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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year:1986-87

SAMPLE SURVEYS
PERIODICAL EXAMINATION

Date : 25.8.86. Maximum Marks : 100 Time: 3 Hours.

Note : Answer all questions.

1.(a) What do you understand by the terms 'inclusion probability π_i of unit U_i' and ' joint inclusion probability π_{ij} of a pair of units U_i and U_j, $i \neq j$ ' for a sampling design. (2+4)=(6)

(b) For a Simple Random Sampling (SRS) With Replacement design of n draws, write down the values π_i and π_{ij}. (2+4)=(6)

(c) Let $N = 3$ and $n = 2$ and $s_1 = \{U_1, U_2\}$, $s_2 = \{U_1, U_3\}$, $s_3 = \{U_2, U_3\}$. Under the SRS design let $p(s_i) = 1/3$ for $i = 1,2,3$. Define the estimator t by

$$t = \begin{cases} y_1/2 + y_2/2 & \text{if } s_1 \text{ occurs} \\ y_1/2 + 2y_3/3 & \text{if } s_2 \text{ occurs} \\ y_2/2 + y_3/3 & \text{if } s_3 \text{ occurs.} \end{cases}$$

Show that t is unbiased for $\overline{Y}$ and that there exist values (+ve) (Y_1, Y_2, Y_3) for which $V(t) < V(\overline{y})$, where $\overline{y}$ denotes the sample mean. What does this example show ? (3+6+2)=(11)

2.(a) How do you select a linear systematic sample of size n from a population of size N ? (3)

(b) Suggest an unbiased estimator for the population mean $\overline{Y}$ of a characteristic y based on the above design. Describe a modification of the above design which makes the sample mean an unbiased estimator of $\overline{Y}$. (4+4)=(8)

(c) When the values of the y-characteristic are known to be of the form $Y_i = \alpha + \beta i$ and when the population size is a multiple of the sample size, would you prefer systematic sampling to simple random sampling ? Give reasons. (11)

p.t.o.

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3.(a) When do you use a Probability Proportional to Size sampling technique ? (4)

(b) Show that the selection of units by 'Lahiri's method' does give the probability of selection for a unit U_i equal to X_i/X where X_i is the size measure of the unit U_i and

$$X = \sum_{i=1}^{N} X_i. \qquad (7)$$

(c) A sample of 6 farms is drawn from a population of 74 farms with probability of selection proportional to the size (x in acres) with replacement and the yield of paddy is observed:

Sampled farm	X Size in acres	Y Yield in bushels per acre
1	21	105.2
2	101	524.3
3	14	73.1
4	6	31.2
5	41	200.9
6	12	64.3

It is also known that the total size of all the 74 farms is

$$X = \sum_{i=1}^{74} x_i = 2949 \text{ acres.}$$

(i) Estimate the average yield of the population. (11)

(ii) Calculate an estimate of the coefficient of variation of the above estimate. (14)

(iii) If, by mistake, a person treats the sample as a without replacement sample obtained in the above order, what would be his estimate of the average yield ? Is it unbiased for $\overline{Y}$? (7+2)=(9)

4. Practical records (to be submitted to the Dean's Office by 27.8.86.) (10)

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B.Stat. (Hons.) III Year:1986-87

STATISTICAL INFERENCE

PERIODICAL EXAMINATION

Date : 27.8.86. Maximum Marks : 80 Time: 3 Hours.

Note : Try two questions from Q.1,Q.2,Q.3. (2x10)
Try two questions from Q.4,Q.5,Q.6. (2x12)
Try Q.7,Q.8 and Q.9. (3x8)
Home-work and Practical work (12)

1. If time is measured in discrete periods a model that is often used for the time X to failure of an item is

$$P_\theta [X = k] = \theta^{k-1}(1 - \theta),\ k = 1,2, \ldots\ldots$$

where $0 < \theta < 1$. Suppose we record only the following events and define

$$Y = k, \text{ if } X = k, \quad k = 1,\ldots\ldots r$$
$$Y = r+1, \text{ if } X > r.$$

(a) Obtain the distribution of Y.

(b) We observe n independent observations on Y, given by $Y_1, \ldots, Y_n$. Show that the maximum likelihood estimate of θ based on $Y_1,\ldots,Y_n$ is
$$\theta = \frac{\sum_1^n Y_i - n}{\sum_1^n Y_i - m},$$
where m is the number of Y_i's which equal r+1.

2. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta,1)$, where $A \leq \theta \leq B$ and A and B are known. Draw the likelihood function and obtain the maximum likelihood estimate of θ.

3. Which of the following families of distributions are exponential families ? (Prove or disprove).

(a) Uniform $(0,\theta)$

(b) $p(x,\theta) = \exp [-2 \log\theta + \log (2x)] \quad I_{(0,\theta)}(x)$

(c) $p(x,\theta) = \frac{1}{9}$, $x = \theta + 0.1, \theta + 0.2, \ldots, \theta + 0.9$

(d) $N(\theta, \theta^2)$

(e) $p(x,\theta) = 2(x+\theta)/(1+2\theta),\ 0 < x < 1,\ \theta > 0$

(f) $p(x,\theta)$ is the conditional frequency function of a binomial $B(n,\theta)$, variable X, given that $X > 0$.

p.t.o.

4. Let 4 items be drawn at random one-by-one without replacement from a shipment of 10 items of which M items are bad, M being unknown ($M = 0,1,\ldots,10$).

Let

$$X_i = \begin{cases} 1, & \text{if the ith item drawn is bad} \\ 0, & \text{if the ith item drawn is good.} \end{cases}$$

(a) Suppose $(X_1,X_2,X_3,X_4) = (1,1,0,0)$. Obtain the likelihood function of M, and the maximum likelihood estimate of M.

(b) Show that $\sum_1^4 X_i$ is sufficient.

(Hint : First obtain the joint p.m.f. of $X_1,\ldots,X_4$. Then use the p.m.f of $\sum_1^4 X_i$).

5.(a) State and prove Rao-Blackwell Theorem.

(b) Obtain the UMVU estimate of θ^m, ($m \leq n$), based on n independent Bernoulli r.v!s with parameter θ.

6. Let $X_1,\ldots,X_n$ be a sample from $U(\theta_1,\theta_2)$ where θ_1 and θ_2 are unknown.

(a) Show that $T(X) = (\min(X_1,\ldots,X_n), \max(X_1,\ldots,X_n))$ is sufficient.

(b) Assuming that T(X) is complete, find a UMVU estimate of $\frac{\theta_1+\theta_2}{2}$.

(Hint : Obtain $EX_{(1)}$ and $EX_{(n)}$).

7. Suppose that T_1 and T_2 are two UMVU estimates of $g(\theta)$ with finite variances. Show that $T_1 = T_2$.

8. Let $X_1,\ldots,X_n$ be a random sample from $N(\xi,\sigma^2)$, with ξ,σ^2 unknown. Obtain the UMVU estimate of ξ^3.
[Hint : Obtain $E\,\overline{X}^3$].

9. Define and illustrate :

(a) Sufficient statistic

(b) Unbiased estimate

(c) UMVU estimate

(d) Complete statistic.

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STOCHASTIC PROCESSES - 2
PERIODICAL EXAMINATION

Date : 29.8.86. Maximum Marks : 50 Time: 2 Hours.

Note : Answer all questions. The maximum one can score is 50.

1. Let $\{X_k, k = 1,2,3,\ldots\}$ be a sequence of independent, non-negative integer-valued random variables with the common distribution $P\{X_k = j\} = f_j$, $j = 0,1,2,\ldots$ and probability generating function $f(s) = \sum_{j=0}^{\infty} f_j s^j$. Let N be a positive integral-valued random variable independent of the X_j's.

   Let $g(s) = \sum_{n=0}^{\infty} g_n s^n$ be the probability generating function of N.

   Consider the random sum $S_N = X_1 + X_2 + \ldots + X_N$.

   (a) Show that the probability generating function of the random sum S_N is the composite function $g(f(s))$.

   (b) If the X_i are Bernoulli variables with $P(X_i = 1) = p$ and $P(X_i = 0) = q$, and N has a Poisson distribution with mean $\lambda > 0$, deduce that the sum S_N has a Poisson distribution with mean λp. [13+5=18]

2. Let $\{X_t, 0 \leq t < \infty\}$ be a continuous time-parameter Markov chain with finite state space $I = \{0,1,2,\ldots, N\}$ and stationary transition probabilities

$$p_{ij}(t) = P(X_{t+s} = j \mid X_s = i), \quad i,j \in I, \; s,t \geq 0.$$

Assume that $\lim_{h \to 0+} \frac{1-p_{ii}(h)}{h} = q_i$

and $\lim_{h \to 0+} \frac{p_{ij}(h)}{h} = q_{ij}$ exist, where

$0 \leq q_{ij} < \infty \; (i \neq j)$ and $0 \leq q_i \leq \infty$, $(i, j \in I)$.

p.t.o.

2. contd....

(a) Define the matrix of infinitesimal generator of the above Markov chain. Prove that $q_i = \sum_{\substack{j=0 \\ j \neq i}}^{N} q_{ij}$ for each $i \in I$.

(b) Assuming the $p_{ij}(t)$ is continuous at $t = 0$ from right prove that $p_{ij}(t)$ is a uniformly continuous function on $T = [0, \infty)$ for every $i, j \in I$. Also deduce that $p'_{ij}(t)$ exists for every $t > 0$ for each $i, j \in I$.

For $i, j \in I$, $i \neq j$, define, for a fixed $h > 0$

$$ {}_j p_{ii}^{(n)}(h) = 1 \quad \text{if } n = 0 $$
$$ = P(X_{nh} = i,\ X_{\nu h} \neq j,\ 1 \leq \nu < n \mid X_0 = i) \quad \text{if } n > 0 $$

and
$$ f_{ij}^{(n)}(h) = 0 \text{ if } n = 0 $$
$$ = P(X_{nh} = j,\ X_{\nu h} \neq j,\ 1 \leq \nu < n \mid X_0 = i) \text{ if } n > 0 $$

(c) Show that
$$ p_{ii}(\nu h) = {}_j p_{ii}^{(\nu)}(h) + \sum_{m=1}^{\nu-1} f_{ij}^{(m)}(h)\, p_{ji}((\nu-m)h) $$

and
$$ p_{ij}(nh) = \sum_{\nu=0}^{n-1} {}_j p_{ii}^{(\nu)}(h)\, p_{ij}(h)\, p_{jj}((n-\nu-1)h) $$

[(2+3)+(5+6)+(5+6)=27]

3. Let $\{X_n, n \geq 0\}$ be a Branching chain with state space $I = \{0,1,2,\ldots\}$ where $X_0 = 1$ and X_n is the number of particles in the nth generation of a certain population of particles, $n \geq 1$. Suppose the number of direct descendants is subject to the geometric distribution with parameter p, $0 < p < 1$ (i.e., the distribution of the random variable X_1 is given by $P(X_1 = k) = q^k p$, $k = 0,1,2,\ldots$ and $p+q = 1$). Given that $q > p$, find out the probability of eventual extinction of the population. State precisely the results you may make use of. [10]

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ELECTIVE - 4 : PHYSICAL AND EARTH SCIENCES
PERIODICAL EXAMINATION

Date : 1.9.86. Maximum Marks : 100 Time : 3 Hours.

Note : Please attempt Question No.1 and any five from the rest.

1. Fill up the blanks(any 10). Only write down one of the four choices for each blank. (2x10 = 20)

(i) Quartz, the most abundant mineral, has a chemical composition ____________ (Fe_3O_4 / SiO_2 / SiO_3 / $CaSO_4$).

(ii) Petroleum is a/an ______ (rocky/mineral/organic/inorganic) substance.

(iii) _______ (Basalt/Feldspar/Mud/Granite) is a fine-grained igneous rock.

(iv) Al is an important element in _____ (magnetite/chalcopyrite/dolomite/andalusite).

(v) Isomorphous replacements are more easy and common at ____ (low/very low/high/moderate) temperature.

(vi) A mineral assemblage which defines a particular P-T environment is called _____ (index/facies/retrogressive/progressive).

(vii) Faults in oceanic trenches are usually ______ (reverse/strike-slip/overturned/normal).

(viii) The bottom surface of a sedimentary layer can be determined from _____ (cross-bedding/fossils/amorphous mineral/migmatite).

(ix) A palaeontologist analyzes remains of ancient organisms to trace their _______ (morphology/stratigraphy/evolution/rock types).

(x) Forams are useful in locating ______ (ore/coal/oil/cement) reserves.

(xi) The _______ (eyes/muscles/blood-cells/teeth) of a dinosaur were most likely to be preserved.

(xii) The principle of Lateral Continuity is due to ________ (Cuvier/Smith/Steno/Hutton).

2. What is a mineral and what is its cleavage ? Describe the physical constraints that guide the packing of atoms in a crystal.

What is a polymorph ? Describe its geological significance.

(4+5+3+4=16)

p.t.o.

3. What is an igneous rock ? How are the igneous rocks classified ? Describe the processes that control the development of different varieties of rocks from the same parent magma. (2+4+10=16)

4. How would you distinguish between an igneous, a sedimentary and a metamorphic rock ?

   Discuss the 'rock cycle'. " Limestones, which have crystalline structures, are classified with the sedimentary rocks"; ____ If true, why ? (6+6+4 = 16)

5. Is granite an igneous or a metamorphic rock ? Discuss the origin of the granitic rocks. (5+11=16)

6. Explain the reasons why entire remains of the presently extinct woolly mammoths are still found in Siberia and Alaska.

   What is petrifaction ? In what kind of sediments would you expect to find fossils of dinosaur ? (6+5+5 = 16)

7. What do we actually mean when we talk about ' geological processes' ?

   " Geological laws are strictly not laws but principles" ____ Elucidate.

   Why the Smith-Cuvier Principle is useful in geological work ? (5+5+6 = 16)

8. Write notes on (any <u>three</u>)

   (i) Metamorphism and Ultrametamorphism ;
   (ii) Physical states of the crust and the upper mantle ;
   (iii) Eutectic system ;
   (iv) Folds and Faults ;
   (v) Folding and Faulting ;
   (vi) Isotropic and Anisotropic minerals.

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B.Stat. (Hons.) III Year: 1986-87

ELECTIVE-4 : ECONOMICS

PERIODICAL EXAMINATION

Date : 1.9.86. Maximum Marks : 100 Time: 3 Hours.

Note : (a) You have to submit Home Exercises (Q.No.7) by 9.9.36.

(b) Answer ANY FOUR questions out of Q.Nos. 1 to 6. All questions carry equal marks.

1. Consider the following linear program :

maximise $x_1 + x_2$

subject to

$$-3x_1 + 2x_2 \leqq -1$$
$$x_1 - x_2 \leqq 2$$
$$-2x_1 + 2x_2 = -2$$
$$x_1, x_2 \geqq 0$$

(a) Graphically show whether the above problem is feasible and whether it has an optimal solution.

(b) Write down the dual of the above problem. In view of the fundamental duality theorem what must be true of this dual problem ? [14+6]=[20]

2. State and prove the equilibrium theorem (i.e., complementary slackness conditions) for a pair of standard linear programs. Give economic interpretations of these conditions. [13+7]=[20]

3. Consider the following linear program :

maximise $4x_1 + 3x_2 + 4x_3 + x_4$

subject to

$$x_1 + 3x_2 + x_4 \leqq 4$$
$$2x_1 + x_2 \leqq 3$$
$$x_2 + 4x_3 + x_4 \leqq 3$$
$$x_1, x_2, x_3, x_4 \geqq 0$$

Show that this problem has an optimal solution :

$$x_1 = 1,\ x_2 = 1,\ x_3 = \tfrac{1}{2} \text{ and } x_4 = 0$$

by finding a solution of the dual problem making use of the equilibrium theorem. [20]

p.t.o.

4.(a) Consider the following transportation problem :

A commodity, coal, is produced at each of m Mines(M_i) with given annual production at each mine, and is required at each of n Towns (T_j), with given annual demand at each town. The shipping cost from M_i to T_j is proportional to the amount of coal shipped.

Formulate a linear programming problem so as to find the cheapest shipping schedule, subject to the various constraints usually supposed to be involved in a transportation problem.

(b) Write the dual of the above problem and give an economic interpretation of the dual problem. [12+8]=[20]

5. Use the " replacement procedure" to solve the following simultaneous equations :

$$\begin{aligned} x_1 + 3x_2 \quad\quad &= 4 \\ 2x_1 + x_2 \quad\quad &= 3 \\ x_2 + 4x_3 &= 3 \end{aligned}$$

[20]

6. Consider a diet problem (with n foods and m nutrients, n > m) in which it is required to satisfy each of the nutritional requirements exactly (as equations rather than inequalities).

Prove that if the above problem has a (cost-minimising) solution, then there is an optimal diet using at most m foods.

[Note : You can use canonical equilibrium theorem without giving a proof; for other results proofs are to be provided.] [20]

7. HOME EXERCISES. [20]

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INDIAN STATISTICAL INSTITUTE
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DIFFERENCE AND DIFFERENTIAL EQUATIONS
PERIODICAL EXAMINATION.

Date : 3.9.86. Maximum Marks : 100 Time: 3 Hours.

Note : Answer as much as you can. Maximum you can score is 100.

1.(a) Use the integrating factor $\frac{1}{y^2}$ to solve

$$y - x\frac{dy}{dx} = \frac{y^2}{1+x^2}$$

(b) Solve

$$y'' - 5y' + 6y = xe^x .$$ [5+15]

2.(a) Compute the Wronskian of four linearly independent solutions of

$$y^{(4)} + 16y = 0.$$

(b) Solve the equation above subject to

$$\phi(0) = 1, \quad \phi'(0) = 0 = \phi''(0) = \phi'''(0).$$ [10+10]

3.(a) Find the value of a for which the following equation is exact and solve the equation for that value of a.

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{(ax+1)}{y^3}\frac{dy}{dx} = 0.$$

(b) Solve

$$\frac{y^2}{2} + 2ye^x + (y+e^x)\frac{dy}{dx} = 0.$$ [10+10]

4.(a) A 192 lb. weight has limiting velocity 16 ft/sec when falling in air which provides a resisting force proportional to the weight's instantaneous velocity. If the weight starts from rest find the velocity of the weight after 1 second.

(b) Given that $g = 32$ ft/sec^2 and e^{-2} is approximately 0.14, compute explicitly the quantity required in part (a).

[7+3]

p.t.o.

5. A parachutist falls from rest towards the Earth. The combined weight of the man and parachute is W lb. Before the parachute opens air resistance is half the instantaneous velocity and after the parachute opens the air resistance is half the square of the instantaneous velocity. Parachute opens 5 seconds after the fall begins. Give explicit formula for the velocity as a function of time. [It is assumed that the parachutist does not hit the Earth before 5 seconds]. [20]

6. Find the general solutions of the following difference equations :

(a) $x_{t+2} + x_{t+1} + x_t = 0$

(b) $x_{t+2} - 4x_{t+1} + 3x_t = 5$

(c) $x_{t+2} - 4x_{t+1} + 4x_t = 25 \sin \frac{\pi}{2}t$.

[5+7+8]

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DIFFERENCE AND DIFFERENTIAL EQUATIONS
SEMESTRAL-I EXAMINATION

Date : 17.11.86. Maximum Marks : 100 Time : 3 Hours

Note : 1. You can answer any part of any question.
2. Maximum you can score is 100 marks.

1.(a) Find the value of the number a which makes the following equation exact and solve it for that value of a.

$$e^{ax+y} + 3x^2y^2 + (2yx^3 + e^{ax+y})\frac{dy}{dx} = 0.$$

(b) Solve the following equation using an integrating factor of the form $e^{-ax}\cos y$; $e^x \sec y - \tan y + \frac{dy}{dx} = 0.$ [7+8]

2.(a) The slope at any point (x,y) of a curve is $1+\frac{y}{x}$ and the curve passes through (1,1). Find the equation of the curve.

(b) Solve by power series method

$$y'' + x^2y' + 2xy = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

For what x is your power series convergent ? [10+10]

3.(a) Derive the normal form of the Bessel's equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0.$$

(b) Show that $y = x^{-1/2}\sin x$ is a solution to the equation

$$x^2y'' + xy' + (x^2 - \tfrac{1}{4})y = 0.$$

Find the general solution of this equation on $(0,\infty)$.
[You must carry out any integrations involved and write the solution explicitly]. [6+4+10]

4.(a) Solve explicitly :

$$y\frac{dy}{dx} + (1+y^2)\sin x = 0,$$
$$y(0) = 1.$$

(b) Solve $y''' + 7y'' + 12y' = 24x$. [10+10]

p.t.o.

5. A particle performs motion on a line as follows : It starts from rest 20 cms from a fixed point O. The only force acting is an attractive force towards O which varies directly as the distance of the particle from O. At O its velocity is 40 cms/sec.

(a) Find its velocity and acceleration 10 cms from O.

(b) Determine the amplitude, period and frequency of the motion.

(c) Find its position, velocity and acceleration at $\pi/3$ secs.

(d) Find the times when the particle passes through O.

[20]

6.(a) Find the stationary (extremal) curve y for the integral

$$\int_0^4 [xy' - y'^2]\, dx,$$

$$y(0) = 0 \;\&\; y(4) = 3.$$

(b) Suppose in a variational problem

$$F(a,b,c) = f(a)\, c^2 + 2g(a)\, bc + h(a)b^2$$

where f,g,h are twice continuously differentiable. Show that the Euler's equation is a second order linear differential equation.

[7+8]

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SEMESTRAL - I EXAMINATION

Stochastic Processes - 2

Date: 21.11.1986 Maximum Marks: 100 Time: $3\frac{1}{2}$ hrs.

Note: The paper carries 115 marks.
Maximum one can score is 100.

1. Define clearly the Wiener process with parameter $\sigma^2 > 0$. Assuming that $\{W_t, -\infty < t < \infty\}$ is the Wiener process with parameter σ^2

(a) show that $E\{(W_s-W_a)(W_t-W_a)\} = \sigma^2 \min\{s-a, t-a\}$, $s \geq a$, $t \geq a$;

(b) find the distribution of

$W_1+W_2+ \dots + W_n$ for a positive integer n.

(5+10+10) = [25]

2. What do you mean by saying that the stochastic process $\{X_t, -\infty < t < \infty\}$ is a Gaussian process ? Show that a Gaussian process is a second order process.

Suppose $\{X_t, -\infty < t < \infty\}$ is a Gaussian process having mean zero. For n distinct real numbers $t_1 < t_2 < t_3 < \dots < t_n$, state the joint distribution of $X_{t_1}, X_{t_2}, \dots, X_{t_n}$ and specify the parameters of the distribution. Find $E(X^4_t)$ in terms of the covariance function of the process.

(3+2+6+4) = [15]

3.(a) Denote by $X_1, X_2, \dots$ mutually independent random variables with the common exponential density $\lambda e^{-\lambda t}$, $\lambda > 0$. Put $S_0 = 0$, $S_n = X_1 + X_2 + \dots + X_n$, n = 1, 2, 3, ... Introduce a family of random variables Y_t as follows:

Y_t is the number of indices $k \geq 1$ such that $S_k \leq t$.

Find the distribution of Y_t for a fixed $t > 0$.

(b) Let $\{X_t, t \geq 0\}$ be a birth and death process on $\{0, 1\}$ with birth and death rates as follows: $\lambda_0 = \lambda = \mu_1 > 0$. Let N_t be the number of times the system has changed states up to time t, $t \geq 0$. Find the distribution of N_t for a fixed $t > 0$. Does it depend on the initial distribution ? [Hint: Use (a)]

(20+10) = [30]

p.t.o.

4. Let $\{X_t, t \geq 0\}$ be a Poisson process with parameter $\lambda > 0$ and $X_o = 0$. Let r, n be positive integers with $1 \leq r < n$. Let $T = \inf\{t > 0 : X_t = r\}$. Find conditional density of T given $X_s = n$. (State carefully the results you make use of).

[15]

5. Formulate the Infinite Server Queuing process $M/M/\infty$ as a birth and death process. Specify the birth and death rates as well as the infinitesimal generator matrix. Find $p_{ij}(t)$ for $M/M/\infty$ and prove that it is positive recurrent. Find the stationary initial distribution for $M/M/\infty$. Also find $E(X_t|X_o = i)$, where X_t is the number of customers being served at time t.

(6+12+4+4+4) = [30]

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STATISTICAL INFERENCE
SEMESTRAL-I EXAMINATION

Date : 24.11.86. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours

GROUP A

Answer any four questions. [4x9]

1. Let $X_1,\ldots,X_n$ be a random sample from a population with mean μ and variance σ^2, both of which are unknown.
   (a) Show that $T(X) = \sum_1^n C_i X_i$ is unbiased for μ, if and only if, $\sum_1^n C_i = 1$.
   (b) Show that $\bar{X} = \frac{1}{n}\sum_1^n X_i$ has uniformly smallest variance among all unbiased estimates of the above form.

2. Let $X_1,\ldots,X_n$ be independent Bernoulli random variables with $P(X_i=1) = \theta = 1-P(X_i=0)$.
   (a) Show that $q(\theta) = \theta/(1-\theta)$ is not estimable with an unbiased estimate based on $X_1,\ldots,X_n$ whatever be n.
   (b) What is the minimum value of n for which $\theta(1-\theta)$ would have an unbiased estimate based on $X_1,\ldots,X_n$?

3. Let $X_1,\ldots,X_n$ be a random sample from $N(\mu,\sigma^2)$ with $\sigma=1$. Find the U.M.V.U. estimate of $P_\mu\,[X_1 \geq 0]$.

4. Let X be distributed as Poisson distribution with mean θ.
   (a) Let $T(X) = 1$, if $X = 0$
   $= 0$, otherwise.
   Calculate the Cramer-Rao lower bound for the variance of and show that it is strictly less than the variance of T.
   (b) Show that T is nevertheless an U.M.V.U. estimate of its expectation.

5. Let X_n be a sequence of i.i.d. random variables with the c... distribution depending on a parameter θ. Let $T_n(X_1,\ldots,X_n)$ be the U.M.V.U. estimate of $g(\theta)$ based on $X_1,\ldots,X_n$. Show that T_n is a consistent sequence for estimating $g(\theta)$.

p.t.o

GROUP B

Answer any three questions. [3x10]

1. Let X be distributed as Binomial (n, θ). Show that the likelihood ratio statistic for testing $\theta = 1/2$ against $\theta \neq 1/2$ is equivalent to $|2X-n|$.

2. Consider the problem of testing $X \sim U(0,1)$ against $X \sim U(1/2, 3/2)$.
   (a) Derive the power of a M.P. size α test as a function of α.
   (b) Obtain the class of all M.P. size α tests.

3.(a) Show that a MP test is unbiased and its power is a nondecreasing function of its size.
   (b) Suppose that the density of X has the MLR property in θ. Show that $E_\theta\, g(x)$ is a nondecreasing function of θ, where $g(x)$ is nondecreasing and $E_\theta\, g(x)$ exists for all θ.

4. A gambler observing a game in which a single die is tossed repeatedly gets the impression that 6 comes up about 18% of the time, 5 comes up about 14% of the time, while the other 4 numbers are equally likely to occur (i.e., with probability 0.17). Upon being asked to play, the gambler asks that he first be allowed to test his hypothesis by tossing the die n times.
   (a) What test statistic should he use if the only alternative he considers is that the die is fair ?
   (b) For n=2, derive the most powerful level .0196 test.

GROUP C

Answer any one question.

1.(a) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be two independent random samples from $\mathcal{E}(\theta)$ and $\mathcal{E}(\lambda)$, respectively, where the density of $\mathcal{E}(\theta)$ is given by

$$f(x, \theta) = \theta e^{-\theta x}, \; x > 0$$
$$= 0, \text{ otherwise.}$$

Derive the U.M.P.U. size α test for $H_0 : \theta = \lambda$ against $H_1 : \theta < \lambda$.

Hint : Show the following steps :

(i) $T_1 = \sum_1^n X_i$ and $T_2 = \sum_1^m Y_i$ are sufficient for (θ, λ).

(ii) When $\theta = \lambda$, $T_1 + T_2$ is sufficient and complete.

contd...

1.(a) contd....

(iii) The density of T_1 given $T_1 + T_2 = t$ is given by

$C(\delta, t)\, K(t_1, t)\, \exp(\delta t_1)$,

where $\delta = \lambda - \theta$.

(iv) When $\delta = 0$, the distribution of $\frac{T_1/n}{T_2/m}$ is $F_{2n,2m}$. [16]

(b) The following are times until breakdown in days of air monitors operated under two different maintenance policies at a nuclear power plant. Experience has shown that the assumption of 'exponential distribution' holds. Test the hypothesis that the mean life for the Policy I is the same as that of Policy II against the alternative that the mean life for the Policy I is greater ($\alpha = 0.05$).

Policy I	x	3	150	40	34	32	37	34	2	31	6	5	14	150	27	30
Policy II	y	8	26	10	8	29	20	10								

[8]

2.(a) Let $X_1, \ldots, X_n$ be a random sample from the Cauchy distribution with median θ. Show that the critical region of the locally most powerful test of size α for testing $\theta=0$ against $\theta>0$ is given by

$$\sum_{i=1}^{n} \frac{2x_i}{1+x_i^2} > k$$

For $n = 100$, $\alpha = 0.05$, find the approximate cut off point k for the size α test. (Hint : $EU = 0$ and $\text{var } U = 1/2$ when $\theta = 0$, and $U = 2X_i/(1+X_i^2)$). [12]

(b) Obtain the uniformly most accurate lower confidence bound for the mean life under Policy II in problem 1(b) above ($1-\alpha = .95$). Support your result with the relevant theory. [12]

Class Practical Work : [1..]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year:1986-87

ELECTIVE-4: PHYSICAL AND EARTH SCIENCES
SEMESTRAL-I EXAMINATION

Date : 26.11.86. Maximum Marks : 100 Time: 3 Hours.

Note : Attempt any three of the following questions. Each question carries a total mark of $33\frac{1}{2}$.

1. Write short notes on (any four) :-
   - (i) Low-Velocity layer ;
   - (ii) Mid-Oceanic ridge ;
   - (iii) Trench (Subduction Zone) ;
   - (iv) Mantle plume ;
   - (v) Palaeomagnetic reversal ;
   - (vi) Polar wandering ;
   - (vii) Generation of a magma ;
   - (viii) P and S waves.
2. Write notes on (any two) :-
   - (i) Geology of the Ocean Floor ;
   - (ii) Geology of the Continental Crust ;
   - (iii) Prediction of Earthquakes.
3. Write an essay on the gravity studies of the earth.
4. Write an essay on the seismic studies of the earth.
5. Write an essay on the Plate Tectonics Model of the earth.

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year:1986-87

SAMPLE SURVEYS
SEMESTRAL-I EXAMINATION

Date : 19.11.86. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours

Note : Answer all questions.

1.(a) Define the terms 'Sampling Design' and 'Inclusion Probability π_i,' of a unit U_i'. [2+2 = 4]

(b) Show that the estimator $\hat{Y} = \sum_{i \in s} \frac{y_i}{\pi_i}$ where the summation is taken over the distinct units in the sample, is an unbiased estimator for the population total $Y = \sum_{i=1}^{N} y_i$, of a study-variate y taking values y_i on the unit U_i, $i=1,2,..,N$. [3]

(c) Write down the above estimator for the designs based on (i) the Midzuno-Sen sampling scheme and (ii) Probability Proportional to Size With Replacement Scheme. [4+4 = 8]

2.(a) What do you understand by 'Combined and Separate Regression estimators' ? [6]

(b) Using the large sample variance formula, discuss which of the two estimators you would prefer. [9]

3.(a) Define the term 'Intra Class Correlation Coefficient'. A population consists of 12 clusters each of size 5. Find the bounds for the intra class correlation coefficient among the elements of the cluster. [2+4 = 6]

(b) Suppose that in a single stage cluster sampling, clusters are of unequal size M_i, $i=1,2,...,N$. Find the expression for the variance of the standard estimator of the population mean when a sample of n clusters is chosen with probability proportional to size M_i and with replacement. [9]

4. A sample survey was conducted to estimate the total yield of paddy in a district. A stratified two-stage sampling design was adopted with villages as first stage units and plots within them as second stage units. From each stratum 4 villages were selected with probability proportional to area and with replacement and

p.t.o.

Q.No.4 contd....

4 plots were selected from each selected village with equal probability and without replacement. The data on yield for the sample plots together with information on selection probabilities are given in the table below :

Stratum	Sample village	Inverse of Probability of selection	Total no. of plots	Yield of sample plots 1	2	3	4
I	1	67.68	189	110	281	120	114
	2	339.14	42	80	61	118	124
	3	100.00	134	121	212	174	106
	4	68.07	161	243	116	314	129
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	138
	3	24.76	222	264	78	144	55
	4	49.99	69	300	114	68	111
III	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119

Using the above data

(a) Obtain an unbiased estimate of the total yield of paddy in the district. [12]

(b) Obtain an unbiased estimate of the sampling variance. [18]

(c) What are the possible sources of non-sampling errors in the above survey and how do you plan to control them ? [9+6 = 15]

5. A survey was conducted in a locality consisting of 625 households by covering a sample of 50 households by Simple Random Sampling (SRS) Without Replacement design to estimate the average weekly household expenditure on toilet goods. The estimate turned out to be Rs.4.20 with a standard error of 0.47. Using this information, determine the sample size needed to estimate the same characteristic in a neighbouring locality on the basis of a sample selected by SRS With Replacement such that the length of the confidence interval at 95% confidence level is 20% of the true value. (You may assume that the coefficients of variation are the same in the two localities). [10]

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B.Stat. (Hons.) III Year : 1986-87

ELECTIVE-4 : ECONOMICS

SEMESTRAL-I EXAMINATION

Date : 26.11.86. Maximum Marks : 100 Time: 3 Hours.

Notes : (a) Home assignments (Q.Nos. 7 and 8) are to be submitted to the respective teachers by 1st Dec.. 1986.
(b) Answer ANY FOUR questions out of Q.Nos.1 to 6.
All questions carry equal marks.
(c) Notations and definitions : I is an identity matrix. A is an input matrix of an input-output system. A matrix is called nonnegatively invertible if its inverse exists and is nonnegative.

1.(a) Distinguish between the 'Commodity Technology' and the 'Industry Technology' assumptions in the context of dealing with secondary products for the purpose of construction of Input-Output tables.

(b) On the basis of the information given below obtain the commodity x commodity I/O tables both on 'Commodity Technology' and ' Industry Technology' assumptions.

Table 1 Make Matrix

Commodities	Industries 1	2	3	Total
1	100	0	0	100
2	10	100	0	110
3	0	0	50	50
Total	110	100	50	260

Table 2 : Commodity X Industry I/O table
(Absorption Matrix)

Commodity	Industry 1	2	3	Final Demand	Total Out
1	20	30	0	50	100
2	30	20	20	40	110
3	10	20	10	10	50
Value added	50	30	20	0	100
Total	110	100	50	100	

[20]

p.t.o.

2.(a) Derive the conditions for no valuation bias while constructing an I/O table.

(b) Discuss the RAS method for updating an I/O table. [20]

3. EITHER

A company has a large water treatment facility located in the flood plain of a river. The construction of a levee to protect the facility during periods of flooding is under consideration. Using historical records which describe the maximum height reached by the river during each of the last 100 years, the frequencies shown in column (2) are ascertained. Data concerning the costs of construction and expected flood damages are shown in column(3) and column(4). Determine the optimum height of the 'Levee' proposed to be constructed.

Feet (x) (1)	No.of years river maximum level was x feet above normal (2)	Loss if river level is x ft. above levee in Rs. (3)	Initial cost of building Levee x ft. high in Rs. (4)
0	48	0	0
5	24	100000	100000
10	16	150000	210000
15	6	200000	330000
20	4	300000	450000
25	2	40000	550000

[20]

OR Write short notes on : -

(a) Choice of appropriate prices for Cost-Benefit analysis

(b) Social Welfare and Private Profitability. [20]

4.(a) Solve the following linear programming problem by the " Simplex Method" :

find $x_1, x_2 \geq 0$ such that $x_1 - x_2$ is a maximum,

subject to
$$-2x_1 + x_2 \leq 2$$
$$x_1 - 2x_2 = 2$$
$$x_1 + x_2 \leq 5.$$

(b) From the optimal tableau of the above problem find also an optimal solution of its dual problem. [20]

(contd....)

5. Suppose, an input-output system is capable of generating some positive income (i.e., value added) in each sector.

Prove that the matrix (I-A) is nonnegatively invertible.

[20]

6.(a) In an input-output system, the matrix (I-A) is non-negatively invertible only if $\lim_{t \to \infty} A^t = 0$.

(b) Consider a two-sector Leontief economy.
Define and derive the marginal rate of transformation between net outputs of the two sectors. Is this rate independent of the sectors' net output levels
(i) when there is a second primary factor in the economy and /or (ii) when there is a capacity constraint in one sector ? [10+10]=[20]

7. HOME ASSIGNMENTS ON LINEAR PROGRAMMING. [10]

8. HOME ASSIGNMENTS FOR THE COURSE 'COST-BENEFIT ANALYSIS'. [10]

INDIAN STATISTICAL INSTITUTE [1986-87|312(b)]

B.Stat. (Hons.) III Year : 1986-87

DIFFERENCE AND DIFFERENTIAL EQUATIONS

SEMESTRAL-I BACKPAPER EXAMINATION

Date : 29.12.86. Maximum Marks : 100 Time: 3 Hours.

Note : Maximum you can score is 100 marks.

I. Consider Hermite's equation

$$y'' - 2xy' + 2py = 0$$

p being a constant.

Get two linearly independent power series solutions of the above equation and show that they are linearly independent and that they are defined on the entire real line.

(i) Show that this equation has a nonzero polynomial solution iff $p = 0,1,2,\ldots$. For $p=n$ integer ≥ 0 show that this equation has exactly one polynomial solution with leading term $2^n x^n$. Denote this polynomial by $H_n(x)$.

(ii) Show
$$H_n(X) = (-1)^n e^{X^2} D^n(e^{-X^2}),$$
$$H_n' = 2 x H_n - H_{n+1} ,$$
$$H_n = 2 x H_{n-1} - 2(n-1) H_{n-2}.$$

(iii) Show that H_{2n} has only even powers of X and H_{2n+1} has only odd powers of X.

(iv) Show $\int_{-\infty}^{\infty} H_m(X) H_n(X) e^{-X^2} dX = 0$ if $m \neq n$

$$= 2^n n! \sqrt{\pi} \text{ if } m = n.$$

[10+5+15+5+10]

II. Consider the problem

$$y' = x+y,$$
$$y(0) = 1.$$

Start with $\varphi_0 = 1$ and calculate the n^{th} Picard approximation φ_n. Calculate the (pointwise) limit φ of these φ_n and verify that φ solves the problem. [20]

p.t.o.

III. If $MdX + Ndy = 0$ is such that $\frac{x^2}{xM+yN}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(y/x)$ then show that an integrating factor is $e^{F(y/x)}$ where F is a function such that $F' = f$. [10]

IV. A parachutist falls from rest towards the Earth. The combined weight of the man and parachute is W lb wt. Before the parachute opens, the air resistance is half the instantaneous velocity and after the parachute opens, the air resistance is half the square of the instantaneous velocity. Parachute opens 5 seconds after the fall begins. Give explicit formula for the velocity as a function of time. [20]

V. Describe the nature of geodesics on a sphere. [15]

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INDIAN STATISTICAL INSTITUTE

B. Stat.(Hons.) III Year : 1986-87

SAMPLE SURVEYS

SEMESTRAL-I BACKPAPER EXAMINATION

Date : 31.12.86. Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours

Note : Answer all questions.

1.(a) Define the terms 'Sampling Design', 'Inclusion Probability π_i, of a unit U_i and 'Joint Inclusion Probability π_{ij} of a pair of units U_i and U_j, $i \neq j$'. [2+2+2]=[6]

(b) For the 'Midzuno-Sen sampling scheme' obtain π_i and π_{ij} and show that the scheme makes the conventional ratio estimator defined by ratio of sample means of two characteristics, unbiased for the population ratio. [3+4+2]=[9]

2.(a) What do you understand by 'Combined and Separate Ratio estimators'. Using the formulae for Bias and Mean Squared Error discuss which of the two estimators you would prefer. [6+9] = [15]

3. Of 105 students sampled using a Simple Random Sampling Without Replacement design from a population of 454 students 11 had to sit for a back-paper examination in 2 or more subjects. Estimate the proportion of students in the population who had to sit for back-paper examination in 2 or more subjects and also obtain an approximate 95% confidence interval for the proportion. [3+7] =[10]

4.(a) Define the term 'intra class correlation coefficient'. A population consists of 14 clusters each of size 7. Find the upper and lower bounds for this coefficient. [2+4]=[6]

(b) Suppose that in a single stage cluster sampling, clusters are of unequal size M_i, $i=1,2,..,N$. Find the expression for the variance of the standard estimator of the population when a sample of n clusters is chosen with probability proportional to size M_i and with replacement. [5]

5. A sample survey was conducted to estimate the total household expenditure in a district. A stratified two-stage sampling design was adopted with villages as first stage units and households within them as second stage units. From each stratum 4 villages were selected with probability proportional to population and with replacement and 4 households were selected from

p.t.o.

Q.5. contd....

each selected village with equal probability and without replacement. The data on expenditure for the sampled households together with information on selection probabilities are given in the table below :

Using the data,

(a) Obtain an unbiased estimate of the total household expenditure in the district. [12]

(b) Obtain an unbiased estimate of the sampling variance. [12]

(c) What are the possible sources of non-sampling errors in the above survey and how do you plan to control them ? [9+6] = [illegible]

Stratum	Sample village	Inverse of probability of selection	Total no. of households	Expenditure of sample households 1	2	3	4
I	1	440.21	28	104	182	148	87
	2	660.43	14	108	64	132	156
	3	31.50	240	100	115	50	172
	4	113.38	76	346	350	157	119
II	1	21.00	256	124	111	135	216
	2	16.80	288	123	177	106	138
	3	24.76	232	264	78	144	55
	4	49.99	69	300	114	68	111
III	1	67.68	189	110	281	120	[illegible]
	2	339.14	42	80	61	110	[illegible]
	3	100.00	134	121	212	174	[illegible]
	4	68.07	161	243	116	314	[illegible]

[1986-87|332(E)]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1986-87
STATISTICAL INFERENCE
SEMESTRAL-I BACKPAPER EXAMINATION

Date : 1.1.1987. Maximum Marks : 100. Time: 3 Hours.

Note : Answer all questions.

1. Let $X_1,\ldots,X_n$ denote the times in days to failure of n similar pieces of equipment. Assume that $(X_1,\ldots,X_n)$ is a random sample from the exponential distribution with parameter λ. Consider the hypothesis H that the mean life $\mu = 1/\lambda \leq \mu_0$.

   (a) Show that the UMP level α test for the alternative $\mu > \mu_0$ is given by the critical region

   $$\bar{X} \geq \mu_0 x_{1-\alpha} / 2n ,$$

   where $x_{1-\alpha}$ is the $(1-\alpha)$th quantile of χ^2_{2n} - distribution.

   (b) Obtain the power function of the above test.

   (c) The following are days until failure of air monitors at a nuclear plant. If $\mu_0 = 25$, use normal approximation to find out whether H is rejected at level $\alpha = 0.05$. [18]

2. Let $X_1,\ldots,X_n$ be a random sample from $N(\mu,\sigma^2)$. We want to test $H : \sigma = \sigma_0$ versus $K : \sigma \neq \sigma_0$ when μ is unknown.

   Show that the size α likelihood-ratio test accepts H iff

   $$c_1 \leq \frac{1}{\sigma_0^2} \sum_1^n (X_i - \bar{X})^2 \leq c_2,$$

   where c_1 and c_2 satisfy

   (i) $F(c_2) - F(c_1) = 1-\alpha$, F being the cdf of χ^2_{n-1}- distributi[on]

   (ii) $c_1 - c_2 = n \log \frac{c_1}{c_2}$. [..

3. Let X and Y be independently distributed as Poisson distributions with means λ and μ respectively. Obtain the UMP unbias[ed] test for $\lambda \leq \mu$ against $\lambda > \mu$. [..

p.t.o.

4. Let $X_1,\ldots,X_n$ be a random sample from $U(0,\theta)$. Let T_1 and T_2 be the MLE and UMVU estimates of θ. Define

$$T = \frac{n+2}{n+1} T_1.$$

Compare the mean-square error i.e. $E_\theta(T-\theta)^2$ of all three estimates T, T_1, and T_2. [15]

5. Suppose n 'identical' pieces of equipment are run and the failure times $X_1,\ldots,X_n$ are observed. We want to estimate the probability of early failure, i.e. $P_\lambda[X_1 \leq x] = 1-e^{-\lambda X}$ for some fixed X.

Use Rao-Blackwell-Lehman-Scheffe Theorem to derive the UMVU estimate of $1-e^{-\lambda X}$. Is it the same as its MLE ? [15]

6. Define and illustrate :
(a) Sufficient statistic
(b) Minimal sufficient statistic
(c) Consistent estimate
(d) Complete statistic
(e) Monotone likelihood ratio property
(f) Uniformly most accurate confidence bound. [15]

7. Class Practical Work. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1986-87

STOCHASTIC PROCESSES-2
SEMESTRAL-I BACKPAPER EXAMINATION

Date : 2.1.1987. Maximum Marks : 100 Time : 3 Hrs.

Note : Attempt all questions.

1.(a) Denote by $X_1, X_2, \ldots$ mutually independent random variables with the common exponential density $\lambda e^{-\lambda t}$, $\lambda > 0$. Put $S_0 = 0$, $S_n = X_1 + X_2 + \ldots + X_n$, $n = 1, 2, \ldots$. Introduce a family of random variables Y_t as follows : Y_t is the number of indices $k \geq 1$ such that $S_k \leq t$. Find the distribution of Y_t for a fixed $t > 0$.

(b) Let $\{X_t, t \geq 0\}$ be a birth and death process on $\{0,1\}$ with birth and death rates as follows : $\lambda_0 = \lambda = \mu_1 > 0$. Let N_t be the number of times the system has changed states upto time t, $t \geq 0$. Find the distribution of N_t for a fixed $t > 0$. Does it depend on the initial distribution ? (Hint: Use (a)).

[20+10=30]

2. Let $\{X_t, t \geq 0\}$ be a Poisson process with parameter $\lambda > 0$ and $X_0 = 0$.

(a) Let $T = \inf \{t > 0 : X_t = r\}$. Find the probability $P\{T \leq u \mid X_s = n\}$ for $0 \leq u \leq s$ and r, n posotive integers with $1 \leq r < n$.

(b) Let U be a random variable that is independent of $\{X_t\}$. Suppose that U has an exponential density with parameter α

$$f_U(u) = \begin{cases} \alpha e^{-\alpha u}, & u > 0 \\ 0, & u \leq 0. \end{cases}$$

Find the distribution of X_U, which can be interpreted as the number of events occurring by time U.

[15+15 =30]

p.t.o.

3.(a) Formulate a pure death process on $\{0,1,2,\ldots\}$

(b) Write the forward differential equations.

(c) Find $p_{ii}(t)$

(d) Solve for $p_{ij}(t)$ in terms of $p_{i,j+1}(t)$

(e) Show that $p_{ij}(t) = 0$ for $j > i$ and $t \geq 0$

(f) Find $p_{i,i-1}(t)$

(g) Show that if $\mu_i = i\mu$, $i \geq 0$, for some constant μ, then

$$p_{ij}(t) = \binom{i}{j}(e^{-\mu t})^j(1-e^{-\mu t})^{i-j}, \quad 0 \leq j \leq i.$$

[4+3+4+3+6+5+5=30]

4. Carefully derive the Kolmogorov Backward differential equations for a conservative continuous time-parameter Markov chain with a countable state space I and stationary transition probability matrix $((p_{ij}(t)))$; $t \geq 0$, $i,j \in I$.

[10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.)III Year: 1986-87

PERIODICAL EXAMINATION

Design of Experiments

Date: 27.2.1987 Maximum Marks: 100 Time: 3 hrs.

Note: Answer any FOUR questions. Marks alloted to a question are indicated in brackets [ ] at the end. Submit your PRACTICAL RECORDS to the course Instructor on or before 4.3.1987. These records carry 20 marks.

1.(a) Explain the terms: plot, treatment, and error. Describe the three fundamental principles of experimental designs explaining their objectives.

(b) Suppose four feeds are to be compared with respect to their growth performances on cows. Suggest two good designs for the experiment explaining which one should be used when. $(3 \times 1+3 \times 3+8)$ = [20]

2. Define a randomised block design with an example. Under the usual fixed-effects additive model, obtain all its estimable functions and their B.L.U.E.'s. Develop also the analysis of variance for this design.

(2+8+10) = [20]

3. Show that there can exist a maximum of (v-1) mutually orthogonal latin squares (MOLS) or order v. Give a method, with proof, of constructing a complete set of MOLS of order v, when v is a prime or power of a prime. Using $x^2 + x + 1$ as a minimum polynomial of GF(4), give an example of a complete set of MOLS of order 4.

(5+10+5) = [20]

4.(a) If $\{L_1, L_2, \ldots, L_u\}$ is a set of MOLS of order v, and $\{L_1^*, L_2^*, \ldots, L_u^*\}$ is a set of MOLS of order v^*, then show that $\{L_1 \otimes L_1^*, L_2 \otimes L_2^*, \ldots, L_u \otimes L_u^*\}$ is a set of MOLS of order vv^*, where $L_i \otimes L_i^*$ stand for the symbolic Kronecker product of the arra L_i and L_i^*, $i = 1, 2, \ldots, u$.

Contd..... 2/-

Contd..... Q.No.4

(b) What is a graeco latin square ? When do you use such a square as an experimental design ? Indicate how you will construct a graeco latin square of order 12.

(12 + (2+3+3)) = [20]

5. Explain the missing plot technique. Show that

(i) $(I - ZC^{-} Z')\underline{w}^{*} = ZC^{-} X'y$ is a consistent system, and has a unique solution of $\underline{w}^{*}$ if and only if $\text{Rank}\left[\cdot\frac{X}{Z}\cdot\right] = \text{Rank } X$;

(ii) $SSE_{\Omega} = \min_{\underline{w}} SSE_{\tilde{\Omega}}(\underline{w}) = SSE_{\tilde{\Omega}}(\underline{w}^{*})$;

where the symbols have their usual significance. Obtain the expression for $\underline{w}^{*}$ for a randomised block design having one missing observation in block j under treatment i

(4 + (5+7) + 4)) = [20]

:bcc:

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INDIAN STATISTICAL INSTITUTE
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PERIODICAL EXAMINATION

Multivariate Distributions and Tests

Date: 23.2.1987 Maximum Marks: 100 Time: 3 hours

Note: Answer any <u>four</u> questions.

1.(a) Give a characterisation of multivariate normal distribution. Find its parameters.

(b) Let

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \longrightarrow N_3\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 6 \end{pmatrix}\right)$$

Find the distribution of u_1 given u_2 and u_3.

2.(a) Let $U_{px1} \longrightarrow N_p(\mu, \Sigma)$. Find the characteristic function of U and hence or otherwise find the distribution of any part of U.

(b) Let $U_{px1} = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}\begin{matrix} qx1 \\ p-qx1 \end{matrix} \longrightarrow N_p\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}\right)$

Find the distribution of

$$(U-\mu)^1\ \Sigma^{-1}(U-\mu) - (U_1-\mu_1)^1\ \Sigma_{11}^{-1}\ (U_1-\mu_1)$$

3.(a) Let $U_{px1} \longrightarrow N_p(\mu_1\ \Sigma)$. Let $D(p \times p)$ be an orthogonal matrix such that $D\Sigma D' = I$. Let $Z = D(U-\mu)$. Then calculate $E(Z'Z)$.

(b) Let us have a random sample of size n from $N_p(\mu, \Sigma)$. Find the distribution of r_{ij} when $\rho_{ij} = 0$.

4.(a) Find the m.l.e. of μ and Σ based on a random sample of size $n > p-1$ from $N_p(\mu, \Sigma)$.

(b) How does the estimate modifies if $\mu = 0$?

p.t.o.

5.(a) Let $X_\alpha(p \times 1)$, $\alpha = 1, \ldots, n$ be a random sample from $N_p(\mu, \Sigma)$.

Let $\overline{X} = \frac{1}{n}(X_1 + \ldots + X_n)$

and $S = (X_1 - \overline{X})(X_1 - \overline{X})' + \ldots + (X_n - \overline{X})(X_n - \overline{X})'$

denote respectively the sample mean vector and correlated sum and product matrix.

Show that:

$$\sqrt{n}\,(\overline{X} - \mu) \longrightarrow N_p(0, \Sigma)$$

(b) $S = Y_1 Y_1' + \ldots + Y_{n-1} Y_{n-1}'$

when $Y_\alpha(p \times 1)$ s are independently and identically distributed as $N_p(0, \Sigma)$.

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PERIODICAL EXAMINATION

Optimization Techniques

Date: 25.2.1987 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. The whole paper carries 105 marks, but the maximum you can score is 100.

1. Explain the simplex algorithm for finding
   (a) an optimal solution to an LP problem. [10]
   (b) Use the method to find an optimal value for the following problem:

$$\begin{aligned} \text{maximise} \quad & x_1 + 2x_2 + 3x_3 - x_4 \\ \text{subject to} \quad & x_1 + 2x_2 + 3x_3 = 15 \\ & 2x_1 + x_2 + 5x_3 = 20 \\ & x_1 + 2x_2 + x_3 + x_4 = 10 \\ & x_i \geq 0 \quad i = 1,2,3,4. \end{aligned}$$

[25]

2. A food store packages a nut sampler consisting of Peanuts, Walnuts and Almonds. Each ounce of peanuts contains 12 units of protein and 3 units of iron and costs 12 paisa, each ounce of Walnuts contains 1 unit of protein and 3 units of iron and costs 9 paisa and each ounce of almonds contains 2 units of protein and 1 unit of iron and costs 6 paisa. If each package of the nut sampler is to contain atleast 24 units of protein and 18 units of iron, how many ounces of each type of nut should be used to minimize cost of nut sampler ? [20]

3. Prove or disprove.
   (a) A linear program is infeasible then its dual must be unbounded. [10]
   (b) Let V be a finite set in E^n and C(V) is its convex hull. The set of extreme points of C(V) is a subset of V. [10]

Contd..... 2/-

Contd..... Q.No.3

(c) X is the set of feasible solutions to a standard LP problem and $\overline{X}$ is the set of its optimal vectors. X and $\overline{X}$ have the same extreme points.

[10]

4. Find an optimal value for the following LP problem:

maximise $8x_1 + 9x_2 + 5x_3$

subject to

$$x_1 + x_2 + 2x_3 \leq 2$$
$$2x_1 + 3x_2 + 4x_3 \leq 3$$
$$6x_1 + 6x_2 + 2x_3 \leq 8$$
$$x_i \geq 0 \quad i = 1,2,3.$$

Check your answer with the optimal value of its dual, if it exists.

[20]

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PERIODICAL EXAMINATION

Nonparametric and Sequential Methods

Date: 2.3.1987 Maximum Marks: 100 Time: $3\frac{1}{2}$ hrs.

Note: Practical note book: [12].

1. Let $X_1, \ldots, X_n$ be i.i.d. random variables with the common distribution f which is assumed to be continuous. Let $R_1, \ldots, R_n$ be the ranks of $X_1, \ldots, X_n$, respectively.

   (a) Prove that

   $$P[(R_1, \ldots, R_n) = (r_1, \ldots, r_n)] = 1/n!$$

   for any permutation $(r_1, \ldots, r_n)$ of $(1, 2, \ldots, n)$. [2]

   (b) Let $L = \Sigma c_i\, a(R_i)$, where c_i's are constants and a is a real-valued function. Show that

   $$EL = n\bar{c}\bar{a}, \quad Var(L) = \frac{1}{n-1}\sum_{i=1}^{n}(c_i - \bar{c})^2 \sum_{i=1}^{n}(a(i) - \bar{a})^2,$$

   where $\bar{c} = \frac{1}{n}\Sigma c_i$, $\bar{a} = \frac{1}{n}\Sigma a(i)$. [10]

   (c) Let r_s be the Spearman's rank correlation coefficient based on a random sample of size n from a continuous bivariate distribution. Use the above result to show that

   $$E(r_s) = 0, \quad V(r_s) = \frac{1}{n-1}$$

   under the hypothesis of independence. [6]

2. Define Tolerance limit. [2]

3. Let $X_1, \ldots, X_n$ be a random sample from continuous distribution F with median m(F) for the problem of testing H_0: m(F) = 0 against H_1: m(F) $\neq$ 0, consider the critical region $|s-n/2| \geq d-n/2$, where s is the number of negative X_i's. Find the size of the test in terms of n and show that the test is unbiased. Use the above result to get a confidence interval for m(F). [12]

p.t.o.

4. The effectiveness of Vitamin C in orange juice and in synthetic ascorbic acid was compared in 20 guinea pigs (divided at random into two groups of 10) in terms of the length of odontoblasts after 6 weeks, with the following result:

Orange juice (x): 8.2 9.4 9.6 9.7 10.0 14.5 15.2 16.1 17.6 21.5

Ascorbic acid(Y): 4.2 5.2 5.8 6.1 7.0 7.3 10.1 11.2 11.3 11.5

(a) Find the significance probability when the hypothesis of no difference is being tested against the alternative that the orange juice tends to give larger values, using Wilcoxon rank-sum test. [10]

(b) Let F and G be the distributions of X and Y, respectively, as defined above. Suppose that F and G are continuous, and $G(x) = F(x + \Delta)$ for all x. Let $D_{(i)}$'s be the ordered difference of $X_i - Y_j$. Show that

$$P_{\Delta}\ [D_{(k)} \leq \Delta \leq D_{(101-k)}] = 1-\alpha,$$

where $P_{\Delta=0}\,[k \leq W_{XY} \leq 100-k] = 1-\alpha$ and W_{XY} is the number of pairs (X_i, Y_j) such that $X_i > Y_j$. Find the confidence interval for Δ using the above data with $1-\alpha$ approximately equal to 0.95. (8+10) = [18]

5. Let $X_1, \ldots, X_n$ be i.i.d. random variables with the common distribution F which is assumed to be continuous. The problem is to test

$$H_0 : F(x) = 1-F(-x) \text{ for all } x.$$

Consider the following statistic $T = \sum_{i=1}^{n} s_i R_i^+$, where $s_i = \text{sign}(X_i)$ and R_i^+'s are the ranks of $|X_i|$'s.

(a) Show that the distribution of T is completely known. Derive E(T) and Var(T) under H_0. [12]

Either

(b) Show that

$$\frac{T}{n(n-1)} = \frac{\Sigma\, s_i}{n(n-1)} - \frac{1}{2} + V_n,$$

where $V_n = \frac{1}{n(n-1)} \sum_{i \neq j} g(X_i, X_j)$, and

$$g(X_1, X_2) = \begin{cases} 1, & \text{if } X_1 + X_2 > 0 \\ 0, & \text{if } X_1 + X_2 \leq 0 . \end{cases}$$

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Now use the result on the large-sample distribution of U-statistic to get the asymptotic distribution of T, properly standardized, under H_0. (6+10) = [16]

Or

(b) The following data report the weight (in lb) that 10 seven-year old children were able to lift before and after an 8-week muscle-training programme.

Before(X)	14.4	15.9	14.4	13.9	16.6	17.4	18.6	20.4	20.4	15.4
After (Y)	20.4	22.9	19.4	21.4	25.1	20.9	24.6	24.4	24.9	19.9

Test the hypothesis that the training has no effect against the alternative that the training has a positive effect. Compare it with the p-values of the paired t-test and the sign test.

Suppose $Z = Y-X$ is symmetrically distributed about a point θ. Obtain the Hodges-Lehmann estimate of θ, and compare it with the mean and median of Z. (10+6) = [16]

Table 1

Wilcoxon rank-sum distrn. $P(W_{XY} \leq a)$; n = m = 10.

a	10	15	20	21	22	23	24
P	.0008	.0034	.0116	.0144	.0177	.0216	.0262

a	25	30	35	40	45	50
P	.0315	.0716	.1399	.2406	.3697	.5147

Table 2

Wilcoxon rank-sign distribution $P[V \leq v]$, n = 10.

v	9	10	11	12	13
P	.0322	.0420	.0527	.0654	.0801
v	14	15	20	25	30
P	.0967	.1162	.2461	.4229	.6152

V = Sum of the R^+ ranks of the positive observations.

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PERIODICAL EXAMINATION

Elective - 5: Economics

Date: 4.3.1987 Maximum Marks: 100 Time: 3 hours

Note: Answer question no.6 and any FOUR from the rest. Marks allotted to each question are given in brackets [ ].

1. Discuss the suitability of the assumption of two-parameter lognormal distribution as a description for the distribution of the size variable income. Obtain the expressions for the mean, median, mode and coefficient of skewness of the two-parameter lognormal distribution and hence comment on the shape of the distribution. (5+15) = [20]

2. Let $y \geq 0$ be a continuous random variable with finite mean μ and having distribution function F. Further, let Δ be the Gini mean difference and L the Lorenz ratio. Then establish the following relations:

(i) $L = \Delta/2\mu$

(ii) $L = 1 - [\int_0^\infty (1-F(t))^2 dt]/\mu$

(8+12) = [20]

3. What is Lorenz curve ? Discuss its properties. Obtain the expression for the Lorenz curve for a two-parameter lognormal distribution. (4+10+6) = [20]

4. Discuss about the universality of Pareto law of income distribution. Obtain the jth moment distribution for the Pareto distribution, and hence obtain the expression for the Lorenz ratio. (6+8+6) = [20]

Distinguish between measures of variablity and measures of inequality for a size variable. What properties would you like an income inequality measure to satisfy ? What is normative approach for measuring income inequality ? (5+10+5) = [20]

p.t.o.

6. The following data gives Lorenz ratios and averages for the distributions of consumer expenditure in rural and urban India for the years 1957-58 and 1967-68 :

	1957-58		1967-68	
	Lorenz ratio	Average expenditure (Rs.)	Lorenz ratio	Average expenditure (Rs.)
Rural India	0.334	100	0.290	125
Urban India	0.359	150	0.336	175

Compute the share of the top 10 per cent of the population in both the sectors for the two periods. Find in each case the percentage of population from bottom enjoying 10 per cent share of the total expenditure. (You may assume that the distribution of consumer expenditure in India is log-normal.) [20]

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PERIODICAL EXAMINATION

Elective - 5: Physical and Earth Sciences

Date: 4.3.1987 Maximum Marks: 100 Time: 3 hours

Note: Attempt all questions.

1. Clearly state the factors on which the resistance of a conductor depend. Derive the expression emerging from these. [5]

What will be the effect of temperature on the resistance ? How do you account it in describing the linear and non-linear resistance ? Draw necessary graph.]8]

Given that the equivalent resistance of parallel combination of 4 resistors are 20 Ω. Find the value of each of them if the currents flowing through them are 0.6, 0.3, 0.2 and 0.1A respectively. Draw the neat circuit diagram. [6]

Two resistance of 4 Ω and 12 Ω respectively connected in parallel and this combination is joined to a 10 Ω resistance in series. Deduce the equivalent resistance. Find the value of potential difference to be maintained across the network, so that a current of 6A proses through the resistance of 12 Ω. Draw the necessary diagrams. [6]

2. What is the utility of grouping of cells ? How do you group them ? [3]

There are N number of cells each which has the emf E and internal resistance r (<u>of some appreciable value</u>). For what combination of cells as well as the relation of external resistance R to the internal resistance r, you can obtain the maximum amount of current. Justify for the method you follow. [14]

36 cells, each of emf 2 Volt and internal resistance of 1.5 Ω, are joined in such a way so that maximum current flows through the external resistance of 6 . Find the mode of combination as well as the magnitude of current through the external resistance. Draw the necessary circuit diagrams. [8]

3. State Thevenin's theorem and explain it "clearly" with suitable diagrams. Derive an expression for current flowing through the resistance on which the theorem has to be applied. [10]

The four arms of a wheat stone bridge have the following resistances (Ω) as R_{AB} = 100, R_{BC} = 10, R_{CD} = 5 and R_{DA} = 60 (variable) respectively. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 Volt is maintained across AC. Sketch the necessary circuit diagrams. [12]

What will be the rearranged value of the variable resistance DA so that you can obtain a null condition. [3]

4. It is given that through the three terminals 1, 2 and 3 respectively connection of three resistances as (a) R_{12}, R_{23} and R_{31} joined as in Delta fashion and (b) R_1, R_2 and R_3 joined as in Star fashion, are electrically equivalent. How do you transform a Star mesh into a delta one. Also find the value of the resistances converted to delta in terms of resistances used in star fashion. [13]

Three resistances (Ω) as R_{AB} = 100, R_{BC} = 40, R_{CA} = 60 are joined in a delta mode of connection. Now these terminals are joined as such: terminal A to the (+)ve terminal of a battery of 160 Volt, terminal B and C to the resistors of 80 Ω and 88 Ω respectively. Also note that the other lone terminals of battery, 80 Ω and 88 Ω resistor are joined together with the ground. Neatly draw the overall circuit diagram and deduce the following (a) overall resistance across the battery, (b) current flowing from the battery and (c) current flowing through the 80 Ω resistors. [12]

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SEMESTRAL-II EXAMINATION

Multivariate Distributions and Tests

Date: 4.5.1987 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions.
Questions are of equal value.

1.(a) Define Hotelling's T^2 statistic. Find its distribution in the null case.

(b) Fifty observations are taken from population Iris versicolor and fifty from population Iris setosa. Let x_1 = sepal length and x_2 = sepal width. The data is summerised as

$$\bar{x}^{(1)} = \begin{pmatrix} 5.936 \\ 2.770 \end{pmatrix}, \quad \bar{x}^{(2)} = \begin{pmatrix} 5.006 \\ 3.428 \end{pmatrix}$$

$$98S = \begin{pmatrix} 19.1434 & 9.0356 \\ 9.0356 & 11.8658 \end{pmatrix}$$

Test whether mean vector of sepal length and sepal width varies for two populations.

2.(a) Let Q_1 and Q_2 be two random variables which are quadratic forms in the items of a random sample of size n from a distribution which is $n(0, \sigma^2)$. Let A and B denote respectively the real symmetric matrices of Q_1 and Q_2. Then show that Q_1 and Q_2 are stochastically independent iff AB = O.

(b) Let x_1, x_2, x_3, x_4 denote random sample of size 4 from $n(0, \sigma^2)$. Let $y = \sum_{i=1}^{4} a_i x_i$ when a_1, a_2, a_3, a_4 are real constants. If y^2 and $Q = x_1 x_2 - x_3 x_4$ are stochastically independent then determine a_1, a_2, a_3, a_4.

3.(a) Define generalised varience. Let $x_1, \ldots, x_2$ be a random sample from $N(\mu, \Sigma)$. Find the distribution of $|S|$ and hence find its moments.

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(b) Find the joint distribution of sets of correlation coefficients when the sample is from $N(\mu, (\sigma_{ij}\ \delta_{ij}))$.

4.(a) State and Prove Cochran's theorem for quadratic forms. State its usefulness in statistical testing problems.

(b) Let μ_1, μ_2, μ_3 be respectively the means of three idependent normal distributions having common but unknown variance σ^2. The table below gives a random sample of size 5 from each of these distributions. Using these data test at 5% significance level the hypothesis H_0: $\mu_1 = \mu_2 = \mu_3$ against H_1: H_0 not true.

Sample	Observations				
1	3	0	-1	0	2
2	2	5	1	3	5
3	4	3	6	8	9

5.(a) Let a_1 and a_2 be apriori probabilities of drawing a distribution from population π_1 and π_2 with densities $p_1(x)$ and $p_2(x)$ respectively. If the cost of misclassifying an observation from π_1 as coming from π_2 is $C(2/1)$ and an observation from π_2 as coming from π_1 is $C(1/2)$, then characterise the region of classification R_1 and R_2 so that expected cost of misclassification is minimised.

(b) Let π_1 be $N(\mu_1, \Sigma)$ and π_2 be $N(\mu_2, \Sigma)$. Let us have an observation x from either π_1 or from π_2 find the appropriate discriminant function when μ_1, μ_2 and Σ are known. Calculate its mean and varience. Hence calculate probabilities of misclassifications.

6. Write notes on any Four:

(i) Sphericity test.
(ii) Likelihood ratio test.
(iii) Wishart distribution.
(iv) Multiple correlation coefficients and its test.
(v) Partial and total correlations.
(vi) Fisher Behran's problem.

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SEMESTRAL-II EXAMINATION

Design of Experiments

Date: 6.5.1987 Maximum Marks: 100 Time: $3\frac{1}{2}$ hrs.

Note: Answer any FOUR questions. Marks allotted to a question are indicated in brackets [ ] at the end. Submit your PRACTICAL RECORDS to the course Instructor on or before 12.5.1987. These records carry 20 marks.

1. What is a latin square design ? Give an example. Postulate a suitable linear model for this design, and identify the estimable functions of interest under the model, and obtain their B. L. U. E.'s. Develop the analysis of variance for this design to test the equality of effects of all the treatments. [(3+2)+(2+4+4)+5] = [20]

2. Prove that there can exist at most (v-1) mutually orthogonal latin squares (MOLS) of order v. What is Euler's conjecture in connection with the MOLS ? What is the current status of this conjecture ?

   Give a method of construction of a complete set of MOLS, proving that the method works in general. Illustrate your method with an example. [(5+2+2)+(8+3)] = [20]

3. Explain the missing plot technique. Show that the error sum of squares obtained through this technique is the valid error sum of squares for the incomplete data. Suppose two observations got lost in a randomised block design for v treatments in r blocks, affecting the treatments i and i', and the blocks j and j'; $i \neq i'$; $j \neq j'$. Obtain the expressions for the missing yields required for the computation of error sum of squares for the incomplete data. (5+10+5) = [20]

4. Give the two definitions of connectedness of a block design and show that they are equivalent. Show also that a connected block design is balanced if and only if the off-diagonal

Contd..... 2/-

Contd..... Q.No.4

elements of its C matrix are all equal. Give an example of a block design which is connected but not balanced.

(10+8+2) = [20]

5. Define a balanced incomplete block design (BIBD) with parameters v, b, r, k, λ, and prove that (i) $\lambda(v-1) = r(k-1)$ and (ii) $b \geq v$, equality holding if and only if every pair of blocks intersect in λ common treatments. Using the second property derive the parameters of the residual and derived designs of a symmetrical BIBD. Give briefly the analysis of variance of a BIBD.

(10+4+6) = [20]

6. Define the main effects and interactions in a 2^n factorial experiment, and show that they form a set of 2^n-1 mutually orthogonal treatment contrasts. Describe the Yates' method of sum and difference to compute the sums of squares due to the various factorial effects.

Identify the confounded effects in a replication of a 2^5 factorial in blocks of size 2^3, in which the following is the Key block :

[(1), abd, acd, bc, de, abe, ace, bcde].

(9+8+3) = [20]

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SEMESTRAL-II EXAMINATION

Optimization Techniques

Date: 11.5.1987 Maximum Marks: 100 Time: 3 hours

Note: You may answer all the questions. The total number of marks you can score is 100.

1. Consider the linear programming problem:

$$\begin{aligned} \text{maximise} \quad & 3x_1 + 2x_2 + 5x_3 \\ \text{subject to} \quad & x_1 + 3x_2 + 2x_3 \le 15 \\ & 2x_2 - x_3 \ge 5 \\ & 2x_1 + x_2 - 5x_3 = 10 \\ & x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0 \end{aligned}$$

Write down its dual and find optimal solutions to both primal and dual problems. [20]

2. Verify whether $(x_1\ x_2\ x_3) = (\frac{5}{26}, \frac{5}{2}, \frac{27}{26})$ is an optimal solution to the following linear programming problem

$$\begin{aligned} \text{maximise} \quad & 9x_1 + 14x_2 + 7x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + 3x_3 \le 6 \\ & 5x_1 + 4x_2 + x_3 \le 12 \\ & 2x_2 \le 5 \\ & x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0 \end{aligned}$$

[15]

3. Prove or disprove:

(a) If an LP problem is feasible and its dual is not feasible, then the objective function of the LP is unbounded. [10]

(b) In a finite bipartite graph on 2n vertices, $n \ge 1$, in which each vertex is incident to the same number of edges, there exist n edges, no two of which are incident to the same vertex. [10]

Contd..... 2/-

Contd..... Q.No.3

(c) Consider an LP problem of finding a vector x, unrestricted in sign., such that

$$c^t x \text{ is a maximum subject to}$$
$$Ax = b.$$

The problem has a solution iff C is a linear combination of the rows of A. [10]

4. Explain a procedure for solving the optimal assignment problem with rating matrix $A = (\alpha_{ij})$, where α_{ij} is the rate of the ith person on the jth job.

For the rating matrix A given below, find an optimal assignment

$$A : \begin{pmatrix} 11 & 9 & 11 & 3 & 8 \\ 6 & 6 & 2 & 2 & 9 \\ 6 & 8 & 10 & 11 & 9 \\ 6 & 3 & 6 & 1 & 1 \\ 11 & 2 & 10 & 9 & 12 \end{pmatrix}$$

Check your result using duality theorem. [20]

5. Formulate the minimum cost - maximum flow problem as a linear programming problem and develop an algorithm to solve it. Verify that the algorithm terminates with a minimum cost - maximum flow. [15]

6. In a certain community, there are 100 boys and 100 girls. Each boy knows atleast 10 girls and each girl knows atmost 10 boys. (Assume that if a boy knows a girl, the girl also knows the boy). Is it possible to arrange a monogermous marriage so that each boy gets married to a girl he knows ? State and prove necessary results. [15]

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SEMESTRAL-II EXAMINATION

Nonparametric and Sequential Methods

Date: 8.5.1987 Maximum Marks: 100 Time: 4 hours

1. Derive the SPRT (A,B), $B < 1 < A$, based on independent observations on X for testing H_0: X $\sim$ uniform (0,2) against H_1: X $\sim$ uniform (1,3).

   Show that $P(N > n) \to 0$ as $n \to \infty$, where N is the effective sample size. Obtain the probabilities of type I and type II error, and E(N) under H_0 and H_1. [16]

2. Let X be a random variables distributed as $N(\theta,1)$. Consider the problem of testing H_0: $\theta = -\delta$ against H_1: $\theta = \delta > 0$, based on independent observations on X.

   (a) Let n_0 be the minimum sample size required so that the probabilities of type I error and type II error are at most α_0 and α_1, respectively, for the fixed sample size most powerful test. Show that

   $$n_0 = \left\{ \frac{(\lambda_1 - \lambda_0)^2}{4\delta^2} \right\},$$

   where $\phi(\lambda_0) = 1 - \alpha_0$, $\phi(\lambda_1) = \alpha_1$, ϕ is the c.d.f. of $N(0,1)$ and $\{x\}$ is the smallest integer $\geq x$. [6]

   (b) Obtain the SPRT (A,B), $B < 1 < A$, for the above testing problem. [4]

   (c) Let α_0 and α_1 be the probability of type I error and type II error, respectively. Show that

   $$\frac{\alpha_1}{1-\alpha_0} \leq B, \quad \frac{\alpha_0}{1-\alpha_1} \leq \frac{1}{A}$$ [4]

Contd..... 2/-

Contd..... Q.No.2

(d) Let $Z = \ln[f(x,\delta)/f(x,-\delta)]$, where $f(x,\theta)$ is the p.d.f. of $N(\theta,1)$. Obtain Z explicitly.

Let $h(\theta)$ be the non-zero root of the equation

$E_\theta\, e^{Z.h(\theta)} = 1$. Show that $h(\theta) = -\theta/\delta$, when $\theta \neq 0$. [4]

(e) In order evaluate the OC function and the ASN function the following approximations are used:

$$L(\theta) \sim \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}}, \quad \text{when } E_\theta Z \neq 0$$

$$\sim \frac{\ln A}{\ln A - \ln B}, \quad \text{when } E_\theta Z = 0$$

$$E_\theta(N) \sim \frac{L(\theta)\ln B + [1 - L(\theta)]\ln A}{E_{(\theta)}(Z)}, \text{ when } E_\theta Z \neq 0$$

$$\sim -\frac{\ln A . \ln B}{E_\theta Z^2}, \quad \text{when } E_\theta Z = 0.$$

State the relevant theorems and briefly indicate the steps to support the above approximations. [10]

(f) Draw the OC function and the ASN function from the above approximations and (c) for

$$\alpha_0 = \alpha_1 = 0.05, \quad \delta = 0.5$$

Compute n_0 and the relative efficiency of the SPRT under H_0 and H_1. [16]

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with the common p.d.f.f. Show that the order statistics and the rank vector are independently distributed. [8]

4. Consider two random variables X and Y with the c.d.f.'s F and G, respectively, such that $G(x) = F(x - \Delta)$. The problem is to estimate Δ when F is unknown (but continuous. Let $X_1, \ldots, X_m$ be m independent observations on X and $Y_1, \ldots, Y_n$ be n independent observations on Y. Define $\hat{\Delta}$ as the median of $(Y_j - X_i)$'s.

Contd..... 3/-

Contd..... Q.No.4

(a) Show that the distribution of $\hat{\Delta} - \Delta$ is free from Δ.

(b) Show that $\hat{\Delta}$ is distributed symmetric about Δ if either $m = n$ or f is symmetric about some point.

[8]

5. Consider an experiment designed to evaluate the effect of pollution on the weight of rats. The x's below are weights of rats kept in a pollution-free environment, while the y's are weights of similar rats kept in a polluted environment. Assuming a location-shift model, compute the level 0.925 confidence interval for the shift parameter.

x	397.4	409.9	419.1	381.9	409.5	323.4
y	369.6	338.1	353.2	356.6	386.7	395.2

[8]

6. Each of ten fields is divided into two similar parts. Two fertilisers A and B are allocated randomly to the parts in each field. The yields are given below (in certain units).

Field	1	2	3	4	5	6	7	8	9	10
Fertilizer A	659	984	397	574	447	479	676	761	617	577
Fertilizer B	452	587	460	787	351	277	234	516	577	513

The problem is to test that the fertilizers have the some effect against the alternative that the fertilizer A is better.

The difference of the yields between two parts in each of the ten fields is computed; absolute values of these differences are ranked. The test statistic proposed is V which equals the sum of these ranks for the negative differences o...

(a) Enumerate all sample points (given the set of absolute differences) for which the V-value is less than the observed V-value.

(b) What is the critical level of the test based on V ?

[8]

7. Practical Class-work. ... [8]

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1986-87	3E2

INDIAN STATISTICAL INSTITUTE
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SEMESTRAL-II EXAMINATION

Elective - 5 : Economics

Date: 14.5.1987 Maximum Marks:100 Time: 3 hours

Note: This paper carries a total of 120 marks. Question no.5 is compulsory. Answer as many questions as you can from the rest. Maximum you can score is 100. Marks allotted to each question are given in brackets [ ].

1.(a) Suppose a size variable X follows a two-parameter lognormal distribution with parameters μ and σ^2. What is then the distribution of $y = \alpha X^{\beta}$, where α and β are two positive constants ? Also obtain the expressions for the mean and the median of y.

(5+6) = [11]

(b) State clearly the law of proportionate effect and show that it leads to a lognormal distribution for income.

(3+5) = [8]

(c) How will you find the Lorenz ratio of a two-parameter lognormal distribution if the coefficient of variation of the distribution is given ? [3]

2.(a) Define Engel curve and Engel elasticity. Distinguish between income and expenditure elasticities.

(5+3) = [8]

(b) Discuss briefly the various criteria used for choosing a suitable form for Engel curve from a given set of cross-section data on consumer expenditure.

[14]

3.(a) What is a production function ? For a generalised CES production function of the form

$$y = a \left\{ \delta L^{-\beta} + (1-\delta) K^{-\beta} \right\}^{-1/\xi},$$

where y, L and K denote the output, labour and capital respectively, show that the elasticity of substitution

Contd..... 2/-

Contd..... Q.No.3.(a)

is equal to $1/(1+\beta)$. Also derive conditions under which the production function will satisfy decreasing, constant and increasing returns to scale.

(3+6+4) = [13]

(b) Describe briefly the statistical problems that may arise in the estimation of a Cobb-Douglas production function from a time-series data.

[9]

4.(a) Justify the use of 'per capita' formulation of an Engel curve. Do you think that such a formulation is always adequate ? Give reasons for your answer.

(4+4) = [8]

(b) Consider the case of a firm in imperfect condition in the product market and perfect competition in the factor (input) markets. Obtain the second-order conditions characterizing profit maximization in such a case. Interpret the conditions thus obtained.

(6+3) = [9]

(c) Show that $\bar{R}^2 \leq R^2$, where R^2 is the coefficient of multiple determination and $\bar{R}^2$ the adjusted coefficient of multiple determination.

[5]

5. EITHER

The table below gives the family-budget data for a few samples of four low-income classes of families of a country for the year 1975-76:

	yearly income per consumer unit in Rs			
	below 600	600-750	750-1050	above 105[illegible]
no. of households in the sample	136	179	111	22
average no. of consumer units per household	2.60	2.57	2.50	2.45
average income per consumer unit	543.1	681.3	861.9	1232.0
average expenditure on food per consumer unit	291.8	331.6	374.4	407.1

Assuming the constant elasticity form of the demand function, estimate the income elasticity of demand for food along with the standard error, by using the necessary information from the above table. How good is the fitted equation ?

(17+5) = [22]

...

Contd..... Q.No.5

OR

The follwing data relate to a sample of six cement industries for 1957:

name of establishment	total invested capital (Rs.lakhs)	total man-hours (no.)	value of production of goods (Rs.lakhs)
A.C.Co. Ltd.	147.24	1631728	144.12
Bhupendra Associated Cements	173.60	2040283	199.37
A.C.C. Ltd. - Kymore	225.94	2737036	244.22
Madukkari Cement Works	111.67	1558660	181.40
Travancore Cements	83.76	861912	45.66
Associated Cement Company	399.96	4054979	297.56

Fit a Cobb -Douglas production function to the above data, and test whether returns to scale are constant.

(17+5) = [22]

Practical records [to be submitted by May 20, 1987]

[10]

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1986-87	3P2

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1986-87

SEMESTRAL-II EXAMINATION

Elective - 5 : Physical and Earth Sciences

Date: 14.5.1987 Maximum Marks: 100 Time: 3 hours

Note: All questions carry equal marks.
Attempt any FOUR questions.
Wherever required draw the necessary sketches and circuit diagrams.

1. State Kirchhoff's laws of electricity. With necessary diagrams clearly explain them. Maxwell's loop current method is more comprehensive than Kirchhoff's branch current method. Why ? How do you get branch currents from loop currents ? (3+5+3+2) = [13]

   Connections of different resistors and battery across a rectangular network ABCD are as follows: $R_{AB} = 6\,\Omega$, $R_{BC} = 5\,\Omega$, $R_{AC} = 2\,\Omega$, $R_{BD} = 2\,\Omega$, $R_{CD} = 6\,\Omega$ and $E_{AD} = 10$ V, $2\,\Omega$ internal resistance with (+)ve terminal of the battery towards A. Find out all the three branch currents meeting at D. Find also potential drop at B and C. [12]

2. With suitable examples, explain Faraday's laws of electromagnetic induction and Lenz's law. With these laws find an expression for the induced emf. in a coil. (8+4) = [12]

   Calculate the emf. induced by the reversal of flux initially at +0.4 mwb within 10 msec. in a coil of 500 turns. [4]

   How does self-induced emf. generated in a coil ? Define self-inductance. Derive an expression for co-efficient of self-inductance L. (2+2+5) = [9]

3. Clearly sketch the series combination of capacitor C and high resistance R connected across a DPDT switch can either join the combination to a battery of Vvolts or can short circuit. Derive the expression for potential v_C across and current i_C flowing through the charging capacitor C. Define the time constant τ of the circuit. (2+5+1) = [8]

Contd..... 2/-

Contd..... Q.No.3

Say at $t = 10\,\tau$, the capacitor is allowed to discharge through the resistor. Find the expression for potential v_d across and current i_d emerging from the discharging capacitor C. Neatly sketch two graphs containing (v_c, v_d) and (i_c, i_d) against time t.

(5+3) = [8]

Under what conditions the output of a CR circuit may be directly proportional to the derivative of the input signal ? Derive an expression for it and clearly sketch the circuit you use. From rectangular input waveform how do you obtain the output as triggering waveform ?

(2+5+2) = [9]

4. An alternating voltage is applied across the combination of a coil (of inductance L and resistance R) and a pure capacitor (of capacitance C). For the series type combination, find the magnitude and phase relation of circulating current to the applied voltage. When series resonance occurs ? What it magnifies ? Deduce the value of frequency of resonance.

(8+1+2+2) = [13]

Derive the expressions of resonant frequency and current if the combination (coil-capacitor) is of parallel type. What is dynamic impedance ? Why the parallel resonant circuit is termmed as rejector circuit ?

(9+1+2) = [12]

5. What is thermionic emission ? Describe clearly the working of a high-vacuum thermionic diode. What is a rectifier ?

(2+5+2) = [9]

Draw a neat circuit diagram of a full-wave rectification of single phase A.C. supply and explain precisely the overall operation of it. [8]

How can you filtered out intermittent pulses of output ? Explain. [4]

Draw the overall circuit diagram and the functional diagram of the circuit to generate smooth unidirectional output.

[4]

MCR

1986-87	3P2(b)

INDIAN STATISTICAL INSTITUTE
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BACKPAPER
SEMESTRAL II EXAMINATION

Elective - 5 : Physical and Earth Sciences

Date: 2.7.1987 Maximum Marks: 100 Time: 3 hours

Note: Attempt any FOUR questions. All questions carry equal marks.

1. How do you account for the rise in temperature on a resistance ? Explain with suitable examples. How this phenomenon can be related to the property of linear and non-linear resistance ? Explain with suitable diagrams.

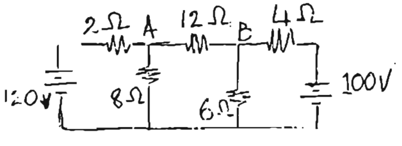
(5+5) = [10]

Prove that R = nr/m where R = external resistance, r = interval resistance of each cell and the total number of cells N (= m x n) are distributed as in m parallel rows and n number of cells in each row. Neatly sketch the circuit. How much power would you expect from this combination ?

(7+1+2) = [10]

Find the minimum number of cells connected in 2 rows in parallel required to pass a current of 6 Ampere through an external resistance of 0.7 Ω. Each cell has emf. of 2.1 volts and internal resistance of 0.5 . [5]

2. With suitable circuit diagrams clearly explain, (using Thevenin's theorem) the successive steps you follow to obtain the value of current flowing through it. [10]



Using Thevenin's theorem calculate the magnitude and direction of flow of current through 12 Ω resistance across AB [9]

Contd..... 2/-

Contd..... Q.No.2

Now if the 100^V battery is connected in reverse, find out the magnitude and direction of current flowing through the 12 Ω resistance across AB. [6]

3. Clearly sketch the series combination of inductance L and resistance R connected across DPDT switch S which can either join the combination to a battery of V volts or short circuit. Derive the expression for the growth of current i_g and potential ϑ_g across L. Define the time constant τ of the circuit. (2+5+1) = [8]

Say at $t = 10\tau$, the current i_d through the inductance is allowed to decay. Find an expression for it. Also derive the value of potential ϑ_d across L at the phase of decaying current. Neatly sketch the graphs of (i_g, i_d) and $(\vartheta_g, \vartheta_d)$ against time t. (5+3) = [8]

For what conditions the output of an LR circuit may be directly proportional to the integration of the input signal? Derive an expression for it and clearly sketch the circuit you use. From the rectangular input waveform how do you obtain the output as triangular waveforms ? (2+5+2) = [9]

4. An alternating current $i = i_m \sin \omega t$ is flowing through a series combination of a coil (of inductance L and resistance R) and pure capacitor C. Derive an expression for impressed voltage across the circuit and also find the phase relation of it to the circulating current. (6+2) = [8]

Under what condition the circuit behaves as (a) inductive, (b) capacitive and (c) resistive ? [5]

What peculiarity you may observe when the circuit is purely resistive ? Find the value of frequency of operation. [4]

3A current is circulating through an LCR series circuit. Given that L = 299.8 in H, R = 80Ω and C = 33.8μF. Impressed voltage is written as E volt F C/S. Find out the value of E and F. [8]

5. An alternating voltage is applied across the parallel combination of a coil (of inductance L and resistance R_1) and a leaky capacitor (of capacitance C and resistance R_2). Find out the magnitude and direction of current flowing from the source. Sketch a neat diagram. [9]

If the leaky capacitor is substituted by a pure capacitor of capacitance C only, then find out the condition for electrical resonance of the parallel ckt-and hence find out the frequency of operation. What will be the value of current (in terms of dynamic impedance) emerging from the source ? [2+3+2) = [7]

400 Volt 50 H_z supply is connected across the parallel combination of a coil (L = 254.65 mH, R = 60 Ω) and a pure capacitor of capacitance C. For what value of C the circuit will be in resonance condition ? Find out the ratio of branch currents and the overall current.

[9]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1986-87

BACKPAPER
SEMESTRAL II EXAMINATION
Nonparametric and Sequential Methods

Date: 3.7.1987 Maximum Marks: 75 Time: 3 hours

1.(a) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution F. Let $R_1, \ldots, R_n$ be the ranks of the observations. Consider the following statistic :
$T = \sum_{i=1}^{n} C_i \, a(R_i)$. Prove that

$E(T) = n\bar{c}\bar{a}$, where $\bar{c} = \sum_{1}^{n} c_i/n$, $\bar{a} = \sum_{i=1}^{n} a(i)/n$
and
$\mathrm{Var}(T) = \frac{1}{n-1} \sum_{1}^{n} (c_i - \bar{c})^2 \sum_{1}^{n} (a(i) - \bar{a})^2$ [7]

(b) Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a random sample from a continuous bivariate distribution. Let $(R_1, \ldots, R_n)$ and $(S_1, \ldots, S_n)$ be the rank vector of the X_i's and Y_i's, respectively. Let r be the sample rank correlation coefficient. Show that

$$E\, r = 0, \quad \mathrm{Var}(r) = \frac{1}{n-1}$$

under the hypothesis H_o of independence between X and Y. [use Q.No.1.(a)]. [5]

(c) For a set of 16 male babies aged 48 weeks, both the height and head circumference measurements were taken with the following results :

Height	77.3	73.0	73.9	71.7	79.6	75.4	77.6
Head circumference	47.5	46.9	45.9	46.3	47.5	47.4	47.1
Height	72.0	76.4	75.6	74.9	70.5	71.6	73.3
Head circumference	47.3	48.2	46.5	46.4	48.2	48.2	45.0
Height	70.9	75.0					
Head circumference	46.1	47.4					

Contd..... 2/-

Contd..... Q.No.1.(c)

Test the hypothesis of independence against positive dependence (use a large-sample test). [8]

2. Let $X_1, \dots, X_n, \dots$ be i.i.d. random variables with the common distribution $N(\xi, \sigma^2)$ where both ξ and σ^2 are unknown. The problem is to find a confidence interval for ξ of length ℓ and confidence coefficient $1 - \alpha$.

A sample of n_o observations $X_1, \dots, X_{n_o}$ is taken, and the sample estimate S^2 of σ^2 is computed by

$$S^2 = \frac{1}{n_o - 1} \left\{ \sum_1^{n_o} X_i^2 - \frac{1}{n_o} \left(\sum_1^{n_o} X_i \right)^2 \right\}.$$

Define n by

$$n = \max \left\{ \left[\frac{S^2}{Z} \right] + 1,\ n_o + 1 \right\},$$

where Z is a specified positive constant.

(a) Show that it is possible to choose $a_1, \dots, a_n$ such that

$$a_1 = \dots = a_{n_o},\quad \sum_1^n a_i = 1,\quad \sum_1^n a_i^2 = Z/S^2. \qquad [5]$$

(b) Define

$$u = \sum_1^n a_i (X_i - \xi)/\sqrt{Z}.$$

Show that the distribution of u is student's t-distribution with $n_o - 1$ d.f. [5]

(c) Use (b) to get a solution of the problem cited above. [5]

3. Consider the SPRT (A , B) of the hypothesis $H_o : f_o(X)$ against the alternative $H_1 : f_1(X)$, where

$$f_o(X) = \begin{cases} 4/5, & \text{if } X = 0 \\ 1/5, & \text{if } X = 1 \end{cases} \qquad f_1(X) = \begin{cases} 2/5, & \text{if } X = 0 \\ 2/5, & \text{if } X = 1 \\ 1/5, & \text{if } X \quad 2 \end{cases}$$

Describe the SPRT (A , B) with $A = 2^{-j}$, $B = 2^k$, where j and k are positive integers. Find the exact values of probabilities of type I error and type II error, $E(N|N_o)$, and $E(N|H_1)$. [15/

4. The following data give the gain in weight of a control group of young rats and a group of rats of the same age kept in an o zone environment for 7 days.

X(control) : 41.0, 38.4, 24.4, 25.9, 21.9, 18.3, 13.1,
27.3, 28.5, -16.9, 26.0, 17.4, 21.8, 15.4, 27.4

Y(treatment): 10.1, 6.1, 20.4, 7.3, 14.3, 15.5, -9.9,
6.8, 28.2, 17.9, -9.0, -12.9, 14.0, 6.6, 39.9

Test, at the 5% level of significance, the hypothesis that the ozone does not affect the gain in weight of young rats.

Apply both the Wilcoxon's test (under normal approximation) and the two sample t-test.

[15]

5. Practical class-work and notebook. ... [10]

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1986-87	572(b)

INDIAN STATISTICAL INSTITUTE
M.Stat. (M-stream) I Year : 1986-87

BACKPAPER
SEMESTRAL II EXAMINATION

Statistical Methods and Inference II : Tests of Hypothesis

Date: 1.7.1987 Maximum Marks: 100 Time: 3 hours

Note: Attempt as many questions as you can.

1.(a) Obtain approximate expresions for the boundaries A and B in a sequential probability ratio test (SPRT).

(b) Describe in detail the SPRT for $H_0: \theta = 0$ against $H_1 : \theta = 1$, in the context of sampling from a N $(\theta,1)$ population.

(16+12) = [28]

2. Describe the run test. [17]

3.(a) Consider a sequence of trials such that in each trial there are k possible mutually exclusive and exhaustive outcomes with respective probabilities $\pi_1, \pi_2, \ldots, \pi_k$. For $i = 1, \ldots, k$, let f_{in} be the observed frequency in the ith class in n trials. Show that

$$\sum_{i=1}^{k} \frac{(f_{in} - n\pi_i)^2}{n\pi_i}$$

is asymptotically distributed as a Chi-square with (k-1) d.f.

(b) Describe the Chi-square test for independence of two attributes.

(20+10) = [30]

4.(a) Consider a lot of N items of which M are of a specific category. Here M is integer-valued and unknown. On the basis of a random sample of size n drawn without replacement from the population, suggest a UMP size α test for $H_0 : M \leq M_0$ against $H : M > M_0$, where M_0 is a given positive integer $(1 \leq M_0 < N)$.
(Any auxiliary result may be stated without proof).

p.t.o.

(b) Let $X_1, X_2, \ldots, X_n$ be iid random variables each having the gamma distribution with pdf.

$$\frac{\theta^p}{\Gamma p} e^{-\theta x} x^{p-1}, \quad 0 < x < \infty$$

where $p(>0)$ is known and $\theta(>0)$ is an unknown parameter. Derive the likelihood ratio test for $H_0 : \theta = \theta_0$ against $H : \theta \neq \theta_0$.

(c) Write a note on the method of confidence belt for interval estimation.

(13+17+7) = [37]

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