INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat. III year

THEORY OF FUNCTIONS (COMPLEX VARIABLE)

Duration: 1 hr. 15 min. Maximum Marks : 100 Date: 4 October 1963

Roll No. ( )

Directions : Answer all the questions. Answers must be brief and to the point. All answers must be written in the spaces provided for the purpose. All scratchwork must be done on the booklet itself. Extra sheets may be taken for scratchwork.

1. What are the real and imaginary parts of the function Cos z ?

2. Carefully define the function Log z. What are the different values of Log i ?

3. What are the different values of $(-1)^i$?

4. State a result using which you can prove that $x^2 e^y$ cannot be the real part of an every where analytic function $f(z)$ where $z = x+iy$.

(Please turn over)

5. Indicate how you would use the Cauchy-Riemann equations to prove that if an everywhere analytic function $f(z)$ takes only real values then $f(z)$ is a real constant.

6. Prove that $\int_C \frac{dz}{z} = \pi i$ where c is a semi-circular arc (centred at the origin) joining the two points $z = 1$ and $z = -1$.

7. Will the value of the above integral be altered if we replace C by any other contour Γ joining the two points $z = 1$ and $z = -1$? Give reasons.

(Please turn over)

8. Is it true that for an arbitrary circle C centred at the origin the integral $\int_C \frac{dz}{1+z^2}$ is equal to zero ? Give reasons.

9. Give a precise statement of a theorem using which you can prove that the integral $\int_C \frac{\sin(\pi z)}{z+1}\,dz = 0$ where C is a square with the four points $2(1+i)$, $2(-1+i)$, $-2(1+i)$ and $2(1-i)$ as vertices.

10. Indicate how you would use Morera's theorem to prove that if $\{f_n(z)\}$ be a sequence of functions converging uniformly (in the domain D) to the function $f(z)$ and if, further, each $f_n(z)$ is analytic in D then so also is $f(z)$.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. III yr.
Periodical Examination

4.10.63. Time: 1 hr.15 mts.

LINEAR ALGEBRA

Answer all questions

1. i) R and S are subspaces of V_n (n-dimensional Vector space)
R+S is the set of all vectors x such that
$x = y+z$; $y \in R$ and $z \in S$.
Show that (R+S) is a subspace of V_n.

ii) If r and s are the dimensions of R and S respectively show that dimension of (R+S) $\leq$ r+s;

iii) Prove that the equality sign in (ii) holds if $R \cap S = [0]$ (i.e. the set containing the null vector only).

2.a) Derive Schwartz's inequality for a finite dimensional Euclidean vector space V_n.

b) If x and y are vectors in V_n show that

i) $|x+y|^2 + |x-y|^2 = 2(|x|^2 + |y|^2)$. Give a geometrical interpretation of the result.

ii) $\big| |x| - |y| \big| \leq |x-y|$

iii) If $|x| = |y|$ then $x+y$ is orthogonal to $x-y$. Give a geometrical interpretation of the result.

3. A linear transformation A from V_n into V_n is said to be idempotent if $A^2 = A$.
Show that

i) A is idempotent iff $Ax = x$ for all $x \in R(A)$ where $R(A)$ is the range of A.

ii) If A is idempotent then there exists a basis $[x_1, x_2, \ldots, x_n]$ for V_n such that
$Ax_i = x_i$ for $1 \leq i \leq \rho(A)$ and $Ax_i = 0$ for $\rho(A) < i \leq n$
where $\rho(A)$ is the rank of the transformation A.

iii) What is the matrix representation of A with respect to the basis discribed in (ii)?

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B.Stat. III Year

PROBABILITY

Duration: $2\frac{1}{2}$ hours | Maximum Marks 100 | Date: 1. 11. 63

1. X and Y are independent random variables.
   X has a Poisson distribution with the parameter λ.

   (a) If Y takes the values 1 and -1 each with probability $\frac{1}{2}$, what is the probability distribution of X+Y ?
   Also find the cumulative distribution function of Y.

   (b) If y follows the Poisson distribution with the parameter , then, using the fact that X+Y follows a Poisson distribution, find the conditional joint distribution of X and Y given that X+Y = 15. (15)

2. The variate X has the p.d.f.

$$f(x) = \text{constant} \ x^{\alpha} \ e^{-x/\beta}, \qquad 0 \leq x < \infty, \ \alpha > -1, \beta > 0.$$

   (a) Find the value of the constant.

   (b) Given $\alpha > 0$, find the mode of the distribution (the point at which f(x) attains its maximum is called the mode of the distribution of X).

   (c) Given that $\alpha = 0$, X is the length of life (i.e., hours of service before burning out) of a certain type of transmitter tube used in aircraft radar sets, with $\beta = 180$. What is the probability that a tube will last less than 90 hours? (20)

3(a) X follows the normal distribution with mean μ and variance σ^2.

   Find the distributions of

   (i) $Y = a + b X \qquad (b \neq 0)$

   (ii) $Y = \dfrac{(X-\mu)^2}{\sigma^2}$

(b) X has the uniform distribution over the interval (0,1).

   Find the distribution of

$$Y = -2 \log_e X$$

   and its relation to the chi-square distribution.
   If $X_1, X_2, \ldots, X_n$ are independent and distributed as X, and

$$Y_i = -2 \log_e X_i \qquad (i = 1, 2, \ldots, n)$$

   <u>State</u> the distribution of $Z = Y_1 + Y_2 + \ldots + Y_n$.

(c) X is said to follow the F distribution with a and b degrees of freedom [$X \sim F(a,b)$ in symbols] if the pdf of X is

$$f(x) = \frac{a^{a/2}\, b^{b/2}}{B(\frac{a}{2}, \frac{b}{2})} \frac{x^{\frac{a}{2}-1}}{(ax+b)^{\frac{a+b}{2}}}, \quad 0 \le x \le \infty$$

Prove that $Y = \frac{1}{X} \sim F(b,a)$.

Hence show how to find $P(Y \le \frac{1}{c})$ given $P(X \ge c)$ for arbitrary $c > 0$.

Deduce that if $a = b$, the median of the F-distribution is unity. (the median of the distribution of X is the number m such that $P(X \le m) = \frac{1}{2}$). (25)

4. The random variables X and Y have the joint p.d.f.

$$f(x,y) = 24y\,(1-x) \qquad 0 \le x \le 1, \quad 0 \le y \le x$$

(a) Find the marginal distributions of X and Y.

(b) Find the conditional distributions of Y given X = x and of X given Y = y.

Verify that $f(x,y)$ is not the product of the marginal probability density functions, nor are the marginal pdf equal to the corresponding conditional pdf.

(c) Are X and Y independent? (25)

5. The pdf $f(x)$ of the variate X is symmetrical about zero [i.e., $f(-x) = f(x)$].

Let $Y = X^2$.

What is the conditional distribution of Y given X = x ?

Compute E(X.Y) and E(X).

Deduce that X and Y are uncorrelated but not independent. (15)

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Periodical Examination

B.Stat. III Year

STATISTICAL METHODS

Duration : 2 hours 30 min. Date: 16.11.63

1. Describe briefly the procedure for testing a statistical hypothesis on the basis of a sample, explaining in this context the following concepts :
   (i) the two kinds of errors associated with a test of significance
   (ii) the critical region
   (iii) the power of a test
   (iv) simple and composite hypotheses.

2. Given below are certain hypotheses about the means and standard deviations of Normal populations. Describe briefly (without proof) how you would proceed to test these hypotheses, on the basis of samples drawn from the population, indicating in each case the form of the statistic that you would use and also the sampling distribution of the statistic when the hypothesis is true.

A. A sample of size n is available from a single Normal population with mean μ and standard deviation σ.

Hypotheses	Free parameters	Alternative
(1) $\mu = \mu_0$	σ known	$\mu \neq \mu_0$
(2) $\mu = \mu_0$	σ unknown	$\mu > \mu_0$
(3) $\sigma = \sigma_0$	μ known	$\sigma > \sigma_0$
(4) $\sigma = \sigma_0$	μ unknown	$\sigma \neq \sigma_0$
(5) $\mu = \mu_0$, $\sigma = \sigma_0$		$\mu \neq \mu_0$, $\sigma \neq \sigma_0$

B. Samples of sizes n_1 and n_2 are available from 2 Normal populations with means and standard deviations (μ_1, σ_1) and (μ_2, σ_2) respectively.

Hypotheses	Free parameters	Alternative
(6) $\sigma_1 = \sigma_2$	μ_1, μ_2 unknown	$\sigma_1 > \sigma_2$
(7) $\mu_1 = \mu_2$	σ_1, σ_2 known	$\mu_1 \neq \mu_2$
(8) $\mu_1 = \mu_2$	$\sigma_1 = \sigma_2$ but common value unknown	$\mu_1 > \mu_2$
(9) $\sigma_1 = 10\,\sigma_2$	μ_1, μ_2 unknown	$\sigma_1 > 10\sigma_2$
(10) $\mu_1 = 5\,\mu_2$	σ_1, σ_2 known	$\mu_1 > 5\,\mu_2$

3. $X_1, X_2, \ldots, X_n$ are independent Normal random variables each with mean μ and standard deviation σ ;

Let $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ and $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2$

Show that (i) $\bar{X}$ and S^2 are independent (ii) $\sqrt{n}(\bar{X}-\mu)/\sigma$ follows the standard Normal distribution and (iii) S^2/σ^2 follows the chi-square distribution with (n-1) degrees of freedom.

4. If U and V are independent random variables, following the chi-square distribution with m and n degrees of freedom respectively, work out the probability density function of

$F = \frac{U/m}{V/n}$.

5. If $\bar{x}$ and s are the mean and the standard deviation in a sample of size n from a Normal population with mean μ ($\mu \neq 0$) and standard deviation σ, show that the sampling distribution of $v = s/\bar{x}$ involves only $V = \sigma/\mu$ as parameter. Work out the asymptotic sampling variance v.

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INDIAN STATISTICAL INSTITUTE
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Periodical Examination

B. Stat. III year

MACHINE TABULATION (Theory)

Duration : 2 hours　　　　　　　　　　　　　　　　　　　　Date: 8.11.63

1. What precautions are normally taken to ensure that information from documents are accurately transferred to punched cards?

2. If cards are fed into a sorter with the printed face up and with Y-edge leading, with the reading brush set on column 17, which cards will go to pocket 5?

3. There is a set of about 25000 cards, arranged in increasing order of serial number which is punched on columns 6-10 of the cards. Each card also has some code number of four digits punched on columns 11-14. The problem is to separate the cards into two decks: (1) cards with the code number 5039 and (2) all other cards; with the cards in each of these two decks arranged as before in increasing order of serial number. How will you do this job?

4. A card is punched for each student of a course, giving the Roll No, sex and the scores in Mathematics (out of a maximum of 100) obtained by the student in the final examination according to the following design :

	item	card columns	remarks
(1)	roll number	1 - 2	
(2)	sex	3	coded 1 for males and 2 for females
(3)	scores in Mathematics	4 - 6	

It is required to get the totals of scores separately for the male and female students and also the number of students of each sex.

Draw the IBM 421 Tabulator control panel wiring diagram for this operation, mentioning the machine set up, switches and keys used etc.

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B. Stat. III year

ECONOMICS

Duration: 3 hours | Total Marks : 100 | Date: 18.10.63

Answer five questions. Attempt at least two questions from each group.

Note: Separate booklet should be used for each Group.

GROUP A

Industrial Revolution

1. What were the main features of the British manorial system? Briefly trace the process of its distintegration.

2. Describe the state of British industries on the eve of the Industrial Revolution. What problems did it pose for the national economy?

3. What do you mean by the term "Industrial Revolution"? In what significant respects did the new economic pattern differ from the pre-revolution pattern?

4. "Even though the two enclosure movements caused widespread sufferings to the British peasants, nevertheless, they made decisive contributions to the growth and eventual transformation of Britain's national economy." Comment.

5. Examine the role played by the foreign trade of Britain in the process of Industrial Revolution.

6. What was the nature of the contribution made by Britain's colonies to her economic development in the second half of the eighteenth century?

GROUP B

Indian Economics

1. Examine the Industrial policy statement issued by the Government of India.

2. "The Indian Fiscal Commission of 1949-50 approached their task from a new angle of vision and laid down new principles of protection". Critically examine the above statement.

3. Discuss the changes that have taken place in the nature, volume and direction of Indias' foreign trade in the last decade and show how far these reflect the changes in the Indian economy.

4. Estimate the causes of India's adverse balance of payments during the plan period. What measures have been adopted to correct the adverse balance?

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INDIAN STATISTICAL INSTITUTE
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Periodical Examination
B. Stat. III year

GEOLOGY

Duration: 3 hours Maximum Marks : 100 Date:27.9.63

To answer ten questions in all;
The questions are of equal marks.

GROUP A

Answer any four of the following:

1. What causes earthquakes? How are the major earthquake epicentres distributed in the world?

2. Confirm or correct the following statements with reasons :

a) Foci of 90% of the earthquakes are located at least 50 km. below the surface of the earth.

b) S-waves do not travel through the core of the earth.

c) Airy's hypothesis of isostasy always gives the best picture of subsurface mass distribution below the mountains.

d) Mountains are formed only by the contraction of the earth's crust.

3. Write short notes on any four of the following :

Tectogene, Geosyncline, Milligal, Mohorovicic discontinuity, Sima, Isostasy.

4. Give a short description of the crust of the earth as has been worked out from seismic evidences.

5. Comment briefly on the following :

a) There are some basic differences between the continental and oceanic crust of the earth.

b) Inner core of the earth is made of very high density material.

c) There is a regular variation of the earth's gravity with latitude.

d) Seismicity, volcanicity and orogenesis are the results of the same natural process.

What are the different stages of mountain building?

(Please turn over)

GROUP B

1. What are the basic principles of Stratigraphy? State main divisions of the subject stratigraphy and the three phases of stratigraphic study.

2. Define and describe briefly different stratigraphic units with examples.

3. Represent a stratigraphic data in a conventional way showing the sequence, interrelations and thickness of stratigraphic units and the palaeontological information side by side.

4. Answer the following :

   a) What are the sedimentary successions in Peninsular India during Cambrian-Devonian time?

   b) Did Peninsular India witness any marine incursion during upper Palaeozoic time and if there be, name the location.

   c) Name the systems of sedimentary rocks that were deposited in India before Cambrian time. What are the principal lithologic types in them?

   d) What is Gondwanaland? Why is the Gondwana system economically important to us? What sediments are found in the Gondwana rocks?

5. Confirm or correct the following statements:

   a) The Trilobites are found in the Gondwana group of rocks in Peninsular India.

   b) Glossopteris flora characterises the lower Gondwana rocks.

   c) Dinosaurian remains have been found by Geological Studies Unit of ISI from the Tertiary of Peninsular India.

   d) Nummulitic Limestone is always a biostratigraphic unit and never a rock-stratigraphic unit.

GROUP C

Answer any one of the following:

1. Answer true or false to the following :

   a) Greywackes indicate a continental environment.
      (If false what does greywacke indicate?)

   b) On a shallow shelf sea we expect to find large boulders associated with clay and limestone.
      (If false what is expected on a shallow shelf?)

   c) Arkose is an early weathering product of mountains.
      (If false under what conditions an arkose is formed?)

   d) Mud-cracks indicate a desiccating environment.
      (If false what do mud-cracks indicate?)

   e) The bathyal environment contains coarse-grained sediment because it is at the foot of mountains.
      (If false what does a bathyal environment contain?)

[illegible] what is meant by palaeo-environment.

b) Write short notes on the following:
   i) Rock-types,
   ii) Rock-associations,
   iii) The continental environments,
   iv) [illegible]

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B.Stat. III year

SOCIOLOGY

Duration : 1 hr. 15 mts. Date: 11 October 1963

Answer any three questions

1. Describe the subject matter of Sociology.

2. Explain the relation, if any, between Sociology and Statistics.

3. What is Kinship? Describe different types of Kinship. Explain the importance of Kinship study in Sociology.

4. Define family and describe different types of family with examples from Indian life. What are the functions of family?

5. Wirte notes on :

   (a) Lineage, (b) Unilateral grouping, (c) Patrilocal,

   (d) Collaterals, (e) Gotra, and (f) Genealogical Method.

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INDIAN STATISTICAL INSTITUTE
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Mid-term examination, 1963

B.Stat. III Year

MATHEMATICS I

Duration: $2\frac{1}{2}$ hours   Date: 3 Dec. 1963

Answer all questions

1. A and B are linear transformations from V_n into V_n. I is the identity transformation. Show that.

   (a) rank A + nullity A = n

   (b) rank (A+B) $\leq$ rank A + rank B

   (c) rank (AB) $\leq$ min {rank A, rank B}

   (d) i) rank A + rank (I−A) $\geq$ n.

   ii) When does the equality hold?

2. A and B are (n x n) matrices, I is the identity matrix and x, y are row vectors.

   (a) Show that $|AB| = |A|\ |B|$

   (b) Show that $|I + x'y| = 1 + xy'$

   (c) Deduce from (b) the value of $|A + x'y|$ where $|A| \neq 0$.

3. A and B are similar matrices. Show that

   (a) A^k and B^k are similar (k is an integer).

   (b) If p(t) is a polynomial in t, then p(A) and p(B) are similar.

   (c) A and B have the same characteristic polynomial.

4(a). A and B are (n x n) matrices. Prove that AB and BA have the same characteristic values.

(b). A is a (n x n) matrix. Such that $a_{ij} \geq 0$ for all i and j and $\sum_{j=1}^{n} a_{ij} = 1$ for i = 1, 2, ... n.

Show that

(i) 1 is a characteristic value of A. What is the corresponding characteristic vector?

(ii) all the characteristic values are $\leq$ 1 in magnitude.

5. A is a (n x n) matrix and T is the linear transformation represented by A with respect to the basis $(\alpha_1, \alpha_2, \ldots \alpha_n)$. x and b are row vectors. Prove that the following statements are all equivalent.

(1) Columns of A are linearly independent.

(2) A is the product of elementary matrices.

(3) There exists a unique matrix A^{-1} such that $A A^{-1} = A^{-1} A = I$.

(4) The system of equations $Ax' = b'$ has a unique solution for every b.

(5) $|A| \neq 0$

(6) Dimension of the range space of T is n.

(7) Dimension of the null space of T is 0.

(8) The linear transformation is one - one.

(9) There is a unique linear transformation T^{-1} such that $T^{-1} T = T T^{-1} = I$.

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Research and Training School

Mid-term examination, 1963

B.Stat. III Year

MATHEMATICS II

Duration: $2\frac{1}{2}$ hours | Maximum Marks: 100 | Date: 3 December 1963

Attempt all the questions

1. Let $f(z)$ be analytic inside and on the simple closed contour C and let us suppose that $f(z) = 3+5i$ for all z on C. Briefly indicate how you would prove that $f(z) = 3+5i$ for all z inside C also. (10)

2a). Define the notion of singularity of a single valued function of a complex variable. (5)

b). Make a classification of singularities that such a function may have. (5)

c). Give an example of a function with a non-isolated singularity at $z = 0$. (5)

3a). Give a definition of the concept of residue. (5)

b). State and prove Cauchy's Residue theorem. (10)

4a). Carefully explain what you mean by the Laurent expansion of a function around one of its isolated singularities. (No proof required). (10)

b). Give an example of a function whose Laurent expansion around the point $z = 3+5i$ is non-termination in both directions. (5)

5a). Explain how you would study the behaviour of a function at the 'point at infinity'. (5)

b). Give an example of a function that may be called regular at the point at infinity. (5)

c). What kind of singularity the function $\sin z$ has at infinity? Give reasons. (5)

d). What is the residue of $\cos z$ at infinity? Give reasons. (5)

6a). Let Γ be the upper semi-circular arc centred at the origin and joining the two points R and $-R$ on the real line. Give a detailed proof of the fact that

$$\int_{\Gamma} \frac{e^{iz}}{z}\, dz \longrightarrow 0 \quad \text{as } R \longrightarrow \infty \qquad (15)$$

b). Indicate where the proof breaks down if we take Γ to be the lower semi-circular arc. (5)

7. Explain how you would use the result stated in the previous question to prove that

$$\int_0^\infty \frac{\sin x}{x}\, dx = \pi \qquad (10)$$

8a). Make a detailed study of the singularities of the function $\pi \operatorname{Cosec}(\pi z)$. (5)

b). What kind of singularity the function has at infinity? (5)

c). What are the residues of the function at its different singularities. Give reasons. (5)

INDIAN STATISTICAL INSTITUTE
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Mid-term examination, 1963

B. Stat. III Year

PROBABILITY

Duration: $2\frac{1}{2}$ hours | Maximum Marks: 100 | Date: 6 December 1963

Answer as many questions as you can.

1(a). The probability of "making the green light" at a certain road intersection is 0.2. In a series of independent trials, what is the number of trials required for the first success? (6)

(b). The probability of obtaining a bulls-eye for a particular person is 1/3. Ten shots are fired. What is the conditional probability of obtaining at least three bulls-eye knowing that at least one bulls-eye has been scored? (6)

2(a). A point is chosen at random inside a circle of radius R. In terms of the distribution function, find the distribution of the distance X of the point from the centre of the circle. (7)

(b). The pdf $f(x)$ of a variable X is symmetrical about zero i.e, $f(x) \equiv f(-x)$. Let $a > 0$. Prove that

$$F(0) = 0.5,$$

$$P(X \geq a) = 0.5 - \int_0^a f(x)\,dx,$$

$$F(-a) + F(a) = 1$$

$$P(-a < X < a) = 2F(a) - 1$$

$$P(|X| > a) = 2F(-a) = 2\left[1 - F(a)\right]$$

where $F(x)$ is the c.d.f. of X. (9)

3. Let $f(x)$ be the pdf of a random variable X.

(a) $$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \leq x < 2 \end{cases}$$

Find the c.d.f. of X.
Also find the distribution of $|X - 1|$. (9)

(b) $$f(x) = \frac{k}{a^2 + (x-b)^2}, \quad -\infty < x < \infty$$

Determine k.

Find the median and the mode by inspection. (7)

(P.T.O.)

4. Given that the waiting time T for some chance event is exponentially distributed, i.e.

$$\{ \text{pdf} = \frac{1}{\beta} e^{-x/\beta}, \quad 0 \leqslant x < \infty, \quad \beta > 0,$$

find (using c.d.f.) the conditional distribution of waiting time under the restriction $T > t_0$ where t_0 is an arbitrary fixed value of T.

Deduce that

$$P(T > t_0 + x \mid T > t_0) = P(T > x) \quad \text{for} \quad x \geqslant 0. \qquad (17)$$

5. The joint distribution of X and Y is given by

$$f(x,y) = \frac{1}{x^2 y^2} \qquad \begin{array}{l} 1 \leqslant x < \infty \\ 1 \leqslant y < \infty \end{array}$$

Let $U = X.Y$ and $V = \frac{X}{Y}$.

By integrating over the appropriate portions of the admissible region, find

(a) $P(U \leqslant 4)$ (7)

(b) $P(V \leqslant 2)$ (7)

(c) the probability of the compound event $U \leqslant 4, \; V \leqslant 2$. (9)

(d) Do any points of the admissible region satisfy the inequalities $U \leqslant 2, \; V > 2$ simultaneously? (4)

(e) Deduce that U and V are not independent. (4)

6. $X_1, X_2, \ldots, X_n$ are independent random variables, i.e., their joint p.d.f. is equal to the product of their marginal density functions.

(a) Let $\phi_1, \phi_2, \ldots, \phi_n$ be functions defined on the real line.

Prove that

$$E[\phi_1(X_1) . \phi_2(X_2) \ldots \phi_n(X_n)] = E[\phi_1(X_1)] . E[\phi_2(X_2)] \ldots E[\phi_n(X_n)] \qquad (5)$$

(b) Each X_i follows a standard normal distribution $N(0,1)$.

Use mgf to obtain the distribution of $\sum_{i=1}^{n} X_i$ (6)

(c) Each X_i has the same distribution as X whose mgf is $M(\theta : X)$. Let $\bar{X} = \frac{1}{n} \sum_{1}^{n} X_i$ be the mean of the random variables. Show that the mgf of $\bar{X}$ is

$$M(\theta : \bar{X}) = M\left[\left(\frac{\theta}{n} : X\right)\right]^n \qquad (7)$$

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B.Stat. III Year

STATISTICS I (Theory)

Duration: $2\frac{1}{2}$ hours | Maximum Marks: 100 | Dated: Dec. 1963

Attempt any FOUR question. Each question carries 20 marks and 20 marks are reserved for assignments.

1. Human blood has been classified into three groups MM, MN and NN according to one system of classification. In a population under pan-mixia the relative frequencies in these three groups are expected to be θ^2, $2\theta(1-\theta)$ and $(1-\theta)^2$ respectively; where the parameter θ, $0 < \theta < 1$ is called the frequency of the gene G_m. In a simple sample of size n from such a population, the frequencies of the groups MM, MN and NN were found to be n_1, n_2 and n_3 respectively; $n_1+n_2+n_3=n$.

   Obtain an unbiased estimate of θ and calculate the variance of the estimate.

2. A gun is aimed at the centre O of a plane target, on which two orthogonal axes of reference XOX' and YOY' are marked out. If a shot fired from the gun hits the target at a point P whose co-ordinates with reference to the above two axes are (x,y); then x and y are said to be the errors of the gun along the two axes. These two errors for a particular gun are independently and Normally distributed random variables, each with mean 0 and standard deviation σ (feet).

   Calculate the chance that a shot from the gun will hit the target within a circular region of radius r feet, with centre O.

3. A statistic X is said to have the chi-square distribution with n degrees of freedom if its probability density function is of the form :

$$p_n(x) = \begin{cases} \dfrac{e^{-\frac{1}{2}x} \, x^{\frac{1}{2}n-1}}{2^{\frac{1}{2}n}\,\Gamma(\frac{1}{2}n)}, & \text{for } 0 \le x < \infty \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate the expected value of X^t.

   If s^2 is the usual unbiased estimator of the variance σ^2 of a Normal population, based on a sample of size n, then the statistic $(n-1)s^2/\sigma^2$ follows the chi-square distribution with $(n-1)$ degrees of freedom. Use this fact to determine the constant c, which will make $s^* = cs$ an unbiased estimate of σ.

(P.T.O.)

4. X and Y are independent random variables distributed Normally with zero mean and unit variance. Work out the sampling distribution of the statistic

$$T = \frac{X+Y}{|X-Y|}.$$

5. A statistician uses the following rule to decide if a coin is unbiased or not : He tosses it 5 times and if he gets either all heads or all tails, he regards the coin as biased; otherwise he takes it as unbiased. What is the 'size' of the test? What is the 'power' of the test when the coin is biased and the probability for getting a head is 0.6?

6. If r is the correlation coefficient computed from a sample of size n from a bivariate Normal population in which the correlation coefficient is ρ, then the asymptotic sampling variance of r for large n is

$$V(r) \sim \frac{(1-\rho^2)^2}{n}$$

Work out the asymptotic sampling variance of the statistic

$$z = \frac{1}{2} \log_e \frac{1+r}{1-r} .$$

Research and Training School

Mid-term examination, 1963

B.Stat. III Year

STATISTICS II (Practical)

Duration : 3 hours | Maximum Marks: 100 | Date: 4 Dec. 1963

Note: Books and Practical records are not to be used during the examination.

1. The standard deviation of a Normal population is known to be unity, but the mean is unknown. How will you test, at the 5% level of significance, the hypothesis H_o that the mean is zero, against the alternative that the mean is different from zero, on the basis of a sample of size n=16 drawn from the population? Calculate the probability $P(\mu)$ that your test procedure will result in the rejection of the hypothesis H_o when the value of the mean is μ for

$\mu = -1.0, -0.5 \ (0.1) + 0.5, + 1.0.$

Plot the graph of $P(\mu)$ against μ. (25)

2. Estimates of the proportion of employed persons in a certain city obtained from two independent random samples are given below. Can the difference between the two estimates be regarded as due to fluctuations of sampling? If so, combine the two estimates and calculate the standard error of the combined estimate.

survey	sample size	estimate of proportion employed
1	$n_1 = 2350$	$p_1 = 0.3069$
2	$n_2 = 2675$	$p_2 = 0.3854$

(20)

3. The following gives the heart weights in grams of 12 female and 15 male cats. Does the heart of a male cat on an average weigh more than that of a female cat?

heart weight of cats in grams.

males : 12.7, 15.0, 9.1, 7.8, 12.8, 8.3, 11.2, 9.4, 8.0, 14.9, 10.7, 13.6, 9.6, 11.7, 9.3

females: 7.4, 7.3, 7.1, 9.0, 7.6, 9.5, 10.1, 10.2, 10.1, 9.5, 8.7, 7.2

(20)

4. The following table gives the observed frequencies of different combinations of colour and pollen shape in sweet pea in a certain plant breeding experiment. Given in parentheses are the relative frequencies expected under Mendel's genetical theory of inheritance. Examine whether the data are in agreement with Mendel's theory.

pollen shape	colour: purple	colour: red
long	296 (9/16)	27 (3/16)
round	19 (3/16)	85 (3/16) 1/16

(25)

5. Class Record (10)

INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examinations 1963

B. Stat. III Year

ECONOMICS – I

Duration: 2 hrs. Date: 2 December 1963

Note: Separate Answer Book should be used for each Group.

Answer four question of which at least one must be from each group.

GROUP A

1. Examine the principal characteristics of the Manorial System. How did it disintegrate?

2. Briefly explain any two of the following :

   i) The Second enclosure movement.
   ii) The Factory System.
   iii) Put out System.

GROUP B

3. "Japan's industrialisation was carried out in a backward setting" – Examine this statement.

4. Discuss the State's role in the process of primary accumulation of capital in Japan's economy.

5. "Agriculture played a vital role in the industrial development of Japan." – Discuss.

6. Examine the role played by the following in the industrial development of Japan at the initial stage (any three):

   i) Textile Industry;
   ii) Traditional consumption pattern;
   iii) Labour-management relation;
   iv) Isolation.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B.Stat. III Year

ECONOMICS- II

Duration : 2 hours | Date: 2 December 1963

Attempt any three

1. Discuss the characteristics of Indian Banking System and its defects.

2. Discuss the steps taken by the Government to solve the problems of Indian Banking System.

3. Examine the role of Reserve Bank of India in the economic development of our country.

4. Critically examine the Deposit Insurance Scheme adopted in India.

5. "Nationalisation of Bank is an immediate need for the rapid and proper growth of our economy." – Discuss.

6. Examine the causes of India's adverse balance of payments and its effects on the economic development of our country.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination, 1963

B. Stat. III

BIOCHEMISTRY (Theory)

Duration: 2 hours | Maximum Marks – 100 | Date: 7 Dec. 1963

1. What is an enzyme? Define enzyme unit. Describe the preparation and properties of the following enzymes.

   i) Pepsin
   ii) Urease (15)

2. Give one example for each of the following :

   Polypeptide, Coenzyme, Aminoacid, Hormone, Nucleotide (10)

3. Describe how pyruvic acid is metabolised in the mammalian system. (15)

4. What is a provitamin? Give example. What are the deficiency symptoms and natural sources for the following vitamins.

   i) Vitamine B_1
   ii) Vitamine B_2
   iii) Vitamine B_{12} (20)

5. How the protein molecule is made up? How can you quantitatively estimate protein in a natural source. Why we need protein in our diet. (15)

6. What is thyroxine? Describe its effect on human body. (10)

7a). How can you identify the individual components in a mixture of amino acids?

b). What is isoelectric point of a protein?

c). What is DPN and TPN?
What are their biological functions? (15)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Mid-term examination 1963

B. Stat, III Year

SOCIOLOGY

Duration : 1 hr. 15 min.   Maximum Marks: 100   Date: 5 December 1963

Note: Answer any three questions, at least ONE from each group.

GROUP A

1. Define clan, moiety and phratry and explain their interrelationships. What are the functions of clan?

2. What is the importance of the study of caste in Sociology?

3. What is age-grade and in what way this influences customary groupings among us?

GROUP B

4. Define "institution". Distinguish between true and natural sexual division of labour. Give examples from primitive and civilized societies.

5. What is incorporeal and corporeal property? Illustrate your answer with examples from both primitive and civilized societies.

6. What is the social significance/value attached behind "lobola", "kula", "potlatch" and "Feasts of Merit"?

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. III Year

ANALYSIS

Duration: 2 hours | Maximum Marks: 100 | Date: 13 April 1964

1. State Wierstrass theorem on approximation of any continuous function by polynomials.

2. Use (1) to prove that

$$\lim_{t \to \infty} \int_a^b f(x) \cos tx \, dx = 0.$$

3. If f and g are two continuous functions on $[0,1]$ such that

$$\int_0^1 e^{tx} f(x)\, dx = \int_0^1 e^{tx} g(x)\, dx$$

for values of t lying in an interval around the origin, show that $f(x) \equiv g(x)$.

4a). State Parceval's theorem on Fourier Series.

b). $f(x)$ is a periodic function of period 2π defined as $f(x) = |\pi - x|$ for $-\pi \leq x \leq \pi$. Find the Fourier Series corresponding to f and discuss its convergence. Deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

c) Find the Fourier Series of the function (of period 1) defined as follows :

$$f(x) = \begin{cases} x & \text{in } 0 \leq x \leq \frac{1}{2} \\ 1-x & \text{in } \frac{1}{2} \leq x \leq 1 \end{cases}$$

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat. III Year

LINEAR ALGEBRA

Duration : $2\frac{1}{2}$ hours | Date: 23 March 1964

Attempt any four questions.

1. A is a self-adjoint linear transformation defined on a Complex Euclidean Vector Space. Show that

   (i) All the characterstic values of A are real.

   (ii) If λ_1 and λ_2 are two distinct characteristic values and x_1, x_2 are characteristic vectors corresponding to λ_1, λ_2, then $(x_1, x_2) = 0$.

   (iii) There exist n independent characteristic vectors of A.

2. C. is an orthogonal linear transformation defined on a real Euclidean vector space. Show that

   (i) C takes an orthonormal basis to another orthonormal basis.

   (ii) All the characteristic value of C are in absolute value equal to unity.

   (iii) The determinant of the matrix of C relative to an orthonormal basis equals ± 1.

3. A is real symmetric matrix. Show that there exist an orthogonal matrix C such that CAC' is diagnol.

   Hence or otherwise show that the quadratic from xAx' can be expressed as sum of squares by a co-ordinate transformation. x is the row vector $(\xi_1, \xi_2, \ldots, \xi_n)$.

4. If A is a symmetric matrix, show that the necessary and sufficient condition for A to be positive definite is that all the principal minors of A are positive.

5. Show that the quadratic form (Ax, x), where A is a self adjoint linear transformation, attains its minimum over all vectors of unit length at a characteristic vector of A and the minimum attained is the corresponding characteristic value of A.

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INDIAN STATISTICAL INSTITUTE
Research and Training School
Periodical Examination
B.Stat. III year
PROBABILITY

Duration : 2½ hours test. Date : 17 Feb., 1964.
Maximum marks : 100

1. X_1 and X_2 are discrete random variables taking two values each. If X_1 and X_2 are uncorrelated, prove that they are independent. (18)

2. X_1 and X_2 have a uniform distribution (i.e., the joint p.d.f. = a constant) over the square whose vertices are the points (1,1), (1,-1), (-1,-1) and (-1,1). Find the joint distribution of $X_1 + X_2$ and $X_1 - X_2$. (22)

   Are X_1, X_2 independent ? (4)

   Are $X_1 + X_2$, $X_1 - X_2$ independent ? (4)

3. $X_1, X_2, \ldots X_n$ are independent standard normal variates. A is a given r×n matrix ($r \leqslant n$) such that AA' = I, where I stands for the identity matrix.

   Let $X = \begin{pmatrix} X1 \\ X2 \\ \vdots \\ Xn \end{pmatrix}$ and $Y = \begin{pmatrix} Y1 \\ Y2 \\ \vdots \\ Yr \end{pmatrix}$. You are given that Y = AX.

   Prove that $Y_1, Y_2, \ldots, Y_r$ are independent standard normal variates. (30)

4. A random sample $X_1, X_2, \ldots, X_n$ of n independent observations is taken from a population where the distribution is as follows: the pdf is f(x) and the cdf is F(x). (Thus each X_i has the pdf f(x)) .

   Let $X_{[1]} = \max \{X_1, \ldots, X_n\}$, be the maximum of the observations.

   (a) Prove that the event $\{X_{[1]} \leqslant x\}$ occurs if and only if the joint even $\{X_1 \leqslant x, X_2 \leqslant x, \ldots, X_n \leqslant x\}$ occurs. (7)

   (b) Hence find the pdf. of $X_{[1]}$ (10)

   (c) Use a similar method to find the pdf. of
   $X_{[n]} = \min. \{X_1, \ldots, X_n\}$. (10)

   (d) Given that $X_i \frown R[\theta, \theta + 1]$, where $R[\theta, \theta+1]$ denotes the rectangular distribution on $[\theta, \theta + 1]$, find $E(X_{[1]})$ (10)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. III Year

PROBABILITY

Duration: $2\frac{1}{2}$ hours | Maximum Marks: 100 | Date: 6 April 19[illegible]

1. It is found that the distribution of income X in a population is given by the Pareto distribution :

$$P(X > x) = \left(\frac{x_0}{x}\right)^{\alpha}, \qquad x > x_0, \quad \alpha > 0,$$

where x_0 and α are the parameters.

Find the median of the distribution; and, given $\alpha > 1$, find the mean. (15)

2(a). X has the pdf

$$f(x) = \text{constant}\left(1 - \frac{x^2}{a^2}\right)^m, \quad -a < x < a.$$

Find the distribution (you need not find the constant in the pdf) of the variable T which is related to X by the equation

$$X = \frac{aT}{\left[2(m+1)+T^2\right]^{\frac{1}{2}}}$$

[Note: The positive value of the square root is taken]

(b). X and Y are independent $N(0,1)$. Find the distribution of $\frac{X}{Y}$. (14+[illegible])

3(a). X follows a rectangular distribution on the interval $[a-h, a+h]$. Find its characteristic function.

(b). The random variable X_n has the cdf

$$F_n(x) = \begin{cases} 0 & \text{for } x < -n \\ \frac{x+n}{2n} & \text{for } -n < x < n \\ 1 & \text{for } x \geq n \end{cases}$$

Find the C.F. $\phi_n(t)$ of X_n.

Also find $\lim_{n \to \infty} \phi_n(t)$.

Is this limit function continuous at $t = 0$? (12+[illegible])

(Please turn over)

4. n independent observations X_1, X_2, ..., X_n are taken at random on a random variable X for which $E(X) = \mu$, $V(X) = \sigma^2$. A function $Y_n = f(X_1, X_2, \ldots, X_n)$ is said to be a consistent estimator of μ if

$$P\left\{ |Y_n - \mu| > \epsilon \right\} \longrightarrow 0 \quad \text{as } n \rightarrow \infty,$$

for each $\epsilon > 0$. Prove that the sample mean $\bar{X}_n = \frac{1}{n} \sum_{1}^{n} X_i$ is a consistent estimator of μ. (15)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B.Stat.III Year

STATISTICS

Duration: 3 hours | Maximum Marks:100 | Date:20 April 1964

Use of books, notes, practical records allowed.

1. A,B,C and D are four points on a straight line. Lengths of the segments AB, AC, AD, BC, BD and CD are measured, and the measurements are denoted by y_1, y_2, y_3, y_4, y_5 and y_6 respectively. The measurements are subject to independently and normally distributed random errors, each with zero mean and a common unknown variance σ^2. The numerical values of the measurements are : $y_1 = 1.98$, $y_2 = 5.13$, $y_3 = 5.89$, $y_4 = 3.06$, $y_5 = 4.13$ and $y_6 = 1.14$.

(i). Denoting by θ_1, θ_2 and θ_3 the true lengths of the segments AB, BC and CD respectively, write down the observational equations in terms of θ_1, θ_2 and θ_3. (3)

(ii). Obtain the least-squares equations for estimating θ_1, θ_2 and θ_3. (5)

(iii). Obtain the 'best' estimates of θ_1, θ_2 and θ_3. (7)

(iv) Estimate σ^2. (5)

(v). Obtain the dispersion matrix of the estimates of θ_1, θ_2 and θ_3. (5)

(vi). Estimate the total length of the segment AD and find the standard error of the estimate. (7)

(vii). Test the hypothesis that the length of the segment AD is 6 units, against both-sided alternatives. (6)

(viii). Test the hypothesis that the points B, and C internally divide the line AD in the ratio 2 : 3 : 1. (12)

2. One evening three persons suspected to be driving under the influence of liquor were stopped and blood samples taken from each were sent to the laboratory. Five determinations of per cent alcohol in blood were made on each sample. According to law, drivers who have more than 0.05% alcohol in their blood should be sent to jail. Do the data suggest that all three drivers were equally inoxicated, as determined by per cent alcohol in blood? Should any or all of them be sent to jail?

driver	per cent alcohol in blood samples .1	.2	.3	.4	.5
1	0.08	0.06	0.06	0.07	0.09
2	0.14	0.18	0.16	0.15	0.12
3	0.00	0.02	0.01	0.02	0.03

[State your assumptions clearly] (50)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Periodical Examination

B. Stat. III Year

ECONOMICS

Duration: 3 hours | Full Marks: 100 | Date: 2 March 1964

Answer five question, of which at least two must be from each group.

GROUP A : Economic Development

1. "The essence of the problem of economic development is the problem of raising labour productivity."

   Amplify the statement and bring out the part played by Capital. In this context discuss also the vicious circle of proverty. (12+7+6)

2. Industrial revolution is an end-product of important changes in the economic and extra-economic spheres. Show that the way of achieving this end-product is not one but many. In this context, compare and contrast the way industrial revolution or the take-off was completed in England and in Japan, with special reference to the position of the factors. (12+13)

3. "The pattern of Japanese growth doffers from the earlier European pattern in several respects, notably in the development of industry at a low income level and the unevenness of technological growth." Explain this statement and describe the peculiar pattern of growth of employment and the three sectors. (25)

4. Statd the salient features of Japan's Doubling the National Income Plan. (25)

GROUP B : Indian Economic Conditions

5. Explain the causes of recent price rise in India and examine the adequacy of the measures so far taken by the Government to curb it. (25)

6. Discuss the characteristics of India's Tax structure and examine the principal features of Kaldor's Tax reform proposals. (25)

7. Examine the role that India's Public Debt has been playing in our economic development. (25)

8. Discuss the importance and functions of the State Trading Corporation of India. (25)

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INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. III Year
Periodical Examination

PHYSICS

Duration: 2 hours   Full marks : 50   Date: 24.2.64

1. What is thermodynamic probability? Show that the number of particles in the i th cell, in the state of maximum thermodynamic probability, is, according to M-B statistics, is

$$N_i = \frac{N}{Z} \exp(-w_i/KT)$$

where N = total number of particles, Z = partition function, w_i = energy of the particle whose phase point is in the i th cell, K = Boltzmann constant and T=temperature in degree absolute. (18)

2. Apply the Maxwell - Boltzmann statistics to the monoatomic ideal gas to find

(i) the velocity distribution function, and

(ii) the equation of state. (16)

3(a). Suppose there are three cells in phase space: 1,2, and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$ and $w_1 = 2$ joules, $w_2 = 4$ joules, $w_3 = 6$ joules. If $\delta N_3 = -2$ find δN_1 and δN_2 such that $\delta N = 0$ and $\delta U = 0$. (10)

(b). Derive Stirling's approximation for finding the factorial of a large number with sufficient precision. (6)

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INDIAN STATISTICAL INSTITUTE
Research and Training School
Periodical Examination
B.Stat. III Year

ENGINEERING

10.2.64. Time : 2 hrs.

Note : (1) Attempt question No.4 and any two questions from the remaining ones.
(2) All questions carry equal marks.

1. State briefly general assumptions made in calculations on reinforced concrete beams.

The moment of resistance of a reinforced concrete beam may be expressed by the formula, $M = R\,bd^2$ where M is the moment of resistance, b the breadth of the beam and d the depth to the reinforcement, R, being a constant, depending on the concrete & steel stresses and the modular ratio.

If the allowable stresses in the concrete and steel are 750 lb. per sq. in. and 18000 lb. per square inch respectively and the value of modular ratio is 18, determine the value of R for economically designed R.C.C. beam. Also calculate the economic percentage of steel. Discuss how the values of R would change in the cases of under-reinforced and over-reinforced beams.

2. A timber beam 25 feet long and 15" square in section floats horizontally in sea water. The weight of timber is 40 lb. per cubic foot and of water 64 lb. Two equal weights just sufficient to immerse the beam are placed on it, 7 feet from each end. Calculate the value of each weight. Draw the shear force and Bending Moment Diagram for the beam. Calculate maximum flexural and shear stresses in the beam.

3. An 18" × 7" × 75 lbs. R.S.B. is simply supported over a span of 18 feet and carries a distributed load which varies uniformly from 1 ton per foot run at one end to 3 tons per foot run at the other. Find the amount of deflection at the mid-span, for the R.S.B., $I_{xx} = 1149\ in^4$, $E = 13000\ tons/in^2$.

4. For a given stress, compare the moments of resistance of a beam of square section placed (i) with two sides horizontal, (ii) with a diagonal horizontal.

A vertical wooden post 25' high tapers from 6½" dia. at the base to 4" at the top. At what point will the post break under a horizontal pull at the top?

If the modulus of rupture of the wood is 4000 lb. per square inch, calculate the pull which will cause failure.

INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

MATHEMATICS I (Analysis)

Duration: 3 hours | Maximum Marks: 100 | Date: 20 May 1964

Answer all questions

1(a). State Wierstrass theorem on approximation of continuous functions by polynomials. (5)

(b). For the function $f(x) = \sqrt{x}$ in $0 \leq x \leq 1$, find the values of n such that $|B_n(f,x) - \sqrt{x}| < 1/10$ uniformly in x, $B_n(f,x)$ denoting the Bernstein polynomial of degree n corresponding to f. (10)

2(a). State Fejer's theorem on Fourier series. (5)

(b). If X and Y are two integral valued random variables having the same characteristic function, deduce from (a) that X and Y must be identically distributed. (5)

(c). If $f(x)$ is continuous and periodic of period 2π, prove the inequality

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f|^2\, dx \geq \sum |a_k|^2$$

where $a_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-ikx}\, dx.$ (10)

(d). Find the Fourier series of the function $f(x) = e^{\alpha x}$, $-\pi \leq x \leq \pi$ α being a complex number. Deduce the value of

$$\sum_{k=0}^{\infty} \frac{1}{1 + k^2\pi^2}$$ (20)

3(a). If $f(x)$ is an integrable continuous function on $(-\infty, \infty)$ show that

$$\lim_{|t| \to \infty} \int_{-\infty}^{\infty} f(x)\, e^{itx}\, dx = 0$$ (10)

(b). Evaluate the Fourier transforms of the following

(i) $e^{-|x|}$ (10)

(ii) $\frac{1 - \cos 2x}{x^2}$ (15)

(iii) $f(x) = 0$ if $x \geq 6$ or $x \leq 4$
$= 6 - x$ if $5 \leq x \leq 6$ (10)
$= x - 4$ if $4 \leq x \leq 5$

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

MATHEMATICS II (Linear Algebra)

Duration : 3 hours | Maximum Marks: 100 | Date: 20 May 1964

Attempt all questions

1. R and S are subsets of an n-dimensional Euclidean Vector Space V. $R^{\perp}$ is the set of all vectors such that every vector in $R^{\perp}$ is orthogonal to every vector in R and $S^{\perp}$ is defined similary.

Show that

(a) if $R \subset S$ then $R^{\perp} \supset S^{\perp}$

(b) $(R+S)^{\perp} \subset R^{\perp} + S^{\perp}$

(c) $R^{\perp} + S^{\perp} \subset (R \cap S)^{\perp}$

(d) If R and S are subspaces of V, state and prove the corresponding results for (b) and (c).

[R+S is the set of all vectors $z=x+y$ such that $x \in R$, $y \in S$]

2(a). Let A be an $(n \times n)$ real matrix.

$x = (\xi_1, \xi_2, \ldots, \xi_n)$ and $b = (b_1, b_2, \ldots, b_n)$

are row vectors. Show that the system of linear equations

$Ax' = b'$ is uniquely solvable for every b if and only if it is solvable uniquely for a particular b.

(b). A is an $m \times n$ real matrix, $b = (b_1, \ldots b_m)$ and x is as in (a). State and prove the necessary and sufficient conditions under which $Ax' = b'$ has a solution.

(c). A, b and x are as defined in (b). Show that even though $Ax'=b'$ may not always have a solution, $A'Ax' = A'b'$ has always a solution.

(Please turn over)

3(a). B is a skew symmetric real matrix (i.e. $b_{ij} = -b_{ji}$) and λ is a characteristic value of B. Show that $-\lambda$ is also a characteristic value of B.

(b). A is a real $n \times n$ matrix such that $I + A$ is nonsingular Then show that A and $(I+A)^{-1}$ commute.

(c). B is as in (a) and $I + B$ is nonsingular. Show that $A = (I-B)(I+B)^{-1}$ is orthogonal. Also show that $(I-A)$ is nonsingular.

4(a). U is a unitary linear transformation defined on a complex Euclidean Vector Space. Show that there exists an orthonormal basis relative to which the matrix of U is diagonal

(b). A and B are linear transformations in complex Euclidean vector space $AB = BA$. Show that they have a common characteristic vector.

(c). Prove that the necessary and sufficient condition for the existence of an orthonormal basis relative to which the matrix of a linear transformation is diagonalis that it should be normal.

5(a). If A is a positive semi-definite linear transformation then show that there exists a positive semi-definite linear transformation H such that $A = H^2$.

(b). Using the above result, prove that any arbitrary linear transformation can be represented as the product of a positive semi-definite linear transformation and unitary transformation.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1961

B. Stat. III Year

PROBABILITY

Duration: 3 hours | Maximum Marks: 100 | Date: 23 May 1961

Answer all questions

1. $X_1, \ldots, X_n$ are independent observations on a random variable X whose distribution is symmetric about 0. Prove that $\sum_{i=1}^{n} X_i/n$ is uncorrelated with $\sum_{i=1}^{n} X_i^{2r}/n$, where r is any positive integer. (6)

2. A random variable X has probability density function (pdf)

$$f(x) = 6x\,(1-x), \qquad 0 \leq x \leq 1.$$

Find a monotically increasing function of X having the pdf

$$g(y) = 3(1 - \sqrt{y}), \qquad 0 \leq y \leq 1.$$ (12)

3. There are two brands of synthetic fibre, A and B. The pdf of the breaking strength (i.e. the force required to break the fibre) for A is

$$f_1(x) = 3ax^2\, e^{-ax^3}, \quad 0 \leq x < \infty, \quad a > 0;$$

and for brand B is

$$f_2(y) = 3by^2\, e^{-by^3}, \quad 0 \leq y < \infty, \quad b > 0.$$

If a piece of each brand of fibre is chosen at random and tested, what is the probability that the fibre of brand A will withstand greater tension than that of brand B? (12)

4. The joint pdf of two variates U and V is given by

$$f(u,v) = \frac{1}{2}\, e^{-u}, \qquad 0 \leq u < \infty, \quad -u \leq v \leq u.$$

Find the marginal distribution of V. (10)

5. X and Y have the joint pdf

$$f(x,y) = \frac{2}{\pi}\,(1 - x^2 - y^2), \quad 0 \leq x^2 + y^2 \leq 1.$$

Find the joint distribution of

$$R = +\sqrt{X^2 + Y^2} \quad \text{and} \quad \theta = \tan^{-1}(Y/X)$$ (10)

(Please turn over)

6. $X_1, \ldots, X_n$ are independent observations on $N(\mu, \sigma^2)$.

(a) Show that the statistics

$$\bar{X} = \sum_{i=1}^{n} X_i/n \quad \text{and} \quad S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

are mutually independent. (10)

(b) Derive the pdf of

$$t = \sqrt{n(n-1)}\ (\bar{X} - \mu)/S$$ (10)

7. Find the characteristic function of a random variable whose pdf is

$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty$$ (10)

8. $X_1, X_2, \ldots$ is an infinite sequence of independent random variables with $P[X_i=1] = p$, $P[X_i=0] = 1-p$.

Show that for each real number a,

$$\lim_{n \to \infty} Pr\left[\frac{X_1 + \ldots + X_n - np}{\sqrt{np(1-p)}} \leq a\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-t^2/2}\, dt.$$ (20)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. STAT. III Year

STATISTICS (Theory)

Duration : 3 hours | Maximum Marks: 100 | Date: 18 May 1964

Attempt any FOUR questions

1. Explain the term Uniformly Minimum Variance Unbiassed Estimate (UMVUE) of a parameter.

Considering statistics with positive finite variance, show that a statistic T is the UMVUE of its expected value, if and only if,

Cov $(T, Z) = 0$ for all statistics Z with $E(Z) = 0$.

2. Consider n independent random variables y_i $(i=1,2,\ldots,n)$ with a common variance and expectations given by

$$E(y_i) = a_{i1}\theta_1 + a_{i2}\theta_2 + \ldots + a_{im}\theta_m$$

where a_{ij}'s are given constants, and θ_j's are unknown parameters. Let $Q_j = a_{1j}y_1 + a_{2j}y_2 + \ldots + a_{nj}y_n$. $(j = 1,2,\ldots,m)$.

(a) Show that for any given constants $\lambda_1, \lambda_2, \ldots, \lambda_m$ the statistic $T = \lambda_1 Q_1 + \lambda_2 Q_2 + \ldots + \lambda_m Q_m$ is the Uniformly Minimum Variance Unbiased Linear Estimate of its expected value.

(b) If $E(T) = \ell_1\theta_1 + \ell_2\theta_2 + \ldots + \ell_m\theta_m$, show that $V(T) = (\ell_1\lambda_1 + \ell_2\lambda_2 + \ldots + \ell_m\lambda_m)\sigma^2$.

3. Let T be the UMVUE for a parameter and T_1 some other unbiassed estimate for it. Let $E = V(T)/V(T_1)$ and ρ = coefficient of correlation between T and T_1.

(a) Determine λ (in terms of ρ and E) for which $T_0 = \lambda T + (1-\lambda)T_1$ has minimum variance.

(b) Evaluate this minimum variance.

(c) Utilizing the fact that T_0 is another unbiassed estimate of the same parameter, show that min $V(T_0) = V(T)$.

(d) Hence obtain ρ in terms of E.

(Please turn over)

4. Given n pairs of observations (y_i, x_i) $i = 1, 2, \ldots, n$; with the set up

$$y_i = \alpha + \beta x_i + e_i$$

where e_i's are (unobservable) random variables, distributed independently and normally, with expectation zero and common but unknown variance σ^2, and α and β are unknown parameters and x_i's are non-stochastic, describe how you will test the hypothesis $\beta = 0$. Write down the test criterion explicitly and obtain its sampling distribution when the hypothesis is true.

Derive Cramer-Rao bound for the variance of an unbiassed estimator of a parameter.

Evaluate this bound for the problem of estimation of the parameter θ on the basis of a sample n from a truncated Poisson population with probability law :

$$p(x, \theta) = (e^{\theta} - 1)^{-1}. \ \theta^x / x! \ , \quad x = 1, 2, \ldots,$$

Write a short note on estimation by the method of maximum likelihood.

Obtain maximum likelihood estimates of the mean and the standard deviation of a normal population on the basis of a sample of size n. Find the asymptotic dispersion matrix of the estimates.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

STATISTICS (Practical)

Duration: 3 hours | Maximum Marks : 100 | Date: 19.5.64

Attempt BOTH the questions.
Accuracy in computation and neat tabular layout of computation will be rewarded.

Three objects A,B,C were weighed ten times on a chemical balance, by putting some objects on the left pan and some on the right and balancing against standard weights put on the pans. The results are given below :

Objects on the right pan	Objects on the left pan	Standard weights on the right pan	Standard weights on the left pan
A,C	-	-	14.15
A	C	4.65	-
A,B	C	-	3.15
A,C	B	-	8.47
B,C	A	-	11.87
-	A,B,C	15.41	-
B,C	A	-	112.10
-	A,B	0.31	-
B	C	0.91	-
C	B	-	3.95

It is suspected that the balance is not accurate and the two pans when empty are not of equal weights.

(a). Introducing one parameter for the weight of each object and one additional parameter for the excess of the weight of the empty right pan over that of the empty left pan, write down the observational equations. (10)

(b) State explicitly the assumptions that you would like to make about the nature of the experimental errors. (10)

(c) Estimate the weights of the three objects together with their standard errors (after correction for bias). (30)

(d) Is correction for bias really necessary? (10)

(Please turn over)

2. The following table gives the observed frequency of different combinations of colour and pollen shape in sweet pea in a certain genetical experiment (Bateson's data)

pollen shape	colour purple	colour red
long	296	27
round	19	85

The theoretical frequencies in the presence of linkage are

pollen shape	colour purple	colour red
long	$\frac{1}{4}(2+\theta)$	$\frac{1}{4}(1-\theta)$
round	$\frac{1}{4}(1-\theta)$	$\frac{1}{4}\theta$

(a) Write down the likelihood function for the data. (5)

(b) Write down the likelihood equation for estimating θ and also the expression for the amount of "information". (7)

(c) Obtain the maximum likelihood estimate for θ and its standard error. (12)

(d) Are the data in agreement with the theory? (8)

(e) Is θ different from $\frac{1}{4}$? (8)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1961
B. Stat. III Year
ECONOMICS I

Duration : 3 hours | Maximum Marks: 100 | Date: 22 May 1964

Separate Answer-book should be used for each Group.

GROUP A – Industrial revolution

1. What do you mean by the term "Industrial Revolution"? What changes did the Industrial Revolution bring about in Britain's economic structure during the period 1760–1860? (20)

Or,

"Without the break-up of the Manorial System, British economy would have remained stagnant and without the Enclosure Movement, the process of dynamic economic growth in England would not have been possible." – Critically examine the above statement. (20)

Or,

Write short notes on any THREE of the following :

i) English Industrial System on the eve of the Industrial Revolution.

ii) The main points of difference between the two enclosure movements.

iii) The Role of colonies in capital accumulation in England, in the early stages of the Industrial Revolution.

iv) Landlord–Tenant Relations under the Manorial System.

v) Internal and external factors, which made England the pioneer in the field of industrial revolution. (20)

2. "If there were any agricultural revolution in Japan, it occured during the period from Restoration to World War I". —Examine the statement. (20)

Or,

Discuss the distinct role played by (a) the textile industry, and (b) Traditional Consumption Pattern in propelling the economic development of Japan. (20)

Or,

Discuss the role of the State in the Economic Planning of Japan. (20)

(Please turn over)

GROUP B : Economic development

Answer any THREE questions

3. Discuss possible alternative definitions of economic development and argue in favour of the one you would like to adopt. Also indicate the main structural changes that in, your view, characterise development. (15+5=20)

4(a). What is gestation lag? Distinguish the major types of gestation lag and briefly discuss their nature.

(b). Can technological lag be eliminated under conditions of planned development? Give reasons for your answer, indicating the course of this lag under conditions of a fairly rapid rise in investment. (10+10=20)

Or,

What is excess capacity? Would you agree with the view that excess capacity is inherent in the capitalist system of production? Is it possible to eliminate such excess capacity under socialism? (4+10+6=20)

5. Discuss the vicious circle of poverty and indicate where to attempt a break-through. In this connection, discuss how unemployed and underemployed labour force could be utilised. (12+8=20)

6. Bring out the inter-relationship between the capital-output ratio, the capital-labour ratio, and the output-labour ratio under (i) static conditions & (ii) dynamic conditions. (20)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

ECONOMICS II

Duration : 3 hours | Maximum Marks: 100 | Date: 22 May 1964

Separate Answer-book should be used in each Group

GROUP - A (Indian Economic Conditions)

Answer any THREE

1. Discuss the principal measures that have been adopted in recent years to place the Indian Banking System on a sounder basis. (22)

2. "The Reserve Bank of India's monetary policy has been a policy of controlled expansion during the Plan period." - Examine the statement and discuss in this connection the main features of the policy. (22)

3. Give a short account of India's balance of payments difficulties in recent years. Discuss the measures adopted to solve these difficulties. (22)

4. How do you explain the rising trend in food prices in India in recent years? Examine the measures so far adopted to stabilize food prices. (22)

5. How far in your view State Trading can solve the problem of distribution of food grainsin India? (22)

6. Write a critical note on (any one) :

   (a) The report of the Third Finance Commission

   (b) Expenditure Tax in India

   (c) Recent trends in Indian Public Debt.

GROUP - B (Planning in India)

Answer any TWO

7. Make a critical assessment of the views of the Bombay Planners on the role of the State in the industrial development of the country. (17)

8. "Not the failures but the successes of the Second Plan are due to the size and the shape of the Plan; the failures and problem are the product of other causes." - Discuss

(Please turn over)

9. Would you agree with the view that compared to the Second Plan, the formulation of the Third Plan embodies a wider and bolder view in the general approach, in the formulation of objectives and in the allocation of investments in the public sector? (17)

10. Write a critical note on the estimates of the financial resources for the public sector in the Third Plan and explain how the experience of the Second Plan period was taken into account. (17)

INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

PHYSICS (Theory)

Duration 1½ hours | Maximum Marks: 100 | Date: 25.5.64

Answer Q. 1 and any TWO of the rest

1. Enumerate the postulates of the Bohr's theory of hydrogen spectra.

Show that the frequency of the line emitted when the electron jumps from state n_1 to state n_2 is

$$\nu_{12} = \frac{2\pi^2 me^4}{h^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where e = electronic charge, m = electronic mass and h = Planck's constant.

Given $e = 4.803 \times 10^{-10}$ e.s.u, $m = 9.108 \times 10^{-28}$ gm., $h = 6.625 \times 10^{-27}$ erg second, calculate the radius of the first Bohr orbit.

(8+20+8=36)

2(a). A light quantum is scattered by an electron at rest. Show that the Compton shift of wavelength is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where ϕ = angle of scattering, h = Planck's constant, m_0 = rest electronic mass, c = velocity of light. (16)

(b). What are Stokes' and anti-Stokes' Raman lines? Give in general terms the explanation of their origin. (8+10=16)

3. Distinguish between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics. Deduce Planck's law of radiation following Bose. (12+20=32)

What is thermodynamic probability? Show that the number of particles in the ith cell, in the state of maximum thermodynamic probability is, according to M-B statistics

$$N_i = \frac{N}{Z}\exp(-\omega_i/KT)$$

where N = total number of particles, Z=partition function, ω_i=energy of the particle whose phase point is in the ith cell, K= Boltzmann constant and T = Temperature in degree absolute.

(8+24=32)

(Please turn over)

5. Write short notes on any <u>three</u> of the following :

a) Macro and Micro state.

b) Spectral series of hydrogen..

c) Ritz combination principle.

d) Intensities of Stokes' and anti-Stokes' Raman lines. $(3 \times 10\frac{2}{3} = 32)$

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

ENGINEERING

Duration : 3 hours | Maximum Marks: 100 | Date: 25 May 1964

Attempt any Five questions. All questions carry equal marks.

1. A simply supported R.S.J. ($I = 800\ in^4$ units) carries two loads each of 5 tons at a distance of 15 ft. and symmetrically situated at distances of $7\frac{1}{2}$ ft. from the ends of the beam.

   (a) Determine the deflection at midspan assuming $E = 13000$ Tons / in^2.

   (b) If the beam is propped at ~~mid-span~~ mid-span so that the mid-span point is level with the ends, find the magnitude of the propping force.

   (c) Calculate the bending ~~and~~ moments at distances of $7\frac{1}{2}$ ft. from the ends of the propped beam.

2(a) Discuss the formula of Rankine in connection with the buckling of struts of moderate lengths and state its limiting conditions.

(b) 10 in.X 6 in. X 40 lbs. R.S.J. is used as a stanchion of 15 ft. in length with one end fixed and the other end hinged. Find, from the data given below, the safe axial load for the stanchion with a factor of safety of 4.

   (i) Area of section = 11.77 sq. in.

   $I_{xx} = 204.80\ in^4$. $I_{yy} = 21.76\ in^4$.

   (ii) The crushing strength of a short strut of this quality steel is 24 Tons/in^2.

   (iii) The constant in the Rankine formula for stanchion with hinged ends is $\frac{1}{7500}$.

3. A timber beam with a rectangular cross-section 4 in. wide and 6 in. deep carries a uniformly distributed load over a span of 10 ft. (a) If the permissible flexural stress is 4000 lb. per sq. in. and the permissible transverse shear stress is 200 lb. per sq. in., calculate the maximum load which it can carry. (b) What other condition is commonly specified for beams? Discuss its significance in design of beams.

4. A single acting force pump has to deliver water against a head of 240 ft. (a) If the diameter of the plunger is 5 inches, what must be the average force exerted on the pump plunger during the delivery stroke if the efficiency of the pump is 85%. (b) If the stroke of the pump is 6 inches and it makes 120 double strokes per minute, what horse power will be required to drive it? (c) If the slip of the above pump is 5% how many gallons will it deliver per minute? (d) Write a short note on double acting force pumps.

(Please turn over)

5(a) Compare the flow through pipes with that through channels.

(b) Deduce the Chezy formula for uniform flow in channels.

(c) A stream is 40 ft. wide at water level. At horizontal intervals of 5 feet the following results are obtained by current-meter :-

Distance from bank (feet)	0	2.5	7.5	12.5	17.5	22.5	27.5	32.5	37.5	40
Depth of water (feet)	0	1.0	2.2	3.2	4.4	5.0	3.4	2.2	1.2	0
mean velocity on vertical (feet per second)	-	1.5	1.9	2.2	2.9	3.3	2.7	1.8	1.4	-

What is the discharge in cusecs?

6(a) Sketch typical stress-strain diagrams for specimens of mild steel and of aluminium alloy subjected to tensile tests.

(b) Briefly compare these and state the difference in properties indicated.

(c) Explain the terms "yield-point" and "proof-stress" and referring to the diagrams, explain how suitable working stresses, may be determined for these materials.

7. Write short notes on :-

(a) Venturi-meter

(b) Hammer blow in pipes

(c) Retaining walls

(d) Beams of uniform strength

(e) Compression test on cement concrete.

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. STAT. III Year

BIOLOGY (Theory)

Duration : 3 hours | Maximum marks : 100 | Date: 23 May 1964

1(a). Reciprocal crosses were made between the 2 rice varieties Dhairal ([illegible]) and Patnai-23 ([illegible]) and between Dhairal ([illegible]) and Bhasmanik ([illegible]). The following results were obtained :

breeding combination	time from sowing to flowering (days)	average plant height (cm.)	average tiller number	grain yield/ear head: dry weight (gm)	grain yield/ear head: number
(1)	(2)	(3)	(4)	(5)	(6)
P selfed	108 ± 2.6	124 ± 3.3	9 ± 0.6	3.2 ± 0.3	125 ± 3.0
B selfed	104 ± 2.9	107 ± 1.6	10 ± 0.7	2.5 ± 0.1	175 ± 3.5
D selfed	87 ± 0.6	93 ± 1.8	4 ± 0.4	3.3 ± 0.4	125 ± 3.6
P ♀ x D♂	144 ± 1.9	129 ± 2.1	12 ± 0.9	6.9 ± 0.6	250 ± 5.6
D ♀ x P♂	136 ± 2.1	143 ± 2.5	18 ± 0.8	15.5 ± 1.1	552 ± 7.2
B ♀ x D♂	138 ± 2.5	130 ± 3.6	25 ± 0.6	16.9 ± 0.9	801 ± 12.5
D ♀ x B♂	139 ± 2.6	119 ± 3.1	21 ± 1.2	6.2 ± 0.7	305 ± 6.4

Note: P is Patnai-23, B is Bhasmanik and D is Dhairal.

Explain the reasons for the differences in the results obtained among different progenies. (8)

(b). The following data were obtained from three populations on the incidence of infant mortality :

country	year	proportions of infant mortality in offspring of parents who are: first cousins	unrelated
(1)	(2)	(3)	(4)
U.S.A.	1865	632/2356	134/837
U.S.A.	1903	113/672	370/3184
France	1953	165/1417	360/5382

Suggest an explanation for the differences in the mortality between offspring of cousins and unrelated parents. (3)

(c). What should be the proportion of heterozygous individuals among descendents of a single Aa plant after 5, 10, 20 and 50 generations of selfing? (9)

(Please turn over)

2. Write a detailed botanical account on any important fibre yielding plant. Name ten species which produce fibre of commercial value and mention in each case the specific organs that yield the fibre. (25)

3. Answer any five of the following :

(a) Write a short note on clonal propagation in Coconut.

(b) Discuss briefly the role of induced mutation in plant breeding.

(c) Write a short note on the importance of Coimbatore sugar canes.

(d) Explain a genotype and a phenotype with suitable examples.

(e) Write an illustrated note on the spikelet of *Triticum vulgare*.

(f) Explain crossing over.

(g) Why selection for vigour in inbred lines may delay attainment of homozygosity. (25)

4. Give the statistics on the area under and production of wheat in the main geographical regions of the world. Mention the figures relating to India.

What are the principal methods adopted for breeding self-pollinated crops? (10+15=25)

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INDIAN STATISTICAL INSTITUTE
Research and Training School

Annual Examination, 1964

B. Stat. III Year

<u>SOCIOLOGY</u>

Duration : 3 hours | Maximum Marks: 100 | Date: 25 May 1964

1(a). What is social structure? Explain the relevance of this concept in social research.

Or,

(b). What is social stratification? How is it different from the concept of social structure? Explain with illustration.

2(a). Discuss briefly about different types of social surveys.

Or,

(b). Illustrate: "traditional or involuntary" social groups and "customary or voluntary" social groups. Examine the necessity of taking into account their distinction in the analysis of data collected over a time period.

A sociologist desires to test the effects of village-to-city migration on people's tendency to set up joint families.

The sociologist may draw his inference from a comparison of the results of random sample surveys conducted separately in the villages as well as in the city areas.

<u>Alternatively</u>, he may (i) draw a random sample of individuals who have actually moved from the villages to the city areas, (ii) collect information from each of them as to the respective familial affiliation <u>before</u> and <u>after</u> migration to the city, and (iii) arrive at the conclusion therefrom.

Which of these two alternative methods would you suggest to the sociologist, and why?

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