with parameters r and p. [5]

b) $X_1, X_2, \ldots, X_r$ are r independent random variables each having Geometric probability distribution with the same parameter p. Show that $X_1 + X_2 + \ldots + X_r$ has negative Binomial distribution. [10]

6. Let q_n be the probability that in n tosses of an ideal coin no run of three consecutive heads appears. Find the generating function of $\{q_n, n \geq 0\}$. [10]

7.a) Define bivariate probability generating function of a pair of non-negative integer-valued random variables X and Y. [2]

b) Derive the generating functions of marginals of X and Y in terms of the bivariate generating function of X and Y. [4]

c) Derive the generating function of X+ Y in terms of the bivariate generating function of X and Y. [4]

d). Derive a necessary and sufficient condition for the independence of X and Y in terms of generating functions. [8]

8. The coefficients of the equation $ax^2 + bx + c = 0$ are determined by rolling a symmetric die three times. Find the probability (i) the equation has real roots and (ii) the equation has equal roots. [10]

9. The probability generating function P(•) of a non-negative random variable is a rational function. Under suitable conditions to be stated clearly and carefully on P(•), derive asymptotic formulae for

$$p_n = Pr. [X = n].$$ [15]

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Statistics-2

Date: 17.3.69 Maximum Marks: 100 Time: 3 hours.

Note: Answer Groups A and B in separate answerscripts
Marks allotted for each question are given in brackets [ ].

GROUP A: Statistics Theory

Maximum Marks: 50 Suggested time $1\frac{1}{2}$ hours

Answer as much as you can.

1. $X_1, X_2, \ldots, X_n$ be n $(n > 1)$ independent samples coming from a normal population with mean μ and variance σ^2. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$.

   a) What is the distribution of $\dfrac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{\sigma^2}$?

   b) What is the distribution of $\dfrac{\sqrt{n}\,(\bar{X} - \mu)}{\sqrt{\dfrac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}}$?

   c) What is the distribution of $\dfrac{(n-1)(X_1 - \mu)^2}{\sum_{i=2}^{n} (X_i - \mu)^2}$? [3×5]=[15]

2.a) Write down the joint density function of the random variable (X, Y) where (X,Y) follows a bivariate normal distribution with mean μ_1 and μ_2 respectively and variance-covariance given by the following

$$\text{Var}(X) = \sigma_1^2, \quad \text{Var}(Y) = \sigma_2^2, \quad \text{Cov}(X,Y) = \sigma_1\sigma_2\rho.$$

   b) Deduce the marginal distribution of X.

   c) Find out the conditional distribution Y given $X = x$.
[6+12+12]=[30]

3. Let $X_0, X_1, \ldots, X_k$ be k+1 random variables. Let $\Sigma = ((\sigma_{ij}))$ denote the variance-covariance matrix of $(X_0, \ldots, X_n)$ [$\sigma_{ij} = \text{Cov}(X_i, X_j)$ $i = 0, \ldots, k$, $j = 0, \ldots, k$].

   a) Define the multiple correlation coefficient of X_0 on $X_1, X_2, \ldots, X_k$. Write down its expression in terms of the elements σ_{ij}'s of Σ.

   b) Define the partial correlation coefficient of X_0 and X_1 after eliminating the effects of $X_2, \ldots, X_k$. Write down its expression in terms of the σ_{ij}'s.
[10+10]=[20]

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GROUP B: Statistics Practical

Maximum Marks: 50 | Suggested time: $1\frac{1}{2}$ hours

Answer all questions.
All questions carry equal marks.

1. Following is the frequency distribution of the number of red blood corpuscles (rbc) per cell of a hemocytometer. Examine whether the data is consistent with the hypothesis that the mean number of (rbc) per cell is 1?

Frequency Distribution of rbc

Number of rbc.	Number of cells
0	143
1	156
2	68
3	27
4	5
5	1
	Total [illegible]

2. Two independent estimates of area under cultivation in Rural India ($\pm$ standard errors) are 348151$\pm$ 7553 (000 acres) and 330402$\pm$ 4934 (000 acres). Are the two estimates consistent?

3. The radii of 15 circular sockets (in cm) are given below. Examine whether the mean differs significantly from 2 cm.

Radii of	2.03	1.98	2.24	2.17	2.09
Sockets	1.97	1.85	1.69	2.23	2.18
	1.88	1.95	1.84	2.01	2.17

4. The following table gives the variance of stature in cms., for Muslims in the undivided Bengal as obtained in the Bengal Anthropometric Survey, 1945. Do you think that the variances are significantly different? If not, obtain a pooled estimate for the common variance.

District	Sample size	Variance of stature (in sq. cms.)
Dacca	337	39.2262
Mymensingh	299	38.3306

5. Practical Records.

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PERIODICAL EXAMINATIONS

Statistics-2: Index Numbers and Time Series
Theory and Practical

Date: 24.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. What are index numbers?
Define simple aggregative price index. Starting from this index obtain Laspeyre's, Paasche's and Marshall-Edgeworth indices, explaining the features of the weights used in deriving them. [3+12]=[15]

2.a) What are the different types of errors that may creep in the construction of index numbers?
Explain how they can be detected and minimised.

b) Explain the terms
(i) Chain index (ii) D-test. [10+5]=[15]

3. The following table gives the data of production of sweet potatoes in the U.S. (1931-1952).

a) Plot the observed production against the time.

b) Fit a linear trend to this data and plot the expected values on the same graph paper.
[10+20]=[30]

Table 1
(Millions of bushels)

Year	Production	Year	Production
1931	67.3	1942	65.5
1932	86.6	1943	71.1
1933	74.6	1944	68.3
1934	77.7	1945	61.3
1935	81.2	1946	60.8
1936	59.8	1947	49.6
1937	68.1	1948	43.1
1938	68.6	1949	45.0
1939	61.7	1950	49.8
1940	51.7	1951	28.8
1941	62.5	1952	28.3

4. Following table gives the farmers prices and quantities produced of a number of grains in U.S. (1950 and 55).

a) Compute Laspeyre's, Paasche's, Marshall-Edgeworth's price indices, for 1955 taking 1950 as base year.

b) Compute the weighted arithmetic mean and the weighted geometric mean of price relative, the weights being the quantities in the base year all expressed in million pounds.

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Table 2

		Quantity			Price		Conversion factor
Grains	Unit	1950	1955	Unit	1950	1955	
Wheat	million bushels	1019	935	bushel	2.00	1.98	1 bushel = 60 lbs.
Rye	''	21	29	''	1.32	1.06	1 bushel = 50 lbs.
Rice	million bags	39	56	bag	5.09	4.80	1 bag = 100 lbs.
Buck wheat	million bushels	4	2	bushel	1.11	1.17	1 bushel = 50 lbs.
Corn	''	2764	2884	''	1.53	1.34	''.
Oats	''	1369	1503	''	0.79	0.60	1 bushel = 32 lbs.
Barley	''	304	401	''	1.18	0.92	1 bushel = 48 lbs.
Sorghums	''	234	243	''	1.05	0.98	1 bushel = 50 lbs.

[25+15]=[40]

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PERIODICAL EXAMINATIONS

General Science-2: Physics Theory

Date: 31.3.69 Maximum Marks: 50 Time: 2 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Establish the equation connecting pressure, volume and temperature of a gas in adiabatic transformation.

A litre of air 76 cm. of mercury and 30°c is adiabatically compressed to a pressure of 120 cm. of mercury. Calculate the new volume and rise of temperature.
[10+4+4]=[18]

2. Describe Carnot's cycle. Deduce the efficiency of a Carnot's engine working between two temperatures. State and prove Carnot's theorem.

A carnot's engine has an efficiency 25 per cent when its high-temperature reservoir is at 127°c. By how many degrees centigrade must the temperature of the sink be lowered to increase the efficiency to 30 per cent?
[4+4+2+4+4]=[18]

3. Develop an equation connecting the instantaneous value of the current with the imposed sinusoidal e.m.f. in an L-C-R circuit. Interpret the equation. What is resonance? Calculate the resonance frequency.
[6+4+2+2]=[14]

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PERIODICAL EXAMINATIONS

General Science-3: Biology: Zoology Theory

Date: 7.4.69 Maximum Marks: 50 Time: 2 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. State the dental as well as the skeletal characters of the class Mammalia. [6+14]=[20]

EITHER

2. State the general characteristics of birds. [15]

OR

Give examples of any five of the following:-

a) An egg-laying mammal
b) An aquatic mammal
c) Volant adaptation in Reptile
d) Arboreal adaptation in Amphibia
e) Cursorial adaptation in mammal
f) Indian salamander. [5 X 3]=[15]

3. Write short notes on:-

a) Dinosaurs; (b) Neoteny and (c) Gastrula. [3 X5]=[15]

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ANNUAL EXAMINATIONS

Mathematics-2: Calculus

Date: 19.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A.

Maximum Marks: 48

Answer question 4 and any two from the rest of this group.

1. Integrate

a) $\int \frac{x-1}{(x^2+2x+3)^2} dx$

b) $\int \frac{x^3}{\sqrt{1-x^2}} dx$

c) $\int \frac{dx}{1+x^3}$ [6+6+4]=[16]

2. Integrate

a) $\int \frac{\sin^3 x}{2+\cos x} dx$

b) $\int_0^{\pi} \frac{dx}{3+2\cos x}$

c) $\int_0^{\pi/2} (a^2\cos^2\theta + b^2\sin^2\theta)d\theta$

[6+6+4]=[16]

3. Evaluate the integral

a) $\int_0^{\infty} \frac{dx}{(x^2+1)^{n+1}}$ [11]

b) $\int_0^1 \frac{x^2}{\sqrt{1+x^2}} dx$ [5]

4. Compute the area bounded by the X- axis and an arc of the the cycloid,

$x = a(t-\sin t), \quad y = a(1-\cos t)$ [8+8]=[16]

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Group B

Maximum Marks: 52

Answer question 3 and <u>any two</u> from the rest of this group.

1. Compute the length of the hypocycloid (astroid)

$$x = a\cos^3 t, \quad y = a.\sin^3 t$$

and find the volume generated by the revolution of this curve about the x - axis. [9+9]=[18]

2.a) Find the Jacobian of the transformation

$$x = r\cos\theta\sin\varphi$$

$$y = r\sin\theta\sin\varphi$$

$$z = r\cos\varphi$$ [8]

b) Evaluate

$$\int_0^a \left[\int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\; dy \right] dx$$ [8]

3. Find the fourier expansion for the following periodic function f(x) with period 2π.

$$f(x) = x^2 \qquad -\pi < x \leq \pi$$

and hence or otherwise deduce that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$ [18]

4. Test for the convergence of

a) $$\int_1^{\infty} \frac{dx}{x^2(1+e^x)}$$

b) $$\int_1^{\infty} \frac{(x+1)\,dx}{\sqrt{x^3}}$$

c) $$\int_0^1 x^{1/4} \log x \, dx.$$ [6+6+4]=[16]

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ANNUAL EXAMINATIONS

Mathematics-2: Matrix Algebra

Date: 20.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1. Carefully explain the meaning of each of the following statements:-
   a) V is a reflexive vector space.
   b) The pair of subspaces (M,N) reduces a linear transformation A.
   c) P is a projection of V.
   d) A^* is the adjoint of A.
   e) $((a_{ij}))_{m \times n}$ is the matrix of T with respect to the bases $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_m\}$.
   f) The nullity of A is 1.
   g) V is an inner product space.
   h) $\{x_1, x_2, \ldots, x_n\}$ is a complete orthonormal set in V.

   $[8 \times 2\frac{1}{2}]=[20]$

2. EITHER
   a) Show that every finite dimensional vector space is reflexive

   OR
   b) Show that any two bases for a finite dimensional vector space have the same number of elements. [12]

3. a) State and prove Bessel's inequality.
   b) Hence, or otherwise, derive the Cauchy-Schwarz's inequality.
   c) If $\{x_1, x_2, \ldots, x_n\}$ is a complete orthonormal set in an inner product space V, then show that for any $x, y \in V$,

$$\langle x, y \rangle = \sum_{i=1}^{n} \langle x, x_i \rangle \langle x_i, y \rangle.$$

[6+3+6]=[15]

4. a) Give an example of two linear transformations A, B on a vector space V into V such that $A \neq 0$, $B \neq 0$ but $AB = 0$.
   b) Give an example of two linear transformations S and T on a vector space W into W such that $ST \neq TS$.
   c) Let V_1, V_2, V_3 be vector spaces over the same field and let $B: V_1 \to V_2$, $A: V_2 \to V_3$ be linear transformations. Let $X = \{x_1, x_2, \ldots, x_n\}$, $Y = \{y_1, y_2, \ldots, y_m\}$ and $Z = \{z_1, z_2, \ldots, z_k\}$ be bases for V_1, V_2 and V_3 respectively. Suppose $[A; Y, Z]=((a_{ij}))_{k \times m}$ and $[B; X, Y]=((b_{ij}))_{m \times n}$. Then show that $[AB; X, Z]=((c_{ij}))_{k \times n}$

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where $c_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$ $i = 1,2,\ldots,k,\ j = 1,2,\ldots,n.$

d) Let A and B be linear transformations on a vector space V into V. Suppose A+B and A-B are invertible. Then show that there exist linear transformations X and Y on V into V such that

$$AX + BY = 0 \quad \text{and} \quad BX + AY = 0.$$

[4 X5]=[20]

Group B

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1. Let V denote the real vector space R^3.

a) If $A = \{(x_1,x_2,x_3) \in V: \ x_1+x_2 = 2\}$ then find span of A.

b) Let $S = \{(x_1,x_2,x_3) \in V: 2x_1 = x_2\}$ and

$T = \{(x_1,x_2,x_3) \in V: 2x_2 + x_3 + 3x_1 = 0\}$.

Show that $S + T = V$.

Find $\dim (S \cap T)$. [4+3+3]=[10]

2. State whether the following statements are true or false. If a statement is true, prove it; if it is false, give a counterexample.

a) Every 5-dimensional vector space over the field Z_2 has exactly 32 elements.

b) Let V be a finite dimensional vector space over a field F. Then it is possible that a linear transformation on V into V is one-to-one but is not invertible.

c) If a linear transformation P on a vector space V into V is idempotent, then P is a projection of V.

d) The union of any two subspaces of a vector space V is a subspace of V.

e) If V_1, V_2 and V_3 are vector spaces over the same field and $B: V_1 \to V_2$, $A: V_2 \to V_3$ are linear transformations, then $(AB)^* = B^*A^*$. [5X6]=[30]

3. Consider the vector space R^2 with usual inner product. Let $S = \{(x,y) \in R^2: \ x+y = 0\}$.

a) Find the orthogonal complement of S.

b) If $u = (3,7)$ find the component of u along S.

c) Find the minimum distance of u from S.

State (without proofs) any theorems you may use. [3+3+6]=[12]

4. Let V and W be finite dimensional vector spaces over the same field and let T be a linear transformation on V into W.

a) Show that i) rank of T equals rank of T^*;

ii) rank of T + nullity of T = dim (V).

b) Indicate briefly how the above results can be used to prove that the row rank and column rank of a matrix are same. [10+5]=[15]

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ANNUAL EXAMINATIONS

Economics-2: Economic Theory

Date: 21.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any two questions.

1.a) Discuss carefully the effects of collective bargaining on wages and employment.

b) Explain how factor prices are determined under different market conditions for factors and products. [25]

2.a) Prove that in a simple monopoly the imposition of a sales tax will induce the monopolist to reduce his output and raise the price but that of a lump sum tax will not affect his output and price.

b) A monopolist in a market with a demand function $p = 48 - q$ can also sell his product in a competitive market where the prevailing price is Rs. 20 per unit. The total cost of production is Rs.$0.5\,Q^2$.

(Q = total number of units produced

q = number of units demanded in the monopoly market at the price of Rs. p per unit).

Show that the monopolist will reap greater profits by selling in both types of markets than by selling in the monopoly market alone. [25]

3. Briefly discuss how different equilibrium price-quantity combinations are possible, depending on the modes of behaviour of the two parties under bilateral monopoly. [25]

4.a) If a firm has a homogeneous production function of degree one, show that its output is indeterminate under perfect competition.

b) Show that under discriminating monopoly price will be lower in the market with a greater elasticity of demand. [25]

Group B

Maximum Marks: 50

Answer any three questions.
Two marks are reserved for neatness.

1. 'We have to consider whether a change in the quantity of money leads to a change in effective demand. Next we have to consider whether a change in effective demand brings about a change in prices'. Examine the Quantity Theory in the light of this statement. [16]

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2.a) Discuss the effect of an increase in resource-using government expenditure on the national income in a closed economy where taxes are a rising function of national income. [16]

b) In a closed economy, the community's planned consumption expenditure is four-fifths of its private disposable income and the government's net tax receipts (i.e., taxes less transfer payments) are one-fourth of the national income at market prices. Initially the resource-using expenditures of the government are just balanced by its net tax receipts. Show that if the government doubles national income by increasing its expenditures while private investment remains unchanged, the total deficits incurred by it will be Rs.1500 crores if the initial level of national income was Rs.10000 crores. [16]

3. In a two-country model, discuss the effects of

i) a change in autonomous exports, and

ii) a change in autonomous investments, (both of country I) on the national income and balance of payments of the two countries. [16]

4.a) Explain the different sacrifice principles of taxation, giving graphical illustration.

b) Suppose the utility of an income y is given by the function

$$u(y) = \log_e y.$$

Discuss whether the tax system would be progressive, proportional or regressive when there is equal absolute sacrifice and when there is equal proportional sacrifice. [16]

5. Briefly discuss the effect of a balanced budget on national income. [16]

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ANNUAL EXAMINATIONS

Economics-2: Indian Economic Problems

Date: 22.5.69 | Maximum Marks: 100 | Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.
Marks allotted for each question are given in brackets [ ].
Answer five questions taking at least two questions from each group.

Group A

1. Analyse the progress of land reforms in India in the light of the recent review of the 'Land Reforms Implementation Committee'. [20]

2. 'Advantages of co-operative farming arise mainly from its large size, joint management and individual proprietorship', - Critically examine the statement.

   Do you agree with the view that the expansion of co-operative farming in India would create more unemployment? Give reasons for your answer. [20]

3. Explain the problems of agricultural finance in India and in this context examine the recommendations of the 'All India Rural Credit Survey Committee'. [20]

4. Write notes on the following:

   a) Concentration of land-ownership; and
   b) Problems of agricultural labourers. [20]

Group B

1. Elucidate the main features of the Industrial Policy Resolution of 1956 indicating the background of the resolution. [20]

2. Indicate the main objectives of the expansion of Public Sector in India.

   Explain how public sector can act as an instrument in achieving the objective of reduction of inequalities in income and wealth. [20]

3. Argue the case for a separate Industrial Development Bank for India in the present times in addition to the existing specialized agencies for industrial finance.

   Do you think that there is an overlapping of functions between the newly formed Bank and the other specialised financial agencies? [20]

4. State the nature and magnitude of the Foreign Assistance in India during the period of Planning. Also indicate the policy of the Government regarding this assistance. [20]

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ANNUAL EXAMINATIONS

General Science-2: Physics Theory

Date: 23.5.69 | Maximum Marks: 100 | Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 40

Answer any two questions.

1. Define the terms: moment of inertia and radius of gyration. What is the physical significance of moment of inertia? Derive an expression for the moment of inertia of a circular disc about a diameter. [4+4+3+9]=[20]

2. Deduce the differential equation for the simple harmonic motion of a particle. Solve the equation and show that the motion is isochronous. [3+5+4]=[12]

   The space-time equation for a S.H.M. is given by $x = a \sin(wt + \epsilon)$. Show that the velocity v and the acceleration f satisfy the relation:

   $$w^2 v^2 + f^2 = a^2 w^4 .$$ [8]

3. Obtain an expression for the time period of a compound pendulum and show that the centres of oscillation and suspension are interchangeable. [6+4]=[10]

   In a Kater's pendulum the time periods about the two knife edges are t and $t+\epsilon$, where ϵ is small. If the knife edges are distant λ and λ' from the centre of gravity show that

   $$\lambda + \lambda' = \frac{gt}{4\pi^2} \left\{ t + \frac{2\lambda'}{\lambda - \lambda'} \epsilon \right\}.$$ [10]

4. Describe with relevant theory how a cantilever may be used to determine Young's modulus of the material of a bar. [12+8]=[20]

Group B

Maximum Marks: 60

Answer any three questions.

State Kirchhoff's laws of distribution of current in a network of conductors. Apply it to deduce the current in the galvanometer in an unbalanced wheatstone's bridge and hence deduce the null-condition. [6+7]=[13]

12 wires each of resistance r are joined to form a skeleton cube and a current enters at one corner and leaves it at the diagonally opposite corner. Find the equivalent resistance. [illegible]

GO ON TO THE NEXT PAGE

3. a) Find an expression for the average power in A.C. circuits. What is meant by 'power factor'? [7+3]=[10]

b) Derive an expression for the instantaneous current during growth in an L-R circuit having a steady e.m.f. E. What is meant by the time constant of the circuit? [7+3]=[10]

3. Describe the Porous-plug experiment. Explain why there is cooling for some gases and heating for some others. Calculate the temperature of inversion for a Vander waal gas. [3+7+3]=[13]

A mass m of water at T_1 is isobarically and adiabatically mixed with an equal mass of water at T_2. Show that the entropy change of the universe is

$$2mc_p \ln \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$$ [6+1]=[7]

where c_p is specific heat of water at constant pressure and prove that this is positive.

4. What are the conditions of interference of light? Describe, with a neat diagram, the production of Newton's rings. How could you determine the unknown wavelength of light by this method? [3+3+7+7]=[20]

5. Describe, with a neat diagram, the construction and the principle of action of a Michelson's interferometer. What are its uses? [5+6+6+3]=[20]

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ANNUAL EXAMINATIONS

General Science-2: Physics Practical

Date: 24.5.69 | Maximum Marks: 100 | Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Perform the experiment as indicated in Card A. [60]

2. Laboratory Note Book. [10]

3. Class work. [20]

4. Oral. [10]

Distribution of Marks of Q.1

Theory (working formula)	6
Accuracy	6
Method	40
Calculation	8

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ANNUAL EXAMINATIONS
Statistics-2: Probability

[75]

Date: 26.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any three questions. Two marks are allotted for clarity in the answers.

1. Peter and Paul, with an initial capital of Rs.20 and Rs.10 respectively, decided to play a series of games under the following rules.
   i) The player who loses a game should pay Rupee one to the winner.
   ii) The play should be continued until one player is ruined completely.

   Suppose the probabilities of Peter's winning and losing a game to Paul are 1/3 and 2/3 respectively. Find the probability of Peter's ultimate ruin. [16]

2.a) Let $N, X_0, X_1, X_2, \ldots$ be a sequence of independent non-negative integer-valued random variables such that
   i) $X_1, X_2, X_3 \ldots$ are identically distributed,
   ii) $P[X_0 = 0] = 1$.

   Define $S_N = X_0 + X_1 + \ldots + X_N$

   Find the probability distribution of S_N, its mean and variance. [6+2+3]=[11]

b) Give an example of a situation fitting into the framework of the problem stated in 2(a). [5]

3.a) When do you say a sequence of random variables $\{X_n:, n \geq 1\}$ converges in probability to a constant C? [3]

b) Examine whether the following sequence of random variables converges in probability or not.

$$X_n: \quad 0 \quad n^2 \qquad n \geq 1$$
$$\text{Pr:} \quad 1-\frac{1}{n} \quad \frac{1}{n}$$
[6]

c) $\{X_n;\ n \geq 1\}$ and $\{Y_n;\ n \geq 1\}$ are two sequences of random variables such that

$$\text{Plim } X_n = c \quad \text{and}$$
$$\text{Plim } Y_n = d.$$

Show that $\text{Plim } (X_n - Y_n) = c-d$. [7]

4.a) Let X_1 and X_2 be two independent random variables each having geometric probability distribution with the same parameter p. Find the conditional distribution of X_1 given $X_1 + X_2 = n$. [6]

4.b) Let $X_1, X_2, \ldots X_n$ be mutually independent random variables each having the uniform distribution $P[X_1=K] = \frac{1}{N}$ for $K = 1,2,\ldots, N$. Let $U_n = \max(X_1, X_2, \ldots, X_n)$ and $V_n = \min(X_1 \ldots X_n)$. Find the individual distributions of U_n and V_n. [5+5]=[10]

Group B

Maximum Marks: 50

Answer any three questions. Two marks are allotted for neatness in the answers.

1. We select one of the integers 1,2,3,4,5 at random. After discarding all integers strictly less than the selected integer, we draw one of the remaining integers at random. Let X and Y denote the numbers obtained on the first and second draws respectively.

   a) Construct the joint probability table of X and Y.

   b) Determine the individual probability distributions of X and Y.

   c) Examine the independence of the two random variables X and Y.

   d) Determine the conditional probability distribution of Y given X = 3.

   e) Find $P(X+Y > 7)$ and $P(Y-X > 0)$ [8+2+1+3+2]=[16]

2. Applying probabilistic methods, show that any real-valued continuous function defined on [0,1] can be approximated by polynomials uniformly to any desired degree of approximation. [16]

3.a) Define

   i) an algebra of subsets of a set Ω

   ii) a σ-algebra of subsets of a set Ω

   iii) a probability measure on a σ-algebra.

b) Let $\Omega = \{1,2,3,\ldots\}$

   Let $\mathcal{C} = \{\{1\}, \{2\}, \{3\}, \ldots\}$

   Find the smallest algebra containing $\mathcal{C}$.

c) Let $\mathcal{Q}_1 \subseteq \mathcal{Q}_2 \subseteq \mathcal{Q}_3 \subseteq \ldots$ be an increasing sequence of algebras on a set Ω.

   Show that $\bigcup_{n \geq 1} \mathcal{Q}_n$ is an algebra.

d) Show that every probability measure is countably sub-additive.

e) State and prove Borel-Cantelli Lemma. [3+3+3+3+4]=[16]

4.a) Define

   i) generating function of a sequence of real numbers $\{a_n; n \geq 0\}$ [2]

GO ON TO THE NEXT PAGE

4.a)

ii) convolution of two sequences of real numbers $\left\{a_n;\ n \geq 0\right\}$ and $\left\{b_n;\ n \geq 0\right\}$. [2]

iii) In a sequence of Bernoulli trials, let U_n denote the probability that the first combination SF occurs at trials number $n-1$ and n. Find the generating function, mean and variance of $\left\{U_n;\ n \geq 0\right\}$. [5+1+2]=[8]

b) Applying convolution techniques, show that if X and Y are independent Poisson variates with parameters λ_1 and λ_2 respectively, then $X + Y$ has Poisson distribution with parameter $\lambda_1 + \lambda_2$. [illegible]

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ANNUAL EXAMINATIONS

Statistics-2: Statistics Theory and Practical

Date: 27.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A:

Maximum Marks: 50

Answer question 4 and two other from questions 1 to 3 in this group.

1. If the joint distribution of (X,Y) is bivariate normal derive the following:

(a) the marginal distribution of X.

(b) $E(Y|X = x)$. [5+5]=[10]

2.a) Define the multiple correlation coefficient between X_0 and $X_1, X_2, \ldots, X_k$.

b) Define the partial correlation coefficient of X_0 and X_k eliminating the effects of $X_1, X_2, \ldots,$ and X_{k-1}. [5+5]=[10]

3. Let X, Y be independent Poisson random variables with parameters λ and μ respectively.

(a) Derive the distribution of X+Y

(b) Derive the conditional distribution of X given X+Y=t. [5+5]=[10]

4. The following table gives the data for 450 students on their intelligence quotient (I), hours of study per week (S) and marks obtained in school (M).

Variable	Mean	Standard deviation	Correlation coefficients	
I	100.6	15.8	IM:	0.60
S	24.0	6.0	SM:	0.32
M	18.5	11.2	IS:	- 0.35

a) Obtain a suitable formula for predicting M based on I and S.

b) Compute the multiple correlation coefficient of M with S and I. [12+8]=[20]

5. Viva Voce. [10]

Group B

Maximum Marks: 50

Answer Q.4 and two other from questions 1 to 3 in this group.

1.a) Derive an expression for the partial correlation coefficient $r_{12.3}$ of X_1 and X_2 eliminating X_3 in terms of the total correlation coefficients r_{12}, r_{23} and r_{31} (r_{ij} = correlation coefficient of X_i and X_j).

GO ON TO THE NEXT PAGE

1.b) If $r_{12} = r_{23} = r_{31} = \rho$ show that

$$1 - R^2_{1.23} = \frac{(1-\rho)(1+2\rho)}{1+\rho}.$$ [5+5]=[10]

2. Let $X_1, X_2, \ldots, X_n$ be independent random samples from a normal distribution with mean μ and variance σ^2.

i) State the distribution of the following random variables:

a) $T_1 = \frac{1}{n} \sum_{i=1}^{n} X_i$

b) $T_2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \bar{X})^2$

c) $T_3 = \dfrac{(X_1-\mu)^2 + (X_2-\mu)^2}{\sum_{i=3}^{n} (X_i - \mu)^2} \cdot \dfrac{n-2}{2}$.

ii) Write down the joint density of the pair (T_1, T_2) [T_1, T_2 are defined in (i)]. [3×2+4]=[10]

3. Let X and Y be independent random variables having the density functions $f_m(x)$ and $f_n(x)$ respectively where

$$f_i(x) = \frac{p^i}{\Gamma(i)} x^{i-1} e^{-px} \quad \text{if } x > 0$$
$$= 0 \quad \text{if } x \leq 0$$

$i = n, m$

$p > 0, \; n > 0, \; m > 0.$

Derive the distribution of $X + Y$. [10]

4.a) A producer of electric bulbs in his desire for putting only good bulbs for sale, rejects all bulbs for which a certain quality characteristic X is less than a certain constant C. It is known that X and the life of the bulb in hours Y are jointly normally distributed with the following parameters

	X	Y
mean	80	1100
standard deviation	10	100
correlation coefficient	0.60	

Compute the proportion P and the average life L of bulbs put for sale for C = 75. [5+5]=[10]

b) The following table gives the additional hours of sleep gained by 10 patients in an experiment to test the efficiency of a certain soporofic drug. Do the data give evidence that the drug produces <u>additional sleep</u>.

Additional hours of sleep gained

patient	1	2	3	4	5	6	7	8	9	10
hours of sleep gained	0.7	-1.1	-0.2	1.2	0.1	3.4	3.7	0.8	1.8	2.0

[10]

5. Practical Records [10]

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ANNUAL EXAMINATIONS

Statistics-C: Index Numbers and Time Series

Date: 28.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets.[ ].

Group A

Maximum Marks: 50

Answer all questions.

1. What is time series? What are its different components? Explain the method of least squares for fitting a linear trend to a time series data. Why is this method called the method of least squares? [2+4+6+3]=[15]

2. For the time series data given below calculate the indices of seasonal variation by the method of moving averages. [25]

Price of gold in Bombay Bullion Market (in Rupees per tola)

1953	1954	1955	1956	1957	1958	Month
87	83	90	96	106	110	Jan.
90	87	93	98	108	113	Feb.
87	89	93	104	106	112	March
87	93	96	104	106	113	April
87	91	94	104	108	112	May
91	85	94	103	106	109	June
88	85	94	101	107	106	July
88	88	95	104	108	107	August
87	87	95	103	108	108	Sept.
82	87	95	104	107	109	Oct.
83	88	96	105	109	113	Nov.
82	88	94	104	108	114	Dec.

3. Viva Voce [10]

Group B

Maximum Marks: 50

Answer all questions.

1. Write short notes on
   i) Use of orthogonal polynomials in time series analysis.
   ii) Method of link relatives for computing seasonal indices.
   iii) Selection of base year in the construction of index number. [5+5+5]=[15]

2. For the following time series data find the trend component by fitting polynomial of appropriate degree using orthogonal polynomials.

National Income in U S.

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ANNUAL EXAMINATIONS

General Science-3: Biology Theory

Date: 29.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer question 1 and any two other questions.

1. Explain the aestivation of corolla in Malvaceae. [10]

2. Write a systematic account of the family Gramineae mentioning the names of eight plants belonging to the family. [12+8]=[20]

3. Compare the morphology of Papilionaceae and caesalpiniaceae with illustrations and examples. [10+10]=[20]

4. Write short notes on any four:
   A. Syngenecious stamens,
   B. Cyathium inflorescence,
   C. Economically important plants of Solanaceae,
   D. Stamens in Scitamineae,
   E. Inflorescence of Palmae. [5 X4]=[20]

Group B

Maximum Marks: 50

Answer question 1 and any two from the rest.

1. Write an account on the principal ecological factors which influence a vegetation. [20]

2. What is meant by physiologically dry soil? Write a brief account on the morphological and anatomical characteristics of hydrophytes and halophytes. [5+10]=[15]

3. Write a critical account of the different theories on ascent of sap in plants. [15]

4. Write short notes on any three:
   A. Tropic movement in plants,
   B. Drought,
   C. Osmosis,
   D. Root pressure . [3 X5]=[15]

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ANNUAL EXAMINATIONS

General Science-3: Biology Practical

Date: 30.5.69 | Maximum Marks: 100 | Time: 3 hours

Note: The number of marks allotted to each question is given in brackets [ ].
Answer all questions.

1. Give a botanical description of specimen A, with detailed labelled sketches. Assign the plant to its family giving valid reasons. [15+5]=[20]

2. Cut a thin transverse section of specimen B and leave the mounted specimen for examination.

   Draw a labelled sketch of the different tissues and comment on the ecological adaptations of the plant. [5+10+5]=[20]

3. Comment on specimens/ experiments C - G. [5 X4]=[20]

4. Identify Specims H - L. [5 X2]=[10]

5. Viva Voce [10]

6. Practical records [20]

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[80]

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ANNUAL EXAMINATIONS

General Science-2: Chemistry Practical

Date: 31.5.69 Maximum Marks: 100 Time: 3 hours

<u>Note</u>: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Determine the Partition co-efficient of Iodine between water and organic layer. [50]

2. Find out the Iodine value of an oil. [25]

3. Viva Voce [15]

4. Laboratory Note Book [10]

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PERIODICAL EXAMINATIONS

81

Statistics-3: Probability

Date: 16.9.68 | Maximum Marks: 100 | Time: 3 hours

Note: The whole paper carries about 110 marks.
Answer as much as you can.
The marks allotted to each question is given in brackets [].

1.a) Define lim sup A_n and lim inf A_n for a sequence $\{A_n\}$ of subsets of a set Ω. When is $\{A_n\}$ said to have a limit? [3+3+3]=[9]

b) If $\{A_n\}$ is a disjoint sequence, show that $\{A_n\}$ converges to $\emptyset$. [5]

c) If $\{A_n\}$ is increasing, show that $\{A_n\}$ converges to $\bigcup_{n=1}^{\infty} A_n$. [5]

d) If $E_n = \begin{cases} (0, 1 - \frac{1}{n}] & \text{if } n \text{ is odd} \\ [\frac{1}{n}, 1) & \text{if } n \text{ is even} \end{cases}$

verify that E_n converges but is not monotone. [6+2]=[8]

2.a) A class $\mathcal{R}$ of subsets of a set Ω is called a ring if $A, B \in \mathcal{R} \Rightarrow A \cap B,\ A \Delta B \in R$. Show that $\mathcal{R}$ is a ring if and only if $A, B \in \mathcal{R} \Rightarrow A \cup B,\ A - B \in \mathcal{R}$. [5]

b) Show that if $\mathcal{R}_\alpha$ is a collection of rings, $\bigcap_\alpha \mathcal{R}_\alpha$ is a ring. Hence define the ring generated by any class $\mathcal{E}$ of subsets of Ω and show that it is the smallest ring containing $\mathcal{E}$. [5+3+6]=[14]

c) Show that for a ring $\mathcal{R}$, the following are equivalent: (i) $\mathcal{R}$ is a σ-ring; (ii) $\mathcal{R}$ is a monotone class. [5]

3. State whether each of the following classes is a (1) semiring (2) ring (3) field (4) σ-ring (5) σ-field (6) monotone class (7) heriditary class (Ω is an uncountable set). [$42 \times \frac{1}{2}$]=[21]

a) all finite sets

b) $\emptyset$, A, $\Omega - A$, Ω where A is a fixed subset of Ω

c) all countable sets and their complements

d) all single point sets.

e) Ω = the real line, all bounded intervals (open, closed, semi-closed).

f) all subsets of A where A is a fixed subset of Ω.

4.a) Define a measure on a ring $\mathcal{R}$. Prove that it is monotone, subtractive, continuous from below and from above at every set of $\mathcal{R}$. [3+5+5+12]=[25]

b) Prove that a finite non-negative additive set function de-defined on a ring $\mathcal{R}$ which is either continuous from below at every set of $\mathcal{R}$ or continuous from above at $\emptyset$ is a measure on $\mathcal{R}$. [16]

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PERIODICAL EXAMINATIONS

General Science-4: Biochemistry Theory

Date: 23.9.68. Maximum Marks:100 Time: $2\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Give one example for each of the followings:

   Basic amino acid, Dehydrogenase, Dipeptide, Phosphoproteine, Oxidase. [10]

2. How the proteins are classified?
   Give one example of each class. [20]

3. What are the functions of proteins. [15]

4. How can you estimate protein from a natural source. [15]

5. Define isoelectric point of amino acids. What is an essential amino acid? Give one colour reaction of amino acids. [15]

6. What is an enzyme? What is transaminase?

   Describe the properties of:

   (a) Amylase
   (b) Catalase [25]

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PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 14.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

R denotes an Archemedian Ordered Field

1.a) Define the terms 'Field' 'Ordered Field' and 'Archemedian Ordered Field'.

b) Can a Field Consist of finite number of elements? Give reasons.

c) Can an Ordered Field consist of finite number of elements? Give reasons. [10+5+5]=[20]

2.a) Prove that any bounded decreasing sequence in R is a Cauchy sequence.

b) Is converse of 'a' true? Give reasons.

c) Prove that any convergent sequence in R is a Cauchy sequence.

d) Is converse of 'c' true? Give reasons. [7+3+7+3]=[20]

3.a) Show that any sequence in R contains a monotone subsequence.

b) Show that if a subsequence of a Cauchy sequence in R converges then the Cauchy sequence itself converges.

c) Deduce from 'a' and 'b' above, that the following two statements are equivalent in R:

1. Any bounded increasing sequence converges.
2. Any Cauchy sequence converges. [8+8+4]=[20]

4.a) Explain the terms 'Open interval' 'Open set' 'limit point' and 'closed set'.

b) Show that a nonempty set A contained in R is open if and only if A^c (the complement of A in R) is closed. [10+10]=[20]

----

5. Assignments [10]

6. For clarity of expression. [10]

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PERIODICAL EXAMINATIONS

Statistics-3: Data Processing

Date: 21.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Name the various types of Punched Card machines known to you and briefly explain their functions. [15]

2. You have a deck of cards which are badly damaged by its 12-Edge.
Is it possible to reproduce a new pack of punched cards by using these cards? If so how. [5]

3. There is a sorted deck of 40,000 cards by considering columns 6-10. It is necessary to pull-out all cards with '4' punched in column 25 of those cards.

How will you get the work done without disturbing the sorting sequence of the cards. [5]

4. It is required to prepare a new set of punched cards from the cards punched according to the design given below. No change of design in the new set of cards is envisaged.

Card design	Card Cols	
cdi	1 - 4	Punch XXX1
department	5	
category	6	
roll no.	7 - 9	
name	10 -34	
daily pay rate	35-38	(2 pl. decimal)

Prepare the reproducer Panel diagram. [12]

5. You are provided with a deck of cards punched according to design XXX1 given in question No.4. Prepare control Panel diagram to prepare the statement given below.

department	category	number of worker	total daily pay
	1		
	2		
department	sub-total		
	1		
	2		
department	sub-total		
department	sub-total		
Total			

[20]

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6. EITHER

Write a general program in FORTRAN II to evaluate the mean and variance of N given numbers $X_1, X_2, \ldots, X_N$.

OR

Write a program in FORTRAN II to find the roots of the quadratic equation

$$AX^2 + BX + C = 0$$

where A, B, C are given real numbers. [15]

7. Write a program in FORTRAN II to obtain N! where N is a positive integer. [8]

8. Class records. [20]

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B. Stat. Part III:1968-69
PERIODICAL EXAMINATIONS
General Science-5: Sociology

Date: 28.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.6 and any four from the rest.
All questions carry equal marks.

1. EITHER

Define and distinguish between family and household. Mention the chief functions of family as a societal institution.

OR

How a nuclear family differs from an extended family? What are the inter-personal relationships you may expect from a nuclear family?

2. EITHER

What are consanguinal and affinal relationships? What are the difference between 'descriptive' and 'classificatory' systems of kinship? Briefly mention the importance of kinship in simple societies.

OR

What do you mean by clan? How it differs from lineage and extended family? What are its main functions ?

3. EITHER

Define 'religion'. Describe the nature of religion and compare it with magic.

OR

What are monogamy and polygamy? Enumerate different types of preferential marriages found in Indian societies.

4. EITHER

Define and distinghish between caste and class. Enumerate the features of caste organization in India.

OR

What are endogamy and exogamy? How do they influence caste formation?

5. EITHER

What is 'incorporeal' and 'corporeal' property? Give examples from simple and complex societies.

OR

What are the social significances attached to (a) lobola (b) kula and (c) pot-latch?

6. Formulate a design of sample survey for studying standard of living of different family groupings in West Bengal with special reference to (a) coverage of universe (b) unit of sampling and (c) unit of enquiry.

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PERIODICAL EXAMINATIONS

Economics-3

Date: 4.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts.

Group A: Economic Development

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Attempt any two questions. Each question carries 25 marks.

1. Describe the significant changes in the national product and its distributions in the advanced economies in the process of long period economic development.

   Could cross section national income data throw any light on the subject?

2. Discuss Abramovitz's analysis of the long period growth of the US economy.

   Can a similar analysis be applied on the Indian data?

3. Bring out the main features of the growth model developed by classical economists.

   Briefly indicate its main differences from Marx's model.

4. Discuss briefly the growth models due to Harrod and Domar and point out the main difference between their approaches.

Group B: Indian Planning

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Note: Answer any three questions.
All questions carry equal marks.

1. Is a planned economy superior to a non-planned one? Give reasons for your answer.

2. Critically discuss the differences between programming, mixed economy planning and centralised planning.

3. Discuss the approach and methods proposed in the Bombay Plan to double the per-capita income in the course of 15 years.

4. Why and what measures were proposed in the Bombay Plan to reduce inequality of income?

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PERIODICAL EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 11.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Marks allotted for each question are given in brackets [ ].

1.a) The following table gives the production of polished plate glass in USA. Fit a suitable polynomial regression for the production of polished plate glass by using the method of orthogonal polynomials.

Year	production (in millions of square-feet monthly)
1933	7.2
1934	7.9
1935	15.0
1936	16.5
1937	16.0
1938	7.1
1939	11.8
1940	13.7
1941	15.9

[16]

b) Suggest a procedure to fit the polynomial regression if some values are missing in the data. (No computations are required). [4]

2. To determine the yield rate (in mounds per acre) of paddy by the method of random sampling ten plots were chosen at random. Within each plot a circle of radius 2 ft. and another of 4 ft. were marked out at random. The crops inside the circles were harvested and then the yield rate was calculated. Obtain a 95 per cent confidence interval for the ratio of the variance of the yield rate as calculated from circle with 2 ft. radius to that calculated from circle with radius 4 ft.

radius of the circle	yield rate
2 ft.	6.1, 5.4, 5.6, 6.3, 5.1, 6.3, 5.9, 5.6, 4.4, 6.1.
4 ft.	5.5, 6.0, 4.7, 5.9, 5.6, 6.1, 5.7, 5.4, 5.2, 5.8.

[20]

3. The table below shows measurements of heights of 11 pairs of twins of opposite sex all of ten years of age.

Sl. No.	Male ht. (in cm.)	Female ht. (in cm.)	Sl. No.	Male ht. (in cm.)	Female ht. (in cm.)
1	136	132	7	130	133
2	141	133	8	139	140
3	137	140	9	128	123
4	137	135	10	132	136
5	134	131	11	133	134
6	134	137			

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Obtain 90 per cent confidence limits for the correlation between the heights of the twins. [20]

4. What are random numbers? Discuss in detail the various requirements you impose for the construction of random number tables. Comment on your requirements. [20]

5. Obtain a random sample of size 25 from the exponential distribution with density function.

$$f(x) = \begin{cases} 2\,e^{-2x} & 0 \leq x < \infty \\ 0 & \text{otherwise} \end{cases}$$

[20]

6. a) Explain clearly what are meant by SRS WR and SRSWOR giving examples of situations in which you use them. [8]

b) Propose suitable estimate for the population mean and give its standard errors in each sampling procedure. [9]

c) How do you use your estimates to obtain population total? What is the standard error of your estimate. [3]

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MID-YEAR EXAMINATIONS

Mathematics: Analysis

Date: 19.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted to each question are given in brackets [ ].

1.a) Define the terms 'Field' 'Ordered Field' and 'Archemedian Ordered Field'.

b) Can a Field consist of finite number of elements? Justify your answer.

c) Can an Ordered Field consist of finite number of elements? Justify your answer. [10+2+3]=[15]

2.a) Show that any closed interval $[a,b]$ where $a < b$ contains a rational number.

b) Hence deduce that given any real number x, there is an increasing sequence of rationals converging to it. [5+5]=[10]

3.a) Define clearly a 'sequence' and a 'series' of real numbers.

b) When do you say that a series is absolutely convergent?

c) Show that any absolutely convergent series is convergent. [3+2+5]=[10]

4.a) Define the Cauchy product of two series.

b) If a series Σa_n converges absolutely to A and a series Σb_n converges to B then show that the Cauchy product convergos to A·B. [2+8]=[10]

5.a) What is meant by rearrangement of a series?

b) State carefully Riemann's theorem about rearrangement of a series. (No proof is needed). [2+3]=[5]

6. Home assignments. [50]

Note: COLLECT YOUR HOME ASSIGNMENT QUESTION PAPER FROM THE DEAN'S OFFICE.

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[68] B

MID-YEAR EXAMINATIONS

Mathematics: Analysis

Home Assignment

Maximum Marks: 50.

Note: 1. You should submit your answerscripts on or before 20th January 1969 in the Dean's Office.

2. Answer all questions.
3. Your answers should be precise, clear and logical.
4. Do not give unnecessary details. But you should clearly explain how one step follows from previous one in your proofs.

1. If Σa_n is a convergent series of positive terms then show that Σa_n^2 is also convergent.

[Hint: After a certain stage $a_n^2 \leq a_n$]

2. If Σa_n^2 is convergent series then show that $\Sigma \frac{a_n}{n}$ is also convergent.

[Hint: $\frac{2a_n}{n} \leq a_n^2 + \frac{1}{n^2}$]

3. If Σa_n is convergent and $a_n \downarrow 0$, then show that $\Sigma 2^n a_{2^n}$ converges.

4. Show that the series $\Sigma \frac{1}{n}$ is not convergent. Find an integer N such that the Nth partial sum of this series is greater than 5.

5. Show that the series Σa_n where

$$a_n = \frac{1}{n^2} \text{ if } n \text{ is not a perfect square}$$
$$= \frac{1}{n} \text{ if } n \text{ is a perfect square}$$

Converges, by showing that the partial sums are Cauchy.
[Note: A positive integer is called perfect square if it is the square of some integer].

6. For any sequence $\{c_n\}$ of real number where $c_n > 0$ for every n show that,

$$\liminf \frac{c_{n+1}}{c_n} \leq \liminf \sqrt[n]{c_n} .$$

7. Show that the series $\Sigma \frac{(-1)^n}{\sqrt{n}}$ is convergent, but the Cauchy product of it with itself is not convergent.

8. Show that for every real number x the series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

is convergent. Denote this by $e(x)$. Show by using the Cauchy product of series, that

$$e(x+y) = e(x).e(y).$$

9. If a power series converges for every real number x, then show that it absolutely converges for every number x.

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10. Find out the limit superior and limit inferior for the following sequence:

$$a_1 = 0,$$
$$a_{2m} = \frac{a_{2m-1}}{2},$$
$$a_{2m+1} = a_{2m} + \frac{1}{2}, \qquad \text{for } m \geq 1.$$

11. Find $\lim\limits_{n \to \infty} \sqrt{n^2 + n} - n$.

[Hint: $\sqrt{n^2+n} - n = \dfrac{n}{\sqrt{n^2+n} + n}$].

12. Let $\{a_n\}$ be the sequence,

$$a_1 = \sqrt{2}, \quad a_n = \sqrt{2+\sqrt{a_n}} \qquad \text{for } n > 1$$

Show that the sequence $\{a_n\}$ is increasing and bounded by 2 and hence converges.

13. If a power series has infinitely many nonzero integer coefficients then show that its radius of convergence is at most 1.

14. You are given a number r such that $0 \leq r \leq +\infty$. Exhibit a power series whose radius of convergence is r.

15. 'Real number system' is defined as a 'Complete Archemedian Ordered Field'. From what you have learned in the class, discuss how far the 'Archemedian' property and 'Completeness' property are essential.

16. What is meant by a 'summability method' in the real number system? Write a few sentences about the reasonable conditions that a summability method should satisfy, according to you. Define and comment on Cauchy's method of summability.

[14×3 + 2×4]

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MID-YEAR EXAMINATIONS

Economics-3

Date: 20.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

Group A

Economic Development

Answer any three questions

Maximum Marks: 60 Suggested time: $1\frac{3}{4}$ hours

1. Mention the important items of Leibenstein's list of characteristics of underdeveloped countries. [20]

   What do you understand by 'vicious circles' in this context.

2. Growth of real national income is not fully explained by changes in labour and capital inputs. What other factors should be considered? Obtain suitable expressions for contributions of different factors to growth.

   Discuss the relevance of the above proposition for analysing the long period growth of either the U.S. or the Indian economy. [20]

3. Why is it important to study transactions in kind in under developed economies?

   Give a descriptive account of various transactions in kind in India.

   State conditions under which the share of transactions in kind (in aggregate of all real transactions) is likely to (i) reduce, (ii) increase. [20]

4. A measure of intersectoral disparity of average sectoral earnings per worker is given by

$$d = \frac{1}{2} \sum_{i=1}^{n} |y_i - \lambda_i|$$

   y_i, λ_i respectively standing for shares of income and labour force in the ith sector. Construct simple examples showing that d increases with increased disparity.

   How could a similar measure of disparity be constructed for size distributions?

   What change in a size distribution you would expect when intersectoral disparity increases? Why is it easier to study changes in size distributions of real income through intersectoral disparities? [20]

5. Write short notes on any two of the following:

   i) backward sloping supply curve of effort;

   ii) employment concept as applied to poorer countries;

   iii) distribution of national income by factor shares in rich and poor countries; and

   iv) linkage coefficients. [20]

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Group B

Indian Planning

Answer <u>any two</u> questions. Maximum Marks: 40

Suggested time: $1\frac{1}{4}$ hours.

1. What influenced the change in the approach in the formulation of the second five year plan? Indicate how this change was reflected in the size, allocation of resources and objectives of the second plan. [20]

2. What are the different sources of finances for the plan in the public sector? Discuss the estimates of these resources for the second five year plan. [20]

3. What were the issues and what were the grounds that led K. T. Shah to differ from the recommendation of the Advisory Planning Board? [20]

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MID-YEAR EXAMINATIONS

General Science-4: Biology: Botany Theory

Date: 21.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. Marks allotted for each question are given in brackets.[ ].

1. The sugarcane variety Co.312 is very successful in the sub-tropical belt of India. What are the factors which contribute to the good performance of this variety? [20]

2. Write an illustrated account on the morphology of the coconut palm. [20]

3. A. Write names of ten economically important Legumes.

   B. Which are the species of wheat cultivated at present? [10+10+20]=[40]

4. Give the statistics on the area under and production of Rice(Oryza sativa) in the various agricultural regions of the world. Mention the corresponding figures for China, India and Pakistan. [14 +6]=[20]

5. EITHER

   Write an essay on the scientific aspect of your visit to the Rice Research Station at Chinsurah.

   OR

   The Jute Agricultural Research Institute, Barrackpore. [20]

6. A: What is the morphology of the economically important part (or organ) in the following plant species?

   (a) *Musa textilis*; (b) *Boehemeria nivea*;

   (c) *Helianthus tuberosus*; (d) *Acer negundo*;

   (e) *Beta vulgaris*; (f) *Pisum sativum*;

   (g) *Manihot utilissima*; (h) *Solanum tuberosum*;

   (i) *Agave sisalana*; (j) *Gossypium barbadense*.

   B: Describe the spikelets of *Sorghum vulgare* with suitable illustrations. [10+10]=[20]

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MID-YEAR EXAMINATIONS

General Science-4: Biochemistry Theory

Date: 23.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe the aerobic glycolysis in mammalian cells. [20]

2. What are epinephrine and vasopressin? Describe their physiological properties. [10]

3. How can you estimate glucose in human blood. [15]

4. Give one example for each of the following:
Nucleoside, Aldose, Triglyceride, Pyrimidine, Steroid. [10]

5. What are the defficiency symptoms of the following vitamins. S
Vitamin A, Pantothenic acid, Niacin, Vitamin K, Folic acid. [15]

6. Discuss the origin of ketone bodies in urine. [10]

7. Describe the biochemical pathways of fatty acid oxidation. [20]

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MID-YEAR EXAMINATIONS

General Science-4: Biochemistry Practical

Date: 24.12.68 Maximum Marks: 100 Time: 3 hours

1. Determine the total amount of glucose in the given sample by Fehling's titration.

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MID-YEAR EXAMINATIONS

Statistics-3: Probability

Date: 26.12.68 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 120 marks. Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) Define an outer measure. [4]

b) How do you prove that given a measure μ on a ring R, it can be extended to a complete measure on a σ-ring containing R? (Give the necessary steps without proofs). [8]

c) Define a Steiltjes measure function F on R^k and the Lebesgue-Steiltjes measure induced by F. [4+6]=[10]

2.a) Define a measurable function f. If f is non-negative, show that it is a limit of a monotone increasing sequence of non-negative simple functions. [4+8]=[12]

b) If f, g are measurable, prove that $f+g$, fg are measurable. [6+6]=[12]

c) If f_n is a sequence of measurable functions, show that $\limsup f_n$ is measurable. [6]

3.a) Define the integral of a non-negative measurable function and prove that it is unique. [4+10]=[14]

b) Define the integral of a measurable function whenever it is possible. [4]

c) Show that any measurable function has integral zero over a set of measure zero. [6]

d) If f is integrable, show that f is finite a.e. [6]

4.a) If A, B are disjoint measurable sets and f is integrable, show that

$$\nu_f(A \cup B) = \nu_f(A) + \nu_f(B)$$

where $\nu_f(A) = \int_A f \, d\mu$. [6]

b) Assuming that if f_n are non-negative, measurable and increasing, then $\int \lim f_n d\mu = \lim \int f_n d\mu$, prove that if f is an integrable function, then for every $\varepsilon > 0$, there corresponds a $\delta > 0$ such that

$$|\nu_f(E)| < \varepsilon \text{ whenever } \mu(E) < \delta.$$ [8]

c) Deduce from the above two results that if f is non-negative and integrable, then ν_f is a measure.
(Hint: it is enough to show that ν_f is continuous from above at $\emptyset$). [8]

d) State Radon-Nikodym theorem. What is its use in the theory of probability? [6+4]=[10]

5. Neatness and clarity. [6]

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[94].

MID-YEAR EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 28.12.68 Maximum Marks: 100 Time: 3 hours

Note:- Answer any four questions. Marks allotted for each question are given in brackets [ ].

1. In a manufacturing company, it is noticed that some defective items are being produced. The authorities wanted to find the number of defective items in the stock they have. Since the stock is very large they cannot afford to check up all the items. As a statistician suggest them a procedure of obtaining an estimate of the number of defective items. What are the properties of your estimates? Derive explicit expressions. Also, suggest a method of obtaining a confidence interval for the true proportion of defectives. [25]

2. It is required to estimate the rate of incidence of a particular disease in a state. Since the incidence of disease depends on the general socio-economic conditions it is decided to take a stratified random sample. Suggest a method of conducting the survey giving details about (a) how you would choose your samples from different strata if you are given a fixed amount of money C to be spent on the survey. (b) The estimate you propose for the incidence rate and an estimate of its variance. [25]

3. The following data show the stratification of all the farms in a Tahsil by farm size and the average acres of wheat per farm in each stratum.

Farm size (acres)	No. of farms N_h	Average acres of wheat $\bar{Y}_h$	Standard deviation σ_h
0 - 40	394	5.4	8.3
41 - 80	461	16.3	13.3
81 - 120	391	24.3	15.1
121 - 160	334	34.5	19.8
161 - 200	169	42.1	24.5
201 - 240	113	50.1	26.0
241 -	148	63.8	35.2

$\Sigma N_h = N = 2010$ $\bar{\bar{Y}} = 26.3$

$$\sigma_h'^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (Y_{hi} - \bar{Y}_h)^2 .$$

For a sample of 100 farms compute the sample sizes in each stratum under (i) Proportional allocation

(ii) Optimum allocation (Neyman allocation)

Compare the variance of the estimate of average acreage of wheat with proportional allocation and with SRSWR. [25]

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4. The following data were collected in an experiment on jute in a village of West Bengal in 1953, in which the weights of green plants and dry jute fibre were recorded for 20 individual plants selected at random.

Weight (in grams) of green jute plant and dry fibre for 20 plants.

Serial No. of plant	Weight in grams	
	green plant	dry fibre
1	93	6.8
2	89	6.3
3	112	7.0
4	8	0.6
5	93	6.5
6	11	0.7
7	16	0.7
8	32	2.9
9	31	2.7
10	37	3.0
11	46	3.3
12	35	2.7
13	30	2.1
14	8	0.5
15	23	1.4
16	33	2.7
17	18	1.7
18	70	5.3
19	87	6.2
20	74	11.5

a) Obtain an estimate of the regression coefficient of the weight of dry fibre on the weight of green plant.

b) Obtain a 90 per cent confidence interval for the regression coefficient.

c) If the weight of the green plant is 50 grams, obtain the 95 per cent confidence interval for the average weight of the dry fibre. [25]

5. Obtain the value of $\int_3^5 \frac{dx}{2x+3}$ by Gauss quadrature formula by taking 10 points in the interval. Give the details of working. [25]

6. Write short notes on the following:-

(a) Two-stage sampling.

(b) Poker test.

(c) Fitting polynomial regression by the method of orthogonal polynomials (No derivations are necessary). [25]

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MID-YEAR EXAMINATIONS

General Science-5: Statistical Mechanics

Date: 30.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. State and derive Stirling's approximation for the factorial of a large number. Suppose there are three cells in phase space 1, 2 and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$ and $w_1 = 2$ joules, $w_2 = 4$ joules, $w_3 = 6$ joules. If $\partial N_3 = -2$, find ∂N_1 and ∂N_2. [8+8]=[16]

2.a) Define the following terms: Phase space, micro and macrostates, thermodynamic probability, Heisenberg's uncertainty relation.

b) Show that the number of particles in the i^{th} cell, in the state of maximum thermodynamic probability, is, according to M-B statistics

$$N_i = \frac{N}{Z} \exp(-w_i/kT),$$

the symbols having their usual meanings. [4 X4+16]=[32]

3. What is meant by an equation of state? Derive, using M-B statistics, the equation of state for an ideal gas. Also show that the molar specific heat capacity at constant volume is 3R/2. [3+13+4]=[20]

4. Enumerate some cases where the M-B statistics have failed. Applying the Bose-Einstein statistics, derive the Planck's radiation formula. [6 +12]=[18]

5. Derive the Fermi-Dirac distribution function for the state of maximum thermodynamic probability. Compare and contrast the three statistics: M-B, B-E and F-D. [9+5]=[14]

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PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 24.2.69 Maximum Marks: 100 Time: 3 hours

Note: i) Answer all questions. ii) Each question carries 10 marks. iii) Home assignment carries 40 marks.

1. State two definitions of continuity of a function and show that they are equal.

2. State and prove the chain rule for differentiable functions.

3. If f_n is a sequence of functions on $[a,b]$ with continuous derivatives f_n' and if $f_n \to f$ and $f_n' \to \varphi$ uniformly then show that f is differentiable and $f' = \varphi$.

4. If f is differentiable function on $[a,b]$ with derivative f', show that the range of f' is again an interval.

5. Show that any continuous function defined on a closed bounded interval is uniformly continuous.

6. State the following theorems carefully (no proofs are needed) -

   i) Mean value theorem

   ii) L' Hospital's rule

   iii) Taylor's formula.

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Mathematics-3 Analysis
Maximum Marks: 40

Date: 24.2.69.

Home assignment

All the functions appearing in the questions below are defined on the whole real line, unless the contrary is specified.

1. A function f is said to be right continuous at a point x_0 iff whenever x_n decreases to x_0, $f(x_n)$ converges to $f(x_0)$. A function f is said to be left continuous at a point x_0 iff whenever x_n increases to x_0, $f(x_n)$ converges to $f(x_0)$. Answer the questions:

   a) Give an f which is right continuous at 0 but not left continuous.

   b) Give an f which is left continuous at 0 but not right continuous.

   c) Show that f is continuous at x_0 iff it is both right and left continuous at x_0.

   d) If f is right continuous at x_0, show that the function g defined as
   $$g(x) = f(-x)$$
   is left continuous at x_0.

2. A function f is said to have right derivative at a point x_0 if there is a real number α such that whenever h_n decreases to zero and $h_n \neq 0$ for all n,
$$\frac{f(x_0 + h_n) - f(x_0)}{h_n} \to \alpha$$
This α is called the right derivative of f at x_0. Similarly f is said to have left derivative at a point x_0 if there is a real number β such that whenever h_n increases to zero, $h_n \neq 0$ for all n,
$$\frac{f(x_0 + h_n) - f(x_0)}{h_n} \to \beta .$$
This β is called the left derivative of f at x_0.
Answer the questions:

   a. Give an f which has right derivative at 0 but not left derivative.

   b. Give an f which has left derivative at 0 but not right derivative.

   c. Give an f which has both left and right derivatives at 0 but they are not equal.

   d. Show that f is differentiable at a point x_0 iff both the left and right derivatives at x_0 exist and equal.

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3. A function f is said to be additive if for any two real numbers x, y

$$f(x+y) = f(x) + f(y).$$

Show that if f is a continuous additive function on the real line then there is a real number α such that

$$f(x) = \alpha x \quad \text{for all } x.$$

Home assignment should be submitted in the Dean's office on or before 28 February 1969. Along with this you must submit previous assignments, if any are due from you. Otherwise you will get zero marks in this.

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PERIODICAL EXAMINATIONS

Economics-3

Date: 3.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A, Group B and Group C in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A: Economic Development

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1. Present Rostow's theory of stages of growth. What are the main lines of criticism of his theory? [25]

2.a) Briefly describe Boeke's theory of sociological dualism.

b) What do you understand by n-Achievement? Discuss briefly McClelland's theory of growth. [25]

3. Write notes on the structural features of low income countries in respect of any two of the following:

i) health services;
ii) education;
iii) scientific research; and
iv) technical man-power. [25]

GROUP B: Socialist Planning

Maximum Marks: 25 Suggested time: 3/4 hour

1. The measures adopted to guide the economy immediately after the Russian Revolution of 1917 Nov., can not be called socialistic - Would you agree with this view? Describe the measures and give reasons for your answer. [25]

GROUP C: Economic Theory

Maximum Marks: 25 Suggested time: 45 minutes

Answer all questions.

1. Bring out (in brief) all the assumptions you require to justify the investment function in Harrod's model of economic growth. [13]

2. If we define the equilibrium of the economy in the sense of equality between desired demand and desired supply in all markets, then what is the condition of short-run equilibrium in Harrod's model? [12]

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PERIODICAL EXAMINATIONS

Statistics-3: Probability

Date: 10.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) Define the joint distribution of two random variables X and Y defined on a probability space $(\Omega, \mathcal{F}, P)$. [4]

b) If X and Y have an absolutely continuous (joint) distribution, show that each marginal distribution is absolutely continuous. Give an example to show that the converse is false. [6+4]=[10]

c) If X and Y are independent and have an a.c. distribution, show that $f(x,y) = f_1(x).f_2(y)$ where f_1 and f_2 are the marginal densities and $f(x,y)$ is the joint density. [6]

d) If $f(x,y) = g(x)\ h(y)$, show that X and Y are independent and X has density $c.g(x)$ where c is a constant. [6]

e) X and Y have the joint density function 2 over the region in the plane bounded by the lines $x=0$, $y=0$ and $x+y=1$, and zero outside. Using (d), can you conclude that X and Y are independent? Give reasons. [6]

2.a) Define the convolution $F*G$ of two distributions F and G. [4]

b) Show that if F is absolutely continuous, then $F*G$ is also absolutely continuous. [8]

c) Find the density of the convolution of the rectangular distribution on [0, 1] with itself. [8]

3.a) Let $(\Omega, \mathcal{F}, P)$ be a probability space and let X be a random variable. Define the conditional probability of an event given $X=x$ using Radon-Nikodym theorem (give the necessary justification). [8]

b) If X and Y have a joint a.c. distribution, show that $\frac{f(x,y)}{f_2(y)}$ serves as a density for the conditional distribution of X given $Y = y$. [8]

4. Prove that if X has density function $f(x)$ and $Y=g(X)$ is a function of X such that g is differentiable and strictly monotone, then the density of Y at a point y is

$$f(h(y)).\left|\frac{dh(y)}{dy}\right|$$

where $h(y)$ is the inverse function of $g(x)$. [8]

b) State the analogue of the above result for a k-dimensional random variable. [4]

5.a) Define the Gamma distribution. Show that if X and Y are independent and have Gamma distributions $G(\lambda; \alpha_1)$ and $G(\lambda, \alpha_2)$, then $Z = \frac{X}{X+Y}$ has a Beta distribution, $U=X+Y$ has a Gamma distribution and Z and U are independent. [4+12]=[16]

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5.b) Deduce from 5(a) or prove otherwise that

$$\int_0^1 x^{\alpha_1 -1}(1-x)^{\alpha_2 -1}\,dx = \frac{\Gamma(\alpha_1)\,\Gamma(\alpha_2)}{\Gamma(\alpha_1+\alpha_2)}.$$ [4]

c) State the relations you know between the following pairs of distributions. [7X3]=[21]

(1) Beta distributions of the two kinds

(2) Normal and Cauchy

(3) Rectangular and Gamma

(4) Chi-square and F

(5) Student's T and F

(6) Beta and F

(7) Normal and chi-square

d) Derive the density of Student's t-distribution (taking the definition as the distribution of the ratio of two random variables) with n degrees of freedom. You may assume the density of the chi-square distribution. Show that student's t distribution with one degree of freedom is Cauchy's distribution. Is this (last) result same as the result of Qn. 5(c)(2)? Why? [8+3+5]=[16]

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PERIODICAL EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 17.3.69 | Maximum Marks: 100 | Time: 3 hours

Note: The whole paper carries 125 marks. Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1. Explain two stage sampling. Obtain an estimate of the population total for a two stage sampling procedure with SRSWR employed at the first stage and SRSWOR employed at the second stage. Also obtain an estimate of the variance of the estimate you propose. [20]

2. What is ratio method of estimation and when is it used? A ratio estimate is said to be a biased estimate in general. Is this statement true? Give a proof. Also obtain an expression for the mean square error of a ratio estimate. State clearly the assumptions you make. [20]

3. A sample of 15 villages was selected from a population of 170 villages with SRSWR for estimating the area under wheat in the region in 1964. Estimate the area under wheat by the method of ratio estimation and estimate its relative standard error. [35]

Village No.	Total area under cultivation	Area under wheat
1	564	515
2	238	209
3	92	85
4	247	221
5	134	133
6	131	104
7	129	103
8	190	175
9	363	335
10	235	219
11	73	62
12	62	59
13	71	60
14	137	100
15	196	141

4. A sample of size 5 is drawn from a normal population with mean 2 and unknown variance. Derive a test criterion for testing the hypothesis that the variance is 4 against the alternatives (1) the variance is 5; (2) The variance is 3; (3) The variance is either 3 or 5.

Compute the powers of the tests you derive in each case against the specified alternatives. [35]

5. Write short notes on:

1) Regression method of estimation. [8]

2) Simple and composite hypotheses. [3]

3) Level of significance of a test. [4]

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PERIODICAL EXAMINATIONS

General Science-5: Psychology Theory and Practical

Date: 24.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. The scores of 5 students on each of the 6 questions in a test are as follows: [30]

Students	Questions 1	2	3	4	5	6
1	6	1	5	0	8	4
2	5	2	6	2	6	3
3	7	2	8	4	3	1
4	3	4	3	2	6	1
5	7	3	9	7	7	5

Obtain (a) question-difficulty-levels

(b) indices of discrimination for each question

and (c) reliability of the test.

2. What is psychological measurement? Explain how you would assess the knowledge of an individual in a subject, say, psychology. [25]

3. Write short notes on [20]

(a) Intelligence

(b) Factor analysis.

4. Practical Record work. [25]

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PERIODICAL EXAMINATIONS

General Science-5: Engineering

Date: 31.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Write short notes on the following:

   a) Bending moment diagram

   b) Shear Force diagram

   c) Moment of Resistance

   d) Free body diagrams. [20]

2. A horizontal beam of 20' span, simply supported at its ends, carries a load which varies uniformly from 1/2 ton/ft at one end and 2 ton/ft at the other. Draw bending moment and shear force diagrams. Find maximum bending moment and bending moment at the mid span. [30]

3. a) Derive the formula $\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$

   b) Write a short note on shapes adopted for mild steel beams used in structure. [20]

4. A teakwood beam 8" X 4" wide in cross section, simply supported at its ends carries a concentrated load of 4000 lbs at 3.33 ft from one support. The effective span is 10 ft. Find the maximum flexural stresses in the beam (a) in the said case and also (b) if the concentrated load of 4000 lb is replaced by a uniformly distributed load at 400 lb /sft, throughout the beam. [30]

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ANNUAL EXAMINATIONS

Mathematics-3: Analysis

Date: 19.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50
Answer any three questions.

1.a) Explain with examples the two concepts 'first kind of discontinuity' and 'second kind of discontinuity'.

b) Show that a monotone function can not have second kind of discontinuities. $[8+8\frac{2}{3}]=[16\frac{2}{3}]$

2.a) State and prove the generalized mean value theorem for differentiable functions.

b) Is there any differentiable function on the real line whose derivative is given by the following:

$$f(x) = 0 \quad \text{if } x \text{ is integer}$$
$$= 1 \quad \text{otherwise.}$$

Give reasons. $[12+4\frac{2}{3}]=[16\frac{2}{3}]$

3.a) Explain the meaning of $\int_0^1 f d\alpha$ where f is a bounded function and α is a monotonically increasing function, both on $[0, 1]$.

b) Suppose α is an increasing function on $[0,1]$ which is continuous at the point $x = \frac{3}{4}$. Define

$$f(x) = 1 \quad \text{if } x = \frac{3}{4}$$
$$= 0 \quad \text{if not.}$$

Then show that f is integrable w.r.t. α and the value of the integral is zero. $[7+9\frac{2}{3}]=[16\frac{2}{3}]$

4.a) Explain the concepts 'orthonormal set' and 'complete orthonormal set' for a collection of functions defined on $[-\pi, +\pi]$.

b) Show that the functions

$$\frac{1}{\sqrt{2\pi}}, \frac{\cos x}{\sqrt{\pi}}, \frac{\sin x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \ldots$$

form an orthonormal set on $[-\pi, \pi]$. You have to evaluate all the integrals involved here. $[10+6\frac{2}{3}]=[16\frac{2}{3}]$

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Group B

Maximum Marks: 50

Answer any three questions.

1.a) If f is a continuous function on the real line such that $f(x+y) = f(x) + f(y)$ for all x and y show that there is a real number α such that

$$f(x) = \alpha x \qquad \text{for all } x.$$

b) If f is a continuous function on the real line such that $f(x+y) = f(x).f(y)$ for all x and y show that either there is a real number α such that

$$f(x) = e^{\alpha x} \qquad \text{for all } x.$$

or $f \equiv 0$. $[8+8\frac{2}{3}]=[16\frac{2}{3}]$

2. State and prove carefully the chain rule for differentiable functions. $[16\frac{2}{3}]$

3.a) If a sequence of integrable (Riemann) functions f_n on $[0,1]$ converge to a function f on $[0,1]$. Can you say that f is also Riemann integrable? Justify your answer.

b) Show that if the convergence in the above question is uniform then f is integrable. $[8+8\frac{2}{3}]=[16\frac{2}{3}]$

4.a) State and prove Abel's theorem for power series.

b) If Σa_n, Σb_n, are two series of real numbers with Cauchy product Σc_n and if we know that these three series donverge to A, B, C respectively then show that AB = C. $[10+6\frac{2}{3}]=[16\frac{2}{3}]$

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ANNUAL EXAMINATIONS

Economics-3

Date: 20.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Economic Development

Maximum Marks: 50

Answer any three questions.

1. Give your understanding of the theory of unbalanced growth and explain briefly the use of linkage coefficients in this context. [16]

2. Describe Arthur Lewis' theory of economic development with unlimited supply of labour. [16]

3. Make very brief comments on any three of the following:
   i) 'big push',
   ii) labour and capital as factors explaining long period growth,
   iii) dualism,
   iv) vicious circles, and
   v) stages of growth. [16]

4. Describe the classical view of growth following Baumol's synthesis. Indicate some significant difference of Marx from this. [16]

5. You have studied how structural features changed in Western countries with economic development. Do you think that the poorer countries will exhibit exactly similar changes with development?

   In your view, what are the main factors inhibiting Indian development? Could you suggest some measures that can promote rapid growth in this country?

   Would you be satisfied with a long period income maximization goal? [16]

Neatness. [2]

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Group B: Socialist Planning

Maximum Marks: 50

Answer any two questions.

1. Discuss the logic and the consequence of soviet price policy as it existed in the fifties. [25]

2. Give a critical review of the agricultural policy pursued during the period biginning with the November Revolution upto the conclusion of the first five year plan. [25]

3. Write short notes on the following:

   a) The scissors crisis of the 1920's.

   b) Allocation of investment in the first five year plan.

   c) Marxian concept of profit and the new role assigned to it now in the Soviet Union. [25]

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ANNUAL EXAMINATIONS

Economics-3

Date: 21.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Answer five questions taking at least two from each group.

Group A: Indian Economics

1. Write short notes on

a) Balance of Trade and Balance of payment.
b) Bill of Exchange.
c) Terms of Trade.
d) Rate of Exchange. [20]

2. The growing adverse balance of payment position of India has not only been due to the emphasis in her Five Year Plans on heavy industries. Comment. [20]

3. What do you mean by the term Public Finance? What role can the fiscal policy play in the achievement of rapid economic growth in a developing economy? [20]

4. Critically review the fiscal policies of the Indian Government, in the context of the triple objectives of the Five-Year Plans. [20]

5. The increasing conflict between the Union Government and the State Governments is essentially a reflection of the crisis in mobilising financial resources. Discuss. [20]

Group B: Economic Theory

1. Describe the two-sector planning model of Prof. Mahalanobis. In an economy visualised in this model, what the planner will do if he is interested in maximising national income at the end of a given planning horizon? [20]

2. Show that the warranted rate of growth in Harrod's model of economic growth is unstable. What assumptions do you require for this result? [20]

3. Show that the income path in Harrod's model corresponding to the warranted rate of growth is unique. Also, show that the long-run equilibrium in a Harrodian economy is, in general, impossible. [20]

4. Describe the way by which a typical neoclassicist (e.g., Solow) can solve the long-run economic problem of Harrod's model. In this connection also give the conditions of existence of steady state in a neoclassical growth model and show that this steady-state is stable. [20]

5. i) Prove that in a neoclassical growth model the steady-state capital-labour ratio and not the steady-state capital output ratio is dependent on the form of the production function.

ii) Verify, with the Cobb-Douglas production function, the neoclassical conclusions about the long-run growth of an economy. [20]

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ANNUAL EXAMINATIONS

Statistics-3: Probability

Date: 23.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) Define a measurable transformation. [4]

b) If $T: (\Omega, \mathcal{F}, \mu) \to (R, \mathcal{B})$ is measurable, then what is the measure induced by T on $(R, \mathcal{B})$? Prove that it is a measure. [4+5]=[9]

c) If g is a real valued measurable function defined on R, then

$$\int g\, d(\mu T^{-1}) = \int (gT)\, d\mu$$

provided one of them exists. Assuming this show that the expectation of a random variable can be defined in two ways. [6]

2. A p-dimensional random variable U is said to have a normal distribution if T'U is normally distributed for all vectors T. Show the following.

(a) The characteristic function of U is $\exp\{i\, T'\mu - \frac{1}{2} T'\Sigma T\}$ where μ, Σ are the mean and the dispersion matrix of U; the distribution of U is uniquely determined by μ and Σ. [6+4]=[10]

(b) any marginal distribution of U is normal. [4]

(c) the conditional distribution of $(U_1, \ldots, U_q)$ given $(U_{q+1}, \ldots, U_p)$ is normal (assuming the dispersion matrix of the second variable is non-singular). [7]

(d) Assuming that the distribution of U is same as that of $\mu + BG$ where B is a $p \times m$ matrix of rank m = rank Σ and G is a vector of independent N(0,1) variates, obtain the density of U when Σ is non-singular. [6]

3.a) State and prove Fisher-Cochran theorem on the distribution of quadratic forms of a sample $X_1, \ldots, X_n$ from N(0,1). [10]

b) Deduce that $\bar{X}^2$ and $\Sigma (X_i - \bar{X})^2$ are independent. What are their distributions? [8]

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Group B

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) Prove that $X'AX \frown \chi^2$ if and only if $A^2 = A$, where $(X_1, \ldots, X_n)' = X$ is a sample from $N(0,1)$. Then, show that the d.f. of χ^2 is $r(A)$. [10]

b) Deduce that if $X \frown N_p(\mu, \Sigma)$ and Σ is non singular then

$$(X - \mu)' \Sigma^{-1}(X - \mu) \frown \chi^2_p$$ [4]

2.a) If $(x - \delta, x + \delta)$ is a continuity interval of a distribution F with characteristic function φ, show that

$$F(x+\delta) - F(x-\delta) = \lim_{T \to \infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sin \delta t}{t} e^{-itx} \varphi(t)dt.$$ [10]

b) Deduce that if $\int_{-\infty}^{\infty} |\varphi(t)|dt < \infty$, then F has a continuous density. [6]

c) State the generalisation of the above inversion theorem for a 2-dimensional random variable (X_1, X_2). If $\varphi(t_1, t_2)$ is the characteristic function of (X_1, X_2) show that $\varphi(t_1, t_2) = \varphi_1(t_1) \cdot \varphi_2(t_2)$ for some functions φ_1, φ_2 implies that X_1, X_2 are independent (using the result you state above). [4+6]=[10]

3.a) State and prove Helly-Bray theorem. [8]

b) Assuming the inversion theorem (Qn. 2(a)) and Helly's lemma, prove that if $f_n(t)$ is the characteristic function corresponding to the distribution F_n and if $f_n(t)$ converges to a function $f(t)$ which is continuous at zero, then F_n converges weakly to a distribution F whose characteristic function is f (Prove any other result you use). [12]

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ANNUAL EXAMINATIONS

General Science-5: Engineering

[105]

Date: 24.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer all questions.

1. EITHER
Write short notes on the following:-

a) Slenderness-ratio
b) Moment of resistance
c) Centre of pressure
d) Venturi-meter. [20]

OR

State and prove Bernoulli's theorem. [20]

2. A vertical gate 10' X 20' high fixed in a vertical dam has its top 10 feet below the water level. Calculate the centre of pressure and total water-pressure. Derive the formula you use. [30]

Group B

Maximum Marks: 50

Answer all questions.

1. A horizontal beam 10' span, simply supported at its ends, carries a load, which varies uniformly from 1 ton per foot at one end to 3 ton per foot at the other end. Draw bending moment and shear force diagrams. [20]

2. EITHER

A 100 inches long hollow circular shaft with external diameter of 10 inches and internal diameter of 5 inches is subjected to torsion of 1,00,000 in lbs. If the modulus of rigidity is

$$5 \times 10^6 \text{ lb./in}^2,$$

calculate the maximum shear stress and angle of twist. Derive the formula you use. [30]

OR

(a) Write short notes on Professor Rankine's formula for struts.

(b) A hollow C.I. column with fixed ends, supports an axial load of 100 tons. If the column is 15' long and has an external diameter of 10 inches, find the thickness of metal required. Use the Rankine's formula, taking a constant of 1/6400 and a working stress of 5 tons per sq. inches. [30]

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ANNUAL EXAMINATIONS

Statistics-3: Statistics Theory

Date: 26.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Answer any five questions.

Group A

1.a) State and prove Neyman-Pearson lemma and show how it can be used to obtain a most powerful test for testing a simple hypothesis against a simple alternative. [10]

b) On the basis of a random sample of size n from a gamma distribution with frequency function

$$f(x, \theta, p_0) = \frac{\theta^{p_0}}{\Gamma(p_0)} \cdot e^{-\theta x} \cdot x^{p_0-1}$$

where $p_0 > 0$ is fixed, $\theta > 0$ and $0 < x < \infty$, obtain a UMP level α test for testing $\theta = \theta_0$ against the alternative $\theta > \theta_0$. Is this test also most powerful for alternatives $\theta < \theta_0$? [10]

2.a) On the basis of n independent tosses of a coin obtain an exact level α most powerful test for testing the hypothesis that the probability of a toss showing head is p_0 against the alternative that it is p_1 ($< p_0$). [10]

b) The number of deaths from road accidents follows a Poisson distribution. $X_1, \ldots, X_n$ are the number of deaths due to road accidents on n days chosen at random. How do you test the hypothesis that the accident rate per day say $\lambda = \lambda_0$ against the alternative that $\lambda \neq \lambda_0$ at level of significance α? [10]

3. Let $\underset{\sim}{Y}$ $(n \times 1)$ denote a vector valued random variable which is normally distributed with

$$E_{n \times 1}(\underset{\sim}{Y}) = \underset{\sim}{X}\,(n \times p)\,\underset{\sim}{\beta}\,(p \times 1) \text{ and}$$

$$V(\underset{\sim}{Y}) = \sigma^2 . I_n \qquad \text{where } \underset{\sim}{X}\,(n \times p)$$

is a matrix of known constants and $\underset{\sim}{\beta}(p \times 1)$ and σ^2 are unknown parameters.

(a) Explain the principle of least squares estimation and obtain the estimates of $\underset{\sim}{\beta}$ by that procedure. Show that they minimize the quadratic form

$$(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\beta})'(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\beta})$$ [10]

(b) Denoting a set of least squares estimates by $\hat{\underset{\sim}{\beta}}$, obtain the distribution of

$$R_0^2 = (\underset{\sim}{Y} - \underset{\sim}{X}\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - \underset{\sim}{X}\hat{\underset{\sim}{\beta}}).$$

Hence obtain an unbiased estimate of σ^2. [10]

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Group B

4.a) In the linear model set up of question (3), obtain a necessary and sufficient condition that a linear function of the parameters $\underset{\sim}{\beta}$, say $\underset{\sim}{P}'(1 \times p)\, \underset{\sim}{\beta}\,(p \times 1)$ to be estimable. [10]

b) Obtain a best linear unbiased estimate of an estimable function $\underset{\sim}{P}'(1 \times p) \cdot \underset{\sim}{\beta}\,(p \times 1)$. [10]

5. What are contingency tables? When you have a sufficiently large sample:

(a) How do you test for given marginal probabilities in a contingency table?

(b) How do you test whether the two attributes are independent with given marginal probabilities.?

(c) How do you test for independence of the two attributes?

(Note: For (a)-(c) the corresponding distributions need not be derived).

(d) In a 2×2 contingency table how do you test for the independence of the two attributes when the number of observations is not large. [4×5]=[20]

6.a) Write short notes on:

(a) Unbiased test

(b) Variance stabilizing transformation for a random variable following a binomial distribution.

(c) Test of equality of means from two normal distributions with same variance.

(d) Analysis of variance table for two way classification.

[4×5]=[20]

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ANNUAL EXAMINATIONS

Statistics-3: Statistics Practical

Date: 27.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions from questions 1 to 5. Marks allotted for each question are given in brackets [ ].

1. It is noticed that the consumer expenditure per person per month on food grains in rural India follows a normal distribution with unknown mean μ and known standard deviation 0.5 Rs. Based on a random sample of 16 individuals it is required to test that $\mu_0 = 8.00$ (rupees)

(a) Obtain the most powerful critical region for testing this hypothesis against the alternative $\mu > \mu_0$.

(b) Obtain the most powerful unbiased critical region for testing this hypothesis against the alternative $\mu \neq \mu_0$. By taking a reasonable number of alternatives draw the power curve of the test in each case. [20]

2. A drug was given to 20 subjects half an hour before bed time while 25 other subjects were kept as controls. The next morning subjects estimated the time taken by them to fall asleep. The following table gives the reported times of the two groups.

(a) Ascertain from the data whether the drug caused a quicker onset of sleep. (Assuming that the variability is the same in both groups of individuals).

Time in minutes:

Controls : 15, 25, 30, 15, 35, 40, 25, 30, 25, 35, 40, 25, 35, 20, 25, 40, 15, 15, 25, 30, 25, 10, 50, 30, 40.

Treated with drug : 25, 30, 40, 45, 15, 15, 20, 25, 30, 25, 20, 15, 10, 25, 15, 25, 35, 10, 10, 15. [12]

(b) Results based on observations on 27 randomly chosen individuals show that the average age of onset of a particular characteristic A is 48.89 years. The standard error is given to be 10.32 years. Another set of 36 randomly chosen individuals show that the average age of onset of the characteristic B is 50 years and the standard error is 8.56 years. Can you conclude that the variance in the age of onset of the characteristic is the same for both the characteristics? [8]

3.a) The following table summarizes the means and corrected sums of squares and products of the weights of green plant and dry jute fibre collected for 40 individual plants selected at random.

Mean weight of green plant $\bar{X} = 52.775$ gms.
Mean weight of dry fibre $\bar{Y} = 3.992$ gms.

$$S_{XX} = 33052.97$$
$$S_{YY} = 209.15$$
$$S_{XY} = 2338.70.$$

Test whether the correlation between the weight of the green plant and the weight of the dry fibre is significantly different from 0.85. [8]

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3.b) The following table gives the means and corrected sums of squares and products of systolic blood pressure (in mm of Hg.) (Y) and the age in years (X) of three groups of subjects.

Group	Sample size	Means $\bar{X}$	Means $\bar{Y}$	Corrected sums of squares and products S_{XX}	S_{XY}	S_{YY}
1	80	26.6	90.4	137.875	74.125	127.875
2	120	28.0	92.2	390.000	124.000	515.670
3	70	28.8	90.6	94.350	54.290	390.860

Test whether the correlation between the blood pressure and age is the same in the three groups. [12]

4. The joint segregation of the two factors, flower colour and pollen shape in Morning glory has been studied by Imar. He records the segregations as follows.

Pollen shape	Flower color: Purple	Flower color: Red	Total
Long	296	27	333
Round	19	85	104
Total	315	112	427

The marginal frequencies are expected to be in the ratio 3:1 and if the two characters, flower colour and pollen shape are independently inherited then the cell frequencies are expected to be in the ratio 9:3:3:1. With this knowledge, analyze the data carefully. [20]

5. Three groups of rats were given the same dose of hypnotic drug. The number of minutes each rat was unconscious was recorded below. Ascertain by the analysis of variance whether the average lengths of time the rats were unconscious are significantly different for the three groups.

Group I	13	16	19	14	16
Group II	20	17	22	24	19
Group III	14	18	13	12	20

[20]

6. Viva Voce [10]

7. Records [10]

----

[109]

INDIAN STATISTICAL INSTITUTE
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ANNUAL EXAMINATIONS

General Science-4: Biology

Date: 29.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets.

Group A

Maximum Marks: 40

Answer any two questions.

1. Explain the handicaps for the improvement of coconut by breeding methods. Mention the possibilities of effecting clonal propagation in coconut. [8+12]=[20]

2. Write an account on the foliar spirals in palms with special reference to cocos nucifera. Which palms are tapped for the sugary juice? [15+5]=[20]

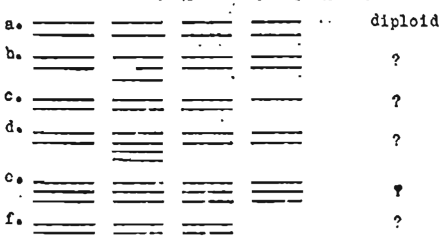
3. Mention names of eight tuber crop plants. Give a morphological account of the plant Solanum tuberosum. [8+12]=[20]

Group B

Maximum Marks: 60

Answer question 1 and any two from the rest.

1. Diagrams representing one normal diploid (a) and five individuals with various aneuploid or polyploid chromosome components (b to f) are given below. Give specific names for each condition.



Explain with suitable illustrations the terms deficiency, deletion, duplication, inversion and translocation. [5+15]=[20]

2. If you are a plant breeder, which breeding methods you would use for producing improved varieties in self-fertilizing plant species? Mention under what circumstances you would use the back cross method in plant breeding. [15+5]=[20]

3. What are inbreeding and heterosis? What is an inbreeding minimum and how it is achieved? Why does homozygosity increase by inbreeding? [5+10+5]=[20]

4. What is polyploidy? How are polyploids induced artificially? Discuss briefly the role of polyploidy in plant breeding. [5+5+10]=[20]

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B. Stat. Part III: 1968-69 [110]

ANNUAL EXAMINATIONS
General Science-3: Psychology Theory and Practical

Date: 31.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer all questions.

Group A

Maximum Marks: 50

1. Explain what is meant by 'phylogenetic differentiation of ability', 'ontogenic differentiation of ability', and 'ontogeny recapitulates phylogeny'. Discuss the important features in the differentiation of ability. [20]

2. Write short notes on the physical stimulus, physiological receptor, and psychological response for

a) vision

b) audition [20]

3. Practical Record Work [10]

Group B

Maximum Marks: 50

1. Specify the three factors which influence whether or not a neuron will respond to a stimulus, and indicate the type of response which takes place. Express statistically how a continuous nerve response occurs if the nerve is composed of neurons differing in terms of these three factors. Consider different levels of stimulus intensity. [20]

2. Discuss human speech in terms of articulation, audition, and localization of cerebral functions. [20]

3. Practical Record Work. [10]

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[122A]

PERIODICAL EXAMINATIONS

Statistics-4: Inference

Date: 14.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any three questions. Marks allotted for each question are given in brackets [ ].

1. For a monotone likelihood ratio family show that one sided most powerful tests exist for all sizes α. State your arguments carefully. (You may use the Neyman-Pearson Lemma.) [16]

2. Let X be a real valued random variable with density $f_0(x)$ or $f_1(x)$ where

$$f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \qquad -\infty < x < \infty$$

and

$$f_1(x) = \frac{1}{2}\left\{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-1)^2} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x+1)^2}\right\}.$$

Find the M.P. test of $H_0(f = f_0)$ vs. $H_1(f = f_1)$ of size $\alpha = .05$. [16]

3. Let X have the Cauchy density

$$f_\theta(x) = \frac{1}{\pi}\,\frac{1}{1+(x-\theta)^2} \qquad -\infty < x < \infty,\ -\infty < \theta < \infty.$$

Find the locally most powerful test of $H_0(\theta = 1)$ vs. $H_1(\theta > 1)$ and show its power $\to 0$ as $\theta \to \infty$. [16]

4. Identify the minimal sufficient statistic in the following cases:

a) $X_1, \ldots, X_n$ are i.i.d. with common density

$$f_\theta(r) = \theta_i \quad \text{if } r = i,\ i = 1, \ldots, k;$$

here $\theta = (\theta_1, \ldots, \theta_k)$ and $\Omega = \left\{\theta;\ \sum_1^k \theta_i = 1, \theta_i \geq 0\right\}$.

b) $X_1, \ldots, X_n$ are as in (a) but Ω contains only two points $(\frac{1}{k}, \ldots, \frac{1}{k})$ and $(\frac{1}{k} - \frac{1}{k^2}, \frac{1}{k} + \frac{1}{k^2}, \frac{1}{k}, \ldots, \frac{1}{k})$.

5. For neatness [2]

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PERIODICAL EXAMINATIONS

Statistics-6: Demography Theory and Practical

Date:14.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted to each question are given in brackets [ ].

1. What are the different columns in an abridged life table? How are they related? Deduce the relationship between ${}_nq_x$ and ${}_nm_x$. Stating clearly the assumption involved. [18]

2. You are given the following data taken from occupational mortality investigation.

Age group	Standard population: population at risk	Standard population: q_x	Occupation X: population	Occupation X: q_x	Occupation Y: population	Occupation Y: q_x
15-24	270,000	.001	4,000	.005	13,000	.001
25-34	310,000	.002	16,000	.002	20,000	.001
35-44	350,000	.003	28,000	.002	33,000	.002
45-54	320,000	.008	33,000	.007	29,000	.008
55-64	250,000	.022	28,000	.021	28,000	.025

The mortality experiences of occupations X and Y may be compared by comparing the standardised death rates by 1) direct method (2) by indirect method. Calculate the death rates by each of these methods and state with reasons to what extent you think a reliable comparison between the occupations is obtained. [17]

3. Briefly discuss any three of the following:

a) 'de facto' and 'de jure' population enumeration,

b) methods of identifying members of economically active population,

c) deficiencies in Indian vital registration statistics and suggestions for improving them,

d) errors in census data. [15]

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PERIODICAL EXAMINATIONS

Statistics-7: Econometrics Theory and Practical

Date: 21.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. How and when the question of multi-collinearity becomes important in connection with the estimation of a demand function. Suggest a method to get over this difficult situation. [10]

2. Give a statement of the Cob-Web model of demand and supply, of one commodity only. Examine the identifiability of the equations of the model. Derive the time path of the equilibrium price and give your comments. [15]

3. From the following data draw the concentration curve for the total per capita consumer expenditure, and compute the Lorenz ratio from the same data.

Table (1): Per capita monthly total consumer expenditure (Rs.) by classes of expenditure level, with percent of persons in each class, all-India, Urban, 1953-54.

per capita monthly expenditure class (Rs.) (1)	percentage of persons (2)	per capita monthly total consumer expenditure in (Rs.) (3)
0 - 8	7.52	6.24
8 - 11	12.09	9.36
11 - 13	8.56	11.92
13 - 15	9.29	14.01
15 - 18	11.36	16.27
18 - 21	10.44	18.98
21 - 24	7.79	22.59
24 - 28	8.32	25.64
28 - 34	5.41	30.67
34 - 43	7.85	38.15
43 - 55	4.86	48.70
55 and above	6.51	80.33

[25]

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[123B]

PERIODICAL EXAMINATIONS

Statistics-7: Planning Techniques

Date: 21.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any two questions. Marks allotted for each question are given in brackets [ ].

1.a) Explain the construction of inter-industry tables and show how they can be used to demonstrate the equivalence of the three concepts of national income.

b) State the basic assumptions of input-output analysis. [18+7]=[25]

2. Discuss the necessary and sufficient conditions for a bill of goods being producible under the Leontief static system. [25]

3. Solve the following linear programming problems graphically and shade the region representing the feasible solutions:

(a)
$$2x_1 + 3x_2 \leq 6$$
$$x_1 + 4x_2 \leq 4$$
$$x_1, x_2 \geq 0$$
max. $Z = x_1 + \frac{3}{2} x_2$

(b)
$$5x_1 + 10x_2 \leq 50$$
$$x_1 + x_2 \geq 1$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0$$
Min. $Z = 2x_1 + x_2$ [25]

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[124A]

PERIODICAL EXAMINATIONS

Statistics-4: Probability

Date: 28.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hour

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) What is a stochastic matrix ? [4]

b) Prove that if A is a stochastic matrix, A^2 is also one. [8]

c) If A^2 is a stochastic matrix, does it follow that A is a stochastic matrix? Give a proof or a counter-example, whichever may be relevant. [8]

d) A is a matrix in which

i) every entry is ≥ 0, and

ii) the sum of the elements in each row is ≤ 1. A^2 is a stochastic matrix.

Prove that A is also a stochastic matrix. [8]

2.a) P is a stochastic matrix such that P^{10} has only positive entries in the last column. Prove that

$$\lim_n P^n = Q$$

exists and has identical rows. State and prove the needed lemma on weighted averages. [10+4]=[14]

b) Give a stochastic matrix P such that every column in every power of P contains at least one zero and such that $[P^n]$ converges to a limit matrix as (as $n \longrightarrow \infty$). [8]

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PERIODICAL EXAMINATIONS

Statistics-4: Sample Surveys Theory and Practical

Date: 28.10.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Prove that the probability of selection of a specified unit at a specified draw is equal to 1/N for s.r.s. without replacement where N is the population size.

b) Prove that the sample mean is the best linear unbiased estimator of the population mean in the subclass $\sum_1^n \alpha_r y'_r$ where α_r ($r = 1,2,\ldots, n$) is the coefficient to be attached to the variate value of the unit appearing at the r-th draw and y'_r is the value of the unit drawn at the r-th draw, for srs without replacement.

c) Assuming that the finite population is a random sample from an infinite, normal super-population, derive exact confidence limits for the population mean. [5+7+5]=[17]

2.a) Derive an exact upper bound for the bias of the classical ratio estimator.

b) Derive an approximately unbiased ratio estimator whose asymptotic bias does not contain terms of order n^{-1} and N^{-1} for srs without replacement.

c) Suppose there are M domains in the population. Derive the estimator of $\sum_{j=1}^{M} \lambda_j \cdot {}_j\bar{Y}$ and derive its variance for s.r.s. without replacement where $\sum_1^M \lambda_j = 0$ and ${}_j\bar{Y}$ is the j^{th} domain population mean. [5+8+5]=[18]

3. From a population of size 120 a sample of 10 is drawn with s.r.s. without replacement. The values of y and x of two characteristics measured on each of them are as follows:

Unit No.	1	2	3	4	5	6	7	8	9	10
y	2	4	2	4	3	6	7	5	4	1
x	65	125	54	120	95	178	230	150	118	40

The population is divided into 4 domains. The sample units in these 4 domains are as given below.

Domain:	1	2	3	4
Unit No.	(1,3,10)	(2,5)	(4,6,8,9)	(7)

Given that the population mean of x for domain 3 is 153 estimate by ratio method the population mean of y for that domain. Estimate its efficiency over the usual unbiased estimate. Give rationale for using ratio method here. [10]

4. Assignments. [5]

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MID-YEAR EXAMINATIONS

Statistics-4: Probability

Date: 18.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) There are 12 states in a Markov chain. In the stochastic matrix, there are positive entries in the following cells and zero in every other cell. Determine the transient states, ergodic classes and cyclically moving sub-classes in each ergodic class.

(1,4), (1,5), (2,1), (3,2), (3,6), (4,6), (5,3), (6,1), (6,7), (7,4), , , (8,1), (8,9), (9,10), (9,11), (10,11), (11,12), (12,9). [12]

b) Same as above but (1,8) and (9,8) also contain positive entries (in addition to the 17 cells of the previous problem). You are advised to draw a completely new network for this second problem, otherwise there may be confusion. [12]

2.a)

$$P = \begin{matrix} .6 & .3 & .1 \\ .2 & 0 & .8 \\ 0 & .6 & .4 \end{matrix}$$

obtain $\lim_n P^n$. Describe your method; proofs (for the theorems used) need not be given. [12]

b) Determine any nonsingular stochastic matrix P such that

$$\lim_n P^n \text{ is } Q = \begin{matrix} .5 & .2 & .3 \\ .5 & .2 & .3 \\ .5 & .2 & .3 \end{matrix}$$ [12]

c) Determine a nonsingular stochastic matrix P such that

i) $\lim_n P^n$ is the Q above, and

ii) the entry in the (1,1)-cell is .3. [8]

3.a) Prove that every real eigen value of a stochastic matrix lies on the interval [-1, +1]. [10]

b) Show that no stochastic matrix of order 2 X 2 can have the eigen values -1, -1. [12]

c) Give an example of a stochastic matrix whose eigen values are

i) +1, +1,

ii) +1, -1. [5+5]=[10]

d) Form the maximum weighted average of 1, 6 and 4 satisfying the following conditions:

i) As usual, the weights w_1, w_2, w_3 are all nonnegative and $w_1 + w_2 + w_3 = 1$;

and (ii) $w_1 \geq w_2$, $w_1 \geq w_3$.

You must prove conclusively that your average is the greatest possible. [10]

4.a) What is a transient state? [2]

b) Prove that if j is a consequent of i and k is a consequent of j, then k is a consequent of ? [7]

c) Prove that every consequent of a nontransient state is nontransient. [7]

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MID-YEAR EXAMINATIONS
Statistics-4: Inference

Date: 19.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer questions 7 and 8 and any three other questions. Marks allotted for each question are given in brackets [ ].

1. Derive the UMP unbiased test of size α for independence in a 2×2 contingency table. [16]

2. Let $X_{i1},\ldots,X_{i,n_i}$, $i = 1,2$ be independent normal with mean μ_i and variance σ^2. Find the MP similar test of size α of $H_0(\mu_1 = \mu_2)$ against $H_1(\mu_1 < \mu_2)$. [16]

3. Let $X_1,\ldots,X_n$ be independent, $N(\mu,\sigma^2)$. Show that a MP test of size α exists for $H_0(\sigma^2 = \sigma_0^2)$ vs. $H_1(\sigma^2 = \sigma_1^2)$ if $\sigma_1^2 > \sigma_0^2$ but no such test exists if $\sigma_1^2 < \sigma_0^2$. Does a MP similar test of size α exist for the latter case? [16]

4. Define Fraser-sufficiency. Show how it can be used to get a MP similar test of size α in the following problem. $X_1,\ldots,X_n$ are i.i.d with a continuous distribution function $F(x)$. Test H_0(Median = 0) vs. H_1(median > 0). [16]

5. Let $X_1,\ldots,X_n$ be i.i.d with common density $f_\theta(x) = k(\theta)e^{\theta x}\psi(x)$. Describe the Bayes solutions in a two action problem if
   i) the loss-difference has one sign-change
   ii) the loss-difference has two sign-changes
   (You have to prove all the results you need). [8+8]=[16]

6. a) Discuss briefly the use of bounded completeness in testing.
   b) Show that a sufficient statistic with boundedly complete family of distributions is minimal sufficient. (You may assume the set-up of discrete distributions.) [6+10]=[16]

7. In the following X is a random variable, real or vector valued, with density $f_\theta(x)$ $a \le \theta \le b$. The statements below are either true or false. Prove the true statements and provide counter examples in other cases.
   a) If $T(X)$ is sufficient for the family of distributions of X and $\phi(T(X))$ is sufficient for the family of distributions of $T(X)$ then $\phi(T(X))$ is sufficient for the family of distributions of X. [4]
   b) If T_1 is sufficient for θ and T_2 is independent of T_1 under each θ, then the distribution of T_2 is free of θ. [4]
   c) The likelihood ratio test depends on the sufficient statistic only. [4]
   d) The likelihood ratio test is unbiased. [4]

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e) If $f_\theta(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta)^2}$ and

$$\phi_{c,d}(x) = 1 \text{ if } x > d \text{ or } < c$$
$$= 0 \text{ otherwise}$$

is an unbiased a test of $H_0(\theta = 0)$ vs. $H_1(\theta \neq 0)$ then $c = -d$. [4]

f) If for some α, and θ_0 there exists a M.P. test of $H_0(\theta = \theta_0)$ vs. $H_1(\theta > \theta_0)$ then the family $\{f_\theta\}$ is an MLR family in x. [5]

g) A necessary and sufficient condition for densities $f_\theta(x)$ to have monotone likelihood ratio, if the mixed second derivative

$$\frac{\partial^2 \log f_\theta(x)}{\partial\theta\partial x}$$

exists, is that this derivative be ≥ 0 for all θ and x. [8]

8. Find the minimal sufficient statistic in each of the following cases and examine whether it is complete

a) $X_1, \ldots, X_n$ are i.i.d and $P_\theta\{X_i = m\} = \frac{1}{\theta}$,

$$m = 1, 2, \ldots, \theta$$
$$\theta = 1, 2, 3, \ldots$$

b) X is a random variable and the parameter space is the family of all symmetric distributions

$$P_\theta\{X = m\} = P_\theta\{X = -m\}, \quad m = 0, 1, 2, \ldots$$

c) $X_1, \ldots, X_n$ are i.i.d,

$$f_\theta(x_1) = \theta \quad \text{if } x_1 = 1$$
$$= 1 - \theta \text{ if } x_1 = 0$$

and Ω consists of three points $\theta = \frac{1}{2}, \frac{1}{3}$ or $\frac{2}{3}$.

[7+6+6]=[19].

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MID-YEAR EXAMINATIONS

Statistics-5: Statistical Methods Theory and Practical

Date: 20.12.68. Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

Group A

Theory

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1.a) Define 'tolerance limits'. [3]

b) If $(x_1, x_2, \ldots, x_n)$ represent a sample of size n from $N(\mu, \sigma^2)$, $\bar{x} = \frac{1}{n} \Sigma x_i$, $s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$ and

$$P(\bar{x}, s) = \int_{\bar{x}-ks}^{\bar{x}+ks} \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dx$$

show how a value of k could be determined such that the expected coverage by the tolerance limits $(\bar{x}-ks, \bar{x}+ks)$, $EP(\bar{x}, s)$ is equal to a preassigned value α $(0 < \alpha < 1)$. [12]

2. Consider random variables $(Y_1, Y_2, \ldots, Y_n) = \underline{Y}'$ and a linear model under which $E(\underline{Y}) = X\underline{\beta}$, dispersion matrix $D(\underline{Y}) = \sigma^2 I$. The matrix X is known and $\underline{\beta}$ is a vector of unknown parameters. Assume further that the variables are normally distributed. Show that

a) If $R_0^2 = \min_{\beta} (\underline{Y} - X\underline{\beta})'(\underline{Y} - X\underline{\beta})$ show that R_0^2/σ^2 has a chi-square distribution with $n-r$ d.f. where r = rank of X. [8]

b) Consider a hypothesis: $H\underline{\beta} = \underline{h}$ and let

$$R_H^2 = \min_{\underline{\beta}: H\underline{\beta} = \underline{h}} (\underline{Y} - X\underline{\beta})'(\underline{Y} - X\underline{\beta})$$

Show that if each component of $H\underline{\beta}$ is individually estimable

$$R_H^2 - R_0^2 = (H\hat{\underline{\beta}} - \underline{h})'(H(X'X)^- H')^- (H\hat{\underline{\beta}} - \underline{h}) .$$ [10]

What can you say about the distribution of $(R_H^2 - R_0^2)/\sigma^2$ if no component of $H\underline{\beta}$ is individually estimable? [2]

3. Obtain the analysis of covariance for a two-way classified data with equal number of observations per cell taking into account a single concomitant variable which has been observed along with the primary variable of interest. [15]

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Group B

Practical

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours.

Answer all questions.

1. In setting confidence intervals for the median of a continuous distribution using the symmetrically spaced order-statistics $x_{(r)}$ and $x_{(n-r+1)}$ on the basis of a sample of size $n = 28$, find the largest values of r yielding a confidence co-efficient not less than

   a) 0.95
   b) 0.99 [10]

2. The measurements of Nasal Length of individuals, belonging to 4 different ethnic groups, as obtained in an Anthropometric study are given below.

   Apply a non-parametric test to examine whether Nasal Length is useful in distinguishing among these groups.

Group	Measurements in mm.	Sample size
KURUMBA	46,51,48,53,44,45,49,42	8
HAKKIPIKKI	40,46,48,45,47,48	6
MUSLIM	48,44,55,49,54,50,56,47,59	9
SOLIGA	54,50,58,44,49,40,47	7

[20]

3. Thirty persons in the income group Rs.1000-Rs.1500 were asked to supply returns of their monthly incomes for purposes of taxation. But only 20 returns were received till the specified last date, and it has to be decided whether these can be accepted as representative of the 30. There are prior reasons to believe that those with larger incomes have natural reluctance to supply the figures and may delay more than the others.

   The following are the figures in the 20 returns received, in that order:-

   1220,1290,1180,1270,1400,1090,1190,1250,1170,1300,1310,
   1280,1350,1320,1380,1420,1390,1470,1360,1460.

   Examine whether there are sufficient evidences for the above claim. [10]

4. Practical records. [10]

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MID-YEAR EXAMINATIONS

Statistics-7: Planning Techniques

Date: 21.12.68 Maximum Marks: 100 Time: 2 hours

Note: Attempt any two questions. Marks allotted for each question are given in brackets [ ].

1.a) Explain how a linear programming problem involving linear inequality constraints can be converted into one involving constraints in the form of a set of simultaneous linear equations so that there will be a one-to-one correspondence between the feasible solutions to the former and those of the latter. How can you ensure that the optimal values of the objective functions, if there are any, will also be the same in both cases?

b) Explain the procedure for the reduction of any feasible solution to a linear programming problem to a basic feasible solution.

c) Consider the set of equations:

$$5\xi_1 - 4\xi_2 + 3\xi_3 + \xi_4 = 3,$$
$$2\xi_1 + \xi_2 + 5\xi_3 - 3\xi_4 = 0,$$
$$\xi_1 + 6\xi_2 - 4\xi_3 + 2\xi_4 = 15$$

A feasible solution is $\xi_1 = 1, \xi_2 = 2, \xi_3 = 1, \xi_4 = 4$. Reduce the solution to a basic feasible solution.

[20+15+15]=[50]

2. Given a basic feasible solution $X_B = B^{-1}b$ to the set of constraints $AX = b$ for a linear programming problem, with the value of the objective function for this solution being $Z = C_B X_B$. If for any column a_j in A but not in B, the condition $Z_j - \gamma_j < 0$ holds, and if at least one $\xi_{ij} > 0$, show that it is possible to obtain a new basic feasible solution by replacing one of the columns of B by a_j, and the new value of the objective function $\bar{Z}$ satisfies $\bar{Z} \geq Z$. If, however, $Z_j - \gamma_j \geq 0$ for every column a_j in A, show that the corresponding value of the objective function will be the maximum that it can attain. [50]

3.a) Explain how you would find an initial basic feasible solution to a linear programming problem.

b) Solve the following linear programming problem by the simplex method:

$$\xi_1 + 3\xi_2 + \xi_4 \leq 4,$$
$$2\xi_1 + \xi_2 \leq 3,$$
$$\xi_2 + 4\xi_3 + \xi_4 \leq 3,$$
$$\xi_1, \xi_2, \xi_3, \xi_4 \geq 0$$
$$\max Z = 2\xi_1 + 4\xi_2 + \xi_3 + \xi_4.$$

[25+25]=[50]

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MID-YEAR EXAMINATIONS

Statistics-7: Econometrics Theory and Practical

Date: 23.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Discuss, how you would formulate the aggregate demand function for a particular commodity starting from the theory of consumer behaviour. Consider both linear and non-linear formulations for this purpose. [15]

2.a) If $X_1, X_2, \ldots, X_n$ are independently and identically distributed as $\Lambda(\mu, \sigma^2)$ then, give the distribution function of $\prod_{i=1}^{n} X_i$. [6]

b) Derive the equation of the Lorenz curve for a random variable distributed as $\Lambda(\mu, \sigma^2)$ and examine the properties of this Lorenz curve. [14]

3.a) What are family budget data? What are the major uses of these data? Briefly describe the method of collection and the main source of such data in India. [15]

b) Examine the role of household size in the formulation of the Engel curve. [10]

4. The following table gives the distribution of Income as obtained from the Indian income Tax Returns (1955). Fit a Pareto distribution to the appropriate number of income classes, by estimating the parameter from the Lorenz-ratio of the distribution.

Table: Statistics of Individual Salaries assessed, 1955.

range of income in rupees (annual)	number of incomes assessed	total income assessed (Rs.)
(1)	(2)	(3)
below - 4200	16361	42266049
4201 - 5000	43498	204483698
5001 - 8400	60032	383320767
8401 - 10000	19835	179451981
10001 - 15000	17583	217018779
15001 - 25000	8550	161809868
25001 - 40000	3779	117902100
40001 - 55000	1178	54294343
55001 - 70000	312	19250598
70001 - 85000	127	9757201
85001 -100000	60	5423397
100001 -150000	89	10623869
150001 -200000	34	6323521
200001 and above	12	2929444

[40]

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MID-YEAR EXAMINATIONS

Statistics-6: Sample Surveys Theory

Date: 25.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Give a general unbiased linear estimator of the population total Y for multistage designs and derive its variance in terms of the primary variances V_1 based on second and higher stage units.

b) Derive a general rule for unbiased variance estimation in multi-stage designs using (a).

c) Consider a stratified two-stage design where both primaries and secondaries within each stratum, are drawn by srs without replacement (srswor). Spell out the formulae for variance of the estimator and unbiased estimator of variance using either (b) otherwise. [5+8+7]=[20]

2.a) Suppose there are two strata. From stratum 1 a sample of n_1 units is selected from the N_1 units in that stratum by srswor. In stratum 2 there are N_2 primaries with M_2 secondaries in each primary. A sample of n_2 primaries and m_2 secondaries from each selected primary is selected by srswor. Suppose the cost function is $C = n_1c_1 + n_2c_2 + n_2m_2c_3$ where c_1 = cost per unit in stratum 1, c_2 = cost per primary in stratum 2 and c_3 = cost per secondary in stratum 2. Derive an unbiased estimator of the population total Y and then derive the optimum value of m_2 which minimises the variance of the estimator for fixed C.

b) Consider a four stage design in which $n > 1$ primaries are drawn with srs with replacement (srswr). Each time a primary is selected, a sample of secondaries is drawn with unequal probabilities without replacement. Within each selected secondary, a sample of third stage units is selected by srswor and finally from each selected third stage unit a systematic sample is selected circular systematically. Suppose we have unbiased estimators of the primary totals selected in the sample. Derive an unbiased estimator of Y and its unbiased variance estimator using the unbiased estimators of sampled primary totals. [13+7]=[20]

3.a) Prove that for Lahiri's method the probability of selection of i-th unit is proportional to its size X_i.

b) Suppose that in a sample of n units the first unit is selected with p.p.s. of X_i's and the remaining (n-1) units with srswor from the remaining (N-1) units in the population. Derive the probability of inclusion of i-th unit in the sample in terms of X_i and prove that the probability of selection of the sample, P(s), is proportional to ΣX_i of the units in the sample.

c) Suppose there are N clusters in the population and the size of i-th cluster is M_i. Suppose n clusters are selected with probabilities proportional to M_i and with replacement. Derive an unbiased estimator of Y, its variance and unbiased variance estimator. [5+7+8]=[20]

4.a) Derive the optimum allocation for stratified srswor using the cost function $C = c_0 + \Sigma c_i n_i$.

b) Suppose there are M domains in the population and the above stratified srswor sample is drawn. Derive an unbiased estimator of i-th domain total and its variance explicitly.

c) Suppose a srswor sample of n units is drawn and the post-stratified estimator

$$\hat{\bar{Y}} = \Sigma N_i \bar{y}_i / N$$

is used, where $\bar{y}_i$ = mean of the $n_i > 0$ units falling in stratum i and $\bar{y}_i = 0$ if $n_i = 0$. Derive the exact bias of $\hat{\bar{Y}}$.

[7+6+7]=[20]

5.a) Describe circular systematic sampling of n units and prove that the sample mean is unbiased for this scheme.

b) Assuming $N = nk$, n units are selected systematically after a random start from 1 to k. Prove that

$$V(\bar{y}) = \frac{\sigma^2}{n}[1 + (n-1)\rho] \quad \text{where } \sigma^2 =$$

population variance and ρ = intraclass correlation between pairs of sample units.

c) From a population of N units, a srswor sample of n' units is selected and their x - values measured. Then a srswor of sample n units from these n' units is selected and their y-values measured. Prove that

$$\hat{\bar{Y}} = \bar{y} + k(\bar{x}' - \bar{x})$$

is an unbiased estimator of $\bar{Y}$, where $\bar{y}$ and $\bar{x}$ are sample means of n units, $\bar{x}'$ is sample mean of n' units and k is a constant. Derive the variance of $\hat{\bar{Y}}$.

[5+6+9]=[20]

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MID-YEAR EXAMINATIONS

Statistics-6: Sample Surveys Practical

Date: 26.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. A survey is to be conducted for estimating the total number of literates in a town having three communities, some particulars of which are given in table 1 based on the results of a pilot study.

Table 1. A rough idea of the total number of persons and proportions of literates in 3 communities.

Community	Total number of persons	Percentage of literates
1	60,000	40
2	10,000	80
3	30,000	60

i) Treating the communities as strata and assuming srswr in each stratum, allocate a total sample size of 2000 persons to the strata in an optimum manner for estimating the overall proportion of literates in the town, using the data in Table 1.

ii) Estimate the efficiency of stratification as compared to unstratified sampling. [25]

2. For studying the living conditions of the working class population residing in an industrial area, a stratified two-stage sampling design is proposed, in which from each stratum a sample of factories is to be drawn systematically with pps*, size being the number of workers in an earlier period (x), and a sample of workers is to be selected from each sample factory linear systematically with a random start using the current pay-roll. The relevant data are given in Table 2.

i) Determine the constant weight to be used in a self-weighting design for ensuring a total sample size of about 1000 workers.

ii) Specify the sampling interval to be used in each sample factory for achieving a self weighting design, using the constant weight determined in (i).

iii) Also find the approximate number of workers expected to be selected from each sample factory, therby determining approximately the total sample size.

Table 2. Number of workers in sample factories.

Stratum	Sample factory	Number of workers past	Number of workers current	Stratum	Sample factory	Number of workers past	Number of workers current
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1	1	99	163	3	1	2697	2839
	2	523	465		2	4667	6255
	3	110	64		3	1423	1158
	4	741	829		4	1064	1150
2	1	4200	3504	4	1	90	91
	2	3187	2527		2	618	416
	3	2215	2186		3	150	131
	4	5322	5285		4	266	282

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(number of workers in an earlier period-stratum 1: 5896; stratum 2: 43098, stratum 3: 31625; stratum 4: 10774). [30]

(* In pps systematic sampling, the cumulative totals T_i, $i = 1,2,\ldots, N$, are determined and the units corresponding to the numbers $\{r + jk\}$, $j = 0,1,2,\ldots(n-1)$ are selected, where $K = T/n = X/n$ and r is a random number between 1 and k. The unit U_i is included in the sample, if $T_{i-1} < r + jk \leq T_i$ for some value of j $(= 0,1,2,\ldots, (n-1))$).

3. In planning a sample survey for estimating the proportion P of area under jute in a region, a pilot study was undertaken, in which independent samples of clusters of different sizes (x) were taken up for estimating the values of σ_b^2 with a view to studying the relationship between x and σ_b^2. The results obtained in this pilot survey are gi given in Table 3.

Table 3. Estimates of $\sigma_b^2/P(1-P)$ for different cluster sizes.

cluster size (acres)	$\sigma_b^2/P(1-P)$	cluster size (acres)	$\sigma_b^2/P(1-P)$
(1)	(2)	(1)	(2)
1.00	0.1120	12.25	0.0454
2.25	0.0813	16.00	0.0419
4.00	0.0659	25.00	0.0398
6.25	0.0577	36.00	0.0342
9.00	0.0505		

i) Examine graphically whether column (2) in Table 3 conforms to the relation

$$\sigma_b^2 = a/x^g ,$$

where a and g are constants to be determined.

ii) Assuming the cost function $C = 1000 + 2.1n + 0.7nx$, determine the optimum values of x and n for estimating P on the basis of a simple random sample of n clusters of size x when the cost is fixed at Rs.10,000. [25]

4. The size of the population in 1920 (x_i) and 1930 (y_i) of 49 united states cities selected with srs without replacement from the population of 196 cities is given in Table 4 Calculate Mickey's unbiased ratio estimator $\hat{t}$ for the choice $k = 4$ and some choice of

$$\{\alpha_i\}$$

and estimate its relative standard error. Given $\bar{X} = 116.933673$. [25]

Table 4: Size of 49 Large United States cities (in 1000's) in 1920 (x_i) and 1930 (y_i).

x_i	y_i	x_i	y_i	x_i	y_i
76	80	2	50	242	291
138	143	507	634	87	105
67	67	179	260	30	110
29	50	121	113	71	79
381	464	50	64	256	288
23	48	44	58	43	61
37	63	77	89	25	57
120	115	64	63	94	85
61	69	64	77	43	50
387	459	56	142	298	317
93	104	40	60	36	46
172	183	40	64	161	232
78	106	38	52	74	93
66	86	136	139	45	53
60	57	116	130	36	54
46	65	46	53	40	58
				48	75

5. Neatness and clarity. [5]

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MID-YEAR EXAMINATIONS

Statistics-8: Demography (Theory and Practical)

Date: 28.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Discuss briefly the method adopted by the census Actuary for the construction of Indian Life Tables from Census returns (1941-50) with special reference to the age groups 0-5 and 60+. [25]

2. Explain clearly the different stages which led to the formulation of the law of population growth

$$P_t = \frac{L}{1 + e^{(\beta - t)/\alpha}}$$

where P_t, L, β, α and t have their usual significance:

Discuss the law of growth of population with reference to India from the following data.

Year	1891	1901	1911	1921	1931	1941	1951	1961
Population in millions	236	235	249	248	276	313	357	436

[20]

3. The proportion of ever-married women observed in successive 5 year age groups are shown in the table below. The average number of female children born on completion of reproductive periods to women (including those who were widowed or divorced before the end of their reproductive periods) married at various ages are also shown in the same table. Calculate the Gross Reproduction rate for the population. Explain clearly the assumptions you make.

Age group in years	Percentage of ever married women	Average no. of female children born to ever married women (completed fertility)
15 - 19	4	1.868
20 - 24	43	1.297
25 - 29	73	0.933
30 - 34	80	0.633
35 - 39	81	0.300
40 - 44	82	0.069
45 - 49	83	0.002

[25]

4. Briefly discuss any three of the following:

a) Hospital records as a source of morbidity data:

b) Incidence and prevalence rates.

c) Uses of vital statistics.

d) Underlying cause of death and the procedure for its selection.

[10+10+10]=[30]

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MID-YEAR EXAMINATIONS

Statistics-8: Educational Statistics Theory and Practical

Date: 30.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer question 8 and any five from the rest. Marks allotted for each question are given in brackets [ ].

1. Briefly describe the computational procedure and properties of any two of the following methods of standardising the raw scores:

a) Percentile scores (b) Stanine grades (c) z-scores. [8+8]=[16]

2.a) What do you understand by 'biserial correlation coefficient'? Derive the formula with the help of which you can estimate 'normal' the correlation coefficient using the biserial correlation coefficient.

b) Test X is reported as having a mean of 68.1, a standard deviation 24.8, a reliability of 0.97 and a validity of 0.75 with the criterion. The mean value of the criterion scores is 117.8 and the corresponding standard deviation is 20.1. If we screen a group using scores on test X and obtain a selected subgroup with standard deviation 15.0 on test X, what will the validity be for this test on this selected group? [10+6]=[16]

3. Describe what you understand by (a) factor loadings (b) factor scores (c) communality (d) uniqueness (e) tetrad difference (f) multiple-factor theory. [3+3+2+2+3+3]=[16]

4.a) Discuss the effects of the following on the variance of the total score.

i) item difficulty (ii) item variance

iii) item validity (iv) item inter-correlation.

b) Determine the reliability of the difference score obtained from the following pair of tests X and Y where

reliability of test X = .95
reliability of test Y = .85
correlation between test X and Y = .40

Assume that the variances of tests X and Y are both equal to 1. [10+6]=[16]

5.a) Show, after stating the underlying assumptions, that the reliability of a test is

$$r_{xx'} = \frac{n}{n-1}\left(1 - \frac{\sum_{i=1}^{n} p_i q_i}{s_x^2}\right) \text{ where}$$

n = the number of items in the test.
p_i = the difficulty value of the i-th item; $q_i = 1-p_i$

s_x^2 = the variance of the total score.

b) Why should the odd-even method of estimating test reliability be used for 'power tests' only? [10+6]=[16]

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6. a) What do you understand by 'item validity'? What are the item validity indices which involve the slope of the regression of item on test?

b) From the equations showing the relationships of test length to validity and to reliability, determine the relationship between test reliability and validity as the length of the test is increased while the criterion remains unchanged i.e. write

$$f(r_{xx'}, r_{xy}, R_{kk'}, R_{ky}) = 0 \quad \text{where}$$

$r_{xx'}$ = the reliability of the original test.

r_{xy} = the validity of the original test.

$R_{kk'}$ = the reliability of the test when the test length is increased k times.

R_{ky} = the validity of the test when the test length is increased k times. [8+8]=[16]

7. a) What are parallel tests? What are the hypotheses H_{mvc}, H_{vc} and H_m?

b) Prove that if a test of n items is a sub-test of a test with m items (n < m) the correlation r_{nm} is

$$r_{nm} = \sqrt{\frac{\frac{1-r}{m} + r}{\frac{1-r}{n} + r}}$$ where r is the reliability of a unit test. [6+10]=[16]

8. Write short notes on any four of the following:

a) Group heterogeneity and Reliability of a test

b) Correction for guessing

c) Coefficient of discrimination of a test

d) Ø-coefficient

e) Rank correlation. [5+5+5+5]=[20]

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PERIODICAL EXAMINATIONS

Statistics-4

Date: 24.2.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A

Probability

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions

1.a) S is the smallest set of positive integers satisfying the following conditions:

i) S is closed under addition,

and ii) S contains 20, 24, 28.

Find the smallest positive integer k such that 4k, 4(k+1), 4(k+2), ... all belong to S. [8]

b) α and β are positive integers such that their G.C.F. is 1. Is the following statement true?

'There is a positive integer m and a negative integer n $\ni (m\alpha + n\beta) = 1$'. If your answer is YES, prove it. If your answer is NO, give a counterexample. [9]

c) α, β are positive integers with G.C.F.1. Prove that there are integers m, n such that $m\alpha + n\beta = 1$. [8]

2. In a finite Markove chain, each state is a consequent of every state. S is the set of positive integers n such that $p_{11}^{(n)} > 0$. d = G.C.F.S.

a) Prove that if $p_{ij}^{(\lambda)} > 0$, $p_{ij}^{(n)} > 0$, then $(\lambda - n)$ is a multiple of d. [8]

b) How are the cyclically moving subclasses $c_1, c_2, \ldots, c_d$ defined? [8]

c) Prove that if states x and y lie in the same subclass,

$$p_{xy}^{(kd)} > 0 \text{ for all large } k.$$ [9]

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GROUP B

Inference

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer all questions.

1.a) Show that if a complete sufficient statistic exists then every estimable parametric function has a best unbiased estimator. [8]

b) Let X_1, X_2 be independent $N(\mu, \sigma^2)$. Let

$\psi(\mu, \sigma^2) = P_{\mu,\sigma^2}(X_1 > 0)$. Find the best unbiased

estimator of ψ. [10]

2.a) $X_1, \ldots, X_m; Y_1, \ldots, Y_n$ are i.i.d., normal with mean μ. Variance of $X_1 = \sigma_1^2$ and that of Y_1 is σ_2^2. If $0 < \sigma_1, \sigma_2 < \infty$, show that no best unbiased estimator for μ exists. Determine the minimal sufficient statistic and show that it is not complete. [12]

b) Assuming above that $\sigma_1 = \sigma_2 = 1$, find the b.u.e. of μ^2. Calculate its variance and show it is strictly greater than the Cramer-Rao lower bound. [6]

3.a) Let X be $N(\mu, 1)$. Let $H_0(\mu = 1)$ be tested against $H_1(\mu = -1)$. Let $gx = -x$. Does g leave the model invariant? Does g leave the testing problem invariant? [2]

b) Let $P_\theta\{X = r\} = n_{c_r}\, \theta^r (1-\theta)^{n-r}$, $\theta = \frac{1}{4}$ or $\frac{3}{4}$ or $\frac{1}{2}$

$r = 0, 1, 2, \ldots, n.$

Let $H_0(\theta = \frac{1}{2})$ be tested against $H_1(\theta \neq \frac{1}{2})$. Let $gx = n-x$.

Does g leave the testing problem invariant? [2]

c) Let X_1, X_2 be i.i.d. $N(\mu, \sigma^2)$, $-\infty < \mu < \infty$, $0 < \sigma^2 < \infty$. Let $H_0(\mu = 0)$ be tested against

$$H_1\left(\frac{|\mu|}{\sigma} = 1\right).$$

Let $g_a x = a.x$. Does $G = \{g_a;\ -\infty < a < \infty,\ a \neq 0\}$ leave the testing problem invariant? Find a maximal invariant statistic. [10]

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PERIODICAL EXAMINATIONS

Statistics-5: Statistical Methods
Theory and Practical

Date: 3.3.1969 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. All questions carry equal marks.

1.a) Show that under certain conditions to be stated in full, the sample third central moment has an asymptotic normal distribution in a sense to be made precise by you. State without proof, all results you may need in this connection.

b) Compute the standard error of the sample third central moment.

2.a) Show that if $O_1, O_2, \ldots, O_k$ are the observed frequencies in the k classes of a multinomial distribution with respective class probabilities

$$\pi_1, \pi_2, \ldots, \pi_k \; (\sum_1^k \pi_i = 1),$$

then the variance of a linear function $\Sigma a_i O_i$ is given by the expression

$$V(\Sigma a_i O_i) = n[\Sigma a_i^2 \pi_i - (\Sigma a_i \pi_i)^2].$$

b) Obtain the limiting distribution of Pearson's goodness of fit chisquare statistic, as computed from a single sample of size n drawn from a multinomial distribution.

3.a) What is a 'U-statistic'?

b) State the conditions under which a U-statistic is asymptotically normally distributed.

c) Compute the standard error of Spearman's rank correlation coefficient when the variates concerned are independently distributed.

4. Consider as in Q.2, a sample of size n from a multinomial distribution and let the class probabilities be known functions of a single unknown parameter θ. Write

$$c(\theta) = \sum_{i=1}^k (O_i - n\pi_i(\theta))^2 / n\pi_i(\theta).$$

Show that if θ is a root of the equation

$$\frac{dc(\theta)}{d\theta} = 0,$$

then, under certain conditions.

(a) $\hat{\theta}$ is Fisher consistent for θ.

(b) $\sqrt{nI(\theta_0)}\,(\hat{\theta} - \theta_0)$ is asymptotically a standard normal variable, where θ_0 is the true value of θ and

$$I(\theta_0) = \sum_{i=1}^k \left(\frac{d \log \pi_i(\theta)}{d\theta}\right)_0^2 \pi_i(\theta_0).$$

(c) $c(\hat{\theta})$ is in the limit distributed as χ^2 on k-2 d.f.

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5.a) Show that to terms of order $O(\frac{1}{n})$, the variance of $\tanh^{-1} r$ is independent of the population parameters where r is the sample correlation coefficient in a sample of size n from the bivariate normal population.

b) Correlations between two variables X and Y were computed from three different samples as follows

Sample No.	Sample size	Value of r
1	215	0.416
2	137	0.531
3	79	0.497

Examine if there are significant differences in values of r. If not, compute the pooled estimate of r.

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PERIODICAL EXAMINATIONS

Design of Experiments (Theory and Practical)

Date: 10.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Marks allotted for each question are given in brackets [ ].

1. Describe the layout and the model of a Randomised Block design.

Explain how you would obtain the analysis of variance table and make use of it to test,

i) equality of all the treatment effects,

ii) equality of two specified treatment effects.

Obtain the expression for the efficiency of the randomised block design compared with that of Completely randomised design. [5+20+5]=[30]

2. Explain with illustration the role of randomisation in the conduct of experiments. Why an experimentor introduces replication while conducting experiments? [16+9]=[25]

3. Describe the use of Analysis of Covariance in increasing the precision of an experiment. Give the model and the method of analysis when a single concomitant variable is used on a Randomised Block design.

a) Can you test the usefulness of the concomitant variable on the basis of the analysis of covariance table obtained above?

b) How will you use the analysis of covariance table obtained above) to test the equality of all the treatment effects when the introduction of the concomitant variable has been found to be useful in (a)? [3+10+12]=[25]

4. Consider the results given in the following table for an experiment involving six treatments in 4 randomised blocks. Suppose that the yield for treatment 1 in block 1 is missing. Analyse the data.

Find an estimate of the variance of the difference between the mean of the treatment with a missing value and that of any other treatment.

Table

Yield for a Randomised-blocks Experiment.
(The treatments are indicated by numbers within parentheses)

Block	Treatment and yield					
1	(1)	(3)	(2)	(4)	(5)	(6)
	-	27.7	20.6	16.2	16.2	24.9
2	(3)	(2)	(1)	(4)	(6)	(5)
	22.7	28.8	27.3	15.0	22.5	17.0
3	(6)	(4)	(1)	(3)	(2)	(5)
	26.3	19.6	38.5	36.8	39.5	15.4
4	(5)	(2)	(1)	(4)	(3)	(6)
	17.7	31.0	28.5	14.1	34.9	22.6

[25+5]=[30]

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5.a) Draw up a randomised blocks layout for an experiment involving 6 treatments A, B, C, D, E, F in 5 blocks.

b) Ten test animals were divided at random into two groups of <u>five</u> each. One group was given a particular type of food A, and the other group a different feed B. Table below gives the increase in weight in pounds of the animals in a six-week period.

Table showing the increase in weight of animals given two different types of feed

Type of feed	Increase in weight (lb.)				
A	1.2	2.4	1.3	1.3	0.0
B	1.0	1.8	0.8	2.6	1.4

Examine whethere there is any difference in the effects of the two types of feed (i) by use of the t-test, (ii) by use of the F-test.

[6+7+7]=[20]

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PERIODICAL EXAMINATIONS
Statistics-7: Industrial Statistics Theory and Practical

Date: 17.3.69 Maximum Marks: 100 Time: 3 hours.

Note: Answer any four questions. Marks allotted for each question are given in brackets [ ].

1. Explain the role of rational sub-grouping in controlling the quality during a production process.

Following are the summarised data on a measurable characteristic.

Sample	$\bar{X}$	R	Sample	$\bar{X}$	R	Sample	$\bar{X}$	R
1	0.7540	.0011	6	.7539	.0009	11	0.7541	.0017
2	0.7542	.0014	7	.7541	.0012	12	0.7542	.0011
3	0.7542	.0009	8	.7543	.0011	13	0.7545	.0011
4	0.7543	.0010	9	.7547	.0007	14	0.7548	.0009
5	0.7550	.0008	10	.7549	.0015	15	0.7551	.0012

Plot an $\bar{X}$ - R chart and examine the data for control using sample size as 4. How would you modify this control chart if the specified limits for the above measurable characteristic are 0.7528-0.7572? Plot the modified charts. [5+20]=[25]

2. Give five examples of industrial situations where Poisson law is applicable.
Following data were collected to establish standard for the defects on an electronic component.

Sample No.	No. inspected	Total No. of defects	Sample No.	No. inspected	Total No. of defects
1	4	17	11	8	31
2	7	23	12	6	39
3	5	24	13	3	29
4	7	27	14	8	30
5	7	32	15	9	31
6	7	33	16	6	21
7	6	18	17	5	26
8	7	28	18	7	20
9	7	29	19	3	24
10	6	31	20	6	29

Analyse by means of control chart technique and recommend standard for average number of defects per item. [5+20]=[25]

3.a) The specification limits on the gross weight of an ink bottle are 110$\pm$3 gms. It was found that the weight of an empty bottle has a mean of 53.9 gms. and a s.d. of 0.7 gms. Empty bottles are fed into the filling machine in a random order. Find out what is maximum allowable standard deviation at the filling stage (i.e. s.d. of the weight of ink filled in a bottle) so that the final product meets the specification limits on the gross weight.

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3.b) Using Dodge-Romig sampling inspection tables select plan to satisfy the following:

i) Single and double sampling plans for lot size 5000, LTPD = 3.5 per cent, process average 1.10 what are AOQL for these plans.

ii) Single sampling plan for lot size 1200 LTPD = 3.0 per cent, process average 0.5 per cent. Find out AOQL for this plan. [10+10+5]=[25]

4.a) Explain the following terms

(i) AOI, (ii) AOQ (iii) AOQL

and also derive mathematical expression for AOQL of a single sampling and O.C of a double sampling plans by attributes.

b) For following double sampling plan

$N = 5000$, $n_1 = 100$, $c_1 = 0$, $n_2 = 100$, $c_2 = 1$.

i) Compute the probability of acceptance of 2 per cent defective lots.

ii) What will be the AOQ if the lots submitted contain 2 per cent defective items and the rejected lots are screened.

iii) What would be the average number of articles inspected per lot in (ii). [10+15]=[25]

5. Write short notes on following:

a) A process in a state of 'Statistical Quality Control'.

b) Process capability and specification

c) Group control charts

d) Screening under sampling plans by attributes

e) 3-sigma limits and Probability limits.

[25]

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Statistics-7

Date: 24.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A

Econometrics

Maximum Marks: 70 . Suggested time: 2 hours

Answer all questions.

1. Explain the concept of 'Economies of scale' in the context of family budget analysis.

   While estimating an Engel Curve from family budget data how would you take into account the possible effect of this phenomena. [20]

2. Describe a method of estimating Engel elasticities from the concentration curves, giving the necessary proofs. [20]

3. Derive the general form of the total cost function under a given production function. How do you think the shape of total cost curve would look like? Give reasons for your answer. [20]

4. Write short note on any one of the following:

   a) Cobb-Douglas Production function

   b) Unit consumer scale. [10]

GROUP B

Planning Techniques

Maximum Marks: 30 Suggested time: 1 hour

EITHER

1. Explain Charnes' perturbation method for resolving the degeneracy problem. [30]

OR

Solve by the simplex method:

Maximize: $Z = 60\,\xi_1 + 60\,\xi_2 + 90\,\xi_3 + 90\,\xi_4$

subject to

$$100\,\xi_1 + 100\,\xi_2 + 100\,\xi_3 + 100\,\xi_4 \leq 1500$$
$$7\,\xi_1 + 5\,\xi_2 + 3\,\xi_3 + 2\,\xi_4 \leq 100$$
$$3\,\xi_1 + 5\,\xi_2 + 10\,\xi_3 + 15\,\xi_4 \leq 100$$
$$\xi_1, \xi_2, \xi_3, \xi_4 \leq 0$$

Also formulate its dual and give its solutions. [30]

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PERIODICAL EXAMINATIONS

Statistics-8: Genetics Theory and Practical

Date: 31.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions. Marks allotted for each question are given in brackets [ ].

1.a) What is genetical linkage? Define cross over ratio. Briefly describe the construction of a chromosome map.

b) In corn, white endosperm (w) is recessive to purple (W) and Shrunken endosperm (s) is recessive to full (S). A pure purple shrunken is crossed to a pure white full. The F_1 is then crossed to a white shrunken, and the offspring are as follows:

Purple shrunken	3149
Purple full	120
White shrunken	115
White full	3334

Test for the presence of linkage. Estimate recombination fraction and the variance of the estimate, if linkage is present. [10+15]=[25]

2.a) What is meant by sex linked inheritance? Describe with suitable examples.

b) In Drosophila, the mutant gene for light eye colour known as 'vermillion' (v) is in the X chromosome and is recessive to red (V). A vermillion female is crossed to a red-eyed male. Give the eye colour of the F_1 (together with their sex) and of the F_2 (when the F_1 are interbred).

c) In humans, red-green colour blindness is controlled by a sex-linked recessive gene (c). Thalassemia is a type of anemia common in Mediterranean populations, but is relatively rare in other peoples. This anemia is inherited autosomally and is expressed in two degrees, severe cases that are usually fatal in childhood (called thalassemia major) and mild cases (called thalassimia minor). Use symbols T T for thalassemia major, Tt for thalassemia minor, and tt for normal individuals.

A woman who is red-green colour blind and afflicted with thalassemia minor marries a man with normal vision but who is afflicted with thalassemia minor.

i) Show the genotypes of these two individuals.

ii) Show the possible genotypes and phenotypes and the proportion of each for all children from this marriage. Designate the sons and daughters.

iii) What proportion of the children would be expected to succumb to thalassemia? [7+8+10]=[25]

3. Calculate the amount of information per individual provided by F_2 coupling and repulsion data regarding linkage between two factors A- a and B- b when

i) classification is complete and

ii) both the factors show complete dominance.

Compare the different results. [25]

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4. For two factors which are individually Mendelian and show dominance,

a) Write down the expected frequencies for the phenotypic classes of F_2.

b) Describe the procedure for the detection of linkage in this case, clearly stating the algebraic expressions to be used.

c) Find the variance of the maximum likelihood estimator of cross over ratio, assuming this to be same for both the parents.

d) Comment on the efficiency of the estimator obtained by equating the linkage function to its expection.

[4+7+7+7]=[25]

5.a) Describe any procedure for detection of linkage from F_2 data when single factor segregations are disturbed.

b) When single factor segregations are subject to viability disturbances, suppose backcross data on both coupling and repulsion are available. Give a suitable method for estimation of recombination fraction and find the large sample variance of the estimator. [10+15]=[25]

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ANNUAL EXAMINATIONS

Statistics-4: Probability

Date: 19.5.69. Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

The whole paper carries 110 marks. You may attempt all questions. The maximum you can score is 100.

WE CONSIDER MARKOV CHAINS WITH A FINITE NUMBER OF STATES. $P = \{p_{ij}\}$ IS THE TRANSITION MATRIX.

Group A

1.a) i and j lie in the same ergodic class.

$$S = E\left\{n: p_{ii}^{(n)} > 0\right\}, \quad T = E\left\{n: p_{jj}^{(n)} > 0\right\}.$$

Prove that S and T have the same greatest common divisor d. [Do not assume the existence of cyclically moving subclasses. If you require any lemma from elementary number theory, state it.] [8]

b) If there are no transient states and only one ergodic class, and $d = 1$ for this class, prove that $p_{ij}^{(n)} > 0$ for all large n. [8]

c) P is such that $Q = \lim_n P^n$ exists. Prove that if there is only one ergodic class, the rows of Q are all the same. (There may be transient states). [12]

2.a) P is such that $Q = \lim_n P^n$ exists. Write down two properties of Q and prove them. [7]

b) Prove that there is no stochastic matrix P such that $\lim_n P^n$ is

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$ [10]

c) P is a stochastic matrix and $\lim_n P^n$

$$= \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{8} & \alpha & \beta \end{bmatrix}$$

Show that α and β are uniquely determined. What are their values? [10]

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Group B

3.a)

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Determine the matrix

$$Q = \lim_{n} \frac{P + P^2 + P^3 + \dots + P^n}{n}$$ [20]

<u>Hint</u>: First obtain the limit of a convergent subsequence of the sequence $\{P^n\}$.

b) Consider a Markov chain whose transition matrix is P above. Determine the absorption probabilities (into ergodic classes) by solving the usual linear equations and also by considering the various paths from the transient states and into the ergodic classes. Show that the answers you get by the two methods agree. [15]

c) The transition matrix of a Markov chain is P and that of another Markov chain is P^k, k being a positive integer. Prove that a state is transient in the second chain if and only if it is transient in the first chain. [10]

d) If $\lim_{n} P^n$ is the identity matrix, prove that the Markov chain has no transient states. [10]

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Statistics-4: Inference

Date: 21.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any two questions.

1.a) Define Fraser and Hajec sufficiency. Explain their use in testing hypotheses. [13]

b) Let X be a discrete random variable with $f_\theta(x) = P_\theta\{X = x\}$, $\theta \in \Omega$. Supposing each $f_\theta(x)$ is symmetrical about the origin show that $|X|$ is sufficient for θ. Generalising this result show that if $G = (g_1, \ldots, g_n)$ is a finite group such that $f_\theta(gx) = f_\theta(x)\ \forall\ \theta, g, x$, then the maximal invariant under G is sufficient for θ. [12]

2.a) State and prove Stein's theorem on invariance and sufficiency and discuss briefly its role in reducing data through invariance and sufficiency. [10+3]=[13]

b) Obtain the one-sided student's t-test as a M.P. invariant test. (You have to prove the MLR property of t-distributions.) [12]

3.a) Let $X_1, \ldots, X_n$ be random variables with joint density $f_\theta(x_1, \ldots, x_n) = f_0(x_1 - \theta, \ldots, x_n - \theta)$ $-\infty < \theta < \infty$. Let $E_0(X_1^2) < \infty$. Find the best translation invariant estimator under the squared error loss. Show it is unbiased. [12+3]=[15]

b) X_1 and X_2 are i.i.d. with common density $f_\theta(x) = f_0(x - \theta)$ specified below. Find the best translation invariant estimator.

i) $f_0(x) = 1$ if $0 \leq x \leq 1$
$= 0$ otherwise.

ii) $f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ $-\infty < x < \infty$. [6+4]=[10]

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Group B

Maximum Marks: 50

Answer any two questions.

1.a) Derive Barankin's lower bound to the variance of an unbiased estimator and show how the Cramer-Rao bound can be obtained from it. [15]

b) Let $P_\theta\{X = -1\} = \theta$, $P_\theta\{X = n\} = (1-\theta)^2\theta^n$

$$n = 0,1,2,\ldots, \qquad 0 < \theta < 1.$$

Find the locally best unbiased estimators of θ.

2.a) Prove the Fundamental Identity for a SPRT assuming $E(Z_1^2) > 0$. [10]

b) Let X_i be i.i.d. with common density -

$f_\theta(x) = \frac{1-\theta^2}{2} \exp(-|x| + \theta x)$ $\theta = \pm\frac{1}{2}$. Construct the SPRT of $H_0(\theta = -\frac{1}{2})$ vs. $H_1(\theta = \frac{1}{2})$ with boundaries $B < 1 < A$ and find its error of first kind exactly. [15]

3.a) Discuss briefly the concept of Pitman efficiency. Let $\{X_i\}$ be i.i.d., normal with mean θ and variance unity. Let ϕ be a test of size α, of $H_0(\theta = 0)$ vs. $H_1(\theta > 0)$, which rejects H_0 for large values of

$$\bar{X}_n = \sum_1^n X_i \mid n;$$

let ϕ_{2n} be a test of size α which rejects H_0 for large values of Median $(X_1,\ldots,X_n)$. Calculate the Pitman efficiency of ϕ_{1n} w.r.t. ϕ_{2n}. [9+3]=[12]

b) Consider two independent samples of size $n = m$ from continuous populations F and G where $G(x) = F(x-\Delta)$ $\Delta \geq 0$ and F has a density. You have to test $H_0(F = G)$ vs. $H_1(G(x) = F(x-\Delta),\ \Delta > 0)$. Obtain the lower bound to the Pitman efficiency of the two sample Fisher-Yates test with respect to the usual t-test. [13]

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ANNUAL EXAMINATIONS

Statistics-5: Statistical Methods Theory

Date: 23.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any two questions.

1.a) Obtain the standard error of the sample correlation coefficient r, in a sample of size n, from a bivariate normal population. [17]

b) Derive the $\tanh^{-1}$ transformation as a variance stabilising transformation of r. [8]

2.a) Let $(Y_{n1}, Y_{n2}, \ldots, Y_{nn})$ be a random permutation of integers $1, 2, \ldots, n$, and $c_n(i,j)$, $i,j = 1(1)n$ be n^2 real numbers. Consider

$$S_n = \sum_{i=1}^{n} c_n(i, Y_{ni}).$$

State sufficient conditions on a sequence $\{c_n\}$, $n = 1,2,\ldots$ for the asymptotic normality of S_n. [8]

b) Let $O_1, O_2, \ldots, O_k$ and $E_1, E_2, \ldots, E_k$ be the observed and the corresponding expected frequencies in k mutually exclusive and exhaustive classes, as recorded in a sample of size n drawn without replacement from a finite population of size N. Use 2(a) to obtain sufficient conditions underwhich

$$\frac{N}{N-n} \sum_{i=1}^{k} (O_i - E_i)^2 / E_i$$

is asymptotically distributed as chisquare on k-1 d.f. [17]

3.a) Show that the null distribution of the Wilk's Λ criterion is the same as that of the product of several independent beta variables. [10]

b) Hence compute the t-th moment of Λ. [10]

c) State without proof a large sample approximation for the null distribution of Wilk's Λ. [5]

Group B

Maximum M[illegible]

Answer any two questions.

1.a) Show that if the variables $Y_1, Y_2, \ldots, Y_p$ have mean zero and a finite dispersion matrix Σ and if $\underset{\sim}{Y}' = (Y_1, Y_2, \ldots Y_p)$ then with probability one $\underset{\sim}{Y} \in \mathcal{M}(\Sigma)$; that is, the equations $\underset{\sim}{Y} = \Sigma \underset{\sim}{X}$ are consistent for $\underset{\sim}{X}$; that is

$$\underset{\sim}{\lambda}' \Sigma = \underset{\sim}{0}' \Rightarrow \underset{\sim}{\lambda}' \underset{\sim}{Y} = 0.$$

[5]

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1.b) $\underset{\sim}{Y}$ is said to have a p-variate normal distribution if for every $\underset{\sim}{\ell}$, $\underset{\sim}{\ell}'\underset{\sim}{Y}$ is univariate normal (with possibly a zero variance). Use this definition to establish the following propositions

i) If $\underset{\sim}{Y}$ is p-variate normal, then $\underset{\sim}{Y}$ has finite mean vector $\underset{\sim}{\mu}$ and a finite dispersion matrix Σ. [5]

ii) The characteristic function of $\underset{\sim}{Y}$ is given by

$$E\, e^{i\,\underset{\sim}{t}'\underset{\sim}{Y}} = e^{i\,\underset{\sim}{t}'\underset{\sim}{\mu} - \underset{\sim}{t}'\Sigma\underset{\sim}{t}/2}$$

(We denote the distribution of $\underset{\sim}{Y}$ by $N_p(\underset{\sim}{\mu}, \Sigma)$). [5]

c) Suppose Rank $\Sigma = r < p$ and let B $(p \times r)$ be such that $\Sigma = B B'$. Write $C = (B'B)^{-1} B'$ and observe $CB = I(r)$. Define $\underset{\sim}{G} = C\underset{\sim}{Y}$. Then show that if $\underset{\sim}{Y} \sim N_p(\underset{\sim}{0}, \Sigma)$ then (i) $\underset{\sim}{G} = N_r(\underset{\sim}{0}, I)$; (ii) With probability one, $\underset{\sim}{Y} = B\underset{\sim}{G}$. [10]

2.a) Define a central Wishart distribution $W_p(n, \Sigma)$. [3]

b) Show that if $S \sim W_p(n, \Sigma)$, and $\underset{\sim}{\ell}$ is a fixed vector such that $\underset{\sim}{\ell}'\Sigma\underset{\sim}{\ell} = 0$, then $\underset{\sim}{\ell}'S\underset{\sim}{\ell} = 0$ with probability one. If $\underset{\sim}{\ell}'\Sigma\underset{\sim}{\ell} = 1$ then $\underset{\sim}{\ell}'S\underset{\sim}{\ell} \sim \chi^2_n$. [2+5]=[7]

c) If a matrix T is such that $\underset{\sim}{\ell}'\Sigma\underset{\sim}{\ell} = 0 \Rightarrow \underset{\sim}{\ell}'T\underset{\sim}{\ell} = 0$ with probability 1 and $\underset{\sim}{\ell}'\Sigma\underset{\sim}{\ell} = 1 \Rightarrow \underset{\sim}{\ell}'T\underset{\sim}{\ell} \sim \chi^2_n$ where Σ is some fixed non-negative definite matrix, is it necessarily true that $T \sim W_p(n, \Sigma)$? If not, give a counter-example. [2+7]=[9]

d) Obtain the characteristic function of $W_p(n, \Sigma)$ when Σ is possibly singular. [6]

3. Let

$$\underset{\sim}{Y} = \begin{pmatrix} \underset{\sim}{Y}_1 \\ \underset{\sim}{Y}_2 \end{pmatrix} \begin{matrix} p \\ q \end{matrix}$$

have a $p + q$ variate normal distribution

$$N_{p+q}\left(\begin{pmatrix} \underset{\sim}{\mu}_1 \\ \underset{\sim}{\mu}_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$$

and let Σ_{11} $(p \times p)$ be nonsingular.

Consider n independent observations on $\underset{\sim}{Y}$ and let T^2_{p+q} and T^2_p be the Hotelling's T^2 based on observations on all the $p+q$ variables and the first p variables respectively for testing the hypothesis that the mean values are all equal to zero. Show that if

$$\underset{\sim}{\mu}_2 = \Sigma_{21}\Sigma_{11}^{-1}\underset{\sim}{\mu}_1 \quad \text{then}$$

$$\frac{n-p-q}{q}\left(\frac{T^2_{p+q} - T^2_p}{n-1+T^2_p}\right)$$

has a variance ratio distribution on d.f. q for numerator and $n-p-q$ for denominator. [25]

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ANNUAL EXAMINATIONS

Statistics-5: Statistical Methods Practical

Date: 24.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any two questions from questions 1 to 3. Marks allotted for each question are given in brackets [ ].

1.a) The quartiles in a sample of size 401 from a certain population, were computed as

$$Q_1 = 3.15, \quad Q_2 = 7.30, \quad Q_3 = 14.56$$

Examine if this could be considered as sufficient evidence to discard the hypothesis that the population distribution is exponential with density

$$\frac{1}{10} e^{-x/10}, \quad 0 \leq x < \infty .$$

The population quartiles for this distribution are 2.88, 6.93 and 13.86.

b) Assuming that the population distribution to be exponential with density

$$\theta e^{-\theta x}, \quad 0 \leq x < \infty \quad (0 < \theta),$$

use the sample quartiles to estimate θ. [40]

2. A study was made of the relationship between oxygen tension and respiratory rate of bacteria. Three Y measurements were obtained for each value of X.

Oxygen tension in mm Hg(X)	Respiratory rate (Y)		
1	9	11	8
3	58	57	64
5	76	76	78
7	82	84	82
9	88	86	90
11	91	89	91
13	93	92	92
15	95	96	95

(a) Fit a linear equation to the regression of Y on X and test for the adequacy of a linear fit.

(b) If the linear fit is inadequate, use a table of orthogonal polynomials to determine the degree and the equation of the best fitting polynomial regression. [40]

3. In an experiment on the effect of radiation on weight loss, total weight losses (in gms.) of a series of twenty four rats were recorded at 1, 3, 6 and 7 days after radiation. Twelve of the rats had been subjected to 500r whole body radiation while twelve others had been subjected to 600r. Given below are the pooled dispersion matrix based on 22 d.f. and the differences in the average weight loss ('higher' minus 'lower' levels of radiation)

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Weight loss to i-th day

	1	3	6	7
1	6.00	3.71	3.33	3.08
3		25.71	7.35	3.60
6			21.42	17.27
7				24.08
Difference in average weight loss:	0.50	2.92	-2.58	-3.50

Examine if there is significant difference between levels of radiation with respect to weight loss of rats. [40]

4. Viva Voce. [10]

5. Practical Records [10]

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ANNUAL EXAMINATIONS

Statistics-6: Design of Experiments

Date: 26.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any two questions from Qns. 1 to 3

1.a) Describe a balanced incomplete block design (BIBD) with parameters b, v, r, k, λ. [2]

b) Establish the following relations among the parameters of a BIBD

i) $bk = rv$

ii) $\lambda(v-1) = r(k-1)$

iii) $b \geq v$ [2+3+7]=[12]

c) In usual notations write down the expression for the estimate of a treatment effect in a BIB experiment in terms of the Q's. (No derivation is necessary). Show that the estimates of the treatment comparisons of the form $\tau_i - \tau_j$ $(i \neq j)$ have the same variance and show that this variance is higher than the variance of the estimate of $\tau_i - \tau_j$ $(i \neq j)$ in a randomised block experiment with the same number of replicates, assuming σ^2 to be same in both cases. [1+2+3]=[6]

2. Explain the roles of randomization and local control in in planned experiments. Illustrate these by discussing some examples. [12+8]=[20]

3. Obtain a plan for conducting a 2^5 Factorial experiment in blocks of 2^3 units each without confounding any main effect or 2-factor interaction. [20]

4. Viva Voce. [10]

Group B

Maximum Marks: 50

Answer all questions

1. EITHER

a) Explain the lay-out for an experiment to compare 5 treatments using a Latin Square design.

b) When is a pair of Latin Squares of order s Said to be mutually orthogonal?

Write down FOUR mutually orthogonal Latin Squares of order 5. [3+3+14]=[20]

OR

a) Write 'true' or 'false' to each of the following statements (Need not give reasons):

i) 'Randomisation is a method by which every experimental unit has an equal chance of receiving a treatment'.

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ii) In a particular experimental design suppose $t_1, t_2, \ldots, t_m$ denote the treatment effects. Two contrasts

$$\sum_{i=1}^{m} a_i t_i \text{ and } \sum_{i=1}^{m} b_i t_i \quad (\text{i.e., } \Sigma a_i = 0 = \Sigma b_i)$$

and said to be mutually orthogonal if $\sum_{i=1}^{m} a_i b_i = 1$.

iii) In a randomised block experiment with r blocks and t treatments where one yield is missing, the variance of the usual estimate of the treatment effect with the missing value is given by

$$\frac{\sigma^2}{r}[1 + \frac{r}{(r-1)(t-1)}]$$

where σ^2 denotes the variance of the experimental error.

iv) Suppose $Z_m = \{0,1,\ldots, m-1\}$ is the residue class modulo m. Z_m is, under usual optations modulo m, a field if and only if m is a prime number.

v) If F is a field with characteristic zero, it must have finite number of elements.

vi) For every integer $n \geq 1$ there exists a polynomial of degree n irreducible over the Galois field GF(p) where p is a prime number.

vii) There exists a Graeco-Latin Square (a pair of mutually orthogonal Latin Squares) of order 6.

viii) There does not exist a Graeco-Latin square of order 10.

ix) In a 2^4 Factorial experiment conducted with 4 plots in a block if the pencils (1,1,1,0) and (0,1,1,1) are confounded then so also the pencil (1,1,1,1).

x) In a Factorial experiment conducted in randomised blocks an effect or interaction is said to be confounded when the effect or interaction gets inextricably mixed up with block difference. [10]

b) The following table gives the plan of a 2^5 Factorial experiment in blocks of size 4.

Block				
1	(1)	abcde	abc	de
2	a	bcde	bc	ade
3	b	acde	ac	bde
4	c	abde	ab	cde
5	d	abce	abcd	e
6	ad	bce	bcd	ae
7	abd	ce	cd	abe
8	acd	be	bd	ace

Identify the confounded effects and interactions. [10]

2. The following table gives the yield of wheat per plot in a manurial experiment carried out in a 4 X 4 Latin Square. The four manurial treatments are denoted by the numbers 1,2,3,4 in parentheses.

Yields in a 4 X 4 Latin Square Experiment.

Row \ Column	1	2	3	4	Total
1	(2) 425	(3) 442	(4) 540	(1) 340	1747
2	(4) 384	(1) 512	(2) 490	(3) 408	1794
3	(3) 506	(4) 508	(1) 536	(2) 600	2150
4	(1) 451	(2) 568	(3) 499	(4) 347	1865
Total	1766	2030	2065	1695	7556

Total S. S. (corrected) = 87,863

Test whether the treatments are significantly different. Write down the estimates of each of the treatment effects in the descending order. Find the variance of the estimates. [14+4+2]=[20]

3. Practical Records [10]

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ANNUAL EXAMINATIONS

Statistics-7: Planning Techniques

Date: 27.5.69. Maximum Marks: 100 Time: 3 hours

Note: Answer any three questions. Marks allotted for each question are given in brackets [ ].

1. Formulate the open static input-output model in terms of linear programming and give an interpretation of its dual. Show that the Leontief solutions for quantities and prices are the optimal solutions of the programming problem. [33]

2. 'The Dynamic Leontief system (with zero final demand) has a unique positive rate of balanced growth with no excess capacity and positive initial stocks'. Discuss the statement.

   Prove that in the set of all balanced growth paths in the Leontief dynamic system (with or without excess capacity), one with no excess capacity is maximal. [33]

3. Use the basic assumptions of the Leontief dynamic system to obtain the schedule of net output possibilities for a two-commodity economy with a one-period production plan and given initial stocks. Explain in this connection the concept of efficiency locus and show how the derivation of such a locus can be viewed as a problem in linear programming. [33]

4. Show that if the primal has an optimal solution, its dual also has an optimal solution. Prove that the corresponding values of the objective functions of the two problems are equal. [33]

5. Solve the following problem by the simplex method:

   Maximize $Z = 8\xi_1 + 19\xi_2 + 7\xi_3$

   subject to

$$3\xi_1 + 4\xi_2 + \xi_3 \leq 25$$

$$\xi_1 + 3\xi_2 + 3\xi_3 \leq 50$$

   Formulate the dual of this problem and solve it. [33]

One mark is allotted for neatness in the answers.

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ANNUAL EXAMINATIONS

Statistics-7: Econometrics

Date: 28.5.69. Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer any three questions. 2 marks are allotted for neatness in the answers.

1. Would you suggest least square estimation if both the dependent and independent variables are subject to errors of observation. Justify your answer. Give without proof Wald's and Bartlets methods of estimation in this connection. [16]

2. Show how a lagged dependent variable can appear as an explanatory variable with reference to the following:
   i) Koyck's model of distributed lag
   ii) Expectational and Adjustment models. [16]

3. State with proof the identifiability conditions in a simultaneous econometric model. [16]

4. Write short notes on any two of the following:
   i) Dummy variables,
   ii) Measurement of quality variation from the family budget data,
   iii) Supply function. [16]

Group B

Maximum Marks: 50

1. The following table gives the percentage distribution of persons by monthly per capita total expenditure classes along with average monthly per capita total expenditure as also average per capita expenditure on cereals and cereal substitutes for rural India during July 1959-June 1960.

exp. class	p.c. of population	per capita exp. on cereal and cereal substitutes (Rs.)	per capita total exp. (Rs.)
(1)	(2)	(3)	(4)
0 - 8	7.15	4.10	6.58
8 - 11	13.83	5.68	9.95
11 - 13	11.14	6.72	12.15
13 - 15	10.45	7.36	13.59
15 - 18	14.04	8.48	16.46
18 - 21	11.24	9.00	19.68
21 - 24	7.66	9.34	21.74
24 - 28	6.65	10.44	25.52
28 - 34	8.07	11.01	31.26
34 - 43	4.87	11.68	37.90
43 - 55	2.60	13.78	48.56
55 - above	2.30	16.40	86.01

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ANNUAL EXAMINATIONS

Statistics-7: Industrial Statistics Theory and Practical

Date: 29.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer question 1 and any other two questions from 2 to 4 in this group.

1.a) Explain the meaning of the following terms

(i) artificial variables (ii) extreme points (iii) basic feasible solution (iv) non-degenerate basic feasible solution.

b) State and prove the optimality condition to obtain the solution for a maximization linear programming problem using simplex method.

c) Show that an optimal solution to a linear programming problem occurs at an extreme point of the convex set of its feasible solutions. Derive the rules for transforming the simplex tableau to improve basic feasible solution while trying to reach the above extreme point at which the optimal solution occurs. [4+4+8]=[16]

2.a) Show that if $a_1, a_2, \ldots, a_n$ is a basis of the vector space V and

$$\beta = \sum_{j=1}^{n} c_j a_j$$

where c_j's are scalars such that $c_n \neq 0$ then $a_1, a_2, \ldots, a_{n-1}, \beta$ form a new basis of V. State the generalised form of the above theorem and indicate its role in solving a linear programming problem.

b) If the vector a_k which is not in the basis matrix B of a linear programming replaces the vector a_r $(r < n)$ in the basis matrix, show that the net decrease affected in the objective function is $\theta(z_k - c_k)$ where $z_k = C_B B^{-1} a_k$. What would you conclude when all $y_{ik} \leq 0$? [9+3]=[12]

3.a) If you are using two phase method, what conclusions can you draw under the following situations at the end of phase I?

i) Max $Z^* < 0$; one or more artificial vectors appear in the basis at positive level

ii) Max $Z^* = 0$; no artificial vectors appear in the basis

iii) Max $Z^* = 0$; one or more artificial vectors appear in the basis at a zero level

where Z^* denotes the objective function of phase I.

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3.b) Show that if the primal problem has an optimal solution then the dual problem also has an optimal solution. Obtain solution to the following problem by solving its dual problem

$$\text{Max} \quad Z = x_1 + 2x_2$$
$$\text{subject to} \quad 3x_1 + 2x_2 \geq 6$$
$$x_1 + 6x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

[3+9]=[12]

4. A company has four territories and four salesmen available for assignment. It is estimated that a typical salesman operating in each territory would bring in the following annual sales.

Territory	Annual sales
I	60,000
II	50,000
III	40,000
IV	30,000

The four salesmen are considered to differ in ability. It is estimated that working under the same conditions their yearly sales would be proportionately as 7:5:5:4 for the salesmen A,B,C and D respectively. If the criteria is the maximum expected total sales, obtain an optimal assignment. [12]

5. Practical Records and Viva Voce. [10]

Group B

Maximum Marks: 50

Answer Question 1 and any other two from 2 to 4 in this group.

1.a) Obtain the steady state probabilities for an unlimited queue served in order of arrival and a single service channel, assuming Poisson arrivals and exponential service. Find also:

i) average number of persons in the queue

ii) probability distribution for the waiting time of an arrival

iii) average waiting time of an arrival. [4+2+4+2]=[12]

b) At what average rate must a clerk at a supermarket counter work inorder to insure a probability of 0.90 that a customer will not have to wait longer than 12 minutes? it is assumed that there is only one counter to which customers arrive in a poisson fashion at an average rate of 15 per hour. The length of the service by clerk has an exponential distribution. [8]

2. In a machine shop there are N machines and stoppages occur completely randomly in running time, at the same rate λ for all machines, and independently for all machines. There is one operator always available for restarting stopped machines. The frequency distribution of service time is exponential with mean $1/\mu$.

Show that the number of machines running follows a truncated Poisson distribution of parameter ρ^{-1}, where ρ the servicing factor = λ/μ. Derive expressions for operative utilisation and machine availability. [15]

3.a) Consider a machine shop where there are 8 automatic machines in operation. From past experience it is known that each machine will operate for an average period of 60 hours and then require an average of 40 hours of maintenance. Machine stoppages are known to occur in a poisson fashion and service is exponential. One hour's work on the machine produces a profit of Rs.10/-. If a service engineer is employed he should be paid wages of Rs.3/-, per hour. Find out how many service engineers should be employed?

b) Suppose it is possible to introduce some sort of preventive maintenance at a fixed cost of Rs.10/-, per hour for all machines. The effect of preventive maintenance is to increase the average running time of machine to 80 hours and decrease the repair time to 20 hours. In this situation how many service engineers are to be appointed? Decide whether it is worthwhile to have preventive maintenance. [9+6]=[15]

4. For a queueing model with random input and general service time, show that when the system is in a state of statistical equilibrium

Average queue length $E(n)=\rho + \dfrac{\rho^2 + \lambda^2 \sigma_v^2}{2(1-\rho)}$ and

Average waiting time $E(w)= \dfrac{\rho^2 + \lambda^2\sigma_v^2}{2\lambda(1-\rho)}$ where

$\rho = \lambda/\mu$; λ = mean arrival rate; μ = mean number of customers serviced in unit time and σ_v^2 is variance of the service time of a customer. [10+5]=[15]

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ANNUAL EXAMINATIONS

Statistics-8: Genetics

Date: 31.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 46

Answer any two questions.

1.a) Describe with examples Mendel's first and second principles in the theory of Genetics and comment on their uses and applicability.

b) In an equilibrium population with regard to blood groups in O-A-B series, let the frequencies of genes O, A, B be respectively 0.72, 0.25 and 0.03. What is the probability that an individual known to be of type A is homozygous? What is the probability that an offspring resulting from the mating of two type A individuals is homozygous? [12+11]=[23]

2.a) Show that under random mating any population will attain equilibrium with regard to one autosomal locus with two alleles in one generation.

b) Prove that for an autosomal locus with n alleles $A_1, A_2, \ldots, A_n$, a population with the following genotypic distribution

$$\sum_i p_i^2 A_iA_i + 2 \sum_{i<j} p_ip_jA_iA_j$$ is in

equilibrium under random mating. [12+11]=[23]

3.a) Define 'Mendelian Population' and give a brief outline of the genetic theory of organic evolution.

b) Consider the case of a simple Mendelian character in which the genotypic array in the present generation is

$$p^2AA + 2pqAa + q^2aa, \qquad p + q = 1.$$

Suppose that the recessives aa are completely eliminated or sterilised, the mating being random otherwise in the present and all subsequent generations. Find the frequency of 'a' gene after n generation.

c) Suppose a recessive trait occurs in 1 in 1000 of a random mating population. How many generations of complete selection against the recessive individuals would be necessary to reduce the proportion to 1 in 1,000,000? [6+12+5]=[23]

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Group B

Maximum Marks: 44

Note: Answer question 2 and any one from the rest.

1.a) Write down the genotypes, phenotypes and phenotypic frequencies in terms of the gene frequencies for O-A-B blood group system under random mating.

b) Show that under repeated sib-mating the heterozygotic frequency in a population with two autosomal alleles 'A' and 'a' at one locus goes to zero at the rate

$$\left[\left(\frac{1+\sqrt{5}}{4}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{4}\right)^{n+1}\right].$$

[4+18]=[22]

2.a) For M-N blood group system write down the different genotypic mating types, their frequencies under random mating and their segregation ratios (i.e., probabilities of producing different types of offsprings).

b) The following are the results of MN testing for two populations, Brahmin and Kaystha. Estimate the gene frequencies and test whether the two populations are homogeneous with respect to M-N blood groups

Genotypes	Observed Frequency	
	Brahmin	Kaystha
MM	79	39
MN	109	67
NN	42	34
Total	230	140

[6+16]=[22]

3.a) State and prove the general equilibrium law in a population with two autosomal alleles 'A' and 'a' at one locus.

b) Show that

$$m + (m-2)F = 0$$

where m denotes the correlation coefficient between the two parents and F, the overall inbreeding coefficient of the population.

c) Outline a few major factors due to which the original distributions are not reproduced in genetic surveys.

[11+7+4]=[22]

Viva Voce. [10]
