INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1967-68
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PERIODICAL EXAMINATIONS

23

General Science-4: Biochemistry Theory

Date: 18.9.67 Maximum marks: 50 Time: $1\frac{1}{2}$ hours

1.(a) Define isoelectric point.

(b) What is the function of lipoproteins?

(c) Describe a colour reaction of protein. [10]

2.(a) What is an Enzyme?

(b) Define turnover number.

(c) Describe the preparation and properties of

(i) Pepsin

(ii) Urease. [25]

3. Give one example of each:

a) Dipeptide, b) Basic amino acid,
c) Polysaccharide, d) Hexose, e) Decarboxylase. [15]

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PERIODICAL EXAMINATIONS

General Science-5: Sociology

Date: 23.10.67 Maximum marks: 100 Time: 3 hours

Note: All questions carry equal marks.

1. EITHER

What is sociology? What is the necessity of studying sociology?

OR

'Statistics is a technology' - How this remark fits with regard to the subject matter of sociology?

2. EITHER

Why is the family an omnipresent social institution? What will be your statistical design in studying such an institution in a city?

OR

What is a 'household'? How it differs from a 'family unit'? Does the urban family differs from rural? If so, what will be your statistical design in studying families in a small town or in a village?

3. EITHER

Define and distinguish between caste, class and rank. Enumerate the features of caste organisation in India.

OR

What are unilindar kingroupings? Describe one of its functions with suitable examples.

4. EITHER

What is 'incorporeal' and 'corporeal' property? 'Primitive economics is based on reciprocity' - Illustrate.

OR

Critically examine the theories of Karl Bucher, F. Engels, Adam Smith and Bruno Hildebrand with respect to preliterate Economy.

5. EITHER

What are levirate and sororate? How do you account for these practices? What are the different methods of acquiring brides prevalent in Indian societies?

OR

Write short notes on (any five):

Twince born, Sect, Social distance, Kula, Social structure, Social stratification, Tecknonymy, Taboo.

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PERIODICAL EXAMINATIONS

Statistics-3: Data Processing

Date: 30.10.67 Maximum marks: 100 Time: $2\frac{1}{2}$ hours

1. What controls would you suggest at the punching stage for accurate transfer of data to punched cards. [8]

2. A sorter machine has suddenly stopped during sorting. What are the probable causes of machine stoppage? [5]

3. Choose any five of the following statements and state which are correct and which are false. [5]

   a) Any of the 4 corners of a Hollerith card can be cut.
   b) ISI made sorter can sort cards at the rate of 650 cards per minute.
   c) Sorted cards are directed to higher pockets when the sorting brush fixed on the brush-holder is longer in length.
   d) Cards are reproduced properly by feeding the original set of cards as well as the blank cards face up and 9-Edge leading.
   e) There are 10 exit hubs of a digit selector on an IBM accounting machine.
   f) Only the adjacent counters of a 402 accounting machine can be coupled.

4. Write notes on (a) Alphabetic listing and (b) substraction in respect to a 402 accounting machine. [8]

5. A new set of XXXI cards are to be obtained from the existing set of cards in the design given in problem No.6.

   Show the Control Panel wiring diagram in respect to IBM 513 type reproducing Punch to get the job done. [10]

6. A list of cards to be obtained in the form given below for cards with daily wage 10.00 to 10.99. The data have been transferred to cards in the design with card design index No.XXXI given below.

   Card-design

	particulars	card columns
1.	Design index number	1 - 4 Punched XXXI
2.	Department	5
3.	Category	6
4.	roll number	7 - 9
5.	name	10 - 34
6.	daily wage	35 - 38 (in Rs.0.00)

   Listing Format:

department	category	roll number	name	daily wage
(1)	(2)	(3)	(4)	(5)

[24]

7. Class records. [40]

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PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 6.11.67 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A

1.a) When do you say that a sequence $\{a_n\}$ of real numbers tends to a limit?

b) If a sequence $\{a_n\}$ tends to a limit l, and if $\bar{s}_n = \frac{a_1 + a_2 + \ldots + a_n}{n}$, show that $\{\bar{s}_n\}$ tends to the same limit. Is the converse true? Justify your answer. [3+8]=[11]

2. Explain clearly the terms, 'metric space'; 'open set', 'compact set in a metric space'. [3+3+4]=[10]

3. Let (R, d) be the real line with usual metric. Let f be a function from R into R.

a) When do you say that f is continuous?

b) If f is continuous and satisfies the functional equation $f(x+y) = f(x) + f(y)$ for every two real numbers x, y; show that $f(x) = ax$ for all x, where a is a constant.

c) Prove that if f is differentiable at a point then it is continuous at that point. Is the converse true? Justify your answer. [3+7+7]=[17]

4. Justify your answer in each of the following questions:

(a) $f(x) = 1$ if x is irrational
$\quad\;\; = 0$ if x is rational
Is f continuous?

(b) Let $A = \{x : 0 \leq x \leq 1\}$

Define $d(x, y) = 1$ if $x \neq y$
$\quad\quad\quad\quad\;\; = 0$ otherwise

Is A compact?

(c) Every infinite set has at least one accumulation point. Is this statement true?

(d) Every uncountable set has at least one accumulation point. Is this statement true. [3+3+3+3]=[12]

Group B

Note: Answer questions 5 and 6 and any one of the remaining two questions.

5.a) When do you say that a function defined on a region is holomorphic? Show that the real and imaginary parts of a holomorphic function satisfy the Cauchy-Riemann equations.

b) Consider the following function on the complex plane into the complex plane:

$$f(x + iy) = u(x+iy) + iv(x+iy)$$

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where $u(x+iy) = \frac{x^3-y^3}{x^2+y^2}$ if $(x, y) \neq (0, 0)$

$= 0$ if $(x, y) = (0, 0)$

and $v(x+iy) = \frac{x^3+y^3}{x^2+y^2}$ if $(x, y) \neq (0, 0)$

$= 0$ if $(x, y) = (0, 0)$.

Show that u and v satisfy the Cauchy-Riemann equations but f is not holomorphic on the complex plane.

(You can assume that u and v have first order partial derivatives).

5.c) Show that, if the derivative of a holomorphic function is zero then it is constant. [7+4+4]=[15]

6.a) Show that the function

$$f(z) = |z|.$$

is every where continuous but nowhere differentiable. Determine the limiting values of the differential quotient $\frac{f(z+h)-f(z)}{h}$ as $h \to 0$ and show that they form a circle in the complex plane. (Do not confuse a circle with a disk).

b) Show that the function

$$f(z) = |z|^2$$

has a derivative at $z = 0$ but nowhere else. [8+7]=[15]

7.a) State and prove the Cauchy-Hadamard's theorem for power series.

b) Determine the radius of convergence of the power series $\Sigma \frac{z^n}{n}$. (You have to evaluate the limit under consideration). Exhibit two complex numbers z_0 and z_1 with same modulus such that the series converges at z_0 but not at z_1. [8+7]=[15]

8.a) When do you say that a subset of the complex plane is an open set? Is the set of all complex numbers with modulus strictly less than one, an open set? Justify your answer.

b) When do you say that a subset of the complex plane is a compact set? Is the set of all complex numbers with modulus greater than or equal to one, a compact set? Justify your answer.

c) What is meant by 'Radius of convergence of a power series series'? State the Cauchy-Hadamard's theorem.

d) Show that every absolutely convergent series of complex numbers is convergent. Is the converse true? Justify your answer.

e) Show that the function

$$f(z) = \bar{z} \quad (= \text{conjugate of } z)$$

is a one to one continuous map of the complex plane onto itself which preserves addition and multiplication. [2+2+3+4+4]=[15]

Assignments. [5]

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PERIODICAL EXAMINATIONS

Economics - 3

ate: 13.11.67 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A, B and C in separate answerscripts

Group A (Max.marks: 30)

Answer any two questions from this group.

What were the broad approach and objectives of the second five year plan? How far were the objectives realised? [15]

Discuss the arguments for and against planning as a means of economic development. [15]

What were the broad estimates of financial resources for the third five year plan? Discuss how these estimates were arrived at. [15]

What is a mixed economy? Evaluate the success of Indian planning in expanding the economic resources of the country. [15]

Group B (Max. marks 30)

(Answer Question No.5 and any one of Question No.6 (a), (b) and (c))

EITHER

Discuss the role Adam Smith assigns to capital accumulation in his theory of economic growth and analyse the process that brings about the stationary state in Smith's model. [20]

OR

Examine the Ricardian theory of factor shares in a growing economy and indicate its bearings on the course of economic development. [20]

Explain briefly any one of the following statements:

a) 'The extent of division of labour which can take place depends on the size of the market'.

b) 'From the standpoint of economic development, the increase in net national product must be a sustained increase'.

c) 'Rent is the payment to the landlord which equalizes the profit rate among capitalists in the employment of equal units of labour and capital on lands of different qualities'. [10]

p.t.o.

Group C (Max. marks: 40)

(Answer any two questions from this group)

7. EITHER

Explain the concept of 'Vicious Circle of Poverty' and examine how far the statement that 'a country is poor because she is poor' is applicable to present day underdeveloped countries. [20]

OR

Discuss the criteria which help you to distinguish a country as an underdeveloped one. [20]

8. EITHER

Examine the factors that are responsible for the under-development of an economy. [20]

OR

Explain the meaning of population explosion with particular reference to India and discuss the demographic characteristics of an underdeveloped economy. [20]

9. Explain the meaning and forms of 'Potential Surplus' that exists in an underdeveloped country and discuss how this surplus can be mobilised. [20]

10. Discuss the different methods of measuring economic development of a country pointing out merits and limitations of these methods. [20]

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PERIODICAL EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 20.11.67 Maximum marks: 100 Time: 3 hours

GROUP A

Note: Answer Q.1 and any 2 other questions from this group.

1.a) Show that in simple random sampling without replacement from a finite population the sample mean has uniformly minimum variance among all linear unbiased estimates of the population mean of a characteristic. [10]

b) In a stratified random sampling scheme without replacement, obtain the optimal allocation (the one which minimises the variance of the estimate of the population mean) of sample size for different strata when the total sample size n is fixed. [10]

2.a) Describe a method of drawing a sample of size n from a population with N units where the probability of choosing any unit is proportional to the size of the unit. [7]

b) Describe a method of drawing a sample of size n from a univariate normal population with specified mean and variance. [8]

3.a) Define the following estimates of the population total of a characteristic y when the population total of an auxiliary characteristic x related to y is known

i) Ratio estimate
ii) Difference estimate
iii) Regression estimate [9]

b) Find out the approximate variance of the ratio estimator, when the sampling scheme is SRS with replacement. [6]

4. In a certain population it is known that the observations y are all zero on a portion qN of the N units $(0 < q < 1)$.

a) If μ and σ^2 are the mean and variance respectively of y's in the original population (including the zero-values) and σ_o^2 is the variance when all the zero-values are excluded, show that

$$\sigma_o^2 = \frac{\sigma^2}{p} - \frac{q}{p}\mu, \text{ where } p = 1 - q.$$ [8]

b) Let V_1 and V_2 be the variances of the sample means when n units are drawn at random with replacement from the original population (including the zero values) and the curtailed population containing only Np units excluding all the zero values, respectively. Show that

$$\frac{V_1 - V_2}{V_1} = \frac{q(1 + c^2)}{c^2} \quad \text{where } c = \frac{\sigma}{\mu}$$ [7]

p.t.o.

GROUP B

EITHER

Consider a sample $(X_1, X_2, \ldots, X_n)$ of size n from a standard normal distribution. Let

$$R(X_1, \ldots, X_n) = \max.(X_1, X_2, \ldots, X_n) - \min.(X_1, \ldots, X_n)$$

Find out the expected value of the random variable R by 3-point Gauss-Hermite formula when the sample size $n = 4$.

[Hint: The density function $T(X_1, \ldots, X_n) = \max.(X_1, \ldots, X_n)$ is given by $g(t) = \frac{1}{\sqrt{2\pi}} n (\Phi(t))^{n-1} e^{-(1/2)t^2}$

$$\text{where } \Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du]$$ [25]

OR

The table below gives the frequency distribution of the number of albino children in families of five children, with at least one ablino child in the family.

number of albino children in the family	1	2	3	4	5	total
number of families	25	23	10	1	1	60

Fit a truncated binomial distribution to the data.

6.a) In a population with $N = 6$ units the values of Y_i's are 8.74, 6.32, 7.29, 2.03, 1.02, 9.20. Calculate the sample mean $\bar{y}$ for all possible random samples without replacement of size $n = 3$. Verify that $\bar{y}$ is an unbiased estimator of the population mean and that the variance of $\bar{y}$ is given by

$$V(\bar{y}) = \frac{N-n}{N-1} \frac{\sigma^2}{n}$$ where σ^2 is the variance in the population. [15]

b) The following data relate to a population divided into 4 strata.

stratum no.	number of units in the stratum	variance of y's in the stratum
1	112	1232.5
2	203	7650.9
3	422	10234.2
4	263	3795.1

Obtain the Neyman's optimal allocation of sample sizes to the different strata, when the total sample size $n = 100$ is fixed. [10]

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PERIODICAL EXAMINATIONS

Statistics-3: Probability

Date: 27.11.67. Maximum marks: 100 Time: 3 hours

1. State and prove the inversion theorem for characteristic functions. [20]

2.a) When do you say that a sequence of distribution functions converges weakly. [4]

b) Examine if F_n converges weakly when defined as follows:

i) F_n is the d.f. of the uniform distribution on $(-n,n)$. [4]

ii) F_n is the d.f. of $X_{n1} + X_{n2} + X_{nn}$ where $X_{n1}, X_{n2}, \ldots X_{nn}$ are i.i.d and $P(X_{ni}=0) = p_n$ and $P(X_{ni} = 1) = 1 - p_n$, $p_n \to 1$, $n(1 - p_n) \to \theta$. [4]

iii) $F_n(x) = \begin{cases} 0 & x < -1 - 1/n \\ 1/4 & -1 - 1/n \le x < 0 \\ 1/2 & 0 \le x < 2 + 1/n \\ 3/4 & 2 + 1/n \le x < 3 \\ 1 & 3 \le x \end{cases}$ [4]

iv) $F_n(x)$ is the d.f. of $\dfrac{X_1 + \ldots + X_n}{n}$ where $X_1, X_2, \ldots$ are i.i.d. and $E(X_1) = 27$. [4]

3. Let θ_1 and θ_2 be independent random variables with distribution functions $\varepsilon_1(x)$ and $\varepsilon_2(x)$.

(a) Obtain the d.f. of $\theta_1 + \theta_2$. [4]

(b) Obtain the c.f. of $\theta_1 + \theta_2$. [4]

(c) Show that the sum of two independent Poisson random variables also has a Poisson distribution. [4]

4.a) X_1, X_2 are random variables with a joint density function const. $e^{-x_1-x_2}(x_1^{\alpha-1} x_2^{\beta-1} + x_1^{\gamma-1} x_2^{\delta-1})$, $x_1 \ge 0$, $X_2 \ge 0$.

i) Find the value of the constant. [4]

ii) Find the conditional distribution of X_1 given X_2. [4]

iii) Find the conditional mean of X_1 given X_2. [4]

b) X and Y are independent normal random variables with means zero and unit variances. Obtain the variance covariance matrix of the random variables $\sigma_1 X$, $\sigma_2(\rho X + \sqrt{1 - \rho^2}\, Y)$. [4]

p.t.o.

5. a) Obtain the characteristic function of the Cauchy distribution. [6]

b) Examine if the following are c.f.'s; give reasons.

i) e^{-t^4}. [4]

ii) $e^{-t^2/2} \sin t / t^2$. [4]

iii) $\sin t/t$. [4]

c) i) How many times can a c.f. be differentiated at o if the distribution admits of n moments? [3]

ii) How many moments will a distribution admit if its c.f. can be differentiated n times at o. [3]

6. State whether the following are true or false.

i) If a sequence of characteristic functions converges to a characteristic function then the corresponding sequence of d.f.'s converge weakly. [3]

ii) If a c.f. is differentiable at one point it is differentiable everywhere. [3]

iii) If two characteristic functions agree on an internal they agree everywhere. [3]

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MID-YEAR EXAMINATIONS

Mathematics-3: Analysis

Date: 18.12.67 Maximum marks: 100 Time: 3 hours.

Note: Answer Groups A and B in separate answerscripts.

Group A Max. marks: 50

1.a) If $a_n = \frac{n^2 + n + 1}{3n^2+1}$, show that $a_n \to 1/3$.

b) If $a_n \to \lambda$, show that $|a_n| \to |\lambda|$. Is the converse true? Justify your answer.

c) Find a necessary and sufficient condition for the convergence of a monotonic sequence. [5+4+8]=[17]

2.a) Let $f(x)$ be continuous in $[-1, 1]$ which assumes only rational values and let $f(0) = 0$. Prove that $f(x) = 0$ everywhere.

b) Let $f(x) = [x]$, in $[0, 4]$.
([x] means the greatest integer less than or equal to x.)
Is f Riemann-integrable? Justify your answer.

c) Let $f(x)$ be a function defined in $[0, 1]$ as follows

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

Show that f is not Riemann-integrable. [6+5+6]=[17]

3.a) When do you say that ξ is an extreme value of a function f defined on a finite closed interval?

b) Show that if $f(c)$ is an extreme value of a function $f(x)$ then $f'(c)$, in case it exists, is zero. Is the converse true? Justify your answer.

c) Give an example to show that $f(c)$ may be an extreme value even when $f'(c)$ does not exist. [5+6+5]=[16]

Group B Max. marks: 50

1.a) State and prove the Cauchy-Hadmard's theorem for power series.

b) Determine the radius of convergence of the power series $\Sigma \frac{z^n}{n}$ (you have to evaluate the limit under consideration). Exhibit two complex numbers with the same modulus such that the series converges at one point and not at the other point. [8+7]=[15]

2. Let R be an open bounded rectangle in the complex plane and f a holomorphic function on R.

a) If the derivative of f is zero at every point of R then f is a constant.

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2.b) If $|f(z)| = 1$ for each z in R then f is a constant.

c) If $f(z) \neq 0$ for any point z in R then $\frac{1}{f(z)}$ is also holmorphic. [5+5+5]=[1

3. Answer whether the following statements are true or false. If true prove them, if false give a counter example.

a) A holomorphic function defined on a region is continuous.

b) Any absolutely convergent series of complex numbers is convergent.

c) If the Radii of convergence of the power series $\Sigma\, a_n z^n$ and $\Sigma\, b_n z^n$ are finite then the Radius of convergence of the power series $\Sigma\, (a_n + b_n) z^n$ is also finite.

d) The set of all complex numbers with modulus equal to 1 is an open set in the complex plane.

e) For any sequence (c_n) of positive real numbers,

$$\liminf \frac{c_{n+1}}{c_n} \leq \liminf \sqrt[n]{c_n} .$$ [3+3+3+5+6]=[20

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MID-YEAR EXAMINATIONS

Economics-3

Date: 20.12.67 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A, B, and C in separate answerscripts.

Group A

1. EITHER

Analyse the possibility of overproduction in a capitalistic economy as brought out in the arguments of Ricardo and Malthus. [25]

OR

Elucidate the Marxian arguments on the problem of underconsumption in the capitalist system. [25]

2. Explain briefly any one of the following statements:

a) In the stationary state there are no profits even though wages are at the subsistence level.

b) Marx's law of the falling rate of profit indicates a potential tendency which, though not realised, may none the less be important. [15]

Group B

3. EITHER

Discuss the view that the present system of prices in the USSR is impeding the growth of Soviet economy. [15]

OR

Discuss the new role of profit in the Soviet economy. [15]

4. EITHER

Discuss the view that without collectivisation of agriculture, the pace of industrialisation in the Soviet Union would have been slower. [15]

OR

Write short notes on any two of the following:

a) NEP [15]
b) war-communism [15]
c) scissor's crisis.

Group C

Answer any two questions.

5. Discuss the problems of measuring National Income in an underdeveloped country. [15]

6. Discuss the ingredients of economic development of a country. [15]

7. 'When unemployment exists, disguised or obvious, the choice of factor proportions may be complicated by a confusion or merging of objectives, between maximum rate of advance in output and maximum employment'. Explain and State whether an underdeveloped country should follow labour intensive technique or capital intensive one in the process of development. [15]

8. Examine the role and limitations of Land Reforms measure in the process of economic development. [15]

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MID-YEAR EXAMINATIONS

Probability

Date: 21.12.67 Maximum marks: 100 Time: 3 hours

Answer any five questions.

1.a) Define clearly random variable, distribution function, discrete distribution, continuous distribution, probability density function for the bivariate case. [10]

b) The conditional distribution of X given Y = y(y> 0) is a Poisson with parameter y. The distribution of Y has density function

$$e^{-\lambda y}\, y^{\alpha-1}\, \lambda^{\alpha} \,/\, \Gamma(\alpha),\ y > 0.$$

Obtain the joint distribution function of (X,Y) and the marginal distribution of X. [10]

2. Let X be any random variable. Denote its central moments by μ_0, μ_1, μ_2, etc; let $\beta_1 = \mu_3^2 / \mu_2^3$, $\beta_2 = \mu_4/\mu_2^2$.

i) Obtain the variance of $aX^2 + bX$. [5]

ii) From the above or otherwise show that
$$\beta_2 \geq \beta_1 + 1.$$ [7]

iii) Let X take the value 0 with probability 1-p and the value 1 with probability p. Calculate μ_2, μ_3, μ_4, β_1 and β_2 for X. Verify that $\beta_2 = \beta_1 + 1$ is true in this case for all p. [8]

3.a) (X,Y) have a uniform distribution on the region bounded by the lines x+y=1, x+y=-1, x-y=1 and x-y=-1. Obtain the marginal distributions and the conditional distributions. Also obtain the variance covariance matrix of (X,Y). [10]

b) If on the same region as above the density function of (X,Y) is $C(x^2+y^2)$ obtain the constant C and all the other entities mentioned in the above question. [10]

4.a) Mention three properties of a characteristic function. [5]

b) F(x) is a function defined for all rational x and is such that $F(x) \leq F(y)$ whenever $x < y$, x,y rational and $\lim F(x) \to 0\ (1)$ as $x \to -\infty(\infty)$ through rational values. Define a distribution function G(x) based on F such that at each y which is a continuity point of G,

$F(x) \to G(y)$ as $x \to y$ through rational values. [15]

5.a) Let X have a Poisson distribution with parameter λ. Show that the distribution of

$$\frac{X - \lambda}{\sqrt{\lambda}}$$

converges to a normal distribution as $\lambda \to \infty$. (Use characteristic functions.) [8]

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b) Show that the sum of two independent normal random variables (means μ_1, μ_2, variances σ_1^2, σ_2^2) has also a normal distribution.

c) The r-th raw moment of a random variable X is $p(p+1) \ldots (p+r-1), (p > 0)$. What is its characteristic function?

6.a) Let X be a random variable with p.d.f. $f(x)$. What is the p.d.f. of

$$\frac{X+3}{5}?$$

b) Let X, Y be independent normal random variables with means 0, 0 and variances 1, 1. What is the joint distribution of $(X \cos \theta + Y \sin \theta, X \cos \theta - Y \sin \theta)$?

c) Let X be a random variable with p.d.f. $f(x)$. State some conditions under which you can readily obtain the p.d.f. of $j(X)$. What is the p.d.f. of $j(X)$ in that case?

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MID-YEAR EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date:23.12.67 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A,B and C in separate answerscripts.

Group A

1. It is desired to test $H: p = .4$ against $A: p < .4$ on the basis of a sample of size 10 from a binomial population. Give a test of exact size .1 and compute its power when $p = .2$. [6 +4]=[10]

2. State and prove Neyman Pearson lemma in the case of continuous distributions. Use this result to obtain UMP test of $H: \mu = 0$ against $A: \mu < 0$ based on a sample of size 12 from a $N(\mu,1)$ population. [12+8]=[20]

3. Let $x_1, \ldots, x_n$ be a sample from a normal population with unknown mean μ and unknown variance σ^2. Use the likelihood ratio method to test $H: \mu = 0$ against $A: \mu \neq 0$. [10]

   (It is not necessary to derive the maximum likelihood estimates of the parameters involved).

Group B

Attempt Q.1 and any one from the rest.

1. From a population divided into k strata, n_i observations are drawn with replacement and with equal probabilities from the i-th stratum, $i=1,2,\ldots,k$.

   a) Obtain an unbiased estimator $\hat{\mu}$ of the population mean μ on the basis of the sample.

   b) Derive the variance of this estimator and an unbiased estimator of this variance.

   c) Suppose that the cost function for the sampling project is of the following form:

   $$\text{cost} \quad C = A + \sum_{i=1}^{k} n_i C_i$$

   where A = overhead cost

   C_i = cost of surveying one unit in the i-th stratum $i = 1,2,\ldots,k$.

   Find out the optimum values of the n_i's $i = 1,2,\ldots,k$ minimising variance of $\hat{\mu}$ subject to a fixed total cost C. [25]

2. Suggest an unbiased estimate of the population total for a two stage sampling scheme in which, units in both the stages are selected with equal probability and without replacement. Assume that each first stage unit contains the same number of second stage units and that from each of the m selected first stage units, a constant number n of second stage units are selected. [15]

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3. Show that if given $X = n$, the conditional distribution of a random variable Y be binomial $b(n,\pi)$ and if marginally the random variable X has a Poisson $p(\lambda)$ distribution then the marginal distribution of X is also Poisson and is precisely $p(\lambda\pi)$.

Group C

Statistics - Practical

Any one question from this group.

1. Consider a random variable X taking values 1,2,..., n, with

$$P(X = x) \propto x.\pi_\theta(x) \quad x = 1,2,\ldots, n$$

where $\pi_\theta(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}, \quad 0 < \theta < 1$.

Find out the distribution of X and fit this distribution to the following data ($n = 5$ in this case).

Distribution of the number of albino children in families of five children, with at least one albino child.

no. of albino children in the family	1	2	3	4	5	total
number of families	25	23	10	1	1	60

[2

2. A population contains 6 units with the following values of y_i and x_i.

unit	1	2	3	4	5	6
y_i	2.39	4.01	5.21	8.61	9.89	12.24
x_i	0.23	1.29	3.02	4.29	5.32	6.14

By working out all cases compare the precisions of the ratio and the difference estimators of population total for simple random samples of size 2, without replacement. [20]

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MID-YEAR EXAMINATIONS

Statistics-3: Data Processing

Date: 25.12.67 Maximum marks: 50 Time: $1\frac{1}{2}$ hours

1. For each of the following statements, state whether it is true or false.

   i) A floating point quantity can never appear in a fixed point expression

   ii) FØRMAT statement can appear anywhere in a FØRTRAN programme.

   iii) CØNTINUE is a dummy statement that causes no additional instruction in the object programme. So it is of no use.

   iv) The statement 'IF (a) n_1, n_2, n_3' transfers the control to n_1, n_2, or n_3 depending on whether the value of a is >, =, or < Zero.

   v) Every input-output statement in a FØRTRAN programme must have a format statement associated with it. [5 X3]=[15]

2. Examine the following statements. Correct errors, if any.

   i) DIMENSIØN, A(10), B(5,5), C(I)

   ii) 4AT = 3(8.* B+C /4.0)

   iii) A(0) = A(1) - A(2)*A(3)/3Y

   iv) GØTØ (4,5,6,7,10)V

   v) DØ05, P = I,M,J. [5 X3]=[15]

3.a)

```
DIMENSIØN A(10),B(5)
READ 100, (A(I), B(I), I = 1,5), (A(J), J= 6,10)
100 FØRMAT (2F5.3/3F5.1).
```

   i) How many cards are read?

   ii) Write down the sequence of the quantities read from each card according to the above format.

b)

```
READ 1, A, X
DIMENSIØN A(5), X(5)
B = 0
DØ 10 I = 1,5
DØ 20 J = 1,5
1 FORMAT (10 F 8.0)
  IF (X(J)) 20, 20, 30
30 A(J) = A(J) + X(J)
10 B = B + X(I)
20 CØNTINUE
```

   i) Is this a valid set of statements?

   ii) Can you rearrange the above instructions to make it a valid set?

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c)
```
DIMENSIØN LM(2)
J1 = 2
DØ 10 I1 = 1,2
LB = 0
I = I1 - 1
J = J1 - 1
12  IF (I) 10, 10, 11
11  IF (J) 10, 10, 13
13  LB = LB + (I- 2*(I/2))*(J-2*(J/2))
    I = I/2
    J = J/2
    GØTØ 12
10  LM (I1) = 1 - 2*(LB - 2*(LB/2))
```

Write down the final output of the computation done with the help of the above programme. [15]

4.a) Write a short note on ISI modification of FØRTRAN II. [5]

b) The values of $x_1, x_2, \ldots, x_n$ of a variable X are punched on cards in a suitable format. Write a FØRTRAN programme to compute the standard deviation of X and to print the result in the form 'STD. DEV. = ... [15]

- ----

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MID-YEAR EXAMINATIONS

General Science-4: Biochemistry Theory

Date: 26.12.67 Maximum marks: 50 Time: $1\frac{1}{2}$ hours

1. Give one example of each:

a) Nucleotide, (b) Fat soluble vitamin,

c) Water soluble vitamin, (d) Antibiotic,

e) Purine. [10]

2. Describe how fatty acid is oxidized in mammelian cells. [15]

3. Describe the functions of different pituitary hormones. [15]

4. How can you estimate vitamin-C in lemon juice? [10]

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MID-YEAR EXAMINATIONS

Gen. Sc. 4
Biochemistry Practical

Date: 27.12.67 Maximum marks: 100 Time: 3 hours

1. Standardize the given 2, 4-dichlorophenol-indophenol dye against the standard ascorbic acid solution.

2. Determine the concentration of ascorbic acid in the unknown sample and express the result as mg. of ascorbic acid per 100 ml.

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MID-YEAR EXAMINATIONS

General Science-5: Statistical Mechanics

Date: 28.12.67 Maximum marks: 100 Time: 3 hours

1. Explain the following terms:

Phase space, Microstates and macrostates, Thermodynamic probability. [3 ×4]=[12]

Establish Stirling's approximation: $\ln (x!) = x \ln x - x$. [7]

Starting from the first principle show that the number of phase points in the i-th cell is given by:

$$N_i = \frac{N}{Z} \exp \; (-w_i/kT)$$

the symbols having their usual meanings. [20]

2. Suppose there are three cells in phase space: 1,2,3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$, and $w_1 = 2$ joules, $w_2 = 4$ joules, $w_3 = 6$ joules. If $\partial N_3 = -2$, find ∂N_1 and ∂N_2 using the two condition equations. [5]

Consider a system of 10^6 particles and a phase space of 5.10^5 cells, in which the energy w_i is the same for all cells. What is the thermodynamic probability of (a) the most probable distribution, (b) the least probable distribution? [5]

3. Apply the Maxwell-Boltzmann statistics to a monoatomic ideal gas ensemble and deduce the familiar equation of state of an ideal gas. [20]

4. What is the essential difference between Bose-Einstein and Fermi-Dirac statistics? Apply Bose-Einstein statistics to derive Planck's formula for the density of radiant energy in an enclosure whose walls are at a temperature T. [5+15]=[20]

Compute on the basis of the Fermi-Dirac statistics, for 4 phase points and 2 cells, and with $n = 4$, the thermodynamic probabilities of the macrostates $N_i = 4, N_j = 0$; $N_i = 2, N_j = 2$; $N_i = 1, N_j = 3$; $N_i = 0, N_j = 4$. Which macrostate has the greatest probability? [2 ×4+3]=[11]

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PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 26.2.68 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A

1.a) Explain clearly the term 'uniform continuity' of a function f. [6]

b) Show that every continuous function on a closed bounded interval is uniformly continuous in that interval. [8]

2.a) If $f'(x) > 0$ for every point x in $[a,b]$, show that $f(x)$ is increasing on $[a,b]$. [5]

b) If $\varphi''(x) \geq 0$ for every point x in $[a,b]$, show that for every set $x_1, x_2, \ldots, x_n$; of points in $[a,b]$

$$\varphi\left[\frac{x_1+x_2+\cdots+x_n}{n}\right] \leq \frac{1}{n}\left[\varphi(x_1)+\varphi(x_2)+\cdots+\varphi(x_n)\right].$$ [8]

3.a) From first principles show that the function $f(x) = x^2$ on $[0, 1]$ is R-integrable and find the value of the integral. [6]

b) Show that the product of two R-integrable functions on $[a,b]$ is R-integrable on $[a,b]$. [7]

c) Let X be the closed unit interval and $r_1, r_2, \ldots, r_n, \ldots$ be an enumeration of rationals in X. Define

$$f(x) = \frac{1}{2^n} \quad \text{if } x = r_n$$
$$= 0 \quad \text{otherwise.}$$

Show that f is R-integrable on X and find the value of the integral. [10]

Group B

4.a) If Σa_n is absolutely convergent series of complex numbers and Σb_n is a rearrangement of it, then show that Σb_n converges to the same sum as Σa_n.

b) Define the product of two series. If $\Sigma a_n = a$ and $\Sigma b_n = b$ and if the two series are absolutely convergent then show that the product series converges to a.b.

c) By using (b) or otherwise show that for any two complex numbers a, b

$$e(a+b) = e(a).e(b)$$

where e is the exponential function. [5+12+4]=[21]

.a) State and prove the chain rule for holomorphic functions. Formulate any such rule for continuous functions.

b) Show that a holomorphic function defined on a region should necessarily be continuous.

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c) Let $f(z)$ be a holomorphic function defined on an open Rectangle R. Define the function $\bar{f}$ on R as follows:

$$\bar{f}(z) = \text{conjugate of the complex number } f(z) \quad \text{for } z \in R.$$

Show that $\bar{f}$ is continuous function.
Show also that $\bar{f}$ is holomorphic if and only if f is constant.

d) Give an example of a power series for which the radius of convergence is zero. [6+3+6+2]=[17]

6. (In answering this question you can assume integration of real valued functions of real variable).

a) Let [a,b] be a closed bounded interval on the real line and f(t) be a complex valued continuous function defined on this interval. Give a suitable definition of

$$\int_a^b f(t)\,dt.$$

b) Let $f(t) = t + it$ for $t \in [0, 1]$.

Then is f a continuous function? Justify your answer.
Is f integrable? Justify your answer.
In case f is integrable, find

$$\int_0^1 f(t)\,dt.$$

c) Explain the terms 'Horizontal Link' 'vertical Link' and 'chain'. [4+5+3]=[12]

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PERIODICAL EXAMINATIONS

Statistics-3: Probability

Date:4.3.68 Maximum marks: 100 Time:3 hours

Note: i) Answer as much as you can.

ii) All questions are to be answered from first principles. However references can be made to solutions of earlier questions whether answered or not.

1. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed with normal distribution with mean μ and variance σ^2.

Let $\bar{X}_n = \sum_1^n X_i/n$, $S_n^2 = \sum_1^n (X_i - \bar{X}_n)^2/n$, $Z_i = (X_i - \mu)/\sigma$.

a) Show that the distribution of Z_1^2 is a χ^2 with 1 d.f. [5]

b) Show that the distribution of $\sum_1^n Z_i^2$ is a χ^2 with n d.f. [10]

c) Show that $\bar{X}_n$ and S_n^2 are independently distributed. [10]

d) Obtain the distributions of $\bar{X}_n$ and S_n^2. [10]

e) Obtain the distribution of

$$\sqrt{n-1}\,(\bar{X}_n - \mu)/S_n.$$ [10]

f) Obtain the distribution of

$$(\bar{X}_n - \mu)^2/(n\sigma^2 \sum_1^n Z_i^2).$$ [12]

2. X_1, X_2 are independently and identically distributed uniformly on (0, 1).

a) Obtain the distribution of $X_1 + X_2$. [8]

b) Obtain the distribution of X_1 / X_2. [8]

3. $X_1, X_2, \ldots, X_n$ are independently and identically distributed uniformly on $(0, \theta)$.

a) Obtain the joint distribution of (Y_n, Z_n) where

$$Y_n = \min_{1\le i\le n} X_i, \quad Z_n = \max_{1\le i\le n} X_i.$$ [10]

b) Obtain the distribution of $Z_n - Y_n$. [10]

4. $\underset{\sim}{X} = (X_1, X_2, \ldots, X_n)'$ has a multivariate normal distribution with mean μ and non-singular variance covariance matrix Σ.

a) Obtain the characteristic function of $\underset{\sim}{X}$? [10]

b) $B_{p\times n}$ is a matrix of rank p. Obtain the distribution of $\underset{\sim}{Y}_{p\times 1}$ where $\underset{\sim}{Y} = B\underset{\sim}{X}$? [15]

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PERIODICAL EXAMINATIONS

General Science-5: Psychology Theory

Date:11.3.68 Maximum marks: 100 Time: 3 hours

1. Answer all three parts.

Part A.: Define or explain the following terms:
Psycho-physical relationship
Absolute threshold
Differential threshold
Psycho-physical scale

Part B.: Give an example of each of the four terms in Part A for the auditory sensory modality.

Part C.: Describe an experiment by which the absolute threshold, the differential threshold, and a psycho-physical scale could be determined for the olfactory sensory modality. [50]

2. Discuss the three classes of effectors and their relevance for behaviour. [25]

3. Carry out a phonetic analysis of your mother tongue. Classify the letters of the alphabet (of your mother tongue) according to the following categories of phonemes: vowels, diphthongs, stops, fricatives, nasals and laterals, and glides. Next to each letter, give the corresponding letter in English and an English word indicating the sound. [25]

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PERIODICAL EXAMINATIONS

General Science-5: Engineering

Date: 18.3.68 Maximum marks: 100 Time: 3 hours

Note: All questions carry equal marks.

1. The reinforced cement concrete foundation footing 10'-long × 3' wide supports at its centre, a column of square cross-section 2' × 2', and is subjected to a soil pressure of one ton per square foot. Calculate the bending moment and shear force at the face of the column.

2.a) Derive the following formula :-

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}.$$

b) A simply-supported beam of rectangular cross-section 10"×5" wide and with a span of 10 feet supports a uniformly distributed load of 1000 lb. per Rft. Calculate the maximum flexural stress.

3.a) Draw the shear force diagram for the beam loaded and described in the question No.2(b).

b) Calculate the maximum shear stress for the said beam.

4. Find the deflection at the mid-span of the simply supported beam described in the question No.2(b) and subjected to a loading varying uniformly from 1000 lb./Rft. at the left hand support to 3000 lb./Rft. at the Right Hand support. Assume $E = 1 \times 10^6$ psi.

5. Write short notes on (a) Rolled steel Joist
(b) Moment of Resistance
(c) Bending moment
(d) Free body diagram

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PERIODICAL EXAMINATIONS

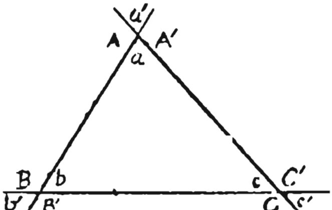
Statistics-3: Statistics Theory and Practical

Date:25.3.68 Maximum marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A

1.



Independent measurements on the angles of the triangle above are given below in degrees:

a = 59.1	b = 60.4	c = 60.1
a'= 58.6	b'= 61.3	c'= 59.2
A = 120.5	B = 119.8	C = 120.7
A'= 122.1	B'= 118.7	C'= 121.5

1) If the true values of the angles a, b and c are denoted by α, β and $\gamma = 180 - \alpha - \beta$ respectively, estimate α, β and γ. (Hint: Use the geometrical properties known).

2) If $\hat{\alpha}$ and $\hat{\beta}$ are the estimates obtained in (1) estimate the dispersion matrix of the vector $(\hat{\alpha}, \hat{\beta})$.

3) Test if $\alpha = \beta = \gamma$. [10+10+5]=[25]

2. The following table gives the means and corrected sums of products matrix for three variates Y, X_1 and X_2.

	Mean	X_1	X_2	Y
X_1	39.2500	984.500	284.030	461.415
X_2	4.1812		212.769	334.456
Y	19.6594			3564.070

1) Obtain the linear regression formula

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2.$$

2) Test the hypothesis H_o: $\beta_1 + \beta_2 = 0.75$. [10+15]=[25]

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Group B

Answer any two questions.

3.a) Show that the method of least squares provides minimum variance linear unbiased estimates for estimable parametric functions in the standard linear estimation set-up.

b) We have observations on height (X) and weight (Y) for children from three localities, there being n_i children from the i-th locality.

i) Derive a test to compare the regression coefficients of Y on X in the three localities.

ii) If the test shows that the regression coefficients are not equal, how would you examine if the difference in regression coefficients in the first two groups has been responsible for your rejecting the hypothesis?

4.a) X follows uniform probability law on [0, A]. Obtain polynomials $P_i(X)$, $i = 0,1,\ldots, 5$ such that

i) $P_i(X)$ is a polynomials in X of degree i

ii) $E[P_i(X)\ P_j(X)] = \delta_{ij}$

where $\delta_{ij} = 0\ (1)$ if $i \neq j$ $(i = j)$.

b) Explain what is meant by interaction between two classifications and derive a test to examine the hypothesis of no interactions. [5+10]=[

5. Write short notes on any two of the following:

(a) Missing plot technique

(b) Analysis of covariance

(c) Orthogonal polynomials.

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PERIODICAL EXAMINATIONS

Economics-3: Indian Economics

Date: 1.4.68 Maximum marks: 100 Time: 3 hours

Note: 1. Answer any four questions.
2. All questions carry equal marks.

1. What do you understand by the term Foreign Trade? How would you distinguish it from Internal Trade?

2. Write short Notes on:
   a) Balance of Trade
   b) Balance of Payment
   c) Terms of Trade
   d) Foreign Exchange and Foreign Exchange Reserves.

3. Critically examine the trends of India's foreign trade during the three 5-year Plans period, particularly with reference to the pattern of the foreign trade, in terms of composition and direction.

4. Discuss the factors behind the slow rate of growth of India's Exports since 1950-1. What role did the movement of internal prices play?

5. How would you explain the growing foreign indebtedness of India? Identify the unavoidable and avoidable factors responsible for the phenomenon.

6. 'The developments in the sphere of India', foreign Trade since 1950-1 reflect the impact of her planned economic development.' Do you agree? Give reasons for your view.

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ANNUAL EXAMINATIONS

Mathematics-3: Analysis

Date: 20.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Group A Max. marks: 50

Answer all questions from this group.

1.a) If $\{b_n\}$ is a positive divergent sequence and $\{a_n\}$ a convergent sequence, show that $\left\{\frac{a_n}{b_n}\right\}$ converges to zero. [5]

b) If $\{a_n\}$ converges to 'a', show that $\{|a_n|\}$ converges to $|a|$. Is the converse true? Justify your answer. [4]

c) Show that the following function φ defined on the closed unit interval, assumes every value between zero and one but is discontinuous everywhere.

$\varphi(\frac{1}{2}) = \frac{1}{3}$,

$\varphi(\frac{1}{3}) = \frac{1}{2}$,

$\varphi(x) = x$, when x is rational different from $\frac{1}{2}$ and $\frac{1}{3}$,

$\varphi(x) = 1-x$, otherwise. [8]

2.a) Give an example of a continuous function which is not differentiable. [4]

b) Show that the following function,

$$f(x,y) = xy\frac{x^2-y^2}{x^2+y^2} \quad \text{if } (x, y) \neq (0, 0)$$

$f(0,0) = 0$,

is continuous at $(0, 0)$. [5]

c) If

$$f(x,y) = \frac{2xy}{x^2+y^2} \quad \text{if } (x, y) \neq (0, 0)$$

$f(0,0) = 0$,

show that $\lim f(x,y)$ as $(x, y) \to (0,0)$ does not exist. [5]

3.a) Show, from first principles, that the function, $f(x) = x$ in the finite closed interval $[a, b]$, is Riemann integrable and find the value of the integral. [7]

b) Discuss the uniform convergence of the following sequence of functions $\{f_n\}$ defined on the closed unit interval

$$f_n(x) = \frac{x}{1+nx^2}, \quad n = 1,2,3,\ldots$$ [6]

c) Find the extreme values of the following function f defined on the closed unit interval

$$f(x) = \frac{5x}{1+25x^2}.$$ [6]

Group B Max. marks: 50

Answer Questions 6 and 7 and any one from Questions 4 and 5.

4.a) Define the product of two series.

b) If the series Σa_n converges to a and Σb_n converges to b, is it true that their product converges to ab? If not, give a counter example.

c) When do you say that a function defined on a region R is continuous?

d) State the chain rule for continuous functions and prove it by using the (ϵ,δ)-definition of continuous functions.

e) State Cauchy-Hadmard theorem for power series. (No proof is needed). [3+4+3+5+3]=[18]

5.a) When do you say that a series is absolutely convergent?

b) Find an integer n such that the n'th partial sum of the following series is greater than 10.

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$$

c) When do you say that a function defined on a region R is holomorphic?

d) State and prove the chain rule for holomorphic functions. [3+5+3+7]=[18]

6. Define

$$f(z) = \frac{z^5}{|z|^4} \quad \text{if } z \neq 0$$
$$= 0 \quad \text{if } z = 0$$

a) Discuss the continuity of f(z) at every point.

b) Discuss the differentiability of f(z) at every point.

c) Write f in the standard form u+iv.

d) Verify whether u,v satisfy the Cauchy Riemann Equations at (0,0). [4+4+4+4]=[16]

7. Define

$$e(z) = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots$$

a) Show that for every complex number this series converges.

b) Show $e(z_1 + z_2) = e(z_1)\, e(z_2)$.

c) Show that the function never vanishes.

d) What is the exact range of this function? (no proof is needed)

e) Is this function one to one?

f) What can you say about the exact range of the function as z varies over complex numbers with zero real parts? [4+4+2+2+2+2]=[16]

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ANNUAL EXAMINATIONS

Economics-3: Indian Economics

Date:21.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Note: Answer any four questions: At least two must be from each group. All questions carry equal marks.

Group A

1. What do you understand by the term Public Finance? What role would you assign to Fiscal Policy in the process of accelerating the rate of growth in a developing economy? [25]

2. 'The failure of Indian fiscal policy during the three plan periods 1951-2—1965-6, lay in the sphere of resource mobilisation rather than in the pattern of Public Expenditure'. Comment. [25]

3. Assess the Public Debt policy of the union Government during the 3rd Plan period. Suggest changes that may make it a more effective instrument for mopping up surplus funds from the public. [25]

4. Critically examine the Union-State financial relationship during the 2nd and 3rd Five Year Plans. [25]

5. 'Deficit financing without effective control over the stocks and distribution of essential consumer and producer goods and raw materials has been responsible for the inflationary trends in Indian economy since the end of the First Five Year Plan'. Discuss. [25]

6. Do you agree that the Indian budgets has become more and more dependent on foreign assistance? Give reasons for your view. [25]

Group B

7. Trace the trend in the evolution of the Indian Banking system and its role in Indian economic development since 1950-1. [25]

8. Critically evaluate the role of the Reserve Bank of India in operating the Monetary Policy of the Government during the three Plans. [25]

9. Do you think Nationalisation of Banks would bring about a more fair and rational credit allocation to the various sectors of the national economy. [25]

10. Examine the background of the devaluation of the Indian rupee in 1949 and 1966 and their respective impact on India's foreign trade and internal prices. [25]

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ANNUAL EXAMINATIONS

Economics-3: Economics Theory

Date: 22.5.68 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

The number of marks allotted to each question is given in brackets [ ].

Note: Attempt any two questions.

1. 'Two conclusions can be made about the Harrod model. The first is that there is a warranted rate of growth in output which, once achieved, will be maintained. The second is that if any other rate of growth is maintained then the adjustment within the system will move the rate, not towards, but further away from the warranted rate'. Critically examine the statement. [25]

2. Explain with the help of Mahalanobis model how larger investment in the producer goods sector is conducive to higher rate of growth of aggregate income in the long run. Discuss the weaknesses of the model. [25]

3. What would be the main common elements of mathematical growth models? Discuss the conditions for the practical applicability of these models particularly for construction of models for planning. [25]

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ANNUAL EXAMINATIONS

Statistics-3: Probability

Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Group A

Note: Answer Q.4 and any two questions of the rest from this group.

1. The conditional distribution of X given Y = y is Poisson with parameter y. The marginal distribution of Y is a Gamma distribution with parameters n and p.

   a) Obtain the joint distribution of X and Y.

   b) Obtain the marginal distribution of X.

   c) Obtain the conditional distribution of Y given X=m, m=0, 1,... . [4+6+5]=[15]

2. $X_1, \ldots, X_n$ $(n \geq 2)$ are independently and identically distributed with p.d.f. e^{-x}, $x \geq 0$. $X_{(1)}$ and $X_{(2)}$ are the smallest and the second smallest among $X_1, \ldots, X_n$.

   a) Obtain the joint distribution of $(X_{(1)}, X_{(2)})$.

   b) Obtain the distribution of $(X_{(2)} - X_{(1)})$.

   c) Obtain the limiting distribution of $nX_{(1)}$. [5+5+5]=[15]

3. Let $X_1, \ldots, X_n, \ldots$ be independently and identically distributed with mean $\mu (\neq 0)$ and finite fourth moment.

   Obtain the asymptotic distributions of the following:

   (a) $\bar{X}_{(n)} = \dfrac{X_1 + \quad + X_n}{n}$

   (b) $(\bar{X}_{(n)} - \mu)/S_n$ where $S_n^2 = \sum_1^n (X_i - \bar{X}_{(n)})^2/n$

   (c) $\bar{X}^2_{(n)}$ [5+5+5]=[15]

What is the definition of convergence of distribution functions usually used in Statistics and Probability? [5]

Let a sequence of d.f.'s F_n converge to a d.f. F as $n \to \infty$ in the above sense. Let g(x) be a bounded continuous function. Show that

$$\int g(x)dF_n(x) \to \int g(x)dF(x)$$

as $n \to \infty$. [12]

Deduce that the characteristic function of F_n converges to the characteristic function of F. [3]

a). Find the distribution of X^2 when X has respectively:

   i) a normal distribution with mean 0 and variance 1. [2]

   ii) a standard Cauchy distribution. [4]

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(iii) an exponential distribution. [3]

b) Obtain the distribution of X/Y when X and Y are independent and have (standard) exponential distributions. [6]

Group B

Note: Answer Q.8 and any two questions of the rest from this group.

6. What is the definition of a multivariate normal distribution? [5]

Show that any linear function of a multivariate normal variable is normally distributed. [4]

Let (X_1,Y_1), (X_2,Y_2),... be independently and identically distributed bivariate random variables with a finite variance-covariance matrix. Let $\bar{X}_n=(X_1+\dots+X_n)/n$ and $\bar{Y}_n=(Y_1+\dots+Y_n)/n$. Write down the asymptotic distribution of $a\bar{X}_n+b\bar{Y}_n$ for any (a, b). [6]

7.a) Construct a sequence of random variables such that.

i) $X_n \to 0$ in probability, EX_n exists for every n, but $E(X_n) \not\to 0$.

ii) $X_n \to 0$ in probability $V(X_n)$ exists for every n, $E(X_n) \to 0$, but $V(X_n) \not\to 0$.

b) Show that

i) $E(|X_n|) \to 0$ implies that $X_n \to 0$ in probability.

ii) $E(X_n) \to 0$, $V(X_n) \to 0$ imply that $X_n \to 0$ in probability. [4+4+4+3]=[15]

8. i) State the inversion theorem for characteristic functions.

ii) State one form of the continuity theorem.

iii) Obtain the distribution of F(X) where X has a continuous distribution function F(x). Also obtain the distribution of $-2 \log F(X)$.

iv) Let $X_1,X_2,\dots,X_k$ be independent variates with continuous distribution functions $F_1(x),\dots,F_k(x)$. What is the distribution of

$$-2 \sum_{1}^{k} \log F_i(X_i)?$$ [5+4+6+5]=[20]

9.a) Let $X_n \to X$ in law. Let $Y_n \to 0$ in probability. Show that $X_nY_n \to 0$ in probability.

b) Let $X_1,\dots,X_n,\dots$ be independently and identically distributed with a d.f. F(x) admitting a continuous non-vanishing p.d.f. f(x). Show that the sample pth fractile has an asymptotic normal distribution. [5+10]=[15]

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10. Let $X_1, \ldots, X_n$ be independently and identically distributed with (A) a normal distribution with mean μ and variance 1, (B) a distribution whose p.d.f. is

$$\lambda e^{-x/\lambda}, \; x \geq 0.$$

Let $\tilde{X}$ = median of $(X_1, \ldots, X_n)$ and

$\bar{X}_n = (X_1 + \ldots + X_n)/n.$

(a) In case (A), $\tilde{X}$ and $\bar{X}_n$ have asymptotic mean μ. What is the ratio of their asymptotic variances?

(b) In case (B), $(\tilde{X}/\log 2)$ and $\bar{X}_n$ have asymptotic mean μ. What is the ratio of their asymptotic variances?

[7+8]=[15]

[illegible]
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ANNUAL EXAMINATIONS
Statistics-3: Statistical Theory and Practical

Date: 27.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks alloted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Group A

Note: Answer any two questions from this group.

1.a) Let Y denote a vector valued random variable with $E(\underset{n\times 1}{Y}) = \underset{n\times p}{X} \; \underset{p\times 1}{\beta}$ and $V(Y) = \sigma^2 I_n$, where X is a known matrix of co-efficients, β and σ^2 being unknown parameters. Let Y^* denote projection of Y on $\mathcal{M}(X)$, the linear manifold spanned by the column vectors of X.

i) Show that $C'Y^*$ is minimum variance linear unbiased estimator (MVLUE) of $E(C'Y)$ for any C in R^n.

ii) Show that for c, d in R^n $\text{Cov}(c'Y^*, d'Y^*) = (c'd^*)\sigma^2$ where d^* is the projection of d on $\mathcal{M}(X)$.

iii) Give an unbiased estimator of $V(c'Y^*)$. [5+3+2]=[10]

b) Three objects O_1, O_2 and O_3 were weighed on a spring balance. The following table gives the weighing scheme and the observations:

Objects	Weight
O_1	2.2
O_2	3.9
O_1, O_2	6.2
O_3	6.3
O_1, O_3	7.8
O_2, O_3	9.8

It is possible that the spring balance may have an additive bias in the sense of always giving higher (or lower) values for weights. Obtain values of MVLUE for this bias and of the true weights of these objects. [10]

2.a) Why does one use transformations of statistics in analysis of variance? Give a general procedure for arriving at appropriate transformations stating carefully any convergence throrems used therein. [3]

b) The following estimates of the correlation coefficients between intelligent test scores were found in an investigation of the relative influences of environmental and heredity factors. Analyse the data and comment on your findings.

	Two brothers		Twins	
	Reared apart	Living together	Reared apart	Living together
Correlation coefficient	.235	.342	.451	.513
Sample size	50	40	45	53

[12]

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3. The data below gives hypothetical probabilities and observed frequencies of offsprings in four classes.

Phenotype Classes	AB	Ab	aB	ab
Probability	$\frac{1}{4}(2+\theta)$	$\frac{1}{4}(1-\theta)$	$\frac{1}{4}(1-\theta)$	$\frac{1}{4}\theta$
Frequency	102	17	36	5

a) Test the hypothesis that the above model fits with $\theta = \frac{1}{4}$ i.e. the probabilities are in the proportion 9:3:3:1. [3]

b) Write down maximum likelihood equation for estimating θ. [7]

c) Test the hypothesis in (a) without specification of θ. (Use a suitable estimate of θ). [?]

Group B

Note: Answer any two from questions 4.5 and 6 of this group.

4.a) Explain what is meant by empirical distribution function. Show that the empirical distribution function converges to the population distribution function (under conditions to be stated explicitly). How would you use the empirical distribution function to test the hypothesis that a given sample arose from a specified distribution? [2+3+5]=[10]

b) Compute asymptotic variance for $\bar{x} + Ks$ where $\bar{x}$ and s denote the mean and the standard deviation in a random sample of size n drawn from a population and K is a constant. Hence or otherwise write down the asymptotic distribution of $\bar{x} + Ks$ for the special case where the sample is drawn from a normal population. [6+4]=[10]

5.a) The following table gives breaking strength y in grams and the thickness x in 10^{-4} inch from tests on 3 types of starch film.

Wheat		Rice		Corn	
y	x	y	x	y	x
263	5.0	556	7.1	731	8.0
131	3.5	552	6.7	710	7.3
393	4.7	397	5.6	604	7.2
302	4.3	532	8.1	508	6.1
213	3.8				

a) Test for differences among the starches in expected breaking strength ignoring the data on thickness. [5]

b) Test if regression of y on x is the same for all three starch types. [5]

c) Obtain estimate of E(Y) at x = 6 for each starch type (without assuming common regression). Obtain 90 per cent confidence limits for this for wheat starch. [5]

d) Compare the starch types after adjusting for the effect of thickness assuming a common regression of y on x. [5]

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6. a) Explain Scheffe's method of simultaneous confidence intervals in a general linear estimation set up. Show how this interval estimation method is related to the usual F-test of a linear hypothesis. [3+5]=[8]

b) Three nozzles A, B, C were compared by four operators selected at random from a large number of operators. Each operator took three observations on each nozzle. Table below gives totals of these observations (suitably coded) for each combination of nozzle with operator

	1	2	3	4
A	-3	43	19	42
B	32	19	44	-8
C	9	-19	-16	6

Within combination mean square = 124.

Analyse the data testing the various hypotheses of interest. Write down expected value of mean squares for each source in your analysis of variance. [10+2]=[12]

7. Viva Voce [10]

8. Practical record [10]

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ANNUAL EXAMINATIONS

General Science-4: Biology Theory

Date: 28.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Group A

Answer question 1 and any two questions from the rest in this group.

1. Write an account of the methods for breeding self-fertilising crops.

Discuss briefly the role of mutation as a breeding tool. [14+6]=[20]

2. What is polyploidy? Describe the methods for inducing chromosome doubling. Mention briefly the physical characteristics of autotetraploid plants. [3+8+4]=[15]

3. What is heterosis? Explain why attainment of homozygosity is delayed by selection for vigour? Write a brief account of heterosis effects in plants and animals. [3+6+6]=[15]

4. What is an isolate? Write some of the important characteristics of human population living in isolates.

The following data were obtained from three populations:

Country	Year	Proportion of children dying at early age — Parents: first cousins	second cousins
U S A	1858	692/2936	134/837
U S A	1908	113/672	370/3182
France	1953	165/1417	306/5382

Suggest an explanation of the differences in mortality between offspring of cousins and of unrelated parents. [3+6+6]=[15]

Group B

Answer question 5 and any two questions from the rest in this group.

5. Answer any two of the following:

i) Draw a labelled diagram of the spikelet of *Triticum vulgare*. [5]

ii) Mention the morphology of the commercially important fibre yielding organs in the following species;

a) *Gossypium hirsutum*,

b) *Corchorus capsularis*,

c) *Agave sisalana*.

d) *Musa Textilis*,

e) *Cocos nucifera*. [5x1]=[5]

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iii) Write the names of ten plants yielding essential oil. [5]

6. Write a detailed account on the clonal propagation in <u>cocos nucifera</u>. [20]

7. Give the area under and production of <u>Oryza sativa</u> for the various agricultural regions of the world. Mention the corresponding figures for India and China. [16+4]=[20]

8. What is a legume? Write the scientific names of eight important pulse crop plants. Give a brief botanical account of any legume plant. [2+8+10]=[20]

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ANNUAL EXAMINATIONS

General Science-5: Psychology

Date: 29.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Group A Max. marks: 50

Discuss A. H. Maslow's theory of self-actualization. What are the characteristics of a self-actualized person? [15]

What are the basic conditions of learning? Discuss. [15]

Define 'reliability' of a psychometric test. Obtain an expression for reliability and explain how it can be estimated from the scores of N students on a sample test. [20]

Group B Max. marks: 50

Write short notes on the following:

Topics from motivation theory:

a. emotion
b. instinct
c. level of aspiration
d. psychological field
d. acquired motives [20]

Topics from learning theory:

a. stimulus generalization
b. stimulus differentiation
c. memory trace
d. curve of retention
e. experimental neurosis [20]

Topics from psychometric theory:

a. item difficulty
b. item discrimination
c. item-ability relation. [10]

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ANNUAL EXAMINATIONS

General Science-5: Engineering

Date: 30.5.68 Maximum Marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer groups A and B in separate answerscripts.

Note: Attempt any five questions, choosing at least two from each group.

Group A

1. a) A 15 ft. 6" diameter steel beam is supported on a central column situated at its mid point. A weight of 1 ton is hung at each of the free ends of the beam. Calculate the maximum flexural stress.

   b) Which is the ideal section for steel beams? Give reasons. [20]

2. a) Explain each terms in the following formulaes:-

   i) $\frac{E}{R} = \frac{f}{y} = \frac{M}{I}$

   ii) $\frac{G}{l/\theta} = \frac{s}{r} = \frac{T}{I_p}$

   b) Compare the weights of equal lengths of hollow and solid shaft to transmit a given torque for the same maximum shear stress if inside diameter is 2/3 of the outside. [20]

3. a) Write a short note on Rankine's formulae for struts.

   b) A hollow Cast Iron column with fixed ends, supports an axial load of 100 tons. If the column is 15' long and has an external diameter of 10" find the thickness of metal required. Assume Rankine's constant = 1/6400 and working stress = 5 tons per square inch. [20]

Group B

4. a) Define 'Centre of pressure'.

   b) A circular plate 4 feet diameter is placed vertically in water so that the centre of plate is 6 feet below the surface. Find the depth of the centre of pressure and the total pressure on the plate. Give your comments if the plate is taken 100 feet below the water surface. [20]

5. a) Define i) Coefficient of contraction
   ii) Coefficient of velocity
   iii) Coefficient of discharge.

   b) Find the time of emptying completely if the 3' diameter tank is filled to a depth of 6 feet is provided with 2" diameter orifice in its bottom. Derive the formula you use. Assume coefficient of discharge is 0.6. [20]

Write short notes: i) Open channels
ii) Venturimeter
iii) River gauging
iv) Factor of safety [20]