INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1969-70 [301]

PERIODICAL EXAMINATION

Mathematics-3: Analysis

Date: 15.9.69 Maximum Marks: 100 Time: 3 hours

Instructions: 1. Answer all questions.
2. Be clear and precise.

Marks alloted for each question are given in brackets [ ].

1. Define a field.

Let $Q = [(\alpha, \beta) : \alpha, \beta \text{ rational}]$

Define $(\alpha, \beta) + (\xi, \eta) = (\alpha + \xi, \beta + \eta)$

$(\alpha, \beta) \cdot (\xi, \eta) = (\alpha\xi - 2\beta\eta ; \alpha\eta + \beta\xi)$

Show that Q is a field with these operations. [5+10]=[15]

2. Let F be an ordered field. Define the concepts 'interval' and 'bounded set'. Explain carefully the meaning of the two statements: 'F satisfies the supremum principle' and 'F is complete'.

Show in detail that F satisfies the supremum principle iff it is Archimedean and complete. [5+5+25+25]=[60]

3. Let R be a complete Archimedean Ordered field. Given $x > 0$; $n > 0$ a natural number; both in R then show that that there is a unique y in R such that [25]

$$y > 0 \text{ and } y^n = x$$

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PERIODICAL EXAMINATION

Economics-3

Date: 29.9.69 Maximum Marks: 100 Time: 3 hours

Note:- Answer Groups A and B in separate answerscripts.

Group A: Economic Development

Maximum Marks: 70 Suggested time: 2 hours

Answer any four questions.
All questions carry equal marks.

1. Mention some important characteristics of poor nations, and describe how some of these are interrelated.

   Explain the idea of 'Vicious circles' in this context.

2. Sketch briefly the available statistical information on the international distribution of national income.

   Describe fully a rigorous procedure for comparing the purchasing powers of currencies of two countries.

3. How can one study the contribution of factor inputs to the growth of national product?

   Describe Abramovitz's analysis of the long period growth of the U.S.economy.

4. What are the principal structural changes in the industrial distribution of labour force and national product accompanying growth of per capita income? Can cross section data be used in this context?

   How can one measure the intersectoral disparity of average income per worker? Does a change in intersector disparity throw any light on the change in size distribution?

5. Write brief notes on any three of the following:

   i) Hagen's adjustment of international income distribution.

   ii) The share of income from assets in national income is independent of the level of per capita income.

   iii) Morgenstern's observation on the error of the rates of growth.

   iv) The share of food in consumption expenditure vs. the share of agriculture in national income.

   v) Long period rates of growth of advanced countries.

   vi) The difference between GNP and national income.

Group B: Economic Theory - Growth Models

Maximum Marks: 30 Suggested time: 1 hour.

All questions carry equal marks. Answer all questions.

1. EITHER

   Show that the capital-output ratio in Harrod's growth model is a constant not because he has assumed it to be so but because it follows from his other assumptions.

   OR

   Define the warranted rate of growth in Harrod's model. Show that there exists a warranted rate of growth of the Harrodian economy.

2. Prove that the warranted rate of growth is unstable.

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PERIODICAL EXAMINATION

General Science-4: Biochemistry Theory

Date: 6.10.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Give an example for each of the following:

   Polypeptide, Dehydrogenase, Disaccharide, Pentose, Ketoacid. [10]

2. What are the different methods of quantitative estimation of amino acids. [10]

3. Describe the anaerobic glycolysis in yeast. [20]

4. Describe the classification of proteins. [20]

5. Give an outline of preparation of enzyme from natural source. How can you estimate the activity of an enzyme preparation? Give example. [20]

6. Describe the quantitative estimation of protein. [20]

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PERIODICAL EXAMINATION

Statistical Mechanics

Date: 13.10.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Define the terms:
Phase space, microstate, macrostate, thermodynamic probability. [4 X5]=[20]

b) Prove that: $\ln (x!) = x \ln x - x$

$$\left(\frac{\partial U}{\partial S}\right)_V = T$$

$$\left(\frac{\partial F}{\partial V}\right)_T = p$$

where p = pressure, T = abs.temp, S = entropy,
F = free energy, U = internal energy and V = volume. [6+3+3]=[12]

2.a) Assuming the number of points in the i^{th} cell N_i as given by

$$N_i = \frac{N}{Z} \exp(- w_i /kT)$$

with usual meanings of the symbols, show that

$$U = NkT^2 \frac{d(\ln Z)}{dT}$$

where U is the internal energy of the system. [7]

b) Apply the Boltzmann statistics to derive the velocity distribution law for a monoatomic ideal gas. [15]

3.a) Explain: Pauli's exclusion principle, Heisenberg's uncertainty principle. [2 X4]=[8]

b) Derive Bose-Einstein's or Fermi-Dirac's distribution function. [14]

4.a) What is black-body radiation? Apply Bose's statistics to derive Planck's radiation formula. [3+15]=[18]

b) Compare and contrast the three statistics:- M-B, B-E and F-D. [6]

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PERIODICAL EXAMINATION

Statistics-3: Probability

Date: 3.11.69 Maximum Marks: 100 Time: 3 hours

Note: The whole paper carries 105 marks. Answer as many questions or parts thereof. The maximum number of marks you can get is 85. 15 marks are allotted for assignments.
Marks allotted for each question are given in brackets [ ].

1. If $\mathcal{E}$ is a countable collection of subsets of a set Ω, show that the smallest algebra containing $\mathcal{E}$ is also countable. [15]

2. For any set $E \subset \Omega$, I_E denotes the characteristic function or indicator function of E. Prove the following:

i) $I_E \leq I_F$ iff $E \subset F$

ii) $I_{E \cup F} = I_E + I_F - I_{E \cap F}$
$= I_E \vee I_F$

(Note: If f and g are two real valued functions defined on a set Ω, $f \vee g$ denotes maximum of f and g, and $f \wedge g$ denotes minimum of f and g).

iii) $I_{E \cap F} = I_E \cdot I_F = I_E \wedge I_F$

iv) $I_{\Omega - E} = 1 - I_E$

v) $I_{E-F} = I_E(1 - I_F)$

vii) $I_{E \Delta F} = |I_E - I_F|$

viii) Let $\{E_n;\ n \geq 1\}$ be a sequence of subsets of Ω. Then

a) $I_{\lim\inf E_n} = \lim\inf I_{E_n}$

b) $I_{\lim\sup E_n} = \lim\sup I_{E_n}$.

[1+2+2+1+1+2+(3+3)]=[15]

3. A collection $\mathcal{L}$ of subsets of Ω is said to be a lattice if

i) $\emptyset \in \mathcal{L}$

ii) $E \in \mathcal{L},\ F \in \mathcal{L} \Rightarrow E \cup F \in \mathcal{L}$

iii) $E \in \mathcal{L},\ F \in \mathcal{L} \Rightarrow E \cap F \in \mathcal{L}$

a) Show that every ring is a lattice.

b) Give an example of a lattice which is not a ring.

[2+8]=[10]

In what follows $(\Omega, \mathcal{A}, P)$ stands for a probability space. [i.e., $\mathcal{A}$ is a σ-algebra of subsets of Ω, and P is a probability measure on $\mathcal{A}$].

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4. Show that if $E \in \mathcal{Q}$, $F \in \mathcal{Q}$ and

$$P(E \Delta F) = 0$$

then $$P(E) = P(F)$$ [4+6]=[1

Is the converse true?

5. Let $\{E_n; n \geq 1\}$ be a sequence of sets in $\mathcal{Q}$. Show that

$$P(\lim\inf E_n) \leq \lim\inf P(E_n) \leq \lim\sup P(E_n)$$
$$\leq P(\lim\sup E_n).$$ [15

6. Let R denote the real line, $\mathcal{B}$ Borel σ-algebra on R, and λ Lebesgue measure on $\mathcal{B}$.

If B is any Borel set, show that

$$\lambda(B + x) = \lambda(B)$$

for every real number x, where

$$B + x = \{y + x;\ y \in B\}$$ [10]

7. Let $(\Omega, \mathcal{Q})$ be a Borel structure. Let f and g be two real valued measurable functions on Ω.

a) Show that

i) $f + g$ is measurable

ii) $f \cdot g$ is measurable

iii) $|f|$ is measurable [5+5+5]=[15]

b) h is a real valued function such that $|h|$ is measurable. Is it true that h is measurable? [5]

8. Let f be a bounded non-negative measurable function on Ω. Show that there exists a sequence of simple functions converging to f uniformly. [10]

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PERIODICAL EXAMINATION

Statistics-3: Statistics (Theory and Practical)

Date: 10.11.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Q. 4 and any two other questions from the rest. Marks allotted for each question are given in brackets [ ].

Q.1.a) A simple random sample of size n is drawn from a population consisting of N units without replacement. How will you estimate the population mean of a numerical characteristic 'y' of the units on the basis of this sample? Find the sampling variance and an unbiased estimate of this variance of this estimator. [5+9+10]=[24]

b) A simple random sample of size 3 is drawn from a population of size N with replacement. Find out the probabilities that the sample contains 1,2 and 3 distinct units. [6]

Q.2.a) In a stratified simple random sampling scheme (with replacement) obtain the optimum allocation of sample sizes, n_i's, $i = 1,2,\ldots, k$, to the different strata minimising the variance of the estimate of the population mean subject to fixed total sample size n. [10]

b) With 2 strata a sampler, using a stratified simple random (with replacement) scheme, would like to have $n_1 = n_2$ for administrative convenience. If V_{equal} and V_{opt} denote the variances of the usual unbiased estimate of the population mean given by (i) $n_1 = n_2$ and (ii) Neyman's allocation respectively show that the fractional increase in variance is given by

$$\frac{V_{equal} - V_{opt}}{V_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

where $r = \frac{\hat{n}_1}{\hat{n}_2}$, $\hat{n}_1$, $\hat{n}_2$ being the values of n_1 and n_2 as given by the Neyman's allocation. [10]

c) The following data has been obtained from a partly complete census of all 460 villages in a district. The villages were divided into 5 strata by size. For the i-th stratum N_i denotes the number of villages, σ_i the standard deviation of the areas under wheat in a village. Obtain the optimum values of n_i's, $i = 1,2,\ldots, 5$, (the sample sizes to be allocated to the 5 strata) minimising the variance of the estimate of the population mean (areas under wheat) subject to the fixed total sample size $n = 46$ (the samples in each stratum is to be drawn with equal probabilities and with replacement).

Stratum No.	N_i	σ_i
1	63	56.1
2	119	116.4
3	53	186.0
4	75	361.3
5	150	401.2

[10]

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Q.3. Suppose that a two stage sampling design is adopted to estimate the population mean of a certain characteristic. Let N be the number of first stage units, M_i be the number of second stage units in the i-th first stage unit, n the number of first stage units to be drawn at the primary stage of sampling and m_i the number of second stage units to be drawn from the i-th first stage unit if the i-th first stage unit happens to be selected at the primary stage of sampling. At both stages samples are drawn by simple random sampling without replacement.

a) Obtain an unbiased estimate $\hat{\bar{Y}}$ of the population mean $\bar{Y}$.

b) Obtain the sampling variance of the estimate $\hat{\bar{Y}}$.

c) Find out an unbiased estimate of the variance of $\hat{\bar{Y}}$.

[5+10+15]=[30]

Q.4 The following table gives the geographical area and the area cultivated under paddy in 60 villages in a district.

a) Select a sample of 2 villages with probability proportional to geographical area without replacement.

b) Estimate the total ~~yield~~ area under paddy in the district by using (i) Desraj's estimator (ii) Horvitz-Thompson's estimator.

c) Find out unbiased estimates of the sampling variance of the estimates obtained in (i) and (ii).

[6+14+20]=[40]

Table

Village-wise Geographical area and area under paddy in a district.

Village No.	Geographical area (acres)	Area under rice (acres)	Village No.	Geographical area (acres)	Area under rice (acres)
1	151	81	31	716	305
2	542	292	32	230	127
3	562	275	33	349	245
4	410	118	34	379	270
5	249	44	35	135	64
6	121	56	36	873	445
7	106	53	37	248	69
8	397	147	38	1034	401
9	453	194	39	503	164
10	710	282	40	206	46
11	176	65	41	682	166
12	730	288	42	653	129
13	327	115	43	166	44
14	280	161	44	445	116
15	287	179	45	1495	164
16	404	273	46	501	96
17	124	58	47	1473	373
18	244	58	48	1117	261
19	370	98	49	389	79
20	541	155	50	716	191
21	153	41	51	274	51
22	658	230	52	570	100
23	805	239	53	251	56
24	435	158	54	691	83
25	418	130	55	147	13
26	235	114	56	864	146
27	364	170	57	803	242
28	515	272	58	924	340
29	586	418	59	601	189
30	246	47	60	396	99

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PERIODICAL EXAMINATION

Statistics-3: Data Processing

Date: 17.11.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. Explain the meaning of Modulus and Mantissa? Assuming standard Modulus and Mantissa, how are the following values stored in internal mode?

a)	573	e)	36.0
b)	-10000	f)	+ 54.3
c)	4932C963.1046	g)	1
d)	-0.00037418	h)	10.328987869

[10]

2. Following Fortran statements form a segment of a program. Explain and correct the mistakes, if any.

```
      DØ  1   I = M,100
      DIMENSION   A(10,10), B(10)
4     B(I) = 0.0
      DØ 2 J = 1,N
1     B(J) = B(J) + A(I,J) * * 2
      PRINT  3, B(J) .
      IF (B(J) - 0.0005) 4,2,2
2     GØ  TØ  5
5     DØ  6   K = M,N
3     FØRMAT (10X, 3F.6.4)
      K = K- 2
      IF (B(K)** 2 - B(1)) 6, 1, 6
6     CØNTINUE
```

[10]

3. Given the values for a,b,c and d punched on a card and a set of values for the variable x punched one value per card, write a fortran program to evaluate the function defined by

$$f(x) = \begin{cases} ax^2 + bx + c & \text{if } x < d \\ 0 & \text{if } x = d \\ -ax^2 + bx - c & \text{if } x > d \end{cases}$$

for each value of x and print the values of x and f(x). [15]

4. Write a FØRTRAN Program to evaluate the function

$$\text{SIN}(x) \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots (-1)^{r+1} \frac{x^{2r-1}}{(2r-1)!}$$

where

$$\left| \frac{x^{2r+1}}{(2r+1)} \right| < 0.000005$$

[25]

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5.
```
      PRINT 1
1     FORMAT (52X, 27HINDIANbSTATISTICALb INSTITUTE//
      16H1969-70, SESSION //
      READ 2, N
2     FORMAT (I5)
      DIMENSION  A(5,N)
      DO 6  J = 1,N
      READ 3, (A(I,J),  I = 1,4)
3     FORMAT (40BLANKS, 4F 10.2)
      DO  4  I = 1,4
      A(5,J) = A(I,J) + A(5,J)
4     A1(I) = A1 (I) + A(I,J)
5     PRINT 6, (A(I,J), I = 1,5)
6     FORMAT (10X, 40H BLANKS, 5F 10.2/)
      DO 7I = 1,5
7     A1(I) = A1(I)/N
      PRINT 6, ((A1(I), I = 1,5))
      STOP
```

The above program is written to tabulate the marks obtained by students in 4 subjects. The average number of marks obtained by each student and the average marks in each subject is also required to be printed. Look for any possible errors and re-write the program correctly. The number of students is read from the 1st data card. (Assume that the number of students is at the most equal to 100). [25]

6. There are 73 departments, about 17000 workers and 27 different scales of pay, in an organisation. The following particulars are maintained for each worker in the organisation.

1) Employee name (max. 30 characters long)
2) Employee number
3) Sex
4) Marital status
5) Date of Birth (assume that all are born in 20th century)
6) Date of appointment
7) Date and month of increment of pay
8) Department
9) Scale of pay
10) Pay (Rs. Ps.)
11) Allowances (Rs.Ps.)
12) Total Emoluments (Rs. Ps.) [12 = 10 + 11]

Suggest a suitable card design and explain any coding scheme that you adopt. [15]

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PERIODICAL EXAMINATION

Mathematics-3

Date: 24.11.69 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ]. Answer groups A and B in separate answerscripts.

Group A: Mathematical Analysis

Maximum Marks: 50

Answer all questions.

1. When do you say that an ordered field satisfies the 'Supremum principle'.

   Show that an ordered field satisfying the supremum principle is Archimedean. [2+5]=[7]

2. Let A be a subset of the real line. Explain what is meant by 'A is closed' 'A is bounded' and 'A is compact'.

   Show that a subset of the real line is compact iff it is closed and bounded. [6+20]=[26]

3. a) Let (x_n) (y_n) be two sequences of real numbers converging to α. Define the sequence (Z_n) as follows:

$$Z_n = \begin{cases} x_n & \text{if } n \text{ is even} \\ y_n & \text{if } n \text{ is odd} \end{cases}$$

   Show that (Z_n) also converges to α.

   b) If a subsequence of a Cauchy sequence converges then show that the Cauchy sequence itself converges.

   c) Let (x_n) be a sequence of real numbers where $x_n \neq 0$ for all n. If $(x_n) \to 0$ show that the sequence $(\frac{1}{x_n})$ does not converge.

   d) If α is limit point of a sequence (x_n) then show that there is a subsequence of (x_n) which converges to α. [5+4+4+4]=[17]

Group B: Linear Algebra

Maximum Marks: 50

Note: Answer Q. No. 1 and as many questions as you can from the rest.

1. Define the following terms:

   (a) Eigen value (b) Geometric multiplicity (c) Algebraic multiplicity (d) Characteristic Polynomial (e) Algebraically closed field (f) Nilpotent transformation. [6 × $2\frac{1}{2}$]=[15]

2. Show, by means of an example, that a linear transformation need not have any eigen values. [8]

3. Show, by means of an example, that the algebraic multiplicity of an eigen value need not be equal to its geometric multiplicity. [8]

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4. Let P be a projection on a vector space V and λ be its eigen value. Then show that either $\lambda = 0$ or $\lambda = 1$. [4]

5.a) Give an expression for the determinant of a linear transformation on a finite dimensional vector space in terms of the elements of its matrix with respect to a basis for V. (No proof needed).

b) Let A be a linear transformation on an n-dimensional vector space V over a field F. Let the matrix of A with respect to a basis for V be $((a_{ij}))_{n \times n}$. Suppose $n^2 - n + 1$ of the scalars a_{ij} are zero. Then show that determinant of A is zero. [5+3]=[8]

6. Let T be a linear transformation on a vector space W over a field F. Suppose W is n-dimensional then show that there exist $n+1$ subspaces $S_0, S_1, \ldots, S_n$ such that

$\dim (S_i) = i$, S_i is invariant under A $(0 \leq i \leq n)$

and $S_0 \subset S_1 \subset \ldots \subset S_n$. [12]

7. If A is a nilpotent transformation of index q on a finite dimensional vector space V, show that there exists a pair of subspaces (H, K) which reduce A such that $\dim (H) = q$. [20]

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MID-YEAR EXAMINATION

Mathematics-3: Analysis

Date: 19.12.69 Maximum Marks: 100 Time: 3 hours

Instructions

1. All questions carry equal marks. Answer ten out of the thirteen questions given below. Marks allotted for each question are given in brackets [ ].
2. Write your answers in a brief, neat, fashion and in a logical order.
3. Be careful in justifying your steps. Conclusions without proper justifications carry no weight.
4. If you want to make use of any theorem proved in the class, state the result carefully and then use it.

1.a) Show that between any two distinct real numbers there is a rational number.

b) Answer with justification whether $\sqrt{12}$ is a rational number. [5+5]=[10]

2.a) Define the term 'Cauchy sequence'.

b) Verify by using the definition, but not any theorems, whether the following are Cauchy sequences.

i) $x_n = \frac{(-1)^n}{n}$; (ii) $x_n = \frac{1}{n}$;

iii) $x_n = n^2$. [3+2+2+3]=[10]

3.a) Explain the two statements

A sequence (x_n) converges to a point x.

A sequence (x_n) does not converge to x.

b) Let (x_n) and (y_n) be two sequences of real numbers converging to 100 and 10 respectively. Put

$$Z_n = \begin{cases} x_n & \text{if } n \text{ is odd} \\ y_n^2 & \text{if } n \text{ is even.} \end{cases}$$

Verify, without using any theorems, whether (Z_n) converges. [3+3+4]=[10]

4.a) Define the term 'subsequence'.

b) Comment on the following: The sequence 1,1,1,... is a subsequence of 1,2,1,2, ...

c) If a sequence (x_n) does not converge to a point x then show that there is an $\epsilon > 0$ and a subsequence (x_{n_k}) of (x_n) such that $|x_{n_k} - x| > \epsilon$ for all k. [3+2+5]=[10]

5.a) Define the terms 'open set' and 'closed set' on the real line.

b) Comment on the following: 'Every subset of the real line is either open or closed'.

c) Is the set of rationals in the real line an open set or a closed set. Give reasons. [4+3+3]=[10]

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6.a) When do you say that a point is a limit point of a set in the real line.

b) Find the set of limit points of S = [All rational numbers]

c) Find the set of limit points of S = [All irrational numbers]

[3+3+4]=[10]

7.a) State without proof the Heine-Borel theorem on the real line.

b) Suppose x^0 is a fixed point and $B_1, B_2, \ldots$ is a decreasing sequence of closed Boxes in R^p each containing x^0. Suppose that length of each side of B_n is $\frac{1}{2^n}$. Suppose $\epsilon > 0$. Show that there is an n such that $B_n \subset S(x^0, \epsilon)$. Recall that $S(x^0, \epsilon) = [\, x : |x - x^0| < \epsilon \,]$.

[4+6]=[10]

8. Given a real number $x > 0$, show that there is a unique $y > 0$ such that $y^2 = x$. [10]

9.a) Let $(x_n), (y_n)$ be two sequences of real numbers. Explain the Landau Symbols $(x_n) = o(y_n)$ and $(x_n) = O(y_n)$.

b) Verify the following:

If $x_n = (-1)^n n^2$ and $y_n = n^2$ and $z_n = n^3$

then $(x_n) = O(n^2)$ and $(x_n) = o(n^3)$.

[6+4]=[10]

10.a) What is meant by 'Cesaro limit of a sequence'.

b) Give an example of a sequence which has a 'Cesaro limit', but not a Cauchy limit. You have to work out all the details.

[3+7]=[10]

11.a) Explain the concepts 'Double sequence' 'Double limit' and 'repeated limits'.

b) Find the double limit and repeated limits of the double sequence (x_{mn}) where

$$x_{mn} = \frac{m \cdot n}{m + n}.$$

[4+6]=[10]

12. Verify the convergence or otherwise of the following sequences.

i) $x_n = n^{\frac{1}{2n}}$; ii) $x_n = \frac{1}{2n+1} + \frac{1}{2n+2} + \ldots + \frac{1}{4n}$.

[5+5]=[10]

13. a) Explain the concepts of 'pointwise convergence' and 'uniform convergence' for a sequence of functions.

b) Let $f_n(x) = \frac{x}{n}$ on the real line. Does (f_n) converge uniformly? Give reasons.

c) Let $f_n(x) = \frac{x}{n}$ on the interval [0, 1]. Does (f_n) converge uniformly? Give reasons.

[4+3+3]=[10]

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MID-YEAR EXAMINATION

Mathematics-3: Linear Algebra

Date: 20.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.1 and as many questions as you can from the remaining. Marks allotted for each question are given in brackets [ ].

1. Carefully explain the following terms:

(a) Alternating multilinear form
(b) Determinant
(c) Algebraically closed field
(d) Algebraic multiplicity of an eigen value. [4×3]=[12]

2. Show that the vector space of all alternating n-linear forms on an n-dimensional vector space is one dimensional. [15]

3. Let V be an n-dimensional vector space and T be a linear transformation on V into V. Let $((a_{ij}))_{n \times n}$ be the matrix of T with respect to some basis for V and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigen values of T. Derive an expression for the determinant of T in terms of a_{ij}'s. Also derive an expression for the determinant of T in terms of λ_i's. [15]

4. When do you say that a linear transformation on a vector space W into W is the direct sum of two linear transformations?

Show that any linear transformation on a finite dimensional vector space is the direct sum of a nilpotent transformation and an invertible transformation. [15]

5. Let V be a finite dimensional vector space over an algebraically closed field F and let A be a linear transformation on V into V. Explain what is meant by the Jordan canonical form of A, stating clearly any theorems you may use in its construction and indicating how you use them. No proofs are needed. [15]

6. Give an example of a nilpotent transformation of index 3. [5]

7. Consider the real vector space R^5 and the linear transformation B on R^5 into R^5 defined by

$$B((x_1, x_2, x_3, x_4, x_5)) = (x_1 - x_2,\ x_3,\ x_4,\ x_5,\ 0)$$

Express B as the direct sum of a nilpotent transformation and an invertible transformation. [20]

8. Find a linear transformation on the real vector space R^2 into R^2 such that its characteristic polynomial is $q(\lambda) = 3 + 2\lambda + \lambda^2$. [5]

9. Let A and B be two linear transformations on a finite dimensional vector space W into W. If A and B are similar, show that the characteristic polynomials of A and B are the same. [6]

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10. If λ_0 is an eigen value of a linear transformation T on a vector space V into V, then show that $a_0 + a_1\lambda_0 + a_2\lambda_0^2 + \cdots + a_n\lambda_0^n$ is an eigen value of the linear transformation $a_0 I + a_1 T + a_2 T^2 + \cdots + a_n T^n$. (Here $a_0, a_1, \ldots, a_n$ are scalars). [6]

11. Let A be the linear transformation on the real vector space R^3 into R^3 defined by

$$A((x_1, x_2, x_3)) = (x_1 + 2x_3, 0, -x_3).$$

(a) Write down the characteristic polynomial of A.

(b) Find the eigen values of A.

(c) Find the Jordan canonical form of A. [4+2+14]=[20]

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MID-YEAR EXAMINATION

Statistics-3: Probability

Date: 22.12.69 Maximum Marks: 100 Time: 3 hours

Note: This paper carries 115 marks. Answer as many questions or parts thereof as you can. The maximum number of marks you can score by answering this paper is 90. 10 marks are reserved for assignments. Marks allotted for each question are given in brackets [ ].

1. Let Ω be any set, $\mathcal{A}$ an algebra of subsets of Ω, μ a non-negative real-valued finitely additive set function defined on $\mathcal{A}$ satisfying the following property.
If $\{A_n ; n \geq 1\}$ is a decreasing sequence of sets in $\mathcal{A}$ converging to empty set, $\mu(A_n) \to 0$.
Show that μ is countably additive. [10]

2.a) Define convergence in probability of a sequence of random variables.

b) Let $\Omega = \{1, 2, 3, \ldots\}$, $\mathcal{A}$ the collection of all subsets of Ω and P a probability measure on $\mathcal{A}$ such that $P(E_n) = \frac{1}{2^n}$ where $E_n = \{n\}$. Show that $\{I_{E_n} ; n \geq 1\}$ converges in probability to the random variable identically zero. [2+8]=[10]

3.a) Let $(\Omega, \mathcal{A}, P)$ be a probability space, $\{f, f_n ; n \geq 1\}$ a sequence of random variables, and
$E_n(\epsilon) = \{w : |f_n(w) - f(w)| \geq \epsilon\}$, ϵ being positive.
Show that $f_n \to f$ a.e. iff

$$P(\limsup E_n(\epsilon)) = 0$$
for every $\epsilon > 0$. [10]

b) Hence or otherwise show that if $f_n \to f$ a.e. then $f_n \to f$ in probability. [5]

c) If $f_n \to f$ in probability, show that there exists a subsequence f_{n_ν} of f_n converging to f a.e. [15]

4.a) Define distribution function. [5]

b) Let $(R, \mathcal{B}, P)$ be a probability space. Define $F(x) = P[(-\infty, x)]$ for every x real number. Show that F is left continuous. [5]

c) Let $\{\gamma_n ; n \geq 1\}$ be an enumeration of rational numbers in R. Define

$$F(x) = \sum_{\gamma_n \leq x} \frac{1}{2^n} .$$

Show that F is a distribution function and determine the set of its continuity points. [10]

GO ON TO THE NEXT PAGE

5. Write down the principal steps involved in showing that there is a 1-1, onto correspondence between the collection of all probability measures on $(R, \mathbb{B})$ and the collection of all distribution functions on R.

[$\mathbb{B}$ is the Borel σ-algebra on R. No proofs are necessary] [20]

6. Let f_n be a sequence of random variables converging to f in probability, and F_n and F be the distribution functions of f_n and f respectively.

Show that

$$F_n(x) \to F(x) \quad \text{for every continuity}$$

point x of F. [15]

7. a) Define weak convergence of a sequence of distribution functions F_n to a distribution function F. [5]

b) Show that the limiting distribution function is always unique. [5]

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MID-YEAR EXAMINATION

Statistics-3: Statistics - Theory

Date: 23.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. EITHER

A population containing N units is divided into k-strata. From the ith stratum which contains N_i units a simple random sample of n_i units is drawn with replacement, $i = 1,2,\ldots, k$.

a) How will you estimate unbiasedly the population mean $\bar{Y}$ of a characteristic 'y' of the units on the basis of y-values of the sampled units? [5]

b) What is the variance of the estimate of $\bar{Y}$? Find an unbiased estimate of this variance. [5+5]=[10]

c) Assume a simple cost function of the type

$$C = c_0 + \sum_{i=1}^{k} n_i c_i$$ for the sampling project

where

C = total cost to be incurred

c_0 = overhead cost

c_i = cost of sampling one unit from the ith stratum, $i = 1, 2, \ldots, k$.

Obtain the optimum values of n_i's which minimise the variance of the estimate of $\bar{Y}$ subject to the condition that the total cost for the project is fixed. [20]

OR

1. a) Describe Lahiri's method of drawing a sample of one unit from a population containing N units with probabilities proportional to the sizes X_i, $i = 1,2,\ldots, N$ of the units. [6]

b) To obtain an estimate of the population total of a characteristic 'y' of the units, a sample of size 2 with probability proportional to size without replacement is drawn from the population. Give the expressions for the following estimates of the population total and show that they are unbiased:

i) Desraj's estimator; (iii) Horvitz-Thompson's estimator
ii) Das' estimator [6+6+10]=[22]

c) Find an unbiased estimate of the variance of Desraj's estimator. [7]

2.a) State the Neyman-Pearson's lemma giving a sufficient condition for a test to be most powerful of level α, $(0 \leq \alpha \leq 1)$ for testing a simple null hypothesis against a simple alternative hypothesis. [5]

b) Give a proof of the lemma when the distribution specified by the null and the alternative hypotheses are both discrete. [10]

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3.a) Let X be a Poisson random variable with parameter λ. Obtain the most powerful test of level α, $(0 \leq \alpha \leq 1)$, for testing $H_0: \lambda = \lambda_0$ against the alternative $H_1: \lambda = \lambda_1$, where $\lambda_1 < \lambda_0$. [10]

b) Hence obtain the uniformly most powerful test of level α, $(0 \leq \alpha \leq 1)$ for testing $H_0: \lambda = \lambda_0$ against the alternative $K: \lambda < \lambda_0$. [5]

4.a) Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed random variables, each being distributed normally with unknown mean μ and standard deviation 2. Obtain the most powerful level α $(0 \leq \alpha \leq 1)$ test for testing $H_0: \mu = \mu_0$ against the alternative $H_1: \mu = \mu_1$ where $\mu_1 > \mu_0$. [10]

b) Hence obtain the uniformly most powerful test of level α, $(0 \leq \alpha \leq 1)$ for testing $H_0: \mu = \mu_0$ against $K: \mu > \mu_0$. [5]

c) What is the standard test used for testing $H_0: \mu = \mu_0$ against $K': \mu \neq \mu_0$? Obtain an expression for the power function of this test in terms of the distribution function $\Phi(x)$ of the standard normal variable. Is this test UMP for testing H_0 against K'? [3+5+2]=[10]

5. Assignments. [10]

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MID-YEAR EXAMINATION

Statistics-3: Statistics - Practical

Date: 24.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. EITHER

The data given in the table below are the number of seedlings for each foot in a bed 60 ft. long.

Number of seedlings in the bed

sl. no. (ft.)	no. of seedlings	sl. no. (ft.)	no. of seedlings	sl. no. (ft.)	no. of seedlings
1	8	21	20	41	26
2	6	22	19	42	26
3	6	23	25	43	10
4	23	24	11	44	41
5	25	25	31	45	30
6	16	26	26	46	55
7	28	27	29	47	34
8	21	28	19	48	56
9	22	29	17	49	39
10	18	30	28	50	41
11	26	31	16	51	27
12	28	32	9	52	20
13	11	33	22	53	25
14	16	34	26	54	39
15	7	35	17	55	24
16	22	36	39	56	25
17	44	37	21	57	18
18	26	38	14	58	44
19	31	39	40	59	55
20	26	40	30	60	39

Find the variance of the mean of a systematic sample consisting of every tenth foot. Compare this variance with the variance of a simple random sample of size 6 without replacement. [15+20]=[35]

OR

The following table gives the total area under cultivation 'X' and the yield of paddy 'Y' for 20 counties in a state.

Area and Yield of Paddy fields

County sl. no.	Area (acres) X	Yield of Paddy (100 lb.) Y	County sl. no.	Area (acres) X	Yield of Paddy (100 lb.) Y
1	870	8521	11	1089	9562
2	883	8554	12	1111	10512
3	894	8783	13	1177	11560
4	901	[illegible]62	14	1203	11631
5	914	9025	15	1224	11913
6	973	8837	16	1229	11517
7	995	9569	17	1260	11955
8	1031	9593	18	1260	11639
9	1043	10316	19	1302	12428
10	1054	10874	20	1319	12126

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Draw a sample of size 6 with replacement with probabilities proportional to area. Obtain an unbiased estimate of the total yield of paddy and an unbiased estimate of the variance of the estimator.

2. Let X denote the number of heads obtained in 12 tosses of a coin where the probability of getting a head in a single toss with the same coin is p. Obtain the uniformly most powerful test of level $\alpha = 0.05$ for testing the hypothesis $H_0: p = 1/2$ against the alternative $H_1: p > \frac{1}{2}$. Compute the power of this test for the following values of p : $p = 0.6(0.1)1.0$. [10+15]=[25]

3. Suppose that $X_1, X_2, \ldots, X_{10}$ are independent and identically distributed normal random variables each with mean μ (unknown) and standard deviation 1.

   a) How will you test the hypothesis $H_0: \mu = 0$ against the alternative $H_1: \mu \neq 0$ at level of significance $\alpha = 0.05$ on the basis of the observations $X_1, X_2, \ldots, X_{10}$?

   b) If the observed value of $\sum_{i=1}^{10} X_i$ is -20.35 which hypothesis will you accept?

   c) Compute the power of the test for $\mu = -2.0(0.2)2.0$.

[5+5+20]=[30]

4. Practical records. [10]

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MID-YEAR EXAMINATION

General Science-5: Sociology

Date: 27.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. All questions carry equal marks.

1. Formulate a design of sample survey for studying standard of living of different families of rural and urban West Bengal. Specify clearly your unit of enquiry and unit of sampling for the said survey.

2. EITHER

What is sociology? How is it different from economics, history and political science?

OR

What is the usefulness of studying sociology, in particular to students of statistics?

3. EITHER

Define unilateral group. Give at least five Indian examples.

OR

What is community? Discuss its various types.

4. EITHER

What is kinship? Compare its importance in simple and modern societies.

OR

'The term family is used to connote varieties of groupings in society' - Illustrate the statement.

5. EITHER

How can the two distinct approaches of sociology and history to social problems be complementary to each other? Is there any difference between the two subjects regarding method of data collection?

OR

Elaborate on a contemporary social problem which has its roots in the 19th Century Indian society.

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MID-YEAR EXAMINATION

Economics-3

Date: 29.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Group A and Group B in separate answer-scripts.

Group A : Economic Development

Maximum Marks: 60 Suggested time 2 hours

Answer any three questions. All question carry equal marks

1. Given

$$\sigma = \beta\mu - \alpha w$$
$$\dot{w} = k\dot{\mu}$$
$$\sigma = \dot{w}$$

where σ, μ and w denote, respectively, saving, income and wealth, a dot denotes the first derivative with respect to time (θ) of the variable it surmounts and α, β and k denote positive constants, show that

$$\mu = \frac{\alpha}{\rho k}(w_0 - k\mu_0) + \frac{1}{\rho k}(\beta\mu_0 - \alpha w_0)e^{\rho\theta}$$

$$w = \frac{\beta}{\rho k}(w_0 - k\mu_0) + \frac{1}{\rho}(\beta\mu_0 - kw_0)e^{\rho\theta}$$

$$\sigma = (\beta\mu_0 - \alpha w_0)e^{\rho\theta}$$

where w_0 and μ_0 denote respectively the values at time $\theta = 0$, and $\rho = (\beta/k) - \alpha$.

Obtain values of $\frac{\partial\rho}{\partial k}\cdot\frac{\alpha}{\rho}$, $\frac{\partial\rho}{\partial\beta}\cdot\frac{\beta}{\rho}$ and $\frac{\partial\rho}{\partial k}\cdot\frac{k}{\rho}$ and use them to show how the above model can be used differently by planners and forecasters.

2. Describe the general features of the classical theory of growth following Baumol's synthesis.

Indicate some major differences between Marx's model and the classical model.

3. Summarize the main points of Rostow's theory of stages of growth. Criticise aspects of the theory which you consider to be inappropriate.

4. Describe briefly any two of the following theories of growth.

i) sociological dualism,
ii) n-Achievement, and
iii) geographical determinism.

Write notes on any two of the following:

i) Schumpeter's ideas on growth,
ii) Alfred Marshall's ideas on growth,
iii) Simple neo-classical growth model.

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Group B: Growth Models

Maximum Marks: 40 Suggested time: 1 hour

All questions carry equal marks. Answer any two questions.

1. Comment on the problem of existence of a warranted rate of growth in a Harrodian economy.

2. EITHER

What is the long-run problem in Harrod's growth model? If you are a typical neoclassicist, how will you solve this problem?

OR

Derive the neoclassical conclusions about the growth (without technical progress) of an economy when it possesses a Cobb-Douglas production function.

3. EITHER

What are the definitions of neutral, capital-saving and capital-using technical progress according to Hicks and also according to Harrod? Compare the definitions of these two authors when the elasticity of substitution between capital and labour assumes different values.

OR

Prove that steady-state growth in a neo-classical economy (in which disembodied technical progress is going on at an externally given rate) is possible if and only if technical progress is Harrod-neutral.

4. Suppose, the economy has only two-sectors consumption-goods sector and investment-goods sector - and it borrows a given fund of investible resources from outside. Suppose, the Chairman of the Planning Commission asks you to allocate the fund between two sectors in such a way that the economy achieves highest possible income at the end of a given planning horizon. How would you proceed to construct a model in order to solve this problem?

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MID-YEAR EXAMINATION

General Science-4: Biochemistry Theory

Date: 30.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe how fat is metabolised in the body. [20]

2. Give an example for each of the following:

Nucleotide, Monoglyceride, Steroidal hormone, Purine, Polynucleotide. [10]

3. How can you estimate Vitamin C in lemon juice. [20]

4. Describe the different hormones present in pituitary gland. [20]

5. What are the deficiency symptoms and sources of the following vitamins.

Vitamin A, Vitamin B_1, Vitamin B_2, Vitamin D. [20]

6. Discuss the origin of ketone bodies in urine. [10]

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B. Stat. Part III

CLASS EXAMINATION- 1

Date: 17.1.70 Time: 50 minutes

Answer all questions.

1. There are two boxes. In the first box, there are 5 white and 5 black balls. In the second box, there are 3 white and 6 black balls. One ball is taken at random from the first box and placed in the second. Then one ball is drawn at random from the second box.

   a) What is the probability that the ball last drawn is white?

   b) Given that the ball last drawn is white, what is the conditional probability that the ball transferred from the first box to the second was white?

2. A random sample of size n is drawn from an infinite population in which the individuals fall into one or the other of the two categories with probability θ and $(1-\theta)$. The numbers of sample falling into the two categories are n_1 and n_2 $(n_1 + n_2 = n)$. Obtain the variance of the difference in the proportions

$$\left(\frac{n_1}{n} - \frac{n_2}{n}\right).$$

3. Bring out the fallacy, if any, in the following statements:

   a) The mean of a Binomial distribution is 4 and its s.d. is 2.

   b) Two normally distributed random variables X and Y are independent if

$$\text{Cov.}(X, Y) = 0.$$

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PERIODICAL EXAMINATION

Mathematics-3 Mathematical Analysis.

Date: 23.2.70 Maximum Marks: 100 Time:3 hours

Note: Attempt Questions 5 and 8 and any five of the remaining. Marks allotted for each question are given in brackets [ ].

1. Let $f_1, f_2, f_3, \ldots$ be a sequence of real-valued functions on an interval $[a,b]$ in R^1 such that $f_n(x') \leq f_n(x'')$ whenever $a \leq x' \leq x'' \leq b$. Show that, if $f_0 = \lim_n f_n$ exists and is continuous on $[a,b]$, the convergence is uniform. [15]

2. Show that if f is bounded on $[0,1]$ and continuous at a point x_0 in $[0,1]$, and $B_1, B_2, \ldots$ are the Bernstein polynomials for f, then $\lim_{n \to \infty} B_n(x_0) = f(x_0)$. [15]

3. If f is a one-to-one continuous real-valued function on a compact set $Z \subset R^1$, prove that f^{-1} is continuous on $f(Z)$. [15]

4.a) If f is any real-valued function on the set N of all integers, show that f is uniformly continuous on N. [5]

b) Give an example of a real-valued function on $[0,1]$ which is discontinuous at each point of $[0,1]$. [10]

5. Prove that the only connected subsets of R^1 are intervals. [15]

6. If $f: Z \to R^1$, Z is compact and f is lower semi-continuous on Z, prove that f assumes its minimum (Give a detailed proof). [15]

7. Write down clearly in mathematical language the meanings of the following expressions.

a) A function $f: Z \to R^1$ is not lower semi-continuous on Z. [5]

b) A function $f: Z \to R^1$ is not uniformly continuous on Z. [5]

c) A sequence of functions $f_n : Z \to R$, $n = 1, 2, \ldots$, does not converge uniformly to a function $f_0: Z \to R^1$ on Z. [5]

8. Prove that a continuous real-valued function on a compact subset of R^1 is uniformly continuous. [10]

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PERIODICAL EXAMINATION

Statistics-3: Probability

Date: 2.3.70 Maximum Marks: 100 Time: 3 hours.

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. X and Y are independent random variables. X has a discrete distribution and takes the values 2 and 3 with probabilities 1/3 and 2/3 respectively. Y has frequency function $2x$ on the interval (0,1) and 0 outside. Obtain the following:

   (a) $E(X+Y)$, (b) $E(X^2 Y^3)$, (c) $E[(X-Y)XY]$,

   (d) range and frequency function of $(X+Y)$,

   (e) range and frequency function of XY.

   [5+5+5+12+12]=[39]

2. In the joint distribution of X and Y, the frequency function is kxy inside the triangle whose vertices are (0,0), (2,0) and (2,1) and 0 outside this triangle.

   (a) Determine the value of the constant k.

   (b) Are X and Y independent? Give reasons.

   (c) What are the range and frequency function of Y?

   (d) Determine the range and frequency function of $(X+Y)$.

   [5+5+6+15]=[31]

3. In the joint distribution of X and Y, the frequency function is 2 inside the triangle whose vertices are (0,0), (1,0) and (1,1) and 0 outside. Obtain the correlation coefficient of X and Y. [15]

4. The random variable X has frequency function Kx^2 on [-1, 2] and 0 outside. Determine the value of the constant K. Obtain the range and frequency function of X^2. [15]

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PERIODICAL EXAMINATION

Statistics-3: Statistics (Theory and Practical)

Date: 9.3.70 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. Marks allotted for each question are given in brackets [].

1. Under the following set up

$$E(Y) = A\theta \quad ; \quad D(Y) = \sigma^2 I$$

where $Y_{n \times n}$ is vector of random variable.
$X_{n \times n}$ matrix of known constants.
$\theta_{m \times 1}$ vector of unknown parameter.

a) Under what condition is $\lambda' \theta$ estimable? Define a best linear unbiased estimator of a given parametric function.
Let $T = c'y$. State and prove the n. s. condition for T to be BLUE for $\lambda' \theta$. [5+5+15]=[25]

b) Show that $Ax = b$ has a solution iff

$$R(A) = R(A \,\vdots\, b).$$

Hence or otherwise show that

$A'A\,\theta = A'Y$ always admits a solution. [20]

c) Define zero function. Show that set of all zero function forms a vector subspace. Show that BLUE is uncorrelated with all zero function. [4+4+7]=[15]

d) Let $L = (Y - A\theta)'(Y - A\theta)$. Show that
$L_{\min_\theta} = (Y - A\hat{\theta})'(Y - A\hat{\theta})$ where $\hat{\theta}$ is the least square estimate of θ and $L_{\min} = Y'Y - Q'\hat{\theta}$ where $Q = A'Y$
Show that $E[L_{\min}] = (n-r)\sigma^2$

where $R(A) = r$. [25]

e) Consider the model

$$y_i = \theta + e_i \quad \text{for } i = 1, \ldots, n.$$

Show that $\frac{1}{n} \Sigma y_i$ is the BLUE for θ. [5]

2. The following is the result of weighing four objects A, B, C and D in a common balance in different combination and putting standard weights so as to obtain equilibrium. Estimate the individual weights of the objects and their standard error assuming the possibility of error in the balance.

Right pan	Left pan	Right pan	Left pan
A,B,C,D	......	....	20.2
B,D	A,C	8.1	....
A,B	C,D	...	9.7
A,D	B,C	....	1.9
...	A,B,C,D	19.9	.....
A,C	B,D	....	8.3
A,B	C,D	....	10.2
B,C	A,D	1.8	.....

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PERIODICAL EXAMINATION

General Science-5: Psychology

Date: 16.3.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all of the following questions. If there are two alternatives to a question, answer only one. Marks allotted for each question are given in brackets [].

1. EITHER
Draw a diagram of the eye and explain the transformation of light to neural impulses by the eye. [25]

OR

Draw a diagram of the ear and explain the transformation of sound waves to neural impulses by the ear.

2. Draw graphs or histograms to explain the following relationships: [25]

a) Distribution of rods and cones from the center to the periphery of the retina.

b) Activation of rods and cones as a function of the intensity of light.

c) Sensitivity of the sensory apparatus as a function of survival value.

d) Distribution of 'hair cells' in the cochlea according to intensity threshold and frequency threshold.

3. Explain how light of different wavelengths is transformed by three types of cones to yield colours differing in quality. [20]

4. Define psychology and discuss its relationship to physics, physiology, sociology and anthropology. [15]

5. Explain briefly how modern psychology represents the convergence of several historically different disciplines. [15]

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PERIODICAL EXAMINATION

General Science-5: Engineering

Date: 30.3.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. EITHER

Write short notes on the following:-

a) Moment of resistance
b) Bending moment
c) Cantilevers
d) Factor of satefy. [20]

OR

Derive the formula $\frac{E}{R} = \frac{f}{y} = \frac{M}{I}$. State clearly the assumptions made by you. [20]

2. A simply supported beam of effective span of 20 ft. carries a uniformly distributed load of 1 ton per foot through out the span and carries 10 tons at a distance of 5 feet from one support.

a) Draw the Bending Moment and Shear force diagrams. [30]

b) Calculate the deflection at the mid-span in terms of E.I. Derive the formulae used by you. [30]

3. A 15 foot long and 6" diameter steel beam is supported on a central column situated at its mid point. A weight of 1 ton is hung at each of the free ends of the beam. Calculate the maximum flexural stress. [20]

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PERIODICAL EXAMINATION

Economics-3: <u>Indian Planning</u>

Date: 6.4.70 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer <u>all</u> questions. Marks allotted for each question are given in brackets [ ].

1. Write short notes on any five of the following. In each case illustrate with figures from Indian plans.

   a) draft on private savings and public savings. [6]

   b) size of a plan [6]

   c) foreign exchange requirement of a plan and its relation with balance of payments [6]

   d) allocation of plan outlay [6]

   e) deficit financing [6]

   f) Marxian socialism. [6]

2. <u>EITHER</u>

   Discuss the view that the second five year plan was qualitatively different from the first five year plan, but realisation of second plan objectives was far from satisfactory. [20]

   <u>OR</u>

   Discuss the arguments for and against planning, and state how the State in India try to regulate the economy. [20]

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B. Stat. Part III: 1969-70

PERIODICAL EXAMINATION

Indian Economics

Date: 13.4.70 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any two of the following.
All questions carry equal marks.

1. What do you understand by the term 'Foreign Trade'? How would you distinguish it from 'Internal Trade'? What is the significance of foreign trade in the economic growth of a developing country?

2. 'The problems of India's foreign trade are the reflection of the poor performance of Indian economy, specially on the agricultural front'.

   Do you agree? Give reasons for your view.

3. 'The rate and pattern of the growth of the foreign trade of any country is a fair indicator of the pace and pattern of the growth of its national economy'.

   Examine this statement with the case of India in view.

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ANNUAL EXAMINATION

Mathematics-3: Mathematical Analysis

Date: 18.5.1970 Maximum Marks: 100 Time: 3 hours

Note: In Group A, attempt question 3 and any two of the remaining. Attempt question 6 and any two of the remaining from Group B. Marks allotted for each are given in brackets [].

Group A

1.a) State and prove Rolle's theorem. Is it possible to relax any one of the hypotheses of Rolle's theorem? Justify your answer. [9]

b) Prove that the function f of the real line defined by $f(x) = e^x - ax - b$, where a and b are real numbers, vanishes at at most two points. [6]

2.a) State and prove the Mean Value Theorem (of differential calculus). [6]

b) Establish the inequality

$$a^{\alpha} b^{1-\alpha} \leq \alpha a + (1-\alpha) b$$

where $0 < \alpha < 1$ and $a, b \geq 0$. Show that equality holds when and only when $a = b$ (Hint: Consider the function $g(x) = \alpha x - x^{\alpha}$, $x \geq 0$). [9]

3.a) Define the concept of equicontinuity for a family of real-valued functions defined on a subset of the real line. [5]

b) If $\{f_n, n \geq 1\}$ is a sequence of uniformly bounded, equicontinuous functions on $[a,b]$, prove that there is a subsequence $\{f_{n_k}\}$ which converges uniformly on $[a,b]$. [15]

4.a) If f is a one-one continuous real-valued function on a compact subset Z of the real line, show that f^{-1} is continuous on $f(Z)$. [8]

b) Show that the image of an interval under a continuous real-valued function is again an interval. [7]

Group B

5.a) If f is a function of bounded variation on $[a,b]$, show that f is a difference of two non-decreasing functions on $[a,b]$. [10]

b) Show that function f defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \in [-1,1] \text{ and } x \neq 0, \\ 0, & x = 0 \end{cases}$$

is of bounded variation. [5]

6.a) Let f be a continuous real-valued function and let g be a function of bounded variation on $[a,b]$. Show that f is Riemann-Stieltjes integrable with respect to g over $[a,b]$. [15]

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6.b) Give an example of a bounded real-valued function on [0, 1] which is Riemann integrable and whose set of discontinuities is countably infinite. [5]

7.a) Define the terms (i) a subset of the line has content zero (ii) a subset of the line has Lebesgue measure zero. [3]

b) State (without proof) necessary and sufficient conditions for a bounded real-valued function f on [a,b] to be Riemann integrable over [a,b]. [5]

c) Let f be a bounded real-valued function on [a,b] and let g be a non-decreasing function on [a,b]. Formulate precisely (without proof) necessary and sufficient conditions for f to be Riemann Stieltjes integrable with respect to g over [a,b]. [7]

[Warning: The necessary and sufficient conditions asked for in (b) and (c) must <u>not</u> be the same as the definitions of Riemann integrability and Riemann-Stieltjes integrability].

8.a) Let g be a non-decreasing function on [a,b] and let $\{f_n\}$ be a sequence of bounded functions which are Riemann-Stieltjes integrable with respect to g over [a, b]. Suppose that the sequence $\{f_n\}$ converges uniformly on [a,b] to a function f. Prove that f is Riemann-Stieltjes integrable with respect to g and

$$\int_a^b f\,dg = \lim_n \int_a^b f_n\,dg$$ [10]

b) Do the conclusions of (a) still hold in case we assume only that the functions $\{f_n\}$ converge to f on [a, b], the other hypotheses remaining the same? Give reasons. [9]

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ANNUAL EXAMINATIONS

Economics-3

Date: 20.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets[].

Group A: Indian Economics

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1. Critically evaluate the Indian Fiscal policy since 1950-51 in the context of the major economic social objectives of the Planners.

   Illustrate your answer with relevant data. [25]

2. The growing share of foreign loans and deficit financing in the Indian budgetary resources reflects the failure on the part of the Government to mobilise adequate investible resources. Comment. [25]

3. Examine the budgetary policy of the State Governments in India, since 1950-51. Do you agree with the criticism that their refusal to tax agricultural incomes is mainly responsible for their increasing dependence on loans and subsidies from the Union Government? [25]

Group B: Indian and Socialist Planning

Maximum Marks: 50 Suggested time: $1\frac{1}{2}$ hours

Answer any two questions.

1. What were the major elements in the economic policy pursued during the period of War.Communism in USSR? How far were they communistic in terms of Marxian concepts? Why were these policies changed in the subsequent period? [25]

2. Discuss the major issues involved in the debate on indus-trialization on the eve of the first Five Year Plan in the U.S.S.R.? [25]

3. Give a brief outline of the first five year plan of the USSR. Discuss the problems faced and the solutions atten-pted in the course of the implementation of the plan. [25]

4. Why was the price policy which had been pursued in the USSR from 1928 to 1955 sought to be changed in the sixties? Discuss, also, the rationale behind the earlier policy. [25]

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ANNUAL EXAMINATIONS

Statistics-3: Statistics Theory

Date: 22.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer all questions.

Group A

1. Describe carefully the roles of randomisation, replication and error control in planning of experiments. [15]

2. Describe a randomised block design and give the structure of the analysis of variance table for such a design.

Assuming only the fact of randomisation and additivity of plot and treatment effects, work out the variance of the usual estimator for a contrast of treatment effects and obtain an unbiased estimator of this variance. Work out the expected values of the components (mean-squares) in the analysis of variance. [30]

Group:B

3. Describe a Latin Square design, and give the structure of the analysis of variance table for such a design.

In a Latin Square experiment with v treatments, the yield from one plot (say the plot in the 1st row, 1st column getting the 1st treatment) was lost. How would you analyse the results of the experiment? [30]

4. Let x_i $i = 1, \ldots, n$ be independent random variables each distributed normally with mean 0 and variance 1. Let $\mu_1, \ldots, \mu_r$ be given constants; $r < n$. Work out the probability distribution of the statistic

$$U = \sum_{i=1}^{r} (x_i + \mu_i)^2 \Big/ \sum_{i=r+1}^{n} x_i^2$$

Describe how this distribution is useful in problems of analysis of variance. [25]

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ANNUAL EXAMINATIONS

Statistics-3: Statistics Practical

Date: 23.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

Group A

1. Three objects A, B, and C were weighed ten times in a common balance by putting some objects on left pan and some on the right pan, and balancing against standard weights put on the pans.

objects on		standard weight in gms.	
right pan	left pan	right pan	left pan
A, C	....	....	14.15
A	C	4.65	.....
A, B	C	....	3.15
A, C	B	....	8.47
B, C	A	....	11.87
......	A, B, C	15.41	......
B, C	A	......	12.10
......	A, B	8.31	......
B	C	0.91	......
C	B	......	3.95

Assuming that there is some bias in the instrument.

(a) Estimate separately the weights of the objects and the instrumental bias.

(b) Find out the standard error of these estimates. [50]

2. EITHER

The following table gives the yields in a comparative trial of 6 treatments in a randomised block experiment. [The number in the parenthesis denote the treatments and figure below it the yield corresponding to that treatment]

Table 1

Yields for Randomised Block design.

Block	Treatments and yields					
	(1)	(3)	(2)	(4)	(5)	(6)
1	24.7	27.7	20.6	16.2	16.2	24.9
	(3)	(2)	(1)	(4)	(6)	(5)
2	22.7	28.8	27.3	15.0	22.5	17.0
	(6)	(4)	(1)	(3)	(2)	(5)
3	26.3	19.6	38.5	36.8	39.5	15.4
	(5)	(2)	(1)	(4)	(3)	(6)
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

(1) Analyse the data. Test for the hypothesis of no differences among the treatment effects.

(2) Let τ_s denote the sth treatment effect. Then test for the hypothesis

$$H_0 : \tau_3 = \tau_5 \quad \text{against} \quad H_1 : \tau_3 > \tau_5 .$$ [30

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OR

The following table gives the yields of wheat per plot in a manurial experiment carried out in a 4 X 4 latin square. The four manurial treatments are denoted by 1,2,3 and 4. in the parenthesis.

Table 2

Yields in a 4 X 4 Latin Square Experiment.

Column / Row	1	2	3	4
1	(2) 425	(3) 442	(4) 540	(1) 340
2	(4) 384	(1) 512	(2) 490	(3) 408
3	(3) 506	(4) 508	(1) 336	(2) 600
4.	(1) 451	(2) 568	(3) 499	(4) 347

(a) Analyse the data.

(b) Verify whether there is any effect of column classification on the inference about the differential effects of treatments. [30]

Group B

Practical Records [10]

Group C

Viva voce [10]

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ANNUAL EXAMINATION

Statistics-3: Probability

Date: 25.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Max. Marks: 50. Answer all questions.

1. In the joint distribution of X and Y, the whole probability is situated inside the triangle whose vertices are (0,0), (1,0) and (1,1). The joint frequency function is 2 inside this triangle.

   a) Find the joint frequency function of $\xi = (X^2 + Y)$ and $\eta = Y$. [10]

   b) What is the range of values of ξ? On the inverval (0,1), completely evaluate the frequency function of ξ. On the remaining interval (or intervals), express the frequency function of ξ as a definite integral (or integrals). [12]

   c) Calculate $\Pr\ (X^2 + Y) \leq c$ for $0 < c < 1$ without using the frequency function in (b). Hence by differentiation obtain the frequency function of $(X^2 + Y)$ on (0,1) and verify that you got the same answer as in (b) above. [12]

2. X has frequency function $2x$ on (0,1) and Y has frequency function 1 on (0,1). The correlation coefficient of X and Y is $+\frac{1}{2}$. What is the correlation coefficient of $(X-Y)$ and $(2X+Y)$? [16]

Group B

Answer as many as you can. Maximum you can score is 50.

3. X takes the values 1,2,3 with probabilities $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$ respectively, Y takes the values 1,2 with probabilities $\frac{1}{3}, \frac{2}{3}$ respectively. What is the greatest possible value of the correlation coefficient of X and Y? Obtain the corresponding joint distribution. [10]

4. What is the spectrum of a probability distribution on $(-\infty, \infty)$? Prove that the spectrum is always a closed set. [12]

5. a) P_n is a sequence of probability distributions on $(-\infty, \infty)$. When do we say that the sequence converges weakly to a probability distribution P? [10]

   b) X is uniformly distributed on [0,1]. Prove rigorously that the sequence of distributions of X^n converges weakly to a distribution ($n = 1,2,3,\ldots$). What is this limit distribution? [6]

6. a) Prove that if the distribution function $F(x)$ is continuous at $x = \alpha$, $\Pr(\alpha) = 0$. [10]

   b) X and Y are independent random variables, α belongs to the spectrum of the distribution of X and β belongs to the spectrum of the distribution of Y. Prove that $(\alpha + \beta)$ bel belongs to the spectrum of the distribution of $(X+Y)$. [15]

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ANNUAL EXAMINATIONS

General Science-4: Biology

Date: 26.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A

Answer Q.1 and any other two from the rest.

1. Give the statistics on the area under and production of rice (Oryza satva) in the various agricultural regions of the world. Mention the respective figures for India. [7+3]=[10]

2. What are the difficulties in the genetic improvement of the coconut palm? Discuss the possibilities of effecting clonal propagation in the coconut. [10+10]=[20]

3. A. State the origin and history of wheat (Triticum Sp.). Give botanical names of the hexaploid wheats [5+5]=[10]

   B. Compare the spikelet of Oryza satva and that of any Triticum Sp. [10]

4. Write a detailed account of the morphology of the coconut palm including the significance of its foliar sprial. [20]

Group B

Answer Q.1 and any other one from the rest.

1. Give an outline scheme of the breeding methods for producing new improved varieties of self-fertilizing plant species. [25]

2. a) What is the purpose of inbreeding?

   b) Why selection for vigour may delay the attainment of homozygosity?

   c) Write some of the important characteristics of human population living in isolates. [5+10+10]=[25]

3. Write short notes on any five of the following:

   i) Heterosis
   ii) Polyploidy
   iii) Chromosomal abberations
   iv) Back-cross method
   v) Induced mutation
   vi) Genotype and phenotype
   vii) Crossing over

[5 X5]=[25]

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ANNUAL EXAMINATIONS

General Science-5: Psychology Theory and Practical

Date: 27.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks: 50
Answer all questions.

1. What are the fundamental problems of test construction? How is probit analysis useful in solving them? [15]

2. Define (a) reliability and (b) validity of a test. Explain how the reliability of a given test can be estimated. [15]

3. The scores of 10 students on each of the 8 questions in a test are given as follows.

Questions

STUDENTS	1	2	3	4	5	6	7	8
1	8	6	0	0	2	6	0	4
2	1	5	8	9	6	7	8	7
3	0	1	6	6	0	8	3	2
4	3	4	2	7	9	2	4	9
5	5	8	5	3	1	8	9	4
6	5	6	7	0	4	5	8	9
7	1	0	9	6	5	5	7	4
8	3	8	2	3	5	4	6	9
9	8	9	1	7	6	4	7	2
10	0	9	7	1	1	3	5	9

Obtain (i) reliability of the above test
and (ii) item difficulty and item discrimination for each question. [20]

Group B: Maximum Marks: 50
Answer all questions.

1. Write short notes on
   (a) Human abilities
   (b) Intelligence
   (c) Measurement in Education and Psychology. [25]

2. Practical Record work [25]

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ANNUAL EXAMINATIONS

General Science-5: Engineering

Date: 28.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks: 50

Answer all questions.

1. EITHER

Write short notes on the following:-

(a) Euler's formula for long columns.

(b) Different types of beams.

(c) Centre of Pressure.

(d) Measurement of discharge of quantity water per second through a river. [20]

OR

State and prove the formula $\frac{T}{I_p} = \frac{s}{r} = \frac{C}{(\lambda/\theta)}$. [20]

2. The difference of head registered in two limbs of a mercury gauge, with water above the mercury, connected to a venture-meter was 12". The diameter of the pipe and the throat of the meter are 12" and 6" respectively. The coefficient of the meter is 0.98. Find the discharge through the meter. Prove the formula you use. [30]

Group B: Maximum Marks: 50

Answer all questions.

1. A horizontal beam 20' span, simply supported at its ends, carries a load, which varies uniformly 1 ton per foot at one end to 3 tons per foot at the other end. Draw Bending diagram. Calculate Bending moment and deflection (in terms of E and I) at the mid span. [25]

2. EITHER

Working conditions to be satisfied by a shaft transmitting power are (a) that the shaft must not twist more than 1 degree in a length of 15 diameter, (b) the shear stress must not exceed 3.5 tons./sq.in. If $C = 12 \times 10^6$ lbs./sq. in, what is the actual working stress and the diameter of the shaft to transmit 1200 horse power at 240 r.p.m.? [25]

OR

(a) Write short notes on Professor Rankine's formula for struts.

(b) 10" X 6" X 40 lb. R.S.J. is used as a stanchion, length 15 ft., both ends fixed. Find the safe axial load for the stanchion. Area of section = 11.77 in^2
$I_{XX} = 204.80$ in^4, $I_{YY} = 21.76$ in^4. Rankine's constant for mild steel stanchions with both ends hinged = $\frac{1}{7500}$. Make your own assumptions. [25]