INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69
QUESTION PAPERS - CONTENTS

Sl. No.	Date	Examination No.	Subject
PERIODICAL EXAMINATIONS			
1	16. 9.68	81	Statistics-3: Probability
2	23. 9.68	82	General Science-4: Biochemistry Theory
3	14.10.68	83	Mathematics-3: Analysis
4	21.10.68	84	Statistics-3: Data Processing
5	28.10.68	85	General Science-5: Sociology
6.	4.11.68	86	Economics-3
7	11.11.68	87	Statistics-3: Statistics Theory and Practical
MID-YEAR EXAMINATIONS			
8	18.12.68	88A	Mathematics: Analysis
9		88B	Mathematics: Analysis Home Assignment
10.	20.12.68	89	Economics-3
11.	21.12.68	90	General Science-4: Biology: Botany Theory
12	23.12.68	91	General Science-4: Biochemistry Theory
13	24.12.68	92	General Science-4: Biochemistry Practical
14	26.12.68	93	Statistics-3: Probability
15	28.12.68	94	Statistics-3: Statistics Theory and Practical
16	30.12.68	95	General Science-5: Statistical Mechanics
PERIODICAL EXAMINATIONS			
17	24. 2.69	96A	Mathematics-3: Analysis
18	24. 2.69	96B	Mathematics-3: Analysis Home Assignment
19	3. 3.69	97	Economics-3
20	10. 3.69	98	Statistics-3: Probability
21	17. 3.69	99	Statistics-3: Statistics Theory and Practical
22	24. 3.69	100	General Science-5: Psychology Theory and Practical
23	31. 3.69	101	General Science-5: Engineering
ANNUAL EXAMINATIONS			
24	19. 5.69	102	Mathematics-3: Analysis
25	20. 5.69	103	Economics-3
26	21. 5.69	104	Economics-3
27	23. 5.69	105	Statistics-3: Probability
28	24. 5.69	106	General Science-5: Engineering
29	26. 5.69	107	Statistics-3: Statistics Theory
30	27. 5.69	108	Statistics-3: Statistics Practical
31	29. 5.69	109	General Science-4: Biology
32	31. 5.69	110	General Science-5: Psychology Theory and Practical

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69
PERIODICAL EXAMINATIONS

[81]

Statistics-3: Probability

Date: 16.9.68 Maximum Marks: 100 Time: 3 hours

Note: The whole paper carries about 110 marks.
Answer as much as you can.
The marks allotted to each question is given in brackets [].

1.a) Define lim sup A_n and lim inf A_n for a sequence $\{A_n\}$ of subsets of a set Ω. When is $\{A_n\}$ said to have a limit? [3+3+3]=[9]

b) If $\{A_n\}$ is a disjoint sequence, show that $\{A_n\}$ converges to $\emptyset$. [5]

c) If $\{A_n\}$ is increasing, show that $\{A_n\}$ converges to $\bigcup_{n=1}^{\infty} A_n$. [5]

d) If

$$E_n = \begin{cases} (0, 1 - \frac{1}{n}] & \text{if } n \text{ is odd} \\ [\frac{1}{n}, 1) & \text{if } n \text{ is even} \end{cases}$$

verify that E_n converges but is not monotone. [6+2]=[8]

2.a) A class $\mathcal{R}$ of subsets of a set Ω is called a ring if $A, B \in \mathcal{R} \Rightarrow A \cap B$, $A \triangle B \in \mathcal{R}$. Show that $\mathcal{R}$ is a ring if and only if $A, B \in \mathcal{R} \Rightarrow A \cup B$, $A - B \in \mathcal{R}$. [5]

b) Show that if $\mathcal{R}_\alpha$ is a collection of rings, $\bigcap_\alpha \mathcal{R}_\alpha$ is a ring. Hence define the ring generated by any class $\mathcal{E}$ of subsets of Ω and show that it is the smallest ring containing $\mathcal{E}$. [5+3+6]=[14]

c) Show that for a ring $\mathcal{R}$, the following are equivalent: (i) $\mathcal{R}$ is a σ-ring; (ii) $\mathcal{R}$ is a monotone class. [5]

3. State whether each of the following classes is a (1) semiring (2) ring (3) field (4) σ-ring (5) σ-field (6) monotone class (7) hereditary class (Ω is an uncountable set). [$42 \times \frac{1}{2}$]=[21]

a) all finite sets

b) $\emptyset$, A, $\Omega - A$, Ω where A is a fixed subset of Ω

c) all countable sets and their complements

d) all single point sets.

e) Ω = the real line, all bounded intervals (open, closed, semi-closed).

f) all subsets of A where A is a fixed subset of Ω.

4.a) Define a measure on a ring $\mathcal{R}$. Prove that it is monotone, subtractive, continuous from below and from above at every set of $\mathcal{R}$. [3+5+5+12]=[25]

b) Prove that a finite non-negative additive set function defined on a ring $\mathcal{R}$ which is either continuous from below at every set of $\mathcal{R}$ or continuous from above at $\emptyset$ is a measure on $\mathcal{R}$. [16]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

82

PERIODICAL EXAMINATIONS

General Science-4: Biochemistry Theory

Date: 23.9.68. Maximum Marks:100 Time: $2\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Give one example for each of the followings:
   Basic amino acid, Dehydrogenase, Dipeptide, Phosphoproteine, Oxidase. [10]

2. How the proteins are classified? Give one example of each class. [20]

3. What are the functions of proteins. [15]

4. How can you estimate protein from a natural source. [15]

5. Define isoelectric point of amino acids. What is an essential amino acid? Give one colour reaction of amino acids. [15]

6. What is an enzyme? What is transaminase?
   Describe the properties of:
   (a) Amylase
   (b) Catalase [25]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69

83

PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 14.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

R denotes an Archemedian Ordered Field

1.a) Define the terms 'Field' 'Ordered Field' and 'Archemedian Ordered Field'.

b) Can a Field Consist of finite number of elements? Give reasons.

c) Can an Ordered Field consist of finite number of elements? Give reasons. [10+5+5]=[20]

2.a) Prove that any bounded decreasing sequence in R is a Cauchy sequence.

b) Is converse of 'a' true? Give reasons.

c) Prove that any convergent sequence in R is a Cauchy sequence.

d) Is converse of 'c' true? Give reasons. [7 +3+7+3]=[20]

3.a) Show that any sequence in R contains a monotone subsequence.

b) Show that if a subsequence of a Cauchy sequence in R converges then the Cauchy sequence itself converges.

c) Deduce from 'a' and 'b' above, that the following two statements are equivalent in R:

1. Any bounded increasing sequence converges.

2. Any Cauchy sequence converges. [8+8+4]=[20]

4.a) Explain the terms 'Open interval' 'Open set' 'limit point' and 'closed set'.

b) Show that a nonempty set A contained in R is open if and only if A^c (the complement of A in R) is closed. [10+10]=[20]

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5. Assignments [10]

6. For clarity of expression. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III:1968-69

[84]

PERIODICAL EXAMINATIONS

Statistics-3: Data Processing

Date: 21.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Name the various types of Punched Card machines known to you and briefly explain their functions. [15]

2. You have a deck of cards which are badly damaged by its 12-Edge.
Is it possible to reproduce a new pack of punched cards by using these cards? If so how. [5]

3. There is a sorted deck of 40,000 cards by considering columns 6-10. It is necessary to pull-out all cards with '4' punched in column 25 of those cards.

How will you get the work done without disturbing the sorting sequence of the cards. [5]

4. It is required to prepare a new set of punched cards from the cards punched according to the design given below. No change of design in the new set of cards is enviesaged.

Card-design	Card Cols	
cdi	1 - 4	Punch XXX1
department	5	
category	6	
roll no.	7 - 9	
name	10 -34	
daily pay rate	35-38	(2 pl. decimal)

Prepare the reproducer Panel diagram. [12]

5. You are provided with a deck of cards punched according to design XXX1 given in question No.4. Prepare control Panel diagram to prepare the statement given below.

department	category	number of worker	total daily pay
	1		
	2		
department	sub-total		
	1		
	2		
department	sub-total		
department	sub-total		
Total			

[20]

GO ON TO THE NEXT PAGE

6. EITHER

Write a general program in FØRTRAN II to evaluate the mean and variance of N given numbers $X_1, X_2, \ldots, X_N$.

OR

Write a program in FØRTRAN II to find the roots of the quadratic equation

$$AX^2 + BX + C = 0$$

where A, B, C are given real numbers.

7. Write a program in FØRTRAN II to obtain N! where N is a positive integer.

8. Class records.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III:1968-69

PERIODICAL EXAMINATIONS

General Science-5: Sociology

Date: 28.10.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.6 and any four from the rest.
All questions carry equal marks.

1. EITHER

Define and distinguish between family and household. Mention the chief functions of family as a societal institution.

OR

How a nuclear family differs from an extended family? What are the inter-personal relationships you may expect from a nuclear family?

2. EITHER

What are consanguinal and affinal relationships? What are the difference between 'descriptive' and 'classificatory' systems of kinship? Briefly mention the importance of kinship in simple societies.

OR

What do you mean by clan? How it differs from lineage and extended family? What are its main functions ?

3. EITHER

Define 'religion'. Describe the nature of religion and compare it with magic.

OR

What are monogamy and polygamy? Enumerate different types of preferential marriages found in Indian societies.

4. EITHER

Define and distinghish between caste and class. Enumerate the features of caste organization in India.

OR

What are endogamy and exogamy? How do they influence caste formation?

5. EITHER

What is 'incorporeal' and 'corporeal' property? Give examples from simple and complex societies.

OR

What are the social significances attached to (a) lobola (b) kula and (c) pot-latch?

6. Formulate a design of sample survey for studying standard of living of different family groupings in West Bengal with special reference to (a) coverage of universe (b) unit of sampling and (c) unit of enquiry.

INDIAN STATISTICAL INSTITUTE
Research and Training School
M.Stat. Part III: 1968-69

PERIODICAL EXAMINATIONS

Economics-3

Date: 4.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts.

Group A: Economic Development

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Attempt any two questions. Each question carries 25 marks.

1. Describe the significant changes in the national product and its distributions in the advanced economies in the process of long period economic development.

   Could cross section national income data throw any light on the subject?

2. Discuss Abramovitz's analysis of the long period growth of the US economy.

   Can a similar analysis be applied on the Indian data?

3. Bring out the main features of the growth model developed by classical economists.

   Briefly indicate its main differences from Marx's model.

4. Discuss briefly the growth models due to Harrod and Domar and point out the main difference between their approaches.

Group B: Indian Planning

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Note: Answer any three questions.
All questions carry equal marks.

1. Is a planned economy superior to a non-planned one? Give reasons for your answer.

2. Critically discuss the differences between programming, mixed economy planning and centralised planning.

3. Discuss the approach and methods proposed in the Bombay Plan to double the per-capita income in the course of 15 years.

4. Why and what measures were proposed in the Bombay Plan to reduce inequality of income?

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69

87

PERIODICAL EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 11.11.68 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Marks allotted for each question are given in brackets [ ].

1.a) The following table gives the production of polished plate glass in USA. Fit a suitable polynomial regression for the production of polished plate glass by using the method of orthogonal polynomials.

Year	production (in millions of square-feet monthly)
1933	7.2
1934	7.9
1935	15.0
1936	16.5
1937	16.0
1938	7.1
1939	11.8
1940	13.7
1941	15.9

[16]

b) Suggest a procedure to fit the polynomial regression if some values are missing in the data. (No computations are required). [4]

2. To determine the yield rate (in maunds per acre) of paddy by the method of random sampling ten plots were chosen at random. Within each plot a circle of radius 2 ft. and another of 4 ft. were marked out at random. The crops inside the circles were harvested and then the yield rate was calculated. Obtain a 95 per cent confidence interval for the ratio of the variance of the yield rate as calculated from circle with 2 ft. radius to that calculated from circle with radius 4 ft.

radius of the circle	yield rate
2 ft.	6.1, 5.4, 5.6, 6.3, 5.1, 6.3, 5.9, 5.6, 4.4, 6.1.
4 ft.	5.5, 6.0, 4.7, 5.9, 5.6, 6.1, 5.7, 5.4, 5.2, 5.8.

[20]

3. The table below shows measurements of heights of 11 pairs of twins of opposite sex all of ten years of age.

Sl. No.	Male ht. (in cm.)	Female ht. (in cm.)	Sl. No.	Male ht. (in cm.)	Female ht. (in cm.)
1	136	132	7	130	133
2	141	133	8	139	140
3	137	140	9	128	123
4	137	135	10	132	136
5	134	131	11	133	134
6	134	137			

GO ON TO THE NEXT PAGE

. Obtain 90 per cent confidence limits for the correlation between the heights of the twins.

4. What are random numbers? Discuss in detail the various requirements you impose for the construction of random number tables. Comment on your requirements.

5. Obtain a random sample of size 15 from the exponential distribution with density function.

$$f(x) = \begin{cases} 2e^{-2x} & 0 \leq x < \infty \\ 0 & \text{otherwise} \end{cases}$$

6. a) Explain clearly what are meant by SRS WR and SRSWOR giving examples of situations in which you use them.

b) Propose suitable estimate for the population mean and give its standard errors in each sampling procedure.

c) How do you use your estimates to obtain population total? What is the standard error of your estimate.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

[88] A

MID-YEAR EXAMINATIONS

Mathematics: Analysis

Date: 18.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted to each question are given in brackets [ ].

1.a) Define the terms 'Field' 'Ordered Field' and 'Archemedian Ordered Field'.

b) Can a Field consist of finite number of elements? Justify your answer.

c) Can an Ordered Field consist of finite number of elements? Justify your answer. [10+2+3]=[15]

2.a) Show that any closed interval [a,b] where $a < b$ contains a rational number.

b) Hence deduce that given any real number x, there is an increasing sequence of rationals converging to it. [5+5]=[10]

3.a) Define clearly a 'sequence' and a 'series' of real numbers.

b) When do you say that a series is absolutely convergent?

c) Show that any absolutely convergent series is convergent. [3+2+5]=[10]

4.a) Define the Cauchy product of two series.

b) If a series Σa_n converges absolutely to A and a series Σb_n converges to B then show that the Cauchy product converges to A·B. [2+8]=[10]

5.a) What is meant by rearrangement of a series?

b) State carefully Riemann's theorem about rearrangement of a series. (No proof is needed). [2+3]=[5]

6. Home assignments. [50]

Note: COLLECT YOUR HOME ASSIGNMENT QUESTION PAPER FROM THE DEAN'S OFFICE.

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INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [85] B

MID-YEAR EXAMINATIONS

Mathematics: Analysis

Home Assignment

Maximum Marks: 50.

Note: 1. You should submit your answerscripts on or before 20th January 1969 in the Dean's Office.

2. Answer all questions.
3. Your answers should be precise, clear and logical.
4. Do not give unnecessary details. But you should clearly explain how one step follows from previous one in your proofs.

1. If Σa_n is a convergent series of positive terms then show that Σa_n^2 is also convergent.

[Hint: After a certain stage $a_n^2 \leq a_n$]

2. If Σa_n^2 is convergent series then show that $\Sigma \frac{a_n}{n}$ is also convergent.

[Hint: $\frac{2a_n}{n} \leq a_n^2 + \frac{1}{n^2}$]

3. If Σa_n is convergent and $a_n \downarrow 0$, then show that $\Sigma 2^n a_{2^n}$ converges.

4. Show that the series $\Sigma \frac{1}{n}$ is not convergent. Find an integer N such that the Nth partial sum of this series is greater than 5.

5. Show that the series Σa_n where

$$a_n = \frac{1}{n^2} \text{ if } n \text{ is not a perfect square}$$
$$= \frac{1}{n} \text{ if } n \text{ is a perfect square}$$

Converges, by showing that the partial sums are Cauchy.
[Note: A positive integer is called perfect square if it is the square of some integer].

6. For any sequence $\{c_n\}$ of real number where $c_n > 0$ for every n show that,

$$\liminf \frac{c_{n+1}}{c_n} \leq \liminf \sqrt[n]{c_n} .$$

7. Show that the series $\Sigma \frac{(-1)^n}{\sqrt{n}}$ is convergent, but the Cauchy product of it with itself is not convergent.

8. Show that for every real number x the series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

is convergent. Denote this by $e(x)$. Show by using the Cauchy product of series, that

$$e(x+y) = e(x).e(y).$$

9. If a power series converges for every real number x, then show that it absolutely converges for every number x.

GO ON TO THE NEXT PAGE

10. Find out the limit superior and limit inferior for the following sequence:

$$a_1 = 0,$$
$$a_{2m} = \frac{a_{2m-1}}{2},$$
$$a_{2m+1} = a_{2m} + \frac{1}{2}, \qquad \text{for } m \geq 1.$$

11. Find $\lim_{n \to \infty} \sqrt{n^2 + n} - n$!

[Hint: $\sqrt{n^2+n} - n = \frac{n}{\sqrt{n^2+n} + n}$].

12. Let $\{a_n\}$ be the sequence,

$$a_1 = \sqrt{2}, \quad a_n = \sqrt{2 + \sqrt{a_n}} \quad \text{for } n > 1$$

Show that the sequence $\{a_n\}$ is increasing and bounded by 2 and hence converges.

13. If a power series has infinitely many nonzero/integer coefficients then show that its radius of convergence is at most 1.

14. You are given a number r such that $0 \leq r \leq +\infty$. Exhibit a power series whose radius of convergence is r.

15. 'Real number system' is defined as a 'Complete Archemedian Ordered Field'. From what you have learned in the class, discuss how far the 'Archemedian' property and 'Completeness' property are essential.

16. What is meant by a 'summability method' in the real number system? Write a few sentences about the reasonable conditions that a summability method should satisfy, according to you. Define and comment on Cauchy's method of summability.

[14×3 + 2×4]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69

89

MID-YEAR EXAMINATIONS

Economics-3

Date: 20.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts. Marks allotted for each question are given in brackets [ ].

Group A

Economic Development

Answer any three questions

Maximum Marks: 60 Suggested time: $1\frac{3}{4}$ hours

1. Mention the important items of Leibenstein's list of characteristics of underdeveloped countries. [20]

What do you understand by 'vicious circles' in this context.

2. Growth of real national income is not fully explained by changes in labour and capital inputs. What other factors should be considered? Obtain suitable expressions for contributions of different factors to growth.

Discuss the relevance of the above proposition for analysing the long period growth of either the U.S. or the Indian economy. [20]

3. Why is it important to study transactions in kind in under developed economies?

Give a descriptive account of various transactions in kind in India.

State conditions under which the share of transactions in kind (in aggregate of all real transactions) is likely to (i) reduce, (ii) increase. [20]

4. A measure of intersectoral disparity of average sectoral earnings per worker is given by

$$d = \frac{1}{2} \sum_{i=1}^{n} |y_i - \lambda_i|$$

y_i, λ_i respectively standing for shares of income and labour force in the ith sector. Construct simple examples showing that 'd' increases with increased disparity.

How could a similar measure of disparity be constructed for size distributions?

What change in a size distribution you would expect when intersectoral disparity increases? Why is it easier to study changes in size distributions of real income through intersectoral disparities? [20]

5. Write short notes on any two of the following:

i) backward sloping supply curve of effort;

ii) employment concept as applied to poorer countries;

iii) distribution of national income by factor shares in rich and poor countries; and

iv) linkage coefficients. [20]

GO ON TO THE NEXT PAGE

Group B

Indian Planning

Answer <u>any two</u> questions. Maximum Marks: 40

Suggested time: $1\frac{1}{4}$ hours.

1. What influenced the change in the approach in the formulation of the second five year plan? Indicate how this change was reflected in the size, allocation of resources and objectives of the second plan. [20]

2. What are the different sources of finances for the plan in the public sector? Discuss the estimates of these resources for the second five year plan. [20]

3. What were the issues and what were the grounds that led K. T. Shah to differ from the recommendation of the Advisory Planning Board? [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part:III: 1968-69 [55]

MID-YEAR EXAMINATIONS

General Science-4: Biology: Botany Theory.

Date: 21.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. Marks allotted for each question are given in brackets.[ ].

1. The sugarcane variety Co.312 is very successful in the sub-tropical belt of India. What are the factors which contribute to the good performance of this variety? [20]

2. Write an illustrated account on the morphology of the coconut palm. [20]

3. A. Write names of ten economically important Legumes.

   B. Which are the species of wheat cultivated at present? [10+10+20]=[40]

4. Give the statistics on the area under and production of Rice(*Oryza sativa*) in the various agricultural regions of the world. Mention the corresponding figures for China, India and Pakistan. [14 +6]=[20]

5. EITHER
   Write an essay on the scientific aspect of your visit to the Rice Research Station at Chinsurah.

   OR

   The Jute Agricultural Research Institute, Barrackpore. [20]

6. A: What is the morphology of the economically important part (or organ) in the following plant species?

   (a) *Musa textilis*; (b) *Boehmeria nivea*;
   (c) *Helianthus tuberosus*; (d) *Acer negundo*;
   (e) *Beta vulgaris*; (f) *Pisum sativum*;
   (g) *Manihot utilissima*; (h) *Solanum tuberosum*;
   (i) *Agave sisalana*; (j) *Gossypium barbadense*.

   B: Describe the spikelets of *Sorghum vulgare* with suitable illustrations. [10+10]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [91]

MID-YEAR EXAMINATIONS

General Science-4: Biochemistry Theory

Date: 23.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe the aerobic glycolysis in mammalian cells. [20]

2. What are epinephrine and vasoprasin? Describe their physiological properties. [10]

3. How can you estimate glucose in human blood. [15]

4. Give one example for each of the following:
Nucleoside, Aldose, Triglyceride, Pyrimidine, Steroid. [10]

5. What are the defficiency symptoms of the following vitamins.
Vitamin A, Pantothenic acid, Niacin, Vitamin K, Folic acid. [15]

6. Discuss the origin of ketone bodies in urine. [10]

7. Describe the biochemical pathways of fatty acid oxidation. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [92]

MID-YEAR EXAMINATIONS

General Science-4: Biochemistry Practical

Date: 24.12.68 Maximum Marks: 100 Time: 3 hours

1. Determine the total amount of glucose in the given sample by Fehling's titration.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

[93]

MID-YEAR EXAMINATIONS

Statistics-3: Probability

Date: 26.12.68 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 120 marks. Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) Define an outer measure. [4]

b) How do you prove that given a measure μ on a ring R, it can be extended to a complete measure on a σ-ring containing R? (Give the necessary steps without proofs). [8]

c) Define a Steiltjes measure function F on R^k and the Lebesgue-Steiltjes measure induced by F. [4+6]=[10]

2.a) Define a measurable function f. If f is non-negative, show that it is a limit of a monotone increasing sequence of non-negative simple functions. [4+8]=[12]

b) If f, g are measurable, prove that f+g, fg are measurable. [6+6]=[12]

c) If f_n is a sequence of measurable functions, show that lim sup f_n is measurable. [6]

3.a) Define the integral of a non-negative measurable function and prove that it is unique. [4+10]=[14]

b) Define the integral of a measurable function whenever it is possible. [4]

c) Show that any measurable function has integral zero over a set of measure zero. [6]

d) If f is integrable, show that f is finite a.e. [6]

4.a) If A, B are disjoint measurable sets and f is integrable, show that

$$\nu_f (A \cup B) = \nu_f (A) + \nu_f (B)$$

where $\nu_f (A) = \int_A f d\mu$. [6]

b) Assuming that if f_n are non-negative, measurable and increasing, then $\int \lim f_n d\mu = \lim \int f_n d\mu$, prove that if f is an integrable function, then for every $\epsilon > 0$, there corresponds a $\delta > 0$ such that

$$|\nu_f(E)| < \epsilon \text{ whenever } \mu(E) < \delta.$$ [8]

c) Deduce from the above two results that if f is non-negative and integrable, then ν_f is a measure.
(Hint: it is enough to show that ν_f is continuous from above at $\emptyset$). [8]

d) State Radon-Nikodym theorem. What is its use in the theory of probability? [6+4]=[10]

5. Neatness and clarity. [8]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [94]

MID-YEAR EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 28.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for each question are given in brackets [ ].

1. In a manufacturing company, it is noticed that some defective items are being produced. The authorities wanted to find the number of defective items in the stock they have. Since the stock is very large they cannot afford to check up all the items. As a statistician suggest them a procedure of obtaining an estimate of the number of defective items. What are the properties of your estimates? Derive explicit expressions. Also, suggest a method of obtaining a confidence interval for the true proportion of defectives. [25]

2. It is required to estimate the rate of incidence of a particular disease in a state. Since the incidence of disease depends on the general socio-economic conditions it is decided to take a stratified random sample. Suggest a method of conducting the survey giving details about (a) how you would choose your samples from different strata if you are given a fixed amount of money C to be spent on the survey. (b) The estimate you propose for the incidence rate and an estimate of its variance. [25]

3. The following data show the stratification of all the farms in a Tahsil by farm size and the average acres of wheat per farm in each stratum.

Farm size (acres)	No. of farms N_h	Average acres of wheat $\bar{Y}_h$	Standard deviation σ'_h
0 - 40	394	5.4	8.3
41 - 80	461	16.3	13.3
81 - 120	391	24.3	15.1
121 - 160	334	34.5	19.8
161 - 200	169	42.1	24.5
201 - 240	113	50.1	26.0
241 -	148	63.8	35.2

$\Sigma N_h = N = 2010$ $\bar{Y} = 26.3$

$$\sigma_h'^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (Y_{hi} - \bar{Y}_h)^2 .$$

For a sample of 100 farms compute the sample sizes in each stratum under (i) Proportional allocation

(ii) Optimum allocation (Neyman allocation)

Compare the variance of the estimate of average acreage of wheat with proportional allocation and with SRSWR. [25]

GO ON TO THE NEXT PAGE

4. The following data were collected in an experiment on jute in a village of West Bengal in 1953, in which the weights of green plants and dry jute fibre were recorded for 20 individual plants selected at random.

Weight (in grams) of green jute plant and dry fibre for 20 plants.

Serial No. of plant	Weight in grams	
	green plant	dry fibre
1	95	6.8
2	89	6.3
3	112	7.0
4	8	0.6
5	93	6.5
6	11	0.7
7	16	0.7
8	32	2.9
9	31	2.7
10	37	3.0
11	46	3.3
12	35	2.7
13	30	2.1
14	8	0.5
15	25	1.4
16	33	2.7
17	18	1.7
18	70	5.3
19	87	6.2
20	74	11.5

a) Obtain an estimate of the regression coefficient of the weight of dry fibre on the weight of green plant.

b) Obtain a 90 per cent confidence interval for the regression coefficient.

c) If the weight of the green plant is 50 grams, obtain the 95 per cent confidence interval for the average weight of the dry fibre. [25]

5. Obtain the value of $\int_{3}^{5} \frac{dx}{2x+3}$ by Gauss quadrature formula by taking 10 points in the interval. Give the details of working. [25]

6. Write short notes on the following:-

(a) Two-stage sampling.

(b) Poker test.

(c) Fitting polynomial regression by the method of orthogonal polynomials (No derivations are necessary). [25]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [95]

MID-YEAR EXAMINATIONS

General Science-5: Statistical Mechanics

Date: 30.12.68 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. State and derive Stirling's approximation for the factorial of a large number. Suppose there are three cells in phase space 1, 2 and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$ and $w_1 = 2$ joules, $w_2 = 4$ joules, $w_3 = 6$ joules. If $\partial N_3 = -2$, find ∂N_1 and ∂N_2. [8+8]=[16]

2.a) Define the following terms: Phase space, micro and macrostates, thermodynamic probability, Heisenberg's uncertainty relation.

b) Show that the number of particles in the i^{th} cell, in the state of maximum thermodynamic probability, is, according to M-B statistics

$$N_i = \frac{N}{Z} \exp(-w_i/kT),$$

the symbols having their usual meanings. [4 X4+16]=[32]

3. What is meant by an equation of state? Derive, using M-B statistics, the equation of state for an ideal gas. Also show that the molar specific heat capacity at constant volume is 3R/2. [3+13+4]=[20]

4. Enumerate some cases where the M-B statistics have failed. Applying the Bose-Einstein statistics, derive the Planck's radiation formula. [6 +12]=[18]

5. Derive the Fermi-Dirac distribution function for the state of maximum thermodynamic probability. Compare and contrast the three statistics: M-B, B-E and F-D. [9+5]=[14]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69 [96] A

PERIODICAL EXAMINATIONS

Mathematics-3: Analysis

Date: 24.2.69 Maximum Marks: 100 Time: 3 hours

Note: i) Answer all questions. ii) Each question carries 10 marks. iii) Home assignment carries 40 marks.

1. State two definitions of continuity of a function and show that they are equal.

2. State and prove the chain rule for differentiable functions.

3. If f_n is a sequence of functions on $[a,b]$ with continuous derivatives f'_n and if $f_n \to f$ and $f'_n \to \varphi$ uniformly then show that f is differentiable and $f' = \varphi$.

4. If f is differentiable function on $[a,b]$ with derivative f', show that the range of f' is again an interval.

5. Show that any continuous function defined on a closed bounded interval is uniformly continuous.

6. State the following theorems carefully (no proofs are needed)

   i) Mean value theorem

   ii) L' Hospital's rule

   iii) Taylor's formula.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69
Mathematics-3 Analysis
Maximum Marks: 40

968

Date: 24.2.69.

Home assignment

All the functions appearing in the questions below are defined on the whole real line, unless the contrary is specified.

1. A function f is said to be right continuous at a point x_o iff whenever x_n decreases to x_o, $f(x_n)$ converges to $f(x_o)$. A function f is said to be left continuous at a point x_o iff whenever x_n increases to x_o, $f(x_n)$ converges to $f(x_o)$. Answer the questions:

a) Give an f which is right continuous at o but not left continuous.

b) Give an f which is left continuous at o but not right continuous.

c) Show that f is continuous at x_o iff it is both right and left continuous at x_o.

d) If f is right continuous at x_o, show that the function g defined as

$$g(x) = f(-x)$$

is left continuous at x_o.

2. A function f is said to have right derivative at a point x_o if there is a real number α such that whenever h_n decreases to zero and $h_n \neq 0$ for all n,

$$\frac{f(x_o + h_n) - f(x_o)}{h_n} \to \alpha$$

This α is called the right derivative of f at x_o. Similarly f is said to have left derivative at a point x_o if there is a real number β such that whenever h_n increases to zero, $h_n \neq 0$ for all n,

$$\frac{f(x_o + h_n) - f(x_o)}{h_n} \to \beta .$$

This β is called the left derivative of f at x_o.
Answer the questions:

a. Give an f which has right derivative at 0 but not left derivative.

b. Give an f which has left derivative at 0 but not right derivative.

c. Give an f which has both left and right derivatives at 0 but they are not equal.

d. Show that f is differentiable at a point x_o iff both the left and right derivatives at x_o exist and equal.

GO ON TO THE NEXT PAGE

A function f is said to be additive if for any two real numbers x, y

$$f(x+y) = f(x) + f(y).$$

Show that if f is a continuous additive function on the real line then there is a real number α such that

$$f(x) = \alpha x \quad \text{for all } x.$$

Home assignment should be submitted in the Dean's office on or before 28 February 1969. Along with this you must submit previous assignments, if any are due from you. Otherwise you will get zero marks in this.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

PERIODICAL EXAMINATIONS

Economics-3

Date: 3.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A, Group B and Group C in separate answerscripts. Marks allotted for each question are given in brackets [ ].

GROUP A: Economic Development

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1. Present Rostow's theory of stages of growth. What are the main lines of criticism of his theory? [25]

2.a) Briefly describe Boeke's theory of sociological dualism.

b) What do you understand by n-Achievement? Discuss briefly McClelland's theory of growth. [25]

3. Write notes on the structural features of low income countries in respect of any two of the following:

   i) health services;
   ii) education;
   iii) scientific research; and
   iv) technical man-power. [25]

GROUP B: Socialist Planning

Maximum Marks: 25 Suggested time: 3/4 hour

1. The measures adopted to guide the economy immediately after the Russian Revolution of 1917 Nov., can not be called socialistic - Would you agree with this view ? Describe the measures and give reasons for your answer. [25]

GROUP C: Economic Theory

Maximum Marks: 25 Suggested time: 45 minutes

Answer all questions.

1. Bring out (in brief) all the assumptions you require to justify the investment function in Harrod's model of economic growth. [13]

2. If we define the equilibrium of the economy in the sense of equality between desired demand and desired supply in all markets, then what is the condition of short-run equilibrium in Harrod's model? [12]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III:1968-69
PERIODICAL EXAMINATIONS
Statistics-3: Probability

Date: 10.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1.a) Define the joint distribution of two random variables X and Y defined on a probability space $(\Omega, \mathcal{F}, P)$. [4]

b) If X and Y have an absolutely continuous (joint) distribution, show that each marginal distribution is absolutely continuous. Give an example to show that the converse is false. [6+4]=[10]

c) If X and Y are independent and have an a.c. distribution, show that $f(x,y) = f_1(x).f_2(y)$ where f_1 and f_2 are the marginal densities and $f(x,y)$ is the joint density. [6]

d) If $f(x,y) = g(x)\, h(y)$, show that X and Y are independent and X has density c.g (x) where c is a constant. [6]

e) X and Y have the joint density function 2 over the region in the plane bounded by the lines x= 0, y= 0 and x+y= 1, and zero outside. Using (d), can you conclude that X and Y are independent? Give reasons. [6]

2.a) Define the convolution F*G of two distributions F and G. [4]

b) Show that if F is absolutely continuous, then F*G is also absolutely continuous. [8]

c) Find the density of the convolution of the rectangular distribution on [0, 1] with itself. [8]

3.a) Let $(\Omega, \mathcal{F}, P)$ be a probability space and let X be a random variable. Define the conditional probability of an event given X= x using Radon-Nikodym theorem (give the necessary justification). [8]

b) If X and Y have a joint a.c. distribution, show that $\frac{f(x,y)}{f_2(y)}$ serves as a density for the conditional distribution of X given Y = y. [8]

4. Prove that if X has density function f(x) and Y= g(X) is a function of X such that g is differentiable and strictly monotone, then the density of Y at a point y is

$$f(h(y)). \left|\frac{dh(y)}{dy}\right|$$

where h(y) is the inverse function of g(x). [8]

b) State the analogue of the above result for a k-dimensional random variable. [4]

5.a) Define the Gamma distribution. Show that if X and Y are independent and have Gamma distributions $G(\lambda; \alpha_1)$ and $G(\lambda, \alpha_2)$, then $Z = \frac{X}{X+Y}$ has a Beta distribution, U= X+Y has a Gamma distribution and Z and U are independent. [4+12]=[16]

GO ON TO THE NEXT PAGE .

5.b) Deduce from 5(a) or prove otherwise that

$$\int_0^1 x^{\alpha_1-1}(1-x)^{\alpha_2-1}\,dx = \frac{\Gamma(\alpha_1)\,\Gamma(\alpha_2)}{\Gamma(\alpha_1+\alpha_2)} .$$ [4]

c) State the relations you know between the following pairs of distributions. [7X3]=[21]

(1) Beta distributions of the two kinds

(2) Normal and Cauchy

(3) Rectangular and Gamma

(4) Chi-square and F

(5) Student's T and F

(6) Beta and F

(7) Normal and chi-square

d) Derive the density of Student's t-distribution (taking the definition as the distribution of the ratio of two random variables) with n degrees of freedom. You may assume the density of the chi-square distribution. Show that student's t distribution with one degree of freedom is Cauchy's distribution. Is this (last) result same as the result of Qn. 5(c)(2)? Why? [8+3+5]=[16]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [94]

PERIODICAL EXAMINATIONS

Statistics-3: Statistics Theory and Practical

Date: 17.3.69 Maximum Marks: 100 Time: 3 hours

Note: The whole paper carries 125 marks. Answer as much as you can. Marks allotted for each question are given in brackets [ ].

1. Explain two stage sampling. Obtain an estimate of the population total for a two stage sampling procedure with SRSWR employed at the first stage and SRSWOR employed at the second stage. Also obtain an estimate of the variance of the estimate you propose. [20]

2. What is ratio method of estimation and when is it used? A ratio estimate is said to be a biased estimate in general. Is this statement true? Give a proof. Also obtain an expression for the mean square error of a ratio estimate. State clearly the assumptions you make. [20]

3. A sample of 15 villages was selected from a population of 170 villages with SRSWR for estimating the area under wheat in the region in 1964. Estimate the area under wheat by the method of ratio estimation and estimate its relative standard error. [35]

Village No.	Total area under cultivation	Area under wheat
1	564	515
2	238	209
3	92	85
4	247	221
5	134	133
6	131	104
7	129	103
8	190	175
9	363	335
10	235	219
11	73	62
12	62	59
13	71	60
14	137	100
15	196	141

4. A sample of size 5 is drawn from a normal population with mean 2 and unknown variance. Derive a test criterion for testing the hypothesis that the variance is 4 against the alternatives (1) the variance is 5; (2) The variance is 3; (3) The variance is either 3 or 5.

Compute the powers of the tests you derive in each case against the specified alternatives. [35]

5. Write short notes on:

1) Regression method of estimation. [8]

2) Simple and composite hypotheses. [3]

3) Level of significance of a test. [4]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

[100]

PERIODICAL EXAMINATIONS

General Science-5: Psychology Theory and Practical

Date: 24.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. The scores of 5 students on each of the 6 questions in a test are as follows: [30]

Students	Questions 1	2	3	4	5	6
1	6	1	5	0	8	4
2	5	2	6	2	6	3
3	7	2	8	4	3	1
4	3	4	3	2	6	1
5	7	3	9	7	7	5

Obtain (a) question-difficulty-levels

(b) indices of discrimination for each question

and (c) reliability of the test.

2. What is psychological measurement? Explain how you would assess the knowledge of an individual in a subject, say, psychology. [25]

3. Write short notes on [20]

(a) Intelligence

(b) Factor analysis.

4. Practical Record work. [25]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1968-69

[101]

PERIODICAL EXAMINATIONS

General Science-5: Engineering

Date: 31.3.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Write short notes on the following:

   a) Bending moment diagram

   b) Shear Force diagram

   c) Moment of Resistance

   d) Free body diagrams. [20]

2. A horizontal beam of 20' span, simply supported at its ends, carries a load which varies uniformly from 1/2 ton/ft at one end and 2 ton/ft at the other. Draw bending moment and shear force diagrams. Find maximum bending moment and bending moment at the mid span. [30]

3. a) Derive the formula $\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$.

   b) Write a short note on shapes adopted for mild steel beams used in structure. [20]

4. A teakwood beam 8" $\times$ 4" wide in cross section, simply supported at its ends carries a concentrated load of 4000 lbs at 3.33 ft from one support. The effective span is 10 ft. Find the maximum flexural stresses in the beam (a) in the said case and also (b) if the concentrated load of 4000 lb is replaced by a uniformly distributed load at 400 lb /sft. throughout the beam. [30]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [102]

ANNUAL EXAMINATIONS

Mathematics-3: Analysis

Date: 19.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50
Answer any three questions.

1.a) Explain with examples the two concepts 'first kind of discontinuity' and 'second kind of discontinuity'.

b) Show that a monotone function can not have second kind of discontinuities. $[8+8\frac{2}{3}]=[16\frac{2}{3}]$

2.a) State and prove the generalized mean value theorem for differentiable functions.

b) Is there any differentiable function on the real line whose derivative is given by the following:

$$f(x) = 0 \quad \text{if } x \text{ is integer}$$
$$= 1 \quad \text{otherwise.}$$

Give reasons. $[12+4\frac{2}{3}]=[16\frac{2}{3}]$

3.a) Explain the meaning of $\int_0^1 f d\alpha$ where f is a bounded function and α is a monotonically increasing function, both on [0, 1].

b) Suppose α is an increasing function on [0,1] which is continuous at the point $x = \frac{3}{4}$. Define

$$f(x) = 1 \quad \text{if } x = \frac{3}{4}$$
$$= 0 \quad \text{if not.}$$

Then show that f is integrable w.r.t. α and the value of the integral is zero. $[7+9\frac{2}{3}]=[16\frac{2}{3}]$

4.a) Explain the concepts 'orthonormal set' and 'complete orthonormal set' for a collection of functions defined on $[-\pi, +\pi]$.

b) Show that the functions

$$\frac{1}{\sqrt{2\pi}}, \frac{\cos x}{\sqrt{\pi}}, \frac{\sin x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \ldots$$

form an orthonormal set on $[-\pi, \pi]$. You have to evaluate all the integrals involved here. $[10+6\frac{2}{3}]=[16\frac{2}{3}]$

GO ON TO THE NEXT PAGE

Group B

Maximum Marks: 50

Answer any three questions.

1.a) If f is a continuous function on the real line such that $f(x+y) = f(x) + f(y)$ for all x and y show that there is a real number α such that

$$f(x) = \alpha x \qquad \text{for all } x.$$

b) If f is a continuous function on the real line such that $f(x+y) = f(x).f(y)$ for all x and y show that either there is a real number α such that

$$f(x) = e^{\alpha x} \qquad \text{for all } x$$

or $f \equiv 0$. [$8+8\frac{2}{3}$]=[$16\frac{2}{3}$]

2. State and prove carefully the chain rule for differentiable functions. [$16\frac{2}{3}$]

3.a) If a sequence of integrable (Riemann) functions f_n on [0,1] converge to a function f on [0,1]. Can you say that f is also Riemann integrable? Justify your answer.

b) Show that if the convergence in the above question is uniform then f is integrable. [$8+8\frac{2}{3}$]=[$16\frac{2}{3}$]

4.a) State and prove Abel's theorem for power series.

b) If Σa_n, Σb_n, are two series of real numbers with Cauchy product Σc_n and if we know that these three series converge to A, B, C respectively then show that AB = C. [$10+6\frac{2}{3}$]=[$16\frac{2}{3}$]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III : 1968-69

ANNUAL EXAMINATIONS

Economics-3

[103]

Date: 20.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Economic Development

Maximum Marks: 50

Answer any three questions.

1. Give your understanding of the theory of unbalanced growth and explain briefly the use of linkage coefficients in this context. [16]

2. Describe Arthur Lewis' theory of economic development with unlimited supply of labour. [16]

3. Make very brief comments on any three of the following:
   i) 'big push',
   ii) labour and capital as factors explaining long period growth,
   iii) dualism,
   iv) vicious circles, and
   v) stages of growth. [16]

4. Describe the classical view of growth following Baumol's synthesis. Indicate some significant difference of Marx from this, [16]

5. You have studied how structural features changed in Western countries with economic development. Do you think that the poorer countries will exhibit exactly similar changes with development?

   In your view, what are the main factors inhibiting Indian development? Could you suggest some measures that can promote rapid growth in this country?

   Would you be satisfied with a long period income maximization goal? [16]

Neatness. [2]

GO ON TO THE NEXT PAGE

Group B: Socialist Planning

Maximum Marks: 50

Answer any two questions.

1. Discuss the logic and the consequence of soviet price policy as it existed in the fifties. [25]

2. Give a critical review of the agricultural policy pursued during the period biginning with the November Revolution upto the conclusion of the first five year plan. [25]

3. Write short notes on the following:

   a) The scissors crisis of the 1920's.

   b) Allocation of investment in the first five year plan.

   c) Marxian concept of profit and the new role assigned to it now in the Soviet Union. [25]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.Part III: 1968-69

[104]

ANNUAL EXAMINATIONS

Economics-3

Date: 21.5.69 Maximum Marks: 100 Time: 3 hours.

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Answer five questions taking at least two from each group.

Group A: Indian Economics

1. Write short notes on

a) Balance of Trade and Balance of payment.

b) Bill of Exchange.

c) Terms of Trade.

d) Rate of Exchange. [20]

2. The growing adverse balance of payment position of India has not only been due to the emphasis in her Five Year Plans on heavy industries. Comment. [20]

3. What do you mean by the term Public Finance? What role can the fiscal policy play in the achievement of rapid economic growth in a developing economy? [20]

4. Critically review the fiscal policies of the Indian Government, in the context of the triple objectives of the Five-Year Plans. [20]

5. The increasing conflict between the Union Government and the State Governments is essentially a reflection of the crisis in mobilising financial resources. Discuss. [20]

Group B: Economic Theory

1. Describe the two-sector planning model of Prof. Mahalanobis. In an economy visualised in this model, what the planner will do if he is interested in maximising national income at the end of a given planning horizon? [20]

2. Show that the warranted rate of growth in Harrod's model of economic growth is unstable. What assumptions do you require for this result? [20]

3. Show that the income path in Harrod's model corresponding to the warranted rate of growth is unique. Also, show that the long-run equilibrium in a Harrodian economy is, in general, impossible. [20]

4. Describe the way by which a typical neoclassicist (e.g., Solow) can solve the long-run economic problem of Harrod's model. In this connection also give the conditions of existence of steady state in a neoclassical growth model and show that this steady-state is stable. [20]

5. i) Prove that in a neoclassical growth model the steady-state capital-labour ratio and not the steady-state capital output ratio is dependent on the form of the production function.

ii) Verify, with the Cobb-Douglas production function, the neoclassical conclusions about the long-run growth of an economy. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [105]

ANNUAL EXAMINATIONS

Statistics-3: Probability

Date: 23.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) Define a measurable transformation. [4]

b) If $T: (\Omega, \mathcal{F}, \mu) \to (R, \mathcal{B})$ is measurable, then what is the measure induced by T on $(R, \mathcal{B})$? Prove that it is a measure. [4+5]=[9]

c) If g is a real valued measurable function defined on R, then

$$\int g\,d(\mu T^{-1}) = \int (gT)\,d\mu$$

provided one of them exists. Assuming this show that the expectation of a random variable can be defined in two ways. [6]

2. A p-dimensional random variable U is said to have a normal distribution if $T'U$ is normally distributed for all vectors T. Show the following.

(a) The characteristic function of U is $\exp\left\{ i\, T'\mu - \frac{1}{2} T'\Sigma T \right\}$ where μ, Σ are the mean and the dispersion matrix of U; the distribution of U is uniquely determined by μ and Σ. [6+4]=[10]

(b) any marginal distribution of U is normal. [4]

(c) the conditional distribution of $(U_1, \ldots, U_q)$ given $(U_{q+1}, \ldots, U_p)$ is normal (assuming the dispersion matrix of the second variable is non-singular). [7]

(d) Assuming that the distribution of U is same as that of $\mu + BG$ where B is a $p \times m$ matrix of rank $m = \text{rank}\ \Sigma$ and G is a vector of independent N(0,1) variates, obtain the density of U when Σ is non-singular. [6]

3.a) State and prove Fisher-Cochran theorem on the distribution of quadratic forms of a sample $X_1, \ldots, X_n$ from N(0,1). [10]

b) Deduce that $\bar{X}^2$ and $\Sigma (X_i - \bar{X})^2$ are independent. What are their distributions? [8]

GO ON TO THE NEXT PAGE

Group B

Answer as many questions or parts of questions as you can. The maximum number of marks you can get from this group is 50.

1.a) Prove that $X'AX \frown \chi^2$ if and only if $A^2 = A$, where $(X_1, \ldots, X_n)' = X$ is a sample from $N(0,1)$. Then, show that the d.f. of χ^2 is $r(A)$. [10]

b) Deduce that if $X \frown N_p(\mu, \Sigma)$ and Σ is non singular then

$$(X - \mu)' \Sigma^{-1}(X - \mu) \frown \chi^2_p$$ [4]

2.a) If $(x - \partial, x + \partial)$ is a continuity interval of a distribution F with characteristic function φ, show that

$$F(x+\partial) - F(x-\partial) = \lim_{T \to \infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sin \partial t}{t} e^{-itx} \varphi(t) dt.$$ [10]

b) Deduce that if $\int_{-\infty}^{\infty} |\varphi(t)| dt < \infty$, then F has a continuous density. [6]

c) State the generalisation of the above inversion theorem for a 2-dimensional random variable (X_1, X_2). If $\varphi(t_1, t_2)$ is the characteristic function of (X_1, X_2) show that $\varphi(t_1, t_2) = \varphi_1(t_1) \cdot \varphi_2(t_2)$ for some functions φ_1, φ_2 implies that X_1, X_2 are independent (using the result you state above). [4+6]=[10]

3.a) State and prove Helly-Bray theorem. [8]

b) Assuming the inversion theorem (Qn. 2(a)) and Helly's lemma, prove that if $f_n(t)$ is the characteristic function corresponding to the distribution F_n and if $f_n(t)$ converges to a function $f(t)$ which is continuous at zero, then F_n converges weakly to a distribution F whose characteristic function is f (Prove any other result you use). [12]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [106]

ANNUAL EXAMINATIONS

General Science-5: Engineering

Date: 24.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 50

Answer all questions.

1. EITHER

Write short notes on the following:-

a) Slenderness-ratio
b) Moment of resistance
c) Centre of pressure
d) Venturi-meter. [20]

OR

State and prove Bernoulli's theorem. [20]

2. A vertical gate 10' X 20' high fixed in a vertical dam has its top 10 feet below the water level. Calculate the centre of pressure and total water-pressure. Derive the formula you use. [30]

Group B

Maximum Marks: 50

Answer all questions.

1. A horizontal beam 10' span, simply supported at its ends, carries a load, which varies uniformly from 1 ton per foot at one end to 3 ton per foot at the other end. Draw bending moment and shear force diagrams. [20]

2. EITHER

A 100 inches long hollow circular shaft with external diameter of 10 inches and internal diameter of 5 inches is subjected to torsion of 1,00,000 in lbs. If the modulus of rigidity is

$$5 \times 10^6 \text{ lb./in}^2,$$

calculate the maximum shear stress and angle of twist. Derive the formula you use. [30]

OR

(a) Write short notes on Professor Rankine's formula for struts.

(b) A hollow C.I. column with fixed ends, supports an axial load of 100 tons. If the column is 15' long and has an external diameter of 10 inches, find the thickness of metal required. Use the Rankine's formula, taking a constant of 1/6400 and a working stress of 5 tons per sq. inch. [30]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [107]

ANNUAL EXAMINATIONS

Statistics-3: Statistics Theory

Date: 26.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Answer any five questions.

Group A

1.a) State and prove Neyman-Pearson lemma and show how it can be used to obtain a most powerful test for testing a simple hypothesis against a simple alternative. [10]

b) On the basis of a random sample of size n from a gamma distribution with frequency function

$$f(x, \theta, p_o) = \frac{\theta^{p_o}}{\Gamma(p_o)} \cdot e^{-\theta x} \cdot x^{p_o - 1}$$

where $p_o > 0$ is fixed, $\theta > 0$ and $0 < x < \infty$, obtain a UMP level α test for testing $\theta = \theta_o$ against the alternative $\theta > \theta_o$. Is this test also most powerful for alternatives $\theta < \theta_o$? [10]

2.a) On the basis of n independent tosses of a coin obtain an exact level α most powerful test for testing the hypothesis that the probability of a toss showing head is p_o against the alternative that it is p_1 ($< p_o$). [10]

b) The number of deaths from road accidents follows a Poisson distribution. $X_1, \ldots, X_n$ are the number of deaths due to road accidents on n days chosen at random. How do you test the hypothesis that the accident rate per day say $\lambda = \lambda_o$ against the alternative that $\lambda \neq \lambda_o$ at level of significance α? [10]

3. Let $\underset{\sim}{Y}$ $(n \times 1)$ denote a vector valued random variable which is normally distributed with

$$E_{n \times 1}(\underset{\sim}{Y}) = \underset{\sim}{X}\,(n \times p)\,\underset{\sim}{\beta}\,(p \times 1) \text{ and}$$

$$V(\underset{\sim}{Y}) = \sigma^2 . I_n \qquad \text{where } \underset{\sim}{X}\,(n \times p)$$

is a matrix of known constants and $\underset{\sim}{\beta}(p \times 1)$ and σ^2 are unknown parameters.

(a) Explain the principle of least squares estimation and obtain the estimates of $\underset{\sim}{\beta}$ by that procedure. Show that they minimize the quadratic form

$$(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\beta})'(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\beta}) \qquad [10]$$

(b) Denoting a set of least squares estimates by $\hat{\underset{\sim}{\beta}}$, obtain the distribution of

$$R_o^2 = (\underset{\sim}{Y} - \underset{\sim}{X}\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - \underset{\sim}{X}\hat{\underset{\sim}{\beta}}).$$

Hence obtain an unbiased estimate of σ^2. [10]

Group B

4.a) In the linear model set up of question (3), obtain a necessary and sufficient condition that a linear function of the parameters $\underset{\sim}{\beta}$, say $\underset{\sim}{P}'(1 \times p)\, \underset{\sim}{\beta}\,(p \times 1)$ to be estimable. [10]

b) Obtain a best linear unbiased estimate of an estimable function $\underset{\sim}{P}'(1 \times p)\, \underset{\sim}{\beta}\,(p \times 1)$. [10]

5. What are contingency tables? When you have a sufficiently large sample:

(a) How do you test for given marginal probabilities in a contingency table?

(b) How do you test whether the two attributes are independent with given marginal probabilities.?

(c) How do you test for independence of the two attributes?

(Note: For (a)-(c) the corresponding distributions need not be derived).

(d) In a 2×2 contingency table how do you test for the independence of the two attributes when the number of observations is not large. [4 × 5]=[20]

6.a) Write short notes on:

(a) Unbiased test

(b) Variance stabilizing transformation for a random variable following a binomial distribution.

(c) Test of equality of means from two normal distributions with same variance.

(d) Analysis of variance table for two way classification.

[4 × 5]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69 [108]

ANNUAL EXAMINATIONS

Statistics-3: Statistics Practical

Date: 27.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions from questions 1 to 5. Marks allotted for each question are given in brackets [ ].

1. It is noticed that the consumer expenditure per person per month on food grains in rural India follows a normal distribution with unknown mean μ and known standard deviation 0.5 Rs. Based on a random sample of 16 individuals it is required to test that $\mu_0 = 8.00$ (rupees)

(a) Obtain the most powerful critical region for testing this hypothesis against the alternative $\mu > \mu_0$.

(b) Obtain the most powerful unbiased critical region for testing this hypothesis against the alternative $\mu \neq \mu_0$. By taking a reasonable number of alternatives draw the power curve of the test in each case. [20]

2. A drug was given to 20 subjects half an hour before bed time while 25 other subjects were kept as controls. The next morning subjects estimated the time taken by them to fall asleep. The following table gives the reported times of the two groups.

(a) Ascertain from the data whether the drug caused a quicker onset of sleep. (Assuming that the variability is the same in both groups of individuals).

Time in minutes:

Controls : 15, 25, 30, 15, 35, 40, 25, 30, 25, 35, 40, 25, 35, 20, 25, 40, 15, 15, 25, 30, 25, 10, 50, 30, 40.

Treated with drug : 25, 30, 40, 45, 15, 15, 20, 25, 30, 25, 20, 15, 10, 25, 15, 25, 35, 10, 10, 15. [12]

(b) Results based on observations on 27 randomly chosen individuals show that the average age of onset of a particular characteristic A is 48.89 years. The standard error is given to be 10.32 years. Another set of 36 randomly chosen individuals show that the average age of onset of the characteristic B is 50 years and the standard error is 8.56 years. Can you conclude that the variance in the age of onset of the characteristic is the same for both the characteristics? [8]

3.a) The following table summarizes the means and corrected sums of squares and products of the weights of green plant and dry jute fibre collected for 40 individual plants selected at random.

Mean weight of green plant $\bar{X} = 52.775$ gms.
Mean weight of dry fibre $\bar{Y} = 3.992$ gms.

$$S_{XX} = 33052.97$$
$$S_{YY} = 209.15$$
$$S_{XY} = 2338.70.$$

Test whether the correlation between the weight of the green plant and the weight of the dry fibre is significantly different from 0.85. [8]

GO ON TO THE NEXT PAGE

3.b) The following table gives the means and corrected sums of squares and products of systolic blood pressure (in mm of Hg.) (Y) and the age in years (X) of three groups of subjects.

Group	Sample size	Means $\bar{X}$	Means $\bar{Y}$	Corrected sums of squares and products S_{XX}	S_{XY}	S_{YY}
1	80	26.6	90.4	137.875	74.125	127.875
2.	120	28.0	92.2	390.000	124.000	515.670
3	70	28.8	90.6	94.850	54.290	390.860

Test whether the correlation between the blood pressure and age is the same in the three groups. [12]

4. The joint segregation of the two factors, flower colour and pollen shape in Morning glory has been studied by Imar. He records the segregations as follows.

Pollen shape	Flower color: Purple	Red	Total
Long	296	27	333
Round	19	85	104
Total	315	112	427

The marginal frequencies are expected to be in the ratio 3:1 and if the two characters, flower colour and pollen shape are independently inherited then the cell frequencies are expected to be in the ratio 9:3:3:1. With this knowledge, analyze the data carefully. [20]

5. Three groups of rats were given the same dose of hypnotic drug. The number of minutes each rat was unconscious was recorded below. Ascertain by the analysis of variance whether the average lengths of time the rats were unconscious are significantly different for the three groups.

Group I	13	16	19	14	16
Group II	20	17	22	24	19
Group III	14	18	13	12	20

[20]

6. Viva Voce [10]

7. Records [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

[109]

ANNUAL EXAMINATIONS

General Science-4: Biology

Date: 29.5.69 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets.

Group A

Maximum Marks: 40

Answer any two questions.

1. Explain the handicaps for the improvement of coconut by breeding methods. Mention the possibilities of effecting clonal propagation in coconut. [8+12]=[20]

2. Write an account on the foliar spirals in palms with special reference to cocos nucifera. Which palms are tapped for the sugary juice? [15+5]=[20]

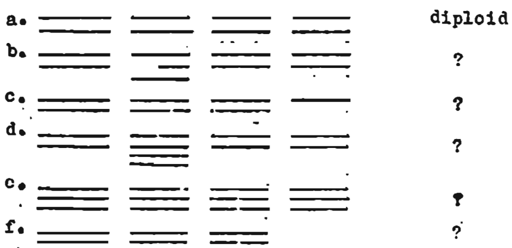
3. Mention names of eight tuber crop plants. Give a morphological account of the plant Solanum tuberosum. [8+12]=[20]

Group B

Maximum Marks: 60

Answer question 1 and any two from the rest.

1. Diagrams representing one normal diploid (a) and five individuals with various aneuploid or polyploid chromosome components (b to f) are given below. Give specific names for each condition.



Explain with suitable illustrations the terms deficiency, deletion, duplication, inversion and translocation. [5+15]=[20]

2. If you are a plant breeder, which breeding methods you would use for producing improved varieties in self-fertilizing plant species? Mention under what circumstances you would use the back cross method in plant breeding. [15+5]=[20]

3. What are inbreeding and heterosis? What is an inbreeding minimum and how it is achieved? Why does homozygosity increase by inbreeding? [5+10+5]=[20]

4. What is polyploidy? How are polyploids induced artificially? Discuss briefly the role of polyploidy in plant breeding. [5+5+10]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1968-69

[110]

ANNUAL EXAMINATIONS
General Science-3: Psychology Theory and Practical

Date: 31.5.69 Maximum Marks: 100 . Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer all questions.

Group A

Maximum Marks: 50

1. Explain what is meant by 'phylogenetic differentiation of ability', 'ontogenic differentiation of ability', and 'ontogeny recapitulates phylogeny'. Discuss the important features in the differentiation of ability. [20]

2. Write short notes on the physical stimulus, physiological receptor, and psychological response for

   a) vision

   b) audition [20]

3. Practical Record Work [10]

Group B

Maximum Marks: 50

1. Specify the three factors which influence whether or not a neuron will respond to a stimulus, and indicate the type of response which takes place. Express statistically how a continuous nerve response occurs if the nerve is composed of neurons differing in terms of these three factors. Consider different levels of stimulus intensity. [20]

2. Discuss human speech in terms of articulation, audition, and localization of cerebral functions. [20]

3. Practical Record Work. [10]
