INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat. II year

ANALYSIS

Duration: Maximum Marks: 200 Date: 3 October 1963

Note: Separate Answer book should be used for each of the Groups

GROUP 1

1. Define a sequence and the limit of a sequence. Explain, the three kinds of sequences, and give examples. Give a graphical interpretation in each case.
Let (a_n), and (b_n) have respective limits, l_1 and l_2. What can be said, about those of (a_n+b_n), and $(k\,a_n)$ where k is a constant? State the condition to be imposed. [15]

2. What is an open set? What is a closed set? Give all possible examples of open sets and closed sets. Establish the inter-relation between these two concepts. State and prove, all results, regarding the union, and intersection of open sets, and similarly for closed sets.
Are there sets, which are neither open, nor closed? If so, cite a few examples.
Mention, (if any), some sets, which are both open and closed. [30]

3. In what way, the concept of "open sets", and "closed sets", is more advantageous than that of "open intervals", and "closed intervals"? (Hint:- Consider the operations of finite intersection, and complementation). Give illustrations [10]

4. Define the limit-point of a set E. Prove that, in this definition, the term "open interval", can be replaced, by the term, "open set". Can it be replaced, by the term "closed set"? [15]

5. Let E be a closed set, and p, a point, not in E. Then, prove that, there exists, at least one open set, containing p, and not containing any point of E. [15]

6. Define left-continuity, and right-continuity of a function at a point. When is a function said to be continuous (a) at a point, (b) in an interval and (c) at the end-points of a closed interval, if the function, is not defined, outside that interval.

Give, an example of a function, which is
(i) left-continuous at a point,
(ii) right-continuous at a point.
(iii) everywhere continuous. [15]

(Please turn over)

GROUP II

1. Show that the series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ is convergent if $p > 1$ and divergent for $p \leq 1$.

Test for convergence the series

(i) $\sum a+n^3/2^n+a.$ (ii) $\sum n^p/(n+1)^q$

(iii) $\sum \frac{n^3}{(n-1)!}$ [6+4+4+4]

2. Sum the series to infinity :

(i) $\frac{2}{1.4.5} + \frac{3}{2.5.6} + \frac{4}{3.6.7} + \ldots$

Show that

(ii) $\frac{1}{(m+1)(m+2)} + \frac{1}{(m+2)(m+3)} + \frac{1}{(m+3)(m+4)} \ldots = \frac{1}{m+1}$

(iii) Sum to infinity of $\frac{1}{(m+1)^2} + \frac{1}{(m+2)^2} + \ldots$

lies between $\frac{1}{m+1}$ and $\frac{1}{m+1} + \frac{1}{(m+1)^2}$ [8+8+10]

3. Find the differential coefficients of the following :

(i) $x\sqrt{a^2+x^2}$ (ii) $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$

(iii) $\log \frac{x}{a^x}$ (iv) $e^x \log x.$

(v) $\sin^m x \cos^n x.$ [6+6+6+6+6]

4(i) If $y^{-2} = 1+2\sqrt{2} \cos 2x$ prove that

$y_2 = y(3y^2+1)(7y^2-1)$

(ii) If $y = A(x+\sqrt{x^2+a^2})^n + B(x+\sqrt{x^2+a^2})^{-n}$

then show that

$(x^2+a^2)y_{m+2} + (2m+1)xy_{m+1} + (m^2-n^2)y_m = 0$

where suffixes denote differentiations in (i) & (ii). [12+14]

INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat. II Year

PROBABILITY

Duration : $2\frac{1}{2}$ hours | Full Marks 100 | Date : 31.10.63

Answer any five questions.

1a) If E_1, E_2 ... E_n are n events, what is the probability that at least one of E_1 ... E_n will happen? (You need not supply the proof). (8)

b) There are five addressed envelopes and five letters are placed at random in those envelopes. What is the probability that none of the letters is placed in the correct envelope? (12)

2a) What is the mathematical expectation a random variable? (5)

b) If X and Y are two random variables show that $E(X+Y)=E(X)+E(Y)$. (15)

3a) When are two random variables said to be independent. (5)

b) If X and Y are two independent random variables show that

i) $E(XY) = E(X).E(Y)$

ii) $V(X+Y) = V(X)+V(Y)$ (15)

4a) Define correlation $\rho(X,Y)$ between two random variables X and Y and show that $|\rho(X,Y)| \leq 1$. (10)

b) If X is a r.v taking values ± 1, ± 2 each with probability $\frac{1}{4}$ and $Y = X^2$, find the correlation between X and Y. (10)

5a) State and prove Bayes' theorem. (10)

b) An urn contains two balls. It is known that the urn was filled by tossing a coin twice and putting a white ball in the urn for each head and a black ball for each tail. A ball is drawn from the urn and is found to be white. Find the probability that the other ball in the urn is also white. (10)

6a) What is Binomial distribution? (5)

b) What is the probability for an event E to occur

i) at least once

ii) at least twice.

in a series of n independent trials with probability p. (15)

INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. II year

STATISTICS

Duration : $2\frac{1}{2}$ hours 7 Nov. 1963

1. A box contains 10 tickets bearing numbers from 1 to 10. Two tickets are drawn at random, without replacement.

   (a) Describe the sample space of the experiment.

   (b) If the following variables ξ write down the alternative values and their respective probabilities :

      (i) ξ_1 = sum of the numbers on the two tickets drawn;

      (ii) ξ_2 = smaller of the numbers on the two tickets drawn;

      (iii) ξ_3 = product of the numbers on the two tickets drawn.

2. Obtain expressions for and draw rough graphs of the cumulative distribution functions of the following random variables :

   (i)

Values of ξ_1 :	-2,	-1,	0,	1,	2,	3
Probability :	$\frac{1}{36}$,	$\frac{7}{36}$,	$\frac{10}{36}$,	$\frac{8}{36}$,	$\frac{7}{36}$,	$\frac{3}{36}$

   (ii) $\xi_2 = \xi_1^2$

   (iii) ξ_3 is a binomial variate $B(5, \frac{1}{5})$.

3(a). The moment generating function of a variable ξ is $M_{\xi}(t)$. Obtain the moment generating function of $\eta = a\xi + b$ where a and b are two constants.

(b). If ξ is a Poisson variate with mean λ, what is the moment generating function of $\eta = \frac{\xi - \lambda}{\sqrt{\lambda}}$? From this function find an expression for μ_3 of ξ.

(c). It is stated that the function $t\,e^{t}$ is the moment generating function of some random variable ξ. Show, with reasons, whether this statement is correct or otherwise.

4(a). Define the "probability density function" of a continuous-valued random variable.

(b) For a r.v. ξ, the p.d.f. is known to be of the form

$$f(x) = k\,|x|, \quad -1 \le x \le +1$$
$$= 0 \quad \text{elsewhere.}$$

Evaluate k.

(c) Find the moment generating function of the r.v. ξ whose p.d.f. is

$$f(x) = \lambda e^{-\lambda x}, \quad \lambda > 0, \quad 0 \le x < \infty$$
$$= 0 \text{ elsewhere.}$$

Also find expressions for β_1 and β_2 for ξ.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. II year

ECONOMICS

Duration :3 hours — Date: 17 October 1963

Note: Separate booklet should be used for each Group.

Attempt three questions from Group A and two from Group B. All questions are of equal value.

GROUP A

1. Describe the different ways of deposit creation by banks. Explain in this connection the different forms of credit instruments.

2. Discuss the relationships between the quantity of money and its value as explained by the Fisher and Cambridge equations.

3. Examine the role of the Central bank as the lender of the last resort.

4. Discuss the economic and social effects of inflation and deflation. Do you think that a mild inflation is beneficial for a developing economy? Give reasons for your answer.

5. Explain the quantity effect and the interest effect of open market operations. Discuss whether open market policy and minimum reserve policy are contradictory or complementary to each other.

6. Write short notes on any three of the following :

   (a) reflation and disinflation, (b) the circular velocity of money, (c) token money and standard money, (d) clearing house, (e) fiduciary issue.

GROUP B

1. "Advantages of co-operative farming arise mainly from its large size, joint management and individual proprietorship" - Critically examine the statement. Do you agree with the view that the expansion of co-operative farming would create more unemployment?

2. Review the progress of co-operative farming in India and point out the main drawbacks of the movement.

3. Appraise the findings and recommendations of the Rural Credit Survey Committee in Inda. What measures have been taken to implement the recommendations?

4. What are the defects in the present system of marketing of agricultural produce in India? Explain how co-operative societies can help the cultivators to remove these defects.

INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat. II year

PHYSICS (Theory) 26.9.83

Duration: 1 hr. 15 min. Maximum Marks 50

1. State the assumptions on which the Kinetic theory of gases is based. Show how the principal gas laws may be accounted for on this theory.

Calculate the r.m.s. velocity of hydrogen molecules at N.T.P., given that the density of hydrogen at N.T.P. is 0.00009 $gm.cm.^{-3}$ and the density of mercury is 13.6 $gm.cm.^{-3}$. (18)

Or

Give an account of Andrews' experiments on the isothermals of carbon dioxide. What is the significance of the critical isothermal? Starting from Vander waal's equation find RT_c/P_cV_c where P_c, V_c, T_c are the critical constants of the gas and R the gas constant. (18)

2. State Kirchhoff's laws for current networks. Twelve equal wires of resistance r ohms are arranged to form the edges of a cube. A battery of e.m.f. E volts and no internal resistance is connected across a diagonal of the cube. Find the current in each conductor.

Define the c.g.s. electromagnetic unit of current. (16)

3. Give the theory of action and the construction of a Helmholtz galvanometer. (16)

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INDIAN STATISTICAL INSTITUTE
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Periodical Examination

B.Stat. II year — 26. 9. 63

CHEMISTRY

Duration: 1 hour — Maximum Marks 50

Answer any two questions

1. (a) State Raoult's law in connection with the relative lowering of vapur pressure. How can it be used for determining molecular weight of a dissolved substance.

   (b) The vapour pressure of a solution containing 13 gms. of solute in 100 gms. of water at 28°c is 27.371 mm. Calculate the molecular weight of the solute. The vapour pressure of water at the same temperature is 28.065 mm.

(6+8+11=25)

2. Distinguish between lyophobic and lyophilic colloids. How may the sign of the charge on colloidialparticle be determined? How does the charge function in determining the stability of a colloid?

   Explain why a colloidal solution is not precipitated in the presence of gelatine?

(6+6+6+7=25)

3. (a) Define the terms 'heat of formation' and 'heat of combustion'. Explain Hess's law of constant heat summation.

   (b) The heats of combustion of ammonia and hydrogen are 9.06 and 68.9 Kcals. respectively. Calculate the heat of formation of ammonia.

(4+4+6+11=25)

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INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. II Yr.

10.10.63

Periodical Examination

Time: 3 hrs.
Full marks: 100

GEOLOGY

To answer six in all.

Group A

Answer Q.2 and any two others.

1. Discuss briefly the abundances of elements in the universe, in the solar system and in the crust of the earth. Point out the significant similarities and dissimilarities. [16]

2. Are the following statements 'true' or 'false'? If 'false', make necessary corrections. [20]

   i) If there was on earth an ocean big enough to drop Juipter (planet) in, it would float because its density is lower than that of sea-water.

   ii) Our earth is the densest of all planets in the solar system.

   iii) All interstellar spaces are absolute vacuum.

   iv) NH_3 is the most abundant gas in the atmosphere of Uranus.

   v) Average surface temperature of the planet Saturn is high enough to hold oceans of water similar to our earth.

   vi) In mass as well as in volume, Venus is closer to earth than any other planet in the solar system.

   vii) The best estimate made so far of the age of the earth, is 4.5 million years.

   viii) The 'inner' planets revolve in the same direction around the sun in elliptical orbits while the 'outer' planets revolve around the sun in elliptical orbits and in a direction opposite to that of the 'inner' planets.

   ix) The sun contains over 99.8 percent of the mass of the solar system but the major part of the angular momentum of the solar system is concentrated in the planets, not in the sun inspite of the concentration of mass in the sun.

   x) Majority of the scientists believe that asteroids are parts of a defunct planet.

3. Write briefly on:-

   a) Density variations within the earth

   b) Temperature within the earth. [16]

4. What is magma? Are there as many magmas as there are igneous rocks? If not, how can one account for the considerable diversity in igneous rock types? [16]

P.T.O.

Group B

Answer any one.

5. Write short notes on :-

a) Delta b) Ox-bow Lake c) Orogeny d) Epeirogeny.

6. Discuss briefly how the surface of the earth is modified by running water.

Group C

Answer any one.

7. Distinguish between :-

a) Gneiss and Schist

b) Regional metamorphism and Cataclastic metamorphism

c) Syncline and Anticline

d) Fault and Joint.

8. Write short notes on :-

a) Primary and Secondary structures

b) Normal and Reverse Faults.

Group D

9.a) What are fossils?

b) Write short notes on :

i) Imprints ; ii) Petrifaction ; iii) Carbonisation ; iv) Cast.

INDIAN STATISTICAL INSTITUTE

Research and Training School

Mid-term examination, 1963

B. Stat. II Year

MATHEMATICS I

Duration: $2\frac{1}{2}$ hours Date: 5 Dec. 1963

Note: Answer any five questions. All questions carry equal marks.

1(a). State and prove D'Alembert's Ratio Test.

(b). For what values of x are the following series convergent and for what values are they divergent?

$$\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \dots\dots\dots \text{ ad inf.}$$

$$\sum_{1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}}\, x^n$$

(c). Find the range of values of x for which the series

$$1 + \left(\frac{5}{2}x - x^2\right) + \left(\frac{5}{2}x - x^2\right)^2 + \left(\frac{5}{2}x - x^2\right)^3 + \dots\dots \text{ ad inf. is convergent.}$$

2(a). By Taylor expansion find the expansion upto 3 terms of tan 46°.

(b). Differentiate $\left\{\log \cot \frac{x}{2}\right\} (a^2+ax+x^2)^n$ with respect to $\tan^{-1}(a \cos bx)$.

(c). Show that $\left(\frac{d}{dx}\right)^r (ax+b)^n = n(n-1)\dots(n-r+1)\, a^r (ax+b)^{n-r}$.

(d). Find the equation to the tangent and normal at the origin to the curve given by $y = ax+bx^2+cx^3$.

3. Find the derivatives of the following (Any four):

(i) $\dfrac{x}{\sqrt{(a^2+x^2)^3}}$ (ii) $\tan^n x$ (iii) $(x-a)^m (x-b)^n (x-c)^k$.

(iv) $e^{\sqrt{\cot x}}$ (v) $\left\{\text{Sin}\ (e^x \log x)\right\} \sqrt{1-(\log x)^2}$

(vi) $\text{Sin}^{-1}\left(\dfrac{a + b \cos x}{b + a \cos x}\right)$.

4. Integrate the following w.r.t. x. (Any four)

(i) $\int \frac{(1-x^2)^2}{x^2}\,dx$ (ii) $\int \frac{dx}{x \log x}$ (iii) $\int \frac{dx}{1-2x+2x^2}$

(iv) $\int \frac{dx}{(x^2+a^2)(x^2+b^2)}$ (iv) $\int x^3 e^{ax}\,dx$ (vi) $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

5. Evaluate the integrals :

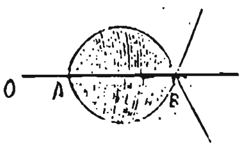
(i) $\int_0^{\pi/2} x \sin x\,dx$ (ii) $\int_2^3 \frac{x\,dx}{1+x^2}$ (iii) $\int_0^{\pi/4} \sin^2\theta\,d\theta$

(iv) $\int_0^1 \frac{dx}{1+x+x^2}$ (v) $\int_0^2 \frac{dx}{x^{3/2}}$

6(a). Evaluate $\int_2^3 x^3\,dx$ as the limit of a sum.

(b). Find the area of the oval of the parabola of the third degree given by the equation

$$cy^2 = (x-a)\,(b-x)^2$$ (Shaded part in the fig.)



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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B. Stat. II Year

MATHEMATICS II

Duration : $2\frac{1}{2}$ hours | Maximum Marks : 100 | Date: 5 December 1963

1. Prove the following theorem :- If a function $f(x)$ is continuous in a finite, closed interval $[a, b]$, then given any $\epsilon > 0$, it is possible to divide (a,b) into a finite number of subintervals, such that, if x_1 and x_2 are any two points in the same sub-interval, then, $|f(x_1) - f(x_2)| < \epsilon$ (25)

2. Define uniform continuity. Prove that, a function continuous in a finite, closed interval (a,b), is also uniformly continuous in it. (10)

Construct a function, which is continuous, but not uniformly continuous, in (a, ∞), $(0 < a < \infty)$ and, explain why it is not uniformly continuous. (7)

Construct a function, which is continuous, but not uniformly continuous, but not uniformly continuous in the open interval (a, b), $0 < a < b < \infty$, and explain why it is not uniformly continuous. (8)

3. If the inverse image, with respect to a function $f(x)$ of every open set is open, then, prove that $f(x)$ is continuous at all points. (12)

Prove the converse result also. (12)

4. Find out, whether the following statement are true, or false. If you feel a particular statement is true, write "true", and give a <u>short</u> proof; whereas, if you feel, a result is false, write "false" and give a counterexample :-

(a) An open set E, will continue to be open, even after the removal of any one of its points. (3)

(b) Any closed set E, will continue to be closed, even after the removal of an arbitrary point of E. (3)

(c) A set E is open, only if, at least one limit-point of E lies outside E. (3)

(d) Any closed set is the intersection of closed intervals. (3)

(e) Even an infinite intersection of open sets is always open. (3)

(f) Even an infinite union of closed sets is always closed. (3)

Consider the following statements. If you feel, a particular statement is true, write, simply "true", whereas, if you feel a statement is false, write "false", and, give either a counterexample, or a short proof, as the case, may be

(a) For a single-valued function f, the inverse function is also always single-valued. (3)

(b) If every open interval $(1 - \epsilon, 1 + \epsilon)$ contains infinitely many points of the sequence (a_n), then, the number 1, is necessarily the limit of the sequence (a_n). (3)

(c) The set $[2, \infty)$ is neither open, nor closed. (3)

INDIAN STATISTICAL INSTITUTE
Research and Training School
Mid-term examination, 1963

B.Stat. II Year

PROBABILITY

Duration: $2\frac{1}{2}$ hours | Maximum Marks: 100 | Date: 7 December 1963

1(a). What is Poisson Distribution. (5)

(b). Find the mean and variance of a Poisson variate. (10)

(c). If X_1 and X_2 are two independent Poisson variates, show that i) $X_1 + X_2$ is also a Poisson variate

ii) the conditional distribution of X_1, given X_1+X_2 is binomial. (10)

2(a). Four numbers are chosen at random from the set of integers (0,1,2, ...,9) and are multiplied. What is the probability that the resulting number does not contain in the units place any one of the integers (0,1,3,5,7,9)? (10)

(b) Suppose that 5 men out of 100 and 25 women out of 10,000 are colorblind. A colorblind person is chosen at random. What is the probability that the person is a male. (Assume that males and females are equal in number). (10)

3(a). State and prove Tchebycheff's inequality. (10)

(b). Prove that if X and Y are two random variables, $E^2(XY) \leq E(X^2).\ E(Y^2)$. Deduce the inequality $\mu_2^2 \leq \mu_4$ (10)

4(a). What is a moment generating function? (5)

(b). Find out moment generating functions for the following random variables.

(i) Normal with mean μ and variance σ^2

(ii) Binomial with parameters n and p. (10)

5(a). A deck of n numbered cards is put into random order. Find the mean and variance of the number of matches (cards in their natural places). (12)

(b). What is the expectation of the number of failures preceding the first success in an indefinite series of trials with probability of success p - ? (8)

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Research and Training School

Mid-term Examination, 1963

B.Stat. II Year

STATISTICS I (Theory)

Duration: 2½ hours | Maximum Marks: 100 | Date: 6 December 196?

Attempt all questions.

1(a). What do you understand by "random number tables"?

(b). If a two-digit number is taken from these tables, what is the probability that this number is divisible by 3?

(c). If 5 two-digit numbers are taken from these tables, how many of these are expected to be divisible by 3?

(d). A large number of groups of 5 two-digit numbers are taken as above. What will be the variance of the frequency of numbers divisible by 3 per group?

2(a). Describe the conditions under which a phenomenon is likely to give rise to a Poisson distribution.

(b). Under specific conditions to be enumerated by you, show that a binomial distribution can be well approximated by a Poisson distribution.

3. In a population there is a proportion p of males. Individuals are selected one by one at random until k males are found. Let ξ denote the number of individuals which it was necessary to take until k males were found.

(a) Derive the probability distribution of ξ.

(b) Calculate the moment generating function of ξ and, from this, calculate the mathematical expectation and variance of ξ.

4(a). Write down the expression for the probability density function of a random variable which has a normal distribution with parameters μ and σ —— $N(\mu, \sigma)$.

(b). Obtain the moment generating function of $N(\mu, \sigma)$ and find an expression for the r th. central moment.

(c). Given the moment generating function of $N(\mu_1, \sigma_1)$, how will you derive from this the moment generating function of $N(\mu_2, \sigma_2)$?

5. ξ_1, ξ_2, ξ_3 are discrete-valued random variables

(a). It is know that $P(\xi_1 = x_1, \xi_2 = x_2, \xi_3 = x_3) = P(\xi_1 = x_1)\, P(\xi_2 = x_2)\, P(\xi_3 = x_3)$ for all x_1, x_2, x_3. Show that ξ_1, ξ_2 and ξ_3 are pairwise independent random variables.

(b). Suppose $P(\xi_1 = x_1, \xi_2 = x_2) = P(\xi_1 = x_1)\, P(\xi_2 = x_2)$ for all x_1, x_2

Verify if $F(x_1, x_2) = F_1(x_1)\, F_2(x_2)$

where $F_1(x_1)$, $F_2(x_2)$ are the c.d.f.'s of ξ_1 and ξ_2 respectively and $F(x_1, x_2)$ the joint c.d.f. of ξ_1, ξ_2.

(P.T.O.)

(c). If ζ_1 and ζ_2 are independent random variables, verify that

$$M(t) = M_1(t)\, M_2(t),$$

where $M_1(t)$, $M_2(t)$ and $M(t)$ are the moment generating functions of ζ_1, ζ_2 and $(\zeta_1 + \zeta_2)$ respectively.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B.Stat. II Year

STATISTICS II (Practical)

Duration 3 hours | Full Marks: 100 | Date: 6 Dec.1963

1. An experiment consists in throwing 12 dice and counting 'fives' and 'sixes' as successes 26,306 such experiments were conducted, the frequency distribution of the number of successes observed is given below. Fit a binomial distribution. [30]

Number of successes in throwing 12 dice

number of success	frequency	number of success	frequency
0	185	7	1331
1	1149	8	403
2	3265	9	105
3	5475	10	14
4	6114	11	4
5	5194	12	0
6	3067	TOTAL	26,306

2. The following table shows the frequency distribution of statures of adult males born in the United Kingdom. Assuming that the distribution is normal, calculate the theoretical frequencies. Estimate the value above which 25% of the distribution is expected to lie. [30]

Height (inches)	No. of men	Height (inches)	No. of men
57–	2	69–	1063
58–	4	70–	646
59–	14	71–	392
60–	41	72–	202
61–	83	73–	79
62–	169	74–	32
63–	394	75–	16
64–	669	76–	5
65–	990	77–	2
66–	1223		
67–	1329	TOTAL	8585
68–	1230		

3. The joint probability distribution of two random variables ξ and η is given below. [20

	20	25	30	35	40	45	50
100				.01			
125		.02	.01	.07	.01		
150	.01	.01	.06	.06	.01	.02	.02
175			.06	.12	.03	.06	.05
200				.03	.08	.11	.03
225			.01	.01	.02	.03	.01
250						.01	.01
275				.01			
300						.01	

Calculate

i) The marginal distributions of ξ and η and their expectations and variances

ii) The conditional expectation of η for each value of ξ

iii) The least squares regression line of η on ξ

For questions (ii) and (iii) show your results graphically also.

Neatness—————————————————————— [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination 1963

B. Stat. II Year

ECONOMICS - I

Duration : 2 hours | Maximum Marks: 100 | Date: 2 December 1963

Note: Answer any four questions.

1. Examine the case for and against co-operative farming in India.

2. Examine, with the help of statistical data, the distinctive features of the Industrial structure of India.

3. Critically discuss the main provisions of the Industries (Development and Regulation) Act of 1951.

4. Indicate the main economic problems facing the country just before the adoption of Industrial Policy Resolution of 1948. Examine whether the provisions of the resolution were suitable to solve these problems.

5. "The 1956 Industrial Policy Resolution charts a fresh course, permitting a freedom of development in the private sector, but with checks and balances to prevent a detrimental concentration of economic power and wealth"Y - Fully examine the statement.

6. "Sectoral inter-dependence constitutes an important feature of Industrial Policy Resolution of 1956"; - Examine the statement. Indicate the economic consequences of such inter-dependence.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B.Stat. II Year

ECONOMICS - II

Duration: 2 hours | Date: 2 December 1963

Attempt Q.1 and any two of the rest. All questions carry equal marks.

1. Distinguish between replacement investment and induced investment and explain how they are related to consumption. Discuss the assumptions necessary for the validity of the acceleration principle. Point out, in this connexion, situations in which this principle does not become operative.

2. State and explain the various canons of taxation. How far do direct and indirect taxes follow these canons?

3. Classify inflation according to its degree of intensity and with respect to its causal factors. Explain the characteristics of different types of inflation.

4. Attempt any three of the following :

(a) State and prove the Haavelmo Theorem.

(b) Show that an increase in government expenditure on goods and services, ceteris paribus, has a stronger expansionary effect on national income than an increase in transfer payments of the same amount.

(c) Define tax and explain its characteristics. Distinguish between incidence and impact of a tax.

(d) What are the merits of direct taxation?

(e) State the assumptions underlying the equal sacrifice principle in taxation and explain why they are necessary.

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INDIAN STATISTICAL INSTITUTE
Research and Training School
Mid-term examination, 1963

B.Stat. II Year

PHYSICS

Duration: 2 hours | Maximum Marks: 100 | Date: 4 December 1963

Either,

1. Define the terms: moment of inertia and radius of gyration. What is the physical significance of moment of inertia? Derive expression for the moment of inertia of a solid sphere about a diameter. (6+6+6+10=28)

Or,

2. What is a compound pendulum? Obtain an expression for the time period of a compound pendulum. Show that the centres of oscillation and suspension are interchangeable. (6+12+10=28)

3. In a Kater's pendulum, the times about the two knife edges are t and $t+\epsilon$, where ϵ is very small. If the knife edges are distant L, L' from the centre of gravity, show that

$$L + L' = \frac{gt}{4\pi^2}\left\{t + \frac{2L'}{L'-L}\,\epsilon\right\}$$

(16)

4. Describe with relevant theory how a cantilever may be used to determine Young's modulus of the material of a rectangular bar. (36)

Either,

5. State the characteristics of a simple harmonic motion.

Suppose a smooth straight tunnel is bored through the centre of the earth and a body is dropped into it. Assuming the earth to be a uniform homogeneous sphere, show that the body will execute S.H.M. Calculate its period. (6+14=20)

Or,

6. Define the following : magnetic moment, magnetic line of force, dip, earth's horizontal intensity.

Given two identical steel bars, one being a magnet. How would you identify the magnet without other accessories? (3 X 4 + 8 =20)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B. Stat. II Year

CHEMISTRY

<u>Duration: 2 hours</u> <u>Date: 4 Dec, 1963</u>

1. State the Law of Mass Action and apply it to calculate the dissociation constant of a weak electrolyte.

   If the degree of ionisation of propionic acid in $\frac{N}{10}$ solution is 0.01133 at 25°c, what would be the degree of ionisation of this acid in $\frac{N}{100}$ solution at the same temperature? 4+5+6=15

2. Explain the terms "equivalent conductivity at infinite dilution" and "absolute velocity of an ion".

   How can you determine the equivalent conductivity at infinite dilution of a weak electrolyte?

   At 15°c, the conductivity at infinite dilution of HCl and CH_3COONa is 380 and 80 respectively. The transport numbers of Hydrogen and acetate ions in these electrolytes are 0.84 and 0.56 respective. Calculate the equivalent conductively for acetic acid at infinite dilution. 3+3+6+7=18

3. <u>Either</u>,

What are the Laws of Thermochemistry? Given the heat of certain reaction ~~expression~~ at constant volume, what correction will you make to convert it into heat of the same reaction at constant pressure?

If the heat of combustion of ethylene at 17°c at constant volume is 332,190 calories, what is the heat of combustion at constant pressure? 6+6+5=17

<u>Or</u>,

What is meant by the term 'molecular elevation constant' of a solvent? From the elevation of boiling point of a solvent by a solute, how can the molecular weight of solute be determined?

If the heat of vaporisation of one gm. of carbon disulphide is 86.72 calories and the boiling point is 46°c, calculate the molecular elevation constant. 5+5+7=17

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B.Stat. II Year

BIOLOGY I (Theory)

Duration: 2½ hours | Maximum Marks: 100 | Date: 3 Dec. 1963

Answer Q.1 and any three of the rest.
All questions carry equal marks.
Illustrate your answers with suitable drawings wherever necessary.

1. Write short notes on any five of the following :

   (a) Archaeopteryx, (b) Economic importance of the family Cruciferae, (c) Metatheria, (d) Androecium of Anonaceae, (e) Symbiosis, (f) Leaves of Rutaceae, (g) Camouflage in animals.

2. What are Protochordates? On what evidence are these included in the phylum Chordata.

3. Mention the important characteristic features of the family Leguminosae. Give a comparative account of the sub-families of Leguminosae, mentioning names of ten plants of the family.

4. What is metamorphosis? Describe the process with the help of suitable diagrams in any animal known to you, mentioning its significance.

5. Describe with illustrations one typical flower of the families Cruciferae and Malvaceae. Give the floral diagram and formula for each flower.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B. Stat. II year

BIOLOGY II (Practical)

Duration: 3 hours | Maximum Marks: 100 | Date: 3 December 1963

1. Draw a fully labelled diagram of specimen A. Record your observations on the specimen. (20)

2. Give a botanical description of specimen B and assign the same to the family, giving reasons. (20)

3. Give floral diagrams and formulae of specimens C and D. (20)

4. Identify and comment on specimens E to N. (20)

5. Practical Records. (20)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

B. Stat. II Year

Periodical Examination

MATHEMATICS

Duration: 1 hr. 15 mins. Date: 24.2.64

All questions carry equal marks

1(a). Find the length of arc from $(0,0)$ to $(x_1\ y_1)$ on $y^2 = 4ax$.

(b). Find the length of the whole cardioide given by the equation $r = a(1+\cos\theta)$.

2(a). Find the volume of the curve given by

$x^2/a^2 + y^2/b^2 = 1$ about y – axis

and ordinates $y = -b$, $y = +b$.

(b). Find the volume of the curve given by

$r^2 = a^2 \cos 2\theta$ about x – axis

and ordinates $x = 0$, $x = a$.

3. Evaluate the integrals

(i) $\int_0^\infty e^{-x}\ x^{p-1}\ dx$ where p+ve integer.

(ii) $\int_0^{\pi/2} \frac{\log\,(1 + \cos\alpha \cos x)}{\cos x}\, dx$

(iii) $\int_0^{\infty} \frac{\tan^{-1}(bx)}{x(1 + x^2)}\, dx$

4. Solve the following differential equations :

(i) $\left(y - x\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$.

(ii) $(x^2 - yx^2)\frac{dy}{dx} + (y^2 + xy^2) = 0$.

(iii) $(x^3 + xy^2)\frac{dy}{dx} = (y^3 - x^2y)$

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INDIAN STATISTICAL INSTITUTE
Research and Training School

B. Stat. II Year
Periodical Examination

ANALYSIS

Duration 1 hr. 30 mins. | Max.Marks: 100 | Date: 24.2.64

1. Of the five statements given below, one does not make sense, while the others are false. Classify them accordingly, giving a detailed explanation in each case :

(a). There are functions $f(x)$, which are uniformly continuous at a point c. (10)

(b). Let $g(x) = \frac{1}{x^2}$. Then $g(x)$ is continuous at $x=0$, since $g(0) = \infty$, and $\frac{1}{x^2} \longrightarrow \infty$, as $x \longrightarrow 0$, from either side. (3)

(c). The function $h(x) = \frac{2x+3}{2x+3}$ is continuous at all points. (20)

(d). If a function $f(x)$ is continuous at a point x_0, then in view of the definition of continuity at a point, it follows that, for some $\delta > 0$, $f(x)$ is continuous at all points of the interval $(x_0 - \delta, x_0 + \delta)$. (Of course, this δ may be very small). ([illegible])

(e). Even if $f(x)$ is continuous in an interval E, $f(x)$ may fail to be derivable at some points of E. However if $f(x)$ is uniformly continuous in E, then $f(x)$ is necessarily derivable at all points of E. (7)

2(a). Is it possible to construct a function, which is continuous everywhere, but not derivable at any point? If so, mention the name of a mathematician, who has given one such example.

Or,

Give two examples, in support of the following statement (no proof is necessary) :

"Even if $f(x,b)$ is continuous at $x = a$, and $f(a,y)$ is continuous at $y = b$, $f(x,y)$, may fail to be continuous at (a,b). (5)

(b). Construct two distinct functions, which are continuous everywhere, but not derivable at a given point 'c', $(c \neq 0)$, and give a simple geometric interpretation in at least one case. (15)

3(a). "Even if $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist, at the point (a,b), $f(x,y)$ may not be continuous at (a,b)." Explain how this possible, and give at least two examples. (15)

(b). Prove that, $f(x,y) = \frac{x^{2k} y^{2k}}{(x^k + y^{2k})^3}$, $(x,y) \neq (0,0)$, $(k > 1)$,

$= 0$, if $x = 0 = y$, is..

discontinuous at the origin, though, $f(x,y) \longrightarrow 0$, as $(x,y) \longrightarrow (0,0)$ along any straight line. (15)

INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. II Year

STATISTICS

Duration: 2 hours | Maximum marks: 100 | Date: 23 March 1964

1. Assume that there are about 4×10^7 households in India. It is required to determine, by means of a complete census of all households, the average size of household and the average yearly income per household for a given year.

   (a). Enumerate as many possible causes as you can, for arriving at inaccurate values of the above averages. (10)

   (b). Enumerate other disadvantages of the method of complete census in the above case. (10)

2. A population consists of 5 families A_1, A_2, A_3, A_4, A_5 whose annual expenditure on food (in some convenient unit) are respectively 10, 15, 13, 18, 24. It is desired to estimate the average annual expenditure on food per family by taking a sample of 2 families and calculating the average expenditure of these two sample families.

   Calculate the sampling variance of the estimate for each of the following 3 procedures of drawing a sample :

   (a) Simple random sample, with replacement. (10)

   (b) Simple random sample, without replacement. (10)

   (c) Take family A_1 and select one of the remaining 4 with equal probability. (10)

3(a). (x_1, y_1), (x_2, y_2) and (x_3, y_3) are three points which do not lie on a straight line. State, with reasons, how many polynomials in x of (i) order 1, (ii) order 2, (iii) order 3 can be made to pass through these points. (12)

(b). Consider the function $f(x) = e^x$ for values of x in the interval $[0, 0.2]$. It is desired to estimate the values of $f(x)$ by means of a polynomial, and the values need be correct up to 4 places of decimal. What should be the smallest order of polynomial to be selected? (13)

4. Let $f(x)$ be a function whose values at n+1 equidistant points $x_0, x_1, \ldots, x_n$ are known. Show how to obtain the value of

   (a) $\Delta^r f(x_0)$ in terms of the given values of the function; (12)

   (b) $f(x_r)$ in terms of $f(x_0)$ and the differences $\Delta f(x_0), \Delta^2 f(x_0), \ldots$ (13)

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INDIAN STATISTICAL INSTITUTE
Research and Training School
Periodical Examination
B. Stat. II Year

ECONOMICS

Duration : 2½ hours | Full Marks: 100 | Date: 2 March 1964

Note: Separate Answer booklet should be used for each group.

GROUP A

Indian Economics

Answer any two questions.

1. Discuss the nature and growth of Public enterprise in India. (25)

2. "Public enterprise may be an important instrument in achieving the objective of reduction of inequalities in income and wealth and a more even distribution of economic power." – Discuss the implications of the statement in the Indian context. (25)

3. Examine critically the recommendations of the Fiscal Commission, 1921-1922. (25)

GROUP B

Economic Theory

Answer any two questions.

4. How does the theory of comparative costs explain international trade? What are the criticisms against this theory? (25)

5. Summarise the principal arguments in favour of free trade and protection. (25)

6. Assuming constant prices and rate of exchange, show how incomes and balance of payments change in a two-country model when

   (a) there is an increase of autonomous exports in one of the countries.

   (b) there occurs an increase of autonomous investment in one of the countries. (25)

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INDIAN STATISTICAL INSTITUTE
Research and Training School
Periodical Examination
B.Stat. II Year

PHYSICS THEORY

10.2.64

Time: 1 hr. 15 min.
Full marks 50

Answer Q. No.5 and any three of the rest.

1. Derive the relation between the pressure P and the volume V of an ideal gas undergoing an adiabatic change. Show that the work done by such a gas in expanding from a state (P_i, V_i) to a state (P_f, V_f) is

$$W = \frac{P_i V_i - P_f V_f}{\lambda - 1},$$

where λ is the ratio of the principal specific heats of the gas. [13]

2. If a working substance in a Carnot Engine is a perfect gas, derive an expression for the work done in each operation of the cycle. Hence obtain the expression for the efficiency of the Engine. [13]

3. State the second law of thermodynamics. What is its significance? Write down the set of Maxwell's thermodynamical relations and obtain the Clausius Clapeyron's equation. [13]

4. What do you understand by Joule-Kelvin effect? Show that for a Vander Waal fluid the inversion temperature is 2a/bR where a, b are the Vander Waal's const. and R is the universal Gas Const. [13]

5. A mass m of water at T_1 is isobarically (const. pressure) and adiabatically mixed with an equal mass of water at T_2. Show that the entropy change of the universe is

$$2mC_p \ln \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$$

where C_p is the specific heat of water at constant pressure. Prove that the expression is positive. [11]

INDIAN STATISTICAL INSTITUTE
Research and Training School
Periodical Examination
B.Stat. II Year

10.2.64. CHEMISTRY (Theory) Full Marks : 50

Time : 1 hr. 15 min

1. What is the influence of a common ion on the dissociation of a weak electrolyte? How would the acidity of an aqueous solution of acetic acid be influenced by adding solid sodium acetate to this solution?

   Given that the dissociation constant of acetic acid at $25^{o}c$ is 1.8×10^{-5}, find the p^{H} value of (a) a solution containing 0.105 gram mole of acetic acid and 0.015 gram mole of sodium acetate per litre and (b) a solution containing 0.019 gram mole of sodium acetate per litre.

2. What do you understand by 'Chemical equilibrium' and 'equilibrium constant? Derive an expression of 'equilibrium constant' for a generalised equation. Establish a relation between K_p and K_c.

   Calculate the value of K_c for the reaction $\frac{3}{2} H_2 + \frac{1}{2} N_2 \rightleftharpoons NH_3$ at $400^{o}c$ from the value of K_p which is equal to 0.0129 atmos.

3. Write notes on: (a) Solubility product, (b) Beer's and Lambert's law and (c) p^{H} of a solution.

INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. II Year

BIOLOGY (Theory)

Duration : 2½ hours | Maximum Marks: 100 | Date: 13 April 1964

All questions carry equal marks

1. Mention the important characteristic features of the family compositae. Give the names of five plants of this family. Describe any one of them with illustrations including floral formula and floral diagram.

2. Write short notes on :-

   a) Aestivation in malvaceae,

   b) Androecium in Scitamineae,

   c) Economic importance of Palmae,

   d) A typical spikelet of Gramineae.

3. Explain very briefly the following :

   a) Replum,

   b) Hesperidium,

   c) Lomentum,

   d) Stylopodium,

   e) Lodicules,

   f) Septifragal capsule,

   g) Sinuous stamen,

   h) Free central placentation,

   i) Tetradynamous stamens,

   j) Thalamiflorae.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examinations, 1964

B. Stat. II Year

MATHEMATICS I (Calculus)

Duration : 3 hours | Maximum Marks: 100 | Date: 21 May 1964

Attempt all questions

1. Show that

(i) $$\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$$

(ii) For what positive values of x the following series is convergent and what values is it divergent?

$$\frac{x}{x+1} + \frac{x^2}{x+2} + \frac{x^3}{x+3} + \ldots\ldots\ldots$$ (7+10)

2. Find $\frac{dy}{dx}$ in the following examples :

(i) $$x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$$

(ii) $$y = \tan^{-1}\sqrt{1+x^2}$$ (6+4)

3. If $y = \left\{\sinh^{-1} x\right\}^2$ show that

$$(1+x^2)\, y_{n+2} + (2n+1)\, x y_{n+1} + n^2 y_n = 0$$

where suffixes denote successive differentiations. (8)

4. Find a reduction formula for $\int e^{ax} \cos^n x \, dx$ where n is a positive integer and evaluate

$$\int e^{ax} \cos^4 x \, dx.$$ (7)

5. Integrate the following :

(i) $\frac{x}{(x^2+a^2)\sqrt{x^2+b^2}}$ (ii) $\sqrt{1+\sin x}$ (iii) a^x. (5+5+5)

(Please turn over)

6. Show that in the catenary

$y = c \operatorname{Cos} h \frac{x}{c}$, the length of arc from the vertex (where $x=0$) to any point is given by $s = c \operatorname{Sin} h \frac{x}{c}$. (7)

7(a). Find the area bounded by any sector of $r^{\frac{1}{2}}\theta = b^{\frac{1}{2}}$ ($\theta = \alpha$ to $\theta = \beta$)

(b). If s be the length of the curve $r = a \tan h \frac{\theta}{2}$ between the origin and $\theta = 2\pi$ and A the area between the same points, show that $A = a(s - a\pi)$ (5+8)

8. Determine the entire volume of the sphere which is generated by the revolution of a circle of radius r around a diameter. (5)

9. Solve the following differential equations :

(i) $y = px + p^3$ where $p = \frac{dy}{dx}$.

(ii) $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = e^x + x^4$

(iii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + q^2y = x \operatorname{Sin}(\log x)$. (6+6+6).

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. II Year

<u>MATHEMATICS II (ANALYSIS)</u>

<u>Duration : 3 hours</u> | <u>Maximum Marks : 100</u> | <u>Date: 22 May 1964</u>

1. Some statements are given below. If you feel that a particular statement is correct, write "true" and give a short proof; whereas if you feel that a particular statement is false, write "false" and produce a counterexample.

   (a) Every closed set contains a non-empty open set.

   (b) Every open set contains a non-empty closed set.

   (c) Any infinite set of positive numbers has at least one limit point. (3+3+3)

2. Define the terms : Perfect set, isolated point. Give at least one example in each case. (10)

3. For a bounded infinite set S, define the concepts of infimum, inferior limit, superior limit and supremum.

   Give examples to show that the following cases can occur :

   (i) infimum < inferior limit < superior limit = supremum

   (ii) infimum < inferior limit = superior limit = supremum

   Prove that for a bounded infinite set the possibility
   infimum = inferior limit = superior limit = supremum
   can never arise. (16)

4(a). Construct a function which is continuous at only one point x_0 $(x_0 > 0)$.

(b). If $f(x)$ is continuous throughout the real line and takes only rational values, prove that $f(x)$ is a constant. What property of continuous functions is made use of in this connection? (5+8)

5. If $f(x) = x^2+3x$ in $[-1, 1]$ and $\epsilon > 0$ be a given number, find the smallest δ' such that $|f(x_2) - f(x_1)| \leq \epsilon$ whenever $|x_2 - x_1| < \delta$, for any x_1, x_2 in $[-1, 1]$. (12)

(Please turn over)

6. State and prove the Brouwer Fixed - Point theorem in one dimension.

7. If (a_n) and (b_n) be two bounded sequences. Such that $(a_n - b_n) \to 0$, then prove that they will have the same limit points.

8(a). Prove that a differentiable function of one variable is necessarily continuous.

(b). Illustrate the following possibility with an example :

(a, b) is a point in the plane and $f(x, y)$ is defined on the plane. Even if

$$\left[\frac{d\, f(x,b)}{dx}\right]_{x=a} \quad \text{and} \quad \left[\frac{d\, f(a,y)}{dy}\right]_{y=b}$$

exist, still $f(x,y)$ may fail to be continuous at (a, b).

9. Prove that the sequence $\sqrt{11}$, $\sqrt{11+\sqrt{11}}$, $\sqrt{11+\sqrt{11+\sqrt{11}}}$, ... converges.

Then prove that the limit is the positive root of $x^2 - x - 11 = 0$.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. II Year

PROBABILITY AND STATISTICS

Duration : 3 hours | Maximum Marks: 100 | Date: 19 May 1964

Separate Answer-book should be used for each Group

GROUP A

1(a). State the central limit theorem of the theory of probability. (3)

(b). Find the approximate probability that among 10,000 random digits the digit 7 appears not more than 968 times. (7)

2. Let $X_1, X_2, \ldots, X_{2n}$ be independent identically distributed random variables taking the values 0 and 1 with probability $\frac{1}{2}$ each. Let $S_n = (X_1 + \ldots + X_{2n}) / n$ and $\phi_n(t) = E(e^{t S_n})$.

Show that $\phi_n(t)$ converges to e^t as $n \to \infty$. Deduce the limiting distribution of S_n. (16)

3. X and Y are two random variables such that $P(X=0) = P(X=1) = \frac{1}{2}$. $P(Y = j / X = 0) = 2^{-(j+1)}$, $P(Y=j / X = 1) = 2 \times 3^{-(j+1)}$, $j = 0, 1, 2, \ldots$. Find the probability distribution of $X+Y$. (7)

4. Two players A and B with capitals a and x−a rupees play a game as follows. Toss a coin; if the result is H, A pays one rupee to B, if T then B pays one rupee to A. This is repeated, the tosses in successive stages of the game being all mutually independent and identically distributed with H and T having probabilities p and q respectively. The game is to terminate as soon as one of them is ruined.

(a) If $a = 10$ and $x = 30$ find the probability distribution of A's net gain at the end of 5th stage of the game. What is the expected value of B's net gain at this time? (10)

(b) If $a = 6$ and $x = 20$, $p = q = \frac{1}{2}$ find the probability that the game will terminate at the 7th stage. (7)

(Please turn over)

GROUP B

1. $\xi, \xi_1, \ldots, \xi_n$ are random variables.

(a). Define a "moment generating function" (m.g.f) of ξ. (3)

(b). Show that if ξ_1 and ξ_2 are independent, the m.g.f. of their sum is the product of their respective m.g.f.'s. (3)

(c). If $P(\xi=1) = p$ and $P(\xi=0) = q(=1-p)$, find the m.g.f. of ξ. (2)

(d). By using the results of (b) and (c) above, obtain the m.g.f. of the number of successes in a sequence of n independent Bernoulli trials with probability p of success at each trial. (3)

(e). Show that under specified conditions the m.g.f. obtained in (d) tends to the m.g.f. of a Poisson variable. (5)

2(a). Discuss, with the help of a specific example how stratified sampling is sometimes preferable to simple random sampling from considerations of physical convenience. (4)

(b). To estimate a characteristic of a population from a sample of given size, several unbiased estimators have been found. Explain why you would prefer one with the smallest sampling variance. (4)

(c). A population of size N is divided into L strata of sizes $N_1, \ldots, N_L$, with stratum means $\bar{X}_1, \ldots, \bar{X}_L$ and stratum variances $S_1^2, \ldots, S_L^2$, respectively, of a particular variable X. For a stratified simple random sampling with replacement, find the allocation of a total sample size n into stratum sample sizes $n_1, \ldots, n_L$ in order to have an unbiased estimator of the population mean with the smallest variance. (9)

3(a). Define a "divided difference" and obtain an expression for a divided difference of order n, which is symmetrical in the arguments. (4)

(b). Show that, if the arguments are in increasing or decreasing order at equal intervals, a divided difference is proportional to an ordinary difference of the same order. (3)

(c). Derive Lagrange's interpolation formula, starting from the definition of a divided difference. (5)

(d). Show that any function whose values are known at $n+1$ points can be written as the sum of Newton's divided difference polynomial of order n and a remainder term which contains a divided difference of $n+1$ as a factor.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. II Year

STATISTICS PRACTICAL

Duration : 3 hours | Maximum Marks: 100 | Date: 20 May 1964

1. The following table show the distribution of heights (in inches) of 500 individuals :

class interval	frequency	class interval	frequency
59.5 – 61.5	1	69.5 – 71.5	113
61.5 – 63.5	10	71.5 – 73.5	52
63.5 – 65.5	37	73.5 – 75.5	15
65.5 – 67.5	104	75.5 – 77.5	6
67.5 – 69.5	160	77.5 – 79.5	2
		Total	500

(a). Calculate β_1 and β_2 of the above distribution and suggest if a normal distribution is likely to fit the data. (20)

(b). Assuming that a normal distribution does fit the data, calculate the expected frequencies in the class intervals 59.5 – 61.5, 65.5 – 67.5, 77.5 – 79.5. (10)

2. The following table gives the frequency distribution of yield of dry bark in ounces (X) and age in years (Y) of 125 cinchona plants.

age in years (Y) \ yield in ozs. (X)	4–7	8–11	12–15	16–19	20–23	24–27
3 – 4	2					
5 – 6	3	6	3			
7 – 8	3	8	10			
9 – 10		2	10	10	6	
11 – 12			8	15	15	4
13 – 14			2	4	10	4

For purposes of studying yield of dry bark (X), it is suggested that the plants be divided into two strata– those of age 8 years or less, and those of age 9 years or more.

(Please turn over)

2(a). Calculate the variance of X for the whole population, as well as for the two strata (S^2 values). (7)

(b). For stratified simple random sampling without replacement, what is the allocation of a sample of size 20 in the two strata, which will give an unbiased estimator of the population mean $\bar{X}$ with minimum variance? (5)

(c). Calculate the variance of

(i) the mean of a simple random sample of size 20 without replacement from the whole population. (5)

(ii) an unbiased estimator $\bar{X}$ from a statified sample of size 20 with proportional allocation in the two strata, sampling being without replacement. (5)

(iii) the same estimator as in (ii), allocation being optimum. (5)

(d). Assuming that the sample averages follow normal distributions, for each of the cases (i), (ii) and (iii) above, obtain intervals around the sample averages within which the population mean can be said to lie with 95% confidence. (8)

3(a). The following table gives the values of $\log_{10}x$ for various values of x.

x	$\log_{10}x$
1.260	.1003705
1.261	.1007151
1.262	.1010594
1.263	.1014034
1.264	.1017471
1.265	.1020905
1.266	.1024337
1.267	.1027766

Find the values of $\log_{10} 1.2614$ and $\log_{10} 1.2667$. (20)

(b). The following values of a certain function $y = f(x)$ are known.

x	$y = f(x)$
0.10087	0.99398
0.22798	0.97367
0.30506	0.95233
0.41687	0.90897
0.51414	0.85771
0.59720	0.80210

Find the value of x if $y = 0.90000$. (15)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. STAT. II Year

ECONOMICS I (Economic Theory)

Duration : 3 hours | Maximum Marks: 100 | Date: 18. 5. 64

Attempt any five questions. All questions are of equal value.

1. Show that every point of tangency between an isoquant and an iso-cost line is the solution of the problems of both constrained output maximization and constrained cost minimization. What inference from this solution can you draw about the shape of isoquants?

2. What are the basic postulates of the marginal productivity theory of distribution? Explain the conditions under which they are fulfilled. What results would you get if the production function is homogeneous of degree one?

3. Explain how collective bargaining affects wages and employment under different market forms.

4. Discuss the arguments against freely fluctuating exchange rates.

5. Under what circumstances does a government prefer over-valuation of the home currency and exchange-control?

6. How does a change in the rate of exchange affect (a) the prices of imports and exports, and (b) the terms of trade?

7. "The way in which the budget affects the quantity of money and liquidity preference is of considerable importance in determining the total effect of a given combination of government expenditure and revenue on national income."
— Critically examine the statement.

8. Attempt any TWO of the following :-

   (i) Show that under perfect competition in the factor market, the quantity of a factor demanded must, ceteris paribus, increase with a reduction in its price.

   (ii) What is scale elasticity of a process? Show that for minimum cost combinations it is equal to the quotient of average costs and marginal costs.

   (iii) Prove that an increase in government expenditure on goods and services, ceteris paribus, has a stronger expansionary effect on national income than an increase in transfer payments of the same amount.

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INDIAN STATISTICAL INSTITUTE
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Annual Examination, 1964

B. STAT. II Year

ECONOMICS II (Indian Economic Conditions)

Duration : 3 hours | Maximum Marks : 100 | Date: 18 May 1964

Answer any FIVE questions.

1. Critically examine the main provisions of the Industries (Development and Regulation) Act of 1951. (20)

2. "The 1956 Industrial Policy Resolution charts a fresh course, permitting a freedom of development in the private sector, but with checks and balances to prevent a detrimental concentration of economic power and wealth." - Fully examine the statement. (20)

3. "The Indian Fiscal Commission of 1949-50 approached their task from a new angle of vision and laid down new principles of protection." - Elucidate the statement. (20)

4. Discuss the nature and growth of public sector enterprises in India since 1950-51. (20)

5. Analyse the factors which led to the formation of special types of financial and development Corporations for Indian industries during the post-independence period. (20)

6. Examine the pattern of foreign business investments in India since 1948. (20)

7. Analyse with the help of statistical data the nature and forms of Industrial Combinations in India. (20)

8. Write notes on any two of the following :-

   (a) The proportion of national income and the distribution of labour force in the Indian industrial sector.

   (b) The role of Cottage and Small-Scale Industries in the Indian economy.

   (c) The National Industrial Development Corporation. (20)

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INDIAN STATISTICAL INSTITUTE
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Annual Examination, 1964

B. Stat. II Year

PHYSICS (Theory)

Duration : 3 hours | Maximum Marks: 100 | Date: 23 May 1964

Answer Question 2 and any FIVE of the rest

1. Distinguish between isothermal and adiabatic changes in a gaseous system. Show that the slope of an adiabatic line on an indicator diagram is steeper than that of an isothermal.

   After detonation of an atom bomb, the ball of fire consisting of a sphere of gas was found to be of 50 ft. radius at 3×10^5 degree absolute. Assuming adiabatic condition to exist, find the radius of the ball after 100 milliseconds when its temperature is 3×10^3 degree absolute. $\gamma = 1.66$. (4+5+7=16)

2(a). A certain reversible heat engine absorbs 8 kilocalories of heat at a temperature of 200°C and rejects its exhaust into a low temperature reservoir at 80°C. Compute (i) the efficiency of the engine, (ii) work done by the engine and (iii) the amount of heat rejected.

(b) A mass m of a liquid at T_1° abs. is isobarically (const. pressure) and adiabatically mixed with an equal mass of the same liquid at T_2° abs. Show that the entropy change of the universe is $2\, m.\, C_p .\, \log_e \dfrac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$, where C_p is the sp. heat of the liquid at constant pressure. Show that it is positive. (9+11=20)

3. What is a Cantilever? Find an expression for the depression at the free end of the Cantilever by a load, in terms of the Young's modulus of the material forming the cantilever and its geometry. (3+13=16)

4. Define the terms: moment of inertia and radius of gyration.
   Obtain an expression for the time period of a compound pendulum. Show that the centres of oscillation and suspension are interchangeable. (3+3+5+5=16)

(Please turn over)

5. A circuit contains an inductance L, a resistance R and a source of steady e.m.f. E. What would be the current at any time t after the circuit is closed?

   Show that in an A.C. circuit, the peak value of the current is $\sqrt{2}$ times the r.m.s. value. (10+6=16)

6. Explain clearly the meaning of the 'resolving power' of an optical instrument. Show that the resolving power of a plane transmission grating is equal to the product of the number of rulings in the grating and the order of the spectra.

   What is the greatest number of order observable using a plane grating with 3000 lines per cm. and normally incident light of $\lambda = 5460$ Å ? (6+6+4=16)

7. Describe Michelson's interfermeter. Explain how circular fringes are produced in it. (8+8=16)

8. Show that a plane polarised light beam incident on a uniaxial crystal cut with its optic axis parallel to its surface may emerge as plane polarised, elliptically polarised or circularly polarised depending on the thickness of the crystal. What is a quarter-wave plate? (14+2=16)

9. Write short notes on any <u>three</u> of the following :

   (a) Nicol prism

   (b) Optical activity and polarimeter

   (c) Newton's rings

   (d) Power factor and its significance

   (e) Simple harmonic motion

   $(3 \times 5\frac{1}{3} = 16)$

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INDIAN STATISTICAL INSTITUTE
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Annual Examination, 1964

B. Stat. II Year

CHEMISTRY

Duration : 3 hours | Maximum Marks: 100 | Date: 23 May 1964

Separate Answer-book should be used for each Group

Answer three questions from Group A and two from Group B. All questions carry equal marks.

GROUP A

1(a). Explain fully what is implied by the statement "substances in dilute solutions obey the gas laws".

Calculate the molecular weight of a substance given that the Osmotic pressure of 2 per cent solution of that substance is 500 mm. of mercury at 10°c.

(b). Will the Osmotic pressure of $\frac{M}{10}$ solution of NaCl be same that of a $\frac{M}{10}$ Cane sugar solution? Give reasons for your answer.

2(a). State the Principle of LeChatelier and discuss with its help the effect of temperature and pressure on the following equilibrium : $N_2 + 3H_2 \rightleftharpoons 2\ NH_3 + Q$ Calories. Calculate the value of Kp in this equation.

(b). A 2 per cent solution of nicotine in water boils at 100.062°C. Calculate the molecular weight of nicotine. (The latent heat of vaporisation of water is 537).

3. What is a Colloidal solution? How does it differ from a true solution? How can you demonstrate the charge on Colloidal particles? Explain how does the charge on Colloidal particle affect the stability of Colloid?

4(a). Distinguish between 'ionic mobility' and 'absolute ionic velocity'. How are they related to each other? State Kohrausch's Law of independent migration of ions.

(b). The equivalent conductivity of sulphuric acid at infinite dilution is 384 reciprocal ohms. If the specific resistance of a solution containing 15 gm. Sulphuric/acid per litre is 18.4 Ohms, calculate the apparent degree of ionisation of this solution.

5. Give a short account of the elementary ideas on physico-chemical methods of analysis.

(Please turn over)

GROUP B

6(a). Write down the structural formula of Sulphuric acid. How can you prove that sulphuric acid contains two hydroxyl groups attached to the same sulphur atom?

(b). State briefly the chemical nature of cement and give an account of the chemical reactions take place during its manufacture.

7. The elements of the second period of the Periodic classification, their atomic weights, and atomic numbers, are given below :

	Na	Mg	Al	Si	P.	S.	Cl	A.
Atomic Wt.	23.0	24.3	27.0	28.0	31.0	32.0	35.46	39.94
Atomic No.	11	12	13	14	15	16	17	18

Explain the typical valencies exhibited by these elements from the standpoint of atomic structure. What explanation is given for the fact that the atomic weight of chlorine is not approximately a whole number?

8. Give an account of the periodic classification of the elements explaining particularly the meaning of the terms, group, short period, long period, atomic number and isotope.

9. Give an account of an electrolytic method by which caustic soda is manufactured.

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INDIAN STATISTICAL INSTITUTE
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Annual Examination, 1964

D.STAT. II Year

BIOLOGY (Theory)

Duration: 3 hours | Maximum Marks: 100 | Date: 25 May 1964

All questions carry equal marks.

1. Write the characteristic features of the family Leguminosae. Give a comparative account of its sub-families mentioning names of 5 plants belonging to each.

2(a). Write a comparative account of the androecium in Malvaceae, Compositae and Euphorbiaceae with illustrations.

(b). Write a short note on the characteristic features of the vegetation of a physiologically dry soil.

3. Write short notes on any five of the following :

(a) Calyciflorae
(b) Fruits of Rutaceae.
(c) Economic importance of Gramineae.
(d) Plant Growth substances.
(e) Grand period of growth.
(f) Ascent of sap.
(g) Photosynthesis.
(h) Root nodule.

4. What are the principal ecological factors that influence a vegetation?

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B.Stat. II Year

BIOLOGY (Practical)

Duration : 3 hours   Maximum Marks: 100   Date: 25 May 1964

1. Give a detailed botanical description and draw a fully labelled sketch of specimen A.

   Identify the specimen upto family mentioning reasons. (10+5+5=20)

2. Cut a transverse section of specimen B and mount the same.

   Draw a labelled sketch showing the different tissues and comment on the ecological adaptation of the species from its anatomical evidence. (5+10+5=20)

3. Comment on C, D, E, F and G. (5 X 4 = 20)

4. Identify the specimens H to Q. (10 X 2 = 20)

5. Practical Records and Field Note. (15+5=20)

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