INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1996-97
SEMESTRAL-I EXAMINATION
GEOLOGY

Date: 15.11.96 Maximum Marks:100 Time: 3 Hours

Note: Attempt Question No.1 and any FIVE from the rest. Answer should be as brief as possible. No over-writing please.

1. Fill up the blanks (any 10). Only write one of the four choices for each blank. (2x10)=20

   (a) The internal structure of muscovite consists of ________ (3-D network/single chain/double-chain/sheet-like arrangement) of silicate tetrahedra.

   (b) The dark colour of an igneous rock is due to the presence of ________ (Fe-Mg/Si-O/Ca-Na/K-Cl) elements in relatively large amount.

   (c) State is a ________ (terrigenous/metamorphic/igneous/nonclastic) rock.

   (d) Petroleum is a/an ________ (crystalline/mineral/organic/inorganic) substance.

   (e) A rock composed of 40%. by volume of angular pebbles is called ________ (breccia/mudstone/eclogite/conglomerate).

   (f) The ________ (thickness/chemical composition/cross bedding/ colour) helps to determine the top surface of a sedimentary layer.

   (g) The ________ (muscles/eyes/teeth /blood cells) of a Dinosaur are best preserved in rocks.

   (h) A horizontal sequence of beds lying over a tilted sequence of beds has a/an ________ conformable unconformable/ sedimentary/metamorphic) contact relationship.

   (i) Presence of herbivorous, four-legged animal fossils indicates a ________ (marine/deep marine/shallow marine/terrestrial) environment of deposition.

   (j) The Indian Gondwana rocks are ________ continental/extra terrestrial /shallow marine/deep marine) sediments.

   (k) The principle of lateral continuity is propounded by ________ (cuvier/smith/steno/Hutton).

   (l) Early Homo-Sapiens appeared in the earth during ________ (Jurassic/Triassic/Pleistocene/recent) time.

contd. ....2.

2. Draw a cross-section of the Earth showing the depth of the boundaries of the crust, lithosphere, asthenosphere and core.
What is elastic rebound theory?
How does the 'chemical-condensation sequence' model explain the differences among the terrestrial planets?
At what depth would you expect the rocks in a region to loose their magnetism where the temperature-increase with depth is 30^{o}C per hundred meters? (4x4)=16

3. The stratigraphic events of an area is as follows:
   i) deposition and lithification of sand into a sandstone
   ii) extrusion of thick lava
   iii) deformation (folding,etc) of i and ii
   iv) erosion
   v) deposition and lithification of sand into another sandstone
   vi) intrusion by igneous dykes
   vii) erosion
   viii) deposition of shales

   Draw a schematic stratigraphic cross-section to illustrate the above events.
4. Why do fossils always provide relative ages?
Is it possible to get fossils of younger horizons in an older rock? If so, explain the sitution. Under what conditions you would expect the soft parts of an animal preserved? (4x4)=16
5. Define erosion and weathering. Indicate their role in the production of sediments. Describe the different weathering processes. What is the role of weathering in the formation of different geomorphological features on the surface of the earth.? (4+8+4)=16
6. What is a fold and its axis? What do you understand by the terms: ductility, competency and incompetency of rocks?
How do you think the great Himalayan mountains have formed? (4+6+6=16)
7. What is the essential characteristic of silicate structure? How can pyroxene, amphibole biotite, feldspar and quartz be differentiated on the basis of silicate structure?
Which are the first and the last common silicate minerals to crystallize out of a magma? (4+10+2)=16
8. Write short notes as (any <u>four</u>) —
Rock-forming minerals; charecteristics of magma;
metamorphism of mudstone; coal and pearl;
law of superposition; P- and S- waves;,sphalerite and cassiterite.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year: 1996-97
BACKPAPER SEMESTRAL - I EXAMINATION
Economics I

Date: 2.1.1997 Maximum Marks: 65 Time: 2 hrs. 15 minutes

Note: Answer ALL the questions.

1. Clearly state and explain the assumptions that ensure the existence of a continuous utility function associated with a consumer's preferences.

(6+7) = [13]

2. A firm has the long-run production function $x = aK^{.5}L^{.5}P^{.25}$, where x = output, a is a positive constant and K, L and P are the inputs of the three factors. The price of the three factors are 1, 9 and 8. In the short-run K and L are variable, P is fixed. Derive an equation in the form $P = f(x)$ showing the optimum quantity of the fixed factor P for the firm to acquire as a function of the intended output x.

[10]

3. Clearly distinguish between:

(a) Cooperative inputs and competitive inputs.

(b) Engel curve and income - consumption curve.

(3 x 2) = [6]

4. Write a short note on a firm's short-run expansion path. Give a sufficient condition for this path to be linear.

(2+4) = [6]

5.(a) If the long-run cost function of firm under perfectly competitive market is $C(x) = x^2 + 1$, what will be its long-run supply curve ?

[4]

(b) A monopolist with cost function $C(Q) = 1.1 + 2Q$ faces a market demand $P = 4 - Q$ (P = price, Q = quantity). Derive its output in equilibrium.

[4]

p.t.o.

6. Suppose the market demand curve is given by $P = 300 - Q$ and the market supply curve by $P = 60 + 2Q$. Find the initial solution of price and output. If the per unit tax of $T = 15$ is imposed, what is the new equilibrium ? Estimate the loss of welfare.

(2+3+7) = [12]

7. Consider an oligopoly market for a homogeneous product. The market demand function is $P = a - \sum_i q_i$, when P is the price of the product and q_i is the ith firm's quantity demand. In the industry there are only two identical firms to supply the product. The ith firm's total cost function is $C_i = c\,q_i$, $c < a$.

   (i) If both firms believe in Cournot conjectures and simultaneously choose their quantities, find the price and industry output in equilibrium.

   [5]

   (ii) If firm 1 acts as Stackelberg leader and firm 2 behaves as Stackelberg follower, what will be the industry output and corresponding price of the product ?

   [5]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1996-97
SEMESTRAL-I BACKPAPER EXAMINATION
PROBABILITY THEORY AND ITS APPLICATIONS-III

Date: 2.1.1997 Maximum Marks:100 Time: 3 Hours

Note: Answer as many question as you can. The maximum you can score is 100. The whole question paper carries 110 marks.

1. Let $\underset{\sim}{X} = (X_1, \dots X_p)$ be a random vector with density

$$f(\underset{\sim}{x}) = C \exp\left(-\frac{1}{2}\underset{\sim}{x}'\, Q\, \underset{\sim}{x}\right), \quad \underset{\sim}{x}' = (x_1, x_2, \dots x_p) \text{ where}$$

$Q = ((q_{ij}))$ is a pxp positive definite matrix.

(a) Find C

(b) Show that the dispersion matrix of $\underset{\sim}{X}$ is Q^{-1}.

(c) Show that the conditional distribution of X_p given $X_1=x_1,\ X_2=x_2, \dots, X_{p-1}=x_{p-1}$ is Normal with expectation

$$-\frac{1}{q_{pp}}(q_{1p}x_1+q_{2p}x_2+\dots+q_{p-1p}\,x_{p-1})$$

and variance $\frac{1}{q_{pp}}$. [2+3+15]

2.(a) Write down the Dirichlet $D_k(r_1, r_2, \dots, r_k ; r_{k+1})$ density. [2]

(b) Let $U_1<U_2<U_3$ be the order statistics based on 3 independent observations from uniform U(0,1)distribution. Show that (U_1, U_2-U_1, U_3-U_2) follows $D_3(1,1,1;1)$ distribution. Hence or otherwise find Cov (U_2, U_3). [4+4]

3.(a) Define convergence in probability. Show that $X_n \longrightarrow 0$ almost surely as $n \longrightarrow \infty$ if and only if for every $\varepsilon > 0$

$$P\left[\limsup_n A_n(\varepsilon)\right] = 0$$

where $A_n(\varepsilon) = [|X_n| > \varepsilon]$. [3+7]

(b) $X_n,\ n \geq 1$ are 0 - 1 valued random variables defined on the same sample space. Show that $X_n \longrightarrow 0$ a.s. if $\sum_1^\infty P[X_n=1] < \infty$. [5]

(c) Define convergence in distribution. Show that if $X_n \xrightarrow{P} X$ then X_n converges to X in distribution. [3+9]

(d) Suppose that $X_n, Y_n,\ n \geq 1$ are random variables defined on the same sample space with $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{P} 0$. Show that $X_n + Y_n \xrightarrow{d} X$. [8]

4. (a) Show that $\varphi(t) = \sum_{k=0}^{\infty} \alpha_k \cos kt$, where $\alpha_k \geq 0$ and $\sum_{0}^{\infty} \alpha_k = 1$, is a characteristic function. What is the corresponding probability distribution ? [5]

(b) Prove that the c.f. $\varphi(t)$ of any random variable is a continuous function. [5]

(c) If X is a random variable with $E(|X|) < \infty$ show that its c.f $\varphi(t)$ is differentiable and $\varphi'(t) = E(i\,X\,e^{itX})$. [8]

(d) Show that $f(x) = \frac{1-\cos x}{\pi x^2}, -\infty < x < \infty$ is a density function. Find its characteristic function. (Hint: consider the density $g(x) = (1 - |x|)\, I_{[|x| \leq 1]}$. What is its c.f ? Use the inversion formula.) [1[illegible]]

5. Let X_n, $n \geq 1$ be Binomial $B(n,p)$ random variables.

(a) Show that $\frac{X_n - np}{\sqrt{npq}} \xrightarrow{\mathcal{L}} N(0,1)$

(b) Show that $\frac{\sqrt{n(1-p^2)}}{\sqrt{pq}} \left(\arcsin \frac{X_n}{n} - \arcsin p\right) \xrightarrow{\mathcal{L}} N(0,1)$. [5+10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1996 - 97
BACKPAPER SEMESTRAL - I EXAMINATION

Statistical Methods III

Date: 2.1.97 · Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1. Let $X_1, \ldots, X_n$ be a random sample from uniform $(-\theta, \theta)$ distribution.

   (a) Find the moment estimator and the m.l.e. of θ.

   (b) Obtain an unbiased estimator of θ based on the m.l.e. for θ.

   (c) Consider estimators of the form $C\hat{\theta}$ in order to estimate θ, where $\hat{\theta}$ is the m.l.e. of θ. Which estimator in this class has the minimum MSE ?

   (8+5+7) = [20]

2. Consider the problem of testing $H_0: \mu = 0$ vs. $H_1: \mu \neq 0$ for $N(\mu, \sigma^2)$, σ being unknown. Derive the likelihood ratio test of size α based on a random sample of size n. Show that this test is based on a statistic which is distributed as student's t with n-1 d.f. under H_0.

   [20]

3. Consider the following data on the blood group distribution of a certain population. Estimate p, q and r by the method of maximum likelihood using the EM algorithm.

Blood group	Frequency	Relative frequency
O	261	r^2
A	226	$p^2 + 2pr$
B	289	$q^2 + 2qr$
AB	62	2pq

(Show computations using 2 iteration cycles).

[20]

4. It is claimed that the mean reaction time of a certain chemical reaction is 3 minutes. The reaction was carried out six times and the reaction times were found to be 2.99, 2.98, 3.01, 2.99, 3.00 and 2.97 minutes.

contd..... 2/-

(a) Using these data, under suitable assumptions, test if the mean reaction time is more than 3 minutes ?

(b) Obtain a 95% confidence interval for the variability in reaction times.

(c) Give a rough sketch of the power function of the test in (a) above.

(8+8+4) = [20]

5.(a) A survey is to be conducted to determine the true proportion p of voters who prefer the first candidate in a given list. We want to estimate p with a 95% confidence interval and a margin of error less than or equal to 0.03. What is the sample size required ?

(b) A random sample of 1000 voters were interviewed and 635 of them were found to prefer the first candidate. Is this enough evidence that more than 60% of the voters prefer the first candidate ? State your assumptions clearly.

(10+10) = [20]

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1996 - 97
BACK-PAPER SEMESTER - I EXAMINATION
Vector and Matrices I

Date: 2.1.1997 Maximum Marks: 100 Time: 3 hours

Note: There are 7 questions carrying 100 marks. Marks allotted to each question are mentioned in brackets.

1. A square matrix $A = ((a_{ij}))$ of order n is called Diagonal if $a_{ij} = 0$ if $i \neq j$.

Suppose that a matrix A of order n with entries from $\mathbb{K}(= \mathbb{R}$ or $\mathbb{C})$ has the property that AB = BA for all $n \times n$ diagonal matrices B with entries from $\mathbb{K}$. Prove : A is a diagonal matrix.

What can you say if AB = BA for all $n \times n$ matrices with coefficients from $\mathbb{K}$?

(8+6) = [14]

2. A linear transformation T on a vector space $\mathcal{V}$ over $\mathbb{K}$ into itself is called Nilpotent if there exists $k \geq 2$ such that T^k is the null linear transformation but T^{k-1} is not so.

Give an example of a nilpotent linear transformation.

[10]

3.(i) A is a square matrix with entries in $\mathbb{K}$. Prove that A^2 is symmetric if either A is symmetric or A is skew-symmetric. Is the converse true ? Justify your answer.

(ii) If A and B are symmetric $n \times n$ matrices, prove that AB is symmetric iff A and B commute i.e., AB = BA.

[(4+6) + 5] = [15]

4.(i) Let $A_{m \times n}$, $B_{m \times k}$ be two real matrices. Suppose the column space of B is a subspace of the column space of A. Prove that there exists a matrix $C_{n \times k}$ such that B = AC.

(ii) Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Compute A^{128}.

(6+9) = [15]

5. If $f \in (\mathcal{M}_n(\mathbb{C}))'$ = Dual of $\mathcal{M}_n(\mathbb{C})$, the set of all $n \times n$ complex matrices, such that $f(AB) = f(BA)$ for all $A, B \in \mathcal{M}_n(\mathbb{C})$, show that $f(A) = c$ trace (A) for some $c \in \mathbb{C}$.

[11]

p.t.o.

6. Let A be a non-singular $n \times n$ real matrix. Show that $A+uv'$ is non-singular iff $v'A^{-1}u \neq -1$. Also show that

$$(A+uv')^{-1} = A^{-1} - \frac{A^{-1}uv'A^{-1}}{1+v'A^{-1}u}.$$

Use this result to discuss the invertibility of the $n \times n$ matrix

$$A_{\alpha,\beta} = \begin{pmatrix} \alpha & \beta & \beta & \cdots & \beta \\ \beta & \alpha & \beta & \cdots & \cdot \\ \beta & \beta & \cdots & \cdots & \alpha \end{pmatrix}$$

and compute $A^{-1}_{\alpha,\beta}$ as $A_{\gamma\delta}$ for some γ, δ whenever $A^{-1}_{\alpha\beta}$ exists.

(15+10) = [25]

7. Find the determinant of the following real matrix:

$$A = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ \vdots & \vdots & \vdots & & \vdots \\ b & b & b & \cdots & a \end{bmatrix}$$

[10]

:DCC:

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1996-97

BACKPAPER SEMESTRAL - I EXAMINATION

Calculus III

Date: 3.1.97 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can.
The maximum marks you can score is 100.

1.(a) Prove that every compact subset of a complete metric space is complete.

(b) A sequence $\{x_n\}$ in a metric space X is convergent, and $\lim_{n \to \infty} x_n = x$. Show that

$$\{x\} \cup \{x_n, \; n = 1, 2, \ldots\}$$

is a compact subset of X.

(10+6) = [16]

2. Let $f(x,y) = \frac{2xy}{x^2 + y^2}$ if $(x,y) \neq (0,0)$ and let $f(0,0) = 0$.

Show that

(a) for each fixed x, f(x,y) is a continuous function of y,

(b) for each fixed y, f(x,y) is a continuous function of x,

(c) Is f(x,y) continuous at (0,0) ?

(d) Do the partial derivatives exist at (0,0) ?

In case $\frac{\partial f}{\partial x}$ or $\frac{\partial f}{\partial y}$ exists at (0,0), is it continuous there ?

[20]

3. Let r and s be differentiable functions of u, v, w which in turn are differentiable functions of x and y. Show that

$$\frac{\partial(r,s)}{\partial(x,y)} = \frac{\partial(r,s)}{\partial(u,v)}\frac{\partial(u,v)}{\partial(x,y)} + \frac{\partial(r,s)}{\partial(v,w)}\frac{\partial(v,w)}{\partial(x,y)} + \frac{\partial(r,s)}{\partial(w,u)}\frac{\partial(w,u)}{\partial(x,y)}$$

[Hint. - Use properties of determinant.]

[10]

4. Find the maximum value of $\log x + \log y + 3 \log z$ on the portion of the sphere $x^2 + y^2 + z^2 = 5r^2$ where $x > 0$, $y > 0$, $z > 0$.

contd..... 2/-

Use the result to prove that for real positive numbers a, b, c

$$abc^3 \leq 27\left(\frac{a+b+c}{5}\right)^5.$$

[15]

5.(a) Find a regular parametric representation of the circular helix C which is described by a point moving around a circular cylinder $x^2 + y^2 = a^2$ rising at a constant speed b ($b > 0$). What is the outer unit normal vector $\vec{n}$ of C ?

(b) If $f(x,y,z) = x^2 + y^2 + z^2 + xyz$ is a scalar field on C, compute the normal derivative $\frac{\partial f}{\partial n}$. If f and g are scalar fields of class C^2 on an open set U of the plane, R is a simply connected region in U bounded by a piecewise smooth jordan curve C, and $\nabla^2 f$ denotes the Laplacian of f, then show that

$$\oint_C \left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right) ds = \iint_R \left(f\,\nabla^2 g - g\,\nabla^2 f\right) dxdy.$$

[20]

6. Use a suitable transformation to evaluate the double integral

$$\iint_R (x-y)^2 \sin^2(x+y)\,dxdy$$

where R is the parallelogram with vertices

$$(\pi, 0),\quad (2\pi, \pi),\quad (\pi, 2\pi),\quad (0, \pi).$$

[12]

7.(a) Let S be the plane surface whose boundary is the triangle with vertices (1, 0, 0), (0, 1, 0), (0, 0, 1), and $\vec{n}$ be the unit normal vector field on S having non-negative ~~z-component~~. If $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ is a vector field on S, evaluate the surface integral

$$\iint_S \vec{F}\cdot\vec{n}\,dS.$$

[12]

(b) Use Stoke's theorem to show that the line integral

$$\int_C (y-z)\,dx + (z-x)\,dy + (x-y)\,dz = 2\pi a(a+b)$$

where C is the intersection of the cylinder $x^2 + y^2 = a^2$ and the plane $\frac{x}{a} + \frac{y}{b} = 1$, $a > 0$, $b > 0$. Explain how to traverse C to arrive at the given answer.

[15]

:bcc:

INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) II YEAR: 1996-97
SEMESTRAL-II EXAMINATION
ECONOMICS-II

Date: 7.5.97 Maximum Marks:100 Time: 3 Hours

Note: Answer any five (5) questions. Maximum marks are indicated in the margin.

1. (a) Explain the idea of "circular flow" with reference to "money circulation". Distinguish between the qualitative and quantitative concepts of the circular flow of money. Comment on the significance of different forms of money in this context.

   (b) State the general idea of circular flow of income and expenditure. How is it related to the circular flow of money? Bring out the various concepts of transfer that arise in this context.

   [(4+2+2)+(3+3+6)]=(8+12) = [20]

2. Consider the difference, "harvest minus seed", in a farm. Discuss comprehensively the significance of different interpretations of the term "seed" in the social accounting of the farm. (Hint: the "farm" must be considered both as a unit of production and as a unit of creation of incomes). (20)

3. Distinguish between the commodity form of production and the service form of production. Give at least three (3) examples of the service form of production each of which defines a "sector" of production and state clearly whether the "sector" so defined falls within what is generally considered the "service sector". Discuss briefly some alternative concepts of the "service sector". (8+8+4) =[20]

4. Quesnary considered "agriculture" as productive and "trade" as unproductive (or "sterile"). Why ? Show how he arrived at the same "boundary" of production starting from either of these two ends (i.e., approaching the problem from "inside" and from "outside"). What scheme of social accounting and what general conception of the structure of society followed from this specification of the boundary of production? Mention some limitations of the approach. (4+6+8+2) = [20]

5. Give a systematic account of Adam Smith's ideas on the social organisation of production stating clearly his ideas on the structure of society as supporting or embedding the organisation of production. What was the basic limitation of his approach? (15+5)=20

p.t.o.

6. What is a "firm" in macro economics? Give an analytical definition of the firm's output with a statement of alternative units of measurement of the "output". Give the corresponding definition of social output clearly specifying the unit of measurement. Interpret the difference between the social output and the sum of the outputs of all firms. When (under what conditions) will the social output be equal to the sumtotal of all incomes created in the firms? Bring out the specific relations of the "circular flow" in this case.
(2+3+3+4+4+4) = (20)

7. What is special about profit as a category of income? State some reasons why the profit of a firm in a given period of time cannot in general be equated to the difference between its sale proceeds and expenses (costs incurred) in the period. Discuss in this context the concept of stationarity and show how the above equation (profit = sale - proceeds - costs incurred) is obtained under stationary conditions. Comment in this context on (a) alternative definitions of "profit" and (b) the distinction between "profits made" and "Profits taken out". (2+5+7+3+3)=(20)

8. (a) Consider a Simple Keynesian Model without government where investment is given exogenously. What is meant by stability of equilibrium in this model? Derive and explain the stability condition in this model.

(b) How is the stability condition modified when the investment function in (a) is replaced by

$$I = \bar{I} + i.y \;,\; 0 < i < 1 \;?$$

Explain your answer.

(c) Do you think that the equilibrium in the model is stable when the consumption and the investment functions are given by

$$C = 100 + .8Y$$

and

$$I = 70 + .3Y$$

Explain your answer. (10+5+5)=(20)

9. (a) Derive the Balanced Budget multiplier in a Simple Keynesian model for a closed economy where investment is given exogenous and explain why it is unity.

(b) Compute the autonomous expenditure multiplier in a S.K.M with government where $T = \bar{T} + t.y$, $0 < t \leq 1$ and $Tr = \bar{R}$. Explain why the multiplier reduces to 1 when t = 1.

Is this the same result as obtained in (a)? Explain your answer. (10)+(8)+(2) = [20]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II and III YEAR : 1996-97
SEMESTRAL-II EXAMINATION
PHYSICS-II

Date: 7.5.97 | Maximum Marks: 100 | Time: 3 Hours

PART-I

Statistical Mechanics

1. A simple harmonic one-dimensional oscillator has energy levels given by $\varepsilon_n = (n + \frac{1}{2})\hbar\omega$, where ω is the characteristic frequency of the oscillator and $\hbar = \frac{h}{2\pi}$ with h as Planck's constant. The quantum number n can take non-negative integral values (n=0,1,2,3,...). The oscillator is in thermal contact with a reservoir at temperature τ (in units of energy) low enough so that $\tau/\hbar\omega \ll 1$.

   (a) Find the ratio of the probability of the oscillator being in the first excited state to the probability of its being in the ground state. [8]

   (b) Assuming only the ground state and the first excited state are appreciably occupied, find the mean energy of the oscillator as a function of temperature τ. [12]

2. (a) Show that the dispersion in numbers of particles $(\bar{\eta}_s)$ in Sth state can be written as

$$\overline{(\Delta \eta_s)^2} \equiv \overline{(\eta_s - \bar{\eta}_s)^2} = \frac{\bar{\eta}_s}{\mu} \tau \frac{\partial \bar{\eta}_s}{\partial \mu}$$

   where μ is the chemical potential. [12]

   (b) Show that for a system of bosons in thermal and diffusive equilibrium with a reservoir retative fluctuation in the number of particles in Sth state is

$$\frac{\overline{(\Delta \eta_s)^2}}{\bar{\eta}_s^{\,2}} = 1 + \frac{1}{\bar{\eta}_s}$$ [8]

3. (a) Show that for thermodynamic systems undergoing transformation at constant temperature and pressure, the Gibbs free energy G given by

   $G \equiv U - \sigma\tau + p\,v$ tends to a minimum. All symbols have their usual meanings. [11]

   (b) Derive the expressions for [9]

   (i) σ and V in terms of Gibbs free energy

   (ii) τ and V in terms of enthalpy H $(\equiv U + p\,V)$

   (iii) p and σ in terms of Helmholtz free energy F $(\equiv U - \tau\sigma)$

p.t.o.

PART-II

Quantum mechanics

1.(a) i) Justify briefly the statement 'quantum mechanics is not merely a probabilistic mechanics in the usual (classical mechanical) sense'. [3]

ii) Although $|\psi|^2$ is associated with measurement possibly the state function ψ (x , t) carries the quantization concept predominately. Analyze this intutively and in the light of fundamental postulates of wave mechanics.

(b) i) What nature of microparticle has been realized from some observational facts and by non-rigorous argument in the ti of old quantum theory? What are the basic hypotheses which incorporate that nature and used for deriving equation of motion of a microparticle.

ii) Write non-relativistic equation of motion of a quantum par cle and consider the potential barrier problem and find ou probability for tunnelling into classically forbidden regi

2.a) Give the physical interpretation of the expectation value and ei values of an observable A in some arbitrary state ψ and also the expansion co-efficients associated with completeness property.

b) Define the adjoint of any operator A and show that A^+A is alw self adjoint where A^+ is adjoint of A and its expectation val is nonnegative. Also show that eigenvalues of a self-adjoint operator are real.

c) Show mathematically that simultaneous measurement of position and momentum of a quantum particle can not be made precisely.

3.(a) Define a unitary operator and show that with the help of thi the time evolution of a quantum system in Schrodinger picture leads to an alternative picture of Heisenberg. Hence derive Heisenberg equation of motion in quantum mechanics. Do you fi any resemblence with any classical equation of motion with it What should be the replacement such that the classical equati of motion goes to Heisenberg equation of motion? Show that th commutation relation between position and momentum can be de with this replacement. [1+6+2+1+3]

(b) Find out the energy spectrum of harmonic oscillator by using Heisenberg equation of motion and basic commutation relation between position and momentum.

INDIAN STATISTICAL INSTITUTE

Final Examination : Semester II (1996-97)

B STAT (Hons) II & III Year

ANTHROPOLOGY

Date : 9.5.1997 Maximum Marks : 100 Time : 3 Hours

Note: Use separate answerscript for Group A and Group B. Answer *all* questions from Group A and any *five* questions from Group B.

GROUP A

1.(a) The uniqueness of Anthropology lies in: (5X2)

(i) Racial classification
(ii) Study of organic evolution
(iii) Holistic approach
(iv) Study of social evolution

(b) Homeostasis essentially involves maintenance of a stable internal environment of an organism: True / False

(c) Pithecanthropus is classified under :

(i) Homo erectus
(ii) Homo habilis
(iii) Homo sapiens
(iv) Homo neandertal

(d) Marriage in human society is a (an) :

(i) Conjugal unit
(ii) Association
(iii) Institution
(iv) Domestic group

(e) Which of the following implies marriage of a woman with more than one man at the same time :

(i) Polygyny
(ii) Polyandry
(iii) Monogamy
(iv) Hypergamy

2. Define adaptation. How would you distinguish between adaptation and acclimatization? Is the elevation of haemoglobin level of the sojourners' (to high altitude) blood an example of adaptation or acclimatization? (15)

OR

Discuss the theory of demographic transition

3. Give an account of the life ways of Homo erectus or Neandertals. (10)

P.T.O

4. Write short notes on any *three* of the following (3X5)

(a) Melanin pigmentation
(b) Lamarckism
(c) Culture
(d) Hypoxia
(e) Clan

GROUP B

1. Point out the similarities and differences between mitosis and meiosis I? What is the significance of meiosis I ? (5+5)

2. What is karyotype? State the number of chromosomes normally found in each group A- G, in the diploid set of : (a) the human male ;(b) the human female ? What are the basis by which the classification has been made? (2+6+2)

3. In man brown eyes (B) are dominant over blue (b), and dark hair (D) over red hair (d). What would be the outcome of marriages between persons of the following genotypes:
(a) BBDd X BbDd
(b) bbDd X BBDD
(c) BbDD X bbdd
State the law that the genotypes follow. (2+2+2+4)

4. A haemophiliac boy was born. Neither of his parents nor his grand parents exhibited the disorder, but his maternal grand uncle (grand mother's brother) did. The boy's elder sister has two daughters, neither of whom suffer from haemophilia. She is worried , however, about having another child. She asks you about it. How would you answer her? (8+2)

5. Define Hardy-Weinberg principle. Assume a population sample of 1000, whose blood group frequencies are A=210, B=340 and O= 450. If marriages occur at random, what will be the frequencies of persons with four blood groups? (4+6)

6. Write short notes on any *two* of the following: (5+5)

(a) Polygenic inheritance
(b) Linkage
(c) Down's syndrome
(d) Genetic drift

—x—

INDIAN STATISTICAL INSTITUTE
B.STAT.(Hons.) II Year
SEMESTRAL-II (1996-97) BACK PAPER EXAMINATION
ELEMENTARY ALGEBRAIC STRUCTURES

Date: 27.6.97 Maximum Marks:100 Time: 3 Hours

Note: Answer all questions.

1. Prove the following:
   (a) If H and K are normal subgroups of G and $K \subset H$ then

   $$(G/_K)/(H/_K) \cong G/_H \qquad [5]$$

   (b) The commutator subgroup G' of a group G is normal, G/G' is abelian and if H is a normal subgroup of G then G/H is abelian iff $G' \subset H$. [5+5+5]

   (c) If G is group with centre Z(G) and if G/Z(G) is cyclic, then G must be abelian. [10]

2. (a) Define maximal ideals and prime ideals in a ring. Prove than in a non-zero commutative ring with identity, an ideal M is maximal iff $R/_M$ is a field. [2+4+4]

   (b) Determine the maximal ideals of the ring $C[0,1]$. [10]

   (c) Let R be a commutative ring with identity in which each ideal is prime. Prove that R must be a field. [10]

3. (a) Let p(x) be an irriducible polynomial in $F[x]$, where F is a field, and let u be a root of p(x) in an extension field E of F. Show that

   $$F(u) \cong F[x]/_{(p(x))}$$

   where F(u) is the subfield of E generated by F and u. What is $[F(u) : F]$? Describe a basis for F(u) over F. [5+5+2]

   (b) Define an algebraic extension of a field F. If E is an extension field of F with $[E:F]<\infty$, is E an algebraic extension? Give reasons for your answer. [2+5+5]

   (c) Let E be the splitting field of a polynomial of degree n over a field F. Show that $[E:F] \leq n!$ [6]

   (d) Let f(x) be a polynomial of degree n over a field F of characteristic $p \neq 0$. Suppose that $f'(x)=0$. Show that $p|n$ and that f(x) has at most $n/_p$ distinct roots. [10]