INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1970-71
PERIODICAL EXAMINATION

[301]

Mathematics-5: Mathematical Analysis

Date: 21.9.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) Define
i) Relation
ii) Injective relation
iii) Surjective relation
iv) Function

b) Show that if R is injective and surjective then R^{-1} is an injective function. [5+5]=[10]

2. F is a function from X into Y. Prove or disprove the following:

i) for subsets $A, B \subset X$, $F(A \cup B) = F(A) \cup F(B)$

ii) for subsets $A, B \subset X$, $F(A \cap B) = F(A) \cap F(B)$

iii) for subsets $C, D \subset Y$, $F^{-1}(C \cap D) = F^{-1}(C) \cap F^{-1}(D)$

iv) for subset $C \subset Y$, $F(F^{-1}(C) = C$

v) for subset $A \subset X$, $F^{-1}(F(A)) = A$

[2+2+2+2+2]=[10]

3.a) Let J be an integral domain with finitely many elements. Show that for integer n, n.1 = 0.

b) Show that a finite integral domain can not be ordered. [5+5]=[10]

4.a) State and prove the division algorithm.

b) Show that if $S \subset Z$ is a subset closed under addition and subtraction then $\exists$ an integer $d \geq 0$ such that $S = \{nd: n \in Z\}$.

c) Define g.c.d. of a set of integers $\{a_1, a_2, \ldots, a_k\}$. Show that if $[a_1, a_2, \ldots a_k]$ = positive g.c.d. of $\{a_1, a_2, \ldots a_k\}$, then $\exists\ n_1, n_2, \ldots, n_k \in Z \ni$

$$[a_1, a_2, \ldots, a_k] = n_1a_1 + n_2a_2 + \ldots + n_ka_k .$$

d) p is a prime and $p|a.b$ then $p|a$ or $p|b$.

e) Factor 100 into its prime factors. [4+4+4+4+2]=[18]

5. $(a_n)_{n=1}^{\infty}$ is a sequence of positive integers such that $a_{n+1} \leq a_n$ for all n. Show that $\exists$ an integer N such that $a_n = a_N$ for all $n \geq N$.
(Use the fact that $\mathcal{P}$ is well ordered). [12]

6. Show that the equation $x^2 + 1 = 0$ has a solution in Z_2 but does not have a solution in the field of rational numbers [12]

7.a) Let x and y be rational numbers such that $x < y$. Then $\exists$ a rational number z such that $x < y < z$.

b) Show that $x^2 = 2$ has no solution in the field of rational numbers.

c) Show that field of rational numbers is not complete.

[6+6+6]=(

8.a) Define equivalence relation.
Define a partition of a set X and show that it gives rise to an equivalence relation on X.

b) Which of the following are equivalence relations on the set of real numbers?

i) $xRy \iff x-y$ is a rational

ii) $xRy \iff x-y$ is a prime

iii) $xRy \iff x-y$ is irrational.

[5+5]=(

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PERIODICAL EXAMINATION

Probability

Date: 28.9.70 Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. X has the uniform distribution in the interval [0, 1]. Find the distribution of $\sin(4\pi X)$. [20]

2. The p.d.f. (probability density function) of X is of the form

$$f(x) = c x^{\alpha-1} e^{-x/\beta}, \quad 0 < x < \infty,$$
$$= 0 \quad \text{elsewhere.}$$

Determine the constant c; find the moment generating function of X, and compute the mean and variance of X. [20]

3. X_1 and X_2 are independent variables; $X_1 + X_2$ and X_1 have chi-square (χ^2) distributions with n and n_1 degrees of freedom, respectively, where $n > n_1$. Prove that X_2 has a χ^2-distribution with $n-n_1$ degrees of freedom. [20]

[Hint: The pdf of X_1 is of the form

$$K\, x^{\frac{n_1}{2}-1} e^{-\frac{1}{2}x} \quad (x > 0).]$$

4. X_1 and X_2 are independent and $E(X_i) = \mu_i$, $V(X_i) = \sigma_i^2$, $(i = 1,2)$. Find the mean and the variance of the variable $X_1 X_2$. [10]

5. X_1 and X_2 are independent and each has the p.d.f. $f(x)$. In each of the following, determine the distribution of the variable Y.

(a) $f(x) = \frac{1}{2} e^{-x/2}, \quad x > 0.$

$Y = \frac{1}{2}(X_1 - X_2).$

(b) $f(x) = \theta^{-x} \log_e \theta, \; x > 0. \quad (\theta > 1).$

$Y = X_1 + X_2.$ [30]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Theory and Practical

Date: 5.10.70 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note:
Marks allotted for each question are given in brackets [ ].

Group A: 72 marks

Answer any three questions.

1. Find the sampling variance of the sample mean for srswor and show how it can be unbiasedly estimated. [24

2. (a) Define linear systematic sampling and circular systematic sampling. Discuss the advantages of the latter over the former.

   b) Mention briefly the major disadvatages of systematic sampling. [24

3. Consider stratified sampling with srswr within strata. Discuss how stratification improves the efficiency of the estimator of population mean. Compare in this connection stratified sampling with proportional allocation and Neyman allocation with unstratified srswr. [24

4. Consider two-stage sampling with srswr at both the stages Suppose n first stage units (fsu's) are selected from N fsu's, and m_i second stage units (ssu's) are selected from the M_i ssu's in the ith selected fsu $(i=1,2,\ldots,n)$.
What is the usual estimator of the population total? Show that it is unbiased and find its sampling variance. [24.

5. Write short notes on any two: (i) Estimation of a proportion (with standard errors) based on a simple random sample without replacement, (ii) BLUE of population mean for stratified sampling with srswr within each stratum, (iii) Lahiri's method of selection for pps sampling. [24.

Group B: 88 marks

6. The following shows the size distribution of factories by number of workers, along with the averages and standard deviations of output for factories in each size class. (These data are based on a pilot enquiry.)

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No. of workers	No. of factories	Average output (Rs. 000)	s.d. of output (Rs.000)
1- 49	18260	100	80
50- 99	4315	250	200
100-249	2253	500	600
250-999	1057	1760	1900
1000-	567	2250	2500

A sample of 3000 factories is proposed to be taken for estimating the total output of all factories. Compare the efficiencies of the following schemes of sampling: (i) (unstratified) srswr, (ii) and (iii) stratified sampling with proportional allocation and Neyman allocation, with the size classes taken as strata. (It is assumed that srswr will be followed for sampling within strata.)

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PERIODICAL EXAMINATION

General Science-4: Bio-chemistry (Theory)

Date: 26.10.70 | Maximum Marks: 50 | Time: $1\frac{1}{2}$ hours

Answer all the questions.

1. Describe the classification of different proteins. [15]

2. Write notes on:

   (a) Essential amino acids

   (b) Zwitterion

   (c) Isoelectric point. [10]

3. Describe the method for the quantitative estimation of protein. [10]

4. What is turn over number of an enzyme? Define enzyme unit Describe the preparation and properties of the following enzymes

   (1) Pepsin

   (2) Urease [15]

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PERIODICAL EXAMINATION

General Science-5: Sociology

Date: 2.11.70. Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Questions carry equal marks.

1. EITHER

Discuss in brief Durkheim's and Weber's ideas on subject-matter of sociology.

OR

Point out in brief the differences and similarities between (i) sociology and psychology and (ii) sociology and political science.

2. EITHER

What do you mean by the term 'family'? How a nuclear family differs from an extended one? What chief interest of the society is served by the social functions of family?

OR

Why typological approach is not always suitable to study a social phenomenon? Illustrate your answer with the help of a concrete example.

3. EITHER

Define marriage. What are the different forms of marriage prevalent among tribal societies? How do you account for such practices?

OR

What are consanguineal and affinal relationships? How classificatory terminology differs from descriptive one? What is the function of classificatory kinship terms?

4. EITHER

Define community. Illustrate the bases for community.

OR

'A phratry is composed of several clans' - Justify.

5. How can you classify the major changes in Indian Society since 1947?

OR

Show how history, inspite of its differences with sociology, can yet help sociological understanding.

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PERIODICAL EXAMINATION

Mathematics-3: Matrix Algebra

Date: 9.11.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Question 6 and any four from Questions 1 to 5. Marks allotted for each question are given in brackets [].

1.a) Define 'eigenvalues' and 'eigenvectors' of a (square) matrix. [5]

b) State and prove a necessary and sufficient condition for a matrix A to be similar to a diagonal matrix, in terms of the eigenvectors of A. [9]

c) Write down the companion matrix A of a polynomial $f(x)$ and prove that $f(x)$ is the minimal polynomial of A. [6]

2.a) Define 'inner product' in a real vector space. [4]

b) State and derive Schwartz's inequality. [9]

c) Show that a set of mutually orthogonal non-null vectors is a linearly independent set. [7]

3.a) Establish the correspondence between real quadratic forms and real symmetric matrices. [10]

b) State and prove Cochran's Theorem on quadratic forms. [10]

4.a) Show that if S is a subspace of an inner product space V, then the orthogonal complement $S^{\perp}$ of S is a subspace and that $S \cap S^{\perp} = \{\emptyset\}$ where $\emptyset$ is the null vector. [7]

b) Show that if A has distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ and if $\lambda_1^p, \lambda_2^p, \ldots, \lambda_n^p$ are distinct then the eigenvalues of A^p, where p is any positive integer, are $\lambda_1^p, \lambda_2^p, \ldots, \lambda_n^p$. [7]

c) Show that for any real matrix A, the form $X^T(A^TA)X$ is positive definite or positive semi-definite. [6]

5.a) Find a 3×3 orthogonal matrix with $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$ in the first column. [7]

b) Express the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

as the sum of a symmetric matrix and a skew-symmetric matrix. [5]

c) Find the minimum polynomial for the matrix

$$\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$$ [8]

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6. State, whether the following statements are true or false.

(a) If A_1 and A_2 are symmetric matrices, then $X^T A_1 X$ and $X^T A_2 X$ are identical in the x's iff $A_1 = A_2$.

(b) The congruence of matrices is an equivalence relation.

(c) Rank of a sum of quadratic forms is equal to the sum of the rank of the components.

(d) If $\alpha_1, \alpha_2, \ldots, \alpha_k$ are linearly independent eigenvectors of a matrix .. corresponding to eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$, then $\lambda_1, \lambda_2, \ldots, \lambda_k$ are distinct..

(e) The dimension of the eigensubspace of an eigenvalue λ is equal to the multiplicity of λ as a root of the characteristic polynomial. [15

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PERIODICAL EXAMINATION

Statistics-3: Data Processing

Date: 23.11.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Give the Card Code and BCD Code for the following

   DIPLØMAbBST [10]

2. You are given a pack of cardsaccording to the following design

	Description	Card Cols.
1)	CDI	1 - 4
2)	Roll No.	5 - 8
3)	Name	9 - 34

   Explain how you will arrange the cards in ascending order of Roll number. [10]

3. Are the following FØRTRAN statements valid? If invalid, give reasons.

i) A = 3* B	ii) A = I* B
iii) GØTØ II	iv) STØP 999
v) PRINT 8	vi) READ 1, 2I, I2
vii) IF (A* B+ C)22. 22	
viii) DOGMATA = AL + SINF(X)	
ix) Z = Z+ 1.	x) Y = B * * C .

[20]

4. Write a FORTRAN program to find the sum $\sum_{i=1}^{n} i!$ [15]

5. You are given a pack of cards as per design below

	Description	Card Cols.	
1)	C.D.I.	1 - 4	
2)	Serial number	5 - 8	
3)	x	9 - 11	(2 pl. dec.)
4)	y	12 - 15	(1 pl. dec.)

   i) Write a FORTRAN program to compute:

   $\bar{x}, \bar{y}, \sigma_x, \sigma_y, r_{xy}$

   ii) Give a flow chart. [45]

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PERIODICAL EXAMINATION

Economics-3

Date: 30.11.70 Maximum Marks: 100 Time: 3 hours

Note: All questions carry equal marks. Answer Groups A and B in separate answerscripts.

Group A: Economic Development

Maximum Marks: 70

Answer any three questions.

1. What is the nature of the distribution of income of nations? How is it possible to compare the national incomes of different countries? Indicate some difficulties of crude comparisons.

   Describe a relatively exact method of comparing purchasing powers of currencies of two countries.

2. Describe the long period growth of national income in the U.S.A. and show how one can study the contributions of different factor inputs to the growth of national product. Do the conventional factor inputs explain the entire growth?

3. Select any two distributions of national income, and describe the structural changes in these distributions associated with economic development.

4. Work out a simple model of economic growth and show how the model is used differently by planners and forecasters.

5. Write brief notes on any four of the following:

   i) vicious circles;

   ii) Morgenstern's observation on the error of rates of growth;

   iii) the relation between intersectral disparity and inequality of the size distribution;

   iv) the requirements of a general theory of economic development; and

   v) backward sloping supply curve of effort.

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Group B: Growth Models

Maximum Marks: 30

Answer <u>any two</u> questions.

6. If you are asked to build up a Harrodlike model of economic growth, what assumptions will you make? Spell out the implications of those assumptions.

7. What is 'warranted rate of growth'? Does such a rate exist in a Harrodian economy?

8. What do you mean by 'stability in the sense of Harrod'? Is the warranted rate stable in this sense?

9. 'Full employment (of capital and labour) in the Harrodian world is accidental'. Elucidate.

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MID-YEAR EXAMINATION

Mathematics-3: Analysis

Date: 21.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer <u>any eight</u> questions. Marks allotted for each question are given in brackets [ ].

1.a) Define: i) Integral domain
ii) Ordered integral domain
iii) Well ordered integral domain. [4]

b) Show that if J is an ordered integral domain and $x \in J$ is not equal to zero then x^2 is positive. [5]

c) Show that $\mathcal{C}$ can not be ordered. [3]

2.a) Prove that there is no rational number r such that
$$r^2 = 5.$$ [4]

b) Prove that if r is a rational number such that $r^2 < 5$ then there is a rational number $s > r$ such that $s^2 < 5$. [4]

c) If x and y are complex numbers then
$$\big|\, |x| - |y| \,\big| \leq |x - y|.$$ [4]

3.a) If z is a complex number such that $|z| = 1$, i.e. $z\bar{z} = 1$, compute $\big|\, |1+z|^2 + |1-z|^2$. [4]

b) Let $z_n = x_n + iy_n$, $x_n, y_n \in \mathbb{R}$. Show that $z_n \to z$ as $n \to \infty \iff x_n \to x$ and $y_n \to y$ as $n \to \infty$, where $z = x + iy$, $x, y \in \mathbb{R}$. [3]

c) Let $(x_n)_{n=1}^{\infty}$ be a sequence of positive real numbers such that
$$\lim_{n \to \infty} \frac{x_{n+1}}{x_n} = \lambda .$$
Show that $\lim_{n\to\infty} x_n^{1/n}$ exists and $= \lambda$. [5]

4.a) Define i) least upper bound
ii) upper limit of a sequence $(a_n)_{n=1}^{\infty}$ of real numbers. [4]

b) Show that if the l.u.b. λ of a bounded set A does not belong to A then λ is a cluster point of A. [3]

c) Let $(a_n)_{n=1}^{\infty}$ be a bounded sequence of real numbers. Prove or disprove the following statements:

i) $\overline{\lim_{n \to \infty}}\, a_n \leq \text{l.u.b.}\{a_1, a_2, a_3, \ldots\}$

ii) $\text{l.u.b.}\{a_1, a_2, a_3, \ldots\} \leq \overline{\lim_{n \to \infty}}\, a_n$. [5]

5.a) Show that every non-decreasing bounded sequence $(a_n)_{n=1}^{\infty}$ is convergent. [4]

b) Calculate $\lim_{n \to \infty} (\sqrt{n^2 + n} - n)$. [3]

c) Let $a_{n+1} = \sqrt{2 + \sqrt{a_n}}$, $a_1 = \sqrt{2}$.
Show that $\lim_{n \to \infty} a_n$ exists and is a solution of
$$\lambda^4 - 4\lambda^2 - \lambda + 4 = 0.$$ [5]

6.a) State and prove any <u>two</u> of the following three theorems:

i) Cantor intersection theorem
ii) Bolzano-Weierstrass theorem
iii) Heine-Borel Theorem. [8]

b) What are the cluster points of the set
$C = \left\{(-1)^k + \frac{1}{m} ; \ k \text{ and } m \text{ positive integers} \right\}$. [4]

7.a) Show that a closed subset of a compact set is compact. [4]

b) A compact set is closed. [4]

c) State which of the following sets in $\mathbb{R}$ are compact:

i) Set of all positive integers -
ii) AB^c where $A = [0,2]$, $B = \langle \frac{1}{2}, 1 \rangle$
iii) Rational number in the interval $0 \leq x \leq 1$.

Give reasons for your answers. [4]

8.a) Define connected set in a metric space (M,d). [5]

b) Show that if $A \subseteq \mathbb{R}$ is connected then $a, b \in A$ and $a < x < b \Rightarrow x \in A$. [7]

9.a) Show that if $(a_n)_{n=1}^{\infty}$ is a convergent sequence of real numbers then $(|a_n|)_{n=1}^{\infty}$ is also convergent. Show that the converse is not true. [6]

b) i) $a_n \to a$, $b_n \to b \Rightarrow a_n b_n \to a.b$ and $a_n + b_n \to a+b$.

ii) What can you say about $\left(\frac{a_n}{b_n}\right)_{n=1}^{\infty}$? [6]

10.a) Let A be the set of all sequences whose terms are the numbers zero or one. Show that A is not countable. [6]

b) Show that collection of rational numbers is countable.. [6]

11. Show that if a metric space is not sequencially complete then it is not compact. [12]

[Neatness] [4]

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MID-YEAR EXAMINATION

Mathematics-3: Matrix Algebra

Date: 23.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer FIVE questions. DO NOT attempt more than TWO out of Questions 1,2, and 3. Marks allotted for each question are given in brackets [ ].

1. Consider ONE of the following topics in matrix algebra:

(a) Determinants;

(b) Eigen values and Eigen vectors;

(c) Quadratic Forms.

Write down the main results on this topic. It is enough if you state the results clearly; no proofs are needed. [20]

2. Define the following terms and give ONE (nontrivial) example of each:

(a) Subspace of a vector space; [4]

(b) Inner product on a vector space; [4]

(c) Generalized inverse of a matrix; [4]

(d) Linear transformation; [4]

(e) Orthogonal matrix. [4]

3.a) State and prove the Cayley-Hamilton theorem. [10]

b) Describe and derive the Gram-Schmidt orthogonalisation process. [10]

4. Attempt any TWO of (a), (b) and (c)

(a) Consider the inhomogeneous eigenvalue problem

$$Ax - \lambda x = b$$

where A is symmetrix and $b \neq 0$.
Show that there exists a solution x if λ is not an eigenvalue of A. Also show that a solution x can be written

$$x = \Sigma \frac{u_j' b}{\lambda_j - \lambda} u_j$$

where the u_j form an orthonormal set of eigenvectors of A, corresponding to eigenvalues λ_j.
(Hint: Write $x = \Sigma a_j u_j$ and evaluate a_j.) [10]

(b) Consider the following equation:

$$|A - \lambda I| = (-\lambda)^n + b_{n-1}(-\lambda)^{n-1} + \ldots + b_1(-\lambda) + b_0$$

where A is a square matrix.
By induction or otherwise find expressions for b_i $(i = 0,1,2,\ldots, n-1)$ in terms of the elements of A. [10]

(c) Evaluate the determinant

$$\begin{bmatrix} 0! & 1! & 2! & \ldots & n! \\ 1! & 2! & 3! & \ldots & (n+1)! \\ 2! & 3! & 4! & \ldots & (n+2)! \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ n! & (n+1)! & (n+2)! & \ldots & (2n)! \end{bmatrix}$$

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5.a) Find the inverse of

$$\begin{bmatrix} 1 - c + c^2 & 1 - c \\ c(1 - c) & c \end{bmatrix}$$

b. Find the canonical form of the quadratic form

$$4x_1^2 - 2x_2^2 + x_3^2 - 2x_1x_2 + 4x_1x_3 - 5x_2x_3.$$

c) Get an orthonormal basis for R^3 with respect to the inner product

$(x_1, x_2, x_3) . (y_1, y_2, y_3) =$

$x_1y_1+x_1y_2+x_1y_3+x_2y_1+x_2y_2+x_2y_3+x_3y_1+x_3y_2+x_3y_3.$

6. State whether the following statements are true or false. Justify your answer.

(a) For an idempotent matrix, the trace equals the rank.

(b) If $AA^-A = A$, then $A^-AA^- = A^-$

(c) For an orthogonal matrix of odd order, at least one eigenvalue is ± 1.

(d) A vector α is an eigenvector of a matrix A if, and only if it is an eigenvector of $g(A)$ for any polynomial g.

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MID-YEAR EXAMINATION

Statistics-3: Statistics Theory

Date: 25.12.70 Maximum Marks: 100 Time: 3 hours

Note: All questions carry equal marks.

Group A: Answer all questions.

1. Suppose one is interested in estimating the mean of a characteristic Y for a population and a stratified sample with srswr within strata will be drawn for this purpose. Show how the stratum sample sizes can be chosen in an optimal manner on a joint consideration of cost and variance functions.

2. EITHER

A pps sample of paddy-fields is drawn with probability of selection proportional to areas of the fields, and the yield-rate of paddy (Y) is observed for each sample field. Establish the formula for the estimator of the total paddy yield of the population of fields and also for the unbiased estimator of its sampling variance.

OR

Consider a population of N units numbered 1 to N. Suppose we have $Y_i = a + bi$, exactly, for $i = 1, 2, \ldots, N$, where Y_i is the value of the characteristic for the i-th unit. Compare, for this population, the sampling variances of the estimators of population mean based on srswr, srswor and linear systematic sampling, assuming that N is an exact multiple of the sample size n.

Group B: Answer any three questions.

3. State and prove the Neyman-Pearson lemma in the theory of testing statistical hypotheses.

4. Let $x_1, x_2, \ldots, x_{25}$ denote a random sample of size 25 from a normal population with mean θ and variance 100 (known). Find the uniformly most powerful critical region of size 0.05 for testing $H_0: \theta = 50$ against $H_1: \theta < 50$.

How does this test perform when θ is actually greater than 50?

5. A r.v. x has a p.d.f. of the form $f(x,\theta) = \theta x^{\theta-1}$, $0 < x < 1$, and $f(x, \theta) = 0$ elsewhere. Find the most powerful test of the null hypothesis $H_0: \theta = 1$ against the alternative hypothesis $H_1: \theta = 2$ on the basis of a random sample of 2 observations x_1, x_2. The level of significance of the test must be 0.05. Find the power of the test when H_1 is true.

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6. Let $x_1, x_2, \ldots, x_m$ form a random sample from a normal population and $y_1, y_2, \ldots, y_n$ an independent random sample drawn from another normal population. Outline the usual procedure for testing the equality of the two population means, stating necessary assumptions. Find also the sampling distribution of the statistic used under the null hypothesis. (You need not prove every result you use.)

7. How is the chi-square test used for examining goodness of fit of theoretical distributions to observed frequency data? Prove that the Pearsonian criterion is asymptotically distributed as χ^2 under the null hypothesis for the case where the theoretical distribution is completely specified.

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MID-YEAR EXAMINATION

Statistics-3: Statistics Practical

Date: 26-12-70 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.1 and any four from Qs. 2-6. Marks allotted for each question are given in brackets [ ].

1. For estimating the average catch of fish landed per operating fishing unit (Y), 5 fish-landing centres in the coast were selected with srswr and from each sample centre 3 operating units were similarly selected. The catches of fish in kg are shown below:

Sample centre	Sample operating unit		
	1	2	3
1	464	204	185
2	160	101	290
3	391	565	150
4	420	186	617
5	144	36	21

Assuming that the total number of operating units is the same for each centre estimate unbiasedly the average Y and also estimate its rse. [18]

2. A psychiatrist claimed that about 40 % of all chronic headaches were of the psychosomatic variety. His disbelieving colleagues mixed some pills of plain flour and water and gave them to all patients suffering from chronic headache as a new headache remedy. Later, the comments of the patients were classified as follows:

comment:	worthless	slower than aspirin	similar to aspirin	better than aspirin
no. of patients:	29	1	3	8

Was the psychiatrist guilty of exaggeration? [18]

3. A metallurgist made 4 determinations of the melting point of manganese: 1269, 1271, 1263, 1265 degrees centigrade. After assessing the various errors that might affect his techniques, he thought his measurements would have a s.d. of two degrees or less. Are the data consistent with this supposition? [18]

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4. The mean and the s.d. of monthly incomes of industrial workers in a certain city were estimated from random sample surveys carried out in 1945 and 1946. The estimates were as follows:

year	sample size	mean (Rs.)	s.d. (Rs.)
1945	230	82.4	18.6
1946	346	85.1	17.2

Was there any significant improvement in the average income between 1945 and 1946?

5. The following shows the additional hours of sleep gained after using each of two drugs by the same group of 8 patients:

Patient:	1	2	3	4	5	6	7	8
Hours gained:								
Drug 1	0.2	0.9	3.8	3.5	-0.2	-1.3	-0.3	-1.7
Drug 2	4.7	1.8	5.5	4.6	-0.3	-0.4	1.9	2.0

Test whether the second drug gives, on the average, an hour more of sleep than the first drug.

6. A surgeon carried out the same operation by two different techniques and found the following results.

Technique	No. of operations		Total
	Successful	Unsuccessful	
1	17	1	18
2	1	2	3

Is there significant difference between the results with the two techniques?

7. Practical Record

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MID-YEAR EXAMINATION

Statistics-3: Probability

Date: Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions.
All questions carry equal marks.

1. X_1 and X_2 are independent and identically distributed with p.d.f.

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 < x < \infty, \quad (0 < \theta < \infty).$$

a) Find the joint p.d.f. of $Y_1 = X_1 + X_2$ and $Y_2 = X_2$.

b) Compute $E(Y_2)$ and $V(Y_2)$.

c) Find $E(Y_2 / y_1) = \phi(y_1)$ and the variance of $\phi(Y_1)$.

2. X_1, X_2, X_3 is a random sample from the distribution having p.d.f.

$$f(x) = e^{-x}, \quad 0 < x < \infty.$$

Show that

$$Y_1 = \frac{X_1}{X_1 + X_2}, \quad Y_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}, \quad Y_3 = X_1 + X_2 + X_3$$

are mutually independent.

3. $X_1, X_2, \ldots, X_n$ is a random sample from a distribution having p.d.f.

$$f(x) = e^{-(x-\theta)}, \quad \theta < x < \infty, \quad (-\infty < \theta < \infty).$$

a) What is the p.d.f. of $Y_1 = \min\{X_1, \ldots, X_n\}$?

b) Compute $E(Y_1)$.

4.a) $X_1, \ldots, X_n$ is a random sample from the normal distribution $N(\mu, \sigma^2)$. Let $0 < a < b$. Find the mathematical expectation of the length of the random interval

$$\left(\sum_1^n (X_i - \mu)^2/b, \ \sum_1^n (X_i - \mu)^2/a\right).$$

b) $X_1, \ldots, X_{32}$ and $Y_1, \ldots, Y_{50}$ are two random samples from two independent normal distributions $N(4, 27)$ and $N(5, 48)$, respectively. Compute $P(16\bar{X} + 11 > 15\bar{Y})$ where

$$\bar{X} = \frac{1}{16}\sum_1^{16} X_i, \qquad \bar{Y} = \frac{1}{25}\sum_1^{25} Y_i .$$

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MID-YEAR EXAMINATION

Economics - 3

Date: 29.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.
All questions carry equal marks.

Group A: Economic Development

Attempt any two questions. Maximum Marks: 40

1. Give an account of the classical theory of growth, mentioning the major variables and relations used.

2. Following Marx, describe the process of transition from capitalism to socialism and note its main differences from the classical theory of growth.

   Write very briefly on Marx's different explanations of business cycles.

3. Attempt any two of the following questions:

   i) Why does the share of agriculture in national product decline more rapidly than the share of food in consumption expenditure along with economic development?

   ii) What are the advantages and disadvantages of choosing nation-states as units in a study of inter-regional variations of incomes?

   iii) What are the main characteristics of underdeveloped countries ?

   iv) What are the advantages and disadvantages of using per capita real national income as a measure of long period economic growth?

Group B: Growth Models

Attempt any three questions. Maximum Marks: 60.

4. What are the neoclassical conclusions about the growth of an economy? Verify these conclusions with the help of a Cobb-Douglas production function.

5. Define 'steady state'. What are the conditions for the existence of steady-state in a neoclassical economy with and without technical progress at an externally given rate?

6. Write down the equations determining the steady-state values of capital-labour ratio, output-labour ratio, real rental, real wage rate and shares of capital and labour in a neoclassical growth model. What should be the possible objections against such a model?

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7. Prove that the production function $Y = F(K, L; t)$ exhibits

a) Hicks-neutral technical progress if and only if

$$F(K, L; t) = A(t)\, G(K, L)$$

and

b) Harrod-neutral technical progress if and only if

$$F(K, L; t) = H(K, B(t)L)$$

where $A(t)$ and $B(t)$ are positive functions.

8. Define 'elasticity of substitution' (σ) in a two-factor neoclassical growth model. Classify technical progress in this model according to Hicks and Harrod in the following cases: (i) $\sigma > 1$; (ii) $\sigma = 1$ and (iii) $\sigma < 1$.

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MID-YEAR EXAMINATION

General Science-5: Statistical Mechanics

Date: 30.12.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1.a) Explain what is meant by microstates and macrostates and the thermodynamic probability. [4 X3]=[12]

b) Show that the number of particles in the i^{th} cell in the state of maximum thermodynamic probability is, according to MB-statistics,

$$N_i = \frac{N}{Z} \exp(-\omega_i / kT)$$

the symbols having their usual significance. [24]

2.a) Derive the following - (symbols having usual meanings)

i) $\ln(x!) = x \ln x - x$, if x is very large. [3]

ii) $T = (\partial U/\partial S)_V$ [3]

iii) $p = -(\partial F/\partial V)_T$. [3]

b) Apply the Maxwell-Boltzmann statistics to get the equation of state for a monoatomic ideal gas. [16]

3.a) What are the essential drawbacks of the MB-statistics? [8]

b) Name the statistics that could remove them. [2]

c) In what respect does the statistics of Maxwell-Boltzmann differ from that of Bose and Einstein. [6]

d) Find an expression for the thermodynamic probability of the BE-statistics. [6]

e) Under what conditions would it degenerate to MB? [3]

4. Show on the basis of BE-statistics, for 4 phase points and 2 cells, and with $n = 4$, which of the following macro-states has the greatest probability.

i) $N_i = 4$, $N_j = 0$ (ii) $N_i = 2$, $N_j = 2$

iii) $N_i = 1$, $N_j = 3$ (iv) $N_i = 0$, $N_j = 4$. [14]

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MID-TERM EXAMINATION

General Science-4: Biochemistry Theory

Date: 31.12.70 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. Describe how fatty acid is metabolised in mammalian system. [15]

2. Describe the nature and action of different hormones present in pituitary gland. [20]

3. Write notes on
   (a) RNA
   (b) Vitamin K
   (c) ATP [15]

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MID-YEAR EXAMINATION

General Science-4: Bio-chemistry Practical

Date: 1. 1. 1971 Maximum Marks: 100 Time: 3 hours

Note: Answer the following question.

Determine the total amount of Glucose present in the unknown sample by Fehling's titration.

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PERIODICAL EXAMINATION

Mathematics-3: Analysis

Date: 29.3.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1.a) Show that if A is connected and $A \subseteq B \subseteq \bar{A}$ then B is connected.

b) Define an interval and show that it is connected. [6+6]=[12]

2.a) Show that if $f: M_1 \to M_2$ is continuous and $A \subseteq M_1$ is connected, then $f(A)$ is connected.

b) Let M be a non-empty set and define

$$d(x,y) = \begin{cases} 0 & \text{if} \quad x = y \\ 1 & \text{if} \quad x \neq y \end{cases} \qquad x,y \in M$$

Show that only connected subsets of (M,d) are singleton sets and the empty set.

c) Describe the connected components of the following subsets of $\mathbb{R}$.

i) Set of irrationals
ii) $[0, 1] \cup \{5\}$
iii) $[0, \frac{1}{2} \rangle \cup \langle \frac{1}{2}, 1]$. [6+4+4]=[14]

3.a) Show that continuous image of a compact set is compact.

b) M_1 is compact and f is a continuous 1-1 map of M_1 onto M_2. Show that M_2 is compact and $f^{-1}: M_2 \to M_1$ is continuous. [6+8]=[14]

4.a) Define: Uniformly continuous function. Show that a continuous function on a compact metric space is uniformly continuous.

b) Give an example of a continuous function which is not uniformly continuous. [6+6]=[12]

5.a) Let f be a continuous function on a closed set $E \subseteq \mathbb{R}$. Show that $\exists$ a continuous function g on $\mathbb{R}$ such that

$$f(x) = g(x) \qquad \text{for} \qquad x \in E$$

(Hint: Note that E^c is open, its connected components are open intervals; define g suitably on these connected components.)

b) Let f and g be two continuous functions on $\mathbb{R}$ such that $f(x) = g(x)$ whenever x is rational. Show that $f = g$, i.e. for all x, $f(x) = g(x)$.

[6+6]=[12]

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6.a) Define: (i) $\underline{S}(f, \pi)$ and $\bar{S}(f,\pi)$. Show that $\pi' < \pi \Rightarrow$

$$\underline{S}(f, \pi) \leq \underline{S}(f, \pi') \leq \bar{S}(f, \pi') \leq \bar{S}(f, \pi).$$

b) Show that if f and g are Riemann integrable on $[a,b]$ then $f+g$ is Riemann integrable on $[a, b]$.

(Hint: Show that $\underline{S}(f,\pi)+\underline{S}(g,\pi) \leq \underline{S}(f+g,\pi)$

$$\leq \bar{S}(f+g, \pi) \leq \bar{S}(f,\pi)+\bar{S}(g,\pi)$$

and use the necessary and sufficient condition for Riemann integrability).

c) Show that a continuous real valued function on $[a,b]$ is Riemann integrable.

[4+4+4]=[12

7.a) Let $[x]$ denote the largest integer $\leq x$, i.e., the intege such that $x-1 < [x] \leq x$, and let $(x) = x - [x]$. What are the discontinuities of the functions $f(x) = [x]$ and $g(x) = (x)$?

b) Show that

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \sin \frac{1}{x} & \text{if } x \neq 0 \end{cases}$$

is continuous everywhere except $x = 0$ and show that $\lim_{x \downarrow 0} f(x)$ does not exist.

[8+4]=[12

8.a) Show that if $|x| < 1$ then $\sum_{n=0}^{\infty} x^n$ converges and the sum is $\frac{1}{1-x}$.

b) Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms such that

$$\frac{a_{n+1}}{a_n} < \alpha < 1. \text{ Then } \sum_{n=1}^{\infty} a_n \text{ converges.}$$

c) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges and the sum is 1.

[4+4+4]=[12

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PERIODICAL EXAMINATIONS

Statistics-3: Probability

Date: 5.4.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ] .

1. X_1, X_2 are independent gamma variates with parameters α_1, α_2, respectively, [pdf of X_1 is $[\Gamma(\alpha_1)]^{-1} x_1^{\alpha_1 - 1} e^{-x_1}$, $x_1 > 0$] where $\alpha_1, \alpha_2 > 0$. Find the distribution of

$$Y = (1 + \frac{X_2}{X_1})^{-1}$$ [19]

2. In each of the following situations, examine whether the probability distribution of the variable (whose p.d.f. f(x) is given) converges to a distribution as $n \to \infty$. If yes, find the pdf of the limiting distribution. Does the pdf converge in each case?

(a) $f(x) = \frac{1}{\sqrt{2\pi/n}} \exp[-\frac{x^2}{2(1/n)}]$, $-\infty < x < \infty$.

(b) $f(x) = \begin{cases} 1 & \text{if } x = 1 - \frac{1}{n} \\ 0 & \text{otherwise.} \end{cases}$

(c) $f(x) = \begin{cases} 1 & \text{if } x = -\frac{n}{2} \\ 0 & \text{elsewhere.} \end{cases}$ [27]

3. $X_1, X_2, \ldots, X_n$ constitute a random sample from a distribution having p.d.f. $f(x) = \frac{1}{\theta}$, $0 < x < \theta$, $(\theta > 0)$. Let $Y = \max(X_1, \ldots, X_n)$ be the largest sample observation.

i) Find the distribution function of Y.

ii) What are the limiting distributions, if any, of Y and of $n(1 - \frac{Y}{\theta})$? [27]

4.a) State, without proof, the Borel-Cantelli lemmas.

b) In a sequence of Bernoulli trials (independent tosses of a coin with probability of success = p), let A_n be the event that a run of n consecutive successes occurs between the 2^nth and 2^{n+1}st trial. Show that, with probability one infinitely many A_n occur, or with probability one only finitely many A_n occur, according as $p \geq \frac{1}{2}$ or $p < \frac{1}{2}$. [27]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Theory and Practical

Date: 12.4.71 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Marks allotted for each question are given in brackets [ ].

Group A : Answer any four questions.

1. Consider the Gauss-Markoff set-up in the theory of linear estimation:

$$y = X\beta + \epsilon, \text{ with } D(\epsilon) = \sigma^2 I.$$

(All symbols have the usual meaning.)

Show that if rank of X is equal to the number of column vectors in X, then the least squares estimator of β is BLUE. [18]

2. Consider the same set-up as in Q.1 but without any assumption regarding the rank of X. Consider a linear parameter function $P'\beta$.

Prove any three of the following:-

i) $P'\beta$ is estimable iff (a) $P \in \mu(X'X)$ or equivalently iff (b) $P'\hat{\beta}$ is the same for all solutions $\hat{\beta}$ of the normal equations.

ii) If $P'\beta$ is estimable, $P'\hat{\beta}$ is BLUE of $P'\beta$.

iii) If $P'\beta$ is estimable, $V(P'\hat{\beta}) = \sigma^2 P'CP$ (where C is any g-inverse of $X'X$) $= \sigma^2 \lambda' P$ (where $P = X'X\lambda$)

iv) If $P'\beta$ is estimable, $Cov\,(P'\hat{\beta}, Y - X\hat{\beta}) = \underset{\sim}{0}$. [18]

3. Consider the Gauss-Markov set-up of Q.1 without any assumption on r = rank of X. Show that $R_0^2/(n-r)$ is an unbiased estimator of σ^2 where $R_0^2 = \min_{\beta} (y - X\beta)'(y - X\beta)$ and n the number of observations.

If ϵ is normally distributed, show further that $R_0^2 \sim \sigma^2 \chi^2_{n-r}$. [18]

4. Consider k normal populations with means $\mu_1, \mu_2, \ldots, \mu_k$ but a common variance σ^2 and suppose that random samples of sizes $n_1, n_2, \ldots, n_k$ have been independently drawn from these populations. How would you test the null hypothesis $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$? (You need not derive the sampling distribution of the test statistic, but you should state the necessary theorems.) [18]

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5. 4 points O, A, B and C lie on a line in that order, so OA = a < OB = b < OC = c. Measurements of OA, OB, OC, AB, AC and BC are made, each only once, independently, having errors which are normally distributed with zero mean and a common variance σ^2. How would you estimate a, b, c and σ^2 and also the sampling variances of the estimators of a, b and c?

Group B

6. Twenty students of a class were divided into 5 homogeneous groups of four students each, according to their background knowledge. Four methods of learning were followed, each by one of the 4 students in each group. The marks obtained by these students in a subsequent test are given below:

Group	Method of learning			
	μ_1	μ_2	μ_3	μ_4
G_1	8	4	7	5
G_2	3	3	7	10
G_3	9	6	5	10
G_4	5	7	8	12
G_5	7	7	12	8

Analyze the data and compare the different methods of learning.

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PERIODICAL EXAMINATION

Economics-3: Indian and Socialist Planning

Date: 19.4.71 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. EITHER

Discuss the statement that planning is a better means of economic development. [20]

OR

Show how the approach and objectives of the second five year plan were conditioned by the results of the first five year plan and the resolution on the 'socialistic pattern of society'. [20]

2. Write notes on any three of the following:

a) different kinds of planning [10]

b) foreign exchange resources of a plan; illustrate with figures [10]

c) proposed organisation of the economy in the Bombay Plan [10]

d) K. T. Shah's note of dissent on the question of nationalisation submitted to the advisory planning board [10]

e) broad results of the second five year plan [10]

f) financial estimates for the third five year plan. [10]

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PERIODICAL EXAMINATION

General Science-5: Psychology Theory

Date: 26.4.71 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ].

Group A

(Answer Question 4 and any two of the rest)

1. What is a nerve impulse? What is meant by the all-or-none law? [10]

2. Describe the cerebral cortex and its basic functions. Explain what is meant by the localisation of cortical functions. [10]

3. Describe the mechanism of hearing. Mention the psychological attributes of auditory experience and their corresponding physical characteristics of sound waves. [10]

4. Write short notes on any six of the following:
(a) neuron (b) refractory period (c) synapse
(d) reflex arc (e) medulla (f) thalamus
(g) retina (h) blind spot (i) colour blindness
(j) negative afterimage (k) physiological zero
(l) paradoxical cold. [30]

Group B

(Answer Question 8 and any two of the rest)

5. Enumerate the characteristics of a self-actualizing person. [15]

6. Name the endocrine glands and write down the functions of the pituitary gland. [15]

7. State the adaptive and maladaptive consequences of frustration. [15]

8. Write short notes on any four of the following:
(a) homoeostasis (b) phobia
(c) super ego (d) projective test
(e) psychopathic personality
(f) approach-avoidance conflict. [20]

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PERIODICAL EXAMINATION

General Science-5: Engineering

Date: 3.5.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. An overhanging beam AB, 25ft. long rests on two supports which are at distances 5 and 19 feet respectively from the end A. The beam carries a load of 2 tons at A, 1 ton at B and 2 tons at the centre of the beam. Draw the shearing force and bending moment diagram for the beam. State the maximum shear and maximum bending moment. [25

2. EITHER

An I-section steel joist, 9" X 4" (wide) moment of Inertia, 81 in. units, has a span of 12 feet. Find the maximum safe distributed load per foot run it will carry with a working stress of 8 tons per square inch. Derive the formula used by you. [25

OR

A 9" deep X 3" wide wooden beam, span 10 ft. supports a uniformly distributed load of 200 lbs. per foot run. Calculate the maximum shear stress. Derive the formula which you use. [25

3. The following observations were made in testing a sample of oak by bending, the load being applied in the centre of the span. With 2.02 in. depth 2.97 in. span 55 in. The loads W and the corresponding deflection scale readings R were as follows:-

W lb.	100	200	300	400	500	600	700	800	900	1000
R in.	0.145	0.218	0.287	0.358	0.430	0.500	0.570	0.643	0.721	0.800

Determine from the observations the modulus of elasticity of the oak and also its limiting elastic (flexural) stress. Derive the formula which you use to find the modulus of elasticity. [50

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ANNUAL EXAMINATION

Mathematics-3: Analysis

Date: 7.6.71 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 144 marks. Attempt as many questions as you like. The maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. Prove or disprove the following statements:

i) A closed bounded interval is compact
ii) A closed set is compact
iii) A bounded set is compact
iv) An open set is compact. [4+4+4+4]=[16]

2.a) Let f be continuous on a closed bounded interval $[a,b]$ and differentiable on $\langle a, b\rangle$. Show that $\exists$ $c \in \langle a,b\rangle$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

b) Give an example to show that the requirement of continuity of the function at the end points of the interval can not be removed. [8+8]=[16]

3.a) Let f be a real valued function defined on an open set $U \subseteq \mathbb{R}^2$. Explain what you mean by 'f has a differential at a point $(x_0, y_0) \in U$'.

b) Show that if f has a differential at (x_0, y_0) then it is continuous at (x_0, y_0) and that it has first order partial derivatives with respect to each variable at (x_0, y_0).

c) Let

$$f(x, y) = \begin{cases} xy \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Show that $f_{12}(0,0) = 1$, $f_{21}(0,0) = -1$.

d) State a non-trivial sufficient condition on a function f defined on an open set

$$U \subseteq \mathbb{R}^2$$

under which $f_{12} = f_{21}$. (No proof needed). [3+3+9+3]=[18]

4. Prove or disprove the following statements:

i) A Riemann integrable function is continuous
ii) A bounded function is Riemann integrable
iii) A monotonic function defined on a closed bounded interval $[a,b]$ is Riemann integrable
iv) Derivative of a function defined on a closed bounded interval is Riemann integrable. [4+4+4+4]=[16]

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5.a) Let f be a Riemann integrable function on $[a,b]$ and let x be a point of continuity of f. Then the function F defined by

$F(t) = \int_a^t f(y)dy$ is differentiable at $t = x$ and

$$\left.\frac{dF}{dt}\right|_{t=x} = f(x)$$

b) Show from first principles that if $n \geq 1$ is an integer, then

$$\int_a^b x^n dx = \frac{b^{n+1} - a^{n+1}}{n+1}.$$

c) Calculate the total variation of the following function

$$f(t) = \begin{cases} \cos t + i\sin t & 0 \leq t \leq \pi \\ 2\cos t + 2i\sin t & \pi < t \leq 2\pi. \end{cases}$$

[8+4+4]=[1

6.a) Given two sequences

$$(a_n)_{n=0}^{\infty}, \quad (b_n)_{n=0}^{\infty}. \quad \text{Put}$$

$$A_n = \sum_{k=0}^{n} a_k \text{ if } n \geq 0; \text{ put } A_{-1} = 0.$$

Show that if $0 \leq p \leq q$ then

$$\sum_{n=p}^{q} a_n b_n = \sum_{n=p}^{q-1} A_n(b_n - b_{n+1}) + A_q b_q - A_{p-1} b_p.$$

b) Let $(a_n)_{n=1}^{\infty}$, $(b_n)_{n=1}^{\infty}$ be sequences of real numbers such that

i) $\forall$ n, $\left|\sum_{i=1}^{n} a_i\right| < K$ where K is a finite constant independent of n,

ii) $b_n \downarrow 0$ as $n \to \infty$,

then $\sum_{n=1}^{\infty} a_n b_n$ converges.

c) Show that the series $\sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n}$ converges. [5+5+6]=[1

7.a) Show that the series $\sum_{n=1}^{\infty} (-1)^n. \frac{1}{n}$ can be rearranged so as to be divergent.

b) Can you rearrange $\sum_{n=1}^{\infty} (-1)^n. \frac{1}{n^2}$ so as to be divergent ?

Give reasons for your answer. [12+4]=[1

8.a) Show that if a series converges absolutely then every rearrangement converges to the same sum.

b) Does every rearrangement of an absolutely convergent series converge absolutely? Give reasons for your answer. [6+6]=[1

a) Test for convergence or divergence of :

i) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{3/2}}$ ii) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$

iii) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \log n}$

b) Let $(f_n)_{n=1}^{\infty}$ be a uniformly convergent sequence of continuous functions defined on a metric space M. Show that the function f to which the sequence converges is also continuous.

c) Give an example of a sequence of continuous functions $(f_n)_{n=1}^{\infty}$ which converges pointwise to a function f which is not continuous. [6+6+6]=[18]

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ANNUAL EXAMINATION

Statistics-5: Statistics Theory

Date: 9.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1. Consider the estimation of a linear parametric function $P'\beta$ in the Gauss-Markoff set up in the theory of linear estimation:

$$\underset{n\times 1}{Y} = \underset{n\times m}{X}\,\underset{m\times 1}{\beta} + \underset{n\times 1}{\varepsilon}, \quad E(\varepsilon) = \underset{\sim}{0}, \quad D(\varepsilon) = \sigma^2 I.$$

(All symbols have their usual meanings.)

a) When is the function $P'\beta$ said to be estimable?

b) If $P'\beta$ is estimable, find its minimum variance linear unbiased estimator.

c) Find an unbiased estimator of the sampling variance of the estimator obtained in (b).

2. If in the set-up mentioned in Q.1, ε is assumed to be normally distributed, and $R_0^2 = \min_\beta (Y - X\beta)'(Y - X\beta)$, and $R_1^2 = \min_\beta (Y - X\beta)'(Y - X\beta)$ subject to $H'\beta = \xi$ (where H is a $m \times k$ matrix of rank k such that $\mathcal{M}(H) \subset \mathcal{M}(X')$, and ξ a $k \times 1$ vector), then prove that

a) $R_0^2 \sim \sigma^2 \chi^2_{n-r}$ where $r = \text{rank}(X)$.

b) R_0^2 and $R_1^2 - R_0^2$ are independently distributed,

c) Under the hypothesis $H'\beta = \xi$, $R_1^2 - R_0^2 \sim \sigma^2 \chi^2_k$.

3. Give a general account of the analysis of variance of two-way classified data with $m > 1$ observations per cell. State the assumptions carefully.

4. Suppose you have n observations on two variables x and y such that for $x = x_i$, there are n_i observations on y denoted $y_{i1}, y_{i2}, \ldots, y_{in_i}$, $(i = 1, 2, \ldots, k)$. Starting from a suitable model, obtain a test of significance of the regression of y on x and also a test of significance of the non-linearity in this regression.

5. Discuss, with suitable illustrations, why randomization and replication are absolutely essential for the validity of an experiment.

What is meant by local control? Explain with reference to the designs you know of.

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6. Give a brief account of the latin square design, stating the model employed and the procedure of analysis.

   Find the expectations of the treatment mean square and the residual mean square.

7. Write short notes on any three:

   i) The Kolmogorov-Smirnov test of goodness of fit,

   ii) The test of significance of the sample (linear) regression coefficient,

   iii) The concept of interaction between the two factors.

   iv) Comparative merits of the randomized block design and the latin square design.

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ANNUAL EXAMINATION

Statistics-3: Statistics Practical

Date: 10.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Three objects O_1, O_2 and O_3 were weighted 6 times in a balance by placing some objects on the two pans and balancing against standard weights placed on the pans. The results of the weighing are given below:

Objects on		Standard weight (gm) on	
right pan	left pan	right pan	left pan
O_1	O_3	-	4.17
O_1	O_2	-	1.84
O_1	O_2, O_3	-	1.02
O_2	O_1, O_3	2.97	-
O_3	O_1, O_2	7.10	-
O_1, O_2, O_3	-	-	9.08

Estimate the weights of the three objects along with their standard errors. Also test whether the sum of the three weights is 10 gm. [30]

2. EITHER

A sample of 20 observations on x and y gives the following sums: $\Sigma x = 186.2$, $\Sigma y = 21.9$, $\Sigma (x-\bar{x})^2 = 215.4$,

$\Sigma (y-\bar{y})^2 = 86.9$, $\Sigma(x-\bar{x})(y-\bar{y}) = 106.4$.

Assuming that the regression of y on x is of the form $y = \alpha + \beta x$, test the following hypotheses: (i) $\alpha = 0$ (ii) $\beta = 0$. Estimate the conditional mean of y when $x = 13$ and find 95 per cent confidence limits for this conditional mean. [25]

OR

Below are given the sums obtained in a regression analysis of data on age in years (x) and chest-girth in inches (y) for two groups of students consisting of 15 and 18 students respectively:

	Σx	Σy	Σx^2	Σy^2	Σxy
Group 1	202.7	23.3	2742.56	44.77	315.07
Group 2	244.1	53.8	3314.01	174.40	729.82

Assuming that the regression of y on x is linear for both the groups, test

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i) whether the two regression lines are parallel,

ii) whether the two regression lines are identical.

3. An experiment on sugarcane conducted in 4 randomized blocks gave the following observations on the weight of cane in Kg. The three treatments compared were introgen (N), phosphbrous (P) and potash (K).

Block	Treatment		
	N	P	K
1	122	81	80
2	120	80	82
3	138	79	65
4	121	75	58

Analyze the data and compare the yields of the three treatments using suitable tests.

4. Practical Records

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ANNUAL EXAMINATION

General Science-5: Engineering

Date: 11.6.70 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ].
Answer all the questions.
Answers should be brief and rough work should be separated from the material meant for the examiner.

1. EITHER

A beam 25 feet long is supported at one end and on a pier at a distance of 5 ft. from the other end. The beam is uniformly loaded from end to end with a load of 1 ton per lineal foot. Draw the bending moment and shearing force diagrams, giving the maximum value in each case. [15]

OR

Prove that the intensity of shear stress q at any point of the cross-section of a beam is

$\frac{SA\bar{y}}{bI}$ where S = shearing force at the section,
I = moment of inertia of the cross-section,
b = breadth of section at the point,
A = area of cross-section on the farther side of the point to the neutral axis,
and $\bar{y}$ = distance of c.g. of this area from the neutral axis.

Show that for a rectangular cross-section the maximum shear stress is one and a half-times the average shear stress.

2. What is the modulus of elasticity of a material and how is it obtained?

A beam of cast iron, 1" broad and 2" deep is tested upon supports 3 ft. apart, and shows a deflection of 1/4" under a central load of 1 ton. Calculate the Young's modulus E. [15]

3. Compare the strength of columns 12 ft.long containing the same volume of metal: (a) the column being rolled steel joist of I-section 10" × 8" (flange width) × 3/4"(thickness of web and flanges), (b) cast iron hollow cylindrical column, the metal being 3/4" thick.

Use Rankine's formula -

$$P = \frac{f_c A}{1 + a(l/k)^2}$$

where f_c for steel = 21 tons/ in^2
f_c for cast iron = 36 tons/in^2
a for steel = $\frac{1}{7500}$
a for cast iron = $\frac{1}{1600}$. [20]

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4. Find the time required to empty a swimming bath through a flat grating at the bottom:-

Depth of water = 5 feet
Length of bath = 80 feet
Breadth of bath = 30 feet
Area of grating = 2 square feet
coefficient of discharge = 0.65

Derive the formula you use. [17]

5. EITHER

Give a full description of the principles underlying the Venturi-meter with details of its construction and method of working. [16]

OR

A circular plate 8 feet diameter is placed vertically in water so that the centre of the plate is 10 feet below the surface. Find the depth of the centre of pressure and the total pressure. If the plate is placed with its centre 300 feet below the surface, find approximately the depth of the centre of pressure.

6. The following observations were made during measurements on a weir whose crest (b) is 3 feet.

Height H in feet	0.2	0.4	0.6	0.8	1.0	1.2	1.5
Q in cusecs	0.846	2.34	4.24	6.48	9.0	11.78	16.35

If the discharge is given by $Q = k\ b\ H^n$ find k and n. [17]

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ANNUAL EXAMINATION

Statistics-3: Probability

Date: 14.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions.
All questions carry equal marks.

1. The variables X_1 and X_2 are independent and each has the p.d.f. $e^{-(x-\theta)}$, $x > \theta$, $(-\infty < \theta < \infty)$, zero elsewhere. Find the joint p.d.f. of $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$. Show that Y_1 and $Y_2 - Y_1$ are independent.

2. X_1, X_2, X_3 is a random sample from a distribution having the p.d.f. $\frac{1}{\theta^2} x e^{-x/\theta}$, $0 < x < \infty$, $(\theta > 0)$, zero elsewhere. Find the joint p.d.f. of $Y_1 = X_1+X_2+X_3$, $Y_2 = X_2$, and $Y_3 = X_3$. Compute the marginal p.d.f. of Y_1.

3. $X_1, X_2, \ldots, X_n$ is a random sample from the normal distribution $N(0, \sigma^2)$. Find the distribution of the variable

$$Y = \frac{X_1}{\sqrt{\frac{1}{n} \sum_1^n X_i^2}}.$$

[You may assume that the p.d.f. of χ_n^2 is const. $x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$, $x > 0$. Since the numerator and denominator are not independent, you may have to relate Y to a variable with $\sum_2^n X_i^2$ instead of $\sum_1^n X_i^2$].

4. Let $Y_n = \max X_1, X_2, \ldots, X_n$, where $X_1, \ldots, X_n$ is a random sample from the rectangular distribution on the interval (0,1). a_n, $n \geq 1$ is an increasing sequence of positive real numbers. Prove that the limiting distribution of $Z_n = a_n(1 - Y_n)$ is degenerate if $\frac{a_n}{n} \to 0$, and non-degenerate if $\frac{a_n}{n} \to c \neq 0$. What happens if $\frac{a_n}{n} \to \infty$?

[You may use the fact: $\lim_{m \to \infty} (1 + \frac{x}{m})^m = e^x$].

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A sequence of random variables $X_1, X_2, \ldots,$ is said to converge in probability to a constant c if for any $\varepsilon > 0$, $P(|X_n - c| > \varepsilon) \to 0$ as $n \to \infty$.

a) Prove that X_n converges in probability to c if, and only if, the distribution of X_n converges to the distribution degenerate at c.

b) Let $\mu_n = E(X_n)$, $\sigma_n^2 = V(X_n)$. Show that if $\sigma_n \to 0$ as $n \to \infty$, then $X_n - \mu_n$ converges in probability to zero.

c) Suppose two sequences X_n Y_n of random variables are given such that the distribution of X_n converges to a distribution with distribution function $F(x)$, and Y_n converges in probability to a constant $c > 0$. Show that the distribution of $(X_n + Y_n)$ tends to the distribution with distribution function $F(x - c)$.

[Hint: For any event S, we have $S = (S \cap A) \cup (S \cap A')$ where A is the event $|Y_n - c| \leq c$].

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ANNUAL EXAMINATION

General Science-5: Psychology

Date: 16.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer Q.4 and any two of the rest.

1. Describe the structure and function of the non-auditory labyrinth. Why is the static sense an indirectly aroused sensation? (No diagram necessary.) [10.

2. How would you define punishment? On what factors does the effectiveness of punishment depend? Explain. [10.

3. Define learning. What are the basic conditions of learning? Discuss. [10.

4. Write short notes on any three of the following:

   a) kinesthetic receptors.
   b) stimulus differentiation.
   c) trace reflex
   d) experimental neurosis.
   e) schedules of reinforcement. [15]

Group B

Answer Q. 8 and any two of the rest.

5. What are the differences between psychoses and neuroses? Give a schematic classification of the major forms of psychoses or neuroses. [10]

6. Enumerate the different methods of measuring personality. [10]

7. Describe the relative influence of heredity and environment on human intelligence. [10]

8. Write short notes on any three of the following:

   a) intelligence quotient
   b) behaviourism
   c) feeble-minded person
   d) shock therapy (electroplexy)
   e) Gestalt psychology [15]

Practical work record. [30]

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ANNUAL EXAMINATION

Economics-3: Indian Economics

Date: 17.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer four of the following questions.
At least one must be from each group.
All questions carry equal marks.

Group A Public Finance

1. Critically examine the Fiscal Policy of the Union and State Governments in the background of the objective of rapid and balanced economic growth.

2. The policy of increasing reliance on indirect taxes for mobilising resources has defeated the social objectives of Indian Plans.

   Comment on the above statement and illustrate your answer with relevant statistics.

3. Do you agree with the complaint of N.A. Palkhivala that India is the most highly taxed nation in the world.?

4. Assess the Indian Taxation Policy in respect of Indian Agriculture during the Post-Independence period.

Group B

Banking and Monetary Policy

5. Briefly outline the structure of the Indian Banking System and the functions of the Reserve Bank of India, the State Bank and the Commercial Banks.

6. Examine the principal features of Indian Monetary Policy since 1960-61 and its impact on the rate of growth of Indian Economy.

7. Discuss the main factors behind the long term tendency of rising prices in India, particularly since 1960-61.

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ANNUAL EXAMINATION

Economics-3: Indian and Socialist Planning

Date: 18.6.71 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: Answer any two questions.
All questions carry equal marks.

1. EITHER

Discuss the changes in agricultural policy at different stages of Soviet economic history.

OR

'Important changes in the economy and decisions on important issues had to precede the launching of the First Five Year Plan in USSR' - Discuss.

2. EITHER

Discuss the view that Soviet price policy needed such a drastic change as occurred in the sixties.

OR

Critically examine the statement that introduction of profit motivation in the USSR industries is unMarxian.

3. Why was the period of War-Communism so called?
Why were the economic policies pursued during this period changed?

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ANNUAL EXAMINATION

General Science-4: Biology Theory

Date: 19.6.71 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Answer any two questions from each Group A and Group B. Marks allotted for each question are given in brackets [].

Group A

1. What are the major obstacles for improving the coconut palm by breeding? Discuss on the possibilities of effecting clonal propagation in *Cocos nucifera*. [25]

2. Mention the names of five improved varieties of wheat. Give the statistics on the area under cultivation and production of wheat in the different agricultural zones of the world. Give an illustrated botanical description of any species of wheat. [25]

3. Write short notes on the following:-
 a) Spikelet of *Oryza sativa*;
 b) Rape seed and mustard;
 c) Important legumes;
 d) Foliar asymmetry in *Cocos nucifera*;
 e) Fibre crops. [25]

Group B

4. Write an illustrated account of the nuclear division in a reproductive cell and state its significance. [20+5]=[25]

5. Give an outline scheme of the breeding methods for improvement of cross-fertilizing crop plants. [25]

6. a) What are inbreeding and heterosis?

 b) What is inbreeding minimum and how is it achieved ?

 Why selection for vigour may delay the attainment of homozygosity?

 What are the important characteristics of human population living as isolates? [10+5+5+5]=[25]
