INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

PERIODICAL EXAMINATION

Statistics-8: Linear Estimation (Theory and Practical)

Date: 11.12.72 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: The paper carries 110 marks. Q.7 is compulsory. Answer as much as you can from the rest of the questions. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. For each of the following five statements, answer either 'TRUE' or 'FALSE'.

   a) For any square matrix A of order n,
   $$\text{rank } A + \text{rank } (I_n - A) \geq n.$$

   b) For any generalized inverse A^- of a matrix A, AA^- is idempotent.

   c) rank A^- > rank A (notations same as Q.1(b)).

   d) If $I = \sum_{i=1}^{k} A_i$, where $A_1, \ldots, A_k$ are all idempotent matrices, then $A_i + A_j$ is idempotent for all $i = 1, \ldots, k$; $j = 1, \ldots, k$.

   e) Let T_1 and T_2 be two unbiased estimators of an unknown parameter θ with $V(T_1) = \frac{1}{2} V(T_2) = \sigma^2$. If T_1 and T_2 are independently distributed, then the BLUE of θ based on T_1 and T_2 is given by $\frac{2T_1 + T_2}{3}$, and the variance of the BLUE is $2\sigma^2/3$. [5]

2. a) Show that $q'\underline{x}$ has a unique value for all $\underline{x}$ satisfying $A\underline{x} = \underline{y}$ if and only if $\underline{q}'A^-A = \underline{q}'$, where A^- is any generalized inverse of the matrix A. [7]

   b) Find a generalized inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$ [7]

3. Let $X_1, \ldots, X_n$ be n independent and identically distributed random variables with a common pdf.

$$f_\theta(x) = \begin{cases} e^{-(x-\theta)} & \text{if } x \geq \theta \\ 0 & \text{, otherwise} \end{cases}$$

Find the maximum likelihood estimator, say T_n of the unknown parameter θ. Show that T_n is a biased estimator of θ. Find an unbiased estimator of θ based on T_n. [3+2+1]=[6]

4. Consider the Gauss-Markoff set up in the theory of linear estimation:
$$E(\underline{Y}) = X\underline{\beta} \quad , \quad V(\underline{Y}) = \sigma^2 I$$
(All symbols have the usual meaning).

Please Turn Over

a) When is a linear parametric function $\underline{\lambda}'\underline{\beta}$ said to be estimable? Show that $\underline{\lambda}'\underline{\beta}$ is estimable if and only if $\underline{\lambda}'\hat{\underline{\beta}}$ is the same for any solution $\hat{\underline{\beta}}$ of the normal equations. [2+6]=[8]

b) If $\underline{\lambda}'\underline{\beta}$ is estimable, show that $\underline{\lambda}'\hat{\underline{\beta}}$ is the BLUE of $\underline{\lambda}'\underline{\beta}$. [7]

c) If $\underline{\lambda}'\underline{\beta}$ is estimable, show that $V(\underline{\lambda}'\hat{\underline{\beta}}) = \sigma^2 \underline{\lambda}'\underline{q}$, where $\underline{\lambda} = X'X\underline{q}$. [4]

d) If $\underline{\lambda}'\underline{\beta}$ is estimable, show that $Cov.(\underline{\lambda}'\hat{\underline{\beta}}, \underline{Y} - X\hat{\underline{\beta}}) = \underline{0}$ [6]

5.a) Let $Y_1, \ldots, Y_n$ be n independent normal (0,1) variables. Define $\underline{Y} = (Y_1, \ldots, Y_n)'$. It is given that

$$\underline{Y}'\underline{Y} = \sum_{i=1}^{k} \underline{Y}' A_i \underline{Y},$$

where $A_1, \ldots, A_k$ are symmetric matrices with ranks $n_1, \ldots, n_k$ respectively.

Show that a necessary and sufficient condition that $\underline{Y}' A_1 \underline{Y}, \ldots, \underline{Y}' A_k \underline{Y}$ are independently distributed chi-square variables with $n_1, \ldots, n_k$ degrees of freedom respectively is $n = \sum_{i=1}^{k} n_i$. [17]

b) Let Y_1, Y_2, Y_3 be three independent normal (0,1) variables. Define $\bar{Y} = \frac{1}{3}(Y_1 + Y_2 + Y_3)$. Show that $\sum_{1}^{3} (Y_i - \bar{Y})^2$ has a chi-square distribution with 2 degrees of freedom. [8]

6. 4 points O, A, B and C lie on a straight line in that order; so $OA = a < OB = b < OC = c$. Measurements of OA, OB, OC, AB, AC and BC are made, each only once, independently having errors which are normally distributed with zero mean and a common variance σ^2. How would you obtain least square estimates of a, b and c? What are the variances of the estimators ? [15]

7. Three objects O_1, O_2 and O_3 are weighed 6 times in a balance by placing some objects on the two pans and balancing against standard weights placed on the pans. The results of the weighing are given below.

Objects on		Standard weight (gm) on	
right pan	left pan	right pan	left pan
O_1	O_3	-	4.17
O_1	O_2	-	1.84
O_1	O_2, O_3	-	1.02
O_2	O_1, O_3	2.97	-
O_3	O_1, O_2	7.10	-
O_1, O_2, O_3	-	-	9.08

Estimate the weights of the three objects along with their standard errors. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

302

PERIODICAL EXAMINATION

Statistics-10: Probability

Date: 18.12.72 Maximum Marks: 50 Time: $1\frac{1}{2}$ hours

Note: The paper carries 60 marks. Attempt as many questions as you can. The maximum one can score is 50. Marks allotted for each question are given in brackets [].

1. Let the joint p.d.f. of X and Y be given by

$$f(x, y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

Find the joint moment generating function. Use it to compute the correlation coefficient between X and Y. [7+8]=[15]

2. Let X and Y be independent and identically distributed uniform random variables in [0, 1]. Find the p.d.f. of X+ Y. [10]

3. Let X and Y be independent χ^2 random variables. Show that $\frac{X}{Y}$ and X+ Y are independent. [10]

4. Find the probability that the roots of the equation $x^2 + 2X_1 x + X_2 = 0$ will be real if X_1 and X_2 are chosen independently and at random between 0 and 1. [10]

5. If a stick of unit length is broken at random into two pieces,

   i) what is the average length of the larger piece?

   ii) What is the average ratio of the length of the smaller to that of the larger? [7+8]=[15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[303]

PERIODICAL EXAMINATION

Mathematics

Date: 1.1.1973 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A: Mathematics-5: Calculus

Maximum Marks: 50

All questions carry equal marks.

1. Define continuity of a function of several variables in two different ways. Check whether the following function is continuous at the point (0,0).

$$f(x, y) = \frac{2xy}{x^2 + y^2}.$$

$$f(0,0) = 0.$$

2. Define derivative of a function of several variables at a point. Evaluate the directional derivative of $f(x,y) = 2x^2 - y^2$ at (1,2) in the direction of the line from (1,2) to (3,5).

3. a) If $u = f(x,y)$ and $x = r \cos \theta$, $y = r \sin \theta$, show that

$$u_x^2 + u_y^2 = u_r^2 + \frac{1}{r^2} u_\theta^2 .$$

b) Define a homogeneous function of degree h, and show that differentiable homogeneous functions satisfy

$$xf_x + yf_y + zf_z + \dots = hf(x,y,z,\dots) .$$

4. Determine the following properties of the curve $r = a \sin (\theta/2)$, (i) symmetry, (ii) extent, (iii) asymptotes, (iv) certain points of the curve; (v) slope at intercepts. Using these information, trace the curve.

Group B: Mathematics-6: Real Analysis

Maximum Marks : 50

Answer all the questions. Marks allotted for each question are given in brackets [].

5. Show that the series: $\sum_{n=1}^{\infty} (-1)^n e^{-n\alpha}$ is convergent for any $\alpha > 0$. [6]

6. EITHER

Show that for any positive real number α

$$\sum_{n=0}^{\infty} \frac{\alpha}{(1+\alpha n)(1+\alpha(n+1))} = 1$$

Please Turn Over

6. OR

Show that

$$\sum_{n=1}^{\infty} a_n$$

converges absolutely if

$$\limsup_{n \to \infty} |a_n|^{\frac{1}{n}} < 1.$$ [8]

7. Show that

$$\lim_{n \to \infty} \int_0^{2\pi} e^x \cos nx.dx = 0.$$ [8]

8. Let f be defined on $[-1, 1]$ and that f'', the second derivative exists in $(-1, 1)$. Further suppose that $f(1) = f(-1) = 0$.

Prove that

$$3 \int_{-1}^{1} f(x)dx = - 2 f''(\xi)$$

for some $\xi \in (-1, 1)$. [10]

9. Let α be a continuous non-decreasing function defined on $[a, b]$. If f is any monotonic function on $[a, b]$, show that f is R-S integrable on $[a, b]$ with respect to α. [8]

10. Assignment-I. [10]

WISH YOU A HAPPY NEW YEAR

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73 [304]

PERIODICAL EXAMINATION

Economics-2

Date: 8.1.73 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Micro Economics

Maximum Marks: 40
Answer any two questions.

1. A firm is the only seller of a normal commodity in a market of competing buyers. Show that the firm will raise its price if the government levies on it a tax proportional to the quantity produced and sold (the constant of proportionality being very small). [20]

2. A firm, enjoying monopoly of the home market in which the price p_1 (in Rupees) and the quantity sold q_1 are connected by the relation $\frac{p_1}{40} + \frac{q_1}{60} = 1$, can also sell in a competitive foreign market at a price of Rs.20/- per unit. If the cost of production of the firm is given by $C = \frac{1}{4} Q^2$, where Q is its total output, find at what price the firm will sell its product in the home market. [20]

3. Explain in what respects Cournot's solution of the duopoly problem differs from Stackelberg's. [20]

Group B:

Maximum Marks: 60
Answer any three questions

1. Explain the different forms of deposit creation by commercial banks. Do you think that bank deposits should be regarded as money? Give reasons for your answer. [20]

2. Show how the factors determining the production decisions of firms and consumption decisions of households ultimately determine the equilibrium level of national income. Give an alternative demonstration of arriving at the same result by using the households' propensity to consume. [20]

3. Explain the policies of minimum reserve and open market operations of the Central bank. Examine the situations where they can be used. [20]

4. What are the different motives behind the demand for money? How is the aggregate demand for money in the economy related to the national income and the rate of interest? [20]

5. 'Statistical studies have shown that the shape of the consumption function differs radically depending upon the type of data used to plot the function'. Discuss the statement. Explain any possible approaches that have been made for reconciling the differently shaped consumption schedules. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73 [305]

PERIODICAL EXAMINATION

Statistics-7: Sample Surveys (Theory and Practical)

Date: 15.1.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 111 marks. Q. No. 6 is compulsory. Answer as many questions as you can from the rest. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

1. Define Horvitz and Thompsons' $\hat{T}_1$, $\hat{T}_2$ and $\hat{T}_3$ subclasses of linear class of estimators for population total T(y) of a character y and show that $\hat{Y}_{HT} = \sum_1^n y_i / \pi_i$ is the best unbiased estimator for T_y in $\hat{T}_2$. [4+4]=[8]

2. Define univariate ratio, regression, difference and product estimators for general sampling designs. In particular give their definitions in case of simple random sampling simple stratified sampling, systematic sampling and ppswr sampling. [10]

3. Show that in case of simple random sampling, regression estimator is at least as precise as simple random mean, ratio, difference and product estimators provided the sample size is large. Is it true for other sampling schemes also? What are these sampling schemes? [10]

4. Define general stratified sampling procedures. State, in brief, the various problems which arise in using stratified sampling. [8]

5. Suppose you want to use either of simple random sampling, simple stratified sampling and systematic sampling for estimating the mean of a finite population. Under what situations you would prefer one sampling procedure to the other and why? [20]

6. The following data show the classification of total number (Ni) of Rakshawallahs in Kanpur City (in year 1970) by their montnly earnings (after making payment to rakshaw owners) The average monthly expenditure ($\bar{Y}_i$) per rikshawallah and the square root of mean square (S_i) are given for each class. (In expenditures the payment of debts has not been accounted for.)

Monthly earnings (in Rs.)	N_i	$\bar{Y}_i$	S_i
30 - 40	102	90	8.3
40 - 60	203	95	13.3
60 - 100	305	102	15.0
100 - 150	1015	125	17.2
150 - 200	210	150	18.3
200 - 225	125	175	25.2

p.t.o.

6. (contd.)
For a stratified sample of 100 rikshawallahs with earning classes as strata how many persons should be selected from each stratum if we want to use (i) proportional allocation (ii) Neyman allocation?

Compare the relative efficiencies of these two sampling methods with that of simple random sampling. [30]

7. A systematic sample of six units is to be drawn from a population of 27 units to estimate the population total of a character. Give an unbiased estimator and its variance. [10]

8. Give cumulative total method and Lahiri's method of selecting the units with PPS with replacement. Discuss their relative advantages and disadvantages. [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73 [306]

PERIODICAL EXAMINATION

Science-5: Statistical Mechanics

Date: 22.1.73 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. All questions carry equal marks.

1. Describe Aston's mass spectrograph and explain its principle of action.

2. Give an elementary theory for the Compton effect. In what respects does Compton effect differ from Raman effect?

3. Give an experimental arrangement for the measurement of the charge of an electron. What is its correct value?

4. Give an elementary treatment of Bohr's theory of hydrogen spectrum. Write the expression for the Balmer series. Explain the physical significance of the series limit.

5. Assuming that the electron is able to behave both as a particle and as a wave, how would you infer that an electron of mass m moving with velocity v should correspond to a wave of wavelength $\lambda = h/mv$?

Give an account of the experiment carried out on the diffraction of electrons which lent support to de Broglie's wave theory of matter.

6. Write short notes on - (any two):

(a) Geiger Muller counter

(b) Wilson cloud chamber

(c) Positive rays.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[307]

MID-YEAR EXAMINATION

Statistics-8: Linear Estimation (Theory and Practical)

19.2.73. Maximum Marks: 100 Time: 4 hours

Note: Answer both the Groups. Group B is compulsory, and carries 30 marks. Answer as much as you can from Group A which carries 85 marks. Maximum you can score in this group is 70. Marks allotted for each question are given in brackets [ ].

Group A

Maximum Marks: 70.

1. Let $Y_1, \ldots, Y_n$ be n independent random variables with $E(Y_i) = \sum_{j=1}^{m} x_{ij}\beta_j$, $V(Y_i) = \sigma^2 (i = 1, \ldots, n)$, where x_{ij}'s are known constants, and β_j's unknown parameters. Define $Q_j = \sum_{i=1}^{n} Y_i x_{ij}$ $(j = 1, \ldots, m)$. Prove the following:

a) Any linear function of Q_j's with zero expectation must identically vanish.

b) The covariance between Q_j and any linear function of Y_i's with zero expectation is zero.

c) Using (a) and (b) or otherwise, show that all non-trivial linear functions of Q_j provide minimum variance unbiased estimators of linear parametric functions.

d) If $\sum_{j=1}^{m} c_j Q_j$ estimates unbiasedly the parametric function $\sum_{j=1}^{m} \lambda_j\beta_j$, then the variance of the estimator is $\sigma^2(\sum_{j=1}^{m} \lambda_j c_j)$. [4+3+5+3]=[15]

2.a) Consider the model

$$E(\underline{Y}) = X\,\underline{\beta},\quad V(\underline{Y}) = \sigma^2 I$$

(The notations have their usual significance).

Suppose $\underline{x}_j' \underline{x}_j \leq c_j^2$, $j = 1, \ldots, m$, where $X = (\underline{x}_1, \ldots, \underline{x}_m)$, c_j^2's are known positive constants. Assuming X to be of full rank, show that $V(\hat{\beta}_j) \geq \frac{1}{c_j^2}$, $(j = 1, \ldots, m)$, and that the minimum is attained when $\underline{x}_j' \underline{x}_j = c_j^2$, $\underline{x}_j' \underline{x}_k = 0 (1 \leq j \neq k \leq m)$. [10]

b) Let $Y_1, \ldots, Y_n$ be iid $N(0,1)$ variables. Consider the quadratic forms $Q_i = \underline{Y}' A_i \underline{Y}$ $(i = 1, \ldots, k)$, where $\underline{Y} = (Y_1, \ldots, Y_k)'$ and A_i's are known symmetric matrices with rank $A_i = n_i$ $(i = 1, \ldots, k)$. Show that for $Q_1, \ldots, Q_k$ to be independently distributed with $Q_i \sim \chi^2_{n_i}$ $(i = 1, \ldots, k)$, either of the following two conditions (i) and (ii) is, necessary and sufficient.

PLEASE TURN OVER

i) $A_1, \ldots, A_k$ are all idempotent matrices.

ii) $A_i A_j = 0$ for all $1 \le i \ne j \le k$.

3.a). Let $\underline{Y}$ $N(X\underline{\beta}, \sigma^2 I)$. Let H be a matrix of order $(m \times k)$ and rank k such that $\mathcal{M}(H) \subset \mathcal{M}(X')$.

Define $R_o^2 = \min_{\underline{\beta}} (\underline{Y} - X\underline{\beta})'(\underline{Y} - X\underline{\beta})$,

$R_1^2 = \min_{\underline{\beta}} (\underline{Y} - X\underline{\beta})'(\underline{Y} - X\underline{\beta})$ subject to $H'\underline{\beta} = \underline{\xi}$.

where $\underline{\xi}$ is given. Prove that if rank $X = r$, then

i) R_o^2 and $R_1^2 - R_o^2$ are independently distributed;

ii) $R_o^2/\sigma^2 \sim \chi^2_{n-r}$;

iii) if $H'\underline{\beta} = \underline{\xi}$ is true, then $(R_1^2 - R_o^2)/\sigma^2 \sim \chi^2_k$.

b) Let Y_i $(i = 1, \ldots, n)$ be n independent normally distributed variables with $V(Y_i) = k_i^2 \sigma^2$, and

$$E(\underline{Y}) = \underline{a} + X\underline{\beta},$$

where $\underline{a}$ is a vector of known constants, X is a known matrix of rank r, β is a vector of unknown parameters, and k_i's are certain known positive constants, $\underline{Y} = (Y_1, \ldots, Y_n)'$. Show that

i) If $R_o^2 = \min_{\underline{\beta}} \sum_1^n (Y_i - E(Y_i))^2/k_i^2$,

then $R_o^2/\sigma^2 \sim \chi^2_{n-r}$.

ii) If $H = PX$ is a known k-rowed matrix of rank k, $\underline{b}$ is a vector of known constants, and

$$R_1^2 = \min_{\underline{\beta}} \sum_1^n (Y_i - E(Y_i))^2/k_i^2,$$

subject to $H\underline{\beta} = \underline{b}$, then under $H\underline{\beta} = \underline{b}$, $(R_1^2 - R_o^2)/\sigma^2 \sim \chi^2_k$ independently of R_o^2.

4. How would you test (i) the equality and (ii) the parallelism of two regression lines. State clearly the assumptions you make and explain in details the test procedures. (No derivations needed).

5. Given four independent stochastic variates Y_1, Y_2, Y_3 and Y_4 having a common variance σ^2 such that $E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_2 + \theta_3$, $E(Y_3) = \theta_3 + \theta_4$, $E(Y_4) = \theta_1 + 2\theta_2 + 2\theta_3 + \theta_4$. Show that $\lambda_1\theta_1 + \lambda_2\theta_2 + \lambda_3\theta_3 + \lambda_4\theta_4$ is estimable if and only if $\lambda_1 - \lambda_2 + \lambda_3 = \lambda_4$. Find an unbiased estimator of σ^2. Test the hypotheses:

i) $2\theta_1 - 7\theta_2 - 5\theta_3 + 4\theta_4 = 10$

ii) $-2\theta_2 + 3\theta_3 + 5\theta_4 = 11$
and $\theta_1 + 4\theta_3 + 5\theta_4 = 18$.

GO ON TO THE NEXT PAGE

Group B

Maximum Marks: 30

6. The following table gives the stature in cm (X) and weight in lbs. (Y) of 15 school-boys from a certain school.

Serial no. of individuals	Stature (cms.)	Weight (lbs.)
1	146	83
2	152	76
3	148	70
4	157	82
5	155	88
6	159	92
7	138	58
8	138	58
9	148	74
10	146	70
11	145	62
12	138	62
13	146	70
14	158	106
15	168	88

a) Obtain the linear regression of Y on X.

b) Test the following hypothesis H_0 about the regression equation $Y = \alpha + \beta X$:

H_0: $\alpha = 2.0$ and $\beta = 0.5$. [18]

7. Three varieties of potato are planted, each on 4 plots of ground of the same size and type, and each variety is treated with 4 different fertilizers. The yields in tons are as follows:

Variety \ Fertilizers	1	2	3	4
1	2.8	1.9	1.7	2.3
2	2.5	1.8	1.8	1.6
3	2.7	2.0	1.7	1.9

Prepare an ANOVA table, and test whether there is any evidence that

a) any difference exists between the yields of varieties independently of the fertilizers;

b) any differential effect is exerted by the fertilizers independently of the varieties. [12]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[308]

MID-YEAR EXAMINATION

Statistics-10: Probability

Date: 20.2.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 115 marks. Attempt as many questions as you can. The maximum you can score is 100. Marks allotted for each question are given in brackets [].

1. Let (X, Y) be a two-dimensional continuous random variable. Define the conditional expectation of Y given X. Show that $E(E(X|Y)) = E(X)$. [4+6]=[10]

2. Find the constant c so that the function f defined by

$$f(x,y) = \begin{cases} cxy & \text{if } 0 < x < y < 5 \\ 0 & \text{otherwise,} \end{cases}$$

is the p.d.f. of a two-dimensional random variable (X, Y). Evaluate (i) $P\{X > 1 | Y < 2\}$ and (ii) $P\{X > 1 | Y = 2\}$. [5+7+8]=[20]

3. Let X and Y be independent χ^2 random variables, each with two degrees of freedom. Find the distribution of

$$U = \frac{1}{2}(X - Y).$$ [15]

4. Show that the joint distribution of (X, Y) is bivariate Normal if and only if, for all real numbers u and v, the distribution of $uX + vY$ is univariate Normal.

(The joint moment generating function of the bivariate Normal distribution is given by

$$\phi(t_1, t_2) = \exp\left[\mu_1 t_1 + \mu_2 t_2 + \frac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]$$

in the usual notation.) [10]

5. A line segment AB is divided by a point C into two parts AC and CB of lengths a and b respectively. Assume that $a > b$. Points X and Y are chosen at random on AC and CB respectively. What is the probability that AX, XY and YB can form a triangle? [15]

6. Players A and B, with Re. 1 and Rs. 2 respectively, agree to play a series of (independent) games. The probability of winning a single game is p for A and $1 - p$ for B. After each game, the loser pays Re. 1 to the winner. What is the probability that B will be ruined (i) exactly at the nth game, (ii) at or before the nth game? [15]

7. The probability of obtaining a head in a single toss with a certain coin is p. Find the probability of not getting two tails in succession in k independent tosses of this coin. [15]

8. Two urns contain, respectively, a white and b black, and b white and a black balls. A series of drawings is made, according to the following rules:
   i) Each time only one ball is drawn and immediately returned to the same urn it came from.
   ii) If the ball drawn is white, the next drawing is made from the first urn.
   iii) If it is black, the next drawing is made from the second urn.
   iv) The first ball drawn comes from the first urn. What is the probability that the nth ball drawn will be black? [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73 [309]

MID-YEAR EXAMINATION

Statistics-7: Sample Surveys (Theory and Practical)

Date: 21.2.73. Maximum Marks: 100 Time: 4 hours

Note: The paper carries 115 marks. Q. No.4 is compulsory. Answer as many questions as you can from the rest. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

1. If sampling is carried out without replacement with equal probabilities at each draw then show that sample mean is the best unbiased estimator for population mean $T_{(y)}$ in the class of linear estimators $\sum_{r=1}^{n} a_r y_r$ where a_r $(r = 1,\ldots,n)$ are the constants to be used as weight to the observative on the unit which appears at r^{th} draw. [15]

2. A simple random sample is to be taken to estimate the number of healthy young plants of flower per foot of length from a 1 foot wide and 400 ft. long bed of plants in a nursery where the sampling unit is 1 ft. of length of the bed. If y is character which gives number of plants per ft. length of the bed then find the desired sample size to estimate $\bar{T}_y$ such that in 95 % cases the sample mean $\bar{y}$ lies in the range $\bar{T}_y \mp 0.01$. From previous experience $S_y^2 = 90$ and $\bar{y}$ may be assumed to be normally distributed. [8]

3. To estimate the population mean $\bar{T}_y$ on the basis of a simple random sample (wtr) of size n, consider an estimator

$$d = \sum_{i=1}^{2} w_i \bar{y}\, \bar{T}_i / \bar{x}_i,$$ where $\bar{x}_i$ and $\bar{T}_i$ are the sample mean and population mean respectively of auxiliary characters x_i $(i = 1,2)$ and w_i are constants with $w_1 + w_2 = 1$.

i) Assuming $\left|\frac{\bar{x} - \bar{T}_i}{\bar{T}_i}\right| < 1$ find bias and MSE of d to the terms of order $1/n$.

ii) Find w_1 and w_2 such that MSE in (i) is minimum and then find the resulting bias and MSE. [15]

4. Following table gives the values of a character y for a population of 4 units and the sizes x_i of the units.

units		U_1	U_2	U_3	U_4
sizes	x_i	10	40	30	20
	y_i	20	25	15	30

i) Select a sample of 2 units with probability proportional to size at each draw and without replacement. Give H-T estimate for $T_{(y)}$ based on your sample.
Find the variance of the estimate and Yates and Grundy unbiased estimate of the variance.

PLEASE TURN OVER

ii) Compare the following sampling strategies with that in (i):

S_1 = {srs wtr, simple estimator}
S_2 = {srs wtr, ratio estimator}
S_3 = {ppx wr, simple estimator}
S_4 = {Midzuno scheme, H-T estimator}

where the samples of size 2 are drawn in each case and estimation of $T_{(y)}$ is under consideration. [25

5. Suppose you want to use Harvitz Thompson estimator, to estimate the total of a finite population and either of simple random sampling and simple stratified sampling. Under what situations you would prefer one sampling strategy over the other and why? [10

6. Suppose you want to use information on a character z to select the units with ppz and with replacement and information on a character x to form a difference estimator for T_y, the population total of a character y. What sampling scheme would you adopt in case neither information on z nor on x is at hand? Give the difference estimator for T_y in this situation and find its variance. [15

7. Mean square error of an estimator d is given by

$$M(d) = \frac{S^2(1-\rho^2)}{m} + \frac{\rho^2 S^2}{n} .$$

Find the values of m and n which minimize M(d) for a given cost of the form $C = a_0 + nC_1 + mC_2$. Find the resulting M(d). [7

8. What do you mean by and why do you need the following:

i) double sampling
ii) circular systematic sampling
iii) cluster sampling. [10

9. A simple random sample of n clusters is drawn without replacement from a population of N clusters each containing M elements. Give an unbiased estimator of the population mean per element, find its variance in terms of intra-class correlation coefficient and discuss its efficiency as compared to the mean of a sample random sample of n M elements drawn without replacement. [10.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1972-73

MID-YEAR EXAMINATION

Economics - 2

Date: 22.2.1973 Maximum Marks : 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts.

Group A: Micro-economics

Maximum Marks: 40
Answer all the questions.

1. A and B are two duopolists assumed to be playing a two-person constant sum game. A's pay-off matrix is as follows:

A's Strategies \ B's Strategies	1	2
1	5	6
2	7	4

Find in what proportions A and B should combine their respective strategies so that there may be a determinate solution of the duopoly problem. [20]

2. In a fully competitive economy a firm produces a single commodity by using two inputs. Show that if the firm is efficient and has a Cobb-Douglas production function, its average and marginal costs of production will be the same at all levels of output. [20]

Group B: Macro-economics

Maximum Marks: 60

3. Give an analytical exposition of the interactions between the multiplier analysis and the principle of acceleration and show that the change in income over time may follow one of four alternative paths. [35]

4. Derive graphically the IS and LM functions and use them to arrive at the general equilibrium of the product and money markets. Examine the situations in which shifts may occur in the two functions and comment on the resulting changes in income and interest rate. [20]

5. Explain the main features of the classical and Keynesian theories of employment, demonstrating carefully how the respective demand and supply functions are derived in the two cases. [25]

6. Examine the view that the central bank's discount policy is not as effective in the matter of controlling the lending potential of the commercial banks as its other two policies. [15]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[311]

MID-YEAR EXAMINATION

Science-5: Physics

Date: 23.2.73 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. Marks allotted for each question are given in brackets [].

1. Describe the essential features of an X-ray tube and explain its mode of action.

Electrons bombarding the anode of a Coolidge tube produce X-rays of wavelength 1 A. Find the energy in electron volt of the bombarding electrons and the velocity at the moment of impact. [8+6+6]=[20]

2. Describe Thomson's method for determining e/m for cathode rays.

What velocity will an electron acquire in moving through a p.d. of 1 volt? Use the standard values for the charge and mass of an electron. [14+6]=[20]

3. Deduce an expression for the change of wavelength in the scattering of Compton type.

An X-ray photon of wavelength 0.1 A is scattered at angle 90' with its original direction after collision with an electron at rest. Find the energy in eV it will lose in collision. [12+8]=[20]

4. Give an elementary theory for the Raman effect and the description of an experimental arrangement for its study in liquids.

When benzene is irradiated with the mercury line 4358 A, a Raman line appears in the same position as the argon line of wavelength 4201 A. Calculate the wavenumber shift (Raman shift) of the parent line. [8+7+5]=[20]

5. Outline the theory of disintegration process of the radioactive nuclei. Define the mean life and show its relation with the half life.

The half life of radioactive potassium is 18.3×10^8 years. Find the number of beta-particles emitted per sec. per gm. of potassium. (decay constant of K is 1.2×10^{-17} per sec., and Avogadro number is 6.023×10^{23}). [7+3+4+6]=[20]

6. Work out, showing full calculations, the following: -

a) the radius of the electron orbit for hydrogen in the ground state

b) the same for singly ionised helium in the ground state

c) the relation between the electron volt and erg

d) the wavelength of de Broglie waves for electron and the voltage by which they are accelerated. [4×5]=[20]

Write short notes on: (a) Uncertainty principle, (b) GM counters, (c) Wilson's cloud chamber. [7+6+7]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III 1972-73

[312]

MID-YEAR EXAMINATION

Mathematics-5 : Calculus.

Date: 24.2.1973 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. If a function of serveral variables is differentiable at a point then the partial derivatives exist. Is the converse true ? If not, then under what condition is it true. Consider the following example.

$$f(x, y) = 0 \qquad \text{if } x = 0 \text{ or } y = 0$$

$$f(x, y) = |x| \qquad \text{if } x-y = 0 \text{ or } x+y = 0$$

Between these lines we define the function in such a way that it is represented geometrically by planes. The surface therefore consists of eight rectangular planes meeting in the roof like edge above the lines $x = 0$, $y = 0$, $y = x$ and $y = -x$. The surface obviously has no tangent plane at origin. Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist. Find their values. [17]

2. Find equations to the tangent plane and the normal to the surface

$$x^2 + 3y^2 + 2z^2 = 6 \quad \text{at} \quad (2, 0, 1)$$

[15]

3. State the mean value theorem for a function of several variables. Find the polynomial of the second degree which best approximates to the function $f(x, y) = \text{Sin } x . \text{Sin } y$ in the neighbourhood of origin. [17]

4. Let $u = f(x_1, x_2, x_3)$ be any differentiable function defined in a region of $x_1\, x_2\, x_3$-space. Define the directional derivative $D^{(\alpha)}f$ at a point (x_1, x_2, x_3) in the direction making angles $\alpha_1, \alpha_2, \alpha_3$ with the coordinate axes. If $\bar{e}$ is a unit vector in this direction, show that

$$D^{(\alpha)} f = e \cdot \text{grad } f.$$

Prove that in the direction of gradient vector, the function increases most rapidly.

[17]

p. t. o.

5. Define vector and scalar fields. Give an example of each of them. Let $u = f(x_1, x_2, x_3)$ be any differentiable function defined in a region of x_1, x_2, x_3 - space. Let $\bar{u}$ be a vector with components $u_1 = f_{x_1}$, $u_2 = f_{x_2}$ $u_3 = f_{x_3}$ in x - system. If we now pass to ζ - system by rotation of axes, then if w_1, w_2, w_3 are the components of $\bar{u}$ now, show that

$$w_1 = f_{\xi_1}, \quad w_2 = f_{\xi_2}, \quad w_3 = f_{\xi_3}$$

where the relation in the coordinate systems is given by the table.

	x_1	x_2	x_3
ζ_1	α_1	β_1	γ_1
ζ_2	α_2	β_2	γ_2
ζ_3	α_3	β_3	γ_3

where α_i is the cos of angle between ζ_i axis with x_1 axis etc.

[17]

6. Define divergence and curl of a vector field.

Find curl $\bar{u}$ where $\bar{u}$ has components $u_1 = x_3^2$, $u_2 = x_1^2$, $u_3 = x_2^2$. $\bar{u}$ is a vector in $x_1\, x_2\, x_3$ space.

Prove (i) div curl $\bar{u}$ = 0 for any $\bar{u}$.

(ii) curl grad $f = \bar{0}$, where f is a function of x_1, x_2, x_3 variables.

[17]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[313]

MID-YEAR EXAMINATION

Mathematics-6: Real Analysis

Date: 26.2.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 115 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

1. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1.3 \ldots (2n-1)}{2.4 \ldots 2n} x^n$$ [10]

2. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. Show that f has second derivative at each x in the interval $(-R, R)$ and

$$f''(x) = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}.$$ [20]

3. Let (M, d) be a metric space. Let A and B be two non-empty subsets of M. Show that

$$\overline{A \cap B} - i(A \cap B) \subset (\bar{A} - i(A)) \cup (\bar{B} - i(B)).$$ [8]

4. Let N denote the set of integers. Show that (N, d) is a matric space, where for any $m, n \in M$

$$d(m,n) = |m-n|.$$

Characterize the connected subsets of (N, d). [12]

5. Let Q be the set of rational numbers. Define for $,y \in Q$

$$d_1(x, y) = |x-y|,$$
$$d_2(x, y) = 0 \quad \text{if } x = y,$$
$$= 1 \quad \text{if } x \neq y.$$

Show that

a) (Q, d_1) and (Q, d_2) are matric spaces.

b) Connocted subsets of (Q, d_1) and (Q, d_2) are same.

c) One of the above two matric spaces is complete and the other is not. [5+15+5]=[25]

6. Let $X =$ (x, y): x and y are real numbers . For points $Z_1 = (x_1, y_1)$ and $Z_2 = (x_2, y_2)$ define

$$d(z_1, z_2) = |y_1 - y_2| \quad \text{if } x_1 = x_2$$
$$= |y_1| + |y_2| + |x_1 - x_2) \quad \text{if } x_1 \neq x_2$$

Show that d is a metric on X . [15]

7.

PLEASE TURN OVER

7. Let (M, d) and (M*, d*) be two metric spaces. Let f be a function on M into M*.

a) Show that the following three statements are equivalent.

i) For any open set U in M*, $f^{-1}(U) = x \in M: f(x) \in U$ is open in M.

ii) For any $x \in M$; $f(x) \rightarrow f(x)$ in M*, whenever x_n is a sequence in M such that $x_n \longrightarrow x$.

iii) For any closed set F in M*, $f^{-1}(F)$ is closed in M.

[Hint: Try to prove (i) $\Rightarrow$ (ii) $\Rightarrow$(iii) $=>$ (i).]

b) Show that if A is any connected subset of (M,d) and if f is any function satisfying (i), then f(A) is connected. [8+7]=[2

INDIAN STATISTICAL INSTITUTE [314]
Research and Training School
B. Stat. Part III: 1972-73

PERIODICAL EXAMINATION

Statistics-8: Multivariate Analysis (Theory and Practical)

Date: 7.5.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Let X_1 and X_2 have a bivariate normal distribution with zero means, variances σ_1^2 and σ_2^2 and correlation coefficient ρ. Find the correlation coefficient between X_1^2 and X_2^2. [7]

b) In Q.1.(a), find the distribution of X_1/X_2 if $\sigma_1=\sigma_2=1$. [7]

c) Suppose X_1, X_2 and X_3 have a trivariate normal distribution with zero means and dispersion matrix

$$\Sigma = \begin{bmatrix} 1 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 2 \end{bmatrix}$$

Find the correlation coefficient between

$$(X_1 + X_2)^2 \text{ and } (X_2 + X_3)^2.$$ [8]

2.a) Write down the pdf of a p-variate normal distribution with mean vector $\underline{\mu}$ and dispersion matrix Σ, Σ being positive definite. [3]

b) Derive the mgf of the distribution in Q.2(a). [6]

c) Let $\underline{X} = (X_1, \ldots, X_p)'$ be a random vector having the pdf given in Q.2(a). Suppose $p = 2q$, $\underline{X}^{(1)} = (X_1, \ldots, X_q)$, $\underline{X}^{(2)} = (X_{q+1}, \ldots, X_{2q})'$. Find the distribution of $\underline{X}^{(1)} + \underline{X}^{(2)}$. [5]

d) Under the same set up as Q.2(a)-2(c), find the distribution of $(\underline{X} - \underline{\mu})' \Sigma^{-1} (\underline{X} - \underline{\mu})$. [6]

e) In Q.2(a), assuming μ and Σ to be unknown, find their maximum likelihood estimates. What will be the maximum likelihood estimate of Σ if $\underline{\mu}$ is known? [13+2]=[15]

3.a) Suppose $X_1, \ldots, X_k, X_{k+1}$ are independently distributed random variables, X_i having the pdf

$$f(x_i) = \begin{cases} \dfrac{\exp(-x_i)\, x_i^{\nu_i - 1}}{\Gamma(\nu_i)}, & x_i > 0,\ \nu_i > 0 \\ 0, & \text{otherwise} \end{cases}$$

$i = 1, 2, \ldots, k+1$. Derive the joint pdf of

$$T_1 = X_1 \Big/ \left(\sum_1^{k+1} X_i\right), \ldots, T_k = X_k \Big/ \left(\sum_1^{k+1} X_i\right).$$ [11]

p.t.o.

3.b) Suppose (X_1, Y_1) and (X_2, Y_2) are independent random vectors, each having the $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ distribution. Show that

$$E[\text{sgn}\,(X_1-X_2)\ \text{sgn}\,(Y_1-Y_2)] = 4P[X_1 > X_2, Y_1 > Y_2]-1 =$$
$$= \frac{2}{\pi}\sin^{-1}\rho,$$

where sgn x = 1, 0 or -1 according as $x >, =,$ or < 0.

4. Draw a random sample of size 10 from the following distribution with pdf

$$f(x) = \begin{cases} x\,e^{-x^2/2} & , \ x > 0 \\ 0 & , \ \text{otherwise} \end{cases}$$

5. The following statistics were obtained from a bivariate frequency distribution of yield of plants (x) and that of offsprings (y) in suitable units.

	mean	variance	correlation coefficient
x	55	15	
y	42	12	0.75

If from the parent plants, only the top 10 % in respect of yield are selected and allowed to propagate, what will be the expected yield in their offsprings? You may assume the joint frequency distribution to be bivariate normal.

6. In a University examination, the frequency distribution of marks in Mathematics and Statistics is assumed to be bivariate normal with means 55 and 50, s.d.'s 5 and 4 and correlation coefficient 0.8.

a) What percentage of students will exceed the mean scores in both the subjects?

b) What percentage of students score more than 140 in the aggregate?

c) Amongst those that score 60 in statistics, what percentage score more than 60 in Mathematics?

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[315]

PERIODICAL EXAMINATION

Statistics-10: Probability

Date: 14.5.73 Maximum Marks: 100 Time: 3 hours

Note: This paper carries 119 marks. Answer as many questions as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1.a) Let Ω be a non-empty set and $\mathcal{A}$ a non-empty class of subsets of Ω. When do you call $\mathcal{A}$ (i) a ring (ii) a field (iii) a σ-ring and (iv) a σ-field?

b) Classify the class $\mathcal{A}$ in each of the following examples into one of the four types mentioned in Question 1(a).

i) Ω is any set. $\mathcal{A} = \{A, A^c, \Omega, \emptyset\}$ where A is any subset of Ω.

ii) Ω is an uncountable set, $\mathcal{A}$ is the class of all finite subsets of Ω.

iii) Ω is an uncountable set, $\mathcal{A}$ is the class of all countable subsets of Ω.

iv) Ω is an uncountable set. $\mathcal{A}$ is the class of all subsets of Ω which either are countable or have countable complements. [8+12]=[20]

2.a) Define the lim inf, lim sup and lim of a sequence of subsets of a space Ω.

b) Let $(\Omega, \mathcal{A}, P)$ be a probability space (in the usual notation). Show that for any sequence $\{A_n\}$ of sets in $\mathcal{A}$,

$$P(\lim\inf A_n) \leq \lim\inf P(A_n) \leq \lim\sup P(A_n) \leq P(\lim\sup A_n).$$

c) Deduce from (b) that if $\{A_n\}$ is an increasing or a decreasing sequence of sets in $\mathcal{A}$, then $\lim A_n$ exists and $P(\lim A_n) = \lim P(A_n)$. [6+10+6]=[22]

3.a) For a sequence $\{A_n\}$ of events in a probability space $(\Omega, \mathcal{A}, P)$, show that $\Sigma P(A_n) < \infty$ implies $P(\lim\sup A_n) = 0$.

b) Show that if $\{A_n\}$ is a sequence of independent events in a probability space, then $\Sigma P(A_n) = \infty$ implies $P(\lim\sup A_n) = 1$.

c) Show by means of an example that without the assumption of independence in (b) the conclusion may not be true. [8+12+8]=[28]

p.t.o.

4.a) Define a random variable on a probability space; define its distribution function.

b) State the important properties of a distribution function. Show that a distribution function can have at most a countable number of discontinuities. [5+15]=[

5.a) Define the following convergence concepts for a sequence of random variables on a probability space:

i) convergence with probability one,

ii) convergence in probability and

iii) convergence in distribution.

b) i) Show that convergence in probability implies convergence in distribution.

ii) Give an example to show that the converse does not always hold. [9+10+10]=[

INDIAN STATISTICAL INSTITUTE [316]
Research and Training School
B. Stat. Part III: 1972-73

PERIODICAL EXAMINATION

Mathematics-6: Analysis

Date: 21.5.73 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1.a) Show that every connected component in a metric space is closed.

b) Find the connected components in (N,d), where N is the set of integers and $d(m,n) = |m-n|$.

c) Find the connected components in the real line, with the usual metric. [5+5+2]=[12]

2.a) Show that in a metric space every connected set is contained in a connected component.

b) Show that any two distinct connected components in a metric space are disjoint. [15+5]=[20]

3.a) Let $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$. Show that

i) $E'(z) = E(z)$ for all complex numbers z

ii) $E(z) \neq 0$ for all complex numbers z

and
iii) $E(z_1 + z_2) = E(z_1)E(z_2)$ for all complex numbers z_1 and z_2.

Also show that, if F is an entire function satisfying i), ii) and iii) above, then $F(z) = E(z)$ for all z.

b) Let f be a holomorphic function in a region Ω. Show that, f' the derivative of f is holomorphic in Ω.

(Hint: You may assume Cauchy's integral formula for rectangular).

c) Let $f = u + iv$ be a holomorphic function in a region Ω, where u and v are real valued function. Show that u and v satisfy Cauchy-Reimann equations and the partial derivatives of u and v are continuous. [18 +12+10]=[40]

4.a) Show that every entire bounded function is a constant.

b) Let f be an entire function and let P be a non-constant polynomial. Suppose g is defined by

$$g(z) = P(f(z))$$

and $\text{Re } g$ is bounded. Show that f is a constant. [6+12]=[18]

5. Define winding number $W(z, \eta)$ of a closed piecewise smooth path η around $z \in \mathcal{C}$ - range η. Show that $W(z, \eta)$ is an integer valued function on $\mathcal{C}$ - range η. [10]

INDIAN STATISTICAL INSTITUTE [317]
Research and Training School
B. Stat. Part III: 1972-73

PERIODICAL EXAMINATION

Statistics-9: Design of Experiments (Theory and Practical)

Date: 28.5.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 115 marks. Answer any part of any question. Maximum marks you can score is 100. Marks allotted for each question are given brackets [ ].

1. What are the basic principles in designing an experiment? Explain with examples their role in achieving the goals of validity and efficiency of experimental results. [20]

2. What is a Latin Square Design?
Give two examples, one from industry and one from agriculture where Latin Square Design will be most suited in your opinion.

How do you get a Graeco-Latin Square and what is its utility?

Briefly indicate the procedures for analysing a Graeco-Latin Square. [20]

3. Describe the General procedure for analysing the data obtained on an experimental design in which observations in some plots are missing.

In particular, supposing an observation is missing in a Randomised Block Design, how will you proceed? Write down the complete procedure along with algebraic expressions wherever necessary. [20]

4. If $N(v)$ denotes the maximum number of mutually orthogonal Latin Squares of order $v \times v$, prove that

(i) $N(v) \leq v-1$ in general

and (ii) $N(v) = v-1$ when v is a prime number or a prime power. [8+12]=[20]

5. Obtain a complete set of mutually orthogonal Lagin Squares of order 8×8. [10]

6. An industrial experimenter wishes to compare the effects of five types of grids A, B, C, D and E on the vacuum of radio tubes. Sealing machines and operators are two factors which might affect vacuum. The experimental data were obtained from the following Latin Square.

Operator	Machines				
	1	2	3	4	5
1	E 98.0	B 95.8	D 97.2	A 97.0	C 97.8
2	C 98.3	D 97.9	B 97.7	E 98.9	A 98.0
3	A 93.6	C 94.5	E 94.6	B 95.3	D 96.8
4	D 97.2	E 95.0	A 93.7	C 97.0	B 97.3
5	B 96.9	A 95.3	C 97.0	D 98.2	E 97.8

Make a suitable analysis of the data and test for the three main effects. [25]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73 [318]

PERIODICAL EXAMINATION

Mathematics-5: Calculus

Date: 4.6.73 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [].

1. State and prove the Implecit function theorem for a function of two independent variables. [20]

2.a) What are the regular and singular points of the curve $F(x,y) = 0$. Give an example of each. Is non-vanishing of $F_x^2 + F_y^2$ at a point (x_0, y_0) necessary for it to be regular. Prove or disprove. [10]

b) Prove that the following equation has unique solution for y near the point (1, 1),
$$xy + \log xy = 1.$$ [10]

3.a) Find the angle between the curves $F(x, y) = 0$ and $G(x, y) = 0$ at a point of intersection. Show that the confocal parabolas
$$y^2 - 2p(x + \frac{p}{2}) = 0$$
for negative and positive values of p intersect at right angles. [10]

b) Find the tangent plane of the surface $\sin^2 x + \cos(y+z) = \frac{3}{4}$ at the point $(\frac{\pi}{6}, \frac{\pi}{3}, 0)$. [10]

4.a) Show that transformation of 'reflection in the unit circle' from xy plane to be $\xi\eta$ plane has an inverse similar to it. Discuss the geometry of the transformation. [10]

b) Define the affine transformation and the transformation to curvilinear coordinates in two variables. Give an example of each of these. [10]

5.a) Define the Jacobian of the functions $\xi = \phi(x, y)$, $\eta = \psi(x, y)$ with respect to the variables x and y and deduce its expression in terms of the Ist partial derivatives of ξ and η. [10]

b) What is a primitive transformation. State and prove the inverse function theorem for a primitive transformation. State also the general theorem on the Inversion of transformations. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1972-73

[319]

PERIODICAL EXAMINATION

Science-5: Geology

Date: 11.6.73 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions.
All questions carry equal marks.

1. Define the terms 'weathering', 'erosion', and 'transportation'. What are the major natural agencies that control these processes?

2. What is a sedimentary rock? How is a sedimentary rock formed from sediments? What do you understand by the 'texture' of a sedimentary rock?

3. What do you understand by 'structure' of a sedimentary rock? What are the major types of bedding commonly found in the sedimentary rocks? Describe two common types of beddings.

4. What are the major environments in which sedimentation takes place? Briefly describe the characters of the river environment.

5. How are the major mountain belts distributed on the surface of the earth? Briefly state the stages of development of these mountain belts.

6. Complete the following statements:

   i) Cross-stratification can help in ....

   ii) 'Bed load' means ....

   iii) A 'formation' is defined as ....

   iv) The major earthquake belts of the world are located in ....

7. Write briefly on : i) the Uniformitarianism concept.

   ii) the laws of sedimentary sequence.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. Part III: 1972-73

[320]

PERIODICAL EXAMINATION

Economics-2

Date: 18.6.73. Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A: Micro-economics

Maximum Marks: 40
Answer any two questions

1. In a simple duopoly, either firm can behave as a leader or follower. Will there always be a determinate market equilibrium? Justify your answer. [20]

2. A firm uses only two inputs, namely capital and labour to produce a single homogeneous commodity, and the production is characterized by constant return to scale. Prove that the firm's elasticity of input substitution is equal to the elasticity of average product of labour with respect to its marginal product. [20]

3. A first degree homogeneous production function involving a single output and two inputs, capital and labour, has a constant elasticity of input substitution $\sigma \neq 1$. Express the output as an explicit function of the inputs. [20]

4. A utility maximizing two commodity consumer buys a collection $(\bar{q}_1, \bar{q}_2)$ in the price income situation (p_1, p_2, y) and a collection $(\bar{q}_1 + \partial q_1, \bar{q}_2 + \partial q_2)$ in the slightly different price income situation $(p_1 + \partial p_1, p_2, y + \partial y)$. If the two collections are equally satisfactory prove that

$$\frac{\partial q_1}{\partial p_1} < 0.$$ [20]

Group B: Indian Economic Problems

Maximum Marks: 60
Answer any three questions

5. Examine the present composition of India's exports and imports. What changes have taken place in the composition during the period of planning? [20]

6. Explain the concepts of 'marketable' and 'marketed' surplus. Indicate the importance of marketed agricultural surplus in the development of the Indian economy. [20]

7. Examine the main aspects and objectives of land reform in India. [20]

8. Indicate the main advantages of co-operative farming in India. [20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B. Stat. (Hons.) Part III: 1972-73

[321]

ANNUAL EXAMINATION

Statistics-8: Statistical Methods Theory

9.7.73 Maximum Marks: 100 Time: 3 hours

Note: Answer five questions, two from group A and three from group B. All questions carry equal marks.

GROUP A

1.a) Show that the random variables $X_1,\ldots,X_p$ have a joint multinormal distribution if and only if for every set $(\lambda_1,\ldots,\lambda_p)$ of constants not all zeroes, $\sum_1^p \lambda_i X_i$ has a univariate normal distribution. Show also that the result holds if instead of every non-zero set $(\lambda_1,\ldots,\lambda_p)$, only those λ_i's (not all zeroes) satisfying $\sum_1^p \lambda_i = 0$ or 1 are considered.

b) Give an example of two random variables X_1 and X_2 each marginally normally distributed with a joint non-normal bivariate distribution. [7+5+8]=[20]

2.a) Show that if $\underline{X} = \binom{\underline{X}^{(1)}}{\underline{X}^{(2)}}$ (where $\underline{X}^{(1)} = (X_1,\ldots,X_q)'$ and $\underline{X}^{(2)} = (X_{q+1},\ldots,X_p)'$ has a p-variate normal distribution with mean vector $\underline{\mu} = \binom{\mu^{(1)}}{\mu^{(2)}}\begin{matrix} q \\ p-q\end{matrix}$, and dispersion matrix $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22}\end{pmatrix}\begin{matrix} q \\ p-q\end{matrix}$ (columns q, $p-q$), where Σ is positive definite, find the conditional distribution of $\underline{X}^{(2)}$ given $\underline{X}^{(1)}$.

b) Suppose Z_1, Z_2 and Z_3 are independent beta variables,

$Z_1 \sim \text{Beta}\,(r_1, \sum_2^4 r_i)$, $Z_2 \sim \text{Beta}\,(r_2, r_3+r_4)$,

$Z_3 \sim \text{Beta}\,(r_3, r_4)$. Let $Y_1 = Z_1$, $Y_2 = Z_2(1-Z_1)$,

$Y_3 = Z_3(1-Z_1)(1-Z_2)$. Show that the joint distribution of Y_1, Y_2 and Y_3 is Dirichlet $(r_1, r_2, r_3; r_4)$. [10+10]=[20]

3.a) For a p-variate normal distribution with mean vector $\underline{\mu}$ and dispersion matrix Σ (Σ positive definite), derive the equation of the concentration ellipsoid.

b) Let $X_1,\ldots,X_p$ have a joint distribution with pdf

$$f(x_1,\ldots,x_p) = \frac{\Gamma(\frac{1}{2}(p+1))}{\pi^{\frac{1}{2}(p+1)}} \frac{1}{(1+\sum_1^p x_i^2)^{\frac{1}{2}(p+1)}}$$

$-\infty < x_1,\ldots,x_p < \infty$.

Find the joint pdf of the subset $X_1,\ldots,X_r$ $(r<p)$. [10+10]=[20]

p.t.o.

GROUP B

4.a) Suppose W_1 and W_2 are independent random variables having chi-square distributions with m and n degrees of freedom respectively. Derive the pdf of W_1/W_2.

b) Let X_1, X_2 and X_3 be iid N(0, 1) variables. Find the distribution of

$$\frac{2X_1^2 + (X_2 + X_3)^2}{(X_2 - X_3)^2}.$$

[10+10]=[20]

5.a) Suppose W has a non-central chi-square distribution with p degrees of freedom and non-centrality parameter λ. Show that $P(W \geq c)$ is non-decreasing in λ, where c is a positive constant. Show also that if V has a central chi-square distribution with p degrees of freedom

$$P(W \geq c) \geq P(V \geq c).$$

b) Consider the same set up as in Q.5(a). Show that if p is <u>even</u>,

$$P(W \leq w) = P(X - Y \geq \tfrac{1}{2}p),$$

where X and Y are independent Poisson variables with parameters $\frac{1}{2}w$ and $\frac{1}{2}\lambda$ respectively.

[<u>Hint</u>: Note that $P(\chi^2_{2m} \leq x) = P(Z \geq m)$, where $Z \sim$ Poisson $(\frac{1}{2}x)$] [7+3+10]=[20]

6.a) On the basis of two independent random samples of sizes n_1 and n_2 from two populations, how would you test the equality of two means? Discuss the test procedure stating clearly the assumptions involved and find the distribution of the test statistic under the null hypothesis.

b) For a random variable X, show that X^2 has a chi-square distribution with 1 degree of freedom if and only if pdf of X is given by

$$f(x) = h(x)\, e^{-\frac{x^2}{2}}$$

where $h(x) + h(-x) = \sqrt{2/\pi}$. [12+8]=[20]

7.a) Let $X_1, \ldots, X_n$ be iid with distribution function F(x), and let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ denote the ordered X_i's. Show that $P(X_{(i)} \leq x) = P(Y \geq i)$, $1 \leq i \leq n$, where $Y \sim \text{Bin}(n, F(x))$.

b) Find the distribution of the sample range in samples of size n from a rectangular (0,1) distribution.

c) Suppose $X_1, \ldots, X_n$ are iid non-negative integer valued random variables. Show that the probability mass function of the X_i's is given by

$$p_\theta(x) = \theta^x(1-\theta),\ x = 0,1,2,\ldots,\ 0 < \theta < 1,$$

<u>if and only if</u> the probability mass function of $T = \min(X_1, \ldots, X_n)$ is given by $f_\theta(t) = \theta^{nx}(1-\theta^n)$, $x = 0,1,2,\ldots,\ 0 < \theta < 1$.

[Hint: $P(X = x) = P(X \geq x) - P(X \geq x+1)$]. [5+5+10]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

[322]

ANNUAL EXAMINATION

Statistics-2: Statistical Methods (Practical)

Date: 10.7.73 Maximum Marks: 50 Time: 3 hours

Note: Answer all the questions. Marks allotted for each question are given in brackets [ ].

1. Draw a random sample of size 5 from the distribution with pdf

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$ [8]

2. The sample correlation coefficient in a sample of size 10 from a bivariate population turns out to be 0.12. Under suitable assumptions to be stated, test whether the population correlation coefficient differs significantly from zero. [5]

3. The standard deviations on the basis of independent samples of sizes 12 and 10 from two populations turn out to be 7.64 and 3.58 respectively. Under suitable assumptions to be stated, test whether the ratio of the corresponding population parameters is 2. [5]

4. A drug was given to 20 subjects half an hour before bed time, while 25 other subjects were kept as controls. The next morning subjects estimated the time taken by them to fall asleep. The following gives the reported time of the two groups.

Time in minutes

Controls: 15, 25, 30, 15, 35, 40, 25, 30, 25, 35, 40, 25, 35, 20, 25, 40, 15, 15, 25, 30, 25, 10, 50, 30, 40.

Treated with drug: 25, 30, 40, 45, 15, 15, 20, 25, 30, 25 20, 15, 10, 25, 15, 25, 35, 10, 10, 15.

Assuming that the variability is the same in both the groups of individuals, test whether the drug caused a quicker onset of sleep. [12]

5. If x and y are independently normally distributed with zero means and unit standard deviations, determine the radius of the circle with centre at the origin having the probability 0.95 that the point (x,y) will fall inside it. [5]

6. The mean of a normally distributed variable X is 2.08 and 9.68 °/. values of X are negative. Find

(i) $P(X \geq 3.2)$, (ii) $P(1.8 \leq X \leq 5.6)$. [5]

7. Practical Records. [10]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

ANNUAL EXAMINATION

Statistics-10: Probability

Date: 11.7.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [].

1.a) Let X and Y be two continuous random variables. Show that $V(Y) = E[V(Y|X)] + V[E(Y|X)]$.

b) A real number X is chosen at random from the interval $[0, 1]$. For a fixed value x of X, another real number Y is chosen at random from the interval $[0, x]$. Write down the marginal p.d.f. of X and the conditional p.d.f. of Y given X. Evaluate $V(Y)$. [8+2+6]=[16]

2. Let X and Y be two independent random variables with uniform distribution on $[0, 1]$. Find the distribution of $X+Y$. [12]

3. Two points are selected at random on a line of length α so as to be on opposite sides of the mid-point of the line. Find the probability that the distance between them is less than $\alpha/3$. [10]

4.a) State the two Borel-Cantelli lemmas.

b) Let $\{X_n\}$ be a sequence of independent random variables that take only the values 0 and 1 and suppose that $P\{X_n = 1\} = p_n$. Prove that $\sum_1^\infty p_n < \infty$ if and only if $P\left\{\sum_1^\infty X_n < \infty\right\} = 1$. [4+8]=[12]

5.a) Prove that convergence with probability one implies convergence in probability.

b) Let $(\Omega, \mathcal{B}, P)$ be the closed unit interval with the Borel σ-field and the Lebesgue measure. For each positive integer n, let f_n be the indicator function of the interval $[\frac{p}{2^q}, \frac{p+1}{2^q}]$ where p and q are the unique integers such that $p+2^q = n$, $0 \le p < 2^q$. Examine whether (i) f_n converges with probability one and (ii) f_n converges in probability. [7+4+4]=[15]

6. Let $\{X_n\}$ be a sequence of independent random variables such that $P\{X_n = \pm 1\} = \frac{1}{2}(1 - 2^{-n})$ and $P\{X_n = \pm 2^n\} = 2^{-n-1}$. Show that the weak law of large numbers holds for $\{X_n\}$. [15]

7. Let $\{X_n\}$ be a sequence of i.i.d. random variables with finite fourth moment. Show that with probability one, the sequence

$$\left\{\frac{1}{n}\sum_1^n X_i\right\} \text{ converges to } E(X_1).$$ [10]

8.a) State Lindeberg's sufficient conditions under which the central limit theorem holds for a sequence of independent random variables.

b) Let $\{X_n\}$ be a sequence of independent random variables such that $P\{X_n = \pm n\} = \frac{1}{2\sqrt{n}}$ and $P\{X_n = 0\} = 1 - \frac{1}{\sqrt{n}}$. Show that $\{X_n\}$ obeys the central limit theorem but does not obey the weak law of large numbers. [5+8+7]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

ANNUAL EXAMINATION

Statistics-9: Design of Experiments
(Theory and Practical)

Date: 12.7.73. Maximum Marks: 100 Time: 4 hours

Note: The paper carries 110 marks. Answer as much as you can. Maximum you can score is 100. Marks allotted for each question are given in brackets [ ].

1. If s is a prime power, show that the following two series of BIBD's always exist:

i) $v = s^2$, $b = s^2 + s$, $r = s+1$, $k = s$, $\lambda = 1$.

ii) $v = b = s^2 + s + 1$, $r = k = s+1$, $\lambda = 1$.

Write down some procedure for constructing them. [24]

2.a) For a BIBD with parameters v, b, r, k and λ show that

(i) $\lambda (v - 1) = r (k - 1)$

and (ii) $b \geq v$.

b) For a symmetrical BIBD prove that the number of treatments common between any two blocks is constant. [10+10]=[20]

3.a) What is a factorial experiment? In what way is a factorial experiment preferable to 'one factor at a time' approach? With the usual notations for treatment combinations and yields in a 2^3 experiment replicated once only, write down the constrasts which estimate the differenent main effects and interactions.

b) Suppose a 2^3 experiment is replicated twice, each replicate being laid out in 2 blocks. In the first replicate BC is confounded, while in the second ABC is confounded. The experiment is carried out and suppose the data are presented to you. Describe clearly the procedure of analysing the data. [12+12]=[24]

4. A 2^5 experiment is to be laid out in 4 blocks of same size. Construct the design so that no main effects or two factor interactions are confounded. [12]

5. The following design was used to test 9 rations fed to rats. The gains in weight of the rats after the feeding experiment were as follows: (The ration numbers are in brackets.)

Replication 1

Block			
1.	(1) 20	(4) 15	(7) 11
2.	(3) 08	(6) 18	(9) 26
3.	(2) 18	(5) 16	(8) 02

Replication 2

Block			
4.	(7) 08	(8) 12	(9) 16
5.	(1) 20	(2) 02	(3) 02
6.	(4) 20	(5) 06	(6) 02

Replication 3

Block			
7.	(1) 13	(9) 19	(5) 14
8.	(8) 14	(4) 34	(3) 02
9.	(6) 14	(2) 20	(7) 14

Replication 4

Block			
10.	(5) 19	(7) 23	(3) 06
11.	(1) 22	(6) 12	(8) 02
12.	(9) 27	(2) 07	(4) 20

Analyse the data and comment. [30]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

[325]

ANNUAL EXAMINATION

Science-5: Physics

Date: 13.7.73 | Maximum Marks: 100 | Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Answer any five questions, taking at least one from each group. Marks allotted for each question are given in brackets [ ].

GROUP A

1. EITHER

Give a brief account of Bohr's theory of hydrogen atom.

Show that the velocity of the electron in the first orbit is about 1/137 times the velocity of light. Given:

$e = 4.8 \times 10^{-10}$ esu; $h = 6.6 \times 10^{-27}$ erg. second. [13+7]=[20]

OR

Explain the physical characteristics of photoelectric emission. Review briefly the failure of the classical electromagnetism to explain the characteristics. Show how Einstein's quantum interpretation gives satisfactory explanation of the effect.

How can Planck's constant be measured from Einstein's photoelectric equation? [6+4+6+4]=[20]

2. Describe Millikan's oil-drop method of measuring the electronic charge, giving a sketch of the apparatus used. What correction did Millikan apply to Stokes' formula?

What was the value of e obtained by Millikan? What is its best modern value? To what factor is the difference ascribed? [9+4+2+2+3]=[20]

3. Assuming that an electron is able to behave both as a particle and as a wave, show that an electron of mass m moving with velocity v corresponds to a wave of length $\lambda = h/mv$, h being the Planck's constant.

Describe the electron diffraction experiment of G.P.Thomson and explain its significance. [5+11+4]=[20]

4. EITHER

Describe with a diagram the Wilson cloud chamber and explain the principle of its operation. Compare and contrast the performances of a cloud chamber and a bubble chamber.

How can the cloud chamber be utilised to detect the α and β particles emitted from a radioelement? [7+5+4+4]=[20]

OR

What is radioactivity? Deduce relations between the decay constant, the half-life and the mean life of a radioactive substance.

The half-life of radon is 3.82 days. What fraction of a freshly prepared sample of radon will disintegrate in 10 days?

[4+10+6]=[20]

5. EITHER

What is Compton effect? Give an elementary theory for its interpretation. What is meant by Compton wavelength? What is its value? [4+12+2+2]=[20]

p.t.o.

5. OR

What is Raman effect? Give the experimental arrangement to obtain Raman lines with liquids. Explain in an elementary way the underlying theory. [4+8+8]=[20]

GROUP F

6. EITHER

State (deduction not necessary) Richardson's equation for the emission of electrons from a hot body and explain what is meant by the work function?

Define the constants of a triode and explain the meaning of each of them in reference to the static characteristics of a triode.

The following data are given for a triode:

Plate Voltage (Volts)	Grid voltage (Volts)	Plate current (amps)
320	- 4.0	1.2×10^{-3}
320	- 3.0	1.6×10^{-3}
280	- 3.0	1.2×10^{-3}

Calculate the constants of the triode. [3+3+10+4]=[20]

OR

Draw the circuit diagram of an R.C. coupled amplifier employing a triode and explain how it amplifies an alternating single voltage.

Explain the function of the coupling capacitance and the grid-leak resistance. [5+9+3+3]=[20]

7. Sketch the energy diagram of a p-n junction and explain the operation of a forward bias p-n junction. What is avalanche breakdown? [7+7+6]=[20]

8. Explain the operation of a forward bias junction transistor. The typical values for an NPN transistor are given as

$\alpha_{cc} = 0.97$ $\qquad \gamma_o = 2 \times 10^6$ ohm

$\gamma_e = 35$ ohm $\qquad \gamma_r = 1.94 \times 10^6$ ohm

$\gamma_b = 100$ ohm

Compute the values of R_{11}, R_{22}, A_i, A_v, and power gain in a common emitter circuit with $R_s = 500$ ohm and $R = 10{,}000$ ohm. [10+10]=[20]

9. Draw the equivalent circuit diagram of a common emitter p-n-p transistor and calculate its power gain. Discuss the relative merits of common base, common emitter and common collector transistor amplifiers. [6+6+8]=[20]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

ANNUAL EXAMINATION

Mathematics-6: Complex Variable Theory

Date: 16.7.73 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Answer as much as you can. Maximum one can score is 100. Marks allotted for each question are given in brackets [ ].

1. Let f be holomorphic in a region Ω and let Re f be constant in Ω. Show that f is a constant function. [12]

2. State and prove Cauchy's theorem for 'rectangles'. [17]

3. Let f be holomorphic in a region Ω. Show that for any $z \in \Omega$, there exists a $z' \in \Omega$ such that
$$|f(z)| < |f(z')|$$
[18]

4. Suppose f is a holomorphic function in a region $\Omega \supset \{z : \text{Im } z \geq 0\}$ and f is real on the real axis. Show that there exists an entire function g such that
$$g(z) = f(z) \quad \text{for all } z \in \Omega .$$
(Hint: You may use Cauchy Riemann equations.) [18]

5. Evaluate the following integrals by the method of residues

i) $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + b^2} dx$, b non-zero real.

ii) $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + b^2} dx$, b real

iii) $\int_{0}^{2\pi} \frac{dx}{4 + \sin x}$. [15]

6. Evaluate the following integrals by the method of residues.

i) $\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 6x^2 + 13} dx$

ii) $\int_{-\infty}^{\infty} \frac{x^2 + 1}{x^3 + ix + 4x + 4i} dx$. [12]

7. Let Ω be a region containing z_0. Suppose f is holomorphic in $\Omega - \{z_0\}$. Show that f has a pole of order k at z_0 if and only if there exists a holomorphic function g on Ω such that
$$f(z) = \frac{g(z)}{(z - z_0)^k}, \quad \text{for } z \in \Omega - \{z_0\}$$
and $g(z_0) \neq 0$. [18]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73 [327]

ANNUAL EXAMINATION

Mathematics-5: Calculus

Date: 17.7.73 Maximum Marks: 100 Time: 3 hours

Note: Answer any eight questions. Marks allotted for each question are given in brackets [].

1. Define partial derivatives, directional derivative and total derivative of a function of several variables at a point. Evaluate the directorial derivative of

$f(x, y) = 2x^2 - y^2$ at $(1, 2)$ in the direction of the line from $(1, 2)$ to $(3, 5)$. [12]

2. Let $f(x,y,z)$ be a function depending only on $r = \sqrt{x^2 + y^2 + z^2}$ i.e. let $f(x,y,z) = g(r)$. Then calculate $f_{xx}+f_{yy}+f_{zz}$. Prove that if $f_{xx}+f_{yy}+f_{zz} = 0$, it follows that

$$f = \frac{a}{r} + b$$

(where a and b are constants). [12]

3. Find the Taylor Series for the function $f(x,y) = 1/(1-x-y)$ and indicate its range of validity. [12]

4. Use Implicit function theorem to find the maximum and minimum values of the function $y = f(x)$ defined by the equation $x^2 + xy + y^2 = 27$. [12]

5. Prove that the equation in t

$$\frac{x^2}{a-t} + \frac{y^2}{b-t} + \frac{z^2}{c-t} = 1 \quad (a > b > c)$$

has three distinct real roots t_1, t_2, t_3 which lie respectively in the intervals

$$-\infty < t < c, \quad c < t < b, \quad b < t < a$$

provided that the point (x, y, z) does not lie on a coordinate plane.

Prove that the three surfaces $t_1 = \text{const.}$, $t_2 = \text{const.}$, $t_3 = \text{const.}$, passing through an arbitrary point are orthogonal to one another. [12]

6. State and prove Lagrange method of undetermined multipliers for the problem:

extremize $u = f(x,y,z,t)$ subject to

$\phi(x, y, z, t) = 0$ and $\psi(x, y, z, t) = 0$. [12]

7. Define the double integral. Prove the Mean Value theorem for double integral. [16]

p.t.o.

8. What is the rule for change of variables in a double integral. Change the order of integration and then solve:

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy.$$ [12]

9. State the rule for transformation of multiple integrals. Evaluate

$$\iint_R (x^2 + y^2)\, dx\, dy \quad \text{where } R = \left\{(x, y) \middle| \begin{array}{l} 0 \le x - y \le 2 \\ 0 \le x + y \le 2 \end{array}\right\}$$ [12]

10. Define a line integral. Evaluate the line integral

$I = \int_c [x^2 y dx + (x - z) dy + xyz dz]$ where C is the arc of the parabola $y = x^2$ in the plane $z = 2$ from

A : (0, 0, 2) to B (1, 1, 2). [12]

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73

[328]

ANNUAL EXAMINATION

Economic-2: Indian Economic Problems

Date: 19.7.73 Maximum Marks: 100 Time: $2\frac{1}{2}$ hours

Note: Answer any four questions. All questions carry equal marks.

1. Discuss the composition of India's exports indicating the changes that have taken place during the period of planning. Do you think that the pattern of change is consistent with the requirements of a developing economy?

2. Indicate the importance of agricultural marketable surplus in the economic development of India. What are the causes for low surplus in the agricultural sector?

3. 'A review of land reforms reveals much that has been achieved as well as a great deal that requires urgent attention. There are many gaps between objectives and legislation and between the laws and their implementation',- Fully examine the statement.

4. Indicate the changes that have taken place in the field of rural credit in India since the findings of the All India Rural Credit Survey Committee. Suggest how the Commercial Banks can play more effective role in providing farm credit to the agriculturists?

5. Explain how the expansion of public sector in India has helped to achieve the declared objectives of planned economic development of the country?

6. Examine the nature of the concentration of economic power in the industrial sector of India.

INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat.(Hons.) Part III: 1972-73 [329]

ANNUAL EXAMINATION

Economics-2: Micro-economics

Date: 19.7.73 Maximum Marks: 40 Time: $1\frac{1}{2}$ hours

Note: Answer any two questions. Marks allotted for each question are given in brackets [ ].

1 EITHER

A utility-maximizing two-commodity consumer chooses a collection $(\bar{q}_1, \bar{q}_2)$ in the price-income situation (p_1, p_2, y) and a collection $(\bar{q}_1, \bar{q}_2 + \partial q_2)$ in the price-income situation $(p_1 + \partial p_1, p_2, y)$ where $\partial p_1 \neq 0$. Show that the consumer regards the first commodity as an inferior good. [20]

OR

Starting from the postulates of revealed preference theory, prove that a consistent consumer's demand for any commodity is unaffected by an equiproportional change in all prices and his spendable income. [20]

2. EITHER

In a duopoly market any increase in one firm's profit is accompanied by an equal decrease in the other firm's profit. If the firms (A and B) think that they are playing a two-person game, what strategies should they choose so that there may be a determinate market equilibrium? A's table of profits (pay-off matrix) is given below:

A's Strategies ↓ \ B's Strategies →	1	2
1	7	5
2	6	3

[20]

3. EITHER

The production function of a firm is given by $Y = F(K, L)$ where Y, K, and L are respectively the amounts of output, capital-input, and labour-input. If the production is subject to constant return to scale, prove that the elasticity of substitution of one input for the other is equal to the elasticity of the average product of labour with respect to its marginal product. [20]

OR

Show that a first degree homogeneous production function involving one output and two inputs is a Cobb-Douglas function if and only if the elasticity of input substitution is equal to unity. [20]
