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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

Stochastic Processes - 2

PERIODICAL EXAMINATION

Date: 27.3.1984 Maximum Marks: 100 Time: 3 hrs.

Note: All Markov Chains considered here are discrete time -parameter and have stationary transition probabilities.

Answer all questions.

1. Prove that there is no stationary initial distribution for a Markov Chain for which

$$p_{i,j} = \frac{\lambda^{j-i} e^{-\lambda}}{(j-i)!} \quad \text{if} \quad j \geq i \geq 0,$$
$$= 0 \quad \text{otherwise}$$
[10]

2. Consider a Markov Chain, the first column of whose one-step transition matrix is $\{q_0, q_1, \ldots\}$ with $p_{i,i+1} = 1-q_i$, $i \geq 0$. When is this Chain irreducible ? Show that if the Chain is irreducible, the states are transient iff $\Sigma q_j < \infty$. Find out when the states are null recurrent and when they are positive recurrent ? [20]

3. Consider a simple random walk on $\{0, 1, \ldots, a\}$, where 0 and a are absorbing states, $p_{i,i+1} = p$, $p_{i,i-1} = q = 1-p$, $i = 1, 2, \ldots, a-1$ and $p \neq \frac{1}{2}$. Find the probability of absorption in 0 given that the random walk starts in i. Also compute the expected time until absorption (in 0 or a) given that the Chain starts in i.

(10+10) = [20]

4. Let $\{(X_n, Y_n), n \geq 0\}$ be a symmetric random walk on the lattice points of $\mathbb{R}^2$.

 (a) Prove that all states are recurrent.

 (b) Let $D_n^2 = X_n^2 + Y_n^2$ and suppose that the random walk starts at the origin. Compute $E(D_n^2)$.

(10+10) = [20]

p.t.o.

5. Let $\underset{\sim}{P}$ be the one-step transition matrix of an irreducible finite Markov chain all of whose states are aperiodic. Prove that there is $n \geq 1$ such all entries in the power $\underset{\sim}{P}^n$ are positive.

[10]

6. (a) Define the period of an essential state.

   (b) If d is the period of an essential state i, prove that

$$d = \text{g. c. d.} \left\{ n \geq 1 : f_{ii}^{(n)} > 0 \right\}.$$

   (c) Show that all states in an essential class have the same period.

   (d) Describe the decomposition of an essential class into cyclic subclasses.

(5+5+5+5) = [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

Sample Surveys
PERIODICAL EXAMINATION

Date: 29.8.84. Maximum Marks : 100 Time: 3 hours.

Note : Questions 1 to 4 carry a total of 110 marks. Answer as much as you can. The maximum you can score is 90. Marks allotted to each question are given in brackets [ ].

1.(a) What do you understand by the terms 'inclusion probability' π_i of a unit U_i and 'joint inclusion probability' π_{ij} of a pair of units U_i and U_j, $i \neq j$, for a sampling design ? (2+3)=[5]

(b) For a Probability Proportional to Size With Replacement (PPSWR) design of sample size n from a population of size N, write down the values of π_i and π_{ij}. Hence or otherwise obtain the π_i, π_{ij} values for a Simple Random Sampling With Replacement design of size n. (3+4+1+2)=[10]

(c) In a recent survey of 205 households in an area selected by SRS without replacement it was found that in 124 households 'Olympics' were watched on the T.V. sets for more than 30 hours. Estimate the proportion of households that watched the programme for this duration in the population and obtain an estimate of its sampling error. Comment on the sampling technique used in this survey. (4+5+3) = [12]

2.(a) How do you select a linear systematic sample of size n from a population of size N ? [4]

(b) Suggest an unbiased estimator for the population total of a characteristic y based on the above design. [6]

(c) When the values of the y-characteristic are known to be of the form $Y_i = \alpha + \beta i$ and when the population size is a multiple of the sample size, would you prefer systematic sampling to simple random sampling ? Give reasons. [10]

(d) On the basis of a single systematic sampling it is known that the variance of the estimator of the population mean is not estimable unbiasedly. Suggest any two methods of obtaining approximate variance estimators. (6+6)=[12]

p.t.o.

3.(a) When do you use Probability Proportional to Size (PPS) sampling technique ? [5]

(b) A sample of 7 square plates is to be drawn from a factory that produced 73 such plates whose areas are all known. It is felt by the quality control engineer that the sample of plates should be selected with probability of selection of a plate proportional to the length of the edge of the plate and with replacement.. Suggest a simple method of selection. [6

The data given below refers to a sample of 7 plates selected from the 78 plates by the PPS WR design, size(x) being the length of the edge of the plate. Column (3) refers to the strength of the plates in suitable units.

(1) sampled plate	(2) length of the edge x	(3) strength of the plat y
1	21	105.2
2	101	524.3
3	14	73.1
4	6	31.2
5	41	200.9
6	12	64.3
7	6	31.2

It is also known that the total of areas of all the plates i 72049 units and the total of circumfrances of all the plate is 9624 units.

(i) Estimate the average strength of the plates in the population. [10

(ii) Calculate an unbiased estimate of the sampling error of your estimate in (i) above. [1

4.(a) A simple random sample of size 3 is drawn from a population of size 4 with replacement. Show that the probabilities that the sample contains i distinct units i = 1,2,3 are $P_1 = 1/16$, $P_2 = 9/16$ and $P_3 = 3/8$ respectively. Show that the variance $\bar{y}'$ is equal to $(7/32)S^2$ where $\bar{y}'$ is the sample mean over

(contd.......

distinct units and $S^2 = \frac{1}{3} \sum_{i=1}^{4} (Y_i - \bar{Y})^2$ and compare this with $\bar{y}$, the conventional estimator for $\bar{Y}$, the population mean.

(3+5+3)=[11]

(b) 2 circular systematic samples of size 4 each are drawn as follows:

y-values

Sample 1 :	104,	203,	178,	165
Sample 2 :	206,	109,	167,	154

Obtain an unbiased estimate for the population mean and an unbiased estimate of the sampling error.

(3+4) = [7]

5. Practical records (to be submitted to the Dean's Office by 29.8.84. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Difference and Differential Equations
PERIODICAL EXAMINATION

Date: 31.8.84. Maximum Marks : 100 Time: 3 hours.

Note : This paper carries a total of 112 marks. You may answer any number of questions or parts thereof.

1. Solve :

(i) $(2x+3y+1)\, dx + (2y-3x+5)\, dy = 0$

(ii) $(y^2 e^{xy} + \cos x)\, dx + (e^{xy} + xy\, e^{xy})\, dy = 0$

(iii) $\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x$

(iv) $\frac{dy}{dx} + xy = x^3 y^3$

(v) $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0.$ [6x5 = 30)

2.(i) Obtain the differential equation of the path 'brachistochrone'.

(ii) Show that the speed v of a planet at any point of its orbit around the sum is given by

$$v^2 = k \left(\frac{2}{r} - \frac{1}{a}\right)$$

where r is the distance of the planet from (the centre of) the sum and a is the semi-major axis of the orbit. [12+12=24]

3. A steel ball is projected vertically upwards from the earth's surface with an initial velocity v_0. (Assume that the acceleration due to gravity (g) is constant, that the air is still, and that the retarding force of air friction is proportional to the velocity of the ball). Show that the ball takes longer to fall back to the earth from its maximum height than to reach that height. [16]

p.t.o.

4. (i) State and prove Picard's theorem for existence and uniquenc of solution to a differential equation.

(ii) Stating necessary results, prove that the equation

$$y'' + P(x)\, y' + Q(x) = R(x) \qquad (P, Q, R \text{ cont. on an interval}$$

has a unique solution on I satisfying any given initial va x_0, y_0, y_0 .

(iii) Show that the equation

$$\frac{dy}{dx} = \frac{y}{1+x^2+y^2} \qquad (x,y) \in \mathbb{R}^2$$

has a unique soln. on $\mathbb{R}$ through any given (x_0, y_0).

[12+8+6=26]

5.(i) Find a solution of the equation

$$y'' - 4y' + 4y = 3\,e^{-t} + 2t^2 + \sin t$$

(ii) A solution of the equation

$$y'' - \frac{2x+1}{x}\, y' + \frac{x+1}{x}\, y = 0 \qquad x \in (0,\infty),\ y \in \mathbb{R}$$

is the function e^x. Find the general solution.

[8+8=16]

INDIAN STATISTICAL INSTITUTE

B. Stat. (Hons.) III Year, 1984-85

Statistical Inference

PERIODICAL EXAMINATION

Date: 3.9.84. Maximum Marks : 100 Time: 3 hours.

1. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent samples from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, respectively.

   (a) Show that

   $$(\bar{X} = \frac{1}{m}\sum_1^m X_i,\ \bar{Y} = \frac{1}{n}\sum_{j=1}^n Y_j,\ S = \sum_1^m (X_i - \bar{X})^2 + \sum_1^n (Y_j - \bar{Y})^2)$$

   is sufficient, when μ_1, μ_2 and σ^2 are unknown.

   Are the above statistics minimal sufficient ?

   (b) Show that the MLE of μ_1, μ_2 and σ^2 are given by $\bar{X}, \bar{Y}$ and $S/(m+n)$, respectively.

   (c) Obtain the UMVUE of μ_1, μ_2 and σ^2.

   (d) Can you find an estimate of the Form C.S. for estimating σ^2 which will have smaller M.S.E. than that of the corresponding UMVUE? [6+8+5+5]

2. Let $X_1, \ldots, X_n$ be independently and identically distributed with density

   $$f(X,\theta) = \frac{1}{\sigma} \exp\left(-\left\{\frac{X-\mu}{\sigma}\right\}\right),\ X \geq \mu$$
   $$= 0, \text{otherwise.}$$

   Where $-\infty < \mu < \infty$, $0 < \sigma$, $\theta = (\mu, \sigma^2)$

   (a) Obtain a non-trivial sufficient statistic for (μ, σ^2).

   (b) Find the M.L.E. of $P_\theta [\, X_1 \geq t \,]$ for $t \geq \mu$. [5+8]

3. Suppose that T_1 and T_2 are two UMVU estimators of $g(\theta)$ with finite variances. Show that $T_1 = T_2$ with probability 1. [8]

p.t.o.

4. Suppose that $X_1, \ldots, X_n$ is a sample from a population with density

$$f(X,\theta) = \theta X^{\theta - 1}, \quad 0 < X < 1, \ \theta > 0.$$

Consider $T(\underset{\sim}{X}) = -\frac{1}{n} \sum_{i=1}^{n} \log X_i$ for estimating $1/\theta$. Use Cramer-Rao inequality to show that T is the UMVUE of $1/\theta$. [10]

5. Let X_i, $i = 1, \ldots, n$, be independently distributed as $N(\alpha+\beta t_i, \sigma^2)$, where α, β and σ^2 are unknown and the t_i's are known constants that are not all equal.

(a) Show that the joint density of X_i's belong to the exponential family.

(b) Use the factorization theorem to show

$$\sum_{1}^{n} X_i, \quad \sum_{1}^{n} X_i t_i, \quad \sum_{1}^{n} X_i^2$$

are sufficient.

(c) Obtain the UMVUE of α, β and σ^2. [5+3+7]

6. Let $X_1, \ldots, X_n$ be i.i.d r.v.'s with the common distribution $U(\theta - 1/2, \theta + 1/2)$

(a) Use Lehmann Scheffe method to show that $(X_{(1)}, X_{(n)})$ is minimal sufficient.

(b) Is the family of distributions of $(X_{(1)}, X_{(n)})$ complete ? [7+4]

7.(a) Define (i) a sufficient statistic for a family of distributions.

(ii) a complete family of distributions.

(b) State Cramer-Rao inequality. [2+2+3]

8. Practical Work. [12]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Elective-4 : Economics
PERIODICAL EXAMINATION.

Date: 5.9.84. Maximum Marks : 100 Time: 2 hours.

Note : Answer any four questions.

1. Do you think that theoretically or in practice, the economic planning for a nation (Socialist or Capitalist) is just an extension of the exercise of economic planning for a business firm. Justify your view.

2. Indicate whether the following are true or false with brief clarification..
   (a) It is not necessary to convert an I/O table in physical terms to an I/O table in value terms for the purpose of theoretical analysis or any empirical use.
   (b) The assumption of 'Fixed coefficients of production' is not an essential but just a simplifying assumption used in Input output analysis.
   (c) Generally in Input-output analysis by 'aggregation' we mean 'clubing' of primary and secondary products of an industry into a single sector.
   (d) To construct a commodity X commodity I/O table from a given 'Make Matrix' and a given 'absorbtion' matrix, it is necessary to start with commodity technology assumption.
   (e) In the dynamic Input-output model the capital coefficient matrix is treated as exogenous.

3. Consider a three order hierarchy of regions, National, State and Local. There are three state regions under each nation and two local regions under each state region. We have only three commodities (1) National (2) State (3) Local. The distribution of final demand among the regions is as follows.

p.t.o.

Regions / Industry	1-1	1-2	1	2-1	2-2	2	3-1	3-2	3	Nation
1										500
2			100			10			200	400
3	50	150	200	120	60	180	100	120	220	600

The I/O matrix is given as

$$\begin{pmatrix} .4 & .1 & .2 \\ .2 & .2 & .2 \\ .1 & .1 & .2 \end{pmatrix}$$

The distribution

$d_{1.1} = .3$ $\quad d_{1.1-1} = .4$ $\quad d_{2,1-1} = .5$

$d_{1.2} = .5$ $\quad d_{1\ 1-2} = .6$ $\quad d_{2.1-2} = .5$

$d_{1.3} = .2$ $\quad d_{1.2-1} = .7$ $\quad d_{2.\ 2-1} = .6$

$d_{1.2-2} = .3$ $\quad d_{2.2-2} = .4$

$d_{1.3-1} = .4$ $\quad d_{2.3-1} = .7$

$d_{1.3-2} = .6$ $\quad d_{2.3-2} = .3$

4. Assume a two region, three commodity world. The same I/O matrix applicable to both the regions Assume fixed trading pattern betw the regions. Develop an interregional I/O model to facilitate interregional impact analysis.

5. Consider the following Make Matrix and the Absorbtion Matrix.

Make Matrix

Industry / Commodity	1	2	3	Total
1	100	30	0	130
2	20	100	0	120
3	0	25	60	85
Total	120	155	60	275

Absorbtion Matrix

Industry / Commodity	1	2		3	To
1	20	30		-50	1(
2	30	20	20	40	11
3	10	20	10	10	5
	50	30	20		
Value added	50	30	20		
Total	110	100	50	100	

Construct the commodity by commodity I/O table on the basis of commodity technology assumption and industry technology assumpti Explain the steps.

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Elective-4: Physical and Earth Sciences
PERIODICAL EXAMINATION

Date: 7.9.84. Maximum Marks : 100 Time: 3 hours.

Note : Attempt question no.10 and any five of the rest.

1. It is believed that the earth had either a hot or a cold beginning. Describe the one which you believe to be the best hypothesis. [16]

OR

Discuss briefly the origin of the earth on the basis of any one of the following hypotheses - [16]

(a) Weizsacker's Nebular Hypothesis,

(b) Kuiper's Nebular Hypothesis,

(c) Urey's Geochemical Hypothesis.

2. What is a mineral and what is the difference between a pyramid and a prism ? In what crystallographic class does the mineral beryl fall ? Describe the characteristic of the class with the help of a sketch. [8+2+6]

3. What is a plutonic rock ? What are its characteristics ? Give one example of a plutonic rock and describe its major mineral constituents. [6+6+1+3]

4. What is a primary magma ? In Bowen's Reaction Series, there are two series of reaction - continuous and discontinuous. Describe the discontinuous series of reaction. [6+10]

OR

Which of these are igneous rocks - basalt, coal, chert, water, petroleum, limestone, marble and granite ?

Describe the formation and the characteristics of an igneous rock. [4+8+4]

5. Define a sedimentary rock. Describe how sediments are transported from one place to another. What is a non-clastic rock ? Give an example each of a clastic and a non-clastic rock. [4+6+4+2]

p.t.o.

6. You perhaps know that sedimentation takes place on continent (non-marine) and in sea. What are the different kinds of environment of deposition in a continent ?

Suppose you are visiting an area where others say that a deposit occurs which is of glacial origin. What are the criteria you would be looking for in such a deposit to prove that it is indeed of glacial origin ? [8+8]

7. Discuss the various criteria that may be used to differentiate a marine sediment from a non-marine sediment. [16]

8. Describe what you know about the folds. What is a geosyncline ? What kind of rocks would you expect to find in geosynclinal troughs ? [8+4+4]

OR

Describe what you know about faults. How do you think the great Himalayan mountain has formed ? [8+8]

9. Write short notes on the following (any four) : - Si - O tetrahedron ; Seismic waves ; Discordant rocks ; Characteristics of a magma ; Volcano ; Heat Flow of the earth ; Earth's magnetism ; Primitive atmosphere ; Primitive hydrosphere. [4x4=16]

10. Fill up the blanks (any 13). You need only to write down one of the four choices for each blank.

$(13 \times 1\frac{1}{2} = 19\frac{1}{2}+\frac{1}{2}=20)$

(i) Asteroids are the product of _____ (Statellite/planet/meteor/comet).

(ii) Jovian planets are _______ (inner/outer/extra-terrestrial /continental) planets of the solar system.

(iii) The exact central position of the earth is _______ (hot solid/ cold solid/ hot liquid/cold liquid).

(iv) Quartz possesses a silicate structure which is ________ (Simple/3-D chain/2-D single chain/ 2-D double chain).

(v) K- feldspar is an important _______ (precious/ economic/rock-forming/ crystalline) mineral.

(contd....3)

10.(vi) The most important element that occurs next to O_2 in the earth's crust is ________ (Fe/Mg/Si/Ca).

(Vii) Basalt is a _______(plutonic/hypabyssal/volcanic/ metamorphic)rock.

(viii) The Tidal Hypothesis was proposed by ____ (Moulton-Chamberlin/ Kant-Laplace/ Jeans-Jeffreys/Ringwood-Alfven)

(ix) The dinosaur bones of the Indian Statistical Institute havebeen obtained from _____ (non-marine/marine/mixed/ turbidite) rocks in the Godavari Valley.

(x) The upper surface of a bedding can be determined from _______ (texture/cross-bedding/fossil/mineral composition).

(xi) The presence of clay-galls in a sedimentary rock indicates its _____(glacial/fluviatile/marine/lacustrine) origin.

(xii) The dark colour of an igneous rock is due to the presence of _____ (Fe-Mg/Si-O/Ca-Na/K-cl) elements in relatively large amount.

(xiii) A marine sedimentary rock can be best differentiated from a non-marine one with the help of ______ (SiO_2/ Cross-bedding/Fe-content/Fossil).

(xiv) A pure sandstone (orthoquartzite) contains mineral constituents which are almost all --- (feldspar/quartz/ quartzuto ./ chert).

(xv) The Gondwana rocks which are now being mapped by the geologists of the Indian Statistical Institute are _____. (terrestrial/extraterrestrial/marine/mixed) sediments.

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Elective - 4: Physical and Earth Sciences
SUPPLEMENTARY PERIODICAL EXAMINATION

Date: 27.10.84.   Maximum Marks : 100   Time: 3 hours.

Note : Attempt Question no.1 and any <u>five</u> of the rest.

1. Fill up the blanks (any 13). You need only to write down one of the four choices for each blank. $[13\times1\frac{1}{2}+\frac{1}{2}=19\frac{1}{2}+\frac{1}{2}=20]$

   (i) The first catastrophic hypothesis about the origin of the earth goes to the credit of ______ (Weizsacker/ Spitzer /Buffon/Laplace).

   (ii) Ca is an important element in ______ (magnetite/ chalcopyrite/dolomite/tourmaline).

   (iii) The sand grains in a sea-beach are _____ (well/poorly/ moderately/somewhat) Sorted.

   (iv) The bottom surface of a bedding can be determined from ______ (cross-bedding/texture/fossil/concretion).

   (v) Mountain-building activity is deeply associated with ______ (Syncline/ anticline/fault/geosyncline).

   (vi) The most important element that occurs in the earth's crust is _______ (Fe / Mg / O / Si).

   (vii) The term texture includes ______ (grain-size/structure/ cross-bedding/ viscosity).

   (viii) A crystal form which has all its faces well-developed is called ______(anhedral/hypidiomorphic/subhedral/ euhedral).

   (ix) First addition of free oxygen in the primitive atmosphere is by means of _____ (volcanic activity/ water vapour/ photochemical dissociation/ photosynthesis).

   (x) Conglomerate is a sedimentary rock which consists of ______ (pebbles only/ pebbles and matrix/ matrix only/ cement only).

p.t.o.

1.(xi) A plutonic rock is made up of ______ (fine-grained/ coarse-grained/both coarse and fine-grained/clastic) minerals.

(xii) A pearl is _____ (mineral/rock/organism/organic) substance.

(xiii) Pyrite is ______ (precious/rock-forming/economic / ordinary) mineral.

(xiv) Presence of plant-eating four-legged animal remains in a sedimentary rock indicates that the environment of deposition of the rocks was most probably _______ (marine/mixed/non-marine/non-terrestrial).

(xv) Quartzite is a _____ (rock/mineral/clast/bedding).

2. Write in short your views about the origin of the earth. [16]

OR

Si-O tetrahedron plays a significant role in the formation of the silicate minerals. Can you describe how it plays its role in the formation of different kinds of igneous rock ? [16]

3. What is the difference between a crystal and a crystalline substance ? Name a mineral and its crystallographic class in which two axes of symmetry are at right angles and the third axis is at an angle with them.

What is meant by texture ? What is a hypidiomorphic texture ? Name a rock or a mineral which shows hypidiomorphic texture.

[6+3+4+2+1=16]

4. Which of these are non-clastic sedimentary rocks - chert, water, petroleum, limestone, granite, chalcopyrite, turbidite. quartzite ?

Describe where and how weathering and erosion take place.

[4+12=16]

5. What is meant by fold axis ? What are dip and strike ?

Suppose you are visiting an area where others say that a syncline occurs. What are the criteria you would look for to establish that it is not a syncline but an anticline ?

[4+6+6=16]

(contd.....3)

6. What do you understand by the terms bedding and lamination ? In what way cross-bedding and graded bedding are useful ? [8+8=16]

7. What is the density of the earth as a whole ? Does it mean that the density in the interior of the earth is different ? If not, why ? If there is any difference, how can you explain that ? [2+2+4+8=16]

8. A violent earthquake takes place at a certain time in a remote place. Describe how would you proceed to determine the time and the location where the earthquake takes place.

In what way earthquakes are useful to the scientists ? [12+4=16]

9. Describe in short the evolution of the earth's atmosphere. [16]

OR

Describe in short the evolution of the seas.

10. Write short notes (any four) : -

Continuous series of reaction; Clastic rock ; Glacier deposits ; the Himalayas; Crystal form; Red sediments; Characteristics of the Solar System. [16]

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Stochastic Processes-2
SEMESTRAL-I EXAMINATION

Date: 19.11.84. Maximum Marks : 100 . Time: $3\frac{1}{2}$ hours.

Note : Answer as many questions or parts thereof as you can. The paper carries 110 marks.

1. Let $\{X_t , t \geq 0\}$ be standard Brownian motion.

   (a) Find the distribution of $X_1 + X_2 + .. + X_n$.

   (b) Find the mean and covariance functions of the process $\{Y_t : t \geq 0\}$, where $Y_t = X_t^2$, $t \geq 0$. [10+10 = 20]

2. Let $\{X_t, t \geq 0\}$ be standard Brownian motion, and let $M_t = \max_{0 \leq s \leq t} X_s$, $t > 0$.

   (a) Prove that, for $0 \leq \xi > x$,

   $$P(M_t \leq \xi, X_t \leq x) = P(X_t \leq x) - P(X_t \leq x - 2\xi).$$

   (b) Hence, or otherwise, write down the joint density function of M_t and X_t.

   (c) Prove that $P(M_t > \xi \mid M_t - X_t = 0) = e^{-\xi^2/2t}$, $\xi > 0$.

   [9+8+8=25]

3. Consider a Yule process $\{X_t : t \geq 0\}$, i.e. a pure birth process with birth rates $\lambda_i = i\lambda$, where λ is a positive constant. Assume that $X_0 \equiv 1$.

   (a) Let $S_1, S_2, \ldots$ be the successive times of birth. Given that $X_t = n+1$, find the conditional joint density function of $S_1, S_2, \ldots, S_n$.

   (b) Suppose that an individual born at time s is robust with probability p(s). Compute the distribution of robust individuals born in (0,t). [10+10=20]

p.t.o.

4. Each individual in a biological population is assumed to give birth at an exponential rate λ and to die at an exponential rate μ. In addition there is an exponential rate of increase θ due to immigration.

   (a) Set up a birth and death model by specifying the birth and death rates.

   (b) Write down the forward differential equations for the corresponding birth and death process.

   (c) If X_t is the number of individuals alive at time t and $m_i(t) = E(X_t | X_0 = i)$, compute $m_i(t)$.

   [7+5+8=20]

5. Let X_t be the number of persons being served at time t in a $M|M|\infty$ queue, $t \geq 0$. Let Y_t be the number of customers arriving in $(0,t]$ who are still in the process of being served at time t ; Z_t will denote the number of those customers present at time 0 who are still being served at time t. Let the number of customers present at time 0 follow the Poisson distribution with parameter ν.

   (i) Find the distribution of Z_t.

   (ii) Find the distribution of X_t.

   (iii) Show that X_t and X_0 have the same distribution iff ~~the~~ $\nu = \dfrac{\text{arrival rate}}{\text{service rate}}$ ~~= the service rate~~.

   [8+9+8 = 25]

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B.Stat. (Hons.) III Year, 1984-85

Sample Surveys
SEMESTRAL-I EXAMINATION

Date: 21.11.84. Maximum Marks: 100 Time: 3 hours.

Note : Answer as much as you can. The paper carries a total of 110 marks. The maximum you can score is 100. Marks allotted to each question are given in brackets [ ].

1.(a) Define 'inclusion probability' of a unit U_1 for a sampling design. Describe a sampling scheme for which the ratio estimator $\hat{R} = \bar{y}/\bar{x}$ is unbiased for the population ratio $R = \bar{Y}/\bar{X}$, where $\bar{y}$ ($\bar{Y}$) and $\bar{x}$ ($\bar{X}$) are the sample (population) means of the y and x characteristics respectively. Also find the probability of inclusion of the unit U_1 under this scheme. [1+4+5]=[10]

(b) A survey was conducted in a village consisting of 625 households by covering a sample of 50 households selected using a simple random sampling without replacement scheme to estimate the average weekly expenditure on toilet goods. The estimate was found to be Rs.4,20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample to be selected by simple random-sampling with replacement scheme such that the length of the confidence interval at 95% confidence level is 20% of the true value. State clearly the assumptions involved in finding out the sample size. [9+2]=[11]

2.(a) What do you understand by the term 'intraclass correlation coefficient'? A population consists of 12 clusters each of size 5. Find the bounds for the intraclass correlation coefficient among the elements of the cluster. [3+3]=[6]

(b) Show that the relative efficiency of sampling a cluster of M units, compared to direct sampling is given by $1/(1+(M-1)\rho)$, where the intra-class correlation coefficient, $\rho = 1 - \left\{ M\sigma_w^2 / (M-1)\sigma^2 \right\}$, σ_w^2 and σ^2 denoting the within and total variances respectively. [7]

p.t.o.

2.(c) For examining the efficiency of sampling households (clusters of persons) instead of persons for estimating the proportion of males in a given area, the following simplifying assumptions are made: (i) each household consists of 4 persons (husband, wife and 2 children) and (ii) the sex of a child is binomially distributed. By considering the 3 classes - households with 2 male children, households with 1 male child, and households with no male child, show that the intra-class correlation coefficient is (-1/6). Hence, show that the efficiency of cluster sampling of households compared to that of sampling persons is 200%.. [7+1]=[8]

3.(a) Explain what you understand by 'combined and separate ratio estimators' in stratified random sampling. Indicate the situations when you use these estimators, justifying your answer with necessary formulae and assumptions.

[6+7] = [13]

(b) Let W_i be the proportion of population units and let $S_i^2 = N_i\sigma_i^2/(N_i - 1)$, where σ_i^2 is the within variance for the ith stratum, i=1,2,...,k. Obtain an expression for the variance of the estimate of the population mean based on an allocation of the sample size proportional to $W_i f(S_i)$, where f is a positive, real function, when sampling in each stratum is done with equal probabilities and without replacement. Hence obtain Neyman's optimum allocation.

[3+5] =[8]

4. A sample survey was conducted to estimate the total household income in an urban area. The design adopted was a stratified two-stage one with census enumeration blocks as first-stage units and households within them as second stage units. From each stratum 4 blocks were selected with probability proportional to population and with replacement and 4 households were selected from each selected block with equal probability and without replacement. The data on household income for the sample households together with information on selection probabilities are given below :

(a) Obtain an unbiased estimate of the total weekly household income. [15]

(b) Obtain an unbiased estimate of the sampling variance of the estimate.

[20]

contd......

4.(c) Compare the efficiency of the above design with that of unistage simple random sampling of households in each stratum. [12]

Stratum	Sample block	Inverse of probability of selection	total no.of households	Weekly household income (in Rs.) of sample households 1	2	3	4
I	1	67.63	189	110	281	120	114
	2	338.12	40	80	60	122	125
	3	101.50	135	122	210	171	105
	4	69.03	160	244	115	312	128
II	1	113.34	73	345	359	160	117
	2	441.00	26	97	179	144	85
	3	31.50	240	100	115	50	172
	4	661.57	14	102	40	126	148
III	1	15.80	287	122	176	108	140
	2	21.00	257	125	110	134	215
	3	48.89	68	300	115	67	110
	4	26.73	218	263	75	142	54

Total no. of households in stratum I = 12848
'' '' '' II = 8422
'' '' '' III = 6354

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Difference and Differential Equations
SEMESTRAL-I EXAMINATION

Date: 23.11.84. Maximum Marks : 100 Time: 3 hours.

Note : This paper carries a total of 110 marks. Answer any number of questions or parts thereof. The maximum you can score is 100.

1. Prove that the equations

(a) $(1-x)y'' + xy' - y = 0$

(b) $2x(2x-1)y'' - (4x^2+1)y' + y(2x+1) = 0$

have a common solution(nonzero). Find a common solution and obtain the general solution to (b). [10]

2. Show that if a particle moving in a plane under a central force-field satisfies (in polar co-ordinates)

$$r^2 \frac{d\theta}{dt} = h$$

and
$$r = \frac{h^2/k}{1+e \cos \theta}$$

where h,k and e are constants, then the particle is attacted towards the origin with a force proportional to $1/r^2$. [12]

3. Solve :

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2z.$$ [8]

4.(a) Show that a nontrivial solution of Bessel's equation on $[1,\infty)$ has infinitely many zeros.

(b) If the parameter p of Bessel's equation is greater than 1/2 and $\lambda_1 < \lambda_2 < \ldots$ are the successive zeros of a nontrivial solution on $[1,\infty)$, prove that

$$\lim_{n\to\infty} (\lambda_{n+1} - \lambda_n) = \pi.$$ [8+12=20]

p.t.o.

..(a) Find the general solution of the equation

$$x(1-x)\,y'' + \left(\tfrac{3}{2} - 2x\right)y' + 2y = 0$$

on (0,1).

(b) If H_n is the nth Hermite polynomial, then find

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x)\, H_n(x)\, dx \quad .$$

for m and n nonnegative integers.

(The Hermite equation with parameter n is
$y - 2xy + 2ny = 0$)

[11+11=22]

6.(a) Show that the solutions of a kth order linear difference equation form a k-dimensional vector space.

(b) Find the general solution of the equation

$$(E-a)^{l}\;(E-b)^{m}\, y = 0$$

where $a \neq b$ and l, m are positive integers.

[8+6=14]

7.(a) Show, stating necessary conditions, that if the vector function $x \to (y_1, \ldots y_n)$ minimises the functional

$$\int_a^b F\,(x, y_1(x), \ldots, y_n(x)\,,\, y_1'(x), \ldots y_n'(x))\, dx$$

subject to the boundary values $y_i(a) = y_i^0$ and $y_i(b) = y_i^{\gamma}$, $i = 1, \ldots n$, then

$$F_{y_i} - \frac{d}{dx} F_{y_i'} = 0 \qquad i = 1, \ldots, n.$$

(b) Find the geodesics on the surface of the unit sphere.

[12+12=24]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

Statistical Inference
SEMESTRAL-I EXAMINATION

Date: 26.11.84 Maximum Marks : 100 Time: 3 hours.

Either

1.(a) Let X be distributed as binomial (n,p), $n>3$. Obtain the UMVU estimate of p^3.

(b) Let $X_1,\ldots,X_n$ be i.i.d $N(\xi,\sigma^2)$ r.v.'S with σ^2 known. Obtain the UMVU estimate of ξ^2. [10]

Or

2. Suppose X,Y are independent r.v.'S, and $X \sim N(\vartheta, 1)$, $Y \sim N(\eta,1)$. Let $\rho = \vartheta/\eta$. Define

$$\delta(X,Y,\rho) = 0, \text{ if } |X-\rho Y| \leq (1+\rho^2)^{1/2} Z_{1-\alpha/2}$$
$$= 1, \text{ otherwise}$$

Where $Z_{1-\alpha/2}$ is the $(1-\alpha/2)$th quantile of $N(0,1)$.

(a) Show that $\delta(X,Y,\rho_0)$ is a size α test for $H : \rho=\rho_0$.

(b) Describe the confidence region obtained by inverting the family $\{\delta(X,Y,\rho)\}$. [10]

3. Let $X_1, X_2, \ldots, X_n$ be the times in months until failure of n similar pieces of equipment. If the equipment is subject to wear, a model often used is the one where $X_1, \ldots, X_n$ is a random sample from a Weibull distribution with density

$$f(x,\lambda) = \lambda C x^{C-1} e^{-\lambda x^C}, \quad x > 0$$

with $C > 0$ known.

(a) Show that the UMP test for $H_0 : \frac{1}{\lambda} \leq 1/\lambda_0$ Vs. $H_1 : \frac{1}{\lambda} > 1/\lambda_0$ is based on $\sum_1^n X_i^C$.

(b) Show that the critical value of the size α test with critical region $\sum_1^n X_i^C \geq K$ is $K = \chi^2_{2n,1-\alpha}/2\lambda_0$ where $\chi^2_{2n,1-\alpha}$ is the $(1-\alpha)$th quantile of χ^2_{2n}. Obtain the power of the test.

p.t.o.

3.(c) Suppose $1/\lambda_0 = 12$. Find the minimum sample size needed for a level 0.01 test to have power at least 0.95 at $1/\lambda = 15$. [25]

4. The number of deaths from road accidents in a city in two consecutive months were 7 and 3, respectively. Can the reduction from the first to the second month be due to chance fluctuation ?
Assume that the number of road accidents in a week follows the Poisson distribution. Derive the UMPU test for the above problem and apply that to the given data. [15]

5. Let X take on the values -1,0,1,... with probabilities

$$P(X=-1) = p,\ P(X=K) = (1-p)^2 p^K,\ K = 0,1,2\ldots(0<p<1).$$

Show that the unbiased estimators if p are given by

$$\delta(X) = \begin{cases} 1-a, & \text{if } X = -1 \\ a, & \text{if } X \neq -1 \end{cases}$$

where 'a' is a real number. Use this result to derive the locally minimum variance unbiased estimate of p at variance $p = p_0$. Does there exist any UMVU estimator of p ?. [15]

6. The following blood pressures were obtained in a sample of size n=5 from a certain population.

120, 110, 114, 100, 190

Assume the normal model.

(a) Using the size $\alpha = 0.05$ one-sample t-test, can we conclude that the mean blood pressure μ in the population is significantly larger than 100 ?

(b) What can you say about any optimum property enjoyed by the above t-test ? Justify your answer.

(c) Compute a level 0.90 level confidence interval for the mean blood pressure by inverting the family UMPU acceptance regions for $\mu = \mu_0$ Vs. $\mu \neq \mu_0$. State the optimum property enjoyed by the above confidence interval. [25]

7. Practical note book. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Elective-4 : Economics
SEMESTRAL-I EXAMINATION

Date: 28.11.84. Maximum Marks : 100 . Time: 3 hours.

Note : Answer any Five questions.

1. Suppose a firm is planning to introduce a new product which is similar to an existing product. After a detailed study of the marketing and production departments estimates are made of possible future cash flows as related to varying market conditions. The estimates of cash flow shown in the table indicate that this new product will produce cash flow A as demand decreases cash flow B if demand remain constant and cash flow C if demand increases

Table

Year	A P(A)=0.1	B P(B)=0.3	C P(C)=0.6
0	-30000	-30000	-30000
1	11000	11000	4000
2	10000	11000	7000
3	9000	11000	10000
4	8000	11000	13000

P(A)=Prob. of A
P(B)= '' '' B
P(C)= '' '' C

How would you proceed to judge the feasibility of the project? Assume that the interest rate is 10%.

2. A machine was purchased 3 years ago for Rs.12000/-. Its present value is Rs.5000 and its operating expenses are expected to continue at Rs.1000 a year. A second hand machine costing Rs.2000 is available and its operating expenses are expected to be Rs. 1600 per year. It is anticipated that both machines will be in service for 6 more years with Rs.1000 salvage value for the present machine and zero for the second hand machine. Assuming that the minimum attractive rate is 15 per cent, find the best course of action.

3. Clarify the following concepts in connection with the construction of an I/O table -
(a) 'Make Matrix' (b) 'Product Mix hypothesis' (c) Market share hypothesis (d) Commodity Technology assumption
(e) Industry Technology assumption.

p.t.o.

4. Consider a privately owned oil operated audiovisual system exhibiting films for one thousand persons. Each ticket costs 2 rupees and the operating cost of the system is 1 rupee per viewer. Now, there is a proposal of construction of a power driven system with the capacity to arrange for films to be exhibited to about ten thousand persons. The power driven audiovisual system involves a capital cost of Rs.5 lakhs (to be financed by taxation). Power is expected to be made available free of cost and other operating costs are negligible. Now, it is estimated that on the average the film auditorium would be fifty per cent full. The capital cost of the new audiovisual system includes the cost of the new auditorium. Indicate how you would proceed to make a Cost-Benefit analysis of the project (i.e. the power driven system).

5.(a) A company makes two products, 1 and 2. To produce each product two different machine types, A and B, are required. There are two varieties of type A machines, which we shall designate by A_1 and A_2, and three varieties of type B machines B_1, B_2 and B_3. Product 1 can be produced in 3 different ways represented by the following combinations of machines (A_2, B_1), (A_2, B_2) and (A_1, B_3). Product 2 can be made in two ways represented by the combinations of machines (A_1, B_1), (A_2, B_1). In the table below is shown the time in minutes required by one unit of each product on each variety of machines along with the total minutes of machine time available per week and the cost of running the machines at full capacity. The cost of raw materials for one unit of each product and its selling price are given in the last two rows of the table. Assume that the total cost of running the machines is directly proportional to the operating time.

contd....

Q.5 (a) contd....

Table

Machine	Product 1	Product 2	Total time available per week (minute)	Cost at full capacity
A_1	10	15	4000	200
A_2	8	[illegible]	8000	300
B_1	15	20	3000	150
B_2	10	5	2000	200
B_3	15	[illegible]	2000	100
Mat.Cost	0.30	0.35	~~0.50~~	
Selling price	1.50	2.50	~~3.50~~	

Derive a Linear Programming formulation for the above problem.

b) Obtain the optimum value of $x_1 + x_2$

For the following problem

$$3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, X_2 \geq 0$$
$$\max Z = 5x_1 + 3x_2$$

. Discuss the main ideas of Mahalanobis' two sector and four sector models of planning designed for economic development of India.

Or

Write short notes on any two of the following :
(a) Effective Rate of Protection (b) Redistributive measures in economic planning (c) Domestic Resource Cost.

. The construction of a levee for protection during periods of flooding is under consideration. Data on the costs of construction and expected flood damages are shown below.

contd....

Q.7 contd....

Historical data on water level of river and cost of various heights of levee.

Feet(x) A	Number of years river's maximum level was x feet above normal(B)	Loss of river is x feet above Lavee	Indicate cost of building levee, x feet high
0	25	0	0
5	10	2000	1500
10	15	3000	2500
15	5	1500	3500
20	3	2000	5000
25	2	4000	6000

Determine the optimum height of the levee (optimum in the sense that the selection of a smaller levee would not provide enough protection to offset the reduced construction costs, while a levee higher than that height requires more investment without providing proportionate savings from expected flood damages).

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1984–85
Difference and Differential Equations

SUPPLEMENTARY PERIODICAL EXAMINATION

Date: 29.11.84. Maximum Marks: 100 Time: 3 hours.

Note : Answer all questions.

1. Solve
(i) $y'' + 2y' = x$ with the initial condition $\phi(0) = 1$, $\phi'(0) = 0$

(ii) $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$

(iii) $(y+xy^2)dx - x\,dy = 0.$ [8+8+8=24]

2. (i) Consider the equation
$$\frac{dy}{dx} = f(x,y)$$
where f and $\frac{\partial f}{\partial y}$ are continuous functions on an open rectangle $I \times \mathbb{R}$. Show that if ϕ_1 and ϕ_2 are solutions of the equation and $\phi_1(x_0) = \phi_2(x_0)$ for some $x_0 \in I$, then $\phi_1(x) = \phi_2(x)$ for all $x \in I$.

(ii) Prove that if ϕ_1 and ϕ_2 are two solutions of the linear nth order equation on I
$$y^{(n)} = a_{n-1}(x)\, y^{(n-1)} + \ldots + a_1(x) y^{(1)} + a_0(x) y$$
with continuous coefficients, then
$$\text{rank}\begin{pmatrix} \phi_1(x), & \phi_1^{(1)}(x), & \ldots, & \phi_1^{(n)}(x) \\ \phi_2(x), & \phi_2^{(1)}(x), & \ldots & \phi_2^{(n)}(x) \end{pmatrix}$$
is constant on I.

(iii) For the system of equations
$$\frac{d\underset{\sim}{y}}{dx} = A\underset{\sim}{y} + \underset{\sim}{b}(x) \qquad (x \in \mathbb{R})$$
where A is a constant matrix, write down an expression for the solution with the initial condition $\underset{\sim}{y}(x_0) = \underset{\sim}{y}_0$.

[10+10+8 = 28]

3.(i) Prove that for a particle moving on a plane under a central force field $h = r^2\dot{\theta}$ is a constant where r and θ have their usual meaning.

p.t.o.

3.(ii) Show that for a particle of mass m attracted towards a centre O by a force of magnitude mr

$$\frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{1}{2} m r^2$$

is a constant C.

(iii) Show that for the particle in (ii)

$$\frac{dr}{d\theta} = \frac{r}{h} \sqrt{\frac{2Cr^2}{m} - h^2 - r^4}$$

(iv) Prove that hm < C in (ii)

(v) Show that the orbit of the particle in (ii) is either an ellipse or a straight line.

[8+4+4+4+10=30]

4.(i) Find the general form of the solution to the equation

$$y'' + y = \operatorname{Sec}^2 x \qquad -\pi/2 < x < \pi/2 .$$

(ii) Given that the equation

$$x y'' - (2x+1) y' + 2y = 0 \qquad (x > 0)$$

has a solution of the form e^{ct} for some c, find the general solution.

[9+9=18]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Statistical Inference
SUPPLEMENTARY PERIODICAL EXAMINATION

Date: 30.11.84. Maximum Marks : 100 Time: 3 hours.

1. Consider the following result of an experiment relating to agriculture. Nine samples of soil were treated with different amounts X of phosphorus. Y is the amount of phosphorus found in corn plants grown for 38 days in the different samples of soil.

X_i	1	4	5	9	11	13	23	23	28
Y_i	64	71	54	81	76	93	77	95	109

Use the following model

$$Y_i = \theta_1 + \theta_2 X_i + \varepsilon_i$$

where ε_i's are independent with 0 mean and the common variance σ^2.

(a) Obtain the least-squares estimate of θ_1 and θ_2 and the variances of these estimates.

(b) What are the MLE's of θ_1 and θ_2 when ε_i's are i.i.d $N(0,\sigma^2)$? [15+5]

2. Let $X_1, X_2, \ldots, X_n$ denote a sample from a population with the following densities or frequency functions. Obtain the MLE of θ and a (non-trivial) sufficient statistic in each case

(a) $f(X,\theta) = \theta\, e^{-\theta X}$, $X \geq 0$, $\theta > 0$.

(b) $f(X,\theta) = \sqrt{\theta}\, X^{\sqrt{\theta} - 1}$, $0 \leq X \leq 1$, $\theta > 0$.

(c) $f(X, \theta) = (x/\theta^2) \exp (-x^2/\theta^2)$, $x > 0$, $\theta > 0$. [24]

p.t.o.

3. Let X be a $N(\Theta, 1)$ variable and consider the estimate $T_{a,b}(X) = aX + b$ of Θ.

   (a) Calculate the M.S.E. of $T_{a,b}$

   (b) Plot the M.S.E. of $T_{1/2,\,0}$ and of the 'natural' estimate $T_{1,0} = X$ as a function of Θ and show that neither estimate improves the other for all Θ.

   (c) Is there any estimate of the form $aX + b$ which improves on X for all Θ ?

   (d) Show that X is the only unbiased estimate of the form $aX + b$. [24]

4.(a) State Rao-Blackwell Theorem.

   (b) Let $X_1, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$, with $\sigma = 1$. Find the U.M.V.U. estimate of $P_\mu[X_1 \geq 0] \equiv \phi(\mu)$.

   (c) Show that $\underset{\sim}{X} = (X_1, \ldots, X_n)$ is sufficient but not complete, if $n \geq 2$. [6+14+5]

5. Practical Note Book. [7]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Elective-4: Physical and Earth Sciences
SEMESTRAL-I EXAMINATION

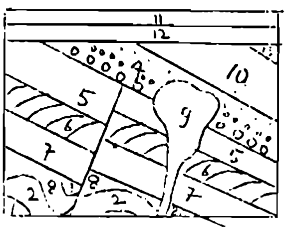
Date: 30.11.84. Maximum Marks : 100 Time : 3 hours.

Note : You may answer any question. Maximum marks for the paper will be 100.

1. What are fossils ? Does the fossil record represent all organisms of the past equally ? Give reasons for your answer. (3+12)

2. Explain the principle of age determination of rocks using radioactive isotopes. What are the general requirements to obtain a correct dating ? (9+6)

3. Below is a section showing the attitude and relationship of the rock formations of an area. The different formations are identified by numbers although not in any specific order. All are sedimentary rocks except 9 which is a granitic intrusion.
   (i) Is there any inconsistency in the diagram? If yes, specify.
   (ii) What can you infer about the geological history of the area ? State all events in their chronological order. If there is any ambiguity, specify giving reasons. (10+2)



p.t.o.

4. Given below are two lists of items A and B. Items in list B provide information to infer about items in list A. For each item in list A, there may be one or more useful items in list B. Pick them out and briefly state the reason for your choice.

A	B
a) Agent of sediment transport	i) Colour of the rock
b) Correlation of beds	ii) Grain-size distribution
c) Unconformities	iii) Fossil assemblages
d) Top and bottom of beds	iv) Strike and dip of beds
e) Environment of deposition	v) Current bedding
	vi) Radioactive isotopes present in the clastic material
	vii) Mudcracks.

(15)

5. What are guide fossils ? Formulate an index to express quantitatively the usefulness of a fossil in correlation.

(5+15)

6. What do you understand by stable regions and unstable regions on the surface of the earth ?
Where was the first concept of geosyncline developed and what were the geological factors that led to such concept ?

(6+3+6)

7. What is meant by 'continental drift'? What are the criteria to support it ? What is supposed to be India's position during early Palaeozoic ?

(3+8+4)

8. In 'plate tectonics', what is understood by 'plate' and 'tetonics' ? Discuss a few of the ways in which the hypothesis is useful to the earth scientists. (5+10)

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

Elective-4 : Economics
SUPPLEMENTARY PERIODICAL EXAMINATION

Date: 1.12.84. Maximum Marks : 100 Time: 3 hours.

Note : All questions carry equal marks. Answer any three questions.

1. Indicate True or False (with reasons):

   (a) Economic planning for a nation is just an extension of the process of planning for a business firm in the private sector.

   (b) The ' Commodity Technology assumption' is an essential assumption for the purpose of construction of an I/O table.

   (c) In the dynamic Input-output Model the capital coefficient Matrix is treated as exogenous.

   (d) It is not necessary to construct an I/O table in physical terms to an I/O table in value terms for the purpose of theoretical analysis or any empirical use.

   (e) Generally in Input-output analysis by 'aggregation' we mean clubbing of primary and secondary products of an industry into a single sector.

2. Assume a two-region three commodity world. The same I/O matrix is applicable to both the regions. Assume a Fixed Grading [illegible] patterns between the regions. Develop an interregional I/O model to facilitate interregional impact analysis.

3. Make ' Industry technology' assumption. Illustrate through example given the 'Make matrix' and an 'Absorbtion matrix' the procedure of construction of an Input-output table.

p.t.o.

4. Consider a three order hierarchy of regions. National state and local. There are three state regions under each nation and two local regions under each state region. We have only three commodities (1) National (2) State (3) Local. The distribution of final demand among the regions is as follows:

Regions Industry	1-1	1-2	1	2-1	2-2	2	3-1	3-2	3	Nation
1										500
2			100			10			100	400
3	50	150	200	120	60	130	100	120	200	600

The I/O matrix is given as

$$\begin{pmatrix} .4 & .1 & .2 \\ .2 & .2 & .2 \\ .1 & .1 & .2 \end{pmatrix}$$

The distribution coefficients are given as follows :

$d_{1.1} = .3$ $\quad d_{1.\,1-1} = .4$ $\quad d_{2.\,1-1} = .5$ $\quad d_{2.\,2-1} = .6$

$d_{1.2} = .5$ $\quad d_{1.\,1-2} = .6$ $\quad d_{2.\,1-2} = .5$ $\quad d_{2.\,2-2} = .4$ $\quad d_{2.3-2} = 3$

$d_{1.3} = .2$ $\quad d_{1.\,2-1} = .7$ $\quad d_{2.\,2-1} = .6$ $\quad d_{2.\,3-1} = .7$

Obtain the output requirement pattern satisfying the inter-sectoral and interregional consistency.

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INDIAN STATISTICAL INSTITUTE
B. Stat. (Hons.) III Year: 1984-85

STATISTICAL INFERENCE
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 26.12.84. Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours.

1. Let $X_1, X_2, \ldots, X_n$ be n independent observations on $X \sim U(0, \theta), \theta > 0$.

   (a) Show that the largest observation $X_{(n)}$ is sufficient for θ. [2]

   (b) Obtain the UMVU estimator of θ. [4]

   (c) For n = 2, derive the class of all UMP size α tests for $H_0: \theta = 1/2$ vs. $H_1: \theta > 1/2$.

   Now consider the following test :

   $$\phi_c(X_1, \ldots, X_n) = 1, \text{ if } X_{(n)} \geq c$$
   $$= 0, \text{ otherwise.}$$ [7]

   (d) Compute the power function of ϕ_c and show that it is a monotonic increasing function of θ. [4]

   (e) In testing $H_0: \theta \leq 1/2$ Vs. $H_1: \theta > 1/2$ what choice of c would make ϕ_c have size exactly 0.05 ? [4]

   (f) How large should n be so that ϕ_c as obtained in (e) has power 0.98 for $\theta = 3/4$?

   (g) If in a sample of size n = 20, $X_{(n)} = 0.48$, what is the p-value ? [2]

2. Define the following and illustrate the concepts with examples.

   (a) Unbiased test
   (b) Similar region
   (c) Test of Neyman Structure
   (d) Monotonic likelihood ratio family.
   (e) Confidence region.
   (f) Size of a test.
   (g) Sufficient statistic.

   [3x7]

p.t.o.

3. A gambler observing a game in which a single die is tossed repeatedly gets the impression that 6 comes up 18% of the time, 5 about 14% of the time, while the other four numbers are equally likely to occur (with probability 0.17). Upon being asked to play, the gambler asks that he first be allowed to test the hypothesis by tossing the die n times.

(a) What test statistic should he use if the only alternative he considers is that the die is fair ?

(b) Show that if $n = 2$ the MP level 0.03 test rejects if, and only if, two 6's are obtained.

(c) Suggest a large-sample test of the above hypothesis.

[6+6+6]

4. The following data is from an experiment to study the relationship between forage production in the spring and mulch left on the ground in the previous autumn . The control measurements (X's) correspond to 0 pounds of mulch per acre, while the treatment measurements (Y's) correspond to 500 pounds of mulch per acre. Forage production is also measured in pounds per acre.

X	794	1800	576	411	897
Y	2012	2477	3498	2092	1303

Assume the two-sample normal model with equal variances.

(a) Find a level 0.95 confidence interval for $\mu_Y - \mu_X$.

(b) Can we conclude that leaving the indicated amount of mulch on the ground significantly improves forage production ? Use $\alpha = 0.05$.

(c) Find a level 0.90 confidence interval for σ.

(d) Comment on any optimum properties of the inference rules used in (a), (b) and (c).

[6+6+6+6]

5. Practical Note-Book. [10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1984–85

DIFFERENCE AND DIFFERENTIAL EQUATIONS

SEMESTRAL-I BACKPAPER EXAMINATION

Date: 27.12.84. Maximum Marks: 100 Time: 3 Hours.

Note : Answer all questions.

1.a) Find the general solution of

$$2xy'' + (1-4x)y' + (2x-1)y = e^x$$

given that e^x is one solution of the homogeneous equation.

b) Write down the general solution of

$$t^2 y'' + ty' + (k^2t^2 - p^2) y = 0$$

in terms of the Bessel functions J_p and Y_p. [10+10=20]

2.a) State and prove Picard's theorem on existence and uniqueness of solution of the equation

$$\frac{dy}{dx} = f(x,y).$$

b) Show that the space of solutions of an nth order linear differential equation with continuous coefficients on an open interval I is an n-dimisional vector space. [15+10=25]

3. Show that

$$e^{\frac{x}{2}(t - \frac{1}{t})} = J_0(x) + \sum_{n=1}^{\infty} J_n(x) [t^n+(-1) t^{-n}]$$

where J_0, J_1, ... are the Bessel functions of the first kind. [20]

4. State and prove Clairaut's principle about geodesics on a surface of revolution. [15]

5. Given that a continuous function of the form $H(x,t) = g(x) \phi(t)$ on $[0,\pi] \times [0,1]$ with $g(0) = g(\pi) = 0$ satisfies the heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ on $(0, \pi) \times (0,1)$, obtain the function H.

6. If $\tan \theta = \sqrt{4 - a^2} / a$ where $|a|<2$, prove that

$$D_n = \begin{vmatrix} a & 1 & 0 & 0 & \dots & 0 \\ 1 & a & 1 & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 0 & 1 \; a & 1 \\ 0 & \dots & \dots & 0 & 0 & 1 \; a \end{vmatrix}$$

$$= \sin (n+1) \theta / \sin \theta.$$ [10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1984-85

SAMPLE SURVEYS

SEMESTRAL-I BACKPAPER EXAMINATION

Date: 28.12.84. Maximum Marks : 100 Time: 3 Hours.

Note : Answer as much as you can. The paper carries a total of 110 marks. The maximum you can score is 100. Marks allotted to each question are given in brackets [ ].

1.a) Explain what you understand by the terms inclusion probability of a unit $U_i(\pi_i)$ and joint inclusion probability of a pair of units U_i, U_j (π_{ij}) for a sampling design. Find the values of π_i and π_{ij} for a Simple Random Sampling With Replacement Design. [3+6]=[9]

b) A survey was conducted in a village consisting of 625 households by covering a sample of 50 households selected using a simple random sampling without replacement scheme to estimate the average weekly expenditure on toilet goods. The estimate was found out to be Rs.4.20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample to be selected by simple random sampling with replacement scheme such that the length of the confidence interval at 95% confidence level is 20% of the true value. State clearly the assumptions involved in finding out the sample size. [10+2]=[12]

2.(a) What do you understand by the term 'Intra class correlation coefficient'? A population consists of 14 clusters of size 6 each. Find the bounds for the intracluster correlation coefficient among the elements of the cluster. [3+3] = [6]

(b) Show that the relative efficiency of sampling a cluster of M units, compared to direct sampling is given by $1/(1+(M-1)\rho)$, where the intra class correlation coefficient, $\rho = 1 - \{M\ \sigma_w^2 / (M-1)\ \sigma^2\}$, σ_w^2 and σ^2 denoting the within and total variances respectively. [7]

p.t.o.

2.(c) Suppose that n clusters are selected at random and without replacement from a population consisting of N clusters of sizes M_i, $i = 1,2, \ldots, N$. Let $\bar{Y}_i = \sum_{j=1}^{M_i} Y_{ij}/M_i$, be the ith cluster mean, where Y_{ij} is the value taken by the study variable y on the jth unit of the ith cluster, $j = 1,2,\ldots,M_i$, $i = 1,2, \ldots, N$. Write down the conventional unbiased estimator and its variance. Also suggest a sampling scheme which makes the estimator $\sum_{i=1}^{n} M_i \bar{Y}_i \big/ \sum_{i=1}^{n} M_i$ unbiased for $\bar{Y}$. [2+2+4]=[8]

3.(a) Explain what you understand by 'Combined and separate ratio estimators' in stratified random sampling. Indicate the situations when you use these estimators, justifying your answer with necessary formulae and assumptions. [6+7]=[13]

(b) The variate y is uniformly distributed in the range (a,a+h). The range is cut in k equal parts to make k strata of equal size. From each stratum, a simple random sample (with replacement) of n|k units is taken. Compare the variance of the estimator of the population mean based on this stratified sample with that of an unstratified sample of size n. Show that the variance in the first case falls off inversely to the square of the number of strata. [5+3]=[8]

4. A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as first stage units and plots within them as second stage units. From each stratum 4 villages were selected with probability proportional to area and with replacement and 4 plots were selected from each selected village with equal probability and without replacement. The data on yield for the sample plots together with information on selection probabilities are given in the table below :

Stratum	Sample village	Inverse of probability of selection	Total no. of plots	Yield of sample plots 1	2	3	4
1	2	3	4	5	6	7	8
I	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119

Contd......

1	2	3	4	5	6	7	8
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	133
	3	24.76	222	264	79	144	55
	4	49.99	69	300	114	68	111
III	1	67.68	189	110	281	120	114
	2	339.14	42	80	61	118	124
	3	100.00	134	121	212	174	106
	4	63.07	161	243	116	314	129

Total no. of plots in stratum I = 8423
" II = 6355
" III =12853

Using the above data

(i) Obtain an unbiased estimate of the total yield of paddy in the district. [15]

(ii) Obtain an unbiased estimate of the variance of the above estimate.

(iii) Compare the efficiency of the above design with that of unistage simple random sampling of plots in each stratum. [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1984-85

STOCHASTIC PROCESSES-2
SEMESTRAL-I BACKPAPER EXAMINATION

Date: 29.12.84. Maximum Marks : 100 Time: 3 Hours.

Note : Answer all questions.

1.(a) Define Brownian bridge.

(b) If $\{X_t, \ t \geq 0\}$ is standard Brownian motion, prove that $\{X_t - t X_1 : 0 \leq t \leq 1\}$ is Brownian bridge. [5+10=15]

2. If $\{X_t, t \geq 0\}$ is standard Brownian motion, find the density of $\left| \min_{0 \leq s \leq t} X_s \right|$. [15]

3. A system is composed of m identical components which act independently. Each component operates for a random length of time until failure. When a component fails it undergoes repair immediately. The failure time and the repair time are both exponential with parameters λ and μ, respectively. Let X_t denote the number of components under repair at time t. Find the infinitesimal parameters of the process X_t. Find the stationary initial distribution. [10+10=20]

4. Suppose that N particles are distributed into 2 boxes labelled 0 and 1. A particle in box i remains in that box for a random length of time that is exponentially distributed with parameter α_i, $i = 0, 1$, before going to the other box. The particles act independently of each other. Let X_t denote the number of particles in box 1 at time t, $t \geq 0$.

(a) Show that $\{X_t : t \geq 0\}$ is a birth and death process. Calculate the birth and death rates.

(b) Find $P(X_t = N \mid X_0 = i)$

(c) Find $E(X_t \mid X_0 = i)$ [10+10+10=30]

p.t.o.

?.(a) Write down the forward differential equations for a birth and death process $\{X_t : t \geq 0\}$ with birth and death rates $\lambda_n = n\lambda$ and $\mu_n = n\mu$, respectively ($\lambda > 0$, $\mu > 0$).

(b) Find $E(X_t \mid X_0 = 1)$ for the above birth and death process.

(c) If $\lambda = \mu$ above, find $P(X_t = 0 \mid X_0 = 1)$.

[5+10+5 = 20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1984-85

MULTIVARIATE DISTRIBUTION AND TESTS
Periodical Examination

Date : 18.2.85. Maximum Marks : 100 Time: 3 Hours.

Note : Attempt as many questions as you can.

1.(a). Let $\underset{\sim}{X}^{(p\times 1)} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is p.d. Partition $\underset{\sim}{X}, \underset{\sim}{\mu}, \Sigma$ as

$$X = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix}, \quad \underset{\sim}{\mu} = \begin{pmatrix} \underset{\sim}{\mu}^{(1)} \\ \underset{\sim}{\mu}^{(2)} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where $\underset{\sim}{X}^{(1)}, \underset{\sim}{\mu}^{(1)}$ are $q\times 1$, Σ_{11} is $q\times q$ ($q\leq p$) and the rest are of appropriate orders. Derive the conditional distribution of $\underset{\sim}{X}^{(1)}$ for given $\underset{\sim}{X}^{(2)} = \underset{\sim}{x}^{(2)}$.

(b) In (a) above specialize to the case $q = 2$ and show that the correlation coefficient between X_1 and X_2 in their conditional distribution for fixed $X_3, \ldots, X_p$ is the same as the partial correlation coefficient between X_1 and X_2 eliminating the effects of $X_3, \ldots, X_p$. [Here $X_1, X_2, \ldots, X_p$ are the components of $\underset{\sim}{X}$]

(c) Under the notations of (a), suppose p is even and equals 2q. Then each of $\underset{\sim}{X}^{(1)}, \underset{\sim}{X}^{(2)}$ has exactly q components. Show that $\underset{\sim}{X}^{(1)} + \underset{\sim}{X}^{(2)}$ and $\underset{\sim}{X}^{(1)} - \underset{\sim}{X}^{(2)}$ are independently distributed if and only if $\Sigma_{11} = \Sigma_{22}$ <u>and</u> $\Sigma_{12} = \Sigma_{21}$. For the (c) part you may use any auxiliary result without proof.

[20+18+12=50]

2. Let $\underset{\sim}{X}_\alpha = (X_{1\alpha}, \ldots, X_{p\alpha})'$, $1 \leq \alpha \leq N$, be a random sample from $N_p(\underset{\sim}{\mu}, \Sigma)$ where $\underset{\sim}{\mu} = (\mu, \mu, \ldots, \mu)'$ and

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \ldots & \rho \\ \rho & 1 & \ldots & \rho \\ \vdots & & \vdots & \\ \rho & \rho & \ldots & 1 \end{pmatrix}$$

Σ being p.d. Define

p.t.o.

$$\bar{X}_\alpha = p^{-1} \sum_{i=1}^{p} X_{i\alpha} \quad (1 \leq \alpha \leq N), \quad \bar{\bar{X}} = \frac{1}{N} \sum_{\alpha=1}^{N} \bar{X}_\alpha$$

$$T = \sum_{i=1}^{p} \sum_{\alpha=1}^{N} (X_{i\alpha} - \bar{\bar{X}})^2, \quad B = p \sum_{\alpha=1}^{N} (\bar{X}_\alpha - \bar{\bar{X}})^2, \quad W = T - B.$$

Show that

$$\frac{B}{W} \cdot \frac{(1-\rho)}{[1+(p-1)\rho]} \cdot \frac{N(p-1)}{N-1}$$

follows F distribution with appropriate degrees of freedom. [25]

3. Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ be iid each $N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is p.d. Define $\bar{\underset{\sim}{X}} = N^{-1} \sum_{\alpha=1}^{N} \underset{\sim}{X}_\alpha$, $A = \sum_{\alpha=1}^{N} (\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})'$. Show that $\bar{\underset{\sim}{X}}$ and A are independently distributed. [32]

4. Let $\underset{\sim}{X}^{(p\times 1)}$ and $\underset{\sim}{Y}^{(p\times 1)}$ be random p-vectors each with null mean vector such that the concentration ellipsoid of $\underset{\sim}{X}$ is wholly contained within that of $\underset{\sim}{Y}$. If $\underset{\sim}{X} = (X_1, \ldots, X_p)'$, $\underset{\sim}{Y} = (Y_1, \ldots, Y_p)'$, show that

$$\text{Var}(X_i) \leq \text{Var}(Y_i)$$

for each i. [13]

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B.Stat.(Hons.) III Year : 1984-85

DESIGN OF EXPERIMENTS

Periodical Examination

Date: 20.2.1985. Maximum Marks : 100 Time: 3 Hours.

Note : Answer any FOUR questions. Marks allotted to a question are indicated in brackets [ ] at the end. 20 marks are allotted to the Practical Records.

1. Explain with illustrations the fundamental principles of experimental designs. Define with examples the three basic experimental designs and indicate their advantages and disadvantages. What is a graeco latin square ? Give an application of such a square. [8+8+4 =20]

2. Let y_i, $i=1,\ldots,5$ be five observations for which we assume the following linear model :

$$y_1 = \theta_1 - \theta_2 + \theta_3 + e_1 \; ; \quad y_2 = \theta_1 + 2\theta_3 + e_2 \; ;$$

$$y_3 = \theta_2 + \theta_3 + e_3 \; ; \quad y_4 = 2\theta_1 - \theta_2 + 3\theta_3 + e_4 ;$$

$$y_5 = \theta_2 + \theta_3 + e_5 \; ;$$

where θ_1, θ_2 and θ_3 are fixed but unknown effects, and e_i's are errors assumed to be i.i.d. with mean zero and variance σ^2.

(a) Identify the class of estimable functions and hence give a necessary and sufficient condition for $l_1\theta_1 + l_2\theta_2 + l_3\theta_3$ to be estimable.

(b) Using two different side restrictions demonstrate that the BLUE of $\theta_2 + \theta_3$ is invariant to the choice of LSE of θ's obtained through the use of the side restrictions mentioned.

(c) Give an unbiased estimator for σ^2 in terms of the y_i's.

[5+7+8 = 20]

3. The effect of five different catalysts on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately $1\frac{1}{2}$ hours, so that only five runs can be made in one day. Taking variation due to days into consideration, suggest an appropriate design for the experiment.

p.t.o.

Q.3 contd......

Propose a suitable model for analysis of the data obtained through the above experimental design. Identify the estimable functions involving only the treatment effects, and give their BLUEs alongwith their variances. Develop the appropriate analysis of variance under the proposed model. [5+2+5+8=20]

4. Derive the expressions for (i) the ''estimates'' of missing values used in the missing plot technique, and (ii) variance of the BLUE of an estimable function involving treatment effects, based on the incomplete data. Suppose in a randomized block design two observations are lost in the same block affecting two treatments i and i' $(i \neq i')$ $(v > 2)$. Obtain the ''estimates'' of the missing values, and the variances of the estimated elementary treatment contrasts based on the incomplete data.

[(7+5)+(4+4) =20]

(a) State and prove a necessary and sufficient condition for orthogonality of a connected block design.

(b) Give the chain definition and the rank definition of a connected block design. Give also an example of a connected non-orthogonal design.

(c) Let C be the C-matrix of a connected block design, and $\hat{\underline{\tau}}$ and $\hat{\hat{\underline{\tau}}}$ be LSE's of $\underline{\tau}$ subject to the side restrictions: $\underline{a}'\hat{\underline{\tau}} = 0$ and $\underline{b}'\hat{\hat{\underline{\tau}}} = 0$ respectively. Show that $(\hat{\hat{\tau}}_i - \hat{\tau}_i)$ is a constant independent of i, and hence, $\hat{\hat{\tau}}_i = \hat{\tau}_i - (\underline{b}'\hat{\underline{\tau}} / \underline{b}'\underline{1})$ where $\underline{1}$ is a vector os unities of appropriate order.

[5+6+9 = 20]

6. SUBMIT YOUR PRACTICAL RECORDS. [20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year: 1984–85

OPTIMIZATION TECHNIQUES
Periodical Examination

Date: 22.2.85. Maximum Marks : 100 Time : 3 Hours.

Note : The whole paper carries 110 marks. Answer all questions. The maximum you can score is 100.

1. Write the dual of the LP

$$\min \quad a^T x + c^T y$$
$$\text{subject to } Ax + By \geq b,$$
$$Cx + Dy = g,$$
$$x \geq 0,$$

and prove the weak duality theorem for these dual LP's. [8+6]

2. State and prove the strong duality theorem for the LP

$$\max \quad c^T x + d^T y$$
$$\text{subject to} \quad 0^T x + 0^T y = 0,$$
$$x \geq 0$$

and its dual. Here c,d are arbitrary real vectors. [15]

3. Consider the transportation problem with two production centres P_1, P_2 and three markets M_1, M_2, M_3. Let the supplies at P_1, P_2 be 10 and 20 respectively and the demands at M_1, M_2, M_3 be 8,5 and 10 respectively. Let the cost matrix be

$$\begin{bmatrix} 10 & 10 & 20 \\ 50 & 20 & 50 \end{bmatrix}$$

(a) Formulate this problem as an LP and write its dual. [8+4]

(b) Find an optimal solution of the primal LP (by guessing', find an optimal solution for the dual and prove that they are optimal by using the weak duality theorem. [14+8+4]

p.t.o.

4. Consider the canonical LP

$$\max c^T x : Ax = b,\ x \geq 0$$

where A is an m x n matrix with full row rank. Let J be a feasible (ordered column) basis and let u,z,Y be defined as usual. Let $k \notin J$, $y_{rk} \neq 0$ and let K be obtained from J by replacing the r^{th} element by k.

(a) Prove that if $z \geq c$ then J is an optimal feasible basis. [6]

(b) Prove that if $z_j - c_j < 0$ and $Y_{*j} \leq 0$ for some j then the objective function of the LP is not bounded above. [12]

(c) If $z_k - c_k < 0$, $y_{rk} > 0$ and

$$\min_i \left(\frac{x^J_i}{y_{ik}} : y_{ik} > 0 \right)$$

is attained when i = r, prove (do <u>not</u> <u>assume</u> any theorem proved in class) that K is a feasible basis and that the value of the objective function corresponding to K is not less than the value corresponding to J. [25]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1984-85

NON-PARAMETRIC AND SEQUENTIAL METHODS
Periodical Examination

Date: 25.2.85. Maximum Marks : 50 Time : 3 Hours.

Note : Answer all questions.

1. Enumerate a few situations where a non-parametric test is preferable to the usual test based on normal theory. Briefly compare the two sample Wilcoxon test with Fisher's t-test. [4+4=8]

2. Prove the unbiasedness of the one-sided two sample Wilcoxon test of H_0: $F=G$ against H_1: $G(x) \leq F(x)$ $\forall$ x (with at least one strict inequality). [12]

OR

Suppose $X_1, \ldots, X_n$ are continuous, i.i.d., symmetric around zero. Prove the asymptotic normality of $\sum_{i=1}^{n}$ (Sign X_i) $\times$ (Rank of $|X_i|$). [12]

3. Suppose in a two sample problem H_0 is true, i.e. $X_1, \ldots, X_n, X_{n+1}, \ldots, X_{n+m}$ are i.i.d., continuous. Find Var $\left(\sum_{1}^{n+m} \ell_i R_i \right)$ where ℓ_i's are constants and R_i is the rank of X_i. Hence or otherwise find Var $\left(\sum_{n+1}^{n+m} R_i \right)$. [5+3=8]

4. Subjects who had received rabies vaccine earlier, but not during the previous six months, were assigned at random to two groups. Subjects in one group received a booster dose, the others did not. The following table shows the antibody responses of the subjects on the fourteenth day of the investigation. Analyse the data formulating hypotheses and carrying out a suitable test.

Group 1(Got booster) 0.26 1.00 1.28 5.00 8.00 16.00 2.56 1.60 8.00 10.20

Group 2(No booster) 0.16 0.08 0.06 0.10 1.28 2.02 0.80 2.56 1.28 5.10

[6]

p.t.o.

5. The following table reports data on mercury levels. All subjects in the exposed group had more than three meals a week of contaminated fish (05.7 mg mercury as methy|mercury per kilogram of fish) for more than three years. None of the controls had a history indicating regular consumption of contaminated fish. State carefully what you would have done if you had to analyse this data. (You don't have to carry at any calculation.) [4]

6. Let X_i's be $N(\mu,1)$. You have to test $\mu=0$ against $\mu = 0.5$. Calculate the relative efficiency of the sign test with respect to the most powerful test, if $\alpha = 0.05$, $\beta = 0.10$. [8]

7. Assignments... [4]

INDIAN STATISTICAL INSTITUTE

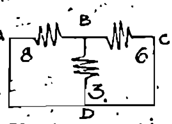
B.Stat.(Hons.) III Year: 1984-85

ELECTIVE-5: PHYSICAL AND EARTH SCIENCES

Periodical Examination

Date : 27.2.85. Maximum Marks : 50 Time : 3 Hours.

Note : Attempt Question No.1 and other three from the rest. Full marks of the Question No.1 is 8 and the rest 14 marks each. Answer should be systametic and to the point.

1. Write down the correct answer as indicated in the letter given below.
   - (a) Resistance is a (A) lateral, (B) element.
   - (b) Diode is a (A) lateral, (B) element.
   - (c) Two resistors of same value are in parallel combination. Its equivalent conductance will be (C) .
   - (d) Two same valued capacitors are combined in series connection so that its equivalent capacitance will be (C).
   - (e) In an ac circuit $E = E_0 \sin wt$, the maximum amplitude will be (D) at wt (E).
   - (f) 

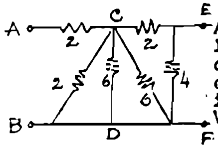
     In the given circuit the resistance across AD will be (F).
   - (g) Electromagnetic energy stored in an inductance of unit value will be (G).
   - (h) Electrostatic energy stored in an unit valued capacitance will be (G) .

HINTS : (A) uni/bi/tri/multi (B) active/passive/neutral (C) zero/half/same/double (D) $0/E_0/-E_0/E$ (E) $0/90^\circ/180^\circ/270^\circ/360^\circ$ (F) $0/10/11/14/\infty$ and (G) $0/0.5I^2/0.5E^2/I^2/E^2$.

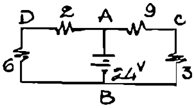
[1x8 = 8]

p.t.o.

2. From Ohm's Law define resistance. How it is related to the conductance? With suitable example name the different resistors used in electronic circuit. [1+1+2=4]



A battery of 24^V with an internal resistance 1 Ω is connected across AB of the given circuit, all resistors in Ω. Find out (i) the equivalent resistance of the circuit (ii) input current I and (iii) output voltage V across 4 Ω. [2.5+1+1.5=6]



Find out the current generated from the battery and voltage across DC i.e. V_{DC}.

[2+2=4]

3. A dc voltage E is applied across a series RL circuit. With nea circuit diagram and appropriate graphs discuss the behaviour o the generated current and induced voltage across L.

[1+3.5+1.5=6]

Discuss with circuit diagram and graphs how the current behaves if the battery is removed and the open path is short circuited. What will be the voltage across L. [1+2.5+1.5=5]

Define the time constant of the circuit. Find the value of charging and discharging current at t = τ. [1+1+1 = 3]

4. For one minute a dc voltage E (=10^V) is applied across a resistance R (=100Ω). For the next minute E has been withdrawn Sketch the behaviour of generated current before and after 1 minute. [1+1 = 2]

In this circuit a capacitor C(0.1F) has been joined in series and is allowed to charge fully for 1 minute. In the next minute battery has been removed and the open path is short circuited. Discuss with the necessary graphs the nature of current flowing in this combination. Also sketch the graph of E_R, the impressed voltage across R. [1+2+1=4]

Calculate the value of time constant τ. Deduce E_c and also accurately work out the value of E_c at t/τ = 0,0.5,1.0,1.5, 2.0,2.5,3.0,4.0, 5.0 and 6.0. Sketch E_c with these values.

[1+1+10x0.5+1 = 8]

contd..........

5. An alternating current $I=I_0 \sin wt$ is flowing through a series R-L-C circuit. Find the phase relationship of the current to the applied voltage. What will be the impedance of the circuit? [1+3+1=5]

Find out the condition for which the given circuit behaves as (i) resistive, (ii) inductive and (iii) capacitive circuit. Also draw the suitable phasor diagram. Draw the phasor diagram of overall RLC circuit. [1+1.5+1.5+2=6]

For what condition series resonance occurs? What will be the resonant frequency and the magnitude of current. [1+1+1=3]

6. An alternating current $I=I_0 \sin wt$ is flowing through an inductive circuit. Find the phase relation to the inductive voltage. What is the inductive reactance? For what condition a choke is used as (i) high frequency and (ii) low frequency choke. [1+3+1+2=7]

An alternating voltage $E = E_0 \sin wt$ is applied across a capacitor. Find the phase relation to the current flowing. What is the capacitive reactance? For what condition a capacitor (i) bypasses and (ii) blocks the current. [1+3+1+2=7]

[1984-85 | 3E1]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1984-85

ELECTIVE -5: ECONOMICS

Periodical Examination

Date: 27.2.85. Maximum Marks : 100 Time: 3 Hours.

Note : Answer Question No.5 and any three of the other questions. Marks for each question are given in brackets.

1. What is Pareto's Law of income distribution ? Obtain the form of the density function of income distribution implied by Pareto's law. Also, obtain the expressions for arithmetic mean, median and mode for the Pareto distribution and hence comment on the shape of this distribution. [20]

2.(a) What do you mean by inequality in the distribution of a "size" variable ? What properties would you like an income inequality measure to possess ?

(b) Obtain the alternative expressions for the Lorenz ratio. Also, examine if this measure of inequality possesses the desirable properties. [6+14]=[20]

3. What are moment distributions ? Obtain the first moment distributions for the Pareto and the two-parameter lognormal distribution, and hence derive the expressions for the Lorenz curve and the Lorenz ratio for these two distributions. [20]

4. Write notes on any two of the following :

(a) Methods of estimation of the parameters of Lognormal distribution;

(b) Universality of Pareto's Law ;

(c) Properties of the Lorenz curve for the Lognormal distribution.

[10x2]=[20]

p.t.o.

5. The following data relate to the size distribution of monthly per capita total consumer expenditure (PCE) for West Bengal during the period July 1977 – June 1978.

monthly PCE(Rs.) class	0-20	20-30	30-40	40-50	50-60	60-70
estimated percentage of population	2.39	10.39	18.74	18.35	15.09	10.11

monthly PCE(Rs.) class	70-80	80-100	100-150	150 and above
estimated percentage of population	7.79	8.08	6.47	2.59

(a) Examine graphically the suitability of a Lognormal distribution for the above data;

(b) Estimate the parameters of Lognormal distribution for the above data by method of quantities;

(c) Based on your estimates in 5(b) above, calculate the Lorenz ratio. [20+15+5]=[40]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

DESIGN OF EXPERIMENTS
SEMESTRAL-II EXAMINATION

Date: 8.5.85. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Note : Answer any FOUR out of questions 1-5. Marks allotted to a question are indicated in brackets [ ] at the end. Question 6 is compulsory.

1.(a) Define variance balance and prove a necessary and sufficient condition for a connected block design to be variance balanced. It is said that simplicity of analysis and variance balance go hand-in-hand. Explain.

(b) Suppose in a randomized block design for v treatments in v blocks, observations under treatment i, in block i, i = 1,2,..., v, got lost. Examine whether the resulting design is connected variance balanced or not. Give the analysis of variance of the resulting design, BLUE of a treatment contrast, and its variance.

[(6+4)+(3+7) = 20]

2.(a) Define a BIBD with parameters v,b,r,k,λ and give an example; prove that for such designs $b \geq v$, equality holding if and only if any two blocks have λ treatments in common.

(b) Show that a set of initial blocks having symmetrically repeated differences generate a BIBD. Give an example of a series of BIBD constructible through this method.

[(3+7)+(7+3) = 20]

3. Explain how analysis of covariance can be looked upon as a device to exercise local control. Show how the analysis of covariance can be worked out easily once the corresponding analysis of variance is known. Apply the results in case of a latin square design with one concomittant variable.

[3+10+7 = 20]

p.t.o.

4.(a) Define a set of mutually orthogonal latin squares (MOLS). Show that there can be at most (V-1) MOLS of order V.

(b) Give a method of construction of a complete set of MOLS of order s, and show how these can be used to construct the following series of BIBD :

$$v = b = s^2+s+1, \; r = k = s + 1, \; \lambda = 1,$$

where s is a prime power. (You have to prove that the method always leads to a BIBD.)

[5+(5+10) = 20]

5.(a) Define main effects and interactions for a 2^n factorial experiment, and show that they are mutually orthogonal contrasts of treatment effects.

(b) Describe a method of construction and analysis of a confounded design for a 2^n factorial in blocks of 2^k plots.

[6+(6+8) = 20]

6. SUBMIT YOUR PRACTICAL RECORDS. [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

NONPARAMETRIC AND SEQUENTIAL METHODS
(THEORY)
SEMESTRAL-II EXAMINATION

Date: 10.5.85. Maximum Marks : 50 Time: 2 Hours.

1.a) Suppose that $X_1, \ldots, X_n$ are i.i.d with common distribution function F and $Y_1, \ldots, Y_n$ are i.i.d with common distribution function G. If $F(x) \leq G(x)\ \forall x$, show that a similar relation holds for the distribution functions of $\bar{X}$ and $\bar{Y}$. [8]

b) $X_1,\ldots,X_n$ and $Y_1,\ldots, Y_m$ are random samples from F and G respectively. Give an example where $F \neq G$ but $P_{F,G}\{X_1<Y_1\} = \frac{1}{2}$. Under this alternative, find the limiting power of the Wilcoxon two sample test of size α, $(0<\alpha<1)$. [8]

2. $X_1,\ldots,X_n$ are i.i.d with continuous distribution function F. The null hypothesis is that F is symmetric around zero. How would you formulate the one sided alternatives ? Prove any result necessary to motivate your formulation. Write down two common test statistics for this problem and describe (without proof) the alternatives for which they provide consistent tests. [1+4+4 = 9]

3.a) State and prove the Fundamental Identity of Sequential Analysis.

b) Let $\{z_i\}$ be i.i.d $N(0,1)$. Let τ = first $m \geq 1$ such that $S_m = z_1 + \ldots + z_m \geq a$ or $\leq b$. Find $E(e^{h\tau})$ for $h = -1/2$, neglecting excess over boundaries. [12+13]

OR

4. Under conditions to be stated by you, prove that the stopping time τ of an SPRT has finite m.g.f. in a neighbourhood of zero. [25]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year, 1984-85

NONPARAMETRIC AND SEQUENTIAL METHODS
(PRACTICAL)

SEMESTRAL-II EXAMINATION

Date: 10.5.85. Maximum Marks : 60 Time: $2\frac{1}{2}$ Hours

Note : Answer all questions.

1. In a pre and post treatment studies of coronary circulation in chronic severe anaemia, the following data (in stroke index ($ml/beat/m^2$) is obtained.

Case	1	2	3	4	5	6	7	8
Before treatment	109	57	53	57	68	72	51	65
After treatment	56	44	55	40	62	46	49	41

Formulate the null hypothesis, alternate hypothesis and carry out a suitable test. [10]

2. Let $X_1, X_2, \ldots X_n$ be iid $N(\theta, 2)$. Find SPRT with $\alpha = .01$ and $\beta = .01$ to test
$H_0 : \theta = 0$ Vs $H_1 : \theta = 1$.
Calculate O.C. of the above SPRT at $\theta = 0, .5$. [14]

3. The following data give the gains in weight of three groups of rats receiving three types of diet. Analyse the data and test whether there is significant difference between the diets.

diet	gain in weight	ranks in the combined sample
A	24.0,25.0,25.1,25.7 28.1,29.0,29.7,30.1	2,7,8,11,22,25,27,30.
B	23.9,24.2,24.3,24.7 25.4,26.1,26.6,27.3 27.6,27.8,30.0,27.0	1,4,5,6,10,12,14,16, 18,19,20,29.
C	24.1,25.2,26.2,26.7 27.2,28.0,28.2,28.7, 29.3,29.8	3,9,13,15,17, 21,23,24,26,28.

[10]

p.t.o.

4. Let $\{X_i\}$ be i.i.d. according to an unknown $F(x)$ which is assumed to be continuous and consider the following testing problem,

$$H_0 : F(x^0) = \xi , \quad F(x^1) = 1 - \frac{\alpha\xi}{1-\alpha}$$

$$H_1 : F(x_0) = \eta, \ F(x^1) = 1 - \frac{(1-\beta)\eta}{\beta} , \text{ where } -\infty < x^0 < x^1 < \infty,$$

$0 < \xi < 1 - \alpha < 1$ and $0 < \eta < \beta < 1$ are specified.

A cartesian sequential test S is defined by the rules : at stage $n \geq 1$, accept H_0 if $x_n \in R^0 = (-\infty, x^0]$ and reject H_0 if $x_n \in R^1 = [x^1, \infty)$ and continue otherwise.

Show that the test S provides a solution with strength (α,β).

In particular for testing $F(x)$ has median at x^0 and a 95% point x^1 or a median at x^1 and a 5% point at x^0, choose α,β suitably so that this becomes a special case of the above problem. Find the ASN of S. [11]

5. In the secretary problem with N=6, find $P(a_3=1,\ y_2=1)$

OR

For N = 6, find the optimal selection rule to maximize the probability of selecting the best candidate.

[5]

6. Assignment. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

OPTIMIZATION TECHNIQUES
SEMESTRAL-II EXAMINATION

Date: 13.5.85. Maximum Marks : 100 Time: 4 Hours.

Note : Answer all questions. The whole paper carries 110 marks but the maximum you can score is 100.

1. Given a problem

   maximize $c^T x$ subject to $Ax = b$, $x \geq 0$

   where it is not even known whether there is a solution, give, without proofs, the complete procedure for solving the problem by the simplex method. [15]

2. Find an optimal transportation schedule and its cost for a problem with unit cost matrix, supplies and demands as given below.

							Supply
	5	10	15	8	9	7	30
	14	13	10	9	20	21	43
	15	11	13	25	8	12	10
	9	19	12	8	6	13	100
demand	50	20	10	35	15	50	

   Briefly indicate how you got your initial basic feasible solution and show the subsequent transportation tableaux (including entering and departing variables etc.) clearly. [20]

3. Give Dijkstra's algorithm for finding a shortest dipath from s to t in a digraph with a (non-negative) length given for each arc. Prove that the algorithm actually gives such a dipath. (You may assume that there is at least one dipath from s to t). [5+20]

4.(a) Prove that a flow f in a network is maximum if and only if there is no s-t augmenting path w.r.t.f. (You may assume that the value of any flow $\leq$ the capacity of any cut). [20]

p.t.o.

4.(b) Seven students $x_1, x_2, \ldots, x_7$ have completed a certain course and are seeking jobs. Four companies C_1, C_2, C_3, C_4 interviewed them and found the following candidates suitable for them. The number of vacancies in each company is also given.

Company	candidates found suitable to it	Number of vacancies
C_1	x_1, x_2, x_3, x_6	3
C_2	x_2, x_4	2
C_3	x_3, x_4, x_6	2
C_4	x_5, x_6, x_7	1

You have to find the maximum number of candidates who can be employed.

(i) Formulate this as a maximum flow problem in a suitable network. Give brief justification. [10]

(ii) Find a maximum flow and hence a solution to the above problem. (You may start with a reasonably good flow instead of 0). [15]

(iii) Find a minimum cut and hence verify that the flow you have got in (ii) is indeed maximum [5]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984-85

ELECTIVE-5 : ECONOMICS

SEMESTRAL-II EXAMINATION

Date: 15.5.85. Maximum Marks : 100 Time: 3 Hours.

Note : Answer Question No.4 and any two of the rest. Marks for each question are given in brackets.

1.(a) Explain the concepts of engel curve and engel elasticity. Distinguish between (i) income and expenditure elasticity, and (ii) value and quantity elasticity of a commodity.

(b) Suppose there are K homogeneous groups of people and the engel elasticity for a commodity for the j^{th} group is η_j, $j = 1,2, \ldots, K$. Obtain the engel elasticity for the commodity for the entire population.

(c) Justify the use of 'per capita' formulation of an engel curve. Do you think that such a formulation is always adequate ? Give reasons for your answer. Discuss a suitable formulation of engel curve that takes into account the demographic characteristics of households. [5+5+20] = [30]

2.(a) What do you mean by price elasticity of demand for a commodity ? Give the alternative classifications of commodities based on price elasticities.

(b) Describe briefly the methodology of statistical analysis of demand for a single commodity based on time series data. Discuss the problem of estimating reliably the income and price elasticities for a commodity in such an analysis. Also, suggest suitable methods for overcoming this difficulty.

(c) Discuss briefly the Identification Problem that may arise in demand analysis based on market data. [5+15+10]=[30]

p.t.o.

3. Write notes on <u>any three</u> :

   (a) Criteria for choosing the algebraic form of engel curve;
   (b) Demand Projection based on engel curves;
   (c) Prais-Houthakker's method of estimating equivalent scales;
   (d) Cobb-Douglas Production Function.

[10x3] = [30]

4. The following expenditure data relate to ten groups of households ranked in ascending order of their per capita monthly total consumer expenditure (PCE) :

PCE group	percentage of persons	per capita monthly expenditure(Rs.) on	
		clothing	all items
1	17.6	16.6	174.4
2	16.2	17.1	198.3
3	5.8	22.6	252.9
4	37.9	24.7	297.4
5	9.3	26.2	342.1
6	2.3	27.6	387.5
7	2.0	33.9	468.2
8	2.4	47.7	570.1
9	3.9	47.2	869.2
10	2.6	71.1	1253.3

(a) Estimate the constant elasticity form of engel curve for clothing on the basis of the above data. Also, calculate (a) the standard errors of the estimated parameters and (b) a measure of goodness of fit.

(b) Suppose the average PCE of this group of households is raised by 25 per cent. Measure the change in the average expenditure on clothing. Clearly mention the assumptions that you will be making in carrying out this measurement.

[25+5] = [30]

5. Practical Records. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year, 1984-85

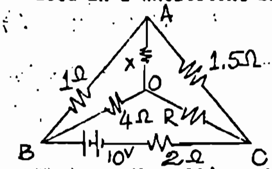
Elective-5: Physical and Earth Sciences
SEMESTRAL-II EXAMINATION

Date: 15.5.85. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Note : Maximum Marks - 100. Each question carries 20 marks. You can answer all the questions but the maximum you can get is 100.

Instruction : Wherever necessary draw the circuit diagram and related graphs.

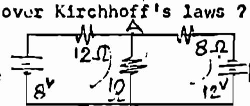
1\. State and explain, in detail, Kirchhoff's laws. Find an expression for the current flowing though the galvanometer used in a Wheatstone bridge. Find null condition. [10]



When the current is zero in branch OA, find the value of R and current flowing through it. [4]

What are Maxwell's modifications over Kirchhoff's laws ?

Find the current flowing through the path AB having 10Ω resistance [6]



2\. An alternating voltage is applied across an RLC series circuit. Derive an expression for current flowing through it by vectorial method or otherwise. For what condition it (a) leads and (b) lags to the applied voltage ? [8]

What is resonance in RLC series circuit ? Why this is referred to as voltage resonance ? Find resonant frequency. [6]

Across 200V, 50 Hz supply a series RLC circuit, with variable inductance, has the maximum current of 314 mA when the voltage across inductor is 300V. Calculate the value of R, L and C. [6]

p.t.o.

3. A choke with an inductance L and small resistance R is in parallel to a capacitor C. Derive the value of the overall and the branch currents assuming that applied voltage across the parallel circuit is of alternating nature. [6]

For what condition this circuit will be in resonance ? Derive (a) resonant frequency, (b) resonance current and (c) dynamic impedance of the circuit. Why this circuit is referred to as rejector circuit ? [10]

A resonant circuit consists of 4 μF capacitor in parallel with an inductor of 0.25 H having a resistance of 50 Ω. Calculate the frequency of resonance. [4]

4. With usual notation, derive an expression for the discharging current through and voltage impressed across C in an RC circuit. [6]

What is differentiating circuit ? Derive an expression for the output voltage of the circuit. For a square wave input sketch the output waveform. [5]

Derive an expression for output voltage of an integrating circuit. [4]

A square wave of peak value + 100V is applied to an RC differentiator of time constant 0.05 of the period of the wave. Plot the output waveform assuming initial value of e_c to be zero. [5]

5. Clearly define, with suitable example, (a) valance band and (b) conduction band. What is hole ? How hole current relates to electron current ? [6]

Differentiate clearly among (a) insulators, (b) conductors and (c) semiconductors. [8]

What consists of semiconductor current ? Derive an expression of intrinsic semiconductor current. [6]

6. What are extrinsic semiconductors ? Give suitable doping elements for them. What are the subdivisions of extrinsic semiconductors ? Describe them with details. [8]

What is transistor ? How junction transistors are formed ? Describe with their symbol, different types of junction transistors ? What are the different sections of the transistors ? How they are related in consideration of doping ? [8]

Derive a relation for the different currents flowing through the transistor. [4]

INDIAN STATISTICAL INSTITUTE
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MULTIVARIATE DISTRIBUTIONS AND TESTS

SEMESTRAL-II EXAMINATION

Dated: 6.5.85. Maximum Marks : 100 Time: 3 Hours.

Note : Attempt as many questions as you can.

1. Discuss in detail the multivariate Behrens Fisher problem for comparing two multinormal mean vectors when the population dispersion matrices are not necessarily identical. (20)

2. (a) Let $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_N$ be iid each $N_p(\underset{\sim}{\mu}, \Sigma)$. Derive the likelihood ratio test for $H_0: \underset{\sim}{\mu} = \underset{\sim}{\mu}_0$ against $H: \underset{\sim}{\mu} \neq \underset{\sim}{\mu}_0$. Examine how the test eventually reduces to an F-test.

   (b) In the set-up of (a) above, let $n = N-1$, $\bar{\underset{\sim}{x}} = \frac{1}{N} \sum_{\alpha=1}^{N} \underset{\sim}{x}_\alpha$,

$$A = \sum_{\alpha=1}^{N} (\underset{\sim}{x}_\alpha - \bar{\underset{\sim}{x}})(\underset{\sim}{x}_\alpha - \bar{\underset{\sim}{x}})', \quad A_0 = \sum_{\alpha=1}^{N} (\underset{\sim}{x}_\alpha - \underset{\sim}{\mu}_0)(\underset{\sim}{x}_\alpha - \underset{\sim}{\mu}_0)',$$

$$T^2 = nN\,(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' A^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})$$

$$T'^2 = nN\,(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' A_0^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})$$

   Find a relation between T^2 and T'^2 and hence find the distribution of T'^2. (15+20=35)

3. Let $\underset{\sim}{x} = (x_1, \ldots, x_p)' \sim N_p(\underset{\sim}{\mu}, \Sigma)$ and β_j be the population partial regression coefficient of x_1 on x_j ($2 \leq j \leq p$). Defining $\underset{\sim}{\beta} = (\beta_2, \ldots, \beta_p)'$, describe a test for $H_0: \underset{\sim}{\beta} = \underset{\sim}{\beta}_0$, when Σ is known. Derive the null distribution of your test criterion. (20)

4. The following data represent the marks obtained by ten students in three papers of an examination

		Student									
		1	2	3	4	5	6	7	8	9	10
	I	70	72	60	65	84	43	42	89	91	54
Papers	II	60	82	63	66	51	70	60	69	73	94
	III	57	63	60	72	65	86	42	49	70	65

p.t.o.

(a) Apply the step-down procedure to examine whether the average marks in each of the three papers may be taken to be 60.

(b) Also examine whether the average marks in all the three papers may be supposed to be equal.

(15+15=30)

5. Assignment. (10)

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1984-85

MULTIVARIATE DISTRIBUTIONS AND TESTS

SEMESTER-II BACKPAPER EXAMINATION

Date: 1.7.85. Maximum Marks : 100 Time : 3 Hours.

Note : Attempt as many questions as you can.

1.(a) Let $A \sim W_p(\Sigma|n)$. Partition A and Σ as $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ where A_{11} and Σ_{11} are q x q. Define $A_{11.2} = A_{11} - A_{12}A_{22}^{-1} A_{21}$, $\Sigma_{11.2} = \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1} \Sigma_{21}$. Show that $A_{11.2} \sim W_q(\Sigma_{11.2}| n-(p-q))$.

(b) If $\underset{\sim}{\ell}$ be a p-component non-null vector, show that with A as in (a),

$$\frac{\underset{\sim}{\ell}' \Sigma^{-1} \underset{\sim}{\ell}}{\underset{\sim}{\ell}' A^{-1} \underset{\sim}{\ell}}$$

follows the Chi-square distribution with n-p+1 d.f.

(30+10=40)

2. Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ be iid $N_p(\mu,\Sigma)$. Let $\underset{\sim}{\mu} = (\mu_1, \ldots, \mu_p)'$. Derive a test for $H_0 : \mu_1 = \mu_2 = \ldots = \mu_p$ and obtain a simplified form of your test statistic. (20)

3. If $\underset{\sim}{X} = (X_1, \ldots, X_p)'$ be p-variate normal show that each of $X_1, \ldots, X_p$ is univariate normal. Give a counterexample (with proof) to demonstrate that the converse of the above result is not true. (25)

4. On the basis of 15 trivariate observations the following correlation matrix is obtained

$$\begin{pmatrix} 1 & 0.1 & 0.2 \\ 0.1 & 1 & 0.1 \\ 0.2 & 0.1 & 1 \end{pmatrix}$$

Assuming trivariate normality and under usual notations test

(a) $H_0 : \rho_{1.23} = 0$ (b) $\rho_{12.3} = 0$, (c) $\rho_{12.3} = 0.08$.

(6+6+8=20)

5. Assignment. (10)

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year, 1984–85

DESIGN OF EXPERIMENTS
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 2.7.85. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Note : Answer any FOUR out of questions 1–5. Marks allotted to a question are indicated in brackets [ ] at the end. Question No.6 is compulsory.

1.(a) Show that for any block design

(i) b + Rank C = v + Rank D ; and

(ii) Adjusted Block S.S. + Unadjusted Treatment S S.
= Adjusted Treatment S.S. + Unadjusted Block S.S.

(b) Show also that $\underline{\ell}'\underline{\tau}$ is estimable if and only if $\underline{\ell} \in$ Column Space of C.

(c) Prove that for any block design :

$$C \text{ Diag } [r_1^{-1}, r_2^{-1} \ldots, r_v^{-1}] N = N \text{ Diag } [k_1^{-1}, k_2^{-1}, \ldots, k_b^{-1}] D.$$

$$= (- \sigma^{-2}) \text{ Cov } (\underline{Q}, \underline{P}').$$

[(4+4)+6+6=20]

2.(a) Define a BIBD (v,b,r,k,λ) with an example, and prove that for such designs $\lambda(v-1) = r(k-1)$.

(b) Give the analysis of variance of a BIBD, BLUE of a treatment contrast and its variance.

(c) Show that for a BIBD ($v = b$, $r=k$, λ) with even v, $(r-\lambda)$ must be a perfect square. Hence give an example of an admissible combination of parameters (v,r,λ) for which a symmetric BIBD is nonexistent. [6+8+(4+2) =20]

3. Give the theory behind missing plot technique. Illustrate with respect to one missing observation in a latin square design. Give also the variances of estimated elementary treatment contrasts. [12+5+3=20]

4.(a) Explain the concepts of connectedness and variance balance with examples.

(b) Consider a BIBD $(v,b,r,k,\lambda=1)$. Suppose the observations of a block got lost. Examine whether the resulting design is connected or not.

p.t.o.

4.(c) Suppose N is the incidence matrix of a BIBD (v,b,r,k,λ) and J_{vb} is a v x b matrix of unities. Define $N^* = N + J_{vb}$. Examine whether N^* is variance balanced or not.

[6+7+7 =20]

5.(a) Explain the concept of confounding with an example.

(b) Is a confounded design always disconnected?? Explain.

(c) Discover the effects confounded in a replication of a $(2^5, 2^3)$ confounded design where the contents of a block are as follows :
(a, abc, ade, cd, abcde, bd, ce, be).

(d) Describe Yates' algorithm of sum and difference and explain its usefulness in the analysis of a $(2^n, 2^k)$ confounded design.

[5+3+4+8 = 20]

6. Submit your practical records. [20]

[1984-85 | 382(b)]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year, 1984 - 85

NONPARAMETRIC AND SEQUENTIAL METHODS

SEMESTRAL-II BACKPAPER EXAMINATION

Date : 3.7.85. Maximum Marks : 50 Time: 2 Hours.

1.(a) Suppose that $X_1, \ldots, X_n$ are i.i.d with common distribution function F and $Y_1, \ldots, Y_n$ are i.i.d with common distribution function G. If $F(x) \leq G(x) \; \forall x$, show that a similar relation holds for the distribution functions of $\bar{X}$ and $\bar{Y}$.

(b) $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are random samples from F and G respectively. Give an example where $F \neq G$ but $P_{F,G}\{X_1 < Y_1\} = \frac{1}{2}$. Under this alternative, find the limiting power of the Wilcoxon two sample test of size α, $(0 < \alpha < 1)$.

(c) Let $\{z_i\}$ be i.i.d $N(0,1)$. Let τ = first $m \geq 1$ such that $S_m = z_1 + \ldots + z_m \geq a$ or $\leq b$. Find $E(e^{h\tau})$ for $h = -1/2$, neglecting excess over boundaries. [10+10+10=30]

2. Prove the efficiency of the SPRT neglecting excess over boundaries. [20]

Or

Prove the monotonicity of the OC function of an SPRT under the MLR condition. [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1984-85

OPTIMIZATION TECHNIQUES
SEMESTRAL-II BACKPAPER EXAMINATION

Date: 4.7.85. Maximum Marks : 100 Time: 3 Hours.

Note : Answer all questions. The whole paper carries 110 marks but the maximum you can score is 100.

1.(a) State the (strong) duality theorem for a general LP. [5]

(b) Deduce from the duality theorem the following two results.

(i) $Ax > 0$ has a solution iff $A^T y$, $y \geq 0 \Rightarrow y = 0$. [10]

(ii) $Ax = b$ has a solution iff $A^T u = 0 \Rightarrow b^T u = 0$. [10]

2. Give Dijkstra's algorithm. Using this find a shortest dipath from each vertex to V_6 in the following digraph G. The vertex set of G is $\{V_1, V_2, \ldots, V_6\}$ and the arc set of G is $\{V_i V_j : i \neq j, c_{ij} < \infty\}$ where $C = ((c_{ij}))$ is

$$C = \begin{bmatrix} 0 & 50 & \infty & 40 & 25 & 10 \\ 50 & 0 & 15 & 20 & \infty & 25 \\ \infty & 15 & 0 & 10 & 20 & \infty \\ 40 & 20 & 10 & 0 & 10 & 25 \\ 25 & \infty & 20 & 10 & 0 & 55 \\ 10 & 25 & \infty & 25 & 55 & 0 \end{bmatrix}$$

Here c_{ij} denotes the length of the arc V_iV_j. [25]

3.(a) State the max-flow min-cut theorem. [5]

(b) From the above theorem, deduce the following result : a family $(S_1, S_2, \ldots, S_m)$ of sets has an SDR iff the union of any k S_i's has at least k elements for all k. [15]

(c) There are m machines and n operators. It is given that there is a positive number p such that (i) for each machine there are exactly p operators who can operate it and (ii) each operator can operate exactly p machines. Prove using the result in (b) that all the machines can be operated simultaneously. [15]

p.t.o.

4. Let A be an m x n matrix and let

$$X = \left\{ x \in \mathbb{R}^m : x \geq 0 \text{ and } \sum x_i = 1 \right\},$$

$$Y = \left\{ y \in \mathbb{R}^n : y \geq 0 \text{ and } \sum y_j = 1 \right\}.$$

(a) Prove using duality theorem that

$$\max_{x \in X} \min_{y \in Y} x^T A y = \min_{y \in Y} \max_{x \in X} x^T A y.$$

[15

(b) If $\max_i \min_j a_{ij} = \min_j \max_i a_{ij}$ then prove the result in (a) without using duality theorem.

[10]