1981-82	362(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Multivariate Distribution and Tests
BACKPAPER EXAMINATION

Date: 9.7.82 Maximum Marks : 100 Time: 3 hours

1. Let $Y \sim N_p(\mu, \Sigma)$.

   (a) If $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, find the conditional distribution of Y_2, given Y_1. (10)

   (b) Show that $(Y-\mu)^T A (Y-\mu)$ follows a central χ^2 iff $\Sigma A \Sigma A \Sigma = \Sigma A \Sigma$. Find the degrees of freedom of this χ^2.

2. Let $S = (s_{ij}) \sim W_p(k, \Sigma)$ where Σ is nonsingular. Let the inverses of S and Σ be denoted by (s^{ij}) and (σ^{ij}). Then show that

   (a) $\frac{\sigma^{pp}}{s^{pp}} \sim \chi^2(k-p+1)$ and is independent of (s_{ij}), $i,j = 1, \ldots, p-1$ (10)

   (b) $l^T \Sigma^{-1} l / l^T S^{-1} l \sim \chi^2(k-p+1)$ for every non-null l; hence derive the distribution of Hotelling's T^2. (5+5)

   (c) Show how Hotelling's T^2 can be used to get simultaneous confidence intervals for all linear function of the mean vector of a multivariate normal population. (12)

P.T.O.

3.(a) Write down the model of analysis of dispersion for multiple measurements. (3)

(b) Explain, in detail, the procedures for estimating the parameters of such a model. (10)

(c) Let $\underset{p\times p}{A}$ be a symmetric matrix and $\underset{p\times p}{C}$ be a positive definite matrix. What is the smallest value of $x^T Ax / x^T Cx$, where x is any p-vector ? Prove your assertion. (10)

4. Observations were obtained on 28 trees for thickness of cork borings in the north (N), east(E), south(S) and west(W) directions. Let

$$y_1 = N-E-W+S, \; y_2 = S-W, \; y_3 = N-S.$$

The vector of means is

$$\bar{y}^T = (8.86, 4.50, 0.86)$$

and the covariance matrix (divisor is 27) for y is

$$\begin{pmatrix} 128.72 & 61.41 & -21.02 \\ & 56.93 & -28.30 \\ & & 63.53 \end{pmatrix}$$

Examine whether the bark deposit is the same in all directions. (25)

1981-82	392(b)

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) Part III, 1981-82

Optimisation Techniques
BACKPAPER EXAMINATION

Date: 8.7.82 Maximum Marks : 100 Time: 3 hours.

Note: The paper carries 105 marks. Maximum one can score is 100. Marks for each question are indicated in brackets.

1. State and prove the Fundamental Duality Theorem for a standard linear programming problem. (20)

2.(a) Consider the linear programme of finding a vector x unrestricted in sign such that

$x\cdot$ is a maximum

subject to $xA = b$.

Show that the problem has a solution only if c' is a linear combination of the columns of A.

(b) Prove : If a canonical linear programme has an optimal vector then it has a basic optimal vector. (State the important results you make use of).

Solve the following canonical maximum problem by comparing all three of its basic solution :

Maximise $2\xi_1+3\xi_2$

subject to $4\xi_1+2\xi_2+\xi_3=4$

$\xi_1+3\xi_2 \quad =5$

Find the value of the problem also. (5+5+5=15)

3. Maximise $8\xi_1+19\xi_2+7\xi_3$

subject to $3\xi_1+ 4\xi_2+\xi_3 \leqq 25$

$\xi_1+ 3\xi_2+3\xi_3 \leqq 50$

$\xi_1,\xi_2,\xi_3 \geqq 0.$ (15)

P.T.O.

4. Consider the following linear programme :

Find $\xi_1, \xi_2 \geqq 0$ such that $\xi_1+\xi_2$ is a maximum

subject to $-3\xi_1+2\xi_2 \leqq -1$

$\xi_1 - \xi_2 \leqq 2$

Write the dual of this problem. Show that the original problem is feasible but has no optimal solution. In view of this result and the duality theorem what must be true of the dual problem ? Verify this directly.

(4+8+3+5=20)

5. State and prove the max flow min cut theorem giving all the underlying definitions clearly. (20)

6. Define a two-person zero-sum game Γ. When do you say that the game Γ has a solution in pure strategies ? Consider the matrix game with the following pay off matrix

$$A = \begin{bmatrix} 0.4 & 0.5 & 0.9 & 0.3 \\ 0.8 & 0.4 & 0.3 & 0.7 \\ 0.7 & 0.6 & 0.8 & 0.9 \\ 0.7 & 0.2 & 0.4 & 0.6 \end{bmatrix}$$

Check whether the matrix A has any saddle point. Find a solution of the above matrix game.

(3+2+5+5=15)

1981-82 | 382(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Nonparametric and Sequential Methods
BACKPAPER EXAMINATION

Date: 7.7.82 Maximum Marks : 100 Time: 3 hours.

Note: Answer as much as you can.

1.(a) Let X be a random variable with density function $f(x,\theta)$. Let $S(b,a)$ be an S P R T of $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. of strength $(\alpha(\theta_0), \beta(\theta_1))$. Show that

$$\log \frac{1-\beta(\theta_1)}{\alpha(\theta_0)} \geq \max(0,a) \text{ and } \log \frac{\beta(\theta_1)}{1-\alpha(\theta_0)} \leq \min(0,b).$$

When do the signs of equality hold?

(b) Using the above result, or otherwise, obtain Wald's approximations for the constants b^* and a^* in the SPRT of given strength (α,β) for $H_0: \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Show that the probabilities of both kinds of errors can not increase by this approximation. (15+10)

2.(a) Let X be a random variable with density function $f(x,\theta)$ $-\infty < \theta < \infty$. Let $S(b,a)$ where $b < 0 < a$, be any SPRT of $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$ with ASN function $E(\underline{n} : \theta) < \infty$. Under suitable conditions to be precisely stated by you, show that for any rival test S one has $E_S(\underline{N} : \theta) \geq E(\underline{n} : \theta)$

(b) If X has the Poisson distribution with unknown parameter λ, show that the SPRT for $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1$ is optimum in the sense described above.

(15+10)

P.T.O.

3.(a) Derive the fundamental identity of sequential analysis stating the conditions clearly.

(b) Hence, or otherwise, derive the formula for the OC function of the S P R T, S(b,a), of strength (α, β), for a simple hypothesis against a simple alternative. (15+10)

4. Let X be a random variable having a normal distribution with unknown mean μ and known variance σ^2. Obtain the S PR T of strength (α, β) for $H_0: \mu \leq \mu_0$ against $H_1: \mu \geq \mu_1 (> \mu_0)$. Show that the test terminates with probability one under both the null and the alternative hypotheses. Obtain the OC and ASN functions of the test and give their values at the conventional values of μ. (25)

5.(a) Define Kendall's τ for a bivariate population. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a sample of size n. Obtain an unbiased and consistent estimator T_n of τ from this sample. If $\tau = 0$ and the marginal distributions of X and Y are continuous, obtain the asymptotic distribution of T_n.

(b) The scores in mathematics and english of 8 randomly chosen students is given below:

Student	A	B	C	D	E	F	G	H
Maths.Score	91	52	69	99	72	78	83	58
English Score	89	72	69	96	66	67	45	35

Examine whether there is any association between the two scores. (15+10)

6.(a) A random sample of size n from a multinomial distribution with class probabilities, $p_1, \ldots, p_k, p_i > 0 \; \forall i$. $\sum_1^k p_i = 1$, has $n_1, \ldots, n_k$ observations in the k classes. Show that

$$\sum_1^k \frac{(n_i - np_i)^2}{np_i}$$ is asymptotically distributed as a χ^2 with (k-1) degrees of freedom. (Prove the general result from which you derive this or give a complete proof of this result).

(Contd.....3).

6.(b) Examine whether the following data come from a normal distribution with mean 0 and variance 1.

-2.5	-1.8	-3.2	-1.9	0.84	-0.32	-0.21	2.58
4.8	-2.1	-1.2	-0.3	1.1	-4.0	-1.8	3.2
0.35	0.48	-0.84	-0.65				

(15+10).

1981-82	372(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Design of Experiments
BACKPAPER EXAMINATION

Date: 6.7.82 Maximum Marks : 100 Time: 3 hours

Note: Answer any FOUR questions. Marks allotted to a question are indicated in brackets ( ) at the end.

1. Define a B.I.B.D. (v,b,r,k,λ) with an example, and give the complete analysis of variance of a B.I.B.D. Describe the method of differences of construction of a B.I.B.D., and prove that the following initial block

   $(x^0, x^2, x^4, \ldots, x^{v-3})$, x being a primitive element of GF(v),

   generates a B.I.B.D. with parameters

   $v=b=4t+3$, $r=k=2t+1$, $\lambda=t$. (5+10+10=25)

2. Describe an m-ple lattice design with an example. Develop the complete analysis of variance of an m-ple lattice design, and obtain the expressions for $V(\hat{\tau}_i - \hat{\tau}_j)$ $(i \neq j)$, where τ_i stand for the effect of treatment i. (5+12+8=25)

3. Explain how the experimental error can be controlled by adopting an analysis of covariance. Consider the data from a latin square design with one observation missing. Obtain (i) the "estimate" of the missing value, and (ii) variances of estimated elementary treatment contrasts, by adopting an analysis of covariance with a dummy variable. (5+12+8=25)

P.T.O.

4. Let $m_d(s,k)$ denote the maximum number of factors that can be accomodated in an s^n factorial experiment, conducted in blocks of s^k plots each so that no effects of order d or less are confounded. Then prove that

   (i) $m_1(2,k) = 2^k - 1$, and (ii) $m_2(2,k) = 2^{k-1}$.

   Give the key blocks of two replications of a $(2^5, 2^3)$ design without confounding main effects and 2-factor interactions, and indicate the analysis of variance of the suggested design. (5+5+15=25)

5. Define main effects and interactions of a 3^n factorial experiment, and show that there are really (3^n-1) mutually orthogonal treatment contrasts. Give a balanced confounding scheme for a $(3^4, 3^2)$ experiment, and indicate the analysis of variance of the suggested balanced confounded design.

   (8+7+10=25)

1981-82	342(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Sample Surveys
BACKPAPER EXAMINATION

Date: 2.7.1982 Maximum Marks: 100 Time: 3 hours.

Note: This paper contains 130 marks. Answer as many questions as you can. Maximum you can score is 100. Maximum mark for each subdivision of a question has been indicated in the margin.

1.(a) Show that in estimating a finite population mean using stratified random sampling (without replacement),

$$V_{opt} \leq V_{prop} \leq V_{ran},$$

when finite population corrections (fpc's) are ignored. Here V_{ran} denotes the variance of the estimator under simple random sampling. V_{prop} and V_{opt} denote the variances of the estimator under stratified random sampling with proportional allocation and optimum allocation, respectively.

(b) A sampler has two strata with relative sizes W_1 and W_2. He believes that the population standard deviations S_1 and S_2 can be taken as equal, but thinks that c_2 may be between $2c_1$ and $4c_1$, when c_i denotes cost per unit sampled from the ith stratum. He would prefer to use proportional allocation but does not wish to incur a substantial increase in variance compared with optimum allocation. For a given cost, $C = c_1n_1 + c_2n_2$, Show (ignoring the f.p.c) that

$$\frac{V_{prop}}{V_{opt}} = \frac{W_1 c_1 + W_2 c_2}{(W_1\sqrt{c_1} + W_2\sqrt{c_2})^2}.$$

(c) In a survey of some characteristic 'y' of a tribal population, the population is stratified according to the type of tribe. Find an estimate of the population mean based on stratified random sampling of the population. Also obtain an expression for its variance.

[8+8+6=22]

P.T.O.

2.(a) Explain and illustrate systematic sampling procedures.

(b) Give unbiased estimators of population mean under these procedures.

(c) Find an expression for the variance of any of these estimator in terms of the intra-class correlation coefficient.

(d) State why it is not (generally) possible to unbiasedly estimate the variance of the estimator of the population mean on the basis of a single sample under this scheme.

(e) Suggest variance estimators of these estimates.

[4+3+6+3+6=22]

3.(a) Give an estimator of the population mean, which utilises information on a variable 'x', closely related with the main variable 'y', under the SRSWOR scheme.

(b) Obtain the bias in it and examine its magnitude wrt its standard error.

(c) Can you find a situation when this estimator is optimum?

[4+11+6=21]

4.(a) Explain the concept and usefulness of multi-stage sampling.

(b) Consider two-stage sampling with SRSWOR at both the stages. Suppose n first-stage units (fsu's) are selected from the N fsu's and m_i second-stage units (ssu's) from the M_i ssu's in the ith selected fsu (i=1, .., n).

(i) Obtain an unbiased estimator of the population total.

(ii) Find the sampling variance of the estimator.

(iii) Find an unbiased estimator of the variance.

[6+5+7+11 =29]

5.(a) A survey is planned to estimate the proportion π of a population belonging to a specified group C. It is, however, feared that the respondents will not co-operate in answering the question involved, which is very personal. Hence, the following procedure is adopted. The respondent is given a spinner with faces marked so that the spinner points to the letter C with probability p ($\neq \frac{1}{2}$) and to $\bar{C}$(not c) with probability q (= 1-p). The respondent is required to spin the spinner unobserved by the interviewer and report only whether or not the spinner has pointed to the letter representing the group to which he belongs (in terms of 'Yes' or 'No') .

(Contd..... 3)

Q.5.(a) contd.....

A WR-simple random sample of n respondents is selected to estimate π. Let x_j be the variable representing the response obtained from the jth individual, where

$$x_j = 1 \quad \text{if the answer is 'Yes'}$$
$$= 0 \quad \text{if the answer is 'No'}.$$

Show that

$$\hat{\pi} = \frac{\bar{x} - q}{p - q}, \quad \bar{x} = \frac{1}{n} \sum_j x_j$$

is an unbiased estimator of π. Find the variance of $\hat{\pi}$.

(b) Write short notes on :

(i) different steps in planning and execution of a large-scale sample survey ;

(ii) interpenetrating sub samples.

[8+12+8 = 28]

6. In a survey of 625 households, a simple random sample of 50 households was surveyed in order to estimate the average monthly household expenditure on toilet items. The estimate (sample mean) came out to be Rs. 0.88 with a standard error of Rs.0.10. Using this information, determine the sample size required to estimate the same characteristic in a neighbouring village such that the permissible margin of error at the 95% probability level is 10 % of the true value. (Assume that the coefficient of variation per unit is the same for both the villages).

[8]

1981-82	332(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Statistical Inference
BACKPAPER EXAMINATION

Date: 1.7.1982 Maximum Marks : 100 Time: 3 hours.

Note : Answer any Five questions. Complete and precise answers carry more weight.

1.(a) Let X have density function of the form
$f(x,\theta) = \exp\{c(\theta)\, T(x) + h(\theta) + S(x)\}.\ I_A(x)$,
where $c(\theta)$ and $h(\theta)$ are differentiable and $c(\theta)$ is an increasing function of θ. Derive the UMPU test of size α for $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ based on a sample of size n. (You can assume the generalized N-P lemma.)

(b) Obtain the UMPU test of size α for testing $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ in $N(\mu, 1)$ based on a sample of size n. (12+8)

2.(a) In a bag there are N balls of which an unknown number m are red. A sample of n balls is drawn by SRSWOR and the no. of red balls, X, is noted. Show that the distribution of X belongs to the monotone likelihood ratio family.

(b) Derive the UMP test for $H_0: m \leq m_0$ against $H_1: m > m_0$. (State precisely the main result you use.) (10+10)

3.(a) Show that the MP test obtained by means of the Neyman-Pearson lemma for testing a simple hypothesis against a simple alternative is essentially unique. Show that the power of the MP test is greater than the size of the test.

(b) Let X be $N(0, \sigma^2)$. Derive the UMP test of size α for $H_0: \sigma^2 \leq \sigma_0^2$ against $H_1: \sigma^2 > \sigma_0^2$, based on a sample of size n. Obtain the power function of the test. How does one determine the sample size to ensure a given power at a given alternative ? Verify also that the power at any value σ^2 in H_1 exceeds α. (8+12)

4.(a) Explain the likelihood ratio principle of deriving a test for $H_0 : \theta \in \Theta_0$ against an alternative $H_1 : \theta \in \Theta - \Theta_0$ based on a sample of size n from a population with density function $f(x,\theta)$, $\theta \in \Theta$.

(b) Let (X,Y) have a bivariate normal distribution with unknown parameters. Obtain the LR Test of size α for testing $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$, based on a sample of size n, where ρ is the population correlation coefficient.

(8 +12)

5. Let X have density function of the form

$f(x,\theta) = \exp\{c(\theta)\, T(x)+h(\theta)+S(x)\} \cdot I_A(x)$

where θ is a scalar ($-\infty < \theta < \infty$), A is independent of θ and $c(\theta)$ and $h(\theta)$ are twice differentiable in θ. Show that there exists a unique function of the parameter θ for which there exists an unbiased estimate whose variance is equal to the Cramér-Rao lower bound and obtain an expression for the lower bound. (20)

6.(a) Describe the method of scoring for simultaneous estimation of K parameters by the method of maximum likelihood.

(b) Let X have a multinomial distribution with probabilities $\left\{\frac{1}{4}(2+\theta^2), \frac{1}{4}(1-\theta^2), \frac{1}{4}(1-\theta^2), \frac{1}{4}\theta^2\right\}$. Obtain the first iterate in the method of scoring with an initial estimate $\hat{\theta}_0$ for θ.

(10+10)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Stochastic Processes - 2
BACKPAPER EXAMINATION

Date: 30.6.82 Maximum Marks : 100 Time: 3 hours

Note : Answer as many questions as you can. Full marks for questions are given in brackets.

1. Suppose that customers arrive for service according to a Poisson process with parameter $\lambda (> 0)$ and that each customer starts being served immediately upon his arrival. Suppose that the service times are independent and exponentially distributed with parameter $\mu > 0$. Let $X(t)$, $t \geq 0$, denote the number of customers in the process of being served at time t.

Formulate the above process as a birth and death process by specifying the birth and death rates. Find $p_{xy}(t), x, y = 0, 1, 2, \ldots, t \geq 0$. Determine $\lim_{t \to \infty} p_{xy}(t)$. [16+9=25]

2. Let $X(t)$, $t \geq 0$, be a Markov Pure jump process on the state space S, a subset of the integers.

(a) Define the infinitesimal parameters of the process. How do you define the embedded chain of the process ?

(b) Consider a birth and death process on $\{0, 1\}$ with birth and death rates as follows : $\lambda_0 = \lambda$, $\mu_1 = \mu$ where $\lambda, \mu > 0$. Obtain $p_{xy}(t), x, y = 0, 1, t \geq 0$ by solving the forward differential equations.

[(2+4)+(7+12)=25]

3. Consider a homogeneous Markov Chain on $\{0, 1, 2, \ldots, d\}$ satisfying $\sum_{y=0}^{d} y p_{xy} = x, \quad x = 0, 1, \ldots, d.$

Show that (i) $E\{X_{n+1} \mid X_0 = x_0, X_1 = x_1, \ldots, X_n = x\} = x$.

(ii) 0 and d are necessarily absorbing states.

P.T.O.

3.(iii) If there is no absorbing states other than 0 and d, the states 1,2, .., d-1 each lead to 0, and hence that each is a transient state. Moreover each stationary distribution of the chain is of the form

$$\pi_\alpha = (1-\alpha)\,\pi_0 + \alpha\pi_1,$$

where $0 \leq \alpha \leq 1$ and π_0 and π_1 are the stationary distributions concentrated at $\{0\}$ and $\{d\}$ respectively.

[4+6+12+8=30]

4. Let $\{X(t),\ t \geq 0\}$ be a Poisson process with parameter $\lambda > 0$ and $X(0) = 0$. Let n be a positive integer. Let $T = \inf\{t > 0 : X(t) = 1\}$. Find the conditional density of T given that $X_s = n$.

[16]

5. Let $\{X(t),\ 0 \leq t < \infty\}$ be an irreducible birth and death process on $\{0,1,2, \ldots\}$. Deduce a necessary and sufficient condition for the existence of a unique stationary distribution π of the process. Find the stationary distribution when it exists. Show that the process described in Q.1 is positive recurrent and find its stationary distribution.

[16+8=24]

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B.Stat.(Hons.) Part III, 1981-82

Difference and Differential Equation

BACKPAPER EXAMINATION

Date: 29.6.1982 Maximum Marks : 100 Time : 3 hours.

Note : This paper carries 110 marks. Answer all questions. The maximum you can score is 100.

1. Find the most general solution of the following difference equation :

$$x_{k+2} - 4x_{k+1} + 4x_k = 1 \qquad (15)$$

2. Find the most general solution of the following differential equation in the interval $(-2, -1)$:

$$x^2y'' + xy' - 4y = 0.$$

Prove that there exists a unique solution φ of this differential equation in the interval $(-2, 2)$ with $\varphi(1) = 1$. (15)

3. Let I be the interval $[0,\infty)$. Consider the differential equation

$$y'' + a_1y' + a_2y = \frac{x-1}{x+1}$$

where a_1, a_2 are constants. You are given that the characteristic polynomial has distinct roots with strictly negative real parts. If ϕ is any solution of the above differential equation, show that there exists $M > 0$ such that $|\varphi(x)| < M$ for all $x \in [0,\infty)$. (20)

4.(a) Let $I = (-\infty, \infty)$. Consider the differential equation

$$y'' - 2xy' + 2ky = 0$$

where k is a positive integer. Find two linearly independent solutions on I.

(b) Let $H_k(x) = e^{x^2}\frac{d^k}{dx^k}(e^{-x^2})$

Show H_k is a polynomial solution of the above differential equation. (15+10)

P.T.O.

5.(a) What does it mean to say

$$y'' + a_1(x)\, y' + a_2(x) y = 0$$

has a regular singular point at $+\infty$?

(b) The following can be considered an example of an equation with regular singular point at ∞ : $x^2y'' + xy' + y = 0$. Justify. (5+10)

6. Let $D = \{(x,y) : x^2 + y^2 \le 1\} \subseteq \mathbb{R}^2$ and let f be the function on $\mathbb{R}^2$ defined by

$$f(\underset{\sim}{y}) = 1 \quad \text{if } \underset{\sim}{y} \in D$$

$$f(\underset{\sim}{y}) = 0 \quad \text{if } \underset{\sim}{y} \notin D.$$

Compute $g(\lambda,\mu) = \iint_{\mathbb{R}^2} e^{i(\lambda x+\mu y)} f(x,y)\, dx\, dy$

explicitly in terms of Bessel's functions. You may assume

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x \sin\theta)\, d\theta$$ (20)

1981-82 | 3P2

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Elective 5 : Physical and Earth Sciences
SEMESTRAL II EXAMINATION

Date: 28.5.82 Maximum Marks : 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1. Show that the fermi level in an intrinsic semiconductor lies at the centre of the forbidden gap between conduction band and valence band.

Prove that the product of the electron and hole densities in any semiconductor is independent of doping concentration.

An n-type semi-conductor specimen has donor impurity density of 10^{16}/c.c. Find the hole density at 300°K, given that the electron density of the intrinsic specimen is 1.5×10^{10}/c.c. at 300°K.

2. Discuss how an abrupt p-n junction acts as a rectifier. Hence find the current voltage relation for the p-n junction.

A p-n junction is forward biased with 0.3 volt. Calculate the current flowing through it at 300°K, given that the reverse saturation current of the junction is 10μA at 300°K.

3. Show that the contact potential across an abrupt p-n junction is given by

$$\phi_o = \frac{kT}{e} \ln \frac{N_D N_A}{n_i^2},$$

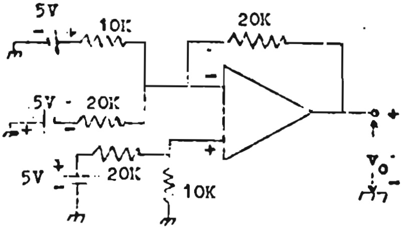
where N_D = donor concentration in n-region, N_A = acceptor concentration in p-region, n_i is the intrinsic carrier concentration, k, T and e have their usual significances.

Calculate the contact potential of an abrupt p-n junction at 27°C for $n_i = 1.5 \times 10^{10}$/c.c., $N_D = 2 \times 10^{15}$/c.c. and $N_A = 1.125 \times 10^{14}$/c.c.

P.T.O.

4. Show with the help of a circuit diagram how an operational amplifier can be used for amplifying a signal with zero phase shift. Derive the formula for the voltage gain in your circuit.

Calculate the output voltage v_o for the following network.



5. Draw the circuit diagram of a full-wave rectifier using a centre-tapped transformer. Show by harmanic analysis that the output of a full-wave rectifier does not contain any odd harmonic components.

Find the r.m.s value of the output voltage of the full-wave rectifier and hence calculate the ripple factor.

6. For a full-wave rectifier with a capacitor filter, calculate the output d-c voltage and the ripple factor after critically explaining the filtering action of the capacitor.

Discuss the filtering performance of a capacitor filter with varying load current and capacitance values.

A voltage regulated power supply designed for an output d-c voltage of 5 volts and full load current of 1 amp has 0.1% regulation. Calculate the actual output voltage of the power supply when the full load current is drawn from it.

7. Find an expression for the voltage gain of a transistor operating in CE mode in terms of the load resistance R_L and the h-parameters in common-emitter mode, if an ideal voltage source is connected at the input terminal.

Calculate the voltage gain of a common-emitter amplifier for a load $R_L = 1000\Omega$, given that $h_{fe} = 50$, $h_{oe} = 25 \times 10^{-6}$ ℧, $h_{ie} = 1100\ \Omega$, $h_{re} = 2.5 \times 10^{-4}$.

1981-82 | 3E2

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Elective 5 : Economics
SEMESTRAL II EXAMINATION

Date: 28.5.82 Maximum Marks : 100 Time : 4 hours

Note: Answer Questions 1,5 and 6, and <u>any two</u> from Qs. 2 to 4.

1.(a) Describe the structure of a two-commodity Leontief open static model with one primary factor of production, stating clearly the assumptions involved.

(b) What do you mean by viability of such a system ? Obtain the viability conditions for the system.

(c) Consider the following technology matrix

	industry 1	industry 2
industry 1	0.10	1.46
industry 2	0.16	0.17
labour	0.04	0.33

(i) Given that the wage rate is Rs. 10 (per unit of time to which the above table refers) obtain the equilibrium prices of the two commodities.

(ii) Suppose the price of commodity 1 is controlled at Re. 1/-per unit. Obtain the wage rate and the price of commodity 2 corresponding to this price of commodity 1.

[6+10+(6+8)]=[30]

2. Explain briefly the problems of identification and least squares bias arising in the estimation of demand functions from time series data.

[15]

P.T.O.

3. Define elasticity of substitution between two factor-inputs in a production function and explain the significance of this measure.

Write a short account of the CES production function and show that it includes the Cobb-Douglas function as a special case.

[7+8]=[15]

4. Write short notes on any two of the following :

(a) ML estimation of parameters of the two-parameter lognormal distribution from grouped data

(b) Prais-Houthakker approach for tackling effects of age-sex composition of households in Engel curve analysis

(c) Economies of scale in household consumption. [15]

5. The following results are based on a family living survey in a working class centre in India during 1958-59 :

monthly income per capita (Rs.)	estimated percentage of families	average family size	average per capita monthly expenditure (Rs.)	
			on all items	on food items
(1)	(2)	(3)	(4)	(5)
below Rs.25	25	6.4	19.2	12.8
25.0 - 49.9	33	5.4	35.8	22.4
50.0 - 74.9	19	4.7	63.4	35.2
75.0 - 99.9	11	4.3	84.5	42.0
100.0 -124.9	6	4.0	112.2	54.7
125.0 -149.9	3	3.7	134.7	62.5
150.0 -	3	3.2	160.9	68.9
Total	100	-	-	-

(a) Obtain the percentage distribution of persons over the different classes of monthly per capita income.

(b) Examine whether the distribution obtained in (a) is approximately lognormal or not, using a graphical test.

(c) Obtain the Engel elasticity for expenditure on food items (y) by regressing it on total expenditure(x). Assume that the relationship is of the form

$$\log y = \alpha + \beta \log x$$

and use the percentages of families given in col.(2) as weights in the weighted least squares method of estimation.

[5+10+15]=[30]

6. Practical Record. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Elective 5 : Mathematics
SEMESTRAL II EXAMINATION

Date: 28.5.82 Maximum Marks : 100 Time: 3 hours.

Note: 1. You can answer any part of any question.
2. Maximum you can score is 100 marks.

1. In $\mathbb{R}^2$ consider,

$$y^2 e^x u_{xx} + (y^2+e^x) u_{xy} + u_{yy} = 0$$

(a) At what points in R^2 is this hyperbolic ? Where is it Elliptic ? Plot the regions

(b) Suppose Ω is a region where the above equation is hyperbolic. What new coordinates $\alpha = \varphi(x,y)$, $\beta = \psi(x,y)$ are to be chosen so that in the new coordinates the principal part is $v_{\alpha\beta}$. Verify that your choice is correct. [6+14]

2. In [0, 1] X [0, ∞) consider

$$u_y - (1-x) u_x = 0$$

(a) What are the characteristic curves ? Plot them. Where does a characteristic curve passing through (x_0, y_0) touch the X-axis ?

(b) Solve the equation subject to the initial condition $u(x,0) = x^5$. [14+6]

3.(a) What are the stationary curves x(t) joining (0,0) to (π/2, 100) for the integral

$$I(x) = \int_0^{\pi/2} [x'^2(t) - x^2(t)] \, dt$$

P.T.O.

3.(b) Calculate $I(x)$ for stationary curves x.

(c) Suppose the last end point is (π , 100) the starting point being same as (0,0) and

$$I(x) = \int_0^{\pi} [x'^2(t) - x^2(t)] \, dt$$

Then what are the stationary curves?

[6+6+8]

4.(a) What are the stationary curves $x(t)$ joining (0,0) to (1,0) for the integral

$$\int_0^1 (x'^2(t) + x'^3(t)] \, dt$$

(b) If in a variational problem,

$F(t,u,v) = a(t)v^2 + 2b(t)uv + c(t)u^2$

where a, b, c are c^2 functions of t, then show that the Euler equation for a stationary curve $x(t)$ is a second order linear differential equation. [10+10]

5. Solve on $[0, \pi] \times [0, \pi]$:

$$u_{xx} + u_{yy} = 0 \qquad 0 < x,\ y < \pi$$

$$u(x,0) = \sin^2 x$$

$$u(x,\pi) = u(0,y) = u(\pi,y) = 0$$

You must evaluate all constants involved. [20]

6. Consider $[0, \pi] \times [0, \infty)$ and the function defined there by

$$u(x,y) = (e^{y} - e^{-y}) \sin x.$$

(a) Show that u is continuous there and is harmonic in $(0,\pi) \times (0,\infty)$.

(b) What is the boundary of this region and what are the values of u on the boundary.

(c) Explain why (b) is not a contradiction to the uniqueness of solution to the Dirichlet problem.

[6+6+8].

1981-82 | 382

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Non-parametric and Sequential Methods
SEMESTRAL II EXAMINATION

Date: 22.5.82 Maximum Marks : 100 Time: 3 hours

Note: Answer any FOUR questions.

1.(a) A random variable X has density function $f(x,\theta)$, where $\theta \in H$ is unknown. Describe a sequential procedure for testing the hypothesis that $\theta \in \textcircled{H}_0 \subset \textcircled{H}$. Define the OC and ASN functions of this sequential procedure and derive a necessary condition for the ASN function to be finite.

(b) Let $\underline{n}$ denote the sample size required for the termination of a sequential procedure S in the above problem and let $T_m(x_1, \ldots, x_m) = \sum_1^m t(x_i), \forall_m$. Let $E(\underline{n} : \theta) < \infty$ and $E(|t(x)| : \theta) < \infty$. Show that $E(T_{\underline{n}} : \theta) = E(\underline{n} : \theta) \times E(t(x) : \theta)$. (13+12)

2.(a) A random variable X has density function $f(x:\theta)$, where $\theta = \theta_0$ or θ_1. Define the sequential probability ratio test (SPRT) of strength (α, β) for testing the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. What are Wald's approximations to the constants you need ? How is the strength of the SPRT affected by these approximations ?

(b) Show that under suitable conditions (to be precisely stated by you) the SPRT you propose above, terminates with probability one under both H_0 and H_1. Show also that the moment generating function $\phi(t)$ of $\underline{n}$ the sample size for termination, is finite for all t less than or equal to some t_0 $(\neq 0)$. (8+17)

P.T.O.

3.(a) Let Z be a random variable whose moment generating function $\phi(h) = E(e^{hZ})$ exists for all real h. Suppose that for some δ $(0 < \delta < 1)$, $P(Z < \log(1-\delta)) > 0$ and $P(Z > \log(1+\delta)) > 0$. Show that the equation $\phi(h) = 1$ has a unique nonzero solution when $E(Z) \neq 0$ and has no nonzero solution when $E(Z) = 0$.

(b) Using the above result, or otherwise, derive the approximate formula for the OC function of the SPRT, S (b,a), of strength (α,β) for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, where the random variable X has density function $f(x,\theta)$, $\theta \in \Theta$.

(14+11)

4. Let X be a Bernoulli random variable with $P(X=1) = p$ which is unknown. Describe the SPRT of strength (α,β) for $H_0: p \leq p_0$ against $H_1: p \geq p_1$ $(> p_0)$. Show that the test is closed. obtain the approximate formulae for the OC and ASN functions of this SPRT. Also obtain their values at the conventional valuesof p.

5.(a) Define Van der Waerden's test for the hypothesis that two random samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ come from the same continuous distribution. Obtain the mean, the variance and the asymptotic distribution of the test statistic under the null hypothesis.

(b) By using a suitable test, examine whether the following observations (which are mutually independent,) come from the same distribution against the alternatives that the distribution of X is stochastically larger than that of Y.

Sample 1 (X) : 79, 33, 138, 129, 59, 76, 75, 53, 83, 96
Sample 2 (Y) : 141, 133, 96, 107, 102, 83, 129, 110, 104.

(15+10)

6.(a) A random sample $(x_1, y_1), \ldots, (x_n, y_n)$ is drawn from a bivariate distribution. Define Spearman's rank correlation R between X and Y and obtain a computational formula for the same under the hypothesis that X and Y are independent. Obtain the mean, the variance and the asymptotic distribution of R.

(contd....3).

6.(b) Two judges are each asked to rank 9 contestants in a beauty contest and the ranks are given below :

Contestants

Judge	A	B	C	D	E	F	G	H	I
1 :	2	1	3	5	4	8	7	6	9
2 :	1	2	4	9	7	6	8	3	5

Test whether the judges ranked the contestants independently.

(15+10).

1981-82	372

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III,'81-82

Design of Experiments
SEMESTRAL II EXAMINATION

Date: 20.5.82 Maximum Marks: 100 Time: 3 hours.

Note: 20 marks are allotted to Practical Records. Submit your Practical Records along with the answerscript. Answer any FOUR questions. Marks allotted to a question are indicated in brackets [ ] at the end.

1. Derive expressions for (i) "estimates" of missing values used in the missing plot technique, and (ii) variance of the best estimator of an estimable function based on the incomplete data. Suppose in a randomised block design two observations are lost, involving, say, treatment i and blocks j and j′ ($j \neq j'$). Obtain the "estimates" of the missing values and the variances of the estimated elementary treatment contrasts for the incomplete data.

[10+5+5=20].

2. Consider the usual linear model for the analysis of covariance, viz , $\tilde{\Omega}$: $\underline{y} = X\underline{\beta}_{\tilde{\Omega}} + H\underline{\gamma} + \underline{e}$, etc.

Let Ω : $\underline{y} = X\underline{\beta}_{\Omega} + \underline{e}$ be the linear model for the corresponding analysis of variance. Show that the analysis under $\tilde{\Omega}$ can be easily done once the analysis under Ω is known, by deriving the following : (i) best estimator of γ under $\tilde{\Omega}$, and its variance-covariance matrix, (ii) the F-statistic for testing $H : \underline{\gamma} = \underline{0}$; and (iii) the best estimator for an estimable function $\underline{l}'\underline{\beta}_{\tilde{\Omega}}$, and its variance. Give the complete table of analysis of variance and covariance for a randomised block design.

[5+5+5+5=20].

P.T.O.

3. Describe a method of construction of a complete set of mutually orthogonal latin squares of order s and hence the following series of B.I.B.D :

$$v = b = s^2 + s + 1, \quad r = k = s^2, \quad \lambda = s\,(s-1).$$

Either construct or prove the non-existence of the following B.I.B.D's :

(i) $v = b = 22, \quad r = k = 7, \quad \lambda = 2$;

(ii) $v = 12, \quad b = 22, \quad r = 11, \quad k = 6, \quad \lambda = 5$;

(iii) $v = b = 67, \quad r = k = 12, \quad \lambda = 2$.

[5+6+(3x3) = 20]

4. Explain confounding with an example. Give a partially confounded design in two replications alongwith its statistical analysis for a 2^5 factorial in blocks of size 8. Give a balanced confounding scheme for the same $(2^5, 2^3)$ experiment.

[5+12+3=20].

5. Identify the confounded interactions in a replicate of a 3^4 factorial experiment in blocks of 9 plots, having the following treatments in one of its 9 blocks :

(2222, 2100, 2011, 1201, 0002, 0121, 0210, 1020, 1112).

Give the key blocks of another 3 replications so that the new design in 4 replications (including the one given above) become a balanced confounded design for the $(3^4, 3^2)$ experiment. Indicate the analysis of variance of the balanced confounded design so constructed.

1981-82 | 362

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, '81-82

Multivariate Distribution and Tests.

SEMESTRAL II EXAMINATION

Date: 17.5.82. Maximum Marks : 100 Time: 3 hours.

1.(a) Find the maximum likelihood estimator of μ and Σ on the basis of a random sample of size n from $N_p(\mu,\Sigma)$, where Σ is known to be nonsingular. Show, also, that $(\bar{X},S)$ is sufficient for (μ,Σ), where $\bar{X}$, S are, respectively, the sample mean vector and sample dispersion matrix. [15+5]

(b) Define Hotelling's T^2 distribution. Show that Hotelling's T^2 test for testing the hypothesis that the mean vector of a p-variate normal population has an assigned value, on the basis of a random sample of size n, is equivalent to the likelihood ratio test. [10]

(c) Under the set-up of (a), describe how you will test the hypothesis that the components of μ are all equal, Σ being unknown. [7]

(d) Let $\{x_\alpha^{(i)},\ \alpha = 1, \ldots, n_i\}$ be independent samples from $N(\mu^{(i)}, \Sigma)$ $i = 1, \ldots, q$. Describe how you will test the hypothesis that $\sum_{i=1}^{q} \beta_i \mu^{(i)} = \mu$, where $\beta_1, \ldots, \beta_q$ are known reals and μ is a known vector. [8]

2.(a) Define Wishart distribution. [3]

(b) Show, in detail, that $\mathcal{U}^T A \mathcal{U} \sim W_p(r, \Sigma)$ iff $Y^T A Y \sim (l^T \Sigma l) \cdot \chi^2(r)$ for every l, where $Y = \mathcal{U} l$

(The symbols have their usual significance). [5+12]

P.T.O.

2.(c) Let $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \sim W_p(k, \Sigma)$, where Σ is nonsingular. Then find the distribution of

$$S_{22} - S_{21} S_{11}^{-1} S_{12}.$$

Also find $E|S|^{\alpha}$. State clearly the results you are using. [10+8].

(d) Under the set up of (c), find the characteristic function of S. [10]

3. Let (X, Y) follow a bivariate normal distribution with zero means, unit variances and correlation coefficient ρ. Find the correlation coefficient between X and F(Y) where F is the marginal distribution function of Y. [7].

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) Part III, 1981-82

Design of Experiments
PERIODICAL EXAMINATION

Date: 8.3.82. Maximum Marks : 100 Time: 3 hours

Note: 15 marks are allotted to Practical Records. Submit your Records alongwith your answerscripts. Attempt Question No.1 and any three out of the rest. Marks allotted to a question are indicated in brackets at its end.

1.(a) Define connectedness of a block design and show that a disconnected design is a disjoint union of several connected designs on subsets of the given set of treatments. State and prove a necessary and sufficient condition for a block design which is connected to be orthogonal. [4+6=10]

(b) Give the two definitions of a connected balanced design and show that they are equivalent. Also show that a connected block design is balanced if and only if all the off-diagonal elements of its C matrix are equal. [6+9=15]

2. Define a latin square design with an example and derive its analysis of variance. [5+15=20]

3. For a general block design with parameters v, b, $\underline{r}$, $\underline{k}$, show that-

(i) the reduced normal equations for the treatment effects turn out to be $C\hat{\underline{\tau}} = \underline{Q}$, where the symbols have their usual significance ;

(ii) v + Rank D = b + Rank C, and Rank C $\leq$ v-1 ;

(iii) the sum of squares for testing the hypothesis of treatment effects is given by

$$\sum_{i=1}^{v} \hat{\tau}_i Q_i ,$$

called the adjusted treatment s.s. [5+7+8=20]

P.T.O.

4. Define a balanced incomplete block design BIBD (v, b, r, k, λ) with an example, and prove that for such a design:

(i) $\lambda(v-1) = r(k-1)$, and

(ii) $v = b \iff$ any two blocks intersect in a common number λ of treatments.

Give the analysis of variance for a BIBD (v, b, r, k, λ).

[2+2x4+10=20]

5. Define an m-ple lattice design for $v = s^2$ treatments and prove that for $m = s + 1$, the lattice becomes a BIBD. Derive for an m-ple lattice design (i) the expression for $\hat{\tau}_i$, a least square estimator for the treatment effect τ_i, and (ii) $V(\hat{\tau}_i - \hat{\tau}_j)$, for $i \neq j$.

[2+5+8+5=20]

6. Write short notes on any two of the followings :

(a) Fundamental principles of experimental designs ;

(b) Randomized block designs ;

(c) Mutually orthogonal latin squares and some of their uses in experimental designs.

[2x10 = 20].

1981-82

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Elective 4 : Mathematics
SEMESTRAL-I EXAMINATION

Date: 26.12.81. Maximum Marks : 100 Time: 3 hours.

Note: Answer any 6 questions.

1. Let G be a group and let $T = G \times G$.

   (a) Show that $D = \{(g,g) \in G \times G : g \in G\}$ is a group isomorphic to G. [4]

   (b) Prove that D is normal in T if and only if G is abelian. [10]

2. If G is a group, define the sequence of subgroups $G^{(i)}$ of G by

   (1) $G^{(1)}$ = commutator subgroup of G = subgroup of G generated by all $aba^{-1}b^{-1}$ where $a,b \in G$.

   (2) $G^{(i)}$ = commutator subgroup of $G^{(i-1)}$ if $i > 1$.

   Prove

   (a) $G^{(1)}$ is a normal subgroup of G and that $G/G^{(1)}$ is abelian. [8]

   (b) Each $G^{(i)}$ is a normal subgroup of G. [6]

3. (a) Prove that there is no simple group of order 56. [8]

   (b) Show that a group cannot be written as the set-theoretic union of two proper subgroups. [6]

4. Characterize all groups of order p^2q where p and q are primes, $q < p$ and $q \nmid p^2-1$. [14]

5. (a) Prove that if the order of an abelian group is not divisible by a square then it must be cyclic. [10]

   (b) Prove that a solvable group always has an abelian normal subgroup $M \neq \{e\}$. [4]

P.T.O.

6. Let F be a field and $f(x) \in F[x]$ be such that $f'(x)=0$. Then prove that

   (a) if characteristic of F is 0 then $f(x) = a \in F$. [7]

   (b) if characteristic of F is $p \neq 0$ then $f(x) = g(x^p)$ for some polynomial $g(x) \in F[x]$. [7]

7. (a) An algebraic number α is said to be an algebraic integer if it satisfies an equation of the form $\alpha^n + a_1\alpha^{n-1} + \dots + a_n = 0$ where $a_1, \dots, a_n$ are integers. If α is an algebraic number, prove that there is a positive integer n such that $n\alpha$ is an algebraic integer. [8]

   (b) Prove that a regular 9-gon is not constructible.

8. (a) An R-module M is said to be irreducible if its only submodules are $\{0\}$ and M. Prove that any unital irreducible R-module is cyclic. [8]

   (b) Let M be an R-module ; if $m \in M$ let $\lambda(m) = \{ x \in R : xm = 0 \}$. Show that $\lambda(m)$ is a left-ideal of R. [6]

9. ASSIGNMENT. [16]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Elective 4 : Economics
SEMESTRAL-I EXAMINATION

Date: 26.12.81 Maximum Marks : 100 Time: 3 hours

Note: Answer 2 questions from group A.
2 questions from group B.
1 question from group C.
Use separate answer books for each group.

GROUP A

1. Solve the following L.P. problem by the simplex method :

$$\text{Max} \quad 3x_1 + 4x_2 + x_3 + 7x_4$$

Subject to

$$5x_1 + 3x_2 + 4x_3 + x_4 \leq 7$$

$$2x_1 + 6x_2 + x_3 + 5x_4 \leq 3$$

$$x_1 + 4x_2 + 5x_3 + 2x_4 \leq 8$$

$$x_j \geq 0, \quad j = 1,2,3,4.$$ [25]

Find also the solution to the dual to the above problem.

2. Consider the following L.P. problem :

$$\text{Minimize} \quad b^T y$$

$$\text{s.t.} \quad Ay = C$$

$$y \geq 0$$

where A is an mxn real matrix.
Suppose the final tableau in the simplex procedure for solving this problem is given by :

	a^1	...	a^m	...	a^n	C	u_1	...	u_m
a^1	1	...	0		τ_{1n}	η_1	λ_{11}	...	λ_{1m}
⋮			⋮		⋮	⋮	⋮		⋮
a^m	0	...	1		τ_{mn}	η_m	λ_{m1}	...	λ_{mm}
$C-b^T$	0	...	0		$\zeta_n-\beta_n$	ζ_0	ξ_1	...	ξ_m

Q.2 (Contd..)

where

a^j is the j^{th} column of A and $u_i = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, i.e.

u_i is a column vector with 1 in the i^{th} place and zeros elsewhere.

Under the assumption that rank (A) = m, show that the solution to the dual problem is given by ($\xi_1, \ldots, \xi_m$). How would your proof change if

rank (A) < m [25]

3. Let a certain commodity, say steel, be produced at each of m plants, $P_1, \ldots, P_m$, and let σ_i be the yearly output of the i^{th} plant. Suppose now that steel is required at each of n markets, $M_1, \ldots, M_n$, and let the annual demand at the j^{th} market be δ_j. Finally, let γ_{ij} be the cost of shipping one unit from P_i to M_j. Assume now that we wish to minimize the total cost of shipping subject to the restrictions that (i) the demand δ_j at market M_j is satisfied (ii) the supply σ_i at plant P_i is not exceeded.

(a) Set up this problem as an L.P. problem. [5]

(b) Set up the dual to this problem. [8]

(c) Interpret the complementary slackness theorem for this problem. [12]

4. Consider the following L.P. problem :

$$\text{Min } y^T b$$

$$\text{Sub. to } Ay = C$$

$$y \geq 0$$

where A is an mxn real matrix.

Let the column vectors $a^1, \ldots, a^p$ be a basis for the problem. Define

$$\zeta_j = \sum_{i=1}^{p} \beta_i \tau_{ij},$$

(Contd......3).

where, the τ_{ij} s are defined by

$$a^j = \sum_{i=1}^{p} \tau_{ij} \; a^i \quad , \quad j = 1, \ldots, n$$

and $b^T = (\beta_1, \ldots \beta_n)$.

(a) Prove that a feasible vector y associated with the basis $a^1, \ldots, a^p$ is optimal if $\xi_j \leq \beta_j$ for $j=1,\ldots,n$. [13]

(b) Suppose it has been decided to introduce a^s into the basis. Moreover, let $\tau_{io} \leq 0$, $i = 1, \ldots, p$. Show that original problem has no solution. [12]

GROUP B

1. The Eastern Bypass in Calcutta is being constructed along the fringe of the city over land mostly reclaimed from swamps. The Bypass would provide an additional channel for flow of traffic between the northern and the southern ends of the city. Discuss how you would apply the methods of Cost Benefit Analysis for the project, taking into consideration the specific items involved. [15]

2. When is Shadow-price used for foreign exchange rates in Cost Benefit Analysis Discuss your reasons fully for not using the international exchange rate. [15]

3. Discuss the distinction between the private and the social rate of return. When do you use social rate of return in project analysis . [15]

GROUP C

EITHER

1.(a) Work out occupation mobility from the given data by considering (i) all the eleven occupation classes named below in order of status and (ii) by grouping occupations as agricultural, labourer and others.

(Contd.....4)

1.(a) Contd...

Status	Occupation description
1.	Superior service workers
2	Money lenders, rent receivers etc.
3	Miscellaneous occupations of literates
4	Traders
5	Shop assistants, brokers, agents, salesmen etc.
6	Owners and proprietors of non-agricultural sector
7	Inferior service workers.
8	Growers of special crops
9	Owner cultivators
10	Non owner cultivators
11	Labourers

Distribution of males by their occupation and that of their fathers.

Occupation of son	Occupation of father										
	1	2	3	4	5	6	7	8	9	10	11
1	0.90	0.05	-	0.83	0.17	0.44	0.53	0.05	2.30	0.38	0.05
2	-	-	-	-	-	-	0.06	-	0.35	-	-
3	-	-	0.47	-	-	-	-	-	0.29	-	-
4	0.16	-	-	1.68	-	0.29	0.05	0.06	0.85	0.18	-
5	-	-	0.05	-	0.24	-	-	-	0.58	0.10	-
6	0.06	-	0.06	-	-	3.23	0.45	0.05	3.26	1.50	0.34
7	0.10	0.05	0.05	0.18	0.29	0.92	1.24	-	2.17	0.15	-
8	-	-	-	-	-	-	-	1.09	1.63	0.23	0.69
9	0.17	-	-	0.05	-	0.39	0.35	1.29	38.05	0.35	0.34
10	0.35	-	-	-	-	0.54	-	-	2.24	8.26	-
11	0.30	-	-	0.23	0.23	0.76	0.39	0.23	4.81	3.06	11.06

(b) How does occupation mobility information of males help planning ? [15+5]

OR

2.(a) What are the classes and the sub classes observed in tabulating census data of 1941 ?

(b) Discuss the need of adopting a changed occupation class distribution in 1951 census. How will you link the two systems of tabulating the occupation data of 1941 and 1951 censuses. [4+16]

[1981-82] [???]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82.

Elective 4 : Physical and Earth Sciences
SEMESTRAL-I EXAMINATION

Date: 26.12.81. Maximum Marks: 100 Time: 3 hours

Note: Attempt any five. All questions carry same marks.

1. What do you understand by erosion and weathering ? Describe the different weathering processes. What are the different physiographic features that we find on the surface of the earth ? [4+10+6]

2. What is a sedimentary rock ? Describe, with sketches wherever applicable, the main varieties of stratification. [5+15]

3. What do you understand by the "texture of a rock" ? Describe how the study of grain-size is used in understanding the depositional environment of a sedimentary rock. [6+14]

4. Define an environment of sedimentation. Describe the lithological and structural characteristics of sediments that were deposited under fluvial environments. [5+15]

5. What is a fossil ? Describe the various ways in which a fossil remain is preserved. Give one important use of fossil in geological study. [4+12+4]

6. What is understood by the Principle of Uniformitarianism and Principle of Superposition ? How is a succession of rock units in an area determined in the course of a geological field work ? What does an unconformity in such a sequence signify ? [6+10+4]

P.T.O.

7. Describe how statistics can help us in understanding the current direction(s) from a large number of directional data collected from the cross-beddings during a geological field work. [20]

Or.

Describe how statistics can differentiate between two sets of sandstones whose mineralogical data and grain-size data are available. [20]

8. Write short notes on (any four) -

Laterite ; Desiccation Cracks ; Feldspar in Sedimentary Rocks ; Metamorphism of Mudstone ; Coal ; Gondwanaland. [20]

[1981-82] [...]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) Part III, 1981-82

Sample Surveys

SEMESTRAL-I EXAMINATION

Date: 24.12.81 Maximum Marks : 100 Time: 3 hours.

Note: This paper contains 112 marks. Answer as many questions as you can. Maximum you can score is 90. 10 marks have been allotted for submission of note books on practical problems done in your class.

1.(a) Explain the concept of multi-stage sampling and indicate its uses.

(b) Consider two-stage sampling with simple random sampling without replacement at both the stages. Suppose n first-stage units (fsu s) are selected from N fsu s and m_i second-stage units (ssu s) from the M_i ssu s in the selected fsu (i=1, .., n).

(i) Obtain an unbiased estimator of the population total.

(ii) Find the sampling variance of the estimator.

(iii) Find an unbiased estimator of the variance.

[6+5+7+11] =[29]

2.(a) Explain and illustrate systematic sampling procedures.

(b) Give unbiased estimates of population mean under these procedures.

(c) State why it is not (generally) possible to unbiasedly estimate the variance of the estimator of population mean on the basis of a single sample under this scheme.

(d) Can you suggest a variance estimator using only one sample drawn under this scheme ?

[6+4+6+6] = [22].

P.T.O.

3.(a) A population consists of 5 units with size measures as given below :

$P_1 = .33,\ P_2 = .19,\ P_3 = .18,\ P_4 = .16,\ P_5 = .14$;

The problem is to estimate the population total by selecting a sample of two units so that the inclusion probabilities are $\pi_i = 2P_i$ for $i = 1, \ldots 5$.

Further, the probability of selecting each of the non-preferred samples $(u_2, u_4), (u_2, u_5), (u_3, u_4), (u_3, u_5)$ and (u_4, u_5) is to be made as small as .2. What would be the probability of selecting the other five preferred samples ?

(b) Consider the following sampling scheme. The first member of the sample is selected with probability P_i, for $i=1, \ldots, N$ ($\sum_1^N P_i = 1$). The remaining (n-1) units in the sample are selected from the remaining (N-1) units in the population by SRSWOR. Show that for this sampling scheme

$$\pi_i = \frac{n-1}{N-1} + \frac{N-n}{N-1} P_i$$

$$\pi_{ij} = \frac{(n-1)(n-2)}{(N-1)(N-2)} + \frac{(N-n)(n-1)}{(N-1)(N-2)} (P_i + P_j).$$

[8+12] = [20]

4.(a)(i) State the linear regression estimator of population total and indicate situations where one should use it.

(ii) Derive a large-sample formula for its variance, and comment on the magnitude of the sample size required to validate this large-sample formula.

(b) Estimate the efficiency of cluster sampling relative to simple random sampling.

[(5+5+5)+6]=[21]

5. Write short notes on :

(i) different steps in the planning and execution of a large-scale sample survey ;

(ii) method of 'collapsed strata'.

[12+8]+[20].

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B.Stat.(Hons.) Part III, 1981-82

Statistical Inference
SEMESTRAL-I EXAMINATION

Date: 21.12.81 Maximum Marks: 100 Time: 3 hours.

Note: Answer any FIVE questions. All questions carry equal marks. Complete and precise answers carry more weight.

1.(a) Let X be a random variable with density function $f(x, \theta)$, where $\theta \in \Theta$ is unknown. Show that there exists a M P test of size α for the simple hypothesis $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ based on a random sample of size n.

(b) Derive the M P test of size α for testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1 \ (> \mu_0)$ based on a sample of size 16 from $N(\mu, 1)$. Describe how you would compute the power of this test at any value of $\mu > \mu_0$ and how you would determine the sample size to ensure given power at any value of $\mu \ (> \mu_0)$. (10+10)

2.(a) Let the density function of a random variable X be $f(x,\theta) = \exp\{c(\theta)\ T(x) + S(x) + h(\theta)\}\ I(x)$, $\theta \in \Theta$. If $\{x_1, \ldots, x_n\}$ is a random sample from this population, obtain a sufficient statistic for the family. Show explicitly that the distribution of the sufficient statistic also belongs to the exponential family when the distribution of X is discrete.

(b) Obtain the UMPU test of size α for $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ based on a sample of size n from a population with density function $f(x,\theta) = \theta.\exp(-\theta x)$, $x \geq 0$, $\theta > 0$. (10+10)

3.(a) Define a monotone likelihood ratio family of distributions and give an example.

P.T.O.

3.(b) Let X be a Bernoulli random variable with P [X = 1] = p, $0 < p < 1$. Obtain the UMP test of size α for $H_0: p \leq p_0$ against $H_1: p > p_0$ based on a sample of size n. Show that the power function of the test is monotonic increasing for all p in (0,1). Obtain the large sample approximation of the test. (4+16)

4.(a) Describe the likelihood ratio principle of deriving a test for a hypothesis $H_0: \theta \in \Theta_0$ against $H_1: \theta \in \Theta - \Theta_0$ based on a sample of size n from a population with density function $f(x, \theta)$, $\theta \in \Theta$.

(b) Derive the likelihood ratio test of size α based on a sample of size n for testing $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 > \sigma_0^2$ in $N(\mu, \sigma^2)$, where (μ, σ^2) are unknown.

5.(a) Describe the method of scoring for simultaneous estimation of k parameters by the method of maximum likelihood estimation.

(b) Let X have a trinomial distribution with probabilities $\{\theta^2, 2\theta(1-\theta), (1-\theta)^2\}$, $0 < \theta < 1$. Obtain the first iterate in the method of scoring, starting with an initial estimator $\hat{\theta}_0$. (8+12)

6.(a) Let $T(x_1, \ldots, x_n)$ be an UMVUE of a parametric function $g(\theta)$. Show that it is unique.

(b) If $T_1(x_1, \ldots, x_n)$ and $T_2(x_1, \ldots, x_n)$ are any two unbiased estimates of $g(\theta)$ having the same variance, show that their correlation is $\geq 2e - 1$, where e is the ratio of the variance of T to the common variance of T_1 and T_2. (8+12).

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B.Stat.(Hons) Part III, 1981-82

Stochastic Processes-2
SEMESTRAL -I EXAMINATION

Date: 11.12.81 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Full marks for questions are given in brackets.

1. Consider the following system: The times when particles arrive into the system constitute a Poisson process on $[0, \infty)$ with parameter $\lambda > 0$ (i.e. $X(t)$, $t \geq 0$, is a Poisson process on $S = \{0,1,2,\ldots\}$ with parameter λ where $X(t)$ is the number of particles arriving in the time interval $(0,t]$.) . Each particle then lives for a certain length of time independent of the arrival times of the particles in the process and independent of the lives of the other particles. Suppose the lengths of life of the particles are exponentially distributed with common parameter $\mu > 0$. Let $M(t)$ denote the number of particles that are alive at time t. Compute the distribution of $M(t)$ and also $E(M(t))$. [16]

2. Let $X(t)$, $t \geq 0$, be a Markov Pure jump process on the state space S, a subset of the integers.

   (a) Define the infinitesimal parameters of the process. How do you define the embedded chain of the process ?

   (b) Describe a birth and death process on the set $S = \{0,1,2,\ldots\}$. Define the birth rates and the death rates of the process. Show that the embedded chain is a birth and death chain. If the birth and death process is irreducible, state a necessary and sufficient condition for the process to be transient.

   (c) Consider a birth and death process on $\{0,1\}$ with birth and death rates as follows: $\lambda_0 = \lambda$, $\mu_1 = \mu$ where $\lambda, \mu > 0$. Obtain $p_{xy}(t)$, $x,y = 0,1$; $t \geq 0$ by solving the backward differential equations.

[(2+4)+(3+3+6+4)+(7+10)=39]

P.T.O.

3. Let $X(t)$, $t \geq 0$, be a Markov pure jump process on the state space S.

   (a) Define a stationary distribution for the given process.

   (b) If S is finite, prove the equivalence of

   (i) $\sum_x \pi(x)\, p_{xy}(t) = \pi(y)$, $y \in S$, $t \geq 0$ and

   (ii) $\sum_x \pi(x)\, q_{xy} = 0$, $y \in S$,

   where $\pi(x)$, $x \in S$, are non-negative numbers adding to unity.

   (c) Suppose that for $y \in S$, the limits $\lim_{t \to \infty} p_{xy}(t)$ exist independently of $x \in S$. Call $\lim_{t \to \infty} p_{xy}(t) = a_y$, $y \in S$.

   If $a_y > 0$ for some $y \in S$, prove that the process admits of a unique stationary distribution π given by $\pi(y) = a_y$, $y \in S$. Apply this result to obtain the unique stationary distribution for the two-state birth and death process mentioned in Q.2(c).

   [2+8+(14+6)=30]

4. Consider the branching process as follows: A collection of particles act independently in giving rise to succeeding generations of particles. Suppose that each particle, from the time it appears, waits a random length of time having exponential distribution with parameter λ (>0) and then splits into two identical particles. Let $X(t)$ be the number of particles present at time t.

   Formulate the branching process $X(t)$, $t \geq 0$, as a pure birth process by specifying the birth rates. Find $p_{xy}(t)$, $x, y = 0,1,2, \ldots$; $t \geq 0$ with the help of the forward differential equations. Determine $E(X(t) \mid X_0 = x)$, $x \geq 0$.

   [4+12+4 = 20]

(Contd....3).

5. Suppose $\{X(t),\ t \geq 0\}$ is a Poisson process with parameter $\lambda > 0$. Fix $t > 0$. Let $Z_t = S_{X(t)+1} - t$, $Y_t = t - S_{X(t)}$

where $S_n = \inf\{s > 0 : X(s) = n\}$, $n \geq 1$

$= 0$, $n = 0$

Compute the distribution functions of (i) Z_t and (ii) Y_t.

Show that Z_t and Y_t are independent.

[Hint : Calculate $P\{Z_t > x,\ Y_t > y\}$.]

[5+6+8 = 20]

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B.Stat.(Hons.) Part III, 1981-82

Differential Equations
SEMESTRAL - I EXAMINATION

Date: 11.12.81 Maximum Marks: 100 Time: 3 hours.

Note: This paper carries 110 marks. Answer all questions. The maximum you can score is 100.

1. Find the most general solution of the following difference equation:

$$x_{k+2} - 3x_{k+1} + 2x_k = 3k \qquad (15)$$

2. Find the most general solution of the following differential equation in the interval $(0, \infty)$:

$$x^2 y'' + xy' - 4y = 0$$

Prove that there exists a unique solution φ of this differential equation such that $\varphi(1) = 1$ and $\varphi(x) \longrightarrow 0$ as $x \longrightarrow \infty$. Find φ. (20)

3. Let I be the interval $[0, \infty)$.
Consider the differential equation

$$y'' + a_1 y' + a_2 y = \frac{1}{x+1}$$

where a_1, a_2 are constants. You are given that the characteristic polynomial has distinct roots with strictly negative real parts. If φ is any solution of the above differential equation, show that $|\varphi(x)| \longrightarrow 0$ as $x \longrightarrow \infty$. (20)

4.a) Let I be the interval $(-1, 1)$.
Consider the differential equation

$$(1-x^2)\, y'' - 2xy' + k(k+1)\, y = 0,$$

where k is a positive integer.

Find two linearly independent solutions ϕ_1, ϕ_2 of this equation satisfying:

$$\phi_1(0)=1 \qquad \phi_2(0)=0$$
$$\phi_1'(0)=0 \qquad \phi_2'(0)=1$$

P.T.O.

4.a) Contd..

These solutions can be expressed as a power series in x, converging in (-1,1). Justify this by quoting precisely an appropriate theorem proved in class.

b) Show that either ϕ_1 or ϕ_2 has to be a polynomial.

(15 + 10)

5.a) What does it mean to say that $(x-x_0)^2\, y'' + a_1(x)y' + a_2(x)y = 0$ has a regular singular point at $x = x_0$?

b) The following can be considered as an example of an equation with regular singular point at $x_0 = 1$:

$$(1-x^2)\, y'' - 2xy' + k(k+1)\, y = 0.$$ Justify.

(5+10)

6. Using standard properties about Bessel's functions that you have seen in class, show that :

There exist an infinite number of continuous functions f on $\mathbb{R}^2$ such that

$$\iint_D f(x,y)\, dx\, dy = 0 \text{ for every disc } D$$

of $\mathbb{R}^2$ of radius 1.
(i.e. $D = \{(x,y) : (x - x_0)^2 + (y - y_0)^2 \leq 1\}$, (x_0,y_0) is arbitrary.)

If you need, you may assume :

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x \sin \theta)\, d\theta \qquad (15)$$

1981-82 | 391(s)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Optimization Techniques
SUPPLEMENTARY PERIODICAL EXAMINATION

Date: 19.4.82 Maximum Marks : 100 Time: 3 hours

Note: The paper carries 105 marks. Maximum one can get is 100. Marks assigned to each question are shown in [ ].

1. Formulate the general maximum L.P.problem and its dual. Obtain a standard maximum problem equivalent to a given general maximum problem. State and prove the Equilibrium Theorem for the general maximum problem.

[5+5+8+12=30]

2. Show that $\xi_1 = 4, \quad \xi_2 = 1$

is an optimal solution of the problem

maximise $\xi_1 - \xi_2$

subject to

$$-2\xi_1 + \xi_2 \leqq 2$$
$$\xi_1 - 2\xi_2 \leqq 2$$
$$\xi_1 + \xi_2 \leqq 5$$
$$\xi_1, \xi_2 \geqq 0$$

by finding a solution of the dual problem making use of the equilibrium theorem. [10]

3. (i) State the Fundamental Duality Theorem for a standard linear programming problem.

(ii) Consider the following linear programme :

Find $\xi_1, \xi_2 \geqq 0$ such that $\xi_1 + \xi_2$ is a maximum

subject to
$$-3\xi_1 + 2\xi_2 \leqq -1$$
$$\xi_1 - \xi_2 \leqq 2$$

Write the dual of this problem. Show that the original problem is feasible but has no optimal solution. In view of this result and the duality theorem what must be true of the dual problem ? Verify this directly.

[5+4+8+3+5=25].

P.T.O.

4.(i) Let $a_1, .., a_m$ be a set of linearly independent vectors, and let $b_1, b_2, .., b_n$ be a set of vectors each of which is a linear combination of the a_i's. Describe the tableau of the vectors b_j with respect to the basis $a_1, a_2, .., a_m$. If $\tau_{rs} \neq 0$ in the above tableau, prove that the vectors $a_1, a_2, .., a_{r-1}, b_s, a_{r+1}, .., a_m$ are linearly independent.

(ii) Let $A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}$ be an m x n matrix and b an n-vector.

In solving the equation $xA = b$ suppose we have arrived at the following tableau :

	a_1	....	a_k	....	a_m	b
a_1	1	....	τ_{1k}	....	τ_{1m}	ξ_1
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
a_r	0	....	τ_{rk}	....	τ_{rm}	ξ_r
u_1	0	.....	σ_{1k}	....	σ_{1m}	δ_1
$\vdots$	$\vdots$					$\vdots$
u_s	0	....	σ_{sk}	....	σ_{sm}	δ_s

Prove that if no further replacements of the u_i are possible then either the numbers $\xi_1, .., \xi_r$ solve the given equation or else there is no solution. Here u_i's stand for the vectors of the standard basis for R^n. [(4+6)+10=20]

5. Let ϕ be a function of two non-negative vectors, x in R^m and y in R^n. The pair $(\bar{x}, \bar{y})$ is called a saddle point of ϕ if

$$\phi(\bar{x}, y) \geq \phi(\bar{x}, \bar{y}) \geq \phi(x, \bar{y}) \text{ for all } x \geq 0 \text{ in } R^m \text{ and } y \geq 0 \text{ in } R^n.$$

Consider a standard L.P. problem and its dual, Let $\bar{x}$ and $\bar{y}$ be the respective optimal vectors. Show that $(\bar{x}, \bar{y})$ is a saddle point of the function ϕ defined by

$$\phi(x, y) = xc + yb - xAy \text{ for } x, y \geq 0.$$

Prove that if $(\underline{x}, \underline{y})$ is a saddle point of the function ϕ defined above then $\underline{x}$ and $\underline{y}$ are optimal vectors for the corresponding pair of dual standard programming problems. (Note: A, b, c have their usual connotations). [10+10=20].

1981-82 | 3·1

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Multivariate Distribution and Tests
PERIODICAL EXAMINATION

Date: 12.4.82 Maximum Marks : 100 Time: 3 hours

Note: State clearly the results you are using. Notations are as usual.

1.(a) Define the $N_p(\mu, \Sigma)$ distribution. Interpret μ and Σ. [3+3]

(b) Obtain the density, when Σ is nonsingular. [5]

(c) Obtain the conditional distribution of X_2 given X_1 where $X = (X_1, X_2)^T \sim N_p(\mu, \Sigma)$. [9]

(d) Obtain the ML estimate of Σ based on a sample of size n from $N_p(\mu, \Sigma)$. μ known. [10]

2.(a) Let $\underset{p\times 1}{x} = (x_1, x_2)^T$ and partition Σ similarly

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

where Σ is symmetric and positive definite. Show that $x_1^T \Sigma_{11}^{-1} x_1 \le x^T \Sigma^{-1} x$; give conditions under which strict inequality holds. [5+2]

(b) $x^T \Sigma^{-1} x = |\Sigma + xx^T| \div |\Sigma| - 1$ [5]

(c) If Σ is nonnegative definite, symmetric and A is symmetric such that

$$\Sigma A \Sigma A \Sigma A = \Sigma A \Sigma A,$$

then show that $\Sigma A \Sigma A \Sigma = \Sigma A \Sigma$ [7]

P.T.O.

3.(a) Define the Wishart distribution. [3]

(b) If $S = ((s_{ij})) \sim W_p(k, \Sigma)$, Σ nonsingular, find the joint distribution of

$$\frac{1}{s^{pp}} \text{ and } s_{ij},\ i, j = 1, \ldots, p-1$$ [9]

(c) If $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \sim W_p(k, \Sigma, M)$ and S_{11} is $r \times r$,

find the distribution of S_{11}. What is the joint distribution of S_{11} and S_{22} when $\Sigma_{12} = 0$? [5+5]

(d) Show how Hotelling's T^2 statistic can be used to get a confindence interval for all linear function of the mean vector of a multivariate normal distribution. [9]

4. Let Σ be the dispersion matrix of the random vector X. Show that

$$\left\{ l : l^T X \text{ is degenerate with probability one} \right\}$$

is a vector space; what is its dimension ? Prove your assertion.

[2+2+6]

5. Class - work. [10]

1981-82	3E1

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B.Stat.(Hons.) Part III, 1981-82

Elective 5 : Economics
PERIODICAL EXAMINATION

Date: 5.4.82 Maximum Marks : 100 Time: $3\frac{1}{2}$ hours

Note: Answer Questions 4 and 5 and any two from Questions 1 to 3.

1.(a) Define Pareto distribution and find its mean and variance.

(b) Examine how the conditional probability $P(X > x+a \mid X > x)$ changes with x if the r.v. X follows a Pareto distribution. (Here a is some constant.) Examine also the behaviour of the conditional probability $P(\log X > x + a \mid \log X > x)$. Comment on your findings.

8+12=(20)

2.(a) State and prove the moment distribution property of the two-parameter lognormal distribution. Use this property to obtain the equation of the Lorenz curve of the distribution.

(b) Discuss Lorenz ratio as a measure of income inequality.

12+8 = (20)

3.(a) Define Engel curve and explain how Engel curves are used to classify commodities into luxuries, necessaries and inferior commodities.

(b) Explain how randomness of residuals is examined to choose the appropriate algebraic form of the Engel curve that may fit a given body of data.

12+8= (20)

4. The following shows estimates of average expenditure on cereals and on all items based on a certain survey in rural India. The households have been classified by per capita household expenditure per 30 days on all items (PCE) for presentation of results. In the table below PCE classes above Rs 24 have been omitted.

P.T.O.

Q.4.(Contd)

Estimate the Engel elasticity for cereals at PCE = Rs 15 and at PCE = Rs 21 assuming that the semi-log form is appropriate.

PCE (Rs)	estimated percentage of population	per capita expenditure per 30 days(Rs)	
		all items	cereals
0- 8	6.37	5.61	3.36
8-11	8.73	9.09	4.93
11-13	7.93	11.44	6.34
13-15	7.33	13.41	6.42
15-18	11.05	16.10	8.24
18-21	11.02	18.70	9.33
21-24	8.98	21.57	9.96

(30)

5.(a) Describe briefly the structure of a two-industry Leontief static open system with one primary input. What do you understand by viability of such a system ? Obtain the conditions for viability.

(b) Consider the following input-output flow table in physical units.

Industry	Industry 1	Industry 2	final use	gross output
1	25	50	50	125
2	50	25	50	125
Labour	10	10	0	20

(i) Obtain the technology matrix from the above data.

(ii) Find out the gross output needed to support final demand of 60 units of each industry's product.

(iii) How much additional labour would be needed to support an additional unit of final demand of industry 1 and 2 ?

[15+15]=(30)

1981-82	3M1

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B.Stat.(Hons.) Part III, 1981-82

Elective-5: Mathematics
PERIODICAL EXAMINATION

Date: 5.4.82 Maximum Marks : 100 Time: 3 hours

Note: Answer as many questions or parts thereof as you can Maximum you can score is 100.

1. (a) Transform the following equation to canonical form

$$u_{xx} - 3\,u_{yy} + 2u_x - u_y + u = 0$$

(b) Is the above equation Elliptic or hyperbolic ?

(c) Solve : $u_x - 3u_y = 0$

$u(x,x) = x^2$

[8+2+10]

2. (a) Consider the eigen value problem :

$$x^2\,\varphi'' + 3x\,\varphi' + \lambda\,\varphi = 0 \qquad 1 < x < e$$

$$\varphi(1) = 0 = \varphi(e)$$

Describe all eigen values and the corresponding eigen functions. Show that the n^{th} eigen function can be written as $\varphi_n(x) = \frac{1}{x}\,\text{Sin}\,(n\,\pi\,\log x)$

(b) Consider the eigen value problem :

$$\varphi'' + \lambda\,\varphi = 0 \qquad 0 < x < 1$$

$$\varphi(0)=0 \;;\; \varphi'(1) + \varphi(1) = 0.$$

Show that there are infinitely many eigen values and that all eigen values are real and positive.

[10+10]

P.T.O.

3.(a) State the Fourier expansion theorem.

(b) Write down the Fourier expansion of

$f(x) = \sin^3 x \qquad -\pi \leq x \leq \pi$

You must evaluate all the constants.

(c) Suppose f is a real C^1 function on $[-\pi, \pi]$ with $f(-\pi) = f(+\pi)$. Using (a) show that there are real numbers α_k $k \geq 1$ and β_k $k \geq 0$ such that the series

$$\frac{1}{2}\beta_0 + \sum_{k=1}^{\infty} (\alpha_k \sin kx + \beta_k \cos kx)$$

uniformly converges to f. [3+7+10]

4.(a) Calculate $u(1,1)$ if

$u_{xx} - u_{yy} = 1$ on $\mathbb{R}^2$

$u(x,0) = x^2, \quad u_y(x,0) = 1$

(b) Show that the function $u \equiv 0$ is the only solution of

$u_{xx} - u_{yy} = 0$ on $\mathbb{R}^2$

$u(x,x) = 0, \quad u(x,-x) = 0$

[5+10]

5.(a) Solve the boundary value problem :

$u_t = u_{xx}$ on $(0, \pi) \times (0, \infty)$

$u(0,t) = u(\pi,t) = 0$ for $t \geq 0$

$u(x,0) = 1 - \cos 4x$ for $0 \leq x \leq \pi$

You must evaluate all the constants involved.

(b) Let f be absolutely integrable C^1 function on $[0,\infty)$ with $f(0) = 0$. Define

$b(\lambda) = \int_0^{\infty} f(\xi) \sin \lambda\xi \, d\xi, \quad \lambda > 0$

Show that $f(x) = \frac{2}{\pi} \int_0^{\infty} b(\lambda) \sin \lambda x \, d\lambda$ for $x \geq 0$.

you can use the Fourier inversion formula. [10+10]

(Contd.....3).

6.(a) Suppose $a < b$ real numbers. g be a continuous function on $[a,b]$ which is piecewise smooth

Show $$\lim_{\alpha \to \infty} \int_a^b g(x) \sin \alpha x \, dx = 0$$

(b) Let g be a continuous function on R which is piecewise smooth. Assume g is absolutely integrable.

Show $$\lim_{\alpha \to \infty} \int_{-\infty}^{\infty} g(x) \sin \alpha x \, dx = 0$$

[15+5]

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B.Stat.(Hons.)Part III,'81-82

Elective 5 : Physical and Earth Sciences

PERIODICAL EXAMINATION

Date: 5.4.82 Maximum Marks : 100 Time: 3 hours

Note : All questions carry equal marks. Answer any five questions.

1. State Coulomb's law.

   Define electric field and derive an expression for it for any arbitrary charge distribution consisting of several point charges only.

   Prove that $\nabla \times \vec{E} = 0$, where $\vec{E}$ is the electrostatic field.

   Consider that two point charges of 2×10^{-9} Coulomb and 5×10^{-9} Coulomb respectively are placed 30 cm apart in free space. What is the force between them ?

2. State and prove Gauss's law in electrostatics. Derive Poisson's equation and Laplace's equation from Gauss's law.

   A conducting sphere of radius 2 cm is charged with a total charge of 400 pico coulomb. Find the magnitude and direction of the electric field at any point on the surface of the sphere.

3. Let $\mathcal{L} f(t) = F(s)$. Hence prove the following .

   (i) $\mathcal{L}[f(t)\, e^{-at}] = F(s+a)$

   (ii) $\mathcal{L}[t\, f(t)] = -\dfrac{dF(s)}{ds}$

   Find the Laplace Transform of the function f(t), where f(t) is a unit impulse train as shown in Fig.1, having a time-period T.

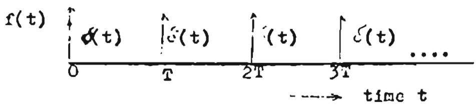

Fig 1

P.T.O.

Q.No.3 contd.

Solve the differential equation using Laplace transform method :

$$2\frac{d^2f(t)}{dt^2} + \frac{df(t)}{dt} - f(t) = 0,$$

the initial conditions being $f(0) = 1$, $f'(0) = 1$.

Find $\mathcal{L} f(t)$ where $f(t) = \int_0^t \frac{\sin t}{t} \, dt$

4. Consider the two series circuits shown in Fig. 2. Given that $v_1(t) = \sin 10^3 t$, $v_2(t) = e^{-1000t}$ for $t > 0$, and $C = 1\ \mu f$. Show that it is possible to have $i_1(t) = i_2(t)$ for all $t > 0$ and determine the required values of R and L for this to hold.

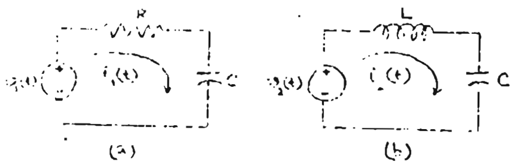

Fig.2.

5. In the network shown in Fig.3, find the current flowing through the branch AB after closing the switch S, using Thévenin's theorem.

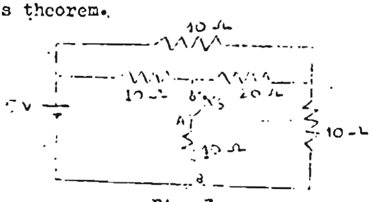

Fig. 3.

6. Consider the circuit shown in Fig. 4.

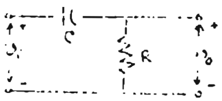

Fig.4

(Contd....3).

Q.No.6 Contd.

a) Find the output voltage v_o for a

i) step input v_i = V u(t)

ii) ramp input v_i = A t u(t)

b) Prove that for any periodic input, the average level of the steady state output is always zero.

c) Show that the circuit can be used as a differentiator. Illustrate your answer with the input and output waveforms when v_i is a pulse input, stating also the reasons why the output waveform deviates from the exact derivative of the input waveform.

7. For the network shown in Fig.5, find the value of the impedance Z_o in terms of Z_1 and Z_2 for which the input impedance Z_{in} seen at the terminals 1-2 is also equal to Z_o.

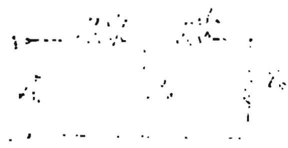

Fig. 5.

If Z_1 is an inductance of 500 mH and Z_2 is a capacitance of 0.5µf, find the values of Z_o at the frequencies $\omega = 0$ and $\omega = 4000$ rad / sec.

1981-82 | 3.1

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1981-82

Optimization Techniques
PERIODICAL EXAMINATION

Date: 29.3.82 Maximum Marks: 100 Time: 3 hours

Note: The paper carries 110 marks. Maximum one can score is 100. Marks for each question are given in [ ].

1. Let A be the m x n matrix $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}$.

Consider the canonical minimum problem of finding non-negative numbers $\xi_0, \xi_1, \ldots, \xi_m$ which

$$\text{minimise } \xi_0$$

$$\text{subject to } \xi_0 b + \sum_{i=1}^{m} \xi_i a_i = b.$$

(i) Formulate the dual of the above minimum problem.

(ii) Show that both the original and the dual problems are feasible.

(iii) Hence by applying the duality theorem obtain a proof of the following: Either the equation $xA=b$ has non-negative solution or the inequalities

$$Ay \geqq 0 \quad yb < 0 \quad \text{have a solution.}$$

(Warning : No other proof is acceptable.)

(iv) In the notation of (iii) if $xA = b$ fails to have any non-negative solution show the L.P. problem above has value 1.

[5+5+10+5=25].

P.T.O.

2. Formulate the general maximum L.P. problem and its dual. Obtain a standard maximum problem equivalent to a given general maximum problem. State and prove the Equilibrium Theorem for the general maximum problem.

[5+5+8+12=30]

3. Consider the following standard maximum problem :

$$\text{maximise } \xi_1 + \xi_2 + \xi_3 + \xi_4$$

$$\begin{aligned} \text{subject to } \quad \xi_1 + \xi_2 \quad & \leq 3 \\ \xi_3 + \xi_4 \quad & \leq 1 \\ \xi_2 + \xi_3 \quad & \leq 1 \\ \xi_1 + \xi_3 \quad & \leq 1 \\ \xi_3 + \xi_4 \quad & \leq 3 \end{aligned}$$

Show that this problem has the optimal solution

$$\xi_1 = 1, \ \xi_2 = 1, \ \xi_3 = 0, \ \xi_4 = 1$$

by finding a solution of the dual problem making use of the equilibrium theorem.

[10]

4. State the general transportation problem. Prove that the transportation problem is feasible if and only if the total supply is at least equal to the total demand.

[10]

5. Show that the following standard maximum problem is not feasible: Find non-negative numbers ξ_1 and ξ_2 which

maximise $3\xi_1 - 2\xi_2$

$$\begin{aligned} \text{subject to } \quad 2\xi_1 + 5\xi_2 & \leq 3 \\ -3\xi_1 + 8\xi_2 & \leq -5. \end{aligned}$$

[10].

(Contd......3).

6. Let the triplet (A,b,c) stand for the canonical maximum problem : Find non-negative x which maximises xc subject to equation $xA=b$.

Consider a canonical maximum problem (A,b,c) with value ω, and let c^k be a sequence of vectors converging to c as $k \longrightarrow \infty$ [i.e., if $c^k = (\alpha_1^k, \alpha_2^k, \ldots, \alpha_m^k)$, $k \geq 1$ and $c = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ we assume that $\alpha_i^k \longrightarrow \alpha_i$ for $1 \leq i \leq m$]. Let ω_k be the value of the problem (A,b,c^k).

Show that ω_k converges to ω as $k \longrightarrow \infty$. (Hint: Use the fact that there are only a finite number of basic feasible vectors.)

[25]

[1981-82] [271]

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B.Stat.(Hons.) Part III, 1981-82

Design of Experiments
PERIODICAL EXAMINATION

Date: 8.3.82. Maximum Marks : 100 Time: 3 hours

Note: 15 marks are allotted to Practical Records. Submit your Records alongwith your answerscripts. Attempt Question No.1 and any three out of the rest. Marks allotted to a question are indicated in brackets at its end.

1.(a) Define connectedness of a block design and show that a disconnected design is a disjoint union of several connected designs on subsets of the given set of treatments. State and prove a necessary and sufficient condition for a block design which is connected to be orthogonal. [4+6=10]

(b) Give the two definitions of a connected balanced design and show that they are equivalent. Also show that a connected block design is balanced if and only if all the off-diagonal elements of its C matrix are equal. [6+9=15]

2. Define a latin square design with an example and derive its analysis of variance. [5+15=20]

3. For a general block design with parameters $v, b, \underline{r}, \underline{k}$, show that-

(i) the reduced normal equations for the treatment effects turn out to be $C\hat{\underline{\gamma}} = \underline{Q}$, where the symbols have their usual significance ;

(ii) $v + \text{Rank } D = b + \text{Rank } C$, and $\text{Rank } C \leq v-1$;

(iii) the sum of squares for testing the hypothesis of treatment effects is given by

$$\sum_{i=1}^{v} \hat{\gamma}_i Q_i ,$$

called the adjusted treatment s.s. [5+7+8=20]

P.T.O.

4. Define a balanced incomplete block design BIBD (v, b, r, k, λ) with an example, and prove that for such a design:

(i) $\lambda(v-1) = r(k-1)$, and

(ii) $v = b \iff$ any two blocks intersect in a common number λ of treatments.

Give the analysis of variance for a BIBD (v, b, r, k, λ).

[2+2x4+10=20]

5. Define an m-ple lattice design for $v = s^2$ treatments and prove that for $m = s + 1$, the lattice becomes a BIBD. Derive for an m-ple lattice design (i) the expression for $\hat{\tau}_i$, a least square estimator for the treatment effect τ_i, and (ii) $V(\hat{\tau}_i - \hat{\tau}_j)$, for $i \neq j$.

[2+5+8+5=20]

6. Write short notes on any <u>two</u> of the followings :

(a) Fundamental principles of experimental designs ;

(b) Randomized block designs ;

(c) Mutually orthogonal latin squares and some of their uses in experimental designs.

[2x10 = 20].

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B.Stat.(Hons.) Part III, 1981-82

Inference
PERIODICAL EXAMINATION

Date: 23.11.81 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions. All questions carry equal marks. Precise and complete answers carry more weight.

1.(a) Let X be a random variable with density function $f(x,\theta)$, where $\theta \in R$ is unknown. A random sample of size n is drawn from the population to estimate a function $g(\theta)$. Obtain a lower bound for the variance of any unbiased estimate of $g(\theta)$. By stating precisely the conditions needed, obtain from the above result or otherwise the Cramér-Rao lower bound for the variance of an unbiased estimate of $g(\theta)$.

(b) If $f(x,\theta) = \frac{\theta^p}{\Gamma(p)} \; e^{-\theta x} \; x^{p-1}$ where p is known and θ is unknown show that there exists an unbiased estimate of $\frac{1}{\theta}$ based on a sample of size one, whose variance is equal to the Cramér-Rao lower bound. (5+7+8)

2.(a) Let $\{f(x,\theta), \theta \in R\}$ be a family of density functions and $g(\theta)$ an estimable parametric function. Show that the UMVUE of $g(\theta)$ based on a sample $x_1, \ldots, x_n$ is a symmetric function of $x_1, \ldots, x_n$.

(b) If U_0 denotes the class of all unbiased estimates of 0 in the above set up show that an unbiased estimate $T(x_1, \ldots, x_n)$ is UMVUE iff $\text{cov}(T,S : \theta) = 0 \;\; \forall \theta$ and $\forall S \in U_0$.

(c) Obtain the UMVUE of μ^2 in $N(\mu,1)$, $-\infty < \mu < \infty$ based on a sample of size n.

(6+6+8).

p.t.o.

3.(a) If T_1 (x_1 ...., x_n) and T_2 (x_1 ...., x_n) are two unbiased estimates with finite variances of a parametric function $g(\theta)$, obtain the best linear combination of T_1 and T_2 (in the usual sense) to estimate $g(\theta)$.

(b) Let $\underset{\sim}{Y}$ be N_n ($X \underset{\sim}{\beta}$, σ^2 I). Show that the least squares estimator of any estimable parametric function $\underset{\sim}{P}' \underset{\sim}{\beta}$ is UMVUE in the class of all unbiased estimators. (8+12)

4.(a) Let X be a random variable with a discrete distribution $\{p(x,\theta), \theta \in R\}$. Show that a statistic T(x) is sufficient for the family iff Neyman factorization holds.

(b) Let X be a Bernoulli random variable with unknown parameter p and x_1, ..., x_n is a random sample of size n. Show that $T = x_1 + \ldots + x_n$ is a sufficient statistic for p. Is it complete ? Justify. (8+12)

5.(a) Let X be a random variable with density function $\{f(x,\theta), \theta \in R\}$ and T(x_1 ...., x_n) a sufficient statistic for θ based on a sample of size n. Let $g(\theta)$ be an estimable parametric function. Show that the UMVUE of $g(\theta)$ is a function of T.

(b) If X has uniform distribution over $[0, \theta]$. obtain the UMVUE of θ based on a sample of size n.

(10+10).

6.(a). Let X have a trinomial distribution with probabilities θ^2, $2\theta(1-\theta)$, and $(1-\theta)^2$. Describe the method of scoring to estimate θ from a sample of size n.

(b) Describe how the method of scoring is used to estimate several parameters by the method of maximum likelihood estimation. (10+10).

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B.Stat.(Hons.) Part III, 1981-82

Elective 4 : Mathematics

PERIODICAL EXAMINATION

Date: 16.11.81 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you like. The maximum marks that one may obtain is 100.

1. Let G be a group and H a subgroup of G. Prove that $N = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G. [6]

2. Let G be a group with $o(G) = p^n$, p a prime and $n \geq 2$. Then show that for all $a \in G$, $o(N(a)) = p^m$ where $m \geq 2$. [12]

3. If G is a non-abelian group, $o(G) = p^n$, $n \geq 3$, then show that $o(Z(G)) = p^m$ for some m, where $1 \leq m \leq n-2$. [12]

4. (a) If a group G has a subgroup of index 2 then prove that that subgroup contains all the elements of odd order in G. [12]

(b) A group of order 4n+2 has exactly 2n+1 elements of odd order. [6]

5. A characteristic subgroup H of G is a subgroup H where $\phi(H) = H$ for all automorphisms ϕ of G.

(a) Let G be an abelian group and let $H = \{x^k : x \in G\}$ where k is a fixed positive integer. Show that H is a characteristic subgroup of G. [6]

(b) If G is a group then Z(G) is a characteristic subgroup of G. [6]

(c) Let G be a group and $H = \{x : x \in G \text{ and } o(x) > 2\}$. Then show that H is a characteristic subgroup of G. [6]

p.t.o.

6. Classify all groups of order $5^2 . 13^2$. [18]

7. If a group G has exactly $(1+p)$ p-Sylow subgroups of order p^n, where p is a prime, then show that these subgroups have p^{n-1} elements in common. [12]

8. Let G be a finite abelian group and s the maximum order of any element in G. Then show that for all $x \in G$, $o(x) \mid s$. [6]

9. Let M be an R-module. If A and B are submodules of M then prove that

   (a) $A \cap B$ is a submodule of M. [2]

   (b) $A + B$ is a submodule of M. [2]

   (c) $(A+B)/B$ is isomorphic to $A/(A \cap B)$ as a module. [8]

10. If L is a left ideal of the ring R and if M is an R-module, show that for $m \in M$, $Lm = \{xm : x \in L\}$ is a submodule of M. [6]

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B.Stat.(Hons.) Part III, 1981-82

Elective 4 : Physical and Earth Sciences

PERIODICAL EXAMINATION

Date: 16.11.81 Maximum Marks: 100 Time:3 hours

Note: Answer any five questions.

1. Broadly classify the different hypotheses on the Origin of the Solar System. What do you understand by 'hot earth theory' and 'cold earth theory' Which one of the two theories seems to you more acceptable and why? (4+6+10)

Or,

Discuss briefly the Origin of the Earth on the basis of the work of one of the following scientists -

(a) Weizsacker
(b) Kuiper
(c) Urey. (20)

2. Write a short essay on the internal constitution of the earth as known from seismic evidence. (20)

3. Write briefly what you know about plate tectonism. (20)

4. Define with examples (any four) -

Crystal, Amorphous Substance, Gemstone, Economic Mineral, Rock-forming Mineral. (5x4=20)

5. Define the terms - Magma and Lava.
Describe the characteristics of the different rock types that are connected with the above two terms.

Which of the following are igneous rocks -
Agate, Basalt, Coal, Petroleum, Limestone, Granite (6+11+3=20)

6. Write brief notes on (any three) -
Island Arcs, Mid-Oceanic Ridge, Bode Law, Primitive Atmosphere of the Earth, Mountain Range. (7x3-1 = 20)

[1981-82] 253]

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B.Stat.(Hons.) Part III,1981-82

Elective 4 : Economics

PERIODICAL EXAMINATION

Date: 16.11.81 Maximum Marks: 100 Time:3 hours

Note: Answer 2 questions from Group A.
1 question from Group B
2 questions from Group C
Use separate answer books for Groups A,B and C.

Group A

1.(a) State and prove the "Complementary slackness" theorem for a Canonical L.P.Problem. Can you give an economic interpretation of the theorem with reference to the diet problem ? [10]

(b) Consider the following L.P.problem :

$$\text{Minimize} \quad y^T b$$
$$\text{Subject to } Ay = C$$
$$y \geq 0.$$

Let y^* be an optimal solution to this problem depending on the first k columns of A. Suppose now that the vector C is changed to $\bar{C}$. Show that if there is a feasible solution $\bar{y}$ to the new problem which depends on the first k columns of A, then $\bar{y}$ is optimal [15]

2. What is meant by the "assumption of nondegeneracy"? Outline (without proof) the simplex procedure for solving a nondegenerate Canonical L.P.problem. [25]

3.(a) Describe a general L.P.problem and its dual. [5]

(b) Prove in detail the following theorem for a general L.P. problem:

If both a programme and its dual are feasible then both have optimal vectors and the optimal values of the objective functions for the two programmes are the same. [20]

p.t.o.

4. Consider the following tableau :

	a^1	a^2	...	a^m	...	a^s	...	a^n
a^1	1	0	...	0	...	τ_{1s}	...	τ_{1n}
a^2	0	1	...	0		τ_{2s}	...	τ_{2n}
⋮	⋮	⋮		⋮		⋮		⋮
a^r	0	0	...	0		τ_{rs}	...	τ_{rn}
⋮	⋮	⋮		⋮		⋮		⋮
a^m	0	0	...	1		τ_{ms}	...	τ_{mn}

(a) Suppose $\tau_{rs} \neq 0$ and it has been decided to introduce a^s into the basis and replace a^r. Prove that under the stated condition, $a^1, \ldots, a^{r-1}, a^s, a^{r+1}, \ldots, a^m$ form a new basis. Moreover, let τ'_{ij} represent the elements of the new tableau. Prove that

$$\tau'_{ij} = \tau_{ij} - \frac{\tau_{is}}{\tau_{rs}} \tau_{rj}, \quad i \neq j$$

$$\tau'_{rj} = \frac{\tau_{rj}}{\tau_{rs}}$$

[13]

(b) Show that if $\tau_{rs} = 0$ then the vectors $a^1, \ldots, a^{r-1}, a^s, a^{r+1}, \ldots, a^m$ are dependent [12]

Group B

5. Discuss the principles of measurement of direct benefits from projects in Cost Benefit Analysis. Illustrate your answer with the case for a project producing consumer goods. [20]

6. Explain the difference between Cost Benefit Analysis and Profit Loss Accounting. Discuss the conditions under which Cost Benefit Analysis is to be preferred. [20]

(Contd....3).

Group C

1. Where the push factors responsible for migration of persons from rural homes ?

Discuss the role of push factors in influencing out migrations in different streams?

[6+9 = 15]

2. How will you measure occupation mobility of males with reference to those of their fathers?

How does occupation mobility information help the planning ?

[10+5 = 15]

3. How many broad subdivisions were observed in tabulating occupation data of Indian census, 1961? What were these subdivisions and indicate the distribution of males by them ?

[3+12 = 15].

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B.Stat.(Hons.) III Year: 1981-82

Sample Surveys
PERIODICAL EXAMINATION

Date : 26.10.81 Maximum Marks : 100 Time: 3 hours

Note : This paper contains 125 marks. Answer as many questions as you can. Maximum you can score is 100. Maximum score of each sub-division of a question is indicated against each question.

1.(a) Define and illustrate the concept of

(i) population (ii) sample (iii) sample space
(iv) sampling design (v) sampling strategy
(vi) unbiased estimation in the context of survey sampling.

(b) When is a sampling strategy said to be better than another sampling strategy? Illustrate.

[20+5] = 25]

2. Show that for any fixed size sampling design with sample size, n

(i) $\sum_{i=1}^{N} \pi_i = n$

(ii) $\sum\sum_{i \neq j=1}^{N} \pi_{ij} = n(n-1)$

(iii) $\pi_{ij} \geq \pi_i + \pi_j - 1$

where π_i and π_{ij} denote first order and second order inclusion-probabilities respectively. [5+5+2 = 12

3.(a) Define and illustrate the concept of simple random sampling without replacement (SRSWOR).

(b) Show that in SRSWOR, sample mean is an unbiased estimator of population mean.

(c) Find expressions for variance of sample mean and an unbiased variance estimator.

p.t.o.

3.(d) A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probabilities that the sample contains 1,2 and 3 different units (for example aaa, aab, abc respectively) are,

$$P_1 = \frac{1}{N^2}, \quad P_2 = \frac{3(N-1)}{N^2}, \quad P_3 = \frac{(N-1)(N-2)}{N^2}$$

[4+5+5+5+5]=[24]

4.(a) Show that in estimating a finite population mean using stratified random sampling (without replacement)

$$V_{opt} \leq V_{prop} \leq V_{ran} ,$$

when finite population corrections (fpc's) are ignored. Here V_{ran} denotes variance of estimator under SRSWOR and V_{opt} and V_{prop} variances under optimum and proportional allocations in stratified random sampling.

(b) Can you find any situation in which V_{prop} exceeds V_{ran}.

(c) In a stratified random sampling intended to use optimum allocation, it was found that $n_{1(opt)} > N_1$, $n_{1(opt)}$ and N_1 being respectively the optimum sample size and population size for stratum 1. Derive an unbiased estimator for population mean and obtain expression for its variance under optimum allocation.

(d) A sampler has two strata with relative sizes W_1 and W_2. He believes that population standard deviations S_1 and S_2 may be taken as equal, but thinks that C_2 may be between $2C_1$ and $4C_1$, where C_i is the cost per unit sampled from the ith stratum. He would prefer to use proportional allocation but does not wish to incur a substantial increase in variance compared with optimum allocation. For a given cost $C = n_1 c_1 + n_2 c_2$ ignoring fpc, show that

$$\frac{V_{prop}}{V_{opt}} = \frac{W_1 c_1 + W_2 c_2}{(W_1\sqrt{c_1} + W_2\sqrt{c_2})^2}$$

[7+4+5+8] = [24]

...

p.t.o.

5.(a)

5.(a) Explain and illustrate the concept of cluster sampling.

(b) Assuming that the clusters are of equal sizes, examine the advantages and disadvantages of cluster sampling vis-a-vis SRSWOR.

[4+8] = [12]

6.(a) Examine the magnitude of bias of ratio estimator relative to its standard error.

(b) In a stratified random sampling using ratio estimates for population mean, clearly esplain when you would use seperate ratio estimates and combined ratio estimates.

(c) When is the ratio estimate best linear unbiased estimate for population total. Explain.

[6+8+6] =[20]

7. In order to estimate the proportion of persons (P) belonging to blood group O, living in a certain island, an anthropologist wants to make a sample survey taking a SRSWOR of 'n' persons from the total number of 3200 individuals in the island. He desires to take a sample large enough to ensure that chance is .95 that the unknown true value of P will be within a range of .05 from the sample estimate on either side. Assuming that a very rough estimate of P is .5, find the desired sample size.

[8]

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B.Stat.(Hons.) Part III 1981-82

1981-82	321

Stochastic Processes

PERIODICAL EXAMINATION

Date: 28.9.81 Maximum marks: 50 Time: $2\frac{1}{2}$ hours

Note: There are 4 questions carrying 65 marks. Maximum one can score is 50. Marks allotted to each question are indicated in [ ].

1. Consider a homogeneours Markov chain on $\{0,1,2,\ldots,d\}$ satisfying

$$\sum_{y=0}^{d} y p_{xy} = x, \quad x = 0,1,\ldots,d.$$

Show that

(i) $E\{X_{n+1} \mid X_0 = x_0, X_1 = x_1, \ldots, X_{n-1} = x_{n-1}, X_n = x\} = x.$

(ii) 0 and d are necessarily absorbing states.

(iii) if there is no absorbing states other than 0 and d, the states 1,2..., d-1 each lead to 0, and hence that each is a transient state.

[4+6+12=22]

2. Let x be a state of a homogeneous Markov Chain. Write p_n and f_n for $p_{xx}^{(n)}$ and $f_{xx}^{(n)}$ respectively. It is given that there exist a subsequence $\{p_{n_k}\}$ of $\{p_n \; n \geq 0\}$, a real number $\lambda \geq 0$ and a positive integer s_0 such that

(i) $\lim_{k \to \infty} p_{n_k} = \lambda$ and

(ii) $\lim_{k \to \infty} p_{n_k - s} = \lambda$ for all $s \geq s_0$.

Prove that $\lim_{k \to \infty} p_{n_k - s} = \lambda$ for all $s \geq 1$.

[10]

P.T.O.

3.(a) Let π_0 and π_1 be distinct stationary distributions for a Markov chain.

(i) Show that for $0 \leqq \alpha \leqq 1$, the function π_α defined by

$\pi_\alpha(x) = (1-\alpha)\,\pi_0(x) + \alpha\,\pi_1(x), \quad x \in S,$

is a stationary distribution.

(ii) Show that distinct values of α determine distinct stationary distributions π_α.

(b) Consider the gambler's ruin chain on $\{0,1,2..d\}$, $d > 0$ with the transition probabilities

$$P_{ij} = \begin{cases} \frac{1}{2} & j = i+1 \\ \frac{1}{2} & j = i-1 \quad \text{if } 1 \leqq i \leqq d-1, \\ 0 & \text{otherwise} \end{cases}$$

and $P_{00} = 1 = P_{dd}$.

Determine the class of all stationary distributions.

(c) Consider the M.C. having the transition matrix

	0	1	2	3	4	5
0	1	0	0	0	0	0
1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0
2	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$
3	0	0	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
4	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
5	0	0	0	$\frac{1}{4}$	0	$\frac{3}{4}$

Find the stationary distribution concentrated on each of of the irreducible closed sets. [3+5+6+12=26]

4. Consider a birth and death chain on the non-negative integers. The transition function is of the form

$$P_{xy} = \begin{cases} q_x & y = x-1 \\ r_x & y = x \\ p_x & y = x+1 \end{cases}$$

where $p_x + q_x + r_x = 1$ for $x \in S$. $q_0 = 0$. Also p_x and q_x are positive for $x > 0$.

Determine the period of this chain.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part III 1981-82
Differential Equations

PERIODICAL EXAMINATION

Date : 14.9.81 Maximum Marks 100 Time : 3 hours

Note : This paper carries 110 marks. Answer all questions. The maximum you can score is 100.

1. Let I be the interval (-1,1).

a) Find the most general form of a real valued function on I that satisfies the differential equation

$$y''' - y'' + y' - y = 0$$

b) Find the most general solution on I of the differential equation

$$y'' - y = |x| \qquad (20)$$

2. Let I be any interval and a_1, a_2 two continuous functions on I. Consider the following differential equation on I :

$$y'' + a_1(x)y' + a_2(x)y = 0$$

Let ϕ_1, ϕ_2 be two solutions of this equation and W the corresponding Wronskian.
If x_0 is a fixed point in I, show that

$$W(x) = W(x_0)\,\exp\left(-\int_{x_0}^{x} a_1(t)\,dt\right), \quad x \in I. \qquad (10)$$

3. Consider the following differential equation in the interval $(0, \infty)$:

$$x^3 y'' - x^2 y' + xy = 0$$

The function $\phi_1(x) = x$ is a solution of this differential equation in $(0,\infty)$. Find another solution ϕ_2 in $(0,\infty)$ such that ϕ_1 and ϕ_2 are linearly independent.
(After having produced ϕ_2, show that ϕ_1 and ϕ_2 are linearly independent.) (20)

4. Let I be the interval $(-1,1)$.
Prove that $\phi_1(x) = x^2$ and $\phi(x) = x^3$ cannot be solutions of the same second order equation of the form $y'' + a_1(x)y' + a_2(x)y = 0$ (in I). Can they be solutions of the same third order equation in I ? - justify your answer. (20)

5. Let I be the interval $(-1,1)$ and ϕ a three times differentiable function on I satisfying the differential equation

$$y''' + x^2 |x| y = x|x|$$

Prove that ϕ is actually four times differentiable on I.
Can ϕ be five times differentiable ? justify your answer. (20)

6. Let I be the interval $[0,\infty)$. Consider the differential equation

$$y'' + 3y' + 2y = b(x)$$

where b is a bounded continous function on $[0,\infty)$ (i.e. $|b(x)| \leq k$ for all $x \in I$). Prove that if ϕ is any solution of this differential equation then there exists a positive number M such that $|\phi(x)| \leq M$ for all $x \in [0,\infty)$. (20)