INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1974-75

[3011]

PERIODICAL EXAMINATION

Sampling distributions and Statistical Methods

Date: 25.11.74. Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

Note: Answer any two questions from Group A and all questions from Group B. Marks allotted for each question are given in brackets [ ].

GROUP A

1. A layman says that it is absurd to try to estimate the average income of all Indian households by collecting data on income from a sample of only a few thousand households. Briefly answer this fundamental objection to the approach of statistical sampling explaining the theoretical considerations in non-technical language as well. [20]

2.a) What is meant by the bias of an estimator ? Is the sample variance an unbiased estimator of the population variance, when a srswr is available ? Is

$$\sqrt{\frac{1}{n-1}\Sigma (x_i - \bar{x})^2}$$

an unbiased estimator of the population s.d.? If not, what is the sign of its bias?

b) What is meant by the bias of a statistical test? Give an example of a biased test. [3+4+5+8]=[20]

3.a) A srswr is to be drawn from a population with s.d. known to be 40 units, approximately. How large must the sample size be in order that the sample mean may lie within ± 2 units of the true mean with probability 0.90? Do you need any additional assumption for computing this size?

b) A die is suspected to be loaded in favour of six . In order to examine this, it is proposed to throw the die 8 times and to infer that it is biased in favour of six if the number of sixes exceeds 3. Calculate the level of significance of the test and also its power against the alternative hypothesis that the probability of occurrence of a six at any throw is 1/4.

GROUP B

4.a) If $x \sim N(0, \sigma^2)$, what is the p.d.f. of x^2 ? How is this result modified if $x \sim N(\mu, \sigma^2)$, where $\mu \neq 0$?

b) Let x and y be two independent N(0, 1)-variates. Find the distribution of the ratio x^2/y^2 and hence that of the ratio x/y, (Prove all the necessary results) [8+12]=[20]

5. Derive the joint sampling distribution of $\bar{x}$ and s^2 based on a random sample from a normal population $N(\mu, \sigma^2)$. Hence obtain the distribution of the statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}},$$ [14+6]=[20]

6. Suppose that x_1, x_2, x_3 are independent N(0, 4) variables. Obtain, using statistical tables, the probabilities

(i) $P(x_1 - 2x_2 + 3x_3 > -1)$, (ii) $P(4x_1^2 + 4x_2^2 > 7)$,

(iii) $P(x_1^2 - 2x_2^2 - 2x_3^2 \leq 0)$.

You may use linear interpolation for the calculations but the procedure with underlying theory must be clearly explained in each case. [20]

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PERIODICAL EXAMINATION

Calculus

Date: 2.12.74 Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

Note: Answer any five. All questions carry equal marks.

1.a) A function is defined as follows:

$$\begin{aligned} \phi(x) &= x^2 && \text{when } x < 1 \\ &= 2.5 && \text{when } x = 1 \\ &= x^2 + 2 && \text{when } x > 1 \end{aligned}$$

Does $\lim_{x \to 1} \phi(x)$ exist ? Is $\phi(x)$ continuous at $x = 1$?

b) Find the values of

i) $\lim_{x \to 0} \frac{x^2 \sin(1/x)}{\sin x}$, (ii) $\lim_{x \to \infty} \frac{\sin x}{x + \cos x}$.

2. Find the differential co-efficients of the following with respect to x

i) $x^n e^x$, (ii) $x \tan x \log x$, (iii) $2^x \sin x$,

iv) $\frac{1 + \sqrt{x}}{1 - \sqrt{x}}$, (v) $\frac{x^3 - 2 + x^{-3}}{x - 2 + x^{-1}}$.

vi) $e^{ax} \sin bx$, (vii) $\sec(\tan^{-1} x)$, (viii) x^{x^x}.

3. The volume of a right circular cone remains constant. If the radius of the base increases at the rate of 3 inches per second, how fast is the altitude changing when the altitude is 8 inches and radius 6 inches ?

4. If $u = \cos^{-1}(x+y)/(\sqrt{x}+\sqrt{y})$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

5. Find the approximate value of $\sin 60^\circ\ 34'\ 23''$ to 4 places of decimals from the expansion of $\sin(x+h)$ in a series of ascending powers of h by putting

$$x = \frac{\pi}{3}\ (= 60^\circ) \quad \text{and} \quad h = \frac{1}{100}$$

of a radian (= 34' 23" nearly).

6. A cylindrical tin can closed at both ends of a given capacity has to be constructed. Show that the amount of tin required will be minimum when the hight is equal to the diameter.

7. Show that the tangents to the curve

$$3x^2 + 4xy + 5y^2 - 4 = 0$$

at the points in which it is intersected by the lines

$$3x + 2y = 0 \quad \text{and} \quad 2x + 5y = 0$$

are parallel to the axes of co-ordinates.

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PERIODICAL EXAMINATION

Date: 6.1.75 Maximum Marks: 50 Time: 2 hours

Note: The paper carries 55 marks. Answer as many questions as you can. Maximum you can score is 50. Marks allotted for each question are given in brackets [ ].

1. Prove that (i) implies (ii) and vice versa

   i) If A is a nonempty set of real numbers which is bounded above, then A has a supremum.

   ii) If A is a nonempty set of real numbers which is bounded below, then A has an infimum. [5+5]=[10

2. Let A and B be two bounded sets of real numbers with a = supremum of A and b = supremum of B. Let C denote the set

$$C = \{x+y : x \in A, \ y \in B\}.$$

   Then prove that a + b = supremum of C. [8

3. Define the set N of natural numbers in terms of the real numbers.

   Prove that N is not bounded above (State clearly the axioms, results you may like to use). [2+4]=[6

4. i) Define an ordered field.

   ii) If F is an ordered field how do you define $x < y$ for $x, y \in F$? Show that $0 < 1$ in F where 0 and 1 have their usual connotations in the field F.

   iii) Prove that the field of complex numbers can not be made into an ordered field. [3+1+3+5]=[12

5. Define the set I of integers.
   Define the set Q of rational numbers.
   When do you call a nonempty subset A of R, the set of real numbers, dense in R. Prove that Q is dense in R.

[1½+ 1½+2+8]=[13

6.a) What is wrong with the following 'proof' that the set of all intervals of positive length is countable?
'Let $\{x_1, x_2, \ldots\}$ denote the countable set of rational numbers and let I be any interval of positive length. Then I contains infinitely many rational points x_n, but among these there will be one with smallest index n. Define a function F by means of the equation $F(I) = n$, if x_n is the rational number with smallest index in the interval I. This function establishes a one-to-one correspondence between the set of all intervals and a subset of the positive integers. Hence the set of all intervals is countable'.

   b) Show that any collection of disjoint intervals of positive length is countable. [3+3]=[6]

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PERIODICAL EXAMINATION

Physics

Date: 13.1.75 Maximum Marks: 60 Time: 2½ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. What is radioactivity ? Deduce the radioactive decay law and hence find an expression for the half-life of a radioelement. What is the relation between half-life and average life?

   The half-life of radium is 1590 years. In how many years will 1 g. of radium be reduced to 1 centigram?
   [4+4+2+4+6]=[20]

2. What are the different processes for the emission of electrons from atoms? Deduce Einstein's photoelectric equation. Show how the relation could be used to determine experimentally the value of Planck's constant.

   The work function of a particular emitter is 2.0 eV and light of wavelength 3000 A is used to cause emission. Find the stopping potential and the velocity of the most energetic electrons.
   [3+4+3+5]=[15]

3. Give an elementary treatment of Bohr's theory of the hydrogen spectrum. Write the expression for the Balmer series. Explain the significance of the series limit.
   [10+3+2]=[15]

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PERIODICAL EXAMINATION

Index Number and Time Series
(Theory and Practical)

Date: 20.1.75 Maximum Marks 100 Time: 3 hours

Note: The paper carries 125 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Discuss the problems of comparing prices between two distant time periods. Also, discuss in this context the suitability of the chain base index number.

b) The table below gives the wholesale prices (p) and quantities (q) of four cereal items in India during 1951-1954.

Item	1951		1952		1953		1954	
	p	q	p	q	p	q	p	q
Rice	16.9	26.97	17.5	22.54	17.5	28.10	16.7	24.21
Wheat	18.6	6.09	23.7	7.38	21.9	7.90	16.4	8.54
Jowar	10.1	5.98	11.9	7.24	12.0	7.95	11.3	9.09
Bajra	10.2	2.31	13.3	3.14	14.5	4.48	11.5	3.66

Calculate the price indices $P_{1951,1954}$, $P_{1952,1954}$

i) using paasche type binary index number; and

ii) using Laspeyre type chain index number. [12+13]=[25]

2.a) Show that both Laspeyre's and Paasche's formula of price index number give biased estimates of the corresponding true cost of living index.

b) The following table gives (i) the group price index numbers and corresponding weights and (ii) average annual money income of working class in Calcutta during 1948-1951. Calculate the nature of change in real income of working class in Calcutta during this period.

Year	Price Index No. (base 1939=100)			Average money income (Rs.)
	Food	Clothing	Others	
1948	370.1	423.3	290.2	650.00
1949	387.2	440.4	305.6	800.00
1950	394.0	432.9	303.4	792.00
1951	397.0	551.4	320.7	820.00
Weights	71.3	2.9	25.8	-

[13+12]=[25]

3.a) Discuss the method of indicators for constructing an index number of quantum of industrial production for a country.

b) Compute the quantum index by method of indicators for the year 1950-51 with 1949-50 as base using the following data.

Sector	Value added in 1949-50	Value of indicator	
		1949-50	1950-51
Sector 1	4137	5306	5570
Sector 2	824	5284	5543
Sector 3	7563	154189	170635

[20+5]=[25]

p.t.o.

4.a) Write notes on.

i) variate difference method, (ii) moving average technique of estimating trend of an observed time series.

b) Fit a polynomial trend of appropriate degree to the following data.

t	1	2	3	4	5	6	7	8	9	10
y_t	10.1	8.2	7.9	9.2	11.3	13.8	16.0	17.9	18.2	15.9

[12+13]=12

5.a) What are seasonal indices? How would you construct seasonal indices when the pattern of seasonality change smoothly over time?

b) Fit a logistic trend curve to the following data.

t	1	2	3	4	5	6	7	8	9	10	11	12
y_t	3.9	5.3	7.2	9.6	12.9	17.1	23.2	31.4	39.8	50.2	62.9	76

{10+15}=12

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PERIODICAL EXAMINATION

Sample Surveys

Date: 27.1.75 Maximum Marks: 100 Time: 3 hours

Note: Answer any two questions from Group A and One question from Group B. Marks allotted for each question are given in brackets [ ].

GROUP A

1.a) If from a simple random sample of n units drawn without replacement, a random sub-sample of n^* units is selected witnout replacement, duplicated, and added to the original sample, prove that the mean t^* based on tne $n+n^*$ units is an unbiased estimator of the population mean. Also show that tne variance of t^* is greater than the variance of the mean based on the original sample of n units by an approximate factor

$(1 + 3n^*n^{-1})(1 + n^*n^{-1})^{-2}$. For what value of n^*/n does the relative loss in efficiency attain its maximum value?

b) A survey was conducted in a village consisting of 625 households by covering a sample of 50 households selected using a simple random sampling without replacement scheme to estimate the average monthly expenditure on toilet goods. Tne estimate was found out to be Rs.4.20 with a standard error of 0.47. Using this information determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample to be selected by a simple random sampling with replacement scheme such that the length of the confidence interval at 95% confidence level is 20% of the true value. State clearly the assumptions involved in finding out the sample size.

[16+12]=[28

2.a) Explain with illustrations what you understand by the terms 'inclusion probability of a unit U_i' and 'inclusion probability of the pair (U_i,U_j)' for a sampling design.

b) Give the unbiased estimator $\hat{Y}_{HT}$ of the population total $Y = \sum_1^N Y_i$ of a characteristic y taking values Y_i on the unit U_i, $i = 1,2,\ldots, N$ as proposed by Horvitz and Thompson.
State its variance.

c) Write down the estimators of tne variance of $\hat{Y}_{HT}$ suggested by Horvitz and Thompson $\hat{V}_{HT}(\hat{Y}_{HT})$; and Yates and Grundy $\hat{V}_{YG}(\hat{Y}_{HT})$, for a fixed sample size n.

d) Suppose that auxiliary information on a characteristic x, taking values X_i on the unit U_i, $i = 1,2,\ldots, N$, highly correlated with y is available. Show that $\hat{V}_{YG}(\hat{Y}_{HT})$ is positive when the sampling design is such that the first unit in the sample is selected with probability proportional to its X-value and the remaining (n-1) units are selected with equal probabilities and without replacement from the remaining (N-1) units of the population.

[3+6+7+12]=[28

3.a) For stratified simple random sampling (without replacement) to estimate the population mean, write down (no derivation required) the optimum allocation of a fixed total sample size n to the strata. How does one use this

p.t.o

allocation in practice? Give a theoretical justification.

b) If we deviate from the optimum allocation by using a sample of size $\tilde{n}_i$ in the ith stratum, show that the proportional increase in variance cannot exceed δ^2, where δ is the maximum deviation $|\tilde{n}_i - n_i|$, expressed as a fraction of $\tilde{n}_i$. Illustrate this result by an example.

c) Let W_i be the proportion of population units and let $S_i^2 = N_i\sigma_i^2 / N_i - 1$, where σ_i^2 is the within variance for the ith stratum. Obtain an expression for the variance of the estimate of the population mean based on an allocation of the sample size proportional to $W_i S_i^\alpha$ where α is a real number in $[0,2]$. Hence show that the allocation in (a) is superior to this allocation with respect to the variance criterion. Comment on the case $\alpha = 2$. [11+8+9]=[28]

4. As a consultant on 'sampling techniques' indicate the suggestions you would offer and the advice you give on the following problems (precise mathematical statements will gain more credit for your answer):

a) A school teacher having planned to estimate the average consumption of sweets in a week for all the children of her school, got hold of the complete list (in alphabetical order) of all the 1217 school children and decided to take a sample of size 120. Using this list the teacher had selected a 'linear systematic sample' with the random start 5. She then computed the sample average of the number of sweets consumed during the week as an estimate of the average consumption for the whole school. Her problem is to know if the estimate is valid and if not, how a valid estimate could be obtained. She is also interested in estimating the sampling error of the estimate from her sample.

b) Square plates of different areas are produced in an industrial process and a quality control engineer collected data on the area for a large number of plates during a shift using a machine that accurately measures the area. It was thought that a sample of plates selected with probability proportional to the length of the edge of the plate (with replacement) would suffice for further inspection on the study variable (number of defects on each plate). You are contacted to suggest a simple method of selecting the sample as required and an estimation procedure.

c) A scientist plans to estimate the average yield of a characteristic on the basis of a sample survey as precisely as possible. Auxiliary information on another characteristic is available before hand and he has reason to believe that there is high positive correlation between the characteristic under study and the auxiliary characteristic. You are approached to suggest suitable sampling methodology. [9+9+10]=[28]

p.t.o.

Group B

The table given below presents the summary of data for all the 112 villages in a certain tehsil in West Bengal. The villages are stratified by size of their agricultural area into 3 strata, as shown in column (2) of the table. The number N_i of villages in different strata is given in column (3). Column (4) gives the number of villages n_i selected from the ith stratum and in column (5) the sampling design used in the stratum is given. The yield of jute for the selected villages and auxiliary information where available are given in column (6) of the table:

Stratum	Size of the village in acres of agricultural area	N_i	n_i	Sampling design	Yield of jute y_{ij}	Auxiliary information agricultural area x_{ij}
(1)	(2)	(3)	(4)	(5)	(6)	
I	$\leq$ 40	51	6	Probability proportional to agricultural area with replacement	75 101 5 78 79 45	25 39 2 26 26 16
II	40 - 80	38	4	Circular systematic sampling	Sub-sample 1 247 238 359 125	Sub-sample 2 256 214 368 141
III	> 80	23	2	Probability proportional to agricultural area without replacement	427 326	147 101

It is also known that the total agricultural area of villages in the first, second and third stratum are 1268, 3443 and 3112 acres respectively.

a) Estimate the total yield of jute for the tehsil.

b) Obtain an estimate of the variance of your estimate in (a). [19+25]=[44]

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PERIODICAL EXAMINATION
Economics

ate:3.2.75 Maximum Marks: 50 Time: 2 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

Group A : Micro-economics

Answer all the questions. Maximum Marks:25

a) Given that the utility of a consumer is a function of quantities consumed of two goods (with given prices), find out the (first-order) condition for the maximisation of the consumer's utility subject to a budget constraint. [5]

b) How the total effect on the demand for a good brought about by a fall in its own price is decomposed into an income effect and a substitution effect? Prove that the substitution effect has a definite sign in this case ? [7+3]=[10]

c) Can you similarly find income and substitution effectson the demand for a good when the price of the other good varies ? Can you say anything about the sign of the substitution effect in this case ? [6+4]=[10]

(Note: For (b) and (c), you may assume a two-good world)

Group B : Macro-economics

Answer all the questions. Maximum Marks: 25

Consider a closed economy with the following consumption function

$$C = 80 + 0.8\ Yd$$

where C is consumption expenditure and Yd is disposable income. Let net investment (I), government expenditure (G), total tax revenue (T_x) and total government transfer payments (T_r) be exogeneously fixed at the levels

$$I = 90$$
$$G = 100$$
$$T_x = 95$$
$$T_r = 5$$

Using the above data and the information given below answer the following questions.

Find the following:

i) the equilibrium level of net national product. [3]

ii) the values of investment, government expenditure and tax collection multipliers. [3]

iii) the 'leakage function' i.e., total leakage [private savings (s) plus net tax ($T_x - T_r$)] as a function of net national product (Y). [2]

p.t.o.

If the full-employment net national product were projected to be 1200 and the government wanted to achieve this level:

i) by how much should government expenditures (G) be increased if other things remain unchanged? [2]

ii) What should be the level of net taxes ($T_X - T_r$) if both I and G are kept at their original levels? [2]

Suppose both G and T_X are increased by 10. Will there be any change in the equilibrium level of net national product? If there is any, calculate its numerical value. What is the value of the multiplier in this case? Give explanation for your result. [5]

Instead of considering T_X and T_r to be autonomously given at the levels 95 and 5 respectively, assume now that net taxes are 25 per cent of net national product i.e., $T = T_X - T_r) = .25Y$. Then calculate

i) the equilibrium level of net national product. [2]

ii) the government expenditure multiplier. [2]

Instead of assuming net investment (I) to be autonomously given at 90 suppose now that it is responsive to the level of income and is given by $I = 10 + 0.1Y$. Then calculate the government expenditure multiplier and explain the reason for the magnitude of this multiplier being different from the corresponding one calculated in A(ii). [4]

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MID-YEAR EXAMINATION

Sampling Distributions and Statistical Methods(T+P)

te: 13.2.75 Maximum Marks: 100 Time: 4 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

Let $\chi^2_n = x_1^2 + x_2^2 + \ldots + x_n^2$ where the x_i's are independent $N(0,1)$ r.v.'s. Derive the expression for the probability density of the χ^2_n-distribution, by the method of induction. [12]

EITHER

Given two samples from two univariate populations, we frequently use (Fisher's) t-test for examining the equality of the two population means. Justify the procedure, stating all underlying assumptions and deriving the sampling distribution of the t-statistic. (State all the theoretical results you use.) [20]

OR

Write a note on the Fisher-Behrens problem, mentioning the different test procedures suggested in this connection. [20]

Assuming that the true correlation coefficient is zero, obtain the sampling distribution of the sample correlation coefficient r based on a random sample of size n from a bivariate normal population. [20]

Write a short note on any one of the following:

i) The combination of probabilities test.

ii) The test of $H_0 : \sigma_x^2 = \sigma_y^2$ where (x,y) $BN(.,.,.,.,.)$ based on a random sample of n observation-pairs. [10]

Two compositors each composed five pages of a book. The numbers of mistakes found in the proof pages were 3 and 8 for the two compositors. Making appropriate assumptions (to be stated) test whether the two workers were equally reliable. [7]

Eight boys and eight girls sat for a certain examination. The scores obtained by them are presented below:

Boys:	56	37	79	22	59	43	62	47
Girls:	29	65	68	34	46	57	63	40

Compare the means and variabilities of the two groups of candidates, applying necessary tests of significance. [14]

In a study on the correlation between degree of industrialization and the extent of inequality in level of living, the observations for the 16 states of India yielded the value : r = 0.37. Test the significance of this finding. [7]

Practical records. [10]

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MID-YEAR EXAMINATION

Sample Surveys (Theory and Practical)

Date: 14.2.75 Maximum Marks: 100 Time: 4 hours

Note: Answer any two questions from Group A and any one from Group B. Marks allotted for each question are given in brackets [ ].

1.a) Distinguish between sampling and non-sampling errors in a survey. List down the various sources of non-sampling errors in a survey. Also discuss briefly the methods of assessment and control of sampling and non-sampling errors.

b) Based on a sample of size n selected from a population of size N using a simple random sampling without replacement scheme estimate the proportion of units in the population which possess a particular attribute. Also obtain an unbiased estimator of the variance of this estimator.

c) If from a simple random sample of n units drawn without replacement, a random sub-sample of n' units is selected without replacement, duplicated, and added to the original sample, prove that the mean t' based on the $(n+n')$ units is an unbiased estimator of the population mean $\bar{Y}$. Also show that the variance of t' is greater than the variance of the mean based on the original sample of n units by an approximate factor $(1+3n'n^{-1})(1+n'n^{-1})^{-2}$. For what value of n'/n does the relative loss in efficiency attain its maximum value? [10+8+10]

2.a) What is the difference between 'selection probability of a unit' and 'inclusion probability of a unit'?

b) Give an unbiased estimator of the population total Y of a characteristic $\underset{\sim}{Y}$, taking values Y_i on the unit U_i, $i = 1,2,\ldots, N$ as proposed by Horvitz and Thompson.

c) Derive an expression for the variance of the above estimator in (b) and show that it cannot be uniformly minimised for all values of $\underset{\sim}{Y} = (Y_1, Y_2, \ldots, Y_N)$.

d) Show further that, under a super-population set up, when auxiliary information on a characteristic $\underset{\sim}{x}$ taking values X_i on the unit U_i, $i = 1,2,\ldots, N$ highly correlated with $\underset{\sim}{y}$ is available, there exists an optimum class of designs best suited for use in conjunction with the Horvitz-Thompson estimator, for which the conditional expectation of the above variance in (c) is uniformly minimised. [3+3+9+13]

3. Let $\underset{\sim}{y}$ be a study variable and $\underset{\sim}{x}$ be an auxiliary variable highly correlated with $\underset{\sim}{y}$ taking values Y_i and X_i respectively on the unit U_i, $i = 1,2,\ldots, N$. For estimating the population total $Y = \sum_{i=1}^{N} Y_i$ consider the ratio type estimator $\hat{Y}_R = \left(\sum_{i\in s} y_i / \sum_{i\in s} x_i\right)\left(\sum_{i=1}^{N} X_i\right)$ where the symbol $\sum_{i\in s}$ denotes summation over the n units of the sample.

a) Show that $\hat{Y}_R$ is not unbiased for Y.

P.t.o.

Suggest a sampling scheme which makes $\hat{Y}_R$ unbiased for Y and prove that for this scheme $\hat{Y}_R$ is unbiased for Y.

As an alternative estimator consider $\hat{Y}' = \sum_{i \in s} y_i/\pi_i$, where π_i is the probability of inclusion of the ith unit in the sample based on the sampling scheme of (b) and show that $\hat{Y}'$ also is unbiased for Y.

Show that the estimator of variance for $\hat{Y}'$ suggested by Yates and Grundy is non-negative. [4+8+4+12]=[2

Suppose that n villages are selected at random and without replacement from a population consisting of N villages each consisting of M_i households, i = 1,2,..., N. Let $\bar{Y}_i = \sum_{j=1}^{M_i} Y_{ij}/M_i$ be the ith village mean, where Y_{ij} is the value taken by the study variable y on the jth household of the ith village, j = 1,2,..., M_i; i = 1,2,..., N. Write down (i) the conventional unbiased estimator and (ii) other 'unbiased' ratio type estimators which you can think of for estimating the population mean

$$\bar{Y} = \sum_{i=1}^{N} M_i \bar{Y}_i \Big/ \sum_{i=1}^{N} M_i$$

Which one of the estimators do you prefer?

Under what conditions is the 'ratio method of estimation' preferable to the 'mean per unit' (conventional) method for estimating the population mean of a characteristic'. (Derive the expressions involved stating clearly the assumptions and justify your claim). [14+14]=[

Group B

Answer any one question.

For estimating the total Y of current population in a region, two sub-samples of 6 villages each are selected (circular) systematically from each stratum with independent random starts. Using the data given in the table obtain a ratio estimate for Y taking the previous census population (x) as the auxiliary information and compare its efficiency with that of the conventional unbiased estimate.

Total number of villages (N) and sample totals of x and y.

Stratum number	No. of villages N	Sub-sample 1		Sub-sample 2	
		x	y	x	y
1	2044	3722	3935	3456	3641
2	1304	3625	4033	4171	4649
3	1265	2769	3050	3746	4043

(total of x for the region = 3,155,680)

An experienced farmer makes an eye estimate of the weight of peaches x_i on each tree in an orchard of N = 200 trees. He finds a total weight of X = 11,600 lbs. The peaches are picked and weighed on a simple random sample (without replacement) of 10 trees and the following results are obtained:

p.t.o.

	Tree Number									
	1	2	3	4	5	6	7	8	9	10
Actual weight y_i	61	42	50	58	67	45	39	57	71	53
Estimated weight x_i	59	47	52	60	67	48	44	58	76	58

(i) Find the regression estimate $\hat{Y}$ of Y, the total weight of peaches in the population and estimate its variance.

(ii) As an alternative estimate if $\hat{Y}' = N \left\{ \bar{Y} + (\bar{G} - \bar{x}) \right\}$ is taken, obtain the precision of $\hat{Y}$ over $\hat{Y}'$.

(25 + 19) = [44]

A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as the first stage units and plots within them as second stage units. From each stratum 4 villages were selected with probabilities proportional to area and with replacement and 4 plots were selected from each selected village with equal probability and without replacement. The data on yield for the sample plots together with information on selection probabilities are given in the table below :

stratum	sample village	inverse of probability of selection	total no. of plots	yield of sample plots			
				1	2	3	4
I	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	115.38	76	346	350	157	119
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	138
	3	24.76	222	264	78	144	55
	4	49.99	69	300	114	68	111
III	1	67.68	189	110	281	120	114
	2	339.14	42	80	61	118	124
	3	100.00	134	121	212	174	106
	4	68.07	161	243	116	314	129

Total number of plots in stratum I, II and III respectively are 8423, 6355, 12853.

Using the above data (i) obtain an unbiased estimate of the total yield of paddy in the district.

(ii) Obtain an unbiased estimate of the sampling variance of the above estimate.

(iii) Also compare the efficiency of the above design with that of unistage simple random sampling with replacement of plots in each stratum.

(12 + 17 + 15) = [44]

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MID-YEAR EXAMINATION

Calculus

Date: 15.2.75 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. All questions carry equal marks.

Find the asymptotes of the following curve

$$x^3 - 2y^3 + xy(2x - y) + y(x-y) + 1 = 0$$

Show that the asymptotes of the curve

$$x^2y^2 - a^2(x^2+y^2) - a^3(x+y) + a^4 = 0$$

form a square two of whose angular points lie on the curve

If the asymptotes of the curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$(h^2 > ab)$$

pass through the origin, prove that $af^2 + bg^2 = 2fgh$.

Find the evolute of the parabola

$$y^2 = 4ax .$$

If (α, β) be the co-ordinates of the centre of curvature of the parabola

$$\sqrt{x} + \sqrt{y} = \sqrt{a}, \quad \text{at } (x, y),$$

then $\alpha + \beta = 3(x + y)$.

Find the radius of curvature at the point 'θ' on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

Show that for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$ the radius of curvature at an extremity of the major axis is equal to half the latus rectum.

Prove that the curves

$$\frac{x^2}{a} + \frac{y^2}{b} = 1 \quad \text{and} \quad \frac{x^2}{a'} + \frac{y^2}{b'} = 1$$

will cut orthogonally if

$$a - b = a' - b' .$$

In the catenary $y = c \cosh\left(\frac{x}{c}\right)$, show that the length of the perpendicular from the foot of the ordinate on the tangent is of constant length.

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MID-YEAR EXAMINATION

ECONOMICS

ate: 17.2.1975. Maximum Marks: 100 Time: 3 hours.

[Note: Answer Group A and Group B in separate answer books]

GROUP A

Answer any two questions. Marks allotted to each question are given in brackets.

Consider a consumer with a given income to be allocated on two goods with given prices. Suppose, his utility function is given by

$$u = q_1^{\alpha} q_2^{\beta} \qquad \alpha, \beta > 0$$

where q_i is the quantity of the ith good consumed by him.

[5]

(a) Derive the (first-order) conditions for the maximisation of the consumer's utility.

(b) Will the (second-order) conditions for utility maximisation be satisfied ? What do these conditions mean in the context of an indifference curve ?

(4 + 6) = [10]

(c) Derive his demand curve for two goods. Can you say anything about the income and price elasticities of demand for two goods ?

(5 + 5) = [10]

What is meant by an 'inferior' good ? Show that a good is a 'Giffen' good only if it is an inferior good. (A good is called a Giffen good if its demand falls (rises) in response to a fall (rise) in its own price).

[25]

On the reverse

GROUP B

Answer any two questions.

how how the theory of risk - avoiding behaviour of individual investors an provide a basis for liquidity preference. Mention the major limitaions of this theory.

[25]

escribe the derivation of IS and LM curves and explain how you would etermine general equilibrium in the product and money markets in a acro-economic model. Examine, in the light of the above, the positions aken by Keynes and the classicists with regard to the effectiveness of onetary and fiscal policies.

[25]

onsider an economy in which the product and money markets are in quilibrium but there exists involuntary unemployment. Discuss the lassical and Keynesian theories about the effects of adjustments in he labour market on the product and money markets. Would these djustments necessarily lead to full employment ?

[25]

A firm produces a single good with the help of two inputs (with given prices). If the firm wants to produce a given amount of output, what are the (first order) conditions for its cost of production to be a minimum ? Can you give an economic interpretation of these conditions? [5+5]=[10]

What is meant by 'elasticity of substitution' between inputs ? Given that the production function is homogeneous of degree one in two inputs, show that the elasticity of substitution is unity only if the production function is Cobb-Douglas. [3+12]=[15]

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MID-YEAR EXAMINATION

PHYSICS

: 18.2.1975. Maximum Marks: 100 Time: 3 hours.

)escribe in detail Millikan's oil-drop method of measuring the electronic :harge, giving a sketch of the apparatus used. What is the present :ccepted value of electronic charge ? What further experiment need be :ade for the measurement of the electronic mass ?

(10 + 5 + 3 + 2) = [20]

hat is Raman effect ? Give an elementary theory for the Raman effect nd describe an experimental arrangement for its study in liquids.. ompare and contrast Raman neffect and Compton effect.

(4 + 6 + 5 + 5) = [20]

n incident photon hits a free electron of mass m at rest. As a result f the collision, the photon loses a part of its energy and is deviated om its original direction by an angle/while the electron recoils with velocity v at an angle/with the direction of the incident photon. ssuming the collision is elastic and is governed by the laws of conser- ation of momentum and energy, derive an expression for the difference tween the wavelengths corresponding to the incident and the scattered oton.

[20]

at is the basic principle underlying the production of x-rays ? scribe a modern x-ray tube. What are the properties of x-rays ? what uses are these radiations put ?

(4 + 6 + 5 + 5) = [20]

ite explanatory notes on the following:

) Photo-electricity

) Natural radioactivity

) Bohr's model of hydrogen atom

(6 + 6 + 8) = [20]

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MID-YEAR EXAMINATION

INDEX NUMBERS AND TIME SERIES

Date: 19.2.1975 Maximum Marks: 100 Time: 3 hours.

[Answer all questions]

1.(a) What is seasonality in time series analysis ? Discuss any one of the methods of estimating seasonal indices ?

(b) Calculate seasonal indices for the following data :

Quarterly Receipts of State Governments in India
(Rs. Crores)

Year	Quarter			
	I	II	III	IV
1952	144	51	49	60
1953	156	62	67	60
1954	161	64	65	75
1955	173	71	74	81

(10 + 15) = [25]

2.(a) Discuss briefly the method of periodogram analysis of cyclical movements of an observed time series.

(b) Fit a Harmonic curve $y_t = \alpha + \beta \sin \frac{2\pi t}{T} + \gamma \cos \frac{2\pi t}{T}$ to the following data taking T = 8 .

t	y_t	t	y_t	t	y_t	t	y_t
1	+0.1	7	-2.3	13	+0.5	19	+2.4
2	+1.5	8	-1.7	14	-1.2	20	+1.3
3	+2.2	9	0	15	-1.8	21	+0.3
4	+1.7	10	+2.0	16	-1.4	22	-1.9
5	+0.2	11	+3.0	17	-0.5	23	-2.7
6	-1.8	12	+2.1	18	+1.0	24	-2.2

(15 + 15) = [30]

3. Write short notes on (any three)

(a) Slutsky - Yule effect.

(b) Sturel's Index Number formula.

(c) Variate - Difference Method.

(d) Deflator method of Calculating Index of Industrial production.

(3 x 13) = [39]

Practical class work. [6]

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MID-YEAR EXAMINATION

Analysis

Date: 20.2.75 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) Find the limit of $n^{-n-1}(n+1)^n$.

b) Find the limit of $a_n = \int_1^{u_n} \frac{1}{x}dx$ where $u_n = (\frac{n+1}{n})^n$ and hence show that $a_n < 1$ for each n. [4+7]=[11]

(State precisely the results you may like to use).

2. Prove that, for every real x, the infinite series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges. Let $f(x)$ stand for the sum $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Show that the convergence is uniform over any bounded interval. Prove that $f(x)$ is continuous everywhere on the real line R. Also prove that $f'(x) = f(x)$ for all $x \in R$. (State precisely the results you are going to use) [5+5+5+8]=[23]

Prove that

$$-\log(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots ; \quad |x| < 1.$$

Sum the series

$$2x\cos y + \frac{3}{2}(x\cos y)^2 + \frac{4}{3}(x\cos y)^3 + \frac{5}{4}(x\cos y)^4 + \ldots ; \quad |x| < 1.$$

[7+5]=[12]

Define two sequences $\{f_n\}$ and $\{g_n\}$ as follows:

$$f_n(x) = x(1 + \frac{1}{n}) \quad \text{if } x \in R, \quad n = 1, 2, \ldots$$

$$g_n(x) = \begin{cases} 1/n & \text{if } x = 0 \text{ or } x \text{ is irrational} \\ b + \frac{1}{n} & \text{if } x \text{ is rational, say } x = \frac{a}{b},\ b > 0. \end{cases}$$

Let $h_n(x) = f_n(x)g_n(x)$. Show that both $\{f_n\}$ and $\{g_n\}$ converge uniformly on every finite interval. Show that $\{h_n\}$ does not converge uniformly on any finite interval. [3+3+4]=[10]

Give illustration in each of the following situations:-

a) A sequence of real-valued functions converging to a limit function uniformly throughout the entire real line.

b) A sequence of real-valued continuous functions converging to a discontinuous function.

c) Let $\{f_n\}$ be a sequence of real-valued function having a derivative at each point of an open interval S. Let $f_n(x)$ converge to $f(x)$ for each $x \in S$. Assume that

p.t.o.

there exists a function g on S such that $f_n' \to$ uniformly on S. If S is unbounded f_n need not converge uniformly to f on S.

d) A function of 2 variables can have both the partial derivatives at a point and yet not be continuous at the point.

e) Let f be defined on an open set S in R^2 and assume that the 2 partial derivatives $D_1 f$ and $D_2 f$ exist. The condition that the 2 partial derivatives be continuous at a point x in S is by no means a necessary condition for the existence of the differential at x. [4+3+6+4+3]

6. Let $f_n(x) = \frac{1}{nx+1}$ if $0 < x < 1$, $n = 1, 2, 3, \ldots$

a) Show that $\{f_n\}$ converges pointwise but not uniformly on $(0, 1)$.

b) Find $f_n'(x)$ and show that $\{f_n'\}$ converges pointwise to some function on $(0, 1)$.

c) From (i) and (ii) above can you conclude whether the convergence in (ii) is uniform ? [3+(2+2)+3]

7. Write 'True' or 'False' for each of the following statements:

a) If $g_n(x) = \frac{x}{nx+1}$, $0 < x < 1$, $n = 1, 2, 3, \ldots$

then $g_n \to 0$ uniformly on $(0, 1)$.

b) There exists no uncountable subset of R whose Lebesgue measure is zero.

c) $\frac{1}{3}$ is a dyadic rational.

d) A proper subset of rationals in $[0, 1]$ may be dense in $[0, 1]$.

e) A rational number may be Transcendental.

f) Contor's ternary set contains no interval. [6x2]

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MID-YEAR EXAMINATION

GEOLOGY

Date: 24.2.1975. Maximum Marks: 100 Time: 3 hours.

[Answer any four questions. All questions carry equal value]

1. Write short notes on any two of the following:

   (a) Origin of the earth's atmosphere.

   (b) Tidal hypothesis of the origin of the earth.

   (c) Nebular hypothesis of the origin of the earth.

2. Write a short essay on the internal constitution of the earth as known from seismic evidence.

3. How are earthquakes caused in nature ? How can they be recorded ? What are the main types of earthquake waves ?

4. Define : mineral, crystal, crystalline substance, amorphous substance. State whether quartz, feldspar, bauxite, pearl, water, pyroxene, mercury are minerals or not.

5. What is an igneous rock ? Describe the common forms in which igneous rocks occur in nature.

6. What is radioactivity ? Name the different radioactive methods generally used for determination of the age of the rocks. What are the major shortcomings of these methods ?

7. Write short notes on any four of the following:

   Magma, Lava, Plutonic rock, Volcanic rock, Sedimentary rock.

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PERIODICAL EXAMINATION

Analysis

e: 7.4.75 Maximum Marks: 50 Time: 2½ hours

Note: There are 4 questions carrying 55 marks
Maximum one can score is 50

a) State and prove Dini's theorem

b) Give an example to show that in Dini's theorem the closed interval cannot be replaced by an open interval

c) If $s(x) = \sum_{n=1}^{\infty} f_n(x)$ is a series of continuous positive terms in a given closed interval, a necessary condition that $s(x)$ should be continuous is that the series should be uniformly convergent over the interval. [8+2+10]=[20]

a) State clearly Abel's theorem for real power series. Demonstrate the application of this theorem by means of a suitable example.

b) Is the direct converse of Abel's theorem true?

Give reasons. State and prove the theorem which is a sort of converse of Abel's theorem. [3+3+2+8]=[16]

Define a real power series about the point zero. State precisely the theorem which asserts conditions of convergence and divergence of a given power series about zero. Also define the radius of convergence of a power series. 2+4+2]=[8]

Let $f_n(x) = (\frac{1}{n})e^{-n^2x^2}$ if $x \in R$, $n = 1,2,$

Show that $f_n \to 0$ uniformly on R, that $f_n' \to 0$ pointwise on R, but that the convergence of $\{f_n'\}$ is not uniform on any interval containing the origin. [4+7]=[11]

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PERIODICAL EXAMINATION

Psychology

Date: 21.4.75 Maximum Marks: 100 Time: 3 hours.

Note: Answer any six questions of which question number 1 is compulsory. Marks allotted for each question are given in brackets ( ).

. What is aptitude ? What is its relation with intelligence, and indicate the difference between aptitude test and achievement test.? (20)

. Examine the respective roles of reward and punishment in learning, using experimental evidences. (16)

. Discuss James-Lange theory of Emotion. In what respect the Cannon-Bard theory differ from the former? Indicate the physiological changes associated with emotion. (16)

. What do you mean by item difficulty and item discrimination? Discuss the possible use of item analysis in test development and for developing scoring keys for inventories. (16)

. Describe some of the important tests for assessment of personality, indicate how far these are suitable in selection situations. (16)

. Discuss the factors involved in efficiency and fatique. Mention some physiological indicators of fatique. (16)

. Discuss the effect of assembled groups on the individual. Indicate the difference between a crowed and a mob. (16)

. What are endocrine glands ? What effect they have on our behaviour ? (16)

. To what extent is behaviour dependent on heredity and maturation ? Which characteristics of the individual are dependent on heredity ? Discuss. (16)

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PERIODICAL EXAMINATION

Calculus

Date: 28.4.75 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

Integrate the following:

i) $\int \frac{e^x - 1}{e^x + 1}\,dx$, (ii) $\int \frac{\sin 2x}{a \sin^2 x + b \cos^2 x}$, (iii) $\int \sqrt{\frac{x}{a-x}}\,dx$

iv) $\int \{f(x)\,\phi'(x) + \phi(x)f'(x)\}\,dx$. [20]

Integrate $\frac{1}{2}f'(x)$ with respect to x^4 where

$$f(x) = \tan^{-1}x + \log \sqrt{1+x} - \log \sqrt{1-x}$$ [15]

Integrate the following

i) $\int \frac{dx}{\sqrt{x^2 + x - 2}}$, (ii) $\int \sqrt{\frac{x-3}{x-4}}\,dx$ (iii) $\int \frac{dx}{x\sqrt{1+x^3}}$

iv) $\int \frac{dx}{(x+2)\sqrt{1+x}}$. [20]

If $u = \int e^{ax} \cos bx\,dx$, $v = \int e^{ax} \sin bx\,dx$ prove that

i) $\tan^{-1}\frac{v}{u} + \tan^{-1}\frac{b}{a} = bx$

ii) $(a^2+b^2)(u^2+v^2) = e^{2ax}$ [10]

Integrate $\int \frac{x^2 + x + 1}{\sqrt{x^2 + 2x + 3}}\,dx$. [10]

Integrate (i) $\int \frac{dx}{a \sin x + b \cos x}$, (ii) $\int \frac{dx}{(x-1)^2(x-2)^3}$ [10]

Find the area above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. [15]

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PERIODICAL EXAMINATION

Sampling Distributions and Statistical Methods (Th.+ P.)

Date: 5.5.75 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

Prove the reproducive property of the non-central χ^2-distribution.

a) State and prove the Fisher-Cochran theorem.

b) Let $y_i \sim N(\mu_i, 1)$, $i = 1, 2, \ldots, n$, be independent random variables and $y = (y_1, \ldots, y_n)'$. Suppose $y'A_1y \sim \chi^2(a)$ and $y'A_2y \sim \chi^2(b)$ are two quadratic forms in y. Prove that they are independently distributed only if $A_1A_2 = 0$. [14+8]=[22]

Starting from the law of large numbers, show that g_1, the sample coefficient of skewness, is a consistent estimator of the corresponding population parameter γ_1. Show also that its asymptotic distribution is normal. (You need not prove any major theorem which becomes necessary.) [10+6]=[16]

Write a brief account of any one: (a) The square root transformation of the Poisson variate (b) The non-central t-distribution.

A random sample of 30 observations from a bivariate population yielded the value of the correlation coefficient $r = 0.5$. Set up a 99 % confidence interval for the true correlation coefficient ρ, stating briefly the theory underlying your procedure.

Random samples were drawn independently from three populations yielding the following estimates of respective popution variance:

population	sample size	estimated variance
1	22	4.82
2	37	6.24
3	28	3.98

Examine whether the three population variances could reasonably be taken as equal. [14]

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PERIODICAL EXAMINATION
Economics

2.5.75 Maximum Marks: 60 Time: 2 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks: 40

Answer any two questions.

Assume that the long-run average cost curve of every firm is 'U' shaped but the average cost curves of some firms are at a higher level than that of the others. Does it imply that the firms with higher average cost functions will necessarily be eliminated in long-run competitive equilibrium? [20]

Show that monopoly equilibrium is possible with decreasing marginal cost curves. With decreasing marginal cost, is it possible for the monopolist to make positive profit by equating price with the marginal cost of production? [20]

Discuss the assumptions under which the game theoretic approach leads to a determinate equilibrium in duopoly. [20]

Group B: Maximum Marks: 20

Answer any one question

Distinguish between different types of planning on the basis of different degrees of state intervention; explain the main features of each type. [20]

Review the aim, approach and the results of the first five Year Plan of India. [20]

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PERIODICAL EXAMINATION

Linear Estimation

Date: 19.5.75 Maximum Marks: 80 Time: 2 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

The underlying model throughout is $(Y, X\beta, \sigma^2 I)$ in the standard notation.

a) Derive the Normal Equations.

b) Show that the Normal Equations are consistent.

c) Characterize all estimable parametric functions.

d) Show that $P'\hat{\beta}$, where $\hat{\beta}$ is any solution of the Normal Equations, is the minimum variance unbiased linear estimator of $P'\beta$, provided $P'\beta$ admits an unbiased estimator.

e) Derive an expression for R_0^2, the residual sum of squares.

[5+5+5+10+5]=[30]

Let $R_0^2 = \min_{\beta} (Y - X\beta)'(Y - X\beta)$

Show that

a) the minimum is achieved when $X\beta$ is the projection of Y on $M(X)$,

a) $R_0^2 \sim \sigma^2 \chi^2_{n-r}$ where r = rank (X)

c) $R_0^2/_{n-r}$ is an unbiased estimator of σ^2. [5+10+5]=[20]

Take $X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

A) Show that

i) $P'\beta$ is estimable iff P is of the form

$$P' = (a, b, b),$$

ii) $P'\beta$ is non-estimable iff P is of the form

$$P' = (c, d+e, d-e) \text{ with } e \neq 0,$$

iii) the minimum variance unbiased estimators are of the form $L'Y$ with

$$L' = (f, g, f-g, -f+g),$$

iv) the zero functions are of the form $L'Y$ with

$$L' = (h, -h, k, h+k).$$

[In the above a, b, c, d, f, g, h, k are arbitrary real numbers and e is a non-zero real number.]

B) Take $Y' = (5, 8, 4, 3)$ and

i) find the estimates of $\beta_1, \beta_2 + \beta_3$,

ii) compute R_0^2 and estimate σ^2,

iii) find the least sum of squares when the β's are subject to the restriction $\beta_2 - \beta_3 = 0$.

Is this same as R_0^2 of (ii)? Why ?

[5+5+5+5]+[2+4+4]=[30]

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PERIODICAL EXAMINATION

Anthropology

: 26.5.75 Maximum Marks: 50 Time: 2 hours

Note: Answer any three questions. All questions carry equal marks.

Discuss the scope, aim and objectives of Anthropology.

Give an outline of human evolution in prehistoric period.

Make a survey of the living races of the world and their main physical characters.

Describe the racial composition of the peoples of India.

Write notes on:

i) Anthropometry,
ii) Endogamy,
iii) Culture,
iv) Proto-Australoid,
v) Genetic markers,
vi) Palaeoanthropology,
vii) Homo Sapiens, and
viii) Artefact.

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PERIODICAL EXAMINATION

Design of Experiments

3.75 Maximum Marks: 100 Time: 3 hours

ter The paper carries 110 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

t are the basic principles a good experimental design uld satisfy? Illustrate your answer with an example. [20]

t are the advantages and disadvantages of a Randomized ck Design over a Completely Randomized Design?

e the complete analysis for a two-way classification el when equal number (c>1) of observations are avai- le in each cell (obtain the Normal equations for esti- ing the effects, Sum of Squares, Mean Squares, Expected n Squares, Blank Anova table, F-tests and Discussion).

ment on the case when c = 1 in (b) above [5+25+5]=[35]

t is a factorial experiment? In what respect is it ferent from a single-factor experiment ?

a 2^3 factorial experiment, define the terms 'main ect' and 'interaction effect'. Give the contrasts to culate these effects and indicate how the number of ares of the effects are calculated from the contrasts. [5+10]=[15]

5x5 latin square experiment was carried out to compare effects of five different manurial treatments A,B,C, D E on the yield of wheat. The crop on one of the plots lost accidentally. The plan and the yields (in certain ts) of the other plots are given below. Analyse the a and draw your conclusions.

Yields in manurial experiment with one plot missing.

B -	D 6.4	E 3.3	A 9.5	C 11.8
C 9.3	A 4.0	B 6.2	E 5.1	D 5.4
D 7.6	C 15.4	A 6.5	B 6.0	E 4.6
E 5.3	B 7.6	C 13.2	D 8.6	A 4.9
A 9.3	E 6.3	D 11.8	C 15.9	B 7.6

o obtain the standard error of the difference between mean of the treatment B and of C. [26+14]=[40]

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PERIODICAL EXAMINATION

Design of Experiments

.75 Maximum Marks: 100 Time: 3 hours

e: The paper carries 110 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

t are the basic principles a good experimental design uld satisfy? Illustrate your answer with an example. [20]

t are the advantages and disadvantages of a Randomized ck Design over a Completely Randomized Design?

e the complete analysis for a two-way classification el when equal number ($c>1$) of observations are available in each cell (obtain the Normal equations for estiing the effects, Sum of Squares, Mean Squares, Expected n Squares, Blank Anova table, F-tests and Discussion).

ent on the case when $c=1$ in (b) above [5+25+5]=[35]

t is a factorial experiment? In what respect is it ferent from a single-factor experiment ?

a 2^3 factorial experiment, define the terms 'main ect' and 'interaction effect'. Give the contrasts to culate these effects and indicate how the number of ares of the effects are calculated from the contrasts. [5+10]=[15]

5x5 latin square experiment was carried out to compare effects of five different manurial treatments A,B,C, D E on the yield of wheat. The crop on one of the plots lost accidentally. The plan and the yields (in certain ts) of the other plots are given below. Analyse the a and draw your conclusions.

Yields in manurial experiment with one plot missing.

B -	D 6.4	E 3.3	A 9.5	C 11.8
C 9.3	A 4.0	B 6.2	E 5.1	D 5.4
D 7.6	C 15.4	A 6.5	B 6.0	E 4.6
E 5.3	B 7.6	C 13.2	D 8.6	A 4.9
A 9.3	E 6.3	D 11.8	C 15.9	B 7.6

o obtain the standard error of the difference between mean of the treatment B and of C. [26+14]=[40]

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ANNUAL EXAMINATION

Sampling Distributions and Statistical Methods: Theory and Practical

7.75 Maximum Marks: 100 Time: 4 hours

ote: Answer all the questions. Marks allotted for each question are given in brackets [].

iven a random sample of observations $(x_1,y_1),\cdots,(x_n,y_n)$ rom a bivariate normal population, how would you test the ill hypotheses

$$H_0: \sigma_x^2 = \sigma_y^2 ?$$

istify the procedure stating necessary theoretical results. [12]

:tempt any two of the following:

If X and Y are independent beta variables with parameters (r_1,δ_1) and (r_2,δ_2) show that the distribution of $U = XY$ is also beta with parameters $(r_2,\delta_1+\delta_2)$ if $r_1 = r_2+\delta_2$.

Prove the reproducive property of the non-central χ^2-distribution.

Let $y_i \sim N(\mu,\sigma^2)$; $i = 1,2,\cdots, n$, be independent r.v's, and let $x_1,x_2,\cdots,x_n$ be constants. Consider

$$r = \sum_i y_i(x_i-\bar{x})/\sqrt{\sum_i (x_i-\bar{x})^2 \sum_i (y_i-\bar{y})^2}\ .$$

Show that $r = Z/\sqrt{Z^2+W}$, where $Z \sim N(0,1)$ and $W \sim \chi^2(n-2)$ are independent. Hence find the sampling distribution of r.

Let $y_i \sim N(\mu_i,1)$, $i = 1,2,\cdots, n$, be independent r.v.'s. Let $y' = (y_1,\cdots, y_n)$ and A be a symmetric nxn matrix. Show that the quadratic form $y'A\,y$ has a chi-square distribution if and only if A is idempotent, and that, in that case, the d.f. of X is rank A = trace A. [2x12]=[24]

nd the asymptotic distribution (a.d.) of the sample rst quartile Q_1 when the population sampled is normal. ate what you know about the joint a.d. of sample first d third quartiles, Q_1 and Q_3. [14]

ite short notes on any one of the following:
The Kolmogorov-Smirnov two-sample tests of homogeneity.
The angular transformation of the binomial proportion. [10]

swer any three of the following:

The following table shows the mathematics scores obtained by a group of students in the Test and School Final examinations:

p t o

td.)

r.no.of tudent	1	2	3	4	5	6	7	8	9	10
est score:	32	58	64	81	54	66	42	48	40	72
F score:	42	56	59	86	68	69	43	57	42	77

et up 95 % confidence limits for the average rise in cores between the two examinations. [1

) The following data are based on a public opinion survey and shows the distribution of one group of sample respondents:

	drinkers	non-drinkers
In favour of local option on the sale of liquor	2	12
against local option	5	7

Do these indicate association between drinking habits and opinion on the subject of local option ? [10

The table below shows the sex distribution of skulls excavated from a locality in three different seasons:

	season		
	first	second	third
male	162	180	210
female	110	125	200

Test whether the sex-ratio is 1 : 1(i) in each season and also (ii) on the whole, taking the three seasons together. Is the sex ratio the same for all the seasons (though not necessarily 1 : 1)? [5+5]=[10

The following are the correlation coefficients between intelligence test scores of brothers/twins obtained in a research investigation. Comment on the relative influences of environmental and hereditary factors applying suitable tests of significance.

	two brothers		twins	
	reared apart	living together	reared apart	living together
sample size	50	40	45	55
correlation coefficient	0.235	0.342	0.451	0.513

[10]

actical Record [10]

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ANNUAL EXAMINATION

Economics

18.7.75 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A: Indian Economic Problems

Maximum Marks: 50

Answer Q.1 and any two from the rest. All questions carry equal marks.

Show that the approach, objectives and perspective of the 2nd Five Year Plan are deducible from the experience of the 1st Five Year Plan and ideas contained in the Avadi resolution on the socialistic pattern of society.

Indicate the main objectives of the expansion of the public sector in India. Explain how the public sector can act as an instrument in achieving the objective of the reductior of inequalities in income and wealth.

Examine the objectives and functions of the Industrial Development Bank of India

Discuss the main features of the recent foreign collaboration agreements as revealed in the Survey of Reserve Bank of India.

Group B: Micro economics

Maximum Marks: 50

Answer any three questions. All questions carry equal marks.

Examine the nature of monopoly equilibrium in the following cases:

a) the elasticity of the demand curve facing the monopolist is always less than unity;

b) the monopolist sells in two different markets.

Examine the relation between average variable cost and marginal cost in a short-run competitive situation. What would be the supply curve of a firm ?

Distinguish between monopoly and monopolistic competition. Compare the long-run equilibrium under monopolistic competition with that under perfect competition.

How will you represent duopoly equilibrium on the assumption that each seller takes his rival's output as given and tries to maximise profit ? How will the equilibrium change if the number of sellers increases ?

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ANNUAL EXAMINATION

Linear Estimation and ANOVA (Theory and Practical)

21.7.75 Maximum Marks: 100 Time: 4 hours

Note: Answer Q.5 and any three of the rest. Marks allotted for each question are given in brackets [].

Consider the model $(Y, X\beta, \sigma^2 I)$.

a) Obtain the normal equations and show that they are consistent.

b) Show that the least squares estimator of any estimable parametric function is uncorrelated with all zero functions.

c) Show that $P'\hat{\beta}$ is linear in Y and unbiased for $P'\beta$, iff $P'\hat{\beta}$ is unique for all solutions $\hat{\beta}$ of the normal equations.

d) Let $\lambda'X'Y$ and $\mu'X'Y$ be the least squares estimators of estimable parametric functions $P'\beta$ and $R'\beta$ respectively. Show that

$$\text{cov}\,(\lambda'X'Y, \mu'X'Y) = \sigma^2\lambda'R = \sigma^2\mu'P. \quad [5+5+5+5]=[20]$$

Consider the model $(Y, X\beta, \sigma^2 I)$. Let $R_0^2 = \text{Min}\,(Y-X\beta)'(Y-X\beta)$ where β's are unrestricted, and let $R_1^2 = \text{Min}\,(Y-X\beta)'(Y-X\beta)$ when β's are subject to the restrictions $h_i'\beta = \xi_i$, $i = 1, 2, \cdots, t$.

a) Assuming that $\Sigma c_i h_i'\beta$ is non-estimable for all non-zero $c' = (c_1, \cdots, c_t)$, show that $R_0^2 = R_1^2$.

b) Assuming that Y_i's are independently and normally distributed, show that $R_0^2 \sim \sigma^2\chi^2_{n-r}$ where $r = \text{rank}\,(X)$. [10+10]=[20]

Consider the setup $(Y, X\beta, \sigma^2 G)$ where X is a known matrix and G is a known positive-definite matrix. Show that by a suitable transformation of the variable Y this setup can be reduced to the setup $(Y, X\beta, \sigma^2 I)$.

Consider the setup $(Y, X\beta, \sigma^2 G)$ with or without restrictions on the parameter β. Let $P_1'\hat{\beta}, \cdots, P_k'\hat{\beta}$ be the individual least squares estimators of the parametric functions $P_1'\beta, \cdots P_k'\beta$, all assumed to be estimable. Let A be the dispersion matrix of these estimators and let B be the dispersion matrix of any other set of unbiased estimators of $P_1'\beta, \cdots, P_k'\beta$. Show that the matrix B-A is non-negative definite.

Discuss the weighing design problem and show that maximum precision is attained when each entry of the observation matrix X is +1 or -1 and the columns of X are orthogonal. [6+7+7]=[20]

p t o

sider the model $(Y, X\beta, \sigma^2 I)$ and assume that Y's are lependently and normally distributed. Let

$= \text{Min } (Y - X\beta)'(Y - X\beta)$ when the β's are unrestricted and

$R_1^2 = \text{Min } (Y - X\beta)'(Y - X\beta)$ when β's are subject to $H'\beta =$

re $\mathcal{M}(H) \subset \mathcal{M}(X')$.

Show that $Z = H'\hat{\beta}$ (where $\hat{\beta}$ is any solution of the nconditional normal equations) and R_0^2 are indepen- lently distributed.

how that $R_1^2 - R_0^2 = (Z - \underline{r})'D^{-1}(Z - \underline{r})$ where $\sigma^2 D$ is he dispersion matrix of Z. [10+10]=[20]

e important measurements from which the log cranial city (y) of a skull may be predicted are the log ella-occipital length (x_1), the log maximum parietal dth (x_2), and the log basio-bregmatic height (x_3). i on the measurements on n = 86 skulls from Farringdon et series obtained by Hooke, the following values are ated:

3·1685, $\bar{x}_1 = 2.2752$, $\bar{x}_2 = 2·1523$, $\bar{x}_3 = 2·1128$

$$(S_{ij}) = \begin{pmatrix} 0·01875 & -0·00848 & 0·00684 \\ & 0·02904 & 0·00878 \\ & & 0·02886 \end{pmatrix}.$$

: 0·03030, $S_{02} = 0.04410$, $S_{03} = 0·03629$

~~0·02781~~ 0·1269

$S_{ij} = \Sigma (x_{ir} - \bar{x}_i)(x_{jr} - \bar{x}_j)$, $S_{0i} = \Sigma(y_r - \bar{y})(x_{ir} - \bar{x}_i)$

$S_{00} = \Sigma(y_r - \bar{y})^2$.

nverse (S^{ij}) of the matrix (S_{ij}) is partly given

$$(S^{ij}) = \begin{pmatrix} 64·21 & -15·57 & * \\ & 41.71 & * \\ & & 39·88 \end{pmatrix}$$

:imate the unknowns in the prediction formula

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 .$$

form a statistical test to see if there is a signi- :ant association between the dependent and indepen- .t variables.

t the hypothesis: $\beta_1 = \beta_2 = \beta_3$. [40]

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ANNUAL EXAMINATION.

Calculus

'.75 Maximum Marks: 100 Time: 3 hours

e: Answer any six questions. All questions are of equal value.

w that the tangents at the ends of a focal chord of the dioide $r = a(1 + \cos\theta)$ intersect at right angles on a cle of radius $3a/2$ and centre

$$(\tfrac{a}{2}, 0).$$

p_1 and p_2 be the perpendiculars from the origin on tangent and normal respectively at the point (x,y) and

$$\tan\psi = \frac{dy}{dx},$$ prove that

$$p_1 = x\sin\psi - y\cos\psi$$
$$p_2 = x\cos\psi + y\sin\psi$$

e prove that $p_2 = \frac{dp_1}{d\psi}$.

the asymptotes of the cubic curve

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0 ;$$

the locus of the centres of curvature of the parabola $4ay$.

the envelope of a line of constant length which slides its extremities upon two fixed rods at right angles to other.

that the area of the greatest rectangle inscribed in ren ellipse and having its sides parallel to the axes ie ellipse is to that of the ellipse as $2 : \pi$.

the area common to the cardioide $r = a(1 + \cos\theta)$ and :ircle $r = \frac{3}{2}a$, and also the area of the remainder of :ardioide.

:urves $y = 4x^2$ and $y^2 = 2x$ meet at the origin O and ie point P, forming a loop. Show that the straight OP divides the loop into two parts of equal area.

that the length of the arc of the parabola $y^2 = 4ax$ is intercepted between the points of intersection e parabola and the straight line $3y = 8x$ is

$2 + \frac{15}{16}$).

n the intrinsic equation of the catenary

$\cosh\frac{x}{c}$ in the form $s = c\tan\psi$.

p t o

olve the following differential equations

a) $\frac{dy}{dx} + \frac{y}{x} = \sin x^2$

b) $\frac{dy}{dx} - 2y \tan x + y^2 \tan^4 x = 0.$

ow that the solution of

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \mu x = 0$$

$x = e^{-\frac{1}{2}kt}(A \cos nt + B \sin nt)$ if $k^2 < 4\mu$,

l $n^2 = \mu - \frac{1}{4}k^2$.

ıd the curve for which the curvature is zero at every nt.

egrate the following

) $\int \sin^2 x \cos^5 x dx$ (ii) $\int \frac{dx}{(x^2+a^2)(x+b)}$

) $\int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$ (iv) $\int \sin^4 x \cos^2 dx.$

, that

$$\int_0^{\pi/2} (a \cos^2 x + b \sin^2 x)dx = \frac{\pi}{4}(a+b)$$

$$\int_0^{\pi/4} \log (1 + \tan \theta)d\theta = \frac{\pi}{8} \log 2$$

$$\int_0^{\pi} \frac{x\, dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi^2}{2ab} \quad (a, b > 0).$$

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}.$$

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ANNUAL EXAMINATION

Design of Experiments (Theory and Practical)

Date: 25.7.75 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ]

1 Discuss the various methods of increasing the accuracy of controlled experiments. Illustrate your answer with examples. [15]

2.a) What are the advantages and disadvantages of the Latin Square Design over the randomized block design.

b) You are required to plan a Latin Square Experiment on Sugar Cane for comparing 5 fertilisers: A = No fertiliser, B = complete inorganic fertiliser, C = 10 tons of manure/acre, D = 20 tons of manure/acre, E = 30 tons of manure/acre. The 1/40 acre experimental plots (25 in number) are laid out in 5 rows and 5 columns. Prepare a randomised lay-out for this experiment, stating explicitly the method of random selection including appropriate references to random number permutations.

c) Explain how you would estimate the missing yield corresponding to the λth row, μth column and νth treatment accidentally lost in a k x k Latin Square Experiment. Give the corresponding ANOVA table (Analysis of variance table) explaining clearly the entries for this design (with one missing value). Also derive the standard error of the difference between the mean of the treatment with the missing value and the mean of any other treatment.

[4+11+6+12+7]=[40]

3. A number of experiments have indicated that electrical stimulation may be helpful in preventing the wasting away of muscles that are denervated. A factorial experiment on rats was conducted in order to learn something about the best type of current and the most effective method of treatment. The factors and levels were:

m :	no. of treatment periods daily	1 and 3
n :	length of treatment (minutes)	1 and 2
p :	type of current	Galvanic and Faradic

The weights of denervated muscle in units of 0.01 gms. for four groups of eight subjects each are given below (in usual notation for the treatments):

p t o

Treatment	Group I	Group II	Group III	Group IV
(1)	32	43	33	45
m	47	41	49	40
n	26	36	28	39
p	29	39	31	42
mn	61	76	64	80
mp	51	34	55	36
np	36	31	38	33
mnp	76	65	81	69

a) Compute the Analysis of Variance table isolating the sum of squares and mean squares attributed to various sources of variation and the corresponding degrees of freedom.

b) Discuss the implications of the significance (or otherwise) of the main effects and interactions.

c) Write a brief report on your findings. [23+12+10]=[45]

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ANNUAL EXAMINATION

Real Analysis

28.7.75 Maximum Marks: 100 Time: 3 hours

Note: [There are 5 questions carrying 102 marks. Maximum one can score is 100]

Let a_n, $n = 0,1,2,\ldots$ be a decreasing sequence of real numbers converging to zero. Define b_n as follows

$$b_n = a_n \quad \text{if } n \text{ is even}$$
$$= -a_n \quad \text{if } n \text{ is odd}$$

Prove that the power series $\sum_{n=0}^{\infty} b_n x^n$ is uniformly convergent on the interval [0, 1]. [10]

Does the convergence of both repeated series $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f(m,n)$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} f(m,n)$ imply their equality? If yes, give reasons, if no, cite an example. State a case in which the two repeated series are equal. Also supply the proof. [3+7]=[10]

Define the Cauchy product of two infinite series. State and prove Mertens' theorem on the convergence of the Cauchy product of two infinite series.

Give an example to show that the Cauchy product of two conditionally convergent series may fail to converge.

Let Σa_n and Σb_n be two convergent series such that their Cauchy product Σc_n also converges. Then establish that

$$\Sigma c_n = (\Sigma a_n)(\Sigma b_n).$$ [2+10+8+8]=[28]

Define a metric space.

i) Let d be a metric on a set X. Define e as follows

$$e(x,y) = d(x,y) \quad \text{if } d(x,y) < 1$$
$$= 1 \quad \text{if } d(x,y) \geq 1.$$

Show that e is also a metric on X.

ii) Let X be a non-empty set. Suppose g is defined as follows

$$g(x,y) = 0 \quad \text{if } x = y$$
$$= 1 \quad \text{if } x \neq y$$

Prove that g is a metric on X.
Also show that any subset of X is open in the metric space (X,g).

i) Define a Cauchy sequence in a metric space (X,d). When do you call the metric space (X,d) complete ? Give an example of a metric space which is not complete.

p t o

4.c) (contd.)

ii) Let R stand for the set of real numbers. Let B(R) = The set of all bounded real valued functions on R.

Define $\partial(f,g) = \sup_{x \in R} |f(x)-g(x)|$; $f, g \in B(R)$.

Show that ∂ is a metric on B(R). Also show that $\partial(f,g) = \partial(f-g, 0)$ where 0 stands for the function which is identically zero.

Prove carefully that $(B(R), \partial)$ is a complete metric space.

iii) Let C(R) denote the set of all bounded continuous functions on R. Using the last part of (ii) supply a proof of the fact that $(C(R), \partial)$ is a complete metric space. [3+5+(2+4)+(3+2+2)+(4+2+8)+7]=[42]

Investigate the existence of the two iterated limits and the double limit of the double sequence f defined by

i) $f(p,q) = \frac{1}{p+q}$

ii) $f(p,q) = (-1)^{p+q}\left(\frac{1}{p} + \frac{1}{q}\right)$ p, q positive integers

iii) $f(p,q) = (-1)^{p+q}$.

iv) $f(p,q) = \frac{(-1)^p}{q}$. [3 x 4]=[12]

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ANNUAL EXAMINATION

Sociology

Date: 30.7.75 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.1 and any four questions from the rest. All questions carry equal marks.

1. EITHER

What is the relation between sociology and (a) history, (b) economics, (c) anthropology ?

OR

Define sociology. 'Sociology is the science of society'- Discuss. To what extent is quantification an essential of science ?

2. What is family ? How does it differ from a household? What chief interest of society is served by the social and economic functions of family ?

3. Analyse the nature of the family and its various types.

4. Elucidate the concept of social stratification. Describe briefly the essential elements of caste. How do you distinguish between a caste and a social class ?

5. Define marriage, and its different forms. What are the causes behind the practices of exogamy and endogamy?

6. What do you mean by the term 'Kinship'? What are classificatory and descriptive kinship terms ? Describe briefly the importance of kinship in simple and modern societies.

7. Write notes on (any four):

Social group, Bow and arrow ceremony, Twice born, Moiety, Lineage, Society.

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ANNUAL EXAMINATION

Demography

1.8.75 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts Answer all questions from Group A and any three questions from Group B. All questions carry equal marks in each group.

Group A: Maximum Marks: 50

Briefly discuss the advantages of sample surveys for collecting demographic data.

Discuss the importance of demographic data in the context of social and economic planning.

Group B: Maximum Marks: 50

Explain the difference between a 'rate' and a 'ratio' in Vital Statistics.

Define the 'mean population' of an area during a given period.

What do you mean by the 'force of morality' at age x ?

How does a 'current' life table differ from a 'generation' one ?

What are the different measures of mortality in a life table? Show how they are inter-related, stating necessary assumptions.

Derive the 'death rate above age x' in a life table population.

Define and interpret the Net Reproduction Rate. State the important drawbacks of this rate as a measure of the prospects of population growth. Suggest a few improvements over this rate.

How do population projections differ from population estimates? Explain the modified G.P. method for obtaining population estimates. Delineate important steps in the component method for population projection.

Starting with suitable assumptions, derive the logistic law of population growth. Interpret the parameters in the law. Indicate how does Rhodes' method for fitting the logistic curve differ from Fisher's method?
