1983-84/311

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1983-84

Difference and Differential Equations

PERIODICAL EXAMINATION

Date: 7.9.1983 Maximum Marks: 100 Time: 3 hrs.

1.(a) Find the general solution of the differential equation

$$x\frac{dy}{dx} - 2y = x^5 \quad \text{on the interval } (0, \infty).$$

(b) Get a differentiable function $y = y(x)$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $y(0) = 2$ and $\frac{dy}{dx} + y \tan x = \sin 2x$. (10+10) = [20]

2. Find the general solution of the following differential equations:

(a) $$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

(b) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 25x^2 + 12$$

(c) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{x^2}$$

(5+10+10) = [25]

3.(a) Find the general solution of the following differential equation on the interval $(0, \infty)$, given that there is a solution of the form x^a for some $a > 0$:

$$4x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y = 0.$$

(b) f is a continuous function on $(0, \infty)$. Find, by inspection, a solution of the differential equation

$$\frac{d^2y}{dx^2} - x f(x) \frac{dy}{dx} + f(x) y = 0,$$

and use this to find the general solution of the above equation.

(10+10) = [20]

4. Find the general solution of

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$$

in term of power series in x. [10]

p.t.o.

5.(a) Show that 0 is a regular singular point of the differenti equation

$$x^3 \frac{d^3y}{dx^2} + (\text{Cos } 2x - 1) \frac{dy}{dx} + 2xy = 0.$$

Find its indicial equation and the roots of the indicial equation.

(b) Verify that the origin is a regular singular point of the differential equation

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$

Find two linearly independent Frobenius series solutions of the above differential equation.

(10+15) = [25]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Difference and Differential Equations
SEMESTRAL-I EXAMINATION

Date: 5.12.83. Maximum Marks: 100 Time: 3 hours.

Note: You can answer any part of any question.
Maximum you can score is 100.

1. Find the general solution on $(0,\infty)$ of the differential equation

$$4x^2 \frac{d^2y}{dx^2} - 8x^2 \frac{dy}{dx} + (4x^2+1)\, y = 0.$$ [15]

2.(a) Let $y(x)$ and $z(x)$ be non-trivial solutions on $(0,\infty)$

of $$\frac{d^2y}{dx^2} + q(x)y = 0$$

$$\frac{d^2z}{dx^2} + r(x)\, z = 0$$

respectively, where $q(x)$ and $r(x)$ are positive continuous functions on $(0,\infty)$ such that $q(x) > r(x)$ for all $x \in (0,\infty)$ show that $y(x)$ has at least one zero between any two zeros of $z(x)$.

(b) Let $y_p(x)$ be a non-trivial solution on $(0,\infty)$ of the equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-p^2)\, y = 0.$$

If $0 \leq p < \frac{1}{2}$, show that any interval of length 2π contains at least one zero of $y_p(x)$.

(c) Show that the general solution on $(0,\infty)$ of the equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right) y = 0 \text{ is :}$$

$$y(x) = \frac{1}{\sqrt{x}}\,(c_1 \cos x + c_2 \sin x)$$

for some $c_1, c_2 \in \mathbb{R}$. [10=10-5=25]

p.t.o.

3. Let $f(x,y)$ be a continuous function that satisfies a Lipschitz condition

$$| f(x,y_1) - f(x,y_2) | \le K |y_1 - y_2|$$

on $[a,b] \times (-\infty,\infty)$ where $-\infty < a < b < \infty$. If $(x_0,y_0) \in [a,b] \times (-\infty,\infty)$, then show that there is a unique function $y(x)$ on $[a,b]$ which is a solution of the problem :

$$\frac{dy}{dx} = f(x,y) \ , \quad y(x_0) = y_0. \qquad [20]$$

4. Find the solution [illegible] of :

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + u = e^{x+2y}$$

$$u(x,0) = 0. \qquad [10]$$

5. Transform the following partial differential equation to canonical form :

$$u_{xx} - 2 u_{xy} + u_{yy} + 3u = 0. \qquad [15]$$

6. Find a continuous function $u(x,y)$ on $[0,\pi] \times [0,\infty)$ such that

$$u_y = u_{xx} \text{ on } (0,\pi) \times (0,\infty)$$

$$u(0,y) = u(\pi,y) = 0 \text{ for } y \ge 0$$

$$u(x,0) = \sin x + 2 \sin 3x \text{ for } 0 \le x \le \pi. \qquad [15]$$

8. Solve

$$u_{xx} - u_{yy} = 0 \qquad \text{on } \mathbb{R}^2$$

$$u(x,0) = e^x \qquad \forall x \in \mathbb{R}$$

$$\frac{\partial u}{\partial y}(x,0) = \frac{1}{1+x^2} \quad \forall x \in \mathbb{R}. \qquad [10]$$

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1983-84
Stochastic Processes - 2
PERIODICAL EXAMINATION

Date: 13.9.1983 Maximum Marks: 100 Time: 3 hrs.

Note: Answer as much as you can. The maximum you can score is 100.

1.(a) For a Poisson process with parameter λ. Let W_r be the random time of occurance of the rth Poisson event. Let $0 \leq s \leq t$ and $1 \leq r \leq n$. Show that

$$P(W_1 \leq s \mid X(t) = n) = 1 - (1 - \tfrac{s}{t})^n$$

and more generally

$$P(W_r \leq s \mid X(t) = n) = \sum_{k=r}^{n} \binom{n}{k} (\tfrac{s}{t})^k (1 - \tfrac{s}{t})^{n-k}$$

[15]

(b) Consider 2 independent Poisson processes $X(t)$, $Y(t)$, where $E(X(t)) = \lambda t$ and $E(Y(t)) = \mu t$, $t \geq 0$. Let 2 successive events of $X(t)$ occur at T and T', $T < T'$, so that $X(t) = X(T)$ for $T \leq t < T'$ and $X(T') = X(T) + 1$. Define $N = Y(T') - Y(T)$, the number of events of the $Y(t)$ process in the time interval $(T, T']$. Find the distribution of N.

[15]

2. Consider a pure death process on $\{0, 1, 2, \ldots\}$ with death rates $\mu_0 = 0$, μ_n, $n \geq 1$.

(a) Write down the forward equations.

(b) Solve for $P_{ij}(t)$ in terms of $P_{ij+1}(t)$.

(c) Find $P_{ii}(t)$.

(d) Find $P_{i\,i-1}(t)$.

(e) If $\mu_n = n\mu$ $n \geq 0$, show that

$$P_{ij}(t) = \binom{i}{j} (e^{-\mu t})^j (1 - e^{-\mu t})^{i-j} \qquad 0 \leq j \leq i$$

(5+7+5+5+3) = [30]

3. Consider a birth and death process with 3 states 0,1 and 2 and birth and death rates given by

$$\lambda_2 = \mu_0 = 0, \quad \lambda_0 = \mu_2 = a, \quad \lambda_1 = b, \quad \mu_1 = c,$$

(a) Write down the forward equations.

(b) Find $P_{00}(t)$, $P_{01}(t)$ and $P_{02}(t) = 1-P_{01}(t) - P_{02}(t)$ using the forward equations.

[illegible]

4. A telephone exchange has m channels. Calls arrive in the manner of a Poisson process with parameter λ; they are accepted if there is an empty channel, otherwise they are lost (no waiting line is formed). The duration of each call is exponential with parameter μ. Let $X(t)$ denote the number of busy channels at time t. Find the birth and death [illegible] for the process $X(t)$. Find the stationary distribution. [15]

5. A system is composed of m identical components which act independently. Each component operates a random length of time until failure. When a component fails it undergoes repair immediately. The failure time and the repair time are both exponential with parameters λ and μ respectively. Let $X(t)$ denote the number of components under repair at time t. Find the infinitisimal parameters of the process $X(t)$. Find the stationary distribution.

[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Stochastic Processes-2
SEMESTRAL-I EXAMINATION

Date: 15.12.83. Maximum Marks: 100 Time: 3 hours.

Note: Answer all questions.

1.(a) Write down the forward differential equations for a linear birth and death process $\{X_t, t \geq 0\}$, with birth and death parameters $\lambda_n = n\lambda$ and $\mu_n = n\mu$, $(\lambda > 0, \mu > 0)$. Find $E(X_t | X_0 = 1)$.

(b) In the case when $\lambda = \mu$; prove that,

$$P_{10}(t) = P(X_t = 0 \mid X_0 = 1) = \frac{\lambda t}{1 + \lambda t}$$

and $P_{1n}(t) = P(X_t = n \mid X_0 = 1) = \frac{(\lambda t)^{n-1}}{(1+\lambda t)^{n+1}}$, $n \geq 1$. [4+6+15]

2. Suppose that N particles are distributed into 2 boxes labelled 0 and 1. A particle in box i remains in that box for a random length of time that is exponentially distributed with parameter α_i, $(i = 0,1)$, before going to the other box. The particles act independently of each other. Let X_t denote the number of particles in box 1 at time t, $t \geq 0$. Then $\{X_t, t \geq 0\}$ is a birth and death process.

(a) Find the birth and death rates.

(b) Find $P_{1N}(t) = P(X_t = N \mid X_0 = 1)$.

(c) Find $E(X_t \mid X_0 = 1)$. [5+10+5]

3.(a) Define a second order stationary process.

(b) Give an example of a second order stationary process which is not strictly stationary.

(c) Let $\{X_t, t \geq 0\}$ be a stationary Gaussian process with mean 0 and let $Y_t = X_t^2$, $t \geq 0$. Show that Y_t is also a second order stationary process. [5+8+7]

p.t.o.

[illegible]. Let W_t, $t \geq 0$ be the standard Brownian motion process with variance parameter $\sigma^2 = 1$.

(a) Show that $X_t = e^{-\alpha t} W(e^{2\alpha t})$, $t \geq 0$ $(\alpha > 0)$, is a stationary Gaussian process.

(b) Show that for any $a > 0$ and $0 \leq t_0 < t_1 < t_2 \dots < t_n = t$,

$$P\left(\max_{0 \leq k \leq n} W(t_k) > a\right) \leq 2\, P(W(t) > a)$$

(c) Let $Y_t = \sup_{0 \leq u \leq t} X(u)$. Using (b) show that for any $a > 0$

$$P(Y_t > a) \leq 2\, P(W(t) > a)$$

(d) Assuming the result that equality holds in the inequality stated in c), write down the density function of Y_t. Using this, prove that the probability p that W_t vanishes at least once in the interval $[t_0, t_1]$, $0 \leq t_0 < t_1$ is $\frac{2}{\pi}$ arc.cos$\sqrt{\frac{t_0}{t_1}}$. [5+10+10+10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Stochastic Processes-2
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 2.7.1984. Maximum Marks : 100 Time: 3 hours.

Note : Answer all questions.

1. Suppose that customers arrive according to a Poisson process and that each customer starts being served immediately upon arrival (infinitely many servers). Suppose that the service times are independent and exponentially distributed random variables with parameter μ. Let X_t, $t \geq 0$ be the number of customers in the process of being served at time t.
   (a) Find the birth and death parameters of this process.
   (b) Let $Y(t)$ denote the number of customers who arrive in the interval $[o,t]$. Prove that the conditional distribution of the actual times of arrival $S_1, S_2, \ldots S_n$ of the n customers given that $Y(t) = n$ is the same as that of the order statistics on the basis n independent observations from the uniform distribution from $[o,t]$.
   (c) Using the result in (b) compute the probability

$$P_{ij}(t) = P(X_t = j \mid X_0 = i).$$ [3+12+15]

2.(a) When is a second order stochastic process X_t, $t \geq 0$ said to be a mean-square continuous process ?
   (b) Let X_t, $t>0$ be a stochastic process with mean function $\mu_X(t) = \lambda$ and covariance function $r_X(s,t)$ given by

$$r_X(s,t) = \lambda\,(1-|t-s|) \quad \text{if } |t-s| < 1$$
$$= 0 \quad \text{if } |t-s| \geq 1$$

   Is X_t a mean-square continuous process ? Give reasons.
   [5+10]

3. Consider a birth and death process having 3 states 0,1 and 2 with birth and death rates such that $\lambda_0 = \mu_2$. Write down the forward equations. Using these find $P_{0j}(t)$, $j = 0,1,2$. [15]

p.t.o.

4. Let $W(t), t \geq 0$ be the standard Brownian motion process with variance parameter $\sigma^2=1$.

(a) Find the mean and covariance function of $X_t = W(t)-t\,W(1)$, $0 \leq t \leq 1$.

(b) Find mean and variance of $\int_0^1 W^2(t)\,dt$.

(c) Show that $\lim_{n \to \infty} \sum_{k=1}^{2^n} [W(\frac{kt}{2^n}) - W(\frac{(k-1)t}{2^n})]^2 = t$ both in mean-square and almost sure sense.

(d) Using (c), prove that

$$\lim_{n\to\infty} \sum_{k=1}^{2^n} \left|W(\frac{kt}{2^n}) - W(\frac{(k-1)t}{2^n})\right| = \infty \text{ almost surely.}$$

[5+10+15+10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1983-84

Statistical Inference

PERIODICAL EXAMINATION

Date: 9.2.1983 Maximum Marks: 100 Time: 3 hrs.

Note: You may answer any part of any question. The maximum you can score is 100.

1.(a) If X be a Poisson random variable with expectation λ, then $1/\lambda$ is not estimable. [7]

(b) Suppose that T_1 and T_2 are two UMVU estimators of $g(\theta)$ with finite variances. Show that

$$P_\theta(T_1 \neq T_2) = 0 \quad \text{for all } \theta \qquad [8]$$

[HINT: Consider $(T_1+T_2)/2$]

(c) Let T be an unbiased estimator of $g(\theta)$ with $V_{\theta_o}(T) < \infty$. Then a necessary and sufficient condition that T has minimum variance at $\theta = \theta_o$ is that $cov_{\theta_o}(T,f) = 0$ for every f such that $E_{\theta_o}(f) = 0$ and $V_{\theta_o}(f) < \infty$. [10]

2.(a) State and prove the Cramér-Rao inequality. (5+10) = [15]

(b) Let $X_1, \ldots, X_n$ be iid random variables following $N(0, \sigma^2)$, σ^2 being unknown. Find the Cramer-Rao lower bound for σ^2. Show that $\Sigma X_i^2/n$ attains the lower bound.

(9+6) = [15]

3. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, μ and σ^2 being both unknown. Show that $\exp(-\mu+\sigma^2/2)$ is estimable. Is μ/σ estimable ?

(6+6) = [12]

4. What do you mean by a statistic being minimally sufficient ? Suggest (without proof) a criterion for minimal sufficiency. Give an example of a statistic which is sufficient but not minimally sufficient.

(3+5+5) = [13]

p.t.o.

5. Show that for random sample of size n from a distribution with p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1}{\theta_2}\exp[-(x-\theta_1)/\theta_2] & \text{if } \theta_1 < x < \infty \\ 0 & \text{otherwise}, \end{cases}$$

where $-\infty < \theta_1 < +\infty$, $0 < \theta_2 < +\infty$, the statistics $X_{(1)}$ and ΣX_i are jointly sufficient for (θ_1, θ_2).

[10]

6.(a) Let $X_1, \ldots, X_n$ be a random sample from the uniform distribution $U(0,1)$. Show that $E(\max_i X_i) = n/(n+1)$; hence deduce that $E(\min_i X_i) = 1/(n+1)$.

(7+3) = [10]

(b) Let $Y_1, \ldots, Y_n$ be a random sample from $U(\theta - \frac{1}{2}, \theta + \frac{1}{2})$, then $(\min_i X_i, \max_i X_i)$ is sufficient for θ but not complete.

(7+8) = [15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Statistical Inference
SEMESTRAL-I EXAMINATION

Date : 13.12.83. Maximum Marks: 100 Time: $3\frac{1}{2}$ hours.

Note : Answer as many questions as you can. Maximum you can score is 100. Marks are given in brackets.

1.(a) State and prove the Bhattacharyya lower bound to the variance of an unbiased estimator. [15]

(b) Let T be an unbiased estimator of $g(\Theta)$ with $V_\Theta(t) < \infty$. Prove that a necessary and sufficient condition that T has minimum variance at $\Theta = \Theta_0$ is that $Cov_{\Theta_0}(T, f) = 0$ for every f such that $E_{\Theta_0}(f) = 0$ and $V_{\Theta_0}(f) < \infty$. [10]

2. Let $X_1, \ldots, X_n$ be n iid observations from normal (μ, σ^2) population, μ and σ^2 being both unknown. Derive the MVUE of $P_\Theta(X \leq c)$ where $\Theta = (\mu, \sigma^2)$ on the basis of the sample [20]

3. Let X be a countable set of points and P_0 and P_1 are two probability distributions on X, $\sum_x P_0(x) = \sum_x P_1(x) = 1$.
Let $0 < \alpha < 1$.

(a) Show that if a test function $\phi_0(x)$ defined on X satisfies the following two conditions (i) and (ii)

(i) $\sum \phi_0(x) P_0(x) = \alpha$

(ii) there is a $k \geq 0$ such that

$$\phi_0(x) = 1 \quad \text{if} \quad P_1(x) > k P_0(x)$$
$$= 0 \quad \text{if} \quad P_1(x) < k P_0(x)$$

then $\phi_0(x)$ maximizes

$$\sum_x \phi(x) P_1(x)$$

in the class of all tests function

$$\mathcal{C}_\alpha = \{\phi : 0 \leq \phi \leq 1, \sum \phi_0(x) P_0(x) \leq \alpha\}$$ [10]

p.t.o.

3.(b) Show that if $\phi_1(x)$ is any test function which maximizes. $\Sigma\, \phi(x)\, P_1(x)$ in the class $\mathcal{C}_\alpha$ and if $\phi_0(x)$ is a test function satisfying (i) and (ii), then $\phi_0(x)=\phi_1(x)$ for all x with $P_1(x) \neq k\, P_0(x)$. [10]

4.(a) Let $X_1, \ldots\ldots, X_n$ be iid observations each with uniform $R(0,\theta)$ distribution $\theta > 0$. Show that the test defined by

$$\phi_0(x) = 1 \text{ if } \max(x_1, \ldots, x_n) > \theta_0 \text{ or } \max(x_1, \ldots, x_n) < \theta_0 \alpha^{\frac{1}{n}}$$
$$= 0 \text{ otherwise}$$

is UMP level α for testing $H: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.

(b) Let $X_1, \ldots, X_n$ be iid observations with X_1 having the density

$$f_\theta(x) = 2\, e^{-2(x-\theta)} \quad \text{for } x > \theta$$
$$= 0 \quad \text{for } x \leq \theta$$

Determine the UMP level α test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$. [10]

5.(a) Give examples to show that the maximum likelihood estimator need not exist and that even if it exists, it may take infinitely many values. [5+5=10]

(b) Estimate all the parameters of a bivariate normal distribution by the method of maximum likelihood.

[20]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year,1983-84

Statistical Inference
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 3.7.84. Maximum Marks: 100 Time:3 hours.

1.(a) State the Neyman-Pearson lemma carefully. [7]

(b) Show that there does not exist a UMP test for testing $H_0: \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ on the basis of sample $X_1, \ldots X_n$ coming from $N(\theta,1)$. [10]

2. State and prove Chapman-Robbins Inequality. [15]

3.(a) State and prove Rao-Blackwell theorem. [15]

(b) Let $X_1, \ldots X_n$ be Poisson with mean θ, $\theta > 0$. Find the MVUE of $e^{-\theta}$. [8]

4.(a) Find an example where the maximum likelihood estimate is inconsistent. [7]

(b) You are given a sample of size n from a bivariate normal distribution all parameters unknown. Test for independence by the method of likelihood ratio statistic. Show that it coincides with the conventional test. [20]

5.(a) Suppose that T_1 and T_2 are two MVUE's of $g(\theta)$ with finite variance. Show that $T_1=T_2$ [6]

(b) Let X follow the distribution with density

$$P_\theta(x) = \frac{\theta^x e^{-\theta}}{x!(1-e^{-\theta})} \quad x = 1,2, \ldots, \quad \theta > 0$$

Find the MVUE of $1 - e^{-\theta}$. [7]

6. Give an example of a sufficient statistic which is not boundedly complete. [5]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1983-84

Sample Surveys

PERIODICAL EXAMINATION

Date: 15.9.1983 Maximum Marks: 100 Time: 3 hrs.

Note: Figures in brackets [ ] indicate marks.
Questions 1-3 carry a total of 100 marks.
The maximum you can score for these questions is 90.

1.(a) What do you understand by the terms 'inclusion probability π_i of unit U_i and 'joint inclusion probability π_{ij} of a pair of units U_i and U_j $i \neq j$, for a sampling design ?

(2+3) = [5]

(b) For any sampling design find the value of $\sum_{i=1}^{N} \pi_i$ and $\sum_{i \neq j}^{N} \sum^{N} \pi_{ij}$ where N is the size of the population.

(4+6) = [10]

(c) For a Probability Proportional to Size With Replacement (PPSWR) design of sample size n from a population of size N, write down the values of π_i and π_{ij}.

(4+6) = [10]

2.(a) For a Simple Random Sampling With Out Replacement (SRSWOR) design, obtain an unbiased estimator of

$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (Y_i - \bar{Y})^2$, where Y_i is the value of the study variate on the unit U_i, $i = 1, 2, \ldots, N$ and $\bar{Y} = \frac{1}{N} \sum_{1}^{N} Y_i$

Hence, obtain an unbiased estimator of Var. ($\hat{\bar{Y}}$).

(4+6) = [10]

(b) Of 105 households sampled using an SRSWOR design from a population of 1241 households in an area 19 households have Colour Television Sets. Estimate the proportion of Colour Television sets in the area. Also obtain an estimate of its sampling error.

(4+6) = [10]

(c) A population consists of N units, the variate-value of one unit being known to be y_1. A simple random sample (s.r.s.) of n units is selected without replacement from the remaining (N-1) units of the population. Show that $y_1 + (N-1)\bar{y}_n$ is more efficient than $N\bar{y}_n$ based on a s.r.s. of size n drawn from the whole population, without replacement.

[12]

3.(a) When do you use a Probability Proportional to Size Sampling Technique ?

[5]

p.t.o.

Contd..... Q.3.

(b) A sample of 6 farms is drawn from a population of 74 farms with probability of selection proportional to the size (x in acres) with replacement and the yield of paddy is observed:

Sampled farm	x Size in acres	y Yield in bushels per acre
1	21	105.2
2	101	524.3
3	14	73.1
4	6	31.2
5	41	200.9
6	12	64.3

It is also known that the total size of all the 74 farms is

$$X = \sum_{1}^{74} x_i = 2949 \text{ acres.}$$

(i) Estimate the average yield of the population.

[10]

(ii) Calculate an unbiased estimate of the sampling error of your estimate in (i) above.

[13]

(iii) Estimate the gain in efficiency of using a PPSWR design compared to a simple random sampling with replacement design.

[15]

4. Practical Records (to be submitted to the Dean's Office by 18.9.1983).

[10]

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Sample Surveys
SEMESTRAL-I EXAMINATION

Date: 9.12.83. Maximum Marks: 100 Time: $3\frac{1}{2}$ hours.

Note : Answer all questions. Figures in brackets( ) indicate the marks allotted to each question.

1.(a) Explain with illustrations what you understand by the terms ' inclusion probability of a unit U_i' and 'joint inclusion probability of a pair of units (U_i, U_j)' for a sampling design.

(b) For a fixed sample size design, obtain the value of
$\sum\sum_{i \neq j}^{N\ N} (\pi_{ij} - \pi_i \pi_j)$.
[Hint : First obtain $\sum_{j \neq i}^{N} \pi_{ij}$].

(c) From a population of size N, one unit is drawn with probability of selection proportional to its size measure (x). The rest of (n-1) units in the sample are selected from the remaining (N-1) units of the population by Simple Random Sampling WithOut Replacement(SRSWOR). Write down p(s), the probability of obtaining the sample for this design. Hence show that the ratio estimator
$\hat{R} = \sum_1^n y_i / \sum_1^n x_i$ is unbiased for $R = \sum_1^N Y_i / \sum_1^N X_i$ the Ratio of population totals of two characteristics y and x for this design.

(2+3) +4+(3+3)=(15)

2.(a) What are the advantages of ' Circular systematic sampling' technique over ' linear systematic sampling'?

(b) Explain how you would estimate the sampling error of your estimator based on a single systematic sample.

(3+4)=(7)

3.(a) For stratified simple random ssampling (without replacement) to estimate the population mean, write down (no derivation required) the optimum allocation of a fixed total sample size n to the strata. How does one use this allocation in practice ? Give a justification to your answer.

p.t.o.

3.(b) If we deviate from the optimum allocation by using a sample size $\tilde{n}_i$ in the ith stratum, show that the proportional increase in variance cannot exceed δ^2, when δ is the maximum deviation $|\tilde{n}_i - n_i|$ expressed as a fraction of $\tilde{n}_i$. Illustrate this result by an example.

(c) Let W_i be the proportion of population units and let $S_i^2 = N_i \sigma_i^2 / (N_i - 1)$, where σ^2_i is the within variance for the ith stratum. Obtain an expression for the variance of the estimate of the population mean based on an allocation of the sample size proportional to $W_i S_i^{\alpha}$, when α is a real number in [0,2]. Hence show that the allocation in (a) is superior to this allocation with respect to the variance criterion. Comment on the case $\alpha = 2$.

(2+2+2)+(7+2)+(4+2+1)=(22)

4.(a) A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as first stage units and plots within them as second stage units. From each stratum 4 villages were selected with probabilities proportional to area and with replacement and the plots are selected by SRSWOR. The data on yield for the sample plots together with information on selection probabilities are given below :

Stratum	Sample village	Inverse of probability of selection	Total no. of plots	Yield of sample plots 1	2	3	4
I	1	67.68	189	110	281	120	114
	2	338.12	40	80	60	122	125
	3	101.50	135	122	210	171	105
	4	69.03	160	244	115	312	128
II	1	113.34	73	325	359	160	117
	2	441.00	26	97	179	144	85
	3	31.50	240	100	115	50	172
	4	661.57	14	102	40	126	148
III	1	15.80	287	122	176	108	140
	2	21.00	257	125	110	134	215
	3	48.89	68	300	115	67	110
	4	26.73	218	263	75	142	54

Total number of plots in stratum I,II,III are respectively 8423, 6355, 12853.

(i) Obtain an unbiased estimate of the total yield of paddy in the district.

(ii) Obtain an unbiased estimate of the sampling variance of the above estimate.

(iii) Discuss briefly the possible sources of non-sampling errors in this agricultural survey and explain how you would assess and control them ?

(12+14+10)=(36)

4.(b) EITHER

An experienced farmer makes an eye estimate of the weight of peaches x_i on each tree in an orchard of N=200 trees. He finds a total weight of X = 11,600 lbs. The peaches are picked and weighed on a simple random sample (without replacement) of 10 trees and the following results are obtained :

	Tree Number									
	1	2	3	4	5	6	7	8	9	10
Actual weight y_i	61	42	50	58	67	45	39	57	71	53
Estimated weight x_i	59	47	52	60	67	48	44	58	76	58

(i) Find the regression estimate $\hat{Y}$ of Y, the total weight of peaches in the population and estimate its variance.

(ii) As an alternative, if $\hat{Y}' = N\{\bar{X} + (\bar{y} - \bar{x})\}$ is used, obtain the gain in precision of $\hat{Y}$ over $\hat{Y}'$.

(12+8)=(20)

OR

For estimating the total Y of current population in a region, two sub-samples of 6 villages each are selected (circular systematically) from each stratum with independent random starts. Using the data given in the table, obtain a ratio estimate for Y taking the previous census population (x) as the auxiliary information and compare its efficiency with that of conventional unbiased estimate.

Total number of villages(N) and sample totals of x and y

Stratum number	no.of villages N	sub-sample 1 x	sub-sample 1 y	sub-sample 2 x	sub-sample 2 y
1	2044	3722	3935	3456	3641
2	1304	3625	4033	4171	4649
3	1265	2769	3050	3746	4043

(Total of x for the region = 3,155, 680)

(8+12)= (20).

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1983-84

Sample Surveys
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 3.7.84. Maximum Marks : 100 Time: $3\frac{1}{2}$ hours.

Note : Attempt all questions. Figures in brackets [ ] indicate marks.

1. A sample survey was conducted to estimate the total monthly household income in an urban area. The design adopted was a stratified two-stage one with census enumeration blocks as first-stage units and households within them as second-stage units. From each stratum 5 blocks were selected with probability proportional to population and with replacement and 4 households were selected from each selected block with equal probability and without replacement. The data on household income for the sample households together with information on selection probabilities are given below :

   (a) Obtain an unbiased estimate of the total monthly household income.

   (b) Obtain an unbiased estimate of the sampling variance of the estimate.

   (c) Compare the efficiency of the above design with that of unistage simple random sampling of households in each stratum. [10+15+9]=[34]

Stratum	Sample block	Increase of probability of selection	total no.of households	monthly household income of sample households 1	2	3	4
I	1	67.68	189	110	281	120	114
	2	338.12	40	80	60	122	125
	3	101.50	135	122	210	171	105
	4	69.03	160	244	115	312	128
	5	113.34	73	345	359	160	117
II	1	441.00	26	97	179	144	85
	2	31.50	240	100	115	50	172
	3	661.57	14	102	40	126	148
	4	15.80	287	122	176	108	140
	5	21.00	257	125	110	134	215

Total no. of households in strata I and II respectively are 12848 and 8422.

p.t.o.

2. An experienced farmer makes an eye estimate of the weight of peaches x_i on each tree in an orchard of N=200 trees. He finds a total weight of X = 11,600 lbs. The peaches are picked and weighed on a simple random sample of 10 trees, selected without replacement with the following results:

		Tree Number									
		1	2	3	4	5	6	7	8	9	10
Actual weight	y_i	61	42	50	58	67	45	39	57	71	53
Estimated weight	x_i	59	47	52	60	67	48	44	58	76	58

Using information on x, obtain an unbiased estimate of Y, the total weight of peaches in the population and obtain an estimate of its variance.

(b) Suggest an alternative estimate which also utilizes the information on X and compare it with the estimate suggested in (a) above. [12+13]=[25

3.(a) Distinguish between sampling and non-sampling errors in a survey. List down the various sources of non-sampling errors and discuss briefly the methods of assessment and control of these errors. [5+8+6]=[19]

(b) A survey was conducted in a village consisting of 625 households by covering a sample of 50 households selected using a simple random sampling without replacement scheme to estimate the average monthly expenditure on toilet goods. The estimate was found to be Rs.4.20 with a standard error of Rs.0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample to be selected by a simple random sampling with replacement scheme such that the length of the confidence interval at 95% confidence level is 20% of the true value. State clearly the assumptions involved in finding out the sample size. [7]

(contd.....3)

4.(a) Explain what you understand by combined and separate ratio estimators' in stratified random sampling. Indicate the situations when you use these estimators. [4+6]=[10]

(b) If the cost function is of the form $C = C_o + \Sigma t_i \sqrt{n_i}$, where C_o and t_i are known numbers, show that in order to minimize $V(\hat{\bar{Y}})$ for a fixed total cost, n_i must be proportional to

$$\left(\frac{W_i^2 \, S_i^2}{t_i}\right)^{2/3}$$

where conventional notations are used. [5]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1983-84

Elective - 4 : Economics

PERIODICAL EXAMINATION

Date: 17.9.1983 Maximum Marks: 100 Time: 3 hrs.

Note: Answer all the questions. All questions carry equal marks.

1.(a) Suppose a dairy farmer believes that each dairy cow must be fed at least 27, 21 and 30 units of the nutritional elements A, B and C respectively per day. Two kinds of feeds, which we call feed 1 and feed 2, are being considered. It is known that one pound of feed 1 contain 3, 1 and 1 unit of the nutritional elements A, B and C respectively. On the other hand, one pound of feed 2 contains 1, 1 and 2 units of these elements. The prices of feed 1 and feed 2 are Rs.4/- and Rs.2/- per pound respectively. The problem of the farmer is to decide whether to purchase only one kind of feed or both kinds and mix them. In this respect his objective must be to minimize the cost of feeding his dairy cow.

State the above as a linear programming problem defining clearly the different variables used and explaining the development and meaning of the constraints in your formulation.

(b) Use the Simplex method to solve the following linear programming problem:

$$\text{maximise} \quad Z = 3x_1 + 2x_2 + x_3$$

$$\text{subject to} \quad -3x_1 + 2x_2 + 2x_3 = 8$$

$$-3x_1 + 4x_2 + x_3 = 7$$

$$x_1, x_2, x_3 \geq 0.$$

You may use the following solution as the initial basic feasible solution:

$$x_1 = 0, \quad x_2 = 1, \quad x_3 = 3$$

$$B = [P_3 \; P_2] = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}.$$

2.(a) While solving a maximising linear programming problem by the Simplex procedure how would you know that an optimum solution has been reached ? Give proof for your answer.

(b) Does the Simplex procedure indicate if a linear programming problem has multiple solutions ? Justify your answer by using the algebraic derivations of the Simplex procedure.

p.t.o.

3. Describe the linear programming formulation of the Transportation problem. Indicate briefly how you can solve a Transportation problem. Write out the dual of the Transportation problem and provide an economic interpretation of the problem.

4. State and prove the complementary Slackness theorem. Give an economic interpretation of the theorem.

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Elective-4 : Economics
SEMESTRAL-I EXAMINATION

Date: 17.12.83. Maximum Marks : 100 Time: 3 hours.

Note : Answer any three questions from group A and any one from group B. Marks allotted to each question are given in brackets.

1. Suppose in a two-sector economy the production structure is as given by the following input-output coefficient matrix

$$A = \begin{pmatrix} 0.2 & 0.2 \\ 0.1 & 0.4 \end{pmatrix}$$

Use the Leontief input-output framework to answer the following :

(a) Find the effect of unit increase in the final demand for commodity one on the gross output requirements of both the sectors. [6]

(b) What would be the requirement of labour statisfying final demands $C_1 = 920$ and $C_2 = 2300$ when the labour coefficients are $a_{01} = 0.2$ and $a_{02} = 0.1$ for sectors 1 and 2 respectively ? [12]

(c) Given capacity restrictions $\bar{X}_1 = 2300$ and $\bar{X}_2 = 500$ in the two sectors, find out the combination of net output C_1 and C_2 of the two commodities that would be available when capacities in both the sectors are fully utilised. Check if this is feasible if the total supply of labour is 450 units. [7]

2. State and prove the nonsubstitution theorem for the Leontief static open input-output model. Explain exactly where the assumptions of constant returns to scale and absence of joint production are used in proving the theorem. Provide an intuitive economic justification for the result. [25]

P.T.C.

3. Bring out clearly the duality relation between quantities and prices in the Leontief Static open input-output model. Also show that the absolute level of prices is completely indeterminate in this system. [25]

4.(a) Show how and when it is possible to get optimal solution of the dual of a linear programming problem from the solution of the primal. [9]

(b) Write down the dual of a standard maximising linear programming problem and provide economic interpretations of the variables, constraints and objective function of the dual problem. [16]

GROUP B

1. In a city an enclosed area with a shed over it has been serving as a market. With increase in the number of customers the market has spilled over on to the pavements surrounding the enclosed structure resulting in disorder to the traffic and inconvenience to pedestrians using the roads and pavements around the market. There is a suggestion to construct a multistoreyed structure over the market land and use it commercially to accomodate the demand for marketing space. How should you go about evaluating the cost and benefit of the project to the society ? [25]

2.(i) Discuss how Social Cost Benefit Analysis is distinguished from commercial Profit and Loss Accounting.

(ii) Discuss the concept of time preference of individuals in a society. How does it affect the rate of interest in the market ? How does it affect the ranking of profits in Social Cost Benefit Analysis ? [25]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Elective-4 : Economics
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 4.7.1984. Maximum Marks: 100 Time: 3 hours.

GROUP A

Note : Answer any three questions. Marks allotted to each question are given in brackets.

1. Suppose in a two -sector economy the production structure is as given by the following input-output coefficient matrix.

$$A = \begin{pmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{pmatrix}$$

(a) Is the system viable ? If so, find the gross output levels which will satisfy final demands of 50 and 70 units of the two commodities respectively. (10)

(b) If the labour coefficients a_{01} and a_{02} for the two sectors are given as 0.2 and 0.1 units respectively, find the direct and indirect labour embodied in a unit of final consumption of each commodity. (15)

2.(a) State the assumptions of the Leontief static open input-output model. (5)

(b) Show that in this model prices of goods relative to wages can be determined from the total labour content of one unit of final output of these goods. (20)

3. Show that every basic feasible solution to a linear programming problem is an extreme point of the convex set of feasible solutions and that every extreme point is a basic feasible solution to the set of constraints in the problem. (25)

4. Formulate any economic optimisation problem as a linear programming problem. Obtain the dual and interpret it in economic terms. (25)

p.t.o.

GROUP B

Note : Answer one question.

1. A metalled road is to be constructed connecting interior villages in a district to the main highway of the district. The villagers were so long using village paths for journey and carrying of products. How would you apply Social Cost Benefit Analysis for the evaluation of the project for the metalled road ? (25)

2. Why is market price considered unsuitable for evaluation of output of a project in Social Cost Benefit Analysis ? Discuss the method for the evaluation of net benefit of a project. (25)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1983-84

Elective - 4 : Physical and Earth Sciences

PERIODICAL EXAMINATION

Date: 17.9.1983 Maximum Marks: 100 Time: 3 hrs.

Note: Answer any five questions. All questions carry equal marks.

1. Briefly discuss about catastrophic theories regarding the origin of the Earth. [20]

2. Briefly describe the interior of the Earth. [20]

3. What is magma ? Discuss about Bowen's Reaction Principle and its implications in magmatic differentiation. (2+18) = [20]

4. What is metamorphism ? What are different agents and kinds of metamorphism ? Briefly discuss about different kinds of metamorphism. (2+6+12) = [20]

5. What is a fault ? Discuss about geometrical classification of fault. (2+18) = [20]

6. Write notes on <u>any two</u> of the following:

   (i) Bouger gravity anomaly.
   (ii) Polymorphism.
   (iii) Volcanic landforms.
   (iv) Eutectic crystallization.
   (v) Silicate minerals.

   (10 x 2) = [20]

7. Distinguish between the following pairs of terms (any five) (Give sketches whenever possible).

   (i) Sill and Dyke.
   (ii) Volcanic rock and Plutonic rock.
   (iii) Prograde metamorphism and Retrograde metamorphism.
   (iv) Hornfelsic texture and Schistose texture.
   (v) Antiform and Synform.
   (vi) Isoclinic lines and Isogonic lines.
   (vii) 'P' wave and 'S' wave.
   (viii) Focus and Epicentre of an earthquake.

   (4 x 5) = [20]

p.t.o.

8. Correct the following statements, if necessary, otherwise retain them as it is (any eight).

(i) Dip of a bed is maximum in the direction at right angle to its strike.

(ii) Sea-level value of 'g' is maximum at equator and gradually decreases toward the poles.

(iii) Limestone when metamorphosed is converted into amphibolite.

(iv) Value of magnetic inclination at Paris has changed considerably during the last three centuries.

(v) Chlorite appears at the earliest stage of regional metamorphism of pelitic rock.

(vi) Bedding is a very common primary structure in sedimentary rock.

(vii) Continental crust is thinner than oceanic crust.

(viii) Quartz is harder than diamond.

(ix) The Sun is moving with respect to the Solar-System barycentre.

(x) Core of an anticline is occupied by the youngest rock in a deformed sequence.

$(2\frac{1}{2} \times 8) = [20]$

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983 -84

Elective 4 : Physical and Earth Sciences
SEMESTRAL-I EXAMINATION

Date: 17-12-83. Maximum Marks: 100 Time: 3 hours.

Note : All questions carry equal marks. Attempt any five questions.

1. What is weathering ? How does it differ from erosion ? Briefly discuss about chemical weathering. 4+4+12=[20]

2. Draw a sketch showing the relation between a set of trough cross-bedding and downcurrent direction. Determine azimuth of the resultant vector, its magnitude and consistency ratio from the following cross-bedding azimuth data collected from a fluvial sand body.

330°, 330°, 356°, 354°, 22°, 25°, 38°, 38°, 33°, 25°, 37°, 35°, 30°, 8°, 18°, 10°, 15°, 12°, 78°, 63°, 70°, 67°, 62°, 78°, 58°, 56°, 55°, 54°, 47°, 55°, 48°, 82°, 85°, 278°, 282°, 292°, 158°. 4+16=[20]

3. What is a fossil ? Are mummies of Egyptian Pharaohs fossils? Justify your answer. Briefly describe various modes of preservation of fossils. 4+2+14=[20]

4. Write notes on <u>any two</u> of the following :

(i) Unconformity (ii) Tillite (iii) Airy's theory of isostasy (iv) Polar wandering (v) Spreading of the sea floor. 10x2=[20]

5. Distinguish between (any two)

(i) Sedimentary rock and Igneous rock
(ii) Channel deposits and Floodplain deposits
(iii) Continental crust and Oceanic crust
(iv) Craton and Geosyncline. 10x2 = [20]

6. What is a mountain ? Describe the major kinds of mountains based on geologic structures. 2+18=[20]

P.T.O.

87. Write a brief essay on the concept of 'Plate Tectonics'. [20]

8. Indicate the correct answer among (a), (b) and (c) in each of the following statements (Attempt any ten):

(i) All sedimentary rocks
   (a) show ripple marks
   (b) have fragmental texture
   (c) have formed at or near the surface of the earth under low pressure-temperature condition.

(ii) Primary classification of mechanically deposited sedimentary rocks is based on
   (a) mineralogical composition
   (b) chemical composition
   (c) grain-size distribution

(iii) Stromatolite is a kind of
   (a) organism
   (b) organo-sedimentary structure
   (c) inorganic sedimentary structure

(iv) A claystone is essentially composed of
   (a) clay minerals
   (b) clay-sized grains
   (c) quartz grains

(v) Well sorted, frosted sand grains are commonly found in
   (a) aeolian dune
   (b) tillite
   (c) alluvial fan

(vi) Fossils are found to be well preserved in
   (a) conglomerate
   (b) shale
   (c) granite

(vii) Fossil remains of dinosaur are found exclusively in rocks of
   (a) Mesozoic Era
   (b) Palaeozoic Era
   (c) Cainozoic Era

(contd....3)

8. (viii) Graded bedding is commonly found in

(a) loess
(b) turbidite
(c) tillite

(ix) In present day oceans no sediments are found to be older than

(a) 50,000 years
(b) 200 million years
(c) 50 million years

(x) Along subduction zones oceanic crusts are

(a) created
(b) destroyed
(c) conserved

(xi) South America and Africa started drifting away from each other during

(a) Middle Cretaceons
(b) Cambrian
(c) Pleistocene

(xii) The fossils useful for correlation of age of rocks should have

(a) wide time range and wide geographic distribution
(b) narrow time range and wide geographic distribution
(c) narrow time range and narrow geographic distribution.

2x10=[20]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year, 1983-84

Multivariate Distributions and Tests

PERIODICAL EXAMINATION

Date: 24.2.84. Maximum Marks : 100 Time: 3 hours.

Note : Group A carries a maximum of 70 marks, while Group B carries a maximum of 30 marks. From each group answer as many questions as you can. Answers to the two groups should be written on separate answer books.

GROUP A

1.(a) Let $\underset{\sim}{Y}_1, \ldots, \underset{\sim}{Y}_n$ be IID each $N_p(\underset{\sim}{0}, \Sigma)$, where Σ is positive definite. Find the characteristic function of $A = \sum_{\alpha=1}^{n} \underset{\sim}{Y}_\alpha \underset{\sim}{Y}_\alpha'$.

(b) State and prove the reproductive property of independent Wishart matrices.

(c) Let $A \sim W_p(\Sigma|n)$, where Σ is positive definite. Then for a vector $\underset{\sim}{l}\ (\neq \underset{\sim}{0})$ such that $\underset{\sim}{l}'\underset{\sim}{l} = 1$, show that

$$\frac{\underset{\sim}{l}'\Sigma^{-1}\underset{\sim}{l}}{\underset{\sim}{l}'A^{-1}\underset{\sim}{l}}$$

follows chi-square distribution with n-p+1 d.f. (In part (c), state all auxiliary results without proof).

10+5+10=25.

2.(a) Show that in a random sample of N IID observations from $N_p(\underset{\sim}{\mu}, \Sigma)$, the joint pdf of the sample correlation coefficients is given by

$$\frac{|R|^{(n-p-1)/2}\ (\Gamma n/2)^p}{\pi^{p(p-1)/4}\ \prod_{i=1}^{p} \Gamma(n-i+1)/2}$$

where R is the sample correlation matrix and n = N-1.

p.t.o.

2.(b) Apply your results in (a) above to show that if r is the sample correlation coefficient in a random sample of size N from a bivariate normal population with populations correlation coefficient zero then

$$\frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \text{ has t distribution with (N-2) d.f.}$$

23+10=33

3. Let $\underset{\sim}{X}_\alpha = (X_{1\alpha}, \ldots, X_{p\alpha})'$, $\alpha = 1, \ldots, N$ represent a random sample of size N from $N_p(\underset{\sim}{\mu} , \Sigma)$ where $\underset{\sim}{\mu} = (\mu, \ldots, \mu)'$ and

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \ldots & \rho \\ \rho & 1 & \ldots & \rho \\ & \vdots & & \\ \rho & \rho & \ldots & 1 \end{pmatrix}$$

Defining $\bar{X}_\alpha = p^{-1} \sum_{i=1}^{p} X_{i\alpha}$, $\bar{\bar{X}} = (Np)^{-1} \sum_{i=1}^{p} \sum_{\alpha=1}^{N} X_{i\alpha}$,

$$W = \sum_{i=1}^{p} \sum_{\alpha=1}^{N} (X_{i\alpha} - \bar{X}_\alpha)^2, \quad B = p \sum_{\alpha=1}^{N} (\bar{X}_\alpha - \bar{\bar{X}})^2,$$

show that $\frac{W}{\sigma^2(1-\rho)}$ and $\frac{B}{\sigma^2[1+(p-1)\rho]}$ are independent chi-square variates with $N(p-1)$ and $(N-1)$ d.f. respectively.

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GROUP B

1. On the basis of a random sample of size 25 from a trivariate normal population, the value of $r_{23.1}$ was obtained as 0.651. Can it be concluded that in the population the partial correlation coefficient between X_2 and X_3 eliminating the effect of X_1 is 0.5 ? 8

2. In a certain experiment, 18 rabbits each received a high dose of insulin and 18 received a low dose. The blood-sugar level (X) of each rabbit was measured at 1,2,3 hours after each dose. The three readings are denoted by X_1, X_2, X_3. The data yielded the following values.

$$\underset{\sim}{d} = (7.59 \quad 19.73 \quad 25.04)',$$

(contd....3)

Group B

Q.No.2 contd.......

$$S = \begin{pmatrix} 2677 & 1278 & 1614 \\ & 2358 & 1956 \\ & & 3223 \end{pmatrix},$$

$$S^{-1} = \begin{pmatrix} 3.735 & -0.553 & -1.765 \\ & 5.337 & -2.945 \\ & & 1.679 \end{pmatrix},$$

where

d_i = mean of low- insulin X_i value - mean of high insulin X_i value,

S = pooled (corrected) SS-SP matrix with 34 d.f.

Is there sufficient evidence in the data to conclude that blood-sugar level falls with rise in the insulin level ? Derive a suitable multivariate test for this purpose. 12

3. Assignment. 10

INDIAN STATISTICAL INSTITUTE.
B.Stat.(Hons.) III Year, 1983-84

Multivariate Distributions and Tests
SEMESTRAL-II EXAMINATION

Date: 23.5.84. Maximum Marks : 100 Time : 3 hours.

Note : Attempt as many questions as you can.

GROUP-A

1.(a) Derive the Hotelling's T^2 test from the likelihood ratio principle.

(b) On the basis of a random sample $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ from $N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is positive definite, define

$$\bar{\underset{\sim}{X}} = N^{-1} \sum_{\alpha=1}^{N} \underset{\sim}{X}_\alpha, \quad A = \sum_{\alpha=1}^{N} (\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})',$$

$$T^2 = N(N-1)(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})' A^{-1} (\bar{\underset{\sim}{X}} - \underset{\sim}{\mu}),$$

$$A^* = \sum_{\alpha=1}^{N} (\underset{\sim}{X}_\alpha - \underset{\sim}{\mu})(\underset{\sim}{X}_\alpha - \underset{\sim}{\mu})',$$

$$T^{*2} = N(N-1)(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})' A^{*-1} (\bar{\underset{\sim}{X}} - \underset{\sim}{\mu}).$$

Find the relation between T^{*2} and T^2 and hence find the distribution of T^{*2}. [15+13=28]

2.(a) Define principal components.

(b) Obtain an expression for the variance of the ith principal component (i=1,2,...).

(c) If $E(\underset{\sim}{X}) = 0$ and $\text{Disp}(\underset{\sim}{X}) = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \rho & \rho & \cdots & 1 \end{pmatrix}_{(p \times p)}$, $-\frac{1}{p-1} < \rho < 1$,

obtain the principal components. [5+5+8=18]

3. Derive the likelihood ratio criteria for testing (a) equality of several dispersion matrices, (b) whether a dispersion matrix is proportional to a given positive definite matrix.
Can you express the test in (a) in terms of an F-statistic in the special case when there are only two populations, each of them being univariate ? [(8+3)+8=24]

p.t.o.

GROUP B

4. The following total and multiple correlation co-efficients have been computed from a sample of size 15 for four variates X_0, X_1, X_2, X_3.

$$r_{01}^2 = 0.32 \qquad r_{0(12)}^2 = 0.49$$
$$r_{02}^2 = 0.39 \qquad r_{0(13)}^2 = 0.78$$
$$r_{03}^2 = 0.55 \qquad r_{0(23)}^2 = 0.83$$
$$r_{0(123)}^2 = 0.86$$

(a) Define SSE – the residual sum of squares of X_0 under a model of linear regression of X_0 on X_1, X_2, X_3.

Also define $(SSE_0)_{(ij)}$ – the residual sum of squares of X_0 under the same model and under the hypothesis $H_{0(ij)}$: $\rho_{0(123)} = \rho_{0(ij)}$ for $i \neq j$, $i,j = 1,2,3$, all symbols having the usual significance.

Use the above to test all the six different hypotheses $H_{0(ij)}$, $i \neq j$, $i = 1,2,3$. State clearly what assumptions you are making and what the alternative hypothesis will be. Give reasons wherever necessary.

(b) Assume the above correlation co-efficients to be population values. It is desired to predict X_0 by observing one or more of the variables and using a linear regression. The cost of observations is made up of a basic cost of

Re. 1 if X_1 is observed
Rs.1.50 if X_2 is observed
Rs.2.50 if X_3 is observed.

The choice of the best combination is to be made by considering the precision of the regression estimate per unit cost. Find the best combination of predictors, i.e., the combination for which the precision per unit cost is maximum.
Make use of the fact that the precision of any prediction formula is measured by the inverse of the residual variance

(contd....3)

Q.No.4 contd....

resulting from it. For example, if X_i and X_j are used for predicting X_0, then an estimated value of the precision of the prediction is $(1 - r^2_{0(ij)})s^2_0$ where s^2_0 = variance of X_0.

[8+8=16]

5. The following values were obtained from a sample of size 9 taken from a population which can be assumed to be bivariate normal. Let m_x, m_y, σ_x, σ_y and ρ stand for the population parameters as usual.

$\bar{x} = 1.34267$ $\quad s_{xx} = 0.014961$ (sample variance of X)

$\bar{y} = 0.71189$ $\quad s_{yy} = 0.000091$ (Sample variance of Y)

$s_{xy} = -0.000036$ (sample covariance of X and Y)

Test the following hypotheses :

(i) $m_x = 1.5$, $m_y = 1.0$

(ii) $m_x = 2m_y$

(iii) $\sigma_x = 2\sigma_y$ (For testing this hypothesis, first observe that it is equivalent to the hypothesis $\rho_{UV} = 0$, where $V = X + 2Y$, $V = X - 2Y$).

[5+5+5=15]

6. In order to investigate whether there were any real differences between two grades A and B of eggs, with respect to the three characteristics

yolk shadow (X_1)

yolk colour (X_2)

and albumen index (X_3)

25 eggs of grade A and 33 of grade B were observed. The following values were obtained.

	Means Grade A	Means Grade B	Pooled Corrected SS-SP Matrix (A)			$100 \times A^{-1}$		
X_1	7.16	10.30	147.33	9.29	150.27	.75684	-.11507	-.06945
X_2	13.92	15.30		150.81	-35.13		.67480	.01985
X_3	21.60	28.33			1669.33			.06686

(a) Find Fisher's discriminant function between the two grades of eggs.

(b) Test whether the variable X_2 is at all useful in discriminating between the two grades.

[9]

7. Assignments. [10]

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Multivariate Distributions and Tests
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 5.7.84. Maximum Marks : 100 Time: 3 hours.

Note : Attempt as many question as you can.

GROUP A

1.(a) Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ be IID each following $N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is positive definite. Writing $\underset{\sim}{\mu} = (\mu_1, \mu_2, \ldots, \mu_p)'$, suggest a test, based on Hotelling's T^2 statistic, for $H_0: \mu_1 = \mu_2 = \ldots = \mu_p$. Show that the appropriate statistic satisfies the relation

$$\frac{T^2}{N(N-1)} = \bar{\underset{\sim}{X}}' A^{-1} \bar{\underset{\sim}{X}} - \frac{(\bar{\underset{\sim}{X}}' A^{-1} \underset{\sim}{\varepsilon})^2}{\underset{\sim}{\varepsilon}' A^{-1} \underset{\sim}{\varepsilon}},$$

where $\bar{\underset{\sim}{X}} = \frac{1}{N} \sum_{\alpha=1}^{N} \underset{\sim}{X}_\alpha$, $A = \sum_{\alpha=1}^{N} (\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})'$, $\underset{\sim}{\varepsilon}^{(p\times 1)} = (1, \ldots, 1)'$.

(b) With the notations as in (a) above, let $\underset{\sim}{X}$ be an additional observation from $N_p(\underset{\sim}{\mu}, \Sigma)$, distributed independently of $X_1, \ldots, X_N$. What can you say about the distribution of

$$(\underset{\sim}{X} - \bar{\underset{\sim}{X}})' A^{-1} (\underset{\sim}{X} - \bar{\underset{\sim}{X}}) ?$$

[20+8=28]

2. Derive the sampling distribution of the sample multiple correlation coefficient on the basis of N IID observations from a multivariate normal population with a positive definite dispersion matrix.

[22]

3. Let
a)
$$\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \\ \vdots \\ \underset{\sim}{X}^{(q)} \end{pmatrix} \sim N_p(\underset{\sim}{\mu}, \Sigma),$$ with Σ positive definite.

Derive the likelihood ratio criterion for testing independence of $\underset{\sim}{X}^{(1)}, \underset{\sim}{X}^{(2)}, \ldots, \underset{\sim}{X}^{(q)}$.

p.t.o.

3.(b) Examine whether one can assert that under the null-hypothesis, a certain function of the above likelihood ratio criterion is distributed as the product of some independent beta random variables. [10+10=20]

GROUP 3

4. An anthropological survey was conducted in West Germany in a study relating to twins . In a specific enquiry on 10 families observed to possess two twin pairs of the same age - male-male and female-female - the mean weights and the (corrected) SS-SP matrix of all the 40 individuals were calculated as follows :

Means

Male Pairs		Female Pairs	
(1)	(2)	(3)	(4)
45.2	46.1	43.5	44.7

Corrected SS-SP Matrix

	(1)	(2)	(3)	(4)
(1)	27.5	16.6	9.6	9.8
(2)		26.8	9.7	10.2
(3)			34.5	16.5
(4)				34.9

Apply suitable multivariate tests to provide answers the following questions raised by the anthropological survey.

(1) Do the mean weights of twins of like sexes differ significantly ?

(2) Assuming that the mean weights are same for two twins of the same sex test if the common mean weight of male twins and the common mean weight of female twin differ significantly. [15]

5.(a) The following table gives the correlation co-efficients between weights of ears of wheat as harvested and weights of grains of wheat after threshing and cleaning, obtained from the results of a crop-cutting experiment in a certain district divided into 6 blocks. Test whether the correlations differ from block to block.

Blocks ->	1	2	3	4	5	6
Sample size ->	274	179	233	54	255	50
Corrl.^{n} Co-eff->	.9055	.8710	.92?C	.9365	.9508	.9780

(contd....3)

5.(a) contd.....

Make use of the z-transform, taking the distribution of z_i to be approx. $N\left(\xi_i, \frac{1}{n_i-3}\right)$, where ξ_i and n_i have the usual significance. Making appropriate assumptions of independence, define a suitable χ^2-statistic for the test.

As an estimate of the z-value of ρ, the common value (under H_0) of the correlation co-efficient, use

$$\xi = \frac{\sum_{i=1}^{6}(n_i-3)z_i}{\sum_{i=1}^{6}(n_i-3)} .$$

(b) The value of the multiple correlation co-efficient $r_0^2(12345)$ was obtained as 0.0856, from a sample of size 25. Test whether the multiple correlation of X_0 with $X_1, X_2, \ldots, X_5$ in the population can be taken to be zero.

[12+6=18]

6. The following table gives variances of stature (in cm) for Muslims in 8 different districts in Bengal, as obtained from the Bengal Anthropometric survey, 1935.

District	Sample size	Variance of stature
1	131	39.5668
2	59	24.4685
3	337	39.2262
4	77	24.8924
5	124	24.3490
6	299	38.3306
7	117	32.4325
8	39	33.4436

Assuming normality of the distribution of stature in each district, derive an LR test for the homogeneity of the variances. Then, making use of the fact that $-2 \log_e \lambda \sim$ approx χ^2 with appropriate degrees of freedom (λ being the LR criterion), draw suitable conslusions. [7]

7. Assignments. [10]

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Design of Experiments
PERIODICAL EXAMINATION

Date: 20.2.84. Maximum Marks: 100 Time: 3 hours.

Note : All questions carry equal marks. Answer any four questions. Assignments and practical carry 16 marks.

1. Explain the terms (a) treatments, (b) Units. What are the characteristics of a good design ? Define (i) Latin Square, (ii) Graeco-Latin Square. When should one use these in experimental design ? Provide the analysis of variance for a Latin Square design.

2. Let N(v) denote the maximum number of mutually orthogonal Latin Squares (MOLS) of order v. Prove that $N(v) \leq v-1$. Suppose $v = \prod_{i=1}^{m} p_i^{e_i}$ and $n(v) = \min \{p_i^{e_i}\} -1$, where p_i are primes. Show that a set of n(v) MOLS of order v can always be constructed. Describe a method of construction of a complete set of MOLS of order v where v is a prime or a prime power. Apply the method to obtain four 5 x 5 MOLS.

3. For a general block design
   (a) write down the normal equations and derive the reduced normal equations for $\hat{\underset{\sim}{\tau}}$,
   (b) define C matrix and show that $C\,E_{v1} = 0$,
   (c) show that $\underset{\sim}{l}'\underset{\sim}{\tau}$ is estimable if and only if it is a contrast,
   (d) prove that if the design is connected then the diagonal elements and the principal minors of all orders (1,2, ..., v-1) of C are positive.

p.t.o.

4. Consider a block design for 6 treatments in 7 blocks

$$\underset{\sim}{N} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Is it a connected design ?
(b) Is it an orthogonal design ?
(c) Is it a balanced design ?

Justify your answer in all the cases. Solve $C \hat{\underset{\sim}{\tau}} = \underset{\sim}{Q}$ for $\hat{\underset{\sim}{\tau}}$.

5. Define a B.I.B. (v,b,r,k,λ) and give an example. Prove that (i) rv = bk, (ii) λ (v-1) = r(k-1) and (iii) $v \leq b$. Show that for B.I.B. (v,b,r,k,λ),

$$C = \frac{vr-b}{v-1} \left(I_v - \frac{1}{v} E_{vv} \right).$$

Prove that for a symmetric B.I.B. design any two blocks have exactly λ treatments in common.

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B. Stat.(Hons.) Part III, 1983-84

Design of Experiments

SEMESTRAL-II EXAMINATION

Date: 21.5.84. Maximum Marks : 100 Time: 3 hours.

Note : Each question carries 25 marks. Attempt as many questions as you can. Maximum marks that you can obtain is 100.

1.(a) Discuss briefly fundamental principles of experimental design. (5)

(b) Describe the model; provide analysis of variance and a discussion on usefulness of each of the following :

(1) Completely Randomized Design (CRD) (4)

(2) Randomized Block Design (RBD) (6)

(3) Latin Square Design (LSD) (10)

2.(a) Define mutually orthogonal latin squares (MOLS) (2)

(b) Discuss the use of MOLS in experimental design (4)

(c) Let N(v) = Max. no. of MOLS of order v. Show that $N(v) \leq v-1$ (5)

(d) Show that a complete set of MOLS of order v can always be constructed where v is a prime or a prime power. Construct a complete set for v = 4. (10+4)

3. For a general block design

(a) State the normal equation and derive the reduced normal equation for $\hat{\tau}$. Define the C matrix and show that $E_{1v} C = 0$. (6)

(b) Define (i) correctedness, (ii) Orthogonality (iii) Balance (4)

(c) Show that if the design is connected then $l'\tau$ is estimable iff it is a contrast. (5)

(d) Prove that a connected design is balanced iff all the non-zero eigenvalues of C are equal. (10)

4. (a) Define a B.I.B.D. (v,b,r,k,λ). Is it orthogonal ? Justify your answer. Prove that $rv = bk$, $\lambda(v-1) = r(k-1)$, $v \leq b$ (4)

p.t.o.

4.(b) Show that

(i) $C = \frac{vr-b}{v-1} \left(I_v - \frac{1}{v} E_{vv} \right)$ for a B.I.B.D. (9)

(ii) for a symmetric B.I.B.D. to exist v is even, it is necessary that $r-\lambda$ be a perfect square. (6)

(iii) for a symmetric B.I.B.D. any two blocks have exactly λ treatments in common. (6)

5.(a) Describe the methods of construction of B.I.B.D. through finite Geometrics. Give examples. (16)

(b) Define and give examples of
(i) Complementary design, (ii) Residual design,
(iii) Derived design. (3+3+3)

6.(a) Discuss usefulness of ' Confounding' and 'Fractional replication' in factorial plans. (7)

(b) Construct a (2^6, 2^3) plan such that none of the main effects and two factor interactions is confounded. (4)

(c) Construct a (2^5,2^2) plan in minimum number of replicates so as to ensure balancing of all 3- and 4-factor interactions (without confounding any main effect or 2-factor interaction). (7)

(d) Explain how the ' identity relations' and ' alias sets' arise in 2^{-p} fractional replicate of a 2^n factorial layout. (7)

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Design of Experiments
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 7.7.84. Maximum Marks: 100 Time: 3 hours.

Note : Each question carries 25 marks. Attempt as many questions as you can. Maximum marks you can obtain is 100.

1.(a) Define Latin Square, Graeco-Latin Square, Mutually Orthogonal Latin Squares (MOLS) and discuss their application in experimental design. (7)

(b) Let $N(v)$ = max. no. of MOLS of order v. Show that $N(v) \leq v-1$. (4)

(c) Suppose $v = \prod_{i=1}^{m} p_i^{e_i}$ and $n(v) = \min \{p_i^{e_i}\} - 1$ where p_i are distinct primes and e_i are positive integers. Show that $N(v) \geq n(v)$. (11)

(d) Construct 4 MOLS of order 5. (3)

2.(a) Derive the ANOVA for a general block design (under the usual assumptions of intra-block analysis). (15)

(b) State and prove the connectedness theorem. (6)

(c) Show that an Incomplete Block Design cannot be orthogonal. (4)

3.(a) Define a B.I.B.D. (v,b,r,k,λ) and prove that $rv=bk$, $\lambda(v-1) = r(k-1)$, $v \leq b$. (4)

(b) Describe the method of symmetrically repeated differences for construction of B.I.B.D.'s. (9)

(c) Show that a B.I.B.D. with $v=b=4t+3$, $r=k=2t+1$ and $\lambda=t$ can always be constructed when $4t+3$ is a prime or a prime power. (6)

(d) Show that B.I.B.D. with $v = 4t+1$, $b = 8t+2$, $r = 4t$, $k=2t$ and $\lambda = 2t-1$ can always be constructed when $4t+1$ is a prime or prime power. (6)

p.t.o.

4.(a) Explain the terms 'Main effects' and 'Interactions' in factorial designs. Give examples. (5)

(b) Describe Yates' Algorithm for the analysis of a 2^n factorial plan. How is it modified for confounded design and fractional replicates? (12)

(c) Analyse the following data of experiments with sprouts. (8)

<u>Yields in Pounds of Salable Sprouts</u>

<u>Repl.1</u>

Block 1 : np = 43.81, nm=58.88, pm=46.11, (1) = 38.62

Block 2 : m = 40.49, npm = 61.55, p = 32.75, n = 55.07

<u>Repl.2</u>

Block 3 : nm = 50.43, pm = 52.31, np = 49.62, (1) 40.26

Block 4 : p = 32.36, npm= 48.49, m = 51.94, n = 53.86

<u>Repl.3</u>

Block 5 : n = 47.37, p = 37.25, npm = 46.87, m = 46.94

Block 6 : pm = 39.30, nm = 49.93, np = 51.43, (1) = 39.23

m = poultry manure, n = sulphate of ammonia, p = superphosphate.

5.(a) Explain the concepts of 'Confounding' and 'Fractional replication' and the relationship between the two. (8)

(b) Obtain a ($2^7, 2^3$) plan such that none of the main effects and 2-factor interactions is confounded. Explain how you develop the other blocks from the key block. (8)

(c) Do you prefer partial confounding to total confounding ? Justify your answer. (2)

(d) What is the definition of 'balance' in factorial experiments? (2)

(e) Obtain a 2^{-2} replication of 2^5 experiment. Is it possible to avoid confounding among the main effects and 2-factor interactions in this case ? (5)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Nonparametric and Sequential Methods
PERIODICAL EXAMINATION

Date: 27.2.84. Maximum Marks : 100 Time: 3 hours.

Note : 1. Answer any FIVE questions.
2. All questions carry equal marks
3. Show that you understood the concepts rather than giving mechanical answers.

1. Define a sequential procedure for testing a null hypothesis against an alternative hypothesis. Obtain the expressions for the OC and ASN functions of the test you propose. Show further that a necessary condition for the ASN function to be finite is that the test terminates with probability one. ([illegible])

2. Define the SPRT of given strength for a simple hypothesis against a simple alternative. Show that the test terminates with probability one under suitable conditions (to be stated by you).
Show further that the moment generating function of the decisive sample number is finite. (5+8+7)

3.(a) Show that the risks of error $(\alpha(\theta_0), \beta(\theta_1))$ associated with any SPRT $S(b,a)$ for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ satisfy, for any fixed $-\infty < b < a < \infty$, the inequalities

$$\log \frac{\beta(\theta_1)}{1-\alpha(\theta_0)} \leq \min.(0,b), \text{ and } \log \frac{1-\beta(\theta_1)}{\alpha(\theta_0)} \geq \max(0,a)$$

When do strict equalities hold ?

(b) Using the result in (a) obtain the approximate values of a and b for the SPRT $S(b,a)$ of given strength (α,β). How do these approximations affect the risks of the resulting SPRT ?

p.t.o.

4.(a) Obtain the Wald's approximation for the OC function of an arbitrary SPRT $S(b,a)$ for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ (state clearly the main result you use in the derivation. Do not prove this result).

(b) Suppose $\{f(x,\theta),\ \underline{\theta} \leq \theta \leq \bar{\theta}\}$ is an MLR family in $T(x)$. Show that any SPRT $S(b,a)$ for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, $\underline{\theta} \leq \theta_0 < \theta_1 \leq \bar{\theta}$ has a non increasing OC function.(State precisely the result you use in proving this). [12+8]

5.(a) Let S be any closed sequential procedure for $H_0: \theta \in w$ against $H_1: \theta \in H - w$. Let $t(x)$ be any arbitrary function with $E(\,|t(x)|\,:\,\theta\,) < \infty$ $\forall\ \theta$. With usual notation for $\underline{n}$, let

$$T_{\underline{n}} = \sum_{1}^{n} t(x_i) \text{ and } E(\underline{n} : \theta) < \infty \quad \forall\ \theta.$$

Then show that $E(T_{\underline{n}} : \theta) = E(t(X) : \theta)\, E(\underline{n} : \theta)$.

(b) Using the above result, show that if $t(x)$ is $\log \dfrac{f(x,\theta'')}{f(x,\theta')}$ and $L(\theta)$ is the OC function of the sequential test S

$$-E(\underline{n}: \theta')\, E(t(x):\theta') \geq L(\theta') \operatorname{Log} \frac{L(\theta')}{L(\theta'')} + (1-L(\theta')) \log \frac{1-L(\theta')}{1-L(\theta'')}$$

[10+10]

6. Consider the family $N(\mu,\sigma^2)$ where μ is known and $\sigma^2 > 0$ is unknown. Obtain the SPRT $S(b,a)$ for $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 = \sigma_1^2$ of strength (α,β). Show that the test terminates with probability one for all σ^2 and obtain the approximate OC and ASN functions of the test.

[20]

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Nonparametric and Sequential Methods
SEMESTRAL-II EXAMINATION

Date: 18.5.84. Maximum Marks: 100 Time: 3 hours.

Note : Answer any FIVE questions.

1. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. observations on a random variable X with p d f $p(x:\theta)$. Show that for any sequential procedure with a given stopping rule and DSN n, for any measurable function $z(x)$,

$$P(n < \infty : \theta) = 1 \Rightarrow E\left\{e^{t U_n} [\phi(t)]^{-n} : H_t\right\} = P(n<\infty : H_t)$$

where under H_t the p.d.f of X is $p(.:H_t) = \frac{e^{-tz(x)}}{\phi(t)} p(.:\theta)$,

$\phi(t) = E(e^{tz(X)} : \theta)$ and $U_n = \sum_1^n z(X_i)$.

(b) Hence deduce the Fundamental identity of Sequential Analysis.

(c) Hence or otherwise obtain Wald's approximate expressions for the OC and A S N functions of the S P R T S(b,a) for testing a simple hypothesis against a simple alternative.

(8+4+4+4)

2. Let X be a random variable with exponential distribution $\exp(\lambda)$, where $\lambda > 0$ is unknown. Derive the S P R T of strength (α,β) for testing the hypothesis $H_0: \lambda \leq \lambda_0$ against $H_1: \lambda \geq \lambda_1$, $0 < \lambda_0 < \lambda_1 < \infty$. Show that the test terminates with probability one for all $\lambda > 0$. Obtain the Wald's approximations for the OC and ASN functions of the test and their values at the usual values of λ. (20)

3. Define the Kolmogorov - Smirnov one sample statistic for testing the hypothesis that $X_1, \ldots, X_n$ have the common c d f $F(x) = F_0(x)$ which is continuous against the alternative that $F(x) \geq F_0(x)$ for all x, with strict inequality for atleast one x. Indicate the distribution of the test statistic for any finite n. Obtain a suitable transformation of the statistic which is asymptotically distributed as $\chi^2(2)$ under H_0. Show that the distribution of the statistic does not depend on $F_0(x)$. (4+8+4+4)

p.t.o.

4.(a) Define a general linear rank statistic that is used in nonparametric testing problems and obtain its mean and variance. Show that the distribution of the statistic is symmetric about its mean under suitable conditions (to be stated by you).

(b) Show that the Van Der Waerden's test for testing the equality of two distributions against the alternative that they differ in their location parameter is a linear rank statistic and obtain its mean and variance under the null hypothesis. (12+8)

5. Four different fertilizers are used on different fields and the yield per acre is calculated for each field. Based on the following data test that there is no difference in the average yields due to different fertilizers.

Fertilizer	Yields
A	80.5, 87, 86.1, 82.1, 79.3, 84.2, 77.1, 77.6,
B	90.1, 83.4, 32.4, 84.9, 87.1, 89.3, 86.2
C	89.4, 93.6, 90.8, 87.6, 93.7, 82.9,
D	81.5, 87.9, 80.4, 83.1, 87.4, 85.0, 84.7, 81.0.

(20)

6. Define Spearman's rank correlation coefficient and obtain a computational formula for the same. Show how you would adjust the formula if some observations have the same values. Indicate how you would obtain its mean and variance and asymptotic distribution.

(6+4+10)

INDIAN STATISTICAL INSTITUTE
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NONPARAMETRIC AND SEQUENTIAL METHODS
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 7.7.84. Maximum Marks : 100 Time: 3 hours.

Note : Answer any FIVE questions.

1. Let X be a random variable with pdf $f(x:\theta)$, where θ is unknown. Show that the SPRT $S(b,a)$ of strength (α,β) for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, terminates with probability tending to one as the sample size increases, under suitable conditions (to be precisely stated by you). Show also that the moment generating function $\psi(t)$ of the decisive sample number $\underline{n}$ is finite in a non degenerate interval of t and consequently the ASN function is finite both under H_0 and H_1.
(10+7+3)

2. Let X be a Bernoulli random variable with $P(x=1) = p$, where p is unknown. Derive the SPRT S of strength (α,β) for testing $H_0: p \leq p_0$ against $p \geq p_1$, $0 < p_0 < p_1 \leq 1$. Show that the test terminates with probability one under any $p > 0$. Obtain the Wald's approximations for the OC and ASN functions and also obtain their values for different values of p. (20)

3. An ordered sequence of n elements of two types, n_1 of the first type and $n_2 (= n-n_1)$ of the second type, is observed. Derive a test for randomness of the sequence based on the number of runs in the sequence. Obtain the exact distribution of the total number of runs under the null hypothesis and indicate its asymptotic distribution. (20)

4. The following data summarize the average monthly electric power in millions of kwh used in two industries. Can you regard that the power consumption has the same distribution in both the industries

	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
A	115.7	103.9	101.4	91.8	85.9	80.9	83.4	90.5	98.1	108.1	116.1	123.7
B	120.4	107.4	104.0	93.3	83.3	82.4	90.5	96.2	88.5	112.5	114.0	120.0

(20)

p.t.o.

5. Derive a linear rank test for testing $H_0: F_X(x) = F_Y(x)$ $\forall x$ against the alternative $H_1: F_X(x) = F_Y(\theta x)$ where $0 < \theta \neq 1$. Obtain the mean, variance and the asymptotic distribution of the test statistic under the null hypothesis. (20)

6. Four different methods of growing corn were randomly assigned to different fields and their yield per acre was computed for each field. Based on the data test whether there is any difference in the median yield of the four methods used.

Method	Yield
A	83, 91, 94, 89, 89, 96, 91.5, 92, 90
B	90.5, 90, 81, 83.5, 84, 83, 88, 93, 89, 84
C	101, 100, 93.5, 95, 96, 92.5, 98.5
D	78, 82, 82, 77, 79, 99, 80.

(20)

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

Optimization Techniques
PERIODICAL EXAMINATION

Date: 29.2.84. Maximum Marks : 100 Time: 3 hours.

Note : This paper carries a total of 120 marks. Answer as much as you can. If you score more than 100, then your actual score will be 100 marks.

1.(a) By geometric interpretation or otherwise solve the following linear programming problem :

$$\text{minimise} \quad 2x_1 + 3x_2$$

$$\text{subject to} \quad x_1 + x_2 \leq 4$$
$$6x_1 + 2x_2 \geq 8$$
$$x_1 + 5x_2 \geq 4$$
$$0 \leq x_1 \leq 3$$
$$0 \leq x_2 \leq 3.$$

(b) Give the dual, with unrestricted variables, of the following LP problem:

$$\text{Maximize} \quad 3x_1 + 4x_2 + 5x_3 + 8x_4$$

$$\text{subject to} \quad 2x_1 + 3x_2 + 4x_3 \leq 5,$$
$$3x_1 + 4x_2 + 5x_3 \leq 6,$$
$$5x_1 - 6x_2 + 7x_3 - 8x_4 \leq 7$$
$$x_i \geq 0, \ 1 \leq i \leq 4.$$

[12+8=20]

2.(a) Prove, for the general maximum LP problem, that if x,y are feasible solutions of the primal and its dual respectively, then $c.x \leq y.b$; further, if equality holds, then x,y are optimal solutions for the respective problems.

p.t.o.

2.(b) Prove that $x_1 = 1 = x_2 = x_4$ and $x_3 = 0$ is an optimal solution for the standard LP problem below

Maximize $x_1+x_2+x_3+x_4$

subject to : $x_1+x_2 \leq 3$,

$x_3+x_4 \leq 1$

$x_2+x_3 \leq 1$,

$x_1+x_3 \leq 1$,

$x_3+x_4 \leq 3$,

$0 \leq x_i$, $1 \leq i \leq 4$.

Find an optimal solution for the dual problem.

[10+10=20]

3.(a) Is it true that every nonempty closed bounded convex subset of $\mathbb{R}^n$ has at least one extreme point ? Justify your answer.

(b) Prove that every extreme point of the set of all feasible solutions of the LP problem

Maximize $c.x$,

subject to $Ax = b$

$x \geq 0$,

and rank $(A) = m$, where A is an $m \times n$ matrix is a basic solution of the LP problem.

[10+10=20]

4.(a) Prove that exactly one of the following two alternatives holds.

Either $Ax \leq b$ has a nonnegative solution.
Or $yA \geq 0$ and $y.b < 0$ have a nonnegative solution.

(b) Exhibit a solution of the inequalities

$$5x_1 - 4x_2 \leq 7,$$
$$-3x_1 + 3x_2 \leq -5.$$

Do they have a nonnegative solution ? Justify your answer.

[10+10=20]

p.t.o.

5.(a) Reduce the solution (2,3,1) of the following LP problems to a basic feasible solution.

$$\begin{pmatrix} 2 & 1 & 4 \\ 3 & 1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 14 \end{pmatrix}$$

(b) Solve the following LP problem by finding all the basic feasible solutions.

Maximize $2x_1 + 3x_2$,

subject to
$$\begin{pmatrix} 4 & 2 & 1 \\ 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

[8+12=20]

6.(a) Describe the simplex method to solve a canonical LP problem without degeneracy.

(b) If x is a basic feasible solution, of the LP problem mentioned in 3(b), based on the basis B and, with usual notation, $Z_j - C_j \geq 0$ for all jth columns of A not in B, then prove that x is an optimal solution for the primal problem. Find an optimal solution of the dual in terms of B^{-1}. [10+10=20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.)III Year, 1983-84

Optimization Techniques
SEMESTRAL-II EXAMINATION

Date: 14.5.84. Maximum Marks: 100 Time : 3 hours.

Note : This paper carries a total of 120 marks. Answer as many questions as you can. If you get more than 100 marks your actual score will be 100.

1.(a) State and prove the equilibrium theorem.

(b) Formulate the transportation problem as a linear programming problem and write down its dual. [12+8=20]

2.(a) Outline the principles involved in the simplex method in the non-degenerate case, explaining the relevant terms and phrases.

(b) Maximize $0.75\,x_1 - 20\,x_2 + 0.5\,x_3 - 6\,x_4$

subject to
$$0.25\,x_1 - 8\,x_2 - x_3 + 9\,x_4 \le 0,$$
$$0.5\,x_1 - 12\,x_2 - 0.5x_3 + 3x_4 \le 0,$$
$$x_3 \le 1,$$
$$x_1,\ x_2,\ x_3,\ x_4 \ge 0.$$
[8+12 =20]

3.(a) Prove that a flow f in a network with a unique source and a unique sink is maximum if and only if there is no f-incrementing path. Also deduce the max-flow min-cut theorem from the above result.

(b) Find the maximum value of a flow in the network of Fig.1 below from the source s to the sink t. Justify your answer.

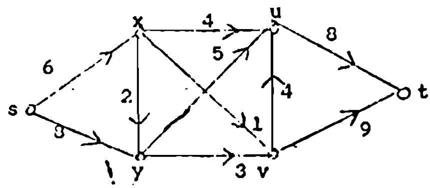


Fig. 1. A net-work with capacities of arcs as shown.

[12+8=20]

p.t.o.

(a) State and prove the Konig-Egervary theorem about matrices with zero-one entries.

(b) Prove that every m×n Latin rectangle with $1 \leq m < n$ based on n symbols can be extended to a Latin square of order n.

[12+8=20]

5.(a) In the network of Fig. 2 below nodes x_1, x_6 are the sources with supplies $a(x_1) = 3$, $a(x_6) = 5$; and nodes x_4 and x_8 are the sinks with demands $b(x_4) = 4 = b(x_8)$. The numbers on the arcs are the capacities. Determine whether if there is a feasible flow in the network satisfying these supplies and the demands. Justify your answer.

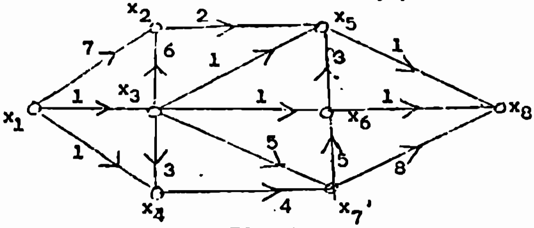


Fig. 2

(b) Five families would like to go for a picnic in 10 cars. There are r_i people in the i-th family and s_j seats in the j-th car, $1 \leq i \leq 5$; $1 \leq j \leq 10$. Can seating be so arranged that no two members of the same family travel in the same car ? [r_i is the i-th term of the vector (8,8,5,5,4) and s_j is the j-th term of the vector (5,5,4,4,4,3,2,1,1,1) ?] Justify your answer.

[10+10=20]

6.(a) State the fundamental theorem of two person zero-sum matrix games and formulate the game problem as an LP problem.

(b) Find the value of the matrix game with payoff matrix A.

$$A = \begin{pmatrix} p & -1 & -1 \\ -1 & p & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad p > 0.$$

[10+10=20]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year, 1983-84

Elective-5: Economics

PERIODICAL EXAMINATION

Date: 2.3.84. Maximum Marks : 100 Time: 3 hours.

Note : Answer questions 1 and 4, and any one of questions 2 and 3. Marks for each question are given in brackets.

1. The following data give Lorenz ratios, percentages of populations and averages for the distributions of consumer expenditure in rural and urban India for the years 1957-58 and 1967-68 :

	1957-58			1967-68		
	Lorenz ratio	Average expenditure (Rs.)	Percentage of population	Lorenz ratio	Average expenditure (Rs.)	Percentage of population.
Rural India	0.334	100	75	0.290	125	70
Urban India	0.359	150	25	0.336	175	30

(i) Compute the share of the top 10 per cent of the population in both sectors for the two periods. Find in each case the percentages of the population from bottom enjoying 10 per cent share of the total expenditure.

(ii) If the poverty line is given as Rs.55, estimate the percentages of poor people in India for the two periods.

You may assume that the distribution of consumer expenditure in India is lognormal. [20+20]

2. What is Pareto's law of income distribution ? Obtain the density function, the equation for the Lorenz curve and the Lorenz ratio for the Pareto distribution. Comment briefly on the universality of Pareto's law. [4+2+9+5+10]

p.t.o.

3. Write short notes on the following :

   (i) Properties of Lorenz curve of lognormal distribution.

   (ii) Measures of concentration in business/industry versus Lorenz ratio.

   (iii) Three parameter lognormal distribution.

[10+10+10]

4. The table below gives the arithmetic means and the corrected sums of squares and products of three variables, x_1, x_2 and y calculated on the basis of 9 observations :

	Corrected sum of Squares and products			Arithmetic means
	x_1	x_2	y	
x_1	650	−112	874	113
x_2		648	−79	106
y			1260	117

(i) Estimate the equation $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ by ordinary least squares. Compute standard errors of the estimates of β_1 and β_2, R^2 and $\bar{R}^2$.

(ii) Set up an analysis of variance table and test the effect of x_1 and x_2 on y. [20+10]

INDIAN STATISTICAL INSTITUTE
B. Stat. (Hons.) III Year, 1983-84
Elective-5 : Economics
SEMESTRAL-II EXAMINATION

Date: 26.5.84. Maximum Marks : 100 Time: 3 hours.

Note : Answer any five questions. Marks allotted to each question are given in brackets.

1.(a) State and prove the moment distribution property of the two-parameter lognormal distribution. Use this property to obtain the expressions for the Lorenz curve and the Lorenz ratio of the distribution.

(b) How will you find the Lorenz ratio of a two-parameter lognormal distribution if (i) the mean (ii) the coefficient of variation of the distribution is given ?

[2+5+3+6+2+2]

2.(a) Examine critically Prais-Houthakker's treatment of household composition in engel curve analysis based on family budget data.

(b) For a group of households the following engel curve for food expenditure is estimated

$$w = .050 + .006 \left(\frac{x}{n} \right) + .100\ C$$

where w : engel ratio for food; x : household total consumer expenditure; n : number of persons in the household, and c: number of children in the household.
A household has x = Rs.500, n = 5 and c = 2. How much additional total expenditure would the household require to maintain its level of living if another child is added to the household ?

[13+7]=[2]

3.(a) Explain the following terms : engel curve, engel ratio, and engel elasticity.

p.t.o.

3.(b) The following data relate to per capita household expenditure on cereals etc ($\bar{y}_j$), total consumer expenditure ($\bar{x}_j$) and the estimated percentage of population (p_j) for a number of monthly per capita total expenditure classes :

Monthly per capita total expenditure class(Rs.)	Percent of population (p_j)	Average per capita expenditure on cereals etc.($\bar{y}_j$)	Average per capita expenditure on all items ($\bar{x}_j$)
(1)	(2)	(3)	(4)
0 - 11	20.98	5.01	8.58
11 - 15	21.59	6.37	12.95
15 - 21	25.28	8.48	17.25
21 - 28	14.31	10.02	23.34
28 - 43	12.94	11.45	35.31
43 - 75	4.90	15.25	62.05

Fit a constant elasticity engel curve to the above data. What can you say about the goodness of fit of the estimated engel curve ? [5+15]=[20]

4.(a) Discuss the 'identification problem' that may arise in the context of estimation of demand function for a commodity on the basis of time series data on aggregate market transactions and market price. How is the problem overcome in practice ?

(b) Consider the following model

$$q_d = \alpha + \beta p + \gamma y + u$$
$$q_s = \theta + \delta p + v$$
$$q_d = q_s$$

where q_d: per capita demand for a commodity; q_s: per capita supply of the commodity; p: price of the commodity, y: per capita income of consumers, u and v are random disturbances.

Examine if the demand equation in this model is identified.

[15+5]=[20]

(contd....3)

5. What is multicollinearity ? Discuss the problem of multicollinearity as faced in the estimation of a demand function from time series data. How can you overcome this problem ? [20]

6. Examine the properties of the Cobb-Douglas Production function. Also, briefly indicate the problems of estimation of an aggregate Cobb-Douglas Production function. [20]

7. Write notes on any two :

(i) Criteria for choice of an engel curve form;

(ii) Cobweb model of market for an agricultural product;

(iii) Least squares inconsistency in the context of demand analysis. [10x2]=[20]

1983-84 | 3S2(b)

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1983-84

Elective-5 : Economics
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 6.7.84. Maximum Marks : 100 Time: 3 hours.

Note : Answer any five questions. Marks for each question are given in brackets.

1.(a) Describe the method of construction of Lorenz curve. How will you estimate the Lorenz ratio graphically from the Lorenz curve ?

(b) Describe different uses of the Lorenz curve.

(c) Discuss the properties of the Lorenz curve of a two-parameter lognormal distribution. [5+3+4+8=20]

2.(a) Give examples of two distributions for each of the following cases satisfying (i) $I_1 < I_2$ and $V_1 < V_2$ (ii) $I_1 < I_2$ and $V_1 > V_2$ and (iii) $I_1 < I_2$ and $V_1 = V_2$ where I_1 and I_2 are Lorenz ratios and V_1 and V_2 are variances of the two distributions respectively.

(b) From the following observations on income of 8 persons

Serial No.	1	2	3	4	5	6	7	8
Income(Rs.)	400	300	100	700	500	200	600	800

Calculate (i) Lorenz ratio

(ii) coefficient of variation

(iii) relative mean deviation. [3+4+2+5+3+3=20]

3.(a) What is the price elasticity of demand for a commodity ? Distinguish between (i) normal and Giffen goods ; (ii) Substitute and complementary goods.

(b) Describe briefly the problems that you may encounter in estimating the demand function for a commodity on the basis of a time series of market statistics relating to the commodity. [5+15=20]

p.t.o.

4.(a) What is a production function ? Explain the concept of elasticity of substitution in the context of production.

(b) What are returns to scale ? How would you infer about the returns to scale in a particular industry on the basis of an estimated Cobb -Douglas Production function for the industry ?

(c) Describe briefly the statistical problems that may arise in the estimation of a Cobb -Douglas production function on the basis of time series data. [6+6+8 = 20]

5. What is an engel curve ? What theoretical properties would you like the algebraic form of an engel curve to possess ? Describe the statistical criteria for choosing the best engel curve form for a given set of data. [20]

6. Write notes on <u>any two</u> :

(i) Universality of Pareto's law ;

(ii) Identification problem in demand analysis ;

(iii) The problem of least squares in consistency in the estimation of a production function.

[10x2 =20]

1983-84 | 3P1

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1983-84

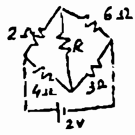
Elective 5: Physical and Earth Sciences
PERIODICAL EXAMINATION

2.3.84. Maximum Marks: 100 Time: $3\frac{1}{2}$ hours.

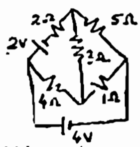
Note : Any five questions to be answered. All questions carry equal marks.

1.(a) State and prove the maximum power transfer theorem assuming sinusoidal voltage sources.

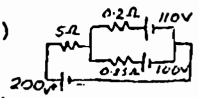
(b) In the circuit shown, for what value of R is maximum power transferred to it ?



2.(a) State and explain Kirchhoff's laws. In the network shown, compute the current through the 3Ω resistor and the potential difference across the 1Ω resistor.



(b)



Using the superposition theorem find the current through the 5Ω resistor.

3.(a) Obtain a star network equivalent to a given delta network and vice-versa.

(b) Find the resistance between points A and B of the curcuit shown.

4.(a) A steady voltage V is suddently applied across a series R-L curciut. After reaching steady state conditions, the voltage source is suddenly replaced by a short-circuit. Derive expressions for the current through the circuit as a function of time for both the cases.

(b) A coil of resistance 10Ω and inductance 0.4H is connected to a 100 v D.C. supply. Compute

(i) The rate of change of current at the instant of connecting the 100 V supply.

(ii) The steady state current.

(iii) The time taken by the current to rise to half its final value.

p.t.o.

5.(a) Show analytically that the current and voltage are not in the same phase when a sinusoidal voltage source is used to excite

(i) a purely capacitive circuit.

(ii) a purely inductive circuit.

(b) A circuit consists of a sinusoidal voltage source, a resistor and a coil all connected in series. The power dissipated inthe resistor is 500 W and the drop across it is 100V. The power dissipated in the coil is 100 W and drop across it is 50 V. Find the reactance and resistance of the coil and also the supply voltage. Draw a vector diagram.

6.(a) Two coils A and B are connected in series across a 240V, 50 C/S supply. The resistance of A is 5Ω and the inductance of B is 0.015H. If the input from the supply is 3KW and 2KVAR, find the inductance of A and resistance of B. Compute the voltage across each coil.

(b) A 100 W, 120V lamp is to be operated from a 240V, 50C/S. supply. Give details of the simplest possible manner in which this could be done using

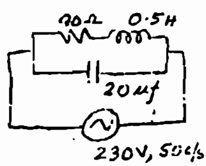
(i) a resistor, (ii) a capacitor, What power factor would be presented to the power supply in each case ? Which method is preferable and why ?

7.(a) For the circuit shown find

(i) the total current (r.m.s.value)

(ii) power factor of the parallel circuit.

(iii) the total power drawn from the source. Draw the vector diagram.



(b) Find analytically the RMS values of

(i) a sinusoidal voltage waveform with peak value of E_m

(ii) a saw-tooth waveform with a peak value of E_m and time period of 2 seconds.

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year, 1983-84

Elective-5 : Physical and Earth Sciences

SEMESTRAL-II EXAMINATION

Date: 26.5.84. Maximum Marks : 100 Time: 3 hours.

Note : Answer any five. All questions carry equal marks.

1. Derive conditions for maximum impedance in a parallel R-L-C circuit (L and R in series and the combination in parallel to C) when
   (i) w is varied
   (ii) L is varied
   (iii) C is varied

   Also show that maximum impedance is obtained at unity power factor when C is varied.

2. (a) Show that the bandwidth of a series resonant circuit is inversly proportional to its quality factor Q. Derive an expression for the impedance of this circuit at a frequency f in terms of Q and δ where $\delta = (f - f_r)/ f_r$.

   (b) A series circuit having a capacitance of negligible resistance and a 120 Mh coil of 18Ω resistance is resonant at 1MHZ, and is connected to a generator of 1 volt at 1MHZ. Compute
   (i) the voltage across the capacitor
   (ii) current at resonance and at 10 MHZ above it
   (iii) bandwidth in cycles.

3. When two coils A and B are connected in series and a current of 5A passed through them, the potential difference across A and B are respectively 80V and 75V, their respective power factors being 0.5 and 0.8 lagging. If these coils are connected in parallel across a 200V, 50HZ supply and a capacitor of 170 μf is connected across them, compute

   (i) the p.f. of the parallel combination.

   (ii) the current taken from the supply.

p.t.o.

4\. A 250/500V transformer gave the following test results :

S.C. test with low voltage winding short circuited
- 20V; 12A; 100W.
O.C.test : 250V, 1A, 80W on low voltage side.

Determine the circuit constants and compute the applied voltage and efficiency when the output is 10A at 500V and 0.8 lagging power factor.

5.(a) Two transformers of unequal secondary voltage are operated in parallel to supply power to a load. Derive expressions for current drawn from the transformer. Make necessary assumptions and draw phasor diagrams.

(b) Two transformers supply in parallel a secondary load of 1000A at 0.8 pf lagging. For each transformer the secondary e.m.f. on open circuit is 3300V and the total leakage impedance refered to secondary are 0.1 + j 0.2 and 0.05 + j 0.4 ohm respectively. Determine the output current of each of the transformers.

6.(a) Draw the circuit of a half wave rectifier. Derive expressions for Idc, Vdc, I_{rms} and % regulation. If a capacitor is connected across the load, find an expression for the voltage across the load using approximate analysis.

(b) A diode bridge sampling gate is to used to sample a signal that can have a maximum peak to peak value of 5V. Design the gate. Derive all necessary expressions.