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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Multivariate Distribution and Tests
PERIODICAL EXAMINATION

Date: 7.3.1983. Maximum Marks: 100 Time : 3 hours.

Note : ANSWER ALL QUESTIONS.

1. U is a p-dimensional random vector with $E(U) = \mu$, $D(U) = \Sigma$. P is a pxp orthogonal matrix and Δ a diagonal matrix such that $P'\Sigma P = \Delta$. Let $Z = P'(U-\mu)$. Find $E(Z'Z)$. [6]

2. Let $\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \sim N_3\left(\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \\ 1 & \frac{3}{2} & 3 \end{bmatrix}\right)$.

Find the conditional distribution of U_3 given U_1, U_2. [6]

3. Let $U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \sim N_p\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right)$, $|\Sigma| \neq 0$.

Find the distribution of

$(U-\mu)' \Sigma^{-1}(U-\mu) - (U_1-\mu_1)' \Sigma_{11}^{-1} (U_1-\mu_1)$. [15]

4. Find the maximum likelihood estimators of μ and Σ based on a random sample of size n (> p-1) from $N_p(0, \Sigma)$ where $\Sigma = ((\sigma_{ij}))$, $\sigma_{ij} = \sigma^2$ (for $i = j$) and $\sigma_{ij} = \rho\sigma^2$ (for $i \neq j$). [20]

5. Let $\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \sim W_2(n, \Sigma)$, $n > 2$.

Find $V(s_{11} + s_{22})$. [9]

P.T.O.

6. Let $(U_1, U_2, U_3, U_4) \sim N_4$ such that the distributions of (U_1, U_2) and (U_3, U_4) are identical. Derive the Likelihood Ratio criterion for testing the identity of distributions of U_1, U_2, U_3, U_4. [10]

7. Develop a test for independence of the components of a vector distributed as N_p, with a nonsingular dispersion matrix. [15]

8. The following data were obtained from collections in a herbari on a genus called Mcesiclla.

   X_1 : length of leaf (cm)

   X_2 : length of inflorescence (cm)

   X_3 : length of fruit (cm)

Mean Vectors

Species A : (4.285, 1.937, 1.067) from 54 specimens

Species B : (5.661, 4.308, 1.043) from 23 specimens

Estimated Common Dispersion Matrix

$$\begin{pmatrix} 1.9255 & 1.0698 & 0.0282 \\ \cdot & 4.2589 & -0.0256 \\ & & 0.0141 \end{pmatrix}$$

Stating assumptions, examine if the two species can be regarded as distinct on the basis of these three variables. [20]

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B.Stat.(Hons.) III Year, 1982-83

Nonparametric and Sequential Methods
PERIODICAL EXAMINATION

Date: 21.3.83. Maximum Marks: 100 Time: 3 hours.

Note : Answer all questions.

1.(a) Discuss briefly the reasons for resorting to sequential analysis.

(b) The yield of a new variety of wheat on available plots may be assumed to be $N(\theta, \sigma^2)$ with known σ^2. You have to test

$$H_0 : \theta = \theta_0 \text{ Vs. } H_1 : \theta > \theta_0 \qquad \theta_0 \text{ being the mean yield}$$

of the variety currently in use. Will you recommend a suitable SPRT or a fixed sample size analysis?

(15+5)=[20]

2.(a) Prove under appropriate assumptions that an SPRT terminates with probability one.

(b) $\{X_i\}$ is a sequence of i.i.d $\sim f_\theta(x) = \frac{1}{\theta}$, $0 < x < \theta$.

Write down the SPRT for $H_0 : \theta = \theta_0$ vs. $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$

Discuss whether the test terminates with probability one under θ.

(16+9)=[25]

3. Let $\{X_i\}$ be i.i.d. $N(\theta, 1)$. You have to test $H_0: \theta = 0$ vs. $H_1: \theta = 1$ with $\alpha = \beta = .01$. Calculate the efficiency of the SPRT with respect to the best fixed sample size test. [30]

4. Assignments. [10]

5. Viva. [15]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Elective-5: Physical and Earth Sciences
PERIODICAL EXAMINATION

Date: 11.4.83. Maximum Marks: 100 Time: 3 hours.

Note: Attempt any five questions. All questions carry equal marks.

1. Find the expressions for electrostatic field and potential due to a point dipole of dipole moment $\vec{p}$ at a point P whose position vector is $\vec{r}$ with respect to the centre of the dipole.

   Hence calculate the electrostatic field and potential at any point outside a dielectric medium having a uniform polarisation $\vec{P}$. [10+10]=20

2. State and prove Gauss's Law in dielectrices.

   A point charge q is placed in an infinitely extent uniform dielectric medium of dielectric constant K. Find the electrostatic field at any point distant r from the point charge. [10+10]=20

3. A perfectly conducting sphere of radius 10 cm. is charged to 1000 volts. Calculate the surface charge density on the sphere.

   The plates of a parallel plate capacitor have length 'l', breadth 'b' and they are kept at a fixed potential difference V by means of a battery. Initially the space between the plates was vacuum. If a dielectric slab of dielectric constant K is now inserted through a distance x between the plates lengthwise and the rest(l-x) is still vacuum, find the force acting on the slab. Assume that the thickness of the slab is equal to the separation 't' between the plates. [8+12]=20

P.T.O.

4. State Biot - Savart's Law.

Prove that div $\vec{B} = 0$

Consider that a current I is flowing through the sides of a regular hexagon, each side being equal to 'a'. Calculate magnetic induction field at the centre of the hexagon. [4+6+10] = 20.

5. State Faraday's law of electromagnetic induction and hence show that

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

Define self-inductance of a coil. What is the unit of inductance in MKS system?

A toroidal coil has 100 no. of turns. The mean circumference of the coil is 4 cm. and the area of cross-section of the coil-former is 1 sq.cm. Calculate the inductance of the coil. Derive the necessary working formula. [6+4+10]=20.

6. A point charge q is placed in front of a perfectly conducting sphere of radius a. The distance of the point charge from the centre of the sphere is 'd'. Calculate, by the method of electrical images or otherwise, the electrostatic potential at any point outside the sphere when

i) the sphere is uncharged

ii) the sphere is kept a potential U_0.

What will be the potential inside the sphere in each of the above two cases. [18+2]=20

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Elective 5 : Economics.
PERIODICAL EXAMINATION

Date: 11.4.83. Maximum Marks: 100 Time: 3 hours.

Note : Separate answer books to be used for groups A and B.

GROUP A

Note: Answer any two questions. Each question will carry 15 marks.

1.(a) How will you measure upward and downward education mobility of daughters in respect of their mothers?

(b) Estimated distribution of women of age group 25-34 years by their education and that of their mothers is given below. Calculate the education mobility of girls.

Education year of women	Education year of mother					
	0	1 - 2	3 - 4	5 - 8	9+	Total
0	58.84	2.16	0.80	0.16	0.00	61.96
1 - 2	4.54	0.24	0.03	0.03	0.00	4.84
3 - 4	11.73	0.56	0.99	0.16	0.00	13.44
5 - 8	3.56	1.45	1.46	1.50	0.27	13.24
9+	2.11	0.31	1.24	2.34	0.52	6.52
	85.78	4.72	4.52	4.19	0.79	100.00

[6+5+4]

2.(a) Distinguish between work Force and Labour Force. Give illustrations.

(b) How will you obtain estimates of labour force from censuses?

P.T.O.

2.(c) You are given the distribution of percentages of males in each age group and social group forming uni-earner households. How will you interpret the data.

Social group	Age group				
	15-24	25-34	35-44	45-59	Total
Upper caste Hindus	7.3	19.3	46.0	43.9	30.5
Scheduled caste Hindus	5.2	20.9	39.5	34.4	23.2
Other Hindus	5.7	23.4	44.6	32.2	25.2
Muslims	12.4	48.9	71.8	31.9	38.9
Combined	7.3	27.8	49.6	34.4	26.5

[4+4+7]

3.(a) Discuss the role of push factors in migration from rural areas in different migration streams?

(b) Define population growth rate? How will you obtain it from two censuses.

(c) Census population of Bihar and West Bengal are given for 1961 and 1981. Obtain estimated population of 2000 AD in respect of those two states?

State	Population (0000)	
	1961	1981
Bihar	4645	6982
West Bengal	3493	5449

GROUP B

Note: Answer all the questions. Marks for each question are given in brackets.

1.(a) What is Pareto's Law of income distribution? Obtain the density function and the equation for the Lorenz curve for the Pareto distribution. Comment briefly on the universality of Pareto's Law.

(contd...3).

Group B (contd..)

1.(b) A given income (x) distribution is found to be approximately lognormal with parameters μ and σ^2. for $0 < x < \infty$. Also, for $x \geq c$ the distribution is approximately of the Pareto type. Given that

(i) the Lorenz ratio for the entire distribution and for $x \geq c$ are L_1 and L_2 respectively ;

(ii) $x = c$ is the median of the entire distribution ;

What is the mean income ($\bar{x}_1$) for the population with $0 < x < c$?

If $\bar{x}$ is the overall mean of the distribution and $\bar{x}_1$ and $\bar{x}_2$ are the means for population with $x < c$ and $x \geq c$ respectively, then using the relation $\bar{x} = \frac{1}{2}(\bar{x}_1+\bar{x}_2)$ show that the following holds approximately

$$\frac{1}{2\,\Phi(1)}\;\frac{(L_2+1)}{(L_2-1)} = \exp\left[\Phi^{-1}\left(\frac{L_1+1}{2}\right)\right]^2$$

where $\Phi(.)$ is the area under $N(0,1)$.

[20+20=40]

2.(a) Define engel curve and engel elasticity. Why would you call a commodity a necessary commodity when its engel elasticity is less than unity?

A commodity is bought by n consumers. The ith consumer has an income y_i and his income elasticity for the commodity is η_i. By how much percent the aggregate demand for the commodity will increase, if the aggregate income of these n consumers is increased by one per cent?

(b) Describe clearly the rationale of using household budget data on consumption for estimating engel curve. Also, describe how would you estimate the constant elasticity engel curve for an item from grouped household budget data.

[15+15=30]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Optimisation Techniques
PERIODICAL EXAMINATION

Date: 18.4.1983. Maximum Marks: 100 Time: 3 hours.

Note : Answer all questions. The whole paper carries 103 marks but the maximum you can score is 100.

1. (a) Write down the dual of the LP :

$\max e^T x + f^T y$ subject to $Ax+By=g, Cx+Dy \leq h, x \geq 0.$ [5]

(b) Show that the objective function at any feasible solution of the above LP is not greater than the objective function at any feasible solution of the dual. [8]

(c) Write down the optimality conditions for feasible solutions of the above LP and its dual to be optimal for the respective programmes. [5]

(d) Write the dual of the LP :

$\max f^T y$ subject to $By = g$

and prove the strong and weak duality theorems for this LP <u>without</u> using the theorems proved in the class (like Tucker's lemma, simplex algorithm etc.). You may use standard results from Linear Algebra like properties of orthogonal complements. [10+5]

2.(a) State Tucker's (complementarity) theorem. [5]

(b) Deduce from the result in (a), the strong duality theorem for the LP:

$\max c^T x$ subject to $Ax \leq b, x \geq 0.$ [12]

P.T.O.

2.(c) Let $z_1, z_2, \ldots, z_n$ be n points in $\mathbb{R}^m$ and let C be the cone generated by them, that is,

$$C = \left\{ \sum_{i=1}^{n} \alpha_i z_i \ : \alpha_i \geq 0 \text{ for all } i \right\}.$$

For any set $S \subseteq \mathbb{R}^m$, the polar of S is defined to be

$$S^P = \left\{ y \in \mathbb{R}^m : y^T z \leq 0 \text{ for all } z \in S \right\}.$$

Prove using the result in (a) or the result in (b) that

$$b \in C \quad \text{iff} \quad C^P \subseteq \{b\}^P.$$ [12]

3. Consider the LP :

$$\max c^T x \quad \text{subject to } Ax = b,\ x \geq 0$$

where A is an m x n matrix of full row rank. Let B be a feasible basis of columns of A and define Y and z_j's in the usual manner.

(a) Prove that if $z_j \geq c_j$ for all j then B is an optimal basis. [10]

(b) Given a j such that $z_j < c_j$ and no entry in the jth column of Y is positive, prove that the objective function has no upper bound. [10]

(c) Suppose $B^{-1}b \geq 0$. Given a j such that $z_j < c_j$ and the j^{th} column of Y has at least one positive element, show how to obtain another feasible basis D such that the objective function corresponding to D > the objective function corresponding to B. Give proof. [15]

(d) Given A, b show that there exists c such that every feasible basis is optimal. [3]

(e) Given A,b and a feasible basis B, show that there exists c such that B is the only optimal basis. [3]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83
Design of Experiments
PERIODICAL EXAMINATION

Date: 14.3.83. Maximum Marks : 100 Time: 3 hours.

Note : Answer any FOUR questions. Each question carries 21 marks. Submit your Practical Records alongwith the answer books. Marks allotted to Practicals are 16.

1. Explain the terms : (a) treatments, (b) plots, (c) experimental error, (d) local control. When would you recommend the use of a latin square design ? Give an example. Develop the analysis of variance of a latin square design. [4x2+2+11 = 21]

2. Let N(s) denote the maximum number of mutually orthogonal latin squares (MOLS) of order s. Show that $N(s) \leq s-1$. Describe a method of construction of a complete set of MOLS (proving that the method really works). Give an example with s=4. [5+12+4 = 21]

3. In the context of general analysis of a block design, prove the following results :

(a) The reduced normal equations for $\hat{\underline{\tau}}$ are $C\hat{\underline{\tau}} = \underline{Q}$.

(b) $\underline{l}'\underline{\tau}$ is estimable if and only if $\underline{l}$ lies in the column space of C.

(c) A connected block design is balanced if and only if C has all its off-diagonal elements equal. [10+4+7 =21]

4. Consider the following incomplete block design for 4 treatments in 10 blocks :

(1,2,3), (1,2), (1,3), (1,2,4), (1,4), (2,4), (1,3,4), (3,4), (2,3,4), (2,3).

(a) Is it a connected design ? Why ?

(b) Is it an orthogonal design ? Why ?

P.T.O.

(c) Is it a balanced design? Why?

(d) Solve $C\hat{\underline{\gamma}} = \underline{Q}$ for $\hat{\underline{\gamma}}$ and obtain an expression for $V(\underline{1}'\hat{\underline{\gamma}})$.

[3×4+5+4 = 21]

5. Define a balanced incomplete block design (BIBD) with an example. State and prove its parametric relations. Let D be a BIBD with parameters v, b, r, k, λ. Let D^* be a new design obtained from D by deleting a block of D. Is the resulting design D^* connected? Why? Is D^* balanced? Why?

[2+(2+2+4)+6+5 = 21]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1982-83

Difference and Differential Equations
PERIODICAL EXAMINATION

Date: 6.9.82. Maximum Marks : 100 Time: 3 hours

Note : Answer all the questions.

1.(a) Solve the following differential equation by finding an integrating factor

$$x\,dy + y\,dx + 3x^3y^4\,dy = 0$$ [8]

(b) Solve any three of the following equations

(i) $x^2 \frac{dy}{dx} = 3(x^2 + y^2)\tan^{-1}\frac{y}{x} + xy$

(ii) $(y + y\cos xy)\,dx + (x + x\cos xy)\,dy = 0$

(iii) $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$

(iv) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$ [3x8=24]

2.(a) For the differential equation

$$y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

show that the singular solution represents the envelope of the family of straight lines represented by the general solution.

(b) Find the curves for which the tangent at any point cuts off from the coordinate axes intercepts whose sum is constant.

(c) Show that the family of parabolas $y^2 = 4c(x + c)$ is self-orthogonal. [3x8=24]

3.(a) Find the Wronskian of the solutions y_1, y_2 of the equation

$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ (n a positive integer) which satisfy the initial conditions

$$y_1(0) = y_1'(0) = 2, \quad y_2(0) = 1, \quad y_2'(0) = -1.$$

P.T.O.

3.(b) Find the general solution of

$$x y'' - (2x + 1) y' + (x + 1) y = 0$$

using the fact that $y_1 = e^x$ is a solution.

(c) Find the Green's function for the linear differential operator

$$L = x^2 D^2 - 2xD + 2$$

(d) For the differential equation

$$(1 - x^2) y'' - 2xy' = 2x, \quad -1 < x < 1$$

verify that $y = c_1 + c_2 \log \frac{1+x}{1-x}$ is the general solution of the associated homogeneous equation, and then find a particular solution of the equation.

[4x7=28]

4. Find the general solution of each of the following linear differential equations (particular solution should be evaluated using Heaviside calculus).

(a) $(D^3 - 7D - 6) y = e^{2x} (1+x)$

(b) $(D^4 + 2D^2 + 1) y = x^2 \cos x$.

[2x8=16]

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B.Stat.(Hons.) Part III, 1982-83

Stochastic Processes-2

PERIODICAL EXAMINATION

Date: 20.9.82. Maximum Marks: 100 Time: 3 hours.

Note : Attempt all questions.

1.(a) When is a stochastic process called a Markov Jump Process?

(b) Show that in a Markov jump process

$$\lim_{t \to 0} p_{ij}(t) = 1 \quad \text{if } i = j$$
$$= 0 \quad \text{if } i \neq j$$

(c) Prove that in a Markov jump process on a finite state space $\left(p_{ij}(t)\right)$ is a nonsingular matrix for all $t > 0$.

(d) In a parking lot with N spaces cars come in according to a Poisson process with parameter λ whenever there is empty space. The occupancy time for each car has an exponential distribution. Formulate the process of number of cars in the parking lot as a Markov jump process(stating the necessary assumptions).

(6+4+6+8)=(24)

2.(a) Obtain Kolmogorov's backward differential equations for a Markov jump process.

(b) Let X(t) be a pure birth process on $\{0,1,2,\ldots\}$

with $\lambda_x = \lambda_1$ if x is odd
$= \lambda_2$ if x is even.

Let $P_1(t) = \Pr\left(X(t) = \text{odd}\right)$ and $P_2(t) = \Pr\left(X(t) = \text{even}\right)$.

Obtain differential equations for P_1 and P_2 and solve them.

(10+12)=(22)

3. Show that the differential equations for a pure birth process with parameters $\lambda_0, \lambda_1, \ldots$ define proper transition probabilities if $\Sigma \frac{1}{\lambda_n} = \infty$

(10)

P.T.O.

4. Let $X_1, X_2, \ldots$ be a Markov chain with the transition probability matrix Q. Let $N(t)$ be a Poisson process independent of $\{X_i.\}$

(a) Write down the transition probability matrix $(p_t(x,y))$ for the process $X_{N(t)}$ (no argument need be given).

(b) Show that $X_{N(t)}$ is a Markov jump process and find the infinitesimal parameter matrix.

(4+12)=(1

5.(a) Show that for an irreducible Markov jump process $\lim_{t \to \infty} p_t(x,y)$ exists (you do not have to prove that $p_t(x,$

is uniformly continuous).

(b) In an M/M/n Queue find the stationary distribution of the queue length.

(8+8)= (16)

6. In a GI/GI/1 queue find necessary and sufficient conditions that infinitely many customers are sure to find an empty line upon arrival. (12)

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III : 1982-83

Statistical Inference
PERIODICAL EXAMINATION

Date: 11.10.82.   Maximum Marks : 100   Time: 3 hours.

Note : This is an OPEN NOTE examination. You may carry your notes, but books will not be allowed.

The paper carries 130 marks. Answer as much as you can. Maximum you can score is 100.

1. Briefly comment on the following statements. Short answers will be appreciated.

(a) Likelihood principle is essentially a Bayesian principle.

(b) A Bayesian is nearly correct if the sample size is large.

(c) Neyman-Pearson theory cannot answer the following question: How confident you are that you have taken a correct decision ? [6x3=18]

2. Prove or disprove the following statements. If a statement is correct, just sketch its proof, and if it is incorrect, give a counter example and merely indicate why it is a counter example.

(a) If $X \sim Poi(\theta)$, $\theta > 0$, then the mle of θ is X.

(b) If $\psi(\theta)$ is uniformly bounded, $T(X)$ is unbiased for $\psi(\theta)$, then $T(X)$ is uniformly bounded.

(c) If X has a density of the form $f(x \mid \theta) = e^{\theta x} c(\theta)\, t(x)$, and $E_\theta(X)$ exists for all θ, then $E_\theta(X) = -\frac{c'(\theta)}{c(\theta)}$.

(d) If $\psi(\theta)$ is such that $\liminf_{\theta \to \infty} \psi(\theta) \geq 0$, $T(X)$ is the UMVUE of $\psi(\theta)$, then $\liminf_{X \to \infty} T(X) \geq 0$.

(e) If there exists a fixed-dimensional sufficient statistic independent of the sample size n, then the underlying distribution is in the exponential family.

P.T.O.

2.(f) If X is complete, and δ_1, δ_2 are two estimates of θ such that $E_\theta[\delta_1(X) - \theta]^2 = E_\theta[\delta_2(X) - \theta]^2$ for all θ, then $\delta_1 = \delta_2$ a.e.

(g) If $X \sim N(\theta, \theta^2)$, $0 < \theta < \infty$, then X is an admissible estimate of θ. [4x7 = 28]

3.(a) Let $X \sim N(\theta, 1)$, and it is suspected that θ is close to zero. Which of X and $\frac{X}{2}$ would you prefer as an estimate of θ?

(b) Formalize the following statement into a proposition and prove it :

The overall bias of a Bayes estimate is zero.

(c) Let $X \sim Bin(1, \theta)$, $0 < \theta < 1$. Let $\delta(x)$ be the Bayes estimate of θ against the prior with density $\pi(\theta) = \frac{4}{\pi(1+\theta}$ Show that there exists θ_0 such that $R(\theta_0, \delta(x)) \leq \frac{1}{4}$.

[6+7+8 = 2

4. Let $X \sim R(0, \theta)$, $\theta > 0$. Let θ have a prior with density

$$\pi(\theta) = \theta e^{-\theta}, \quad \theta > 0.$$

(a) Is it a conjugate prior?

(b) Find the Bayes estimate of θ. [5+7 = 12.

5. Let $X_1, X_2, \ldots, X_n$ be iid $R(\theta - \frac{1}{2}, \theta + \frac{1}{2})$.

(a) Show that $T(X_1, \ldots, X_n) = (X_{(1)}, X_{(n)})$ is sufficient for θ,

(b) Show that T is not complete.

(c) Show that θ is a location parameter for the distribution of T.

(d) Is $\bar{X}$ a reasonable estimate of θ?

(e) Is it admissible (proof not needed)?

(f) Find an admissible minimax estimate of θ,

[6+6+4+4+2+12=34]

co

6. (a) Define a randomized test.

(b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, 1)$. Write down exactly the UMP level α test for testing $H_0 : \theta \leq \theta_0$ vs. $H_1 : \theta > \theta_0$.

(c) Suppose in (b) θ is either 0 or 10. Let n = 10,000 and suppose you have data $\bar{X} = 0.02$. Do you think θ is 0 or 10?

(d) With $\alpha = 0.05$, use the MP level α test in (b) to test $H_0: \theta = 0$ vs. $H_1 : \theta = 10$. What is your conclusion? (The 95th percentile of N(0,1) distribution is 1.65).

(e) Comment on the conclusion in (d).

[3+3+3+5+3=17]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1982-83

Sample Surveys
PERIODICAL EXAMINATION

Date: 13.10.82. Maximum Marks : 100 Time: 3 hours.

Note : Marks allotted to each question are given in brackets.

1.(a) Explain what you understand by the terms inclusion probability of a unit U_i (π_i) and joint inclusion probability of a pair of units U_i , U_j (π_{ij}) for a sampling design. [4]

(b) Find the values of π_i and π_{ij} for a Simple Random Sampling (SRS) with replacement design of n draws. [2+3 = 5]

2.(a) A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probabilities that the sample contains 1,2,3 distinct units (for example, $U_1U_1U_1$, $U_1U_1U_2$, $U_1U_2U_3$, respectively) are $P_1 = 1/N^2$, $P_2 = 3(N-1)/N^2$, $P_3 = (N-1)(N-2)/N^2$. Suppose that $\bar{Y} = \Sigma Y_i/N$ is estimated by $\bar{y}'$ the unweighted mean over the distinct units in the sample. Show that the variance of $\bar{y}'$ is

$$V(\bar{y}') = (2N-1)(N-1)\, S^2/\, 6N^2.$$

Hence or otherwise show that $V(\bar{y}') < V(\bar{y})$, where $\bar{y}$ is the conventional estimator based on all the observations in the sample. [10+5= 15]

(b) A survey was conducted in a locality consisting of 625 households by covering a sample of 50 households by simple random sampling(srs) without replacement design to estimate the average weekly household expenditure on toilet goods. The estimate turned out to be Rs. 4.20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring locality on the basis of a sample selected by srs with replacement such that the length of the confidence interval at 95%

P.T.O.

2.(b) contd....

confidence level is 20% of the true value. (You may assume that the coefficients of variation (C.V.) are the same in the two localities). [10]

3.(a) What are the differences between a Linear and a Circular Systematic sample ? [5]

(b) Explain why the variance of an estimator of the population total based on a single systematic sample is not estimable? [5]

4.(a) What are the advantages of stratified sampling ? [5]

(b) A population of 112 villages is divided into 3 strata. Col.(2) in the table below gives the stratum sizes and Col.(3) gives the sample size. The sampling design used in the stratum is indicated in Col.(4). Col(5) gives the y-values of a study variate y and information on an auxiliary variate x where available. It is also known that the total of x-values for the third stratum is 3149.

Stratum (1)	Stratum size (2)	Sample size (3)	Sampling design (4)	y - values.
1	61	6	SRSWR	64, 78, 67, 58, 65, 63
2	29	3	Circular Systematic Sampling	2 independent samples: a. 424, 489, 459 b. 449, 487, 443
3	22	2	PPSWR	y-values : 324, 439 x-values (size) : 115, 212

(i) Estimate the population mean $\bar{Y}$. [10]

(ii) Estimate the sampling error of your estimate in(i). [12]

(iii) If the administrators are interested in obtaining an estimate of the proportion of units whose y-values are larger than 450, how do you obtain this estimate and its estimated sampling error ? [3+7 = 10]

(iv) Suppose that a surv y is planned next year in the same area for estimating the population mean. Suggest a suitable allocation of a total sample size of n=11. [5]

5. Evaluation of Practical work.

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III : 1982-83

Elective 4 : Physical and Earth Sciences
PERIODICAL EXAMINATION

Date: ?.11.82. Maximum Marks : 100 Time: $2\frac{1}{2}$ hours.

Note: All questions carry equal value.

1. Write short notes on (any three) :-
   (a) The generation of a magma
   (b) Magnetic reversal
   (c) Earthquake waves
   (d) Index fossil and fossil assemblage
   (e) Polymorphism in minerals.

2. Distinguish between (any three) :-
   (a) Polar wandering and Polar reversal
   (b) 'Sial' and 'Sima'
   (c) Igneous rock and ultrametamorphic rock
   (d) Mohorovicic discontinuity and Gutenburg discontinuity
   (e) Continental drift and Plate tectonics

3. Write short essays on (any two) :-
   (a) Geological time scale and rates of geological processes
   (b) Creation and destruction of the sea floor
   (c) Evolution of the atmosphere
   (d) Gravity studies of the earth.

4. Write a short essay on (any one) :-
   (a) The interior of the earth
   (b) The geological laws and geological methodology
   (c) Plate tectonics.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1982-83

Elective 4 : Economics
PERIODICAL EXAMINATION

Date: 8.11.82. Maximum Marks: 100 Time: 3 hours.

Note : Answer 4 questions attempting atleast one from each group. All questions carry equal marks.

GROUP A

1. Given a basic feasible solution to the L.P. problem max $Z = c'X$ subject to $AX = b$, $X \geq 0$ such that $z_j - c_j < 0$ for some column a_j in A, describe, using the simplex algebra, how you can generate a ~~new~~ basic feasible solution. Discuss, in this context, how and under what condition one would conclude that the solution is unbounded. (The symbols have their usual meanings).

2. Solve the following L.P. problem by the simplex method :

$$\max \quad Z = 3x_1 + 2x_2$$

$$\text{subject to} \quad 3x_1 + 2x_2 \leq 18$$

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$x_1 , x_2 \geq 0.$$

Is the solution unique ? If not, find another solution.

P. T. O.

GROUP B

3. Consider the following system of equations :

$$p_j = (1+r)\sum_{i=1}^{n} a_{ij}\, p_i + w\, a_{oj} \quad , \; j = 1, \ldots, n,$$

where :

a_{ij} = the $(i-j)^{th}$ element of an indecomposable input-output matrix A ;

a_{oj} = the number of labour hours necessary for producing one unit of the j^{th} commodity ;

p_j = the money price of the j^{th} commodity ;

w = the hourly money wage rate prevailing in all sectors

r = the (uniform) rate of profit for all sectors of production.

(a) Suppose that the composition of the basket of commodities consumed by the labourers is given by $\bar{c} = (\bar{c}_1, \ldots, \bar{c}_n)$ i.e. a labourer's consumption vector is always $\lambda\bar{c}$, where $\lambda \in R+$ is a variable to be chosen by the labourer. What interpretation should we give to $\dfrac{w}{\sum_{i=1}^{n} p_i \bar{c}_i}$, if labourers do not save any part of their income ?

(b) Show that a strictly inverse relationship exists between r and $\dfrac{w}{\sum_{i=1}^{n} p_i \bar{c}_i}$. (In your answer, you may assume, without proof, the result to be proved in Question 4(b). However you must prove any other result that you may require to establish the relationship).

(contd.....3)

4.(a) Consider an extremely simple society which produces only three commodities, X_1, X_2 and X_3. Suppose, moreover, that this economy is observed to be in a self-repeating state for which the input-output flows are given by :

$X_{11} = 0$; $X_{21} = 36$; $X_{31} = 426$; $X_1 = 184$

$X_{12} = 120$; $X_{22} = 0$; $X_{32} = 34$; $X_2 = 126$

$X_{13} = 64$; $X_{23} = 90$; $X_{33} = 0$; $X_3 = 460$

Would this society be capable of maintaining the above flows by working on a capitalist basis ?

(b) Can you establish a necessary and sufficient condition which guarantees that the system described in Question 3 admits a meaningful solution ?

GROUP C

5. (a) Discuss why the market prices are considered inadequate in Social Cost Benefit Analysis (SCBA).

(b) How are prices to be determined for SCBA under such conditions.

6. (a) Discuss how the rate of interest indicates time preference of individuals in a society.

(b) Explain why the social rate of interest differs from the market rate of interest in the discounting of values over time.

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B.Stat.(Hons.) III Year, 1982-83

Stochastic Processes-2
SEMESTRAL-I EXAMINATION

Date: 10.12.82. Maximum Marks: 100 Time: 3 hours.

Note : Answer all questions.

1.(a) Prove that in a Markov jump process the transition probability functions $p_{ij}(t)$ are uniformly continuous functions of t on $[0,\infty]$ for each i and j.

(b) A telephone exchange has m channels. Calls arrive in the pattern of Poisson process with parameter λ ; they are accepted if there is an empty channel, otherwise they are lost. The duration of each call is a random variable whose distribution function is exponential with parameter μ. With standard assumptions find the stationary distribution of the no. of busy channels.

(c) Let $\{X_t , t \geq 0\}$ be a Markov jump process and $\{\tau_n\}$ the transition times. Denote by $p_{ij}(t)$ the transition probabilities for the X_t process and by $q_{ij}^{(n)}$ the transition probabilities for the embedded profess $\{X_{\tau_n}\}$. Is it necessary that

$$\lim_{n\to\infty} q_{ij}^{(n)} = \lim_{t\to\infty} p_{ij}(t) ?$$

[8+12+7=27]

2.(a) Define the virtual waiting time in a queue. Give (not necessarily regorous) arguments to show that, in an M/GI/1 queueing process, the virtual waiting times constitute a Markov process.

(b) Suppose that in an M/GI/∞ queuing process the parameter for the arrival times is λ and the service time distribution is given by the distribution function H. If there are initially no customers present, find

(i) $P_k(t)$ = Prob. (at time t there are exactly k customers being served).

2.(i)(ii) $\lim_{t \to \infty} P_k(t) = \pi_k$. [8+15 = 23].

3.(a) Let $\{X_t\}$ be a process where each X_t has zero mean and finite variance. Define

$$\int_a^b X_t \, dt$$

(b) Show that, if $\{X_t\}$ is as above,

$$\int_a^b X_t \, dt$$

exists if $\int_a^b \int_a^b r(t,s) \, dt \, ds$ exists

where $r(t,s) = \mathrm{cov}(X_t, X_s)$, $a \leq t$, $s \leq b$.

(c) Give a reasonable definition of

$$\int_a^b X_t \, dt$$

where, now, each X_t has finite variance but not necessarily zero mean. Decide if the formula

$$E\left(\int_a^b X_t \, dt\right) = \int_a^b E(X_t) \, dt$$

holds with your definition. [5+10+10=25]

4. Consider the stochastic differential equation

$$X''(t) + X'(t) + X(t) = W'(t)$$

(a) Write down the general solution of this equation on an interval $[t_0, t]$ (explaining your notation - but without proof).

(b) Consider the solution $X(t)$ of the above equation on $[t_0, \infty)$ which satisfies $X(t_0) = 0$ and $X'(t_0) = 0$. For $t_0 < t_1 < t_2$ find the optimal linear predictor for X_{t_2} based on $\{X_t, t_0 \leq t \leq t_1\}$.

(c) Show that the equation above has a solution on $\mathbb{R}$ which is weakly stationary.

[5+12+8=25].

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B.Stat.(Hons.) III Year, 1982-83

Statistical Inference
SEMESTRAL-I EXAMINATION

Date: 15.12.82. Maximum Marks: 100 Time: 3 hours.

Note : The paper carries 131 marks. Answer as much as you can. Maximum you can score is 100.

1. All answers must be brief and precise.
   (a) What is meant by an estimate being unbiased? Give an example of a UMVUE which is inadmissible. Proof not needed.
   (b) What makes the use of conjugate priors in a Bayesian analysis appealing?
   (c) Give an example which exposes the danger of blindly using the minimax principle. Only main features are to be mentioned, but no proofs.
   (d) What makes the MLR family the natural family to be studied in one-sided testing problems?
   (e) State in one sentence the moral of the Neyman-Pearson lemma. [5+4+5+3+3=20]

2. (a) State and prove the Cramer-Rao inequality. Carefully state the assumptions needed and indicate where exactly in the proof each assumption is used.
   (b) Hence or otherwise, show that if $X \sim N(\theta,1)$, $-\infty < \theta < \infty$, then X is a minimax estimate of θ. What other properties of X do you know? [10+12=22]

3.(a) Define a MLE.
   (b) If $X_1, X_2, \ldots, X_n$ are iid $N(\theta, \theta^2)$, $\theta > 0$, find the MLE of θ. [3+10=13]

4. Let $X \sim Bin(6,p)$, $0 < p < 1$. For testing $H_0 : p \leq \frac{1}{2}$ vs. $H_1 : p > \frac{1}{2}$, write down explicitly the UMP test of level $\alpha = 0.04$ and find the power of this test against the alternatives p = 0.6 and 0.9. Deriving the form of the test is not necessary. [12]

P.T.O.

5.(a) Explain briefly how the problem of testing a composite null hypothesis can often be reduced to that of testing a simple null hypothesis by taking a "least favourable" mixture of the null distributions.

(b) Prove a relevant theorem in this context.

(c) If $X_1, \ldots, X_m$ is a random sample from the $N(0, \sigma^2)$ distribution and $Y_1, \ldots, Y_n$ is an independent random sample from the $N(0, \tau^2)$ distribution, find the UMP level α test of $H_0 : \sigma^2 \leq \tau^2$ Vs. $H_1 : \sigma^2 > \tau^2$.

[5+10+10=2_]

6.(a) Define a uniformly most accurate family of confidence sets for a parameter θ.

(b) Explain how such a family of confidence sets can be constructed from the UMP tests of appropriate hypotheses, by proving a relevant theorem.

(c) If $X_1, X_2, \ldots, X_n$ are iid $R(0,\theta)$, $\theta > 0$, obtain a UMA family of confidence sets for θ of confidence $1-\alpha$, $0 < \alpha < 1$.

[3+8+6=17]

7.(a) Define a HPD Bayesian credible region of confidence $1-\alpha$.

(b) Explain why this makes a reasonable definition and how a Bayesian credible region differs from a usual Neyman-Pearson confidence set in interpretation.

(c) If x_0 is an observation from the $N(\theta, 1)$ distribution and θ has a conjugate $N(\mu, \tau^2)$ distribution, obtain a 95% Bayesian credible region for θ,

(d) If data $x_1, x_2, \ldots, x_n$ are available from distributions $N(\theta_1, 1), N(\theta_2, 1), \ldots, N(\theta_n, 1)$ respectively, where θ_i's are believed to have been generated by the same $N(\mu, \tau^2)$ distribution, μ, τ^2 not completely known, find an empirical Bayes estimate of θ. Why would you call it empirical Bayes?

[3+5+7+7=22]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Sample Surveys
SEMESTRAL-I EXAMINATION

Date: 20.12.82. Maximum Marks : 100 Time: 3 hours

Note : The paper is divided into Group A and Group B. You may answer any part of any question from Group A. However, the maximum you can score is 40. The maximum marks for Group B is 60. Marks allotted to each question are given in brackets [ ].

GROUP: A Maximum marks 40

1.(a) Define the terms 'Sampling Design' and 'Unbiased Estimator'. [4]

(b) Let $N = 3$ and $n = 2$ and $s_1 = \{U_1, U_2\}$, $s_2 = \{U_1, U_3\}$, $s_3 = \{U_2, U_3\}$. Under the Simple Random Sampling design let $p(s_i) = 1/3$ for $i = 1,2,3$. Define the estimator t by

$$t = \begin{cases} t_1 = y_1/2 + y_2/2 & \text{if } s_1 \text{ occurs} \\ t_2 = y_1/2 + 2y_3/3 & \text{if } s_2 \text{ occurs} \\ t_3 = y_2/2 + y_3/3 & \text{if } s_3 \text{ occurs.} \end{cases}$$

Show that t is unbiased for $\bar{Y}$ and that there exist values (Y_1, Y_2, Y_3) for which $V(t) < V(\bar{y})$, where $\bar{y}$ denotes the sample mean. What does this example show? [3+6+3=12]

(a) How do you select a linear systematic sample of size n from a population of size N ? [5]

(b) Suggest an unbiased estimator for the population total Y of a characteristic y based on the above design. [4]

(c) When the values of the y-characteristic are known to be of the form $Y_i = \alpha+\beta i$ and when the population size is a multiple of the sample size, would you prefer systematic sampling to simple random sampling ? Give reasons. [9]

P.T.O.

3.(a) Explain what you understand by 'Combined and Separate Ratio Estimators' in stratified random sampling. Indicate the situations when you use these estimators, deriving the necessary formulae.

[You may assume, $B\left(\frac{\hat{Y}}{\hat{X}}\right) = [\,R V(\hat{X}) - \text{Cov.}(\hat{Y}, \hat{X})\,]/X^2$ and

$M\left(\frac{\hat{Y}}{\hat{X}}\right) = [\,V(\hat{Y}) - 2R\,\text{Cov.}(\hat{Y}, \hat{X}) + R^2 V(\hat{X})\,]/X^2$ [5+9] = [14]

with the usual notations.]

(b) The variate y is uniformly distributed in the range (a, a+h). The range is cut in k equal parts to make k strata of equal size. From each stratum, a simple random sample (with replacement) of n/k units is taken. Compare the variance of the estimator of the population mean based on this stratified sample with that of an unstratified sample of size n. Show that the variance in the first case falls off inversely to the square of the number of strata. [4+2=6]

GROUP : B Maximum Marks: 60.

4.(a) A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as first units and plots within them as second stage units. From each stratum 4 villages were selected with probabilities proportional to area and with replacement and 4 plots were selected from each selected village with equal probability and without replacement. The data on yield for the sample plots together with information on selection probabilities are given in the table below:

Stratum	Sample village	Inverse of probability of selection	Total no. of plots	Yield of sample plots 1	2	3	4
I	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	133
	3	24.76	222	264	78	144	55
	4	49.99	69	300	114	68	111
III	1	67.63	189	110	281	120	114
	2	339.14	42	80	61	118	124
	3	100.00	134	121	212	174	106
	4	68.07	161	243	116	314	129

P.T.

Total number of plots in stratum I : 8423
Total number of plots in stratum II : 6355
Total number of plots in stratum III :12853

Using the above data

(i) Obtain an unbiased estimate of the total yield of paddy in the district. [15]

(ii) Obtain an unbiased estimate of the variance of the above estimate. [20]

(iii) What are the possible sources of non-sampling errors in the above study? [8]

(b) An experienced farmer makes an eye estimate of the weight of peaches x_i on each tree in an orchard of $N = 200$ trees. He finds a total weight of $X = 11,600$ pounds. The peaches are picked and weighed on a simple random sample (without replacement) of 10 trees and the following results are obtained:

	Tree Number									
	1	2	3	4	5	6	7	8	9	10
Actual weight y_i	61	42	50	58	67	45	39	57	71	53
Estimated weight x_i	59	47	52	60	67	48	44	58	76	58

(i) Find the regression estimate $\hat{Y}$ of Y, the total weight of peaches in the population and estimate its sampling error. [10]

(ii) As an alternative, if $\hat{Y}' = N\left\{\bar{X} + (\bar{y} - \bar{x})\right\}$ is used, obtain the gain in precision, if any of $\hat{Y}$ over $\hat{Y}'$.

[7]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III year

1982-83/55

Elective 4: Physical and Earth Sciences

SEMESTRAL-I EXAMINATION

Date: 24.12.82 Maximum marks: 100 Time: 3 hours.

Note: Answer any five questions. All questions carry equal marks.

1. What is weathering? How does it differ from erosion? Describe how different kinds of mechanical weathering can take place in nature. 4+4+12=20

2. What is meant by texture of a sedimentary rock? Discuss how grain size analysis of clastic rocks are utilised in understanding transportational processes. 5+15 =20

3. What is cross-bedding? Why dip direction of foresets in cross-beddings generated by migration of bed forms indicate down current direction? Determine the azimuth of the resultant vector, its magnitude and consistency ratio from the cross-bedding azimuth data given below:

$3^\circ, 7^\circ, 5^\circ, 19^\circ, 12^\circ, 33^\circ, 26^\circ, 38^\circ, 22^\circ, 29^\circ, 24^\circ, 44^\circ,$

$58^\circ, 48^\circ, 51^\circ, 64^\circ, 79^\circ, 75^\circ, 70^\circ, 85^\circ, 89^\circ, 105^\circ, 358^\circ,$

318° 3+2+15=20

4. What is a sandstone? Discuss how the mineralogical property of a sandstone reflects the tectonic control. 4+16=20

5. What is diagenesis? Describe the diagenetic processes responsible for conversion of carbonate sediments into limestone. 4+16=20

6. Enumerate briefly the salient features of meandering river deposits. 20

Contd. ...2/

7. What is turbidity current? What is a turbidite? State the characteristic features of turbidite. 5+5+10=20

8. Write notes on any two of the following:
(i) Hjulstrom's diagram (ii) Walthers Law
(iii) Unconformity (iv) Tillite (v) Flysch
(vi) Loess 10x2=20

9. Discuss with the help of sketches the distinction between (any two) 10x2=20

(i) turbulent flow and laminar flow
(ii) sphericity and roundness of grains
(iii) flute casts and load casts
(iv) Mechanically deposited sediment and chemically precipitated sediment.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III year

1982-83/35

Elective 4: Economics
SEMESTRAL-I EXAMINATION

Date: 24.12.82 Maximum marks: 100 Time: 3 hours.

Note: Answer four questions, attempting atleast one question from each group. All questions carry equal marks.

Group A.

1.(a) Bring out clearly the distinction between primary inputs and produced inputs in the context of input-output analysis. $(12\frac{1}{2})$

(b) Given a viable (nxn) input-output matrix A and the corresponding (1xn) primary input coefficient vector a_0, discuss the economic meaning of the series

$$\underset{t\to\infty}{\mathrm{Lt}}\ (a_0Ac + a_0A^2c + \dots + a_0A^tc)$$

where, c is an exogenously specified final use vector realized by the given input-output system. $(12\frac{1}{2})$

2. State, prove and interpret the nonsubstitution theorem of input-output analysis. (25)

Group B

3. State and prove the complementary slackness theorem. Also give an economic interpretation of the theorem. (25)

4.(a) Write out the dual of the following linear programming problem

$$\text{Minimise} \quad Z = x_1+x_2+x_3$$
$$\text{subject to} \quad x_1-3x_2+4x_3=5$$
$$x_1-2x_2 \leq 3$$
$$2x_2-x_3 \geq 4$$

$x_1\geq 0$, $x_2\geq 0$ but x_3 is unrestricted in sign. (10)

(b) Find the optimal solution of the dual of the following linear programming problem by solving the primal by the simplex method

$$\text{Maximise} \quad Z = 2x_1 + 3x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \leq 80$$
$$x_1 + x_2 \leq 50$$
$$x_1 + 2x_2 \leq 80$$
$$x_1\geq 0,\ x_2\geq 0$$

(15)

p.t.o.

5. Discuss the approach taken in Social Cost Benefit Analysis to the pricing of the product of a project. Discuss the reasons for which the approach to the same is different in commercial accounting. (25)

6. A project for housing is to be taken up by the government. Discuss the approach you should adopt in evaluating the benefits from the project. (25)

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B.Stat.(Hons.) III Year, 1982-83

Difference and Differential Equations
SEMESTRAL-I EXAMINATION

Date: 17.1.83. Maximum Marks: 100 Time: 3 hours

Note : Answer all the questions. Figures in the margin indicate marks.

1. Solve any three of the following finite difference equations.

(a) $3y_{n+2} - 7y_{n+1} - 6y_n = 0$ subject to $y_0 = 0,\ y_1 = 1.$

(b) $y_{n+3} - 3y_{n+1} + 2y_n = 3.4^n$

(c) $y_{n+2} + y_{n+1} - 56y_n = 2^n(n^2-3)$

(d) $y_{n+3}\, y_n^{60} = y_{n+2}^4 + y_{n+1}^{17}$. [3x10 =30]

2.(a) Use the convolution formula to find the inverse Laplace transform of the function

$$\frac{3s^2}{(s^2+1)^2}$$

(b) Use Laplace transform to solve the following Volterra equation

$$\varphi(t) + \int_0^t e^{-(t-\xi)}\,\varphi(\xi)d\xi = \cos t.$$

[10+10=20]

3.(a) Prove that

$$\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$$

(b) Find two linearly independent solutions about $x = 0$ of the following equation

$$xy'' - (x+3)y' + 2y = 0$$

[8+14=22]

P.T.O.

;.(a) Obtain the general solution of any two of the following partial differential equations

(i) $p - 3q = \sin x + \cos y$

(ii) $x^2 p + y^2 q = axy$

(iii) $x(y-z)p + y(z-x)q = z(x-y)$

(b) Use D'Alembert's method to construct the solution of the initial - value problem for the homogeneous wave equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}, \quad -\infty < x < \infty, \quad t \geq 0$$

$$f(x,0) = A \sin a x, \quad \frac{\partial f}{\partial t}(x,0) = B \cos b x, \quad -\infty < x < \infty$$

where a, b, A, B are constants.

[2x10+8=...

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III, 1982-83

Stochastic Processes-2
BACKPAPER EXAMINATION

Date: 31.1.83. Maximum Marks : 100 Time: 3 hours.

Note : Answer all questions.

1.(a) Define what you mean by a Markov jump process.

(b) Obtain Kolmogorov's backward equations for a Markov jump process. [10+15=25]

2.(a) Consider an M/GI/1 queue. Find the expected queue length (at the times of departure of customers) under the assumption that the queing process has attained a stationary state.

(b) Consider an M/M/1 system with the queue discipline of last come first served type. Let $X(t)$ be the queue size at time t. Show that the process $\{X_t, t \geq 0\}$ is a birth and death process and determine its parameters. [12+13=25]

3.(a) Let $\{X_n\}$ be a sequence of square integrable random variables with zero mean. Show that $\{X_n\}$ is Cauchy if

$$\text{Cov}(X_n, X_m) \longrightarrow \rho$$

as n, m $\longrightarrow \infty$ where ρ is a real number.

(b) Let X_t be a Gaussian process, continuous in quadratic mean where each X_t has zero mean. Show that

$$\int_a^b X_t \, dt$$

is normally distributed. [8+12=20]

4. Consider the stochastic differential equation $a_1 X'(t) + a_0 X(t) = W'(t)$.

(a) What do you mean by saying that $X(t)$ is a solution of the above equation on an interval $[c,d]$?

P.T.O.

4.(b) Consider the solution of this equation on [0,1] for which $X(0)=0$. Is this solution differentiable (in q.m.) on [0,1]?

(c) Decide when the above equation has a solution on $\mathbb{R}$ which is weakly stationary.

(d) Show that this equation can have at most one weakly stationary solution on $\mathbb{R}$.

[6+6+10+8=30]

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Elective-5 : Economics
SEMESTRAL-II EXAMINATION

Date: 13.5.83. Maximum Marks : 100 Time: 3 hours.

Note : Answer all questions. Marks for each question are given in brackets.

1.(a) What is Lorenz curve ? Obtain the expressions for Lorenz curve and Lorenz ratio of a random variable having a two-parameter Lognormal distribution. Indicate the properties of Lorenz curve in this case. (5+10+5)=[20]

(b) Consider the following size distribution of per capita total consumer expenditure (x) for urban India :

x-class (Rs.)	0 - 11	11 - 15	15 - 21	21 - 28	28 - 43	43 -
per cent population	15.80	16.40	30.80	17.40	16.70	13.70

From this data, obtain the estimate of the parameter μ of the Lognormal distribution by method of quantiles. [16]

2.(a) Discuss briefly the various criteria for choosing a suitable form of engel curve for a given set of cross-section data on consumer expenditure. [16]

(b) The following quantities were computed from a grouped cross-section data for estimating engel elasticity for sugar for urban consumers :

$\Sigma\, p_j(\log \bar{y}_j)^2 = 1.4864$ $\quad \Sigma\, p_j \log \bar{y}_j = -0.7937$

$\Sigma\, p_j \log \bar{x}_j = 2.9695$ $\quad \Sigma\, p_j(\log \bar{x}_j)^2 = 9.1903$

$$\Sigma\, p_j \log \bar{x}_j \log \bar{y}_j = -1.8007$$

where $\bar{x}_j$: average per capita total expenditure for the j^{th} per capita total expenditure class ;

$\bar{y}_j$: average per capita expenditure on sugar for the j^{th} class ;

p_j : estimated proportion of persons in the j^{th} class ;

p.t.o.

2.(b) contd...

there being 12 expenditure classes ($j = 1,2,12$) and all logarithmic values are based on natural logarithm.

(i) Obtain the estimate of the engel elasticity for sugar. What is the standard error of this estimate ?

(ii) What can you say about the goodness of fit of the fitted engel curve ?

[16].

3. Write notes on the followings :

(a) Pareto's Law of Income Distribution ;

(b) Problems of estimation of demand function for an item from time series data. [10x2=20]

4. Practical records. [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Elective-5 : Physical and Earth Sciences
SEMESTRAL-II EXAMINATION

Date: 13.5.83. Maximum Marks: 100 Time: 3 hours.

Note : Answer any *five* questions. All questions carry equal marks.

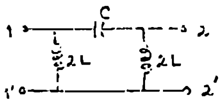
1. Define 'image impedances' and 'characteristic impedance' as applied to a two-terminal pair network.

   Find the expressions for image impedances of a two-terminal pair network in terms of A,B,C,D parameters.

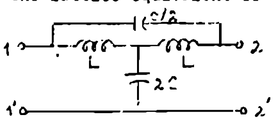
   Discuss the different possible techniques of evaluating the characteristic impedance of a two-terminal pair network.

   Find the characteristic impedance of the following network.

   [4+8+4+4=20]



2. State Bartlett's bisection theorem.
   Determine the lattice equivalent of the following network.



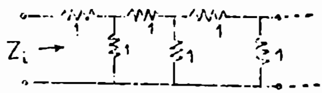
   Given any lattice network with series and shunt arm impedances Z_a and Z_b respectively, find its equivalent T-section network and comment on its physical realizability.
   Is the equivalent T-configuration of the above circuit physically realizable with R,L and C elements at $\omega = \frac{1}{\sqrt{LC}}$?

   [4+6+6+4=20]

p.t.o.

3.(a) What do you mean by Q-factor of a coil ? A series resonant circuit consists of a capacitor C and a coil having an inductance L and resistance R. Find an expression for the current flowing through the circuit at the resonant frequency. Is the circuit capacitive or inductive at a frequency ω less than the resonant frequency ? Derive an expression for the 3-db bandwidth of the circuit in terms of the Q-factor of the coil.

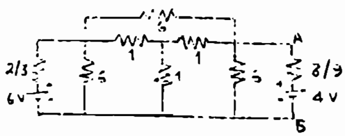
(b) Find the input impedance Z_i of the following network. All impedance values are in ohms.



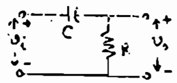
⟶ to infinity

[4+4+2+4+6=20]

4.(a) Find the current flowing through the branch AB in the following circuit. All resistance values are in ohms.



(b) A ramp voltage $v_i(t) = \alpha t$ is applied to the following circuit, α being a constant. Find the output voltage v_o at i) very small t (t < < RC) and ii) very large t (t > > RC).



5. Find an expression for the width of an unbiased abrupt p-n junction at 300°k given that the doping concentrations in p - and n - regions are 2×10^{14}/c.c. and 1.125×10^{15}/c.c. respectively, the electron density in the intrinsic specimen is 1.5×10^{10}/c.c. at 300°k, dielectric constant of the semiconducting material is 16. Derive the necessary working formula.

[6+14=20]

(contd......3)

6.(a) Draw the circuit diagram of a full-wave bridge rectifier.

Calculate the d-c output voltage and the ripple factor of the output waveform when a capacitor filter is used with a full-wave rectifier, after critically explaining the filtering performance. Define percentage regulation of a power supply. [6+12+2=20]

7.(a) A transistor has a common base current gain $\alpha = 0.98$ and leakage current $I_{CBO} = 10\mu A$. What will be its common emitter current gain β and leakage current I_{CEO} ? Derive the necessary working formulae.

(b) The reverse saturation current of a p-n junction rectifier is $10\mu A$ at $300^{\circ}k$. Find out the value of the current through it at $300^{\circ}k$ when the junction is forward biased with 0.258 volt.

(c) Show how a positive logic AND gate can be realized using diodes. Assume that logic 1 = +5 volts and logic 0 = 0 volt.

[10+5+5=20]

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Optimisation Techniques
SEMESTRAL-II EXAMINATION

Date: 16.5.83. Maximum Marks: 100 Time: 3 hours.

Note: Answer all questions. The whole paper carries 108 marks but the maximum you can score is 100.

1. Given a problem

$$\max c^T x \text{ subject to } Ax=b,\ x \geq 0,$$

explain how you would solve it. It is not even known whether $Ax=b$ is consistent. Give the complete procedure without proofs. [15]

2.(a) Let A be totally unimodular and b be integral. If the objective function of the LP

$$\max c^T x \text{ subject to } Ax=b,\ x \geq 0$$

has an upper bound (the problem is assumed to be feasible), prove that there exists an integral optimal solution. [10]

(b) Formulate the maximum flow problem in a network as an LP in the form considered in (a) and prove that the matrix is totally unimodular. Write the dual LP in the same form and hence show that it (the dual) has an integral optimal solution. [5+10+ 10]

(c) Assuming the max-flow min-cut theorem, show that the weak duality theorem holds for the LP in (b). [5]

(d) An $r \times n$ Latin rectangle is an $r \times n$ matrix $A = ((a_{ij}))$ with entries from $\{1,2,\ldots,n\}$ such that $a_{ij} \neq a_{ik}$ whenever $j \neq k$ and $a_{ij} \neq a_{hj}$ whenever $i \neq h$. Given an $r \times n$ Latin rectangle with $r < n$, we want to add a row to get an $(r+1)\times n$ Latin rectangle. Formulate this as a problem of finding an SDR for a suitable family of sets. Explain how you would use networks (give the complete procedure without proof) to find such an SDR. [6+15]

p.t.o.

3.(a) State and prove von Neumann's minimax theorem. [12]

(b) A matrix game is said to be completely mixed w.r.t. row (resp. column) player if every optimal strategy for the row (resp.column) player uses every row (resp.column) with positive probability. Show that a 2 x 2 matrix game is completely mixed w.r.t. the row player iff it is completely mixed w.r.t. the column player. [8]

(c) Suppose a 2 x 2 matrix game with pay-off matrix A is completely mixed w.r.t. either player. Let x_0 be an optimal strategy for the row player. Prove that

$x_0^T A = w \underset{\sim}{1}^T$ where w is the value of the game and deduce that the row player has a unique optimum strategy.

[6+6]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Nonparametric and Sequential Methods
SEMESTRAL-II EXAMINATION

Date: 20.5.83. Maximum Marks : 100 Time: 3 hours

Note: Answer all questions.

1. In each of the following problems
   (a) write down the appropriate null and alternative hypotheses and
   (b) indicate what would be your test statistic and which values of it are significant.

   (i) A psychologist wanted to known whether the mother's judgment about the personality of the eldest son is better than the father's. He interviewed husbands and wives of ten families which he knew well and which had at least one son. The following data gives the ratings given by the psychologist Higher rating indicates better judgement.

Family

	1	2	3	4	5	6	7	8	9	10
Husband	6	5	3	5	1	7	4	8	2	3
Wife	4	8	4	6	5	3	7	6	7	8

   (ii) An IQ test was given to each member of ten couples by a psychologist who wanted to know whother people choose spouses having similar IQ as their own. Below are the test scores.

Couple

	1	2	3	4	5	6	7	8	9	10
Husband	120	125	140	120	118	144	110	144	130	120
Wife	138	110	142	128	122	152	121	138	110	141

p.t.o.

1.(iii) A behavioral scientist investigating whether women are faster than men in learning skilful jobs conducted an experiment on nine couples. The following data gives the timings (in minutes) for learning a particular skilful job.

	1	2	3	4	5	6	7	8	9
Husband	20	25	28	24	22	21	18	25	22
Wife	22	23	16	18	20	25	24	20	24.

[4+4+4=12]

2. Carry out analysis for any one of the problems of question 1. (Take α = .05 and do not use normal approximations). [8]

3. If F_0 is a continuous d.f. and if D_n is the Kolmogorov statistic based on a sample of size n for testing $H : F = F_0$ VS. $H_1 : F \neq F_0$, show that

$$D_n = \max_{1 \leq i \leq n} \operatorname{Max} \left\{ \frac{i}{n} - F_0 (X_{(i)}), F_0 (X_{(i)}) - \frac{(i-1)}{n} \right\}$$ [4]

4. With notations as in class notes show that if F is continuous so that ties may be ignored, then

$$W_s = \sum_{1 \leq i} \sum_{\leq j \leq n} I [z_i + z_j > 0]$$

Hence or otherwise get $E_{H_1}(W_s)$. [4+1 = 5]

5. Obtain the distribution of the Kolmogorov-Smirnov statistic $D_{3,1}$ (in notations of class notes) under the null hypothesis that both the samples come from the same continuous d.f.

Compute the power of the test with critical region $D_{3,1} \geq 3/4$ at the following alternative

$H_1 : X \sim F , Y \sim F^3$, F has density f.

[8+7=15]

6. (a) State the optimum property of an SPRT.
(b) Do you know of any sequential test which minimises the random sample size among all tests with same or lower error probabilities ? Is it an SPRT ?

6.(c) Define a GSPRT. Give sufficient conditions for a GSPRT to have monotone OC function (NO proof needed).

[3+5+4 =12]

7. In the secretary problem consider the rule : Pass the first $[\frac{N}{2}]$ candidates and then choose the first candidate with relative rank one, (when there is only the last candidate you must choose him.) Write down probability of selecting the best candidate and find its limit as $N \longrightarrow \infty$. Compare this limit with the limiting probability for the optimal rule which (in the limit) passes by $N e^{-1}$ candidates and then picks the candidate with relative rank one

or

Let Xi's be i.i.d $N(\mu, \sigma^2)$. Let $\phi\ (X_1, \ldots, X_n)$ be a test of $H_0 : \mu = 0$ Vs. $H_1 : \mu > 0$, based on n observations ; let $E_{0,\sigma}(\phi) = \alpha\ \forall \sigma$. Prove that $\inf_{\sigma} E_{\mu,\sigma}(\phi) = \alpha\ \forall\ \mu \neq 0$.

[If you use any result proved in class, give a proof.] [10]

8. A municipality has to provide light daily for ten hours at 400 sites. The life (in hours) of a bulb may be assumed to have exponential distribution $\theta e^{-\theta x}$ with $\theta^{-1} = 120$. Determine N such that the municipality will not need more than N bulbs with probability approximately equal to .95.

[Hint : Use Wald's first and second equation and the central limit theorem]. [24]

9. Assignments. [10]

INDIAN STATISTICAL INSTITUTE
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Design of Experiments
SEMESTRAL-II EXAMINATION

Date: 23.5.83. Maximum Marks : 100 Time: 3 hours.

Note : Answer any FOUR questions. Each question carries 21 marks. Submit your Practical Records alongwith the answer books. Marks allotted to Practicals are 16.

1. Let N be the incidence matrix of a block design D. Define the dual design D^* of D which has the incidence matrix $N^* = N'$ (transpose of N). Prove that D^* is connected if and only if D is so (proving the intermediate results used). Show also that if D is a balanced incomplete block design (BIBD) with parameters (v,b,r,k,λ) then D^* is also a BIBD if and only if $v = b$. [10+11=21]

2. In an experiment with a randomized block design for v treatments in v + t blocks , v observations got lost affecting all the v treatments with a block having at most one observation missing. Is the resulting design

   (a) connected ? why ?
   (b) orthogonal ? why ?
   (c) balanced ? why ?

   Solve for $C\hat{\underline{\tau}} = \underline{Q}$ and obtain an expression for $V(\underline{l}'\hat{\underline{\tau}})$ where $\underline{l}'\hat{\underline{\tau}}$ is the B.L.U.E. of an estimable treatment contrast $\underline{l}'\underline{\tau}$.

   [3x4+(6+3)=21]

3.(a) Describe a method of construction (proving that the method works in general) of the following series of BIBD :

   $v = b = s^2+s+1,\ r = k = s^2,\ \lambda = s(s-1)$, where s is a prime or power of a prime.

   (b) Prove the non-existence of the following BIBD's :

   (i) $v = b = 22,\quad r = k = 7,\quad \lambda = 2$;
   (ii) $v = b = 67,\quad r = k = 12,\quad \lambda = 2$.

   [15+2x3=21]

p.t.o.

4. Derive the expressions for (i) '' estimates'' of missing values used in the missing plot technique, and (ii) variance of the best estimator of estimable function based on the incomplete data. Suppose in a randomized block design two observations are lost in the same block ($v > 2$) affecting the treatments i and i' ($i \neq i'$). Obtain the '' estimates'' of the missing values, and the variances of the estimated elementary treatment contrasts based on the incomplete data.

[(7+5)+(4+5)=21]

5.(a) Describe with an illustration the method of construction of a replication of a confounded design for 2^n factorial experiment in blocks of size 2^k. Indicate the analysis of variance of the above design in r randomized blocks.

(b) Let A,B,C,D,E,F and G stand for seven factors of 2^7 experiment. The following treatment combinations form a block of a replication of a confounded design for this experiment: (bcdefg, abcd, cdg, bdf, bce, acdef, abdeg, abcfg, de, cf, bg, adfg, aceg, abef, a, efg). Determine all the confounded effects in this replication.

[(8+7)+6=21]

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Multivariate Distributions and Tests
SEMESTRAL-II EXAMINATION

Date: 27.5.83. Maximum Marks : 100 Time: 3 hours.

1. Let $\bar{X}$ and S be respectively the mean vector and covariance matrix formed from a random sample of size n+1 from a non-singular $N_p(\mu,\Sigma)$. Find the joint distribution of

$$\bar{X} \quad \text{and} \quad \frac{n\bar{X}'S\bar{X}}{\bar{X}'\Sigma\bar{X}}.$$ [10]

2. Show that based on a random sample from a nonsingular $N_p(\mu,\Sigma)$, the sample mean vector and sample dispersion matrix are jointly sufficient for μ,Σ. [10]

3. Consider two populations $N_p(\mu, \sigma_1^2 I)$ and $N_p(\mu, \sigma_2^2 I)$. Find the Union-Intersection criterion for testing the equality of the two distributions. Further, assuming the two types of losses to be the same, find the discriminant function between the two populations. [25]

4. Consider the linear model

$$\underset{n\times 1}{E(Y)} = \underset{n\times m}{X}\ \underset{m\times 1}{\beta}$$

where $D(Y)=\Sigma$ (a positive definite matrix).
Show that a value of β minimising

$$(Y-X\beta)'\Sigma^{-1}(Y-X\beta)$$

is $\hat{\beta} = (X'\Sigma^{-1}X)^{-}X'\Sigma^{-1}Y$. [15]

p.t.o.

5. A researcher at a school for the deaf gave several motor tests to the resident students, and also tested a group of hearing children, paired child for child on the basis of sex, age and height with the deaf children. Scores for 10 deaf and their hearing counterparts on a test of grip (X_1) and a test of balance (X_2) are shown below. Examine if the deaf and hearing children are different in grip and balance.

Pair No.	X_1		X_2	
	Deaf	Hearing	Deaf	Hearing
1	25	26	2.0	2.3
2	22	22	2.0	1.0
3	28	27	2.7	3.7
4	35	39	2.7	3.3
5	37	34	3.0	10.0
6	48	51	1.7	4.3
7	49	52	2.0	4.7
8	54	54	2.0	7.0
9	65	77	2.7	3.3
10	57	68	1.0	1.7

[20]

6. Measurements on the lengths of humerus, ulna, femur and tibia bones of 276 leghorn fowl gave the following correlation matrix,

$$\begin{pmatrix} 1 & 0.94 & 0.94 & 0.94 \\ & 1 & 0.94 & 0.94 \\ & & 1 & 0.94 \\ & & & 1 \end{pmatrix}$$

each of the variances being 16 $cm.^2$

Carry out a principal component analysis and interpret your analysis. [10]

7. Show that in a stepwise regression problem, the following two criteria for the addition of a predictor are equivalent :

(a) partial correlation of the predictor with the dependent variable given the previously selected predictors is maximum ;

(b) the increase in the multiple correlation between the dependent variable and the predictor is greatest. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1982-83

Nonparametric and Sequential Methods
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 15.7.83. Maximum Marks:100 Time: 3 hours.

Note : Answer all questions.

1. Explain how one can get a confidence interval for the shift parameter in a two sample shift problem. (20)

2. Show that

$$\lim_{n\to\infty} P_{H_0} (\sqrt{n}\, D_n \quad \lambda) = 2 \sum_{i=1}^{\infty} (-1)^{i+1} e^{-i^2 x^2}$$

where D_n is the Kolmogorov -Smirnov statistic with $n_1=n_2=n$ and under H_0 the underlying common distribution function of X's and Y's is continuous. (20)

3. Under suitable assumptions derive lower founds for the ASN of a sequential test under H_0 and H_1. Hence show the approximate optimality of an SPRT. (25)

4. Explain how Stein's two stage sampling leads to founded length confidence intervals for the mean μ of $N(\mu, \sigma^2)$. Show that this is impossible to achieve with fixed sample size. (25)

5. Assignments. (10)