INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.): II YEAR 1997-98
SEMESTRAL-I EXAMINATION
CALCULUS-III

Date:3.11.97 Maximum Marks: 60 Time: $2\frac{1}{2}$ hours.

Note: Answer as many question as you like.
But the maximum marks you can score
is 60.

1. Define directional derivative of a function $f: U \to \mathbb{R}^m$, where U is an open subset of $\mathbb{R}^n$.

For the function $f(x,y) = \begin{cases} x+y & \text{if } x=0 \text{ or } y=0 \\ 1 & \text{otherwise} \end{cases}$

determine the partial derivatives at (0,0), if they exist. Does the directional derivative of the above function exist in any arbitrary direction? Justify. [10]

2. Define differentiability of a function defined on an open subset of $\mathbb{R}^n$. Suppose $f: U \to \mathbb{R}$, U being an open subset of $\mathbb{R}^n$, is a differentiable function such that there exists a point P in U with the property $f(P) \geq f(x)$ for all x in U. Prove that grad $f(P)=0$.

Prove that the function $f(x,y)=\sin x$ is differentiable. [10]

3. Let B(a) be an open ball in $\mathbb{R}^n$ with center at a. Let $f: B(a) \to \mathbb{R}$ be a function with continuous second order partial derivatives. Assuming 'a' to be a critical point state conditions for 'a' to be a point of relative minima, a point of relative maxima or a saddle point for f.

For the function $f(x,y)=3x^4-4x^2y+y^2$ show that on every line $y=mx$ the function has a minimum at (0,0). However show that (0,0) is a saddle point. [10]

4. State inverse function theorem.
Determine the points for which the function $F(x,y)=(e^x \cos y, e^x \sin y)$ is locally invertible.
Is f a diffeomorphism? Justify. [10]

5. Define exterior differentiation of differential forms on $\mathbb{R}^n$.
For a vector field $F=(F_1,F_2,F_3)$ on $\mathbb{R}^3$ define the forms

$$w_F^1 = F_1\, dx + F_2\, dy + F_3\, dz$$

$$w_F^2 = F_1\, dy \wedge dz + F_2\, dz \wedge dx + F_3\, dx \wedge dy$$

Prove that $dF = w^1_{\text{grad } F}$

$$d(w_F^1) = w^2_{\text{Curl } F}$$

Hence deduce Curl grad F = 0. [10]

contd. ....2.

6. Let ϕ be a smooth scalar field with a continuous gradient grad ϕ on an open set U of R^n. Let $c:[0,1] \to U$ be a smooth path in U from a to b. Prove that the line integral of grad ϕ along c is $\phi(b) - \phi(a)$.

Find the line integral of the vector field $F(x,y)=(y+3x,2y$ counter clockwise around the ellipse $4x^2+y^2=4$. [10]

7. State Stoke's theorem.

Let S be the surface defined by $z=x^2+y$, $0 \leq x \leq 1$, $-1 \leq y \leq 1$.

Find the integral of the vector field $F(x,y,z)=(x,y,0)$ over the sphere $x^2+y^2+z^2=a^2$, $a>0$. [10]

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B.Stat. (Hons.) II and III Year : 1997 - 98

SEMESTRAL-I EXAMINATION

Biology I (Theory)

Date: 5.11.1997 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer ALL the questions.

1. Discuss briefly the structure of DNA. How does it differ from RNA ? State where it is localised and how does it regulate cellular functions ? (7+5+3) = [15]

Or

State and explain Mendel's laws of inheritance. Cite the number of cases where Mendel's laws of inheritance does not hold good in truest sense.

Broad leaved red flowered plant (BBRR) are crossed with narrow leaved white flowered plant (bbrr). Both the characters are incompletely dominant in F_1. Give the genotype and phenotypic expression of the progenies in F_2 if the F_1 progenies are inter-crossed among themselves. (5+5+5) = [15]

2. What is taxonomy ? What do you mean by Binomial Nomenclature and Trinomial Nomenclature ? What is meant by author's citation ? What are the different types of plant classification ? Discuss briefly Bentham and Hooker's system of classification. (2+3+2+3+5) = [15]

Or,

Write notes on the followings:

(i) Plant hormones in relation to different physiological functions.

(ii) Mode of action of a) Adrenocortico trophic hormone (ACTH) b) Vasopressin c) Insulin d) Cholecystokinin - pancreozymin. (8+7) = [15]

3. Distinguish between the followings: (any five)

(i) Simple and conjugated protein.

(ii) Storage polysaccharide and structural polysaccharide.

(iii) Trypsin and Steapsin.

(iv) Gymnosperm and Angiosperm.

contd..... 2/-

(v) Sporophytes of Anthoceros and Funaria.

(vi) Fat Soluble Vitamins and Water soluble vitamins.

(2x5) = [10]

4. Supply the missing words:

(i) The seed bearing plant according to Eugler and Prantl's system of classification is known as ..... .

(ii) ..... and ..... received Noble Prize for artificial synthesis of nucleic acid.

(iii) Carolus Linnaeus is famous for his publications namely ....., ..., ...

(iv) Vitamin B_{12} or ..... is essential for maturation of ..... .

(v) Synthetic plant hormones such as ..... are used in rooting and cutting while ..... is used to control weeds.

(vi) Functional units of gene are ..... .

(vii) Various pigments such as ..... and ..... are derived from haemoglobin after disintegration of red cells.

(viii) The most common protozoan parasite that causes malaria is ..... .

(ix) ..... is the characteristic feature of echinoderms.

(x) The member of class Mammalia possesses ..... on skin an special glands named ..... .

5. What is meristem ? Describe with diagrams the various types of meristematic tissues with their functions that occur in angiosperms.

(3+12) = [15]

Or,

Define tissue. Describe with illustrations the different types of permanent tissues found in plants and mention their functions

(3+12) = [15]

6. What is photosynthesis ? Describe briefly with diagram the pathways of photosynthesis in C_4 plants.

(3+12) = [15]

Or,

contd..... 3/-

What is respiration ? Describe with diagram the Krebs cycle that takes place in plant cells.

(3+12) = [15]

7. What is ecological succession ? How the succession proceeds towards a climax community ? What are the generalizations that hold true for most cases where both autotrophes and heterotrophes are involved ?

(3+5+7) = [15]

Or,

What is ecological group ? Describe briefly the different types of ecological groups of plants with their adaptations.

(3+12) = [15]

8. Distinguish between: (any four) $(2\frac{1}{2} \times 4)$ = [10]

(a) Fibres and sclereids.

(b) Vessel elements and tracheids.

(c) Actinocytic and cyclocytic type of stomata.

(d) Homologous and analogous structures.

(e) Autotrophe and heterotrophe hypotheses.

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B.Stat. (Hons.) II Year : 1997 - 98

SEMESTRAL-I EXAMINATION

Probability III

Date: 10.11.1997 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions.

1. Let $a > 0$ and $f(x,y) = [(1+ax)(1+ay) - a]\, e^{-x-y-axy}$ for $x > 0$, $y > 0$ and $f(x,y) = 0$ elsewhere.

(a) Prove that f is a density of a pair of random variables (X,Y). Find the marginal density of X.

(b) Find the conditional density $u_x(y)$ and $E(Y \mid X)$.

(10+10) = [20]

2.(a) Prove that $E(E(Y|X)) = E(Y)$.

(b) Let X and Y be independent where X has the uniform distribution on [0,1] and Y has the exponential distribution exp(1). Let $Z = X/Y$. Find the conditional distribution of X given Z. (No justification is necessary.)

(10+10) = [20]

3.(a) Let $X_1, \ldots, X_n$ be identically and independently distributed with the common distribution $N(0, \sigma^2)$. Show that

$$\bar{X} = \frac{X_1 + \ldots + X_n}{n} \quad \text{and} \quad S^2 = \frac{1}{n-1} \Sigma (X_i - \bar{X})^2$$

are independent random variables.

(b) Show that if n_1, n_2, n_3, n_4 are positive integers, then the function

$$f(x_1,x_2,x_3) = \frac{\Gamma(n_1+n_2+n_3+n_4)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)\Gamma(n_4)} x_1^{n_1-1} x_2^{n_2-1} x_3^{n_3-1} (1-x_1-x_2-x_3)^{n_4-1}$$

if $x_i \geq 0$, $i = 1,2,3,4$ and $\Sigma x_i \leq 1$

$= 0$ otherwise

is a density function. (10+10) = [20]

p.t.o.

4. Show that if $X_n \longrightarrow X$ in probability,

(a) then $X_n \longrightarrow X$ in distribution

(b) then $X_n^2 \longrightarrow X^2$ in probability.

(10+10) = [20

5.(a) Show that if $X_n \longrightarrow X$ in distribution and if f is a bounded continuous function, then $E(f(X_n)) \longrightarrow E(f(X))$.

(b) Find the characteristic function of the Cauchy distribution.

(12+8) = [20

6.(a) If X is a random variable such that $EX = 0$ and $\mathrm{Var}\, X = 1$, then show that the characteristic function φ of X can be written as

$$\varphi(t) = 1 - \frac{t^2}{2} + \frac{t^2}{2} h(t)$$

where $h(t) \longrightarrow 0$ as $t \longrightarrow 0$.

(b) Let $X_1, X_2, \ldots$ be independent random variables such that

$$P(X_n = -\frac{1}{2^n}) = P(X_n = \frac{1}{2^n}) = \frac{1}{2},$$

$n = 1, 2, \ldots$. Let $X = \sum_{n=1}^{\infty} X_n$. Find the distribution of X.

(8+12) = [20]

:bcc:

INDIAN STATISTICAL INSTITUTE

Final Examination : Semester I (1997-98)

B. Stat. 2nd Year

Economics - I

Date: 7.11.97 Maximum Marks: 65 Duration: 3 Hours

Note: The paper carries 75 marks. Attempt ALL questions. The maximum you can score is 65.

1. What is an income-compensated demand curve? How does it differ from the ordinary demand curve? For a normal good (i.e. good with positive income elasticity) which of the two curves is more elastic (say, at a given price)? [5 + 3 + 2 = 10]

2. "The long-run supply curve of a competitive industry with free entry and exit should be essentially flat at a price equal to the minimum long-run average cost." Explain in detail.

   What is the role of industry demand curve in this context? [8]

3. A monopolist in a certain country faces a domestic demand curve:

$$P_d = 120 - \frac{q_d}{10}$$

   where P_d and q_d are the price and quantity sold in the domestic market.

   The firm can also sell in the more competitive world market and can sell any amount q_r at a given world price $P_r = 80$. Thus it faces a horizontal demand curve in the world market:

$$P_r = 80$$

   Marginal cost is given by

$$MC = 50 + \frac{Q}{10}$$

   (where $Q = q_d + q_r$).

   (a) Find the equilibrium overall output Q and and its division between two markets.

   (b) Compare the prices and demand elasticities of the domestic market and the world market (corresponding to the equilibrium output). [7 + 3 = 10]

4. Suppose a consumer initially has monetary wealth W. There is some probability p that he will lose an amount L (for example, his car may be stolen or a fire may damage his house). Suppose an insurance company offers him to pay R rupees if the loss occurs in exchange for a πR amount of premium (π is the premium per rupee of insurance coverage).

   Show that if fair insurance is offered, a risk-averse and expected utility maximising consumer will optimally choose to be fully insured. [8]

contd..... 2/-

5. (a) If an individual's demand curve for a good x cuts the vertical price axis at finite "choke price" P_x^0, show the equivalent situation in terms of the individual's indifference curves and the budget line (involving x and say, 'all other goods', y).

   (b) Consider a pair of commodities like bread and butter which are strong complements, versus another pair like butter and margarine, which are close substitutes. Which pair is more likely to have a member that is an inferior good? Explain. [2 + 2 = 4]

6. A family of short-run total cost curves are generated by

$$C(y, k) = 0.04y^3 - 0.9y^2 + (10 - lnk)y + 8k^2$$

   where $k > 1$ denotes the fixed factor, plant size. Determine the firm's long-run total cost curve. [4]

7. A firm produces one output with two inputs using the production function

$$y = Ax_1^a x_2^{1-a}$$

   $(0 < a < 1)$. He buys the inputs and sells the output at fixed prices (p, w_1, w_2). He is subject to a quota which allows him to purchase no more than x_1^0 units of x_1. He would have purchased more in the absence of quota. Determine the firm's conditions for profit maximization. What is the optimal relation between the value of the marginal product of each input and it's price? What is the optimal relation between the TRS and the input price ratio? [5 + 3 + 2 = 10]

8. State the Weak and Generalized Axioms of Revealed Preference explaining the notation. [6]

9. In the Cournot duopoly game, even if the goods y_1 and y_2 are strategic complements, an equilibrium pair of quantity choices (y_1^*, y_2^*) may exist! (Here the reaction curves $f_1(y_2)$ and $f_2(y_1)$ are positively sloped.) Find the mathematical restriction on the slopes of $f_1(y_2)$ and $f_2(y_1)$ for this equilibrium to exist. [5]

10. Find the Nash equilibria of the following game: [3]

		Player 2	
		a	b
Player 1	A	-3, 3	2, 0
	B	0, 2	1, 1

11. Consider a two-person two-good pure exchange economy with the following structure:

$U_1 = x_1^1 x_1^2 + 12x_1^1 + 3x_1^2 \quad w_1 = (8, 30)$

$U_2 = x_2^1 x_2^2 + 8x_2^1 + 9x_2^2 \quad w_2 = (10, 10)$

Determine excess demand functions for the two consumers. Determine an equilibrium price ratio for the economy. [4 + 3 = 7]

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B.Stat. (Hons.) II Year : 1997 - 98

SEMESTRAL-I EXAMINATION

Statistical Methods III

Date: 12.11.1997 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1. Let $X_1, X_2, \ldots, X_n$ be independent observations where X_i has $N(\mu, i\sigma^2)$ distribution $(1 \leq i \leq n)$. Here $-\infty < \mu < \infty$ and $0 < \sigma^2 < \infty$ are unknown parameters.

   (a) Derive (with adequate justification) the maximum likelihood estimates of μ and σ^2 as well as the best linear unbiased estimate of μ based on the data.

   (b) Are the estimates derived in (a) consistent as the sample size n tends to ∞ ? Justify your answer.

   (15+15) = [30]

2. Consider the following data:

X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Y	5.6	8.9	14.7	18.9	21.3	23.8	26.9	29.1	31.5	34.7

   Assume that the X's are deterministic and the Y's are random satisfying the linear model $Y_i = \alpha + \beta X_i + e_i$ $(1 \leq i \leq 10)$, where $e_1, e_2, \ldots, e_{10}$ are i.i.d. $N(0, \sigma^2)$ random variables. Here α, β, σ^2 are all unknown. Compute 95% confidence intervals for α and β based on the data.

   (8+8) = [16]

3. A random sample of 400 observations has been drawn from a Cauchy distribution with density $\frac{1}{\pi\{1 + (x-\theta)^2\}}$. Let the data be $X_1, \ldots, X_{400}$. Describe explicitly how you will set up a 95% confidence interval for the unknown parameter θ based on sample median in this case. [9]

4.(a) Let X be a 0-1 valued random variable with $Pr[X = 1] = p$, where $0 < p < 1$ is an unknown parameter. Suppose that we have n i.i.d. observations on X that are denoted by $X_1, X_2, \ldots, X_n$ $(n \geq 5)$.

contd..... 2/-

Consider the hypotheses H_o: $p = 0.5$ against H_A: $p > 0.5$. Describe explicitly and with adequate justification a statistical test that will have level exactly equal to 0.05, will be unbiased and have its power monotonically increasing with increase in the true value of p.

(b) Describe how you will modify your test if n in (a) above is very large and we only need the level to be approximately 0.05 and the test to be a consistent test as n to ∞. Justify your answer.

(15+15) = [30]

5. Assignments. [15]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.)II YEAR:1997-98
SEMESTRAL-I EXAMINATION
PHYSICS-I

Date:13.11.97 Maximum Marks:50 Time: 3 Hours

Note: Answer any Two from Gr.A, any one from Gr.B and any Two from Gr.C. All questions carry equal marks.

GROUP-A
(Classical Mechanics)

1.(i) Distinguish between 'inertial' and 'non-inertial' frames of reference with suitable examples.

(ii) Discuss about the co-ordinate transformations between inertial frames.

(iii) What is the 'Galilean' viewpoint of classical space-time? Comment on its acceptability.

(iv) In the mechanical descriptions, how do 'fictitious' forces arise? How are they different from the 'real' forces acting on the system? (10)

2. Consider an N-particle, closed, mechanical system, acted upon by an external, conservative force $\vec{F}$. Using Newtonian mechanical concepts, discuss about the motion of the system (of particles) and all the possible associated conservation theorems. (10)

3.(i) Define 'virtual' work of a force.

(ii) Define and classify 'constraints' of a mechanical system.

(iii) Deduce, using the D'Alembert's principle of virtual work, the Lagrange's equations of motion for a mechanical, conservative system of N-particles having K number of holonomic contraints. (10)

GROUP-B
(Classical Thermodynamics)

1. State and explain the 1st law of thermodynamics. Apply it to derive the adiabatic equation of state for a perfect gas. Also, for one gram-mole of a perfect gas, deduce the relation $C_p - C_v = R$ where, the symbols C_p, C_v, R have their usual meanings in the thermodynamic context. How does this relation modify in case of real gases? (10)

contd. .....2.

2.(i) Give an account of the working principle of a heat engine. Can you design a heat engine which is 100% efficient? Justify your answer. State, how we can have the most efficient heat engine.

(ii) If entropy is a measure of the 'orderliness' of a system, substantiate the statement - "our world is gradually evolving (in time) into states of higher and higher disorder". (16

GROUP-C

(Geometrical and Physical Optics)

1.(i) State and explain Fermat's Principle of least time in geometrical optics. Using it, deduce the laws of specular reflection.

(ii) Give a short account of the construction and working of an "achromatic doublet". (1

2. Describe the interference phenomena, as observed in the Young's double slit experiment. What will happen, if one of the two slits is shut off? (1

3.(i) Describe, with necessary details, Fizeau's method of determination of velocity of light in air.

(ii) Given a beam of unpolarized light, describe a method of producing a beam of plane-polarized light. (

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B.Stat. (Hons.) II and III Years: 1997 - 98

SEMESTRAL-I EXAMINATION

Biology I (Practical)

Date: Maximum Marks: 20 Time: 3 hours

Note: Answer ALL the questions.

1. Identify the specimens supplied with reasons. [5]

2. Estimate the concentration of albumin in the given sample. [5]

3. Cut transverse section of the specimen supplied. Stain, mount and draw labelled sketches. (2+3) = [5]

4. Submit your practical record book. [5]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year . 1997 - 98
BACKPAPER SEMESTRAL-I EXAMINATION
Statistical Methods III

Date: 14.1.1998 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1. 500 i.i.d. observations $X_1, X_2, \ldots, X_{500}$ have been drawn from a Laplace distribution with density $\frac{1}{2}\exp(-|x-\theta|)$, where $-\infty < \theta < \infty$ is an unknown parameter. Describe explicitly and with adequate justification how you will set up a 90% confidence interval for θ based on sample median. [10]

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. with a common uniform $[0, \theta]$ distribution. Here $0 < \theta < \infty$ is an unknown parameter. Construct the shortest expected length confidence interval for θ of the form $[X_{(n)}/c_1, X_{(n)}/c_2]$ with confidence coefficient $(1-\alpha)$, where $X_{(n)}$ is the maximum order statistic. Justify your answer. [20]

3. Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be i.i.d. with common density $\frac{1}{\lambda}\exp(\theta - x)$ for $x \geq \theta$. Here $0 < \theta < \infty$ and $0 < \lambda < \infty$ are unknown parameters.

   (a) Derive explicitly and with adequate justification the maximum likelihood estimates of θ and λ.

   (b) Are the maximum likelihood estimates unbiased for the corresponding parameters ? Justify your answers.

   (c) Are the maximum likelihood estimates consistent as the sample size n grows to ∞ ? Justify your answers.

(10+10+10) = [30]

4. Consider the heights (in meters) of 15 trees selected randomly from a population:

4.6	4.3	5.1	5.4	4.8
5.5	5.7	4.2	5.0	4.9
5.9	4.4	4.7	5.8	4.5

Construct a 95% confidence interval for the median height of the entire population based on the data. You need to

contd..... 2/-

explain your construction with justification wherever necessary.

[20]

5. Suppose that we observe X distributed as Poisson (λ) and Y distributed as Poisson (μ), where X and Y are independent and $0 < \lambda < \infty$ and $0 < \mu < \infty$ are unknown parameters. We want to test $H_0: \mu = \lambda$ against $H_A: \mu > \lambda$. Describe explicitly and with adequate justification a statistical test with level exactly 0.05, which will be unbiased and have its power monotonically increasing with increase in the value of

$$\frac{\mu - \lambda}{\mu + \lambda}.$$

[20]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1997-98
SEMESTRAL-I BACK PAPER EXAMINATION
PROBABILITY THEORY AND ITS APPLICATIONS III

Date:14.1.98 Maximum Marks:100 Time: 3 hours

Note: Answer all questions.

1.(a) Let $F(x,y) = x$ if $x \le y$ and $0 \le x,y \le 1$

y if $x \ge y$ and $0 \le x,y \le 1$

be a distribution function concentrated on the unit square. Show that if F is the distribution function of the pair (X,Y) then P(X=Y)=1.

(b) Let f be a density function of a random variable concentrated on $(0,\infty)$. Prove that

$$g(x,y) = \frac{f(x+y)}{x+y} \quad x>0,\ y>0$$

$$= 0 \quad \text{otherwise}$$

is a density function. [10+10=20]

2.(a) Let X and Y be independent random variables with the common distribution exp(1). Find the conditional distribution of X given X+Y.

(b) If X and Y are independent random variables having the Gamma distributions $\Gamma(k_1,1)$ and $\Gamma(k_2,1)$ (k_1 and k_2 being the shape parameters), show that X+Y and X/X+Y are independent random variables. [10+10=20]

3.(a) If the random variables X_n are defined on the same probability space and $X_n \longrightarrow 0$ in distribution, then show that $X_n \longrightarrow 0$ in probability.

(b) If $X_n \longrightarrow 0$ in probability and if h is a continuous function, then prove that $h(X_n) \longrightarrow h(0)$ in probability.

(c) Give an example to show that convergence in probability does not imply convergence with probability one. [6+6+8=20]

4.(a) Prove the Parseval relation

$$\int_G (e^{-iat}\, \varphi_F(t)) = \int_F (\varphi_G(x-a))$$

where F and G are distribution functions with the characteristic functions φ_F and φ_G respectively and $a \in \mathbb{R}$ is fixed.

(b) Prove that distinct probability distributions have distinct characteristic functions. [8+12=20]

contd. .....2.

5. (a) If the characteristic function φ of a random variable X is such that $\varphi(a)=1$ for some nonzero real number a, then show that X is a discrete random variable.

(b) If X_n is a random variable having the Poisson distribution with the parameter λ_n and $\lambda_n \to \infty$, then show that $\frac{X_n-\lambda_n}{\sqrt{\lambda_n}}$ converges in distribution to the standard normal random variable

(c) Show that $\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}$ [5+10+5=20]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR
OPERATION'S RESEARCH AND SQC
CLASS TEST

Date:17.4.98 Time: 3 Hours

Note: Answer all questions. For Operations Research Max[Q1,Q2] and for SQC Max [Q3,Q4] will be considered.

Operations Research

1.(a) A precision Engineering Factory consumes 50000 units of a component per year. The ordering, receiving and handling costs are Rs.3 per order while the trucking costs are Rs.12 per order. Further details are as follows:
Interest cost Rs.0.06 per unit per year. Deterioration and obsolescence cost Rs.0.004 per unit per year.
Storage cost Ps.1000 per year for 50000 units; calculate the economic order quantity. ($7\frac{1}{2}$)

(b) An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs.100 per set up and holding cost is Rs.0.01 per unit of item per day, find the economic lot size and optimum period of replenishment for one run, assuming that the shortages are not permitted. ($7\frac{1}{2}$)

2. A company producing three items has a limited storage space of 750 items of all types. Determine the optimal production quantities for each item separately, when the following information are given:

	Product 1	Product 2	Product3
holding cost per unit per unit time	0.05	0.02	0.04
set up cost per order	50	40	60
demand rate	100	120	75

(15)

SQC

3. A factory manufactures power sockets for use in the refining of oil. The hexagonal inside of the socket is prepared by an electrolytic method, and the resulting diameter of the hexagon is very critical. Rather than measuring every possible diagonal in the socket, one diagonal is selected at random from each socket and recorded. A sample of 5 consecutively produced sockets is taken each day.

contd. ....2.

The following table contains the values of $\bar{X}$ and R from 22 samples.

Day	$\bar{X}$(cm)	R(cm)	Day	$\bar{X}$(cm)	R(cm)
1	20.813	0.035	12	20.827	0.028
2	20.821	0.021	13	20.808	0.032
3	20.816	0.038	14	20.824	0.037
4	20.810	0.016	15	20.816	0.024
5	20.822	0.028	16	20.818	0.018
6	20.828	0.019	17	20.829	0.033
7	20.814	0.039	18	20.823	0.039
8	20.817	0.025	19	20.813	0.019
9	20.822	0.016	20	20.814	0.023
10	20.809	0.019	21	20.828	0.027
11	20.824	0.037	22	20.811	0.024

(a) Draw the $\bar{X}$-R control chart and comment about the state of process control. (10)

(b) If LSL=20.80 and USL=20.85, then how much is the Off-specification at the current process centering and at the mid-specification assuming normality ? ($2 \times 2\frac{1}{5} = 5$)

4. Define

(a) ISO definition of QUALITY

(b) ISO definition of QUALITY CONTROL

(c) Juran's definition of QUALITY

(d) Taguchi's definition of QUALITY

(e) Philip Crosby's definition of QUALITY. ($5 \times 3 = 15$)

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1997-98
ECONOMIC AND OFFICIAL STATISTICS
SEMESTRAL-II EXAMINATION

Date:27.4.98 Maximum Marks:60 Time: 3 Hours

Note: This question paper carries a total of 70 marks. Answer all questions. Maximum marks that you can score is 60.

1.(a) Briefly explain the price index number problem and indicate how this can be resolved.

(b) State and explain briefly the axioms that a price index number formula should satisfy.

(c) Indicate the problems of comparing price levels of two time periods which are sufficiently apart. Suggest an alternative to fixed base price comparison for such a situation. When would the two price index numbers be same? (4+6+5)=[15]

2.(a) Define a size variable and give some examples of such a variable.

(b) Explain briefly how will you examine graphically the suitability of lognormal distribution for a given set of grouped income distribution data.

(c) Derive the Lorenz curve for the lognormal distribution and explain its properties. (3+6+6)=[15]

3.(a) Briefly distinguish between the concepts of absolute and relative poverty.

(b) Mention the three aspects that a satisfactory measure of absolute poverty should incorporate. Derive Sen's poverty measure from his axioms of absolute poverty.

(c) Justify briefly the use of per capita formulation of an engel curve in statistical analysis of cross sectional consumer expenditure data. Indicate how you would examine adequacy of such a formulation in actual application. (3+6+6)=[15]

4.(a) Explain briefly what do you mean by additive and multiplicative models of time series. When would you use a multiplicative model?

(b) Briefly explain the moving average method of finding trend curve of an observed time series. Will this method always give satisfactory estimate of the trend values? Justify your answer.

(c) Define weak stationarity. Is the time series $x_t, t=1,2,\ldots$, where $x_t=\alpha+\rho\, x_{t-1}+\varepsilon_t$, $\alpha>0$, $0<\rho<1$, $E(\varepsilon_t)=0 \quad t$, $E(\varepsilon_t^2)=\sigma^2 \quad t$, $E(\varepsilon_t\, \varepsilon_{t'})=0 \quad t\neq t'$, a weakly stationary series? (3+6+6)=[15]

p.t.o.

5. Write short notes on any two of the following:

(a) Divisia Price Index Number;

(b) Demand projection formula when engel curve is of constant elasticity form and PCE follows lognormal distribution;

(c) Index number of industrial production. (5x2)=[10]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1997-98
SEMESTRAL-II EXAMINATION
DEMOGRAPHY

Date:30.4.98 Maximum Marks:50 Time: $1\frac{1}{2}$ Hours

Note: Answer Q.1 and ANY THREE from the rest.

1. Write a short note on any one
   (a) Infant Mortality Rate
   (b) Chiang's Method for estimating ${}_nq_x$.
   (c) Hajnal's Method for estimating mean age at marriage. [5]

2. (a) Define vital statistics
   (b) Derive a formula for evaluating errors in reporting ages over (10-99) years from census data.
   (c) Assume that the preference for digits in age reporting spreads around six year age intervals. Show that

$$C_x = \frac{1}{7}\left(\frac{6}{AR_x}+1\right),$$

   where

   C_x = Correction factor for age x and

   AR_x = Age ratio at age x for a 6-year age interval.

   (2+7+6)=[15]

3. (a) Derive the expression for instantaneous growth rate of a population.
   (b) Show that the tripling time of a population is approximately $1.0986/r$,

   where r = annual growth rate.

   (c) Show that the geometric growth rate is equal to the exponential growth rate up to a certain level of approximation. (5+5+5)=[15]

4. (a) Describe a method to estimate the parameters k, s, g and c in the relation

$$l_x = ks^x g^{c^x}$$

   (b) An aviary of birds which has a constant intake of 1500 new-born birds per year experiences the following mortality rates:

Age(x)	q_x
0	.3
1	.1
2	.2
3	.4
4	.7
5	1.0

   (i) What is the total number of birds in the aviary at any time?

p.t.o.

4.(b) contd.....

(ii) If the owner wanted the population to be a steady 5000, how many extra new-born birds would he have to add each year? (7+8)=[15]

5.(a) Discuss: "Total fertility rate is preferred over crude birth rate in comparing fertility between two countries".

(b) Find the mathematical relationship between crude birth rate and total fertility rate.

(c) Interpret the situations:
(i) NRR = 1, (ii) NRR > 1 and (iii) NRR < 1. (3+6+6)=[15]

SQC and OR Max.Marks:50 Time:2 Hours

Note: Answer all questions

1.(a) Prove that if arrivals occur at random over time with an exponential inter-arrival time distribution, then the number of arrivals occuring in a fixed time interval follows a Poisson distribution. [5]

(b) Show that the distribution of the time interval between successive arrivals is exponential if the number of arrivals in a given time interval follows a Poisson distribution. [6]

(c) There is congestion on the platform of a railway station. Trains arrive at a rate of 30 trains per day. The waiting time for any train to hump is exponentially distributed with an average of 36 minutes. Calculate the following

(i) The mean queue size.

(ii) The probability that queue size exceeds 9. [$2\frac{1}{2}$x2=5]

2. Derive the expressions for EO and reorder point in a continuous review inventory model, both for back-order and lost-sales cases, assuming that demand during lead time follows a Normal distribution. [9]

3.(a) State the objectives in constructing a control chart. [5]

(b) What are the differences between common causes of variation and special causes of variation? [6]

(c) What is meant by rational subgrouping? Give some examples. [5]

4.What do you understand by process-capability? Discuss about the process-capability indices C_p,C_{PK} and C_{pm} when the underlying distribution of a quality characteristic is Normal. [1+3+3+2=9]

:ss:

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year: 1997-98

SEMESTRAL-II EXAMINATION

Elements of Algebric Structures

Date: 4.5.1998 Maximum Marks: 100 Time: 3 hrs.

Note: You may answer ALL questions.
Maximum Marks 100. Paper carries 110 marks.

1.(a) Show that a group of order p^2 (p - prime) is Abelian. [10]

(b) In a finite group G, if H is a subgroup such that it is the only subgroup whose order is equal to O(H), show that H is normal in G. [10]

2. Find a polynomial of degree 3 irreducible over $\mathbb{Z}_3$, the ring of integers modulo 3. Explain how to construct a field of 27 elements, using this polynomial. Find a primitive element in this field. (3+7+5) = [15]

3. Prove or disprove

(a) If G is an Abelian group in which the equation $x^n = e$ has atmost n solutions in G for every positive integer n. (e is the identity element of G), then G is cyclic. [9]

(b) A, B are cyclic groups of orders m and n respectively, then A×B is a cyclic group of order m.n. [9]

(c) R is an integral domain with multiplicative identity; R[x] the polynomial ring over R. If U is the set of invertible elements of R[x] then $U \subseteq R$. [9]

4.(a) Find the number of nonisomorphic Abelian groups of order 36. [10]

(b) Show that a group of order 28 with a normal subgroup of order 4 is Abelian. [10]

5.(a) Let $\alpha = \sqrt{2} + \sqrt[3]{5}$. Show that α is algebraic over the field Q of rational numbers. Find the dimension of $Q(\alpha)$ over Q. (5+8) = [13]

p.t.o.

(b) Let ω be a root of the polynomial $x^2 + x + 1$ lying in some extension field of the field Q of rational numbers.

Find $G(K,Q)$ if 1) $K = Q(\sqrt[3]{5})$, 2) $K = Q(\sqrt[3]{5}, \omega)$.

Which of these extensions is normal over Q — substantiate.

(5+5+5) = [15]

INDIAN STATISTICAL INSTITUTE

B H Semester II

Statistical Methods IV

Semestral Examination

Total points 100

Date: 6.5.98 **Time: 3:00 hours**

Clearly explain your assumptions and notations used so that all answers are self-contained and to the point

1.(a) If $F(.)$ is a cumulative distribution function, justify whether

(i) $F(x,y) = F(x) + F(y)$ is a joint cumulative distribution function. [3]

(ii) $F(x,y) = F(x)F(y)$ is a joint cumulative distribution function. [3]

(b) Let X have a Bernoulli distribution with parameter p, *i.e.*, $P[X = 1] = p = 1 - P[X = 0]$. Further, for another random variable Y, let $E[Y|X = 0] = 1$ and $E[Y|X = 1] = 2$. Obtain $E[Y]$. [4]

2.(a) Prove that a bivariate normal random variable may be expressed as a linear combination of two independent standard normal variables. [6]

(b) Suppose that two random variables X_1 and X_2 have a bivariate normal distribution and that $Var(X_1) = Var(X_2)$. Show that the sum $X_1 + X_2$ and the difference $X_1 - X_2$ are independent. [6]

(c) Suppose that the joint distribution of heights of husbands and wives is a bivariate normal distribution with $\mu_W = 1.67m, \sigma_W = 5cm, \mu_H = 1.75m, \sigma_H = 5cm$ and $\rho_{HW} = 0.68$. Find the probability that the wife is taller than the husband. [8]

3.(a) Derive the characteristic function of a multivariate normal distribution. [6]

(b) Suppose $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)'$ follows a multivariate normal distribution with dispersion matrix $\Sigma = diag(\sigma_1, \sigma_2, \ldots \sigma_p)$.

(i) Prove that the individual components of $\boldsymbol{X}$ are independently distributed. [6]

(iii) Derive the distribution of sample correlation coefficient between X_1 and X_2. [12]

P.T.O

contd..... 2/-

4.(a) Define Hotelling's T^2 statistic. [2]

(b) What is its relation to F distribution? [2]

(c) Suppose you want to test that the mean of one normal population is equal to the mean of a second normal population, where the covariance matrices of both populations are assumed equal but unknown. Using the definition of T^2 write down a proper test statistic for this problem and the corresponding critical region, indicating the correct degrees of freedom. [6]

5. Let $X \sim N_n(\mu, I_n)$. Define $Q_i = X'A_iX$ where A_i is a symmetric matrix of order n and rank r_i for $i = 1, \ldots, k$. Show that if $A_iA_j = 0, i \neq j, i, j = 1, \ldots, k$ then Q_i, Q_j are independently distributed for all pairs of i, j. [10]

6. Let $(X_1, \ldots, X_n)$ be a random sample from the uniform distribution on $(0, \theta)$. Show that, for large n, $\sqrt{12}\log(2\bar{X})$ is an asymptotic normal $(\sqrt{12}\log\theta, \frac{4}{n})$ variable. [8]

7. The frequencies of the four blood groups O, A, B and AB for 140 army cadets and 295 other individuals are given below

	O	A	B	AB	Total
Army cadets	56	60	18	6	140
Others	120	122	42	11	295

Test that the blood group distributions in the two groups are the same. [10]

8. Show that the multiple correlation coefficient $R_{1.23\ldots p}$ is the maximum correlation coefficient between X_1 and a linear combination of $(X_2, \ldots, X_p)$, where $(X_1, X_2, \ldots X_p)$ has a joint distribution. [8]

INDIAN STATISTICAL INSTITUTE

Final Examination Semester II (1997-98)
B Stat (Hons) II & III Year
Elective : ANTHROPOLOGY

Date : May 8, 1998 Maximum Marks : 100 Time : 3 Hours

Note: Use separate answerscript for Group A and Group B. Answer *any five* questions from each Group

GROUP A

1. Choose the correct answer [5x2]

A The ongoing evolution of modern human populations is otherwise known as :
(a) Biological evolution (b) Microevolution (c) Macroevolution

B Cultural evolution and biological evolution are two separate phenomena that do not influence each other : True / False

C Clans are groups where members cannot claim descent from a common ancestor True / False

D Monogamy means marrying within one's own group : True / False

E When a woman marries her deceased husband's brother, the marriage is known as
(a) Polyandry (b) Sororate (c) Levirate

2. Describe in brief the anatomical, physiological and behavioral changes that have occurred in the evolutionary development of mankind [10]

3.(a) Who are man's closest primates ? [2]
(b) Describe the morphological similarities that are found between man and his closest primates [8]

4. Choose the correct answer : [5x2]

A Adaptability includes more than just the ability to survive and reproduce : True / False

B The actual state of deviation from homeostasis in an individual organism or population is known as : (a) Hypoxia (b) Stress (c) Strain

C. "Populations living in warm, humid areas have more melanin than do populations in cooler areas" -- is stated in the :
(a) Bergmann's rule (b) Allen's rule (c) Gloger's rule

D Adaptability occurs at the cell and tissue levels as well as at the system level
True / False

contd..... 2/-

E Univariate analysis is more useful than multivariate analysis in adaptational studies and the vice-versa is true in population comparison studies : True / False

5.A Identify the correct statement : [3]

(i) Acclimation means gradual reduction of responses to, or perception of repeated stimulation

(ii) Acclimation means adaptive biological response to single , experimentally induced stress

B The condition -- low birth weight, high placental weight, slow growth and hypertrophy of the thorax -- is typically associated with : [2]

(a) Nutritional stress (b) Hypoxia (c) Physiological maladjustment

C Which of the following would support the statement that "modern man is a creature adapted to the tropical climate" : [3]

(i) People who live in cold climates generally have lighter skins than people who live in hot climates

(ii) Humans need about one gram of protein to maintain stable body weight

(iii) Humans cannot endure cold temperatures comfortably without clothing

D In the field of epidemiology it is necessary to interrelate the biological and cultural information in the prevalence of disease : True /False [2]

6. Write short notes on any two of the following : [5x2]

(a) Stress

(b) Medical Anthropology

(c) Growth

GROUP B

1 What is the significance of Meiosis? How does it differ from Mitosis? If two daughter cells resulting from meiosis, do they have the same amount of genetic material, do they also have the same genotype? (5+3+2)

2 Describe the normal human karyotype At each conception there is certain probability for a child being a male or a female. If a couple have four girls, what is the probability that their fifth child will be a boy? How many Barr bodies would you predict, in an individual with XXXXX sex chromosome constitution? (6+2+2)

3 Discuss Mendel's law of segregation. A married couple both unable to taste PTC, have a taster child. (a) what is the probability that their 2nd child is also a taster? (b) what are the probabilities of every possible configuration of four children in this family ? (4+2+4)

4. Name a rare disease in human which is caused by single, autosomal recessive gene Draw a pedigree to show the pattern of inheritance If a couple who are both heterozygous produce a large number of children, what is the expected ratio of normal to affected children ? (2+6+2)

5 What is meant by population genetics? There are several factors that can cause allele frequency to change over time Can you identify any factor which has a large impact on changing the gene frequencies in a small population compared to a large one? Illustrate your answer with suitable examples (3+7)

6 The following table shows the allele frequencies of I^A, I^B and I^O in four different populations.

Population	p^A	q^B	r^O	$p^A+q^B+r^O$
English	0 268	0 052	0 680	1 000
Bulgarian	0 271	0 108	0 621	1 000
Arabs	0 209	0 129	0 662	1 000
Hindus	0 149	0 291	0 560	1 000

(A) Determine the frequencies of the four ABO blood groups in each population (B) If equal numbers of Arabs and Hindus intermarry at random, what will be the frequencies of the four blood groups in the resulting population? (6+4)

7 Write short notes on any two of the following? (5x2)

A Sex limited trait
B Nature and Nurture
C Polymorphic
D Mutation

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1997 - 98

SEMESTRAL-II EXAMINATION

Economics II

Date. 8.5.1998 Maximum Marks: 100 Time: 3 hrs.

Note: Answer TWO questions from each of Group - A and Group - B. Maximum marks for each question indicated on the margin.

GROUP - A

1. Consider an economy with "gold" as the only form of money. Outline two possible developments in the economy that lead respectively to (a) an increase in the velocity of money circulation, and (b) the emergence of a new form of money. Show how these two developments may combine to lead to an increase in the total quantity of money in circulation. Obtain an expression this total quantity of money in terms of the quantity of gold-money by suitable parametrisation of the monetary system. Briefly indicate in what way this "formula" may apply in a monetary system where "gold" is replaced by liabilities of the central bank as the primary form of money.

(5+5+5+10+5) = [30]

2. Define the concepts of "value added" and "final use" (in value) in a general system of production without introducing any commodity classification. Show that the sum total of value added in all firms in the economy is equal to the sum total of final use of all commodities (in value). Hence derive the three alternative ways of viewing the national income of the economy. Briefly discuss in this context the basis and significance of the distinction between "capital goods" and "intermediate goods".

(10+10+5+5) = [30]

3. Starting from the identity that any expenditure on goods and services mean an equal sale-proceeds for firms producing these goods and services, show how - by what process - a given amount of final expenditure in an economy creates equal national income. Comment briefly on the notion of "national income" in this statement. Show how the process of determination of national income in (a) Kalecki and (b) Keynes can

contd..... 2/-

be seen as extensions of this "process" where a suitable part of final expenditure (different in (a) and (b)) is treated as "dependent" upon national income and the rest is taken as "given".

(8+?+10+10) = [30]

GROUP - B

4. Explain the general idea of the multiplier process. Give some reasons why the multiplier process may take time to work itself out. Define the notion of "truncated multiplier" and bring out its significance in this context.

(5+7+8) = [?0]

5. Give an account of the interaction of the multiplier process and prices on the assumption that all production is carried out by firms maximising their profits taking prices as given

[?0]

6. Show how monetary factors can put constraints upon the working of the multiplier process. By what mechanism can the economy be brought to an "equilibrium" in this context?

(14+6) = [?0]

7. Write short notes on any two of the following:

(a) Monetisation of public debt.

(b) Balanced budget multiplier.

(c) Inflationary gap.

(d) Three lags in the circular flow of income.

(10 x ?) = [?0]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1997-98
SEMESTRAL-II EXAMINATION
BIOLOGY-II

Date:8.5.98 Maximum Marks:100 Time: 3 Hours

Note: Answer question number One and any seven from the rest.

1. Justify the following statements (any four) [4x4]=[16]

(i) $r = \frac{2.303(\log_{10} w - \log_{10} w_0}{(t_2 - t_1)}$

when:

r = growth rate; w_0 = initial growth; w=final growth;
t_1=initial time; t_2 = final time.

(ii) Enzyme action depends on substrate/enzyme concentration.
(iii) Structural analogue of a substrate inhibits the enzyme action.
(iv) In resistant variety of wheat prevent the 'rust disease' by functional registance
(v) Accumulation of toxic materials within the infected tissue is one of the defense mechanisms of plant.

2. Write the phasic distributions of 120 days duration rice plant. [12]
3. What are Phytoalexines? How they are synthesized in the infected cells? Give salient feature of Phytoatexines. [6x2]=[12]
4. Write short notes on:

a) Melting [illegible]temperature) of DNA
b) [illegible]binant DNA and its use in genetic engineering.

5. State the following: [6x2]=[12]

a) Buffer action
b) Calculate pH of 1 litre solution containing 0.5 mole Na_2HPO_4 and 0.1 mole NaH_2PO_4. ($NaH_2PO_4 \rightleftharpoons Na_2HPO_4 + H^+$, $pK_a = 7.2$)

6. What are the different agrometeorological parameters influencing the crop yield? Also mention their measuring devices with units. What would be the amount of water received by 1 ha. rice crop if the rainfall is 20 mm? [12]

p.t.o.

7. What is moisture availability index? Draw a suitable rice calender with the help of the following data. [12]

Week No.	23	24	25	26	27	28, 29	30	31	32	33
ET(mm)	38	40	32	30	25	20	22	25	19	14	20
Rainfall (at 0.7Prob)	0	4	8	15	22	26	24	30	45	48	50

Week No.	34	35	36	37	38	39	40	41	42
ET(mm)	17	15	23	18	30	37	32	38	36
Rainfall (at 0.7 Prob)	68	72	80	43	30	10	0	0	5

8. What are the different types of fertilizers? [12]
Find out the amount of FYM, Urea, Single Superphosphate and Murriate of potash of rice if the recommendation of fertilizers are 120Kg N + 80 Kg P_2O_5 + 80Kg K_2O. 50% of the recommended N should be given through FYM.

9. What are the different cultural operations in direct and transplanted rainfed rice? Why Boro rice gives higher yield then Aus and Aman rice. [12

10. Write short notes on: (any four) [4x3]=[12]
a) Start of rain
b) Water requirement of crops.
c) Growing degree days
d) Capilary water
e) Rohan rain
f) Soil texture
g) Vermicompost.