INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1987-88

OPTIMIZATION TECHNIQUES
Semestral-II Backpaper Examination

Date : 1.7.88. Maximum Marks : 100 Time : 3 Hours

1. Let $\bar{x}$, $\bar{y}$ be optimal solutions of the following problem (P) and its dual respectively.
(P) Find $x \geqq 0$ such that xc is maximum subject to $xA \leqq b$.

Suppose $c \in \mathbb{R}^m$, $b \in \mathbb{R}^n$ and A is an $m \times n$ matrix. Show that for any non negative x, y in $\mathbb{R}^m$, $\mathbb{R}^n$ respectively
$\bar{x}c + by - \bar{x}Ay \geqq \bar{x}c + b\bar{y} - \bar{x}A\bar{y} \geqq xc + b\bar{y} - xA\bar{y}$. [15]

2. Consider the problem :
Maximize $\xi_1 + 3\xi_2 + \xi_3$ subject to

$$5\xi_1 + 3\xi_2 \leqq 3$$
$$\xi_1 + 2\xi_2 + 4\xi_3 \leqq 4$$

Either find an optimal solution or show that no such solution exists. [15]

3. Define a flow from s to s' in a network (N,k). For any such flow, show that $f(s,N) = f(N, s')$. [10]

4. Solve the following simple assignment problem if possible. If not, prove that there is no solution. There are 5 individuals and 5 jobs. Individual I_i is suitable for job J_j if and only if $\alpha_{ij} = 1$ in the following matrix:

$$((\alpha_{ij})) = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$ [15]

p.t.o.

5. Let (N,k) be a network with source s and sink s'. Let x_1, x_2 be nodes of N satisfying the following :

(a) If $k(s,x_1)$ is increased by δ_1, the maximal flow increases by Δ_1.

(b) If $k(s, x_2)$ is increased by δ_2, the maximal flow increases by Δ_2.

(c) If $k(s,x_1)$ increases by δ_1 and $k(s,x_2)$ increases by δ_2 simultaneously, the maximal flow increases by Δ_{12}.

(δ_1, δ_2, Δ_1, Δ_2, Δ_{12} are positive numbers)

Show that $\Delta_{12} \leqq \Delta_1 + \Delta_2$. [20

6. Show that a matrix game always has a solution in mixed strategies. [15

7. Write the pay off matrices for the following game for different values of j and k.

P_2 chooses either of two integers j,k and P_1 chooses an integer i between 1 and 3. P_1 earns an amount i if $i = j$ or k, otherwise he loses an amount i. [10

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NONPARAMETRIC AND SEQUENTIAL METHODS
Semestral II Examination

Date : 6.5.88. Maximum Marks : 100 Time : 3 Hours

Note : Answer all questions.

1. Briefly explain the concept of Pitman's asymptotic relative efficiency. [6]

2. (a) Describe briefly how the projection principle and the central limit theorem are used to prove asymptotic normality of a U-statistic. [6]

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution which is symmetric about zero. Find the asymptotic null distribution of the Wilcoxon signed-rank statistic. [12]

OR

Find the asymptotic distribution of Kendall's sample Tau coefficient under the hypothesis of independence. [12]

3. Suppose we have two independent samples with

$$X_1, \ldots, X_m \text{ i.i.d. } F(x) \text{ and } Y_1, \ldots, Y_n \text{ i.i.d. } F(x-\Delta)$$

where F is the c.d.f. of a continuous distribution.
We want to test

$$H_0 : \Delta = 0 \quad \text{Vs.} \quad H_1 : \Delta > 0.$$

Suppose that we reject H_0 for large values of a rank statistic of the form

$$S = \sum_{i=1}^{n} a(R_i)$$

where $a(1) \leq \ldots \leq a(m+n)$ are known values (not all the same) and R_i is the rank of Y_i among all (m+n) observations.

(i) Show that the test based on S will have a monotone power function in Δ for this problem.

(ii) What are the implications of the above result. [9+3]

4. Suppose $X_1, X_2, \ldots$ are i.i.d. random variables and H_0 and H_1 are two simple hypotheses concerning $(X_1, X_2, \ldots)$. Show that the SPRT for testing H_0 against H_1 terminates with probability one under both H_0 and H_1. [14]

p.t.o.

5. State the fundamental identity of sequential analysis and using this obtain approximate expressions for the O.C. and A.S.N. functions of the SPRT. [16]

OR

Let $X_1, X_2, \ldots$ be i.i.d. $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Describe Stein's double sampling procedure to find a bounded length confidence interval for μ with a given confidence level $1-\alpha$.

Obtain a confidence interval for μ of exact length 2ℓ with confidence coefficient not less than $1-\alpha$.

Also obtain an unbiased estimator of μ (using Stein's double sampling procedure) for which the variance is bounded by some number not depending on σ. [16]

6.(a) Derive under suitable conditions the Cramer-Rao inequality for the variance of an unbiased estimator in the sequential case. [9]

(b) Suppose $X_1, X_2, \ldots$ are i.i.d $\sim$ Bin $(1, \theta)$, $0 < \theta < 1$. Suggest a sequential procedure to obtain an unbiased estimator of $\frac{1}{\theta}$ which attains the corresponding Cramer-Rao lower bound. [9]

7. Assignments and class performance . [16]

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B.Stat.(Hons.) III Year : 1987-88

ELECTIVE-5 : PHYSICAL AND EARTH SCIENCES
Semestral II Examination

Date : 11.5.88. Maximum Marks : 100 Time : 3 Hours

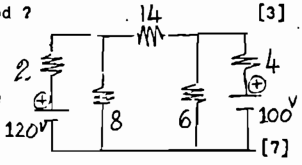
Note : Each question carries 20 marks. Answer as much as you can but maximum obtainable marks is 100.
Wherever necessary draw the circuit diagrams and related graphs.

1. A battery consists of 'm' number of parallel rows, each of which has 'n' number of cells joined in series. If each of the cell has an internal resistance 'r' and open circuit emf 'e' find the overall internal resistance and emf of the battery. Draw the necessary diagrams. [4]

With suitable circuit diagram, find for what value of the external resistance 'R', this mixed combination of cells will yield maximum current, the magnitude of which you have to determine. [6]

Why Maxwell's loop current method is more comprehensive than Kirchhoff's branch current method ? [3]

Calculate the magnitude and the direction of current flowing through the different resistance (in Ω) of the network DABC (Apply Maxwell's Law) [7]



2. State and explain Faraday's laws of electromagnetic induction and Lenz's law. Hence find an expression for the induced emf generated in a coil. Define self inductance L of a coil. [6+4+2 = 12]

A coil of self inductance 'L' and an appreciable resistance 'R' is being put across a battery of V volts (dc). After the inductive current reaches to its maximum value the battery is withdrawn and the open terminals (of the battery) is joined together. Derive an expression for the decaying current and draw the graph of the voltage impressed across L. [8]

p.t.o.

3. A series combination of capacitor 'C' and high resistance 'R' is joined across a battery of 'V' volts (dc) through a DPDT switch. Draw the circuit diagram. Briefly describe with suitable derivation how the capacitor charges to the maximum value of the battery voltage and the current flows through it. Draw the nature of voltage across 'R'. [10]

   It is possible to get an output which is the derivative of the input signal applied to this RC circuit. Clearly state, with suitable derivation how and under what conditions this output can be obtained. Draw the necessary circuit diagram. How trigerring output can be obtained from the rectangular input waveform ? [10]

4. An alternating voltage (Vm Sin wt) is applied across the combination of a practical coil (of inductance 'L' and appreciable resistance 'R') and a pure capacitor (of capacitance 'C'). Using the vector method or otherwise, derive the related expressions and describe the different parameters (as condition for resonance, resonant frequency, the impedance, magnitude of the current, power factor and the special name of the circuit) associated to the circuit when the choke-capacitor combination is in (a) series type and (b) parallel type. [8+8 = 16]

   Make a table for the comperative study of the parameters associated to the two type of combination. [4]

5. What do you mean by thermionic emission ? Clearly explain, with suitable diagrams, the function of a high-vacuum thermionic diode. [6]

   Describe, with neat and labelled circuit diagram, the overall operation of a full wave single phase ac. rectification. Draw all the necessary figures. [9]

   With suitable circuit diagram, describe how the intermittent pulses of output derived from the above mentioned full wave rectifier can be filtered. [5]

6. In the light of electronic theory, clearly differentiate among insulators, semiconductors and conductors. [6]

   What consists of semiconductor current ? Derive an expression for intrinsic semiconductor current. [5]

   What is extrinsic semiconductor ? Name the suitable doping elements for this. Describe briefly the different sub-divisions of extrinsic semiconductors. [9]

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ELECTIVE - 5 : ECONOMICS
Semestral II Examination

Date : 11.5.88. Maximum Marks : 100 Time : 3 Hours

Note : This paper carries a total of 110 marks. Answer as many questions as you can. You can answer any part of any question. But the maximum you can score is 100. Marks allotted to each question are given in brackets [ ]. Answers should be brief and to the point.

1. Give a broad account of the method of quantiles for estimation of parameters of a two-parameter lognormal distribution. Mention, in particular, the choice of quantiles which give estimates with the highest asymptotic efficiency. (10+5)=[15]

2. How would you compare different algebraic forms of the Engel curve in respect of goodness of fit when fitting them to empirical data based on budget enquiries ? Give a detailed account mentioning in particular the Durbin-Watson statistic. [20]

3. Discuss the 'identification problem' that may arise in the context of estimation of the demand function for a commodity on the basis of time-series data on aggregate market transactions and market prices. How is the problem overcome in practice ? (7+8) =[15]

4. Examine the problem of multicollinearity arising in the context of estimation of demand functions from time-series data. Do you feel that ridge regression is an ad-hoc solution to the multicollinearity problem ? Justify your answer. (9+6) = [15]

5. Define elasticity of substitution between two factor-inputs in a production function, and explain the significance of this measure. Show that the CES production function includes the Cobb -Douglas production function as a special case. (8+7)=[15]

p.t.o.

6. The following results are based on a family budget survey in a working class centre in India during 1958-59. Obtain the Engel elasticity for expenditure on food items by regressing it on total expenditure by a suitable form.

monthly income per capita (Rs.)	estimated % of families	average family size	average per capita monthly expenditure (Rs.)	
			on all items	on food items
below - 25.0	25	6.4	19.2	12.8
25.0 - 49.9	33	5.4	35.8	22.4
50.0 - 74.9	19	4.7	63.4	35.2
75.0 - 99.9	11	4.3	84.5	42.0
100.0 - 124.9	6	4.0	112.2	54.7
125.0 - 149.5	3	3.7	134.7	62.5
150.0 and above	3	3.2	160.9	68.9

[20]

7. Practical Records (to be submitted by May 16, 1988). [10]

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OPTIMIZATION TECHNIQUES
Semestral II Examination

Date : 9.5.88. Maximum Marks : 100 Time : 3 Hours

Note : The paper carries 115 marks. The maximum you can score is 100.

1. Let A be an mxn matrix, $c \in \mathbb{R}^m$, $b \in \mathbb{R}^n$. $\bar{x}$, $\bar{y}$ are non negative vectors such that

$\bar{x}c + by - \bar{x}Ay \geq \bar{x}c + b\bar{y} - \bar{x}A\bar{y} \geq xc + b\bar{y} - xA\bar{y}$ for all non negative x,y with $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$.

Show that $\bar{x}$, $\bar{y}$ are optimal solutions of the following problem (P) and its dual respectively.

(P) Find $x \geq 0$ such that xc is maximum subject to $xA \leq b$. [20]

2. Suppose $\bar{x} = (\xi_1, \ldots \xi_m)$ maximises xc subject to some linear constraints where $c = (\gamma_1, \ldots \gamma_m)$. Suppose $x' = (\xi_1', \ldots \xi_m')$ maximises xc' subject to the same constraints where $c' = (\gamma_1', \ldots \gamma_m')$, $\gamma_1' > \gamma_1$ and $\gamma_i' = \gamma_i$ for $i \geq 2$. Show that $\xi_1' \geq \xi_1$. [10]

3. Find $\xi_1, \xi_2, \xi_3 \geq 0$ such that $8\xi_1 + 19\xi_2 + 7\xi_3$ is maximum subject to

$$3\xi_1 + 4\xi_2 + \xi_3 \leq 25$$
$$\xi_1 + 3\xi_2 + 3\xi_3 \leq 50$$ [20]

4. Let (N,k) be a network with source s and sink s'. Let x_1, x_2 be distinct nodes of N different from s and s'. Suppose $\delta_1, \delta_2, \Delta_1, \Delta_2, \Delta_{12}$ are positive numbers satisfying the following :

(a) If $k(s,x_1)$ is increased by δ_1, then the value of the maximal flow is increased by Δ_1.

(b) If $k(x_2, s')$ is increased by δ_2, then the value of the maximal flow is increased by Δ_2.

p.t.o.

4.(c) If $k(s,x_1)$ is increased by δ_1 and $k(x_2, s')$ is increased by δ_2 simultaneously, then the value of the maximal flow is increased by Δ_{12}.
Show that $\Delta_{12} \geqq \Delta_1 + \Delta_2$. [20]

5. What is a minimal cut in a network (N,k). Prove that the value of a maximal flow equals the capacity of a minimal cut. [15]

6. Let $(S , T; \phi)$ be a matrix game with value w. Let α, β be real numbers with $\alpha \geqq 0$. Calculate the value of the game $(S , T; \alpha \phi + \beta)$. [10]

7. Write down the pay off matrix and find the solution for the following game:

P_1 and P_2 independently choose integers between 1 and 4. If P_1 chooses i and P_2 chooses j, P_2 pays P_1 an amount $|i-j|$. [20]

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DESIGN OF EXPERIMENTS
Semestral II Examination

Date : 4.5.88. Maximum Marks : 100 Time : $3\frac{1}{2}$ Hours

Note : Answer any four questions. Marks allotted to each question are given within parentheses.

1. Give the details of the analysis of covariance for the following model :-

$$y_{ij} \sim N(m_{ij}, \sigma^2) \; ; \quad \text{all independent,}$$

$$m_{ij} = \mu + \alpha_i + \gamma x_{ij} \; ; \quad 1 \leq i \leq b \; , \; 1 \leq j \leq n_i .$$

Indicate the changes in the analysis appropriate to the following revised model :-

$$y_{ij} \sim N(m_{ij}, \sigma^2) \; ; \quad \text{all independent,}$$

$$m_{ij} = \mu + \alpha_i + \gamma x_{ij}^2 \; ; \quad 1 \leq i \leq b, \; 1 \leq j \leq n_i .$$

(12+8) =[20]

2.(a) Consider a 2^3 factorial experiment involving 3 factors A, B and C each at two levels 0 and 1. Write down the expressions for the main effects and interactions and also the sum of squares due to them in terms of the yields of different treatment combinations.

(b) Enumerate the different types of confounding possible in an arrangement of these 8 treatment combinations into two blocks of 4 plots each.

(c) Consider the following unequal arrangement in two blocks (in standard notation)

Block I (1) a
Block II b ab c ac bc abc.

What main effects and interactions are not confounded at all ?

(14+3+3)=[20]

3.(a) Below is given an incomplete key-block of a 2^4 factorial experiment conducted in two 8-plot blocks :-

Incomplete Key-block : (1) ac bc acd ... ...

Search out the other 4 treatment combinations for the Key-block and also the confounded interaction.

p.t.o.

3.(b) Consider a 3^3 factorial experiment involving 3 factors A, B and C each at the levels 0,1 and 2. Construct two replicates of this experiment in blocks of 9 plots each confounding the interactions ABC^2 and AB^2C^2 respectively. Give the analysis of the resulting design. (5+5+10)=[20

4. A laboratory investigation was conducted to examine the effe of two inorganic materials A and B and a base C on the outpu of cotton dye-stuff. It was decided to examine all combinati of the levels of each of these factors and the whole experiment was repeated to give sufficient precision. Levels of ea factor are at equal intervals of the variables. Analyse the data to test if the treatment combinations produce significa tly different yields. Also estimate the mean squares due to the linear and the quadratic components of the three main effects.

Yields of a direct cotton dye-stuff

	Levels of factor A								
	A_0			A_1			A_2		
	Levels of B			Levels of B			Levels of B		
	B_0	B_1	B_2	B_0	B_1	B_2	B_0	B_1	B_2
C_0	74	13	69	112	46	130	71	56	125
	85	12	115	148	52	107	75	47	70
C_1	211	110	199	166	218	220	201	216	227
	184	145	164	288	204	142	216	239	265
C_2	74	147	195	47	146	198	90	102	164
	75	104	183	65	124	165	60	70	114

(14+6) =[20]

5. In a randomised block experiment for testing the difference among seven varieties A,B,C,D,E,F and G of guayule, the follow ing table gives the layout plan adopted and the data on resin percentage and shrub weight (gms.) for a randomly selected pla in each plot. The upper figures denote the resin percentage and the lower figures in brackets, the shrub weight in gms.

Cont.....

Q.5 cont.....

Block - I	B 5.24 (61)	F 4.85 (84)	G 5.99 (86)	E 3.97 (34)	D 5.50 (65)	A 4.49 (88)	C 5.74 (39)
Block - II	E 5.71 (58)	G 5.15 (89)	A 5.15 (28)	F 4.86 (67)	D 6.00 (96)	B 4.49 (142)	C 6.15 (88)
Block - III	G 5.88 (58)	C 4.88 (61)	F 5.82 (125)	D 4.75 (125)	B 5.60 (95)	A 6.22 (111)	E 6.05 (87)
Block - IV	B 6.13 (70)	E 5.64 (101)	A 7.24 (115)	D 6.30 (101)	G 5.36 (99)	C 6.20 (127)	F 5.17 (104)
Block - V	G 5.80 (93)	B 5.78 (98)	D 5.85 (88)	A 5.67 (132)	F 5.86 (144)	E 3.85 (105)	C 6.18 (169)

Perform the analysis of variance and covariance and compare varieties for resin percentage adjusted for shrub weight. Find the standard error of the difference between the adjusted mean resin percentages for two varieties.

(5+12+3)=[20]

6. Practical Assignments. [20]

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NONPARAMETRIC AND SEQUENTIAL METHODS
Periodical Examination

Date : 26.2.88. Maximum Marks : 100 Time : 3 Hours

Note : Answer all questions.

1.(a) Show that the one-sample Kolmogorov-Smirnov statistics D_n, D_n^+, D_n^- are all distribution-free if the underlying distribution is continuous.

(b) Why do you expect the Kolmogorov-Smirnov test to perform well as a goodness of fit test ? [8+3]

2. Consider the paired-sample problem. Let $(X_1,Y_1),(X_2,Y_2),\ldots,(X_n,Y_n)$ be a random sample of n pairs.

(a) Construct an upper confidence bound for the median θ of the difference X-Y on the basis of the Wilcoxon signed-rank statistic.
Show that the distribution of the difference $\hat{\theta} - \theta$, the error of the estimator, is independent of θ where $\hat{\theta}$ denotes the Hodges-Lehmann estimator of θ. [12+3]

OR

(b) Show that the null distribution of Wilcoxon signed-rank statistic T_+ is symmetric about its expectation

$E(T_+) = N(N+1)/4,$
and $\mathrm{Var}(T_+) = \frac{1}{24} N(N+1)(2N+1).$ [15]

3.(a) Describe the sign test for one-sample location problem.

(b) Find the asymptotic null distribution of the sign test statistic. [3+4]

4.(a) Suppose we have two independent random samples from two populations with continuous and strictly increasing d.f.'s F and G. What is the level α test based on the Mann-Whitney U statistic for the null hypothesis H_0: F=G against the one-sided alternative

$$H : F(x) \geq G(x) \text{ for all } x, \text{ but } F \neq G$$

Show that the same test is also a level α unbiased test for the null hypothesis H_0': $F(x) \leq G(x)$ for all x against the same alternative. [10]

OR

4.(b) Consider the shift model $G(x) = F(x - \theta)$ for all x; in a two-sample location problem where F and G are d.f.'s for the two populations and θ is the amount of shift.

Show that the Hodges-Lehmann estimator $\hat{\theta}$ is distributed symmetrically about θ if either of the following two conditions hold :

(i) The distribution F is symmetric about some point μ.
(ii) The two sample sizes are equal. [10]

5.(a) Show that the Wilcoxon rank-sum test and the Mann-Whitney U test for the two-sample location problem are equivalent. Calculate the mean and variance of one of these test statistics under both the null and alternative hypotheses.

OR [10]

(b) Define a simple linear rank statistic and give an example. Find the expectation and variance of a simple linear rank statistic under the hypothesis that the observations are i.i.d. with a common continuous d.f. and use this result to calculate the expectation and variance for your exampl..

[10]

6. Consider the two-sample problem where the populations sampled may be assumed to be normal. Suppose that only the ranks of the observations are preserved, the original observations having been lost. How would you test the hypothesis that the two populations are identical against a location alternative

[5]

7. In each of the following problems state the model, the null hypothesis and the alternative and suggest a suitable test.

(a) The following are the weights in pounds, before and afte of 8 persons who stayed on a certain reducing diet for four weeks :

Before	147.0	183.5	232.1	161.6	197.5	206.3	177.0	215.4
After	137.9	176.2	219.0	163.8	193.5	201.4	180.6	203.2

Test whether the weight-reducing diet is effective.

(b) The following table gives the estimated values of θ, the ratio of the mass of the earth to that of the moon, obtained from seven different spacecraft.

contd....

7.(b) contd....

Spacecraft	Θ
Mariner 2	81.3001
Mariner 4	81.3015
Mariner 5	81.3006
Mariner 6	81.3011
Mariner 7	81.2997
Pioneer 6	81.3005
Pioneer 7	81.3021

Is this compatible with the previous Ranger spacecraft findings, on the basis of which scientists had considered the value of Θ to be approximately 81.3035 ?

(c) A chemical compound containing 12.5% of iron was given to two technicians A and B. The results are given below.

DETERMINATION BY A	12.46, 12.40, 12.76, 11.95, 12.77, 12.43, 12.60, 11.98, 12.49, 12.39, 12.94, 12.65
DETERMINATION BY B	12.33, 12.22, 12.70, 12.49, 12.61, 12.43, 12.59, 12.53, 12.41, 12.64

Assuming that the analyses are free from bias test whether B is more reliable than A.

(d) Consider the following data where the y's are exam. scores for students who pre-enrolled in a course and the x's are similar scores for students who did not pre-enroll. Can you conclude that the pre-enrolled students did significantly better than the others ?

x	73	68	82	62	75
y	86	81	91	76	84

[16]

8.(a) Assuming shift model for the data given in Question 7(d) compute the 95% confidence interval for the shift parameter Θ.

(b) Use this confidence interval to test the hypothesis $\Theta = 0$ versus the two-sided alternative $\Theta \neq 0$. [12]

9. Practical assignments and class performance. [14]

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OPTIMIZATION TECHNIQUES
Periodical Examination

Date : 29.2.88. Maximum Marks : 100 Time : 3 Hours

Note: The paper carries 110 marks. The maximum you can score is 100.

1. Write a general linear programming problem. Show that such a problem is equivalent to a problem in canonical form. [10]

2. If a canonical linear programming problem has an optimal solution, show that both it and its dual have basic optimal solutions. [15]

3. Write the duals of the following problems. Solve the problems as well the duals.

   (a) Find $\xi_1, \xi_2, \xi_3 \geqq 0$ such that $2\xi_1 + 3\xi_2$ is maximum subject to

$$4\xi_1 + 2\xi_2 + \xi_3 = 4$$
$$\xi_1 + 3\xi_2 = 5$$
[15]

   (b) Find ξ_1, ξ_2 such that $3\xi_1 + \xi_2$ is maximum subject to

$$3\xi_1 \leqq 3$$
$$2\xi_1 + 4\xi_2 \leqq 4.$$
[15]

4. Find $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5 \geqq 0$ such that

$$5\xi_1 + \xi_2 + 6\xi_3 - \xi_5 = 2$$
$$-7\xi_1 - \xi_2 - 2\xi_3 + \xi_4 + 2\xi_5 = -5.$$
[15]

5. Using the simplex method, find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 0 \\ 3 & -2 & 0 \end{pmatrix}$$
[15]

6. Suppose the problem of finding x, unrestricted in sign, such that xc is maximum subject to xA = b, has a feasible solution. Show that it has an optimal solution if and only if c is a linear combination of the columns of A. [10]

7. Define the lexicographic order $\prec$ on m-vectors.
   Show that
   (a) If $x \prec y$ and $y \prec z$, then $x \prec z$.
   (b) If $x \prec y$ and $z \prec w$, then $x+z \prec y+w$.

[2+5+8]

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B.Stat.(Hons.) III Year : 1987-88

MULTIVARIATE DISTRIBUTIONS AND TESTS
Semestral II Examination

Date : 2.5.88. Maximum Marks : 100 Time : 3 Hours

Note : Answer any five questions. Questions carry equal marks.

1.(a) Define Wishart matrix. State and prove the additive property of Wishart distribution.

(b) Let $A \sim W_p(I,n)$ then prove that
(i) $a_{ii} \sim \chi^2$, (ii) $\Sigma a_{ii} \sim \chi^2$, (iii) $\sum_{ij} a_{ij} \sim \chi^2$
Find the appropriate d.f. in each case.

2.(a) Let $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\begin{matrix} q \\ p-q \end{matrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$
Find the distribution of x_1 given x_2.

(b) Find the distribution of
$$X'\Sigma^{-1}X - x_2' \Sigma_{22}^{-1} x_2$$
when $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ as given above in (a).

3.(a) Let Θ denote a r.v. which is Q.f. in the items of a random sample of size n from a distribution which is $n(0,\sigma^2)$. Let A denote the symmetric matrix in Θ and let r, $0 \leq r \leq n$, denote the rank of A.
Prove that
$$\Theta/\sigma^2 \sim \chi^2(r) \text{ iff}$$
$A^2 = A$ and rank of $(A) = r$.

(b) Let $x_1,\ldots,x_2$ denote a random sample of size n from a distribution which is $n(0,\sigma^2)$. Prove that Σx_i^2 and every quadratic form which is non identically zero in $x_1,\ldots,x_n$ are stochastically dependent.

4.(a) Let q_1 and q_2 be apriori probability of drawing an observation from population π_1 with density $P_1(x)$ and π_2 with density $P_2(x)$ respectively and if the cost of mis-classifying an observation from π_1 as from π_2 is $c(2|1)$ and an observation from π_2 as from π_1 is $c(1|2)$, define the region of classification so that the expected cost of misclassification is minimised.

Cont....

4.(b) If the two populations π_1 and π_2 happen to be multivariate normal with means μ_1 and μ_2 and variance covariance matrices Σ then find the best region of classifications.

Calculate the two probabilities of mis-classifications.

5.(a) Let $X_1, \ldots, X_n$ be independent, each with distribution $N(\mu, (\sigma_{ij} \delta_{ij}))$. Find the density of sample correlation matrix.

(b) Let the components of X correspond to scores on tests in arithmetic speed (x_1), arithmatic power (x_2) and memory for words (x_3). The observed correlation matrix in a sample of size 140 is

$$\begin{pmatrix} 1.000 & .425 & .042 \\ .425 & 1.000 & .149 \\ .042 & .149 & 1.00 \end{pmatrix}$$

Find the multiple correlation between x_1 and the set x_2 and x_3. Test the hypothesis at 5% of significance that arithmetic speed (x_1) is independent of arithmetic power (x_2) and memory for words (x_3).

6. Write notes on (any four)

(i) Behren Fisher Problems

(ii) Multiple Correlation Coefficient

(iii) Hotelling's T^2 statistic

(iv) Test for independence of two sets of multinormal vectors

(v) Cochran's Theorem and its applications.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1987-88

ELECTIVE-5 : ECONOMICS
Periodical Examination

Date : 2.3.88. Maximum Marks : 100 Time : 3 Hours

Note : Answer question no. 6 and any four from the rest. Marks allotted to each question are given in brackets [ ].

1.(a) Find the equation of the Lorenz curve for an exactly Paretean income distribution over the income range (c,∞) where $c(>0)$ is the subsistence income. [8]

(b) In (a) above, what would be the equation of the Lorenz curve for the truncated income distribution over (c',∞) where $c' > c$? [6]

(c) Suppose you are given some income data and you plot a graph showing $\log T_x$ against $\log N_x$, where N_x is the number of earners earning x or more and T_x is the total income of these N_x persons. What would be the equation of the graph if the income distribution is Paretean ? [6]

2.(a) Discuss the general properties of a Lorenz curve. [8]

(b) Explain clearly the significance of Lorenz curve comparisons of different income profiles. [6]

(c) Obtain the expressions for the Lorenz ratio of a two-parameter lognormal distribution. [6]

3. Suppose a size variable X follows a two-parameter lognormal distribution with parameters μ and σ^2. What is then the distribution of $Y = \alpha X^{\beta}$, where α and β are two positive constants? Also obtain the expression for the mean, median, mode and coefficient of skewness of Y. (5+15)=[20]

4.(a) Clearly bring out the distinctions between the two approaches viz., "conventional statistical" and normative, for measuring income inequality. [10]

(b) Take any two measures based on each of the two approaches and discuss their suitability as appropriate measures of income inequality. [10]

p.t.o.

5. Write short notes on any two of the following :

   (a) Sen's poverty measure.
   (b) The moment distribution property of the lognormal distribution and its uses.
   (c) Universality of Pareto law of income distribution.

(10x2) = [20]

6. Examine the size distribution of income given below and fit a Paretean distribution over the appropriate range.

Income (Rs.)	No. of earners
10001 - 20000	6286
20001 - 30000	1404
30001 - 50000	696
50001 - 75000	218
75001 - 100000	82
100001 - 200000	74
200001 and above	25

(6+14)=[20]

1987-88 | 3P1

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1987-88

ELECTIVE-5 : PHYSICAL AND EARTH SCIENCES
Periodical Examination

Date : 2.3.88. Maximum Marks : 100 Time : 3 Hours

Note : All questions carry equal marks. 100 is the maximum marks you can attempt/obtain. Whenever necessary draw the figure/circuit diagram/graph for complete explanation.

1. Clearly state and explain the effects of temperature rise in a resistance, made of different materials. [4]

In this connection what do you meant by temperature co-efficient of a resistance. [3]

The currents through an electrical conductor are 1^A and 0.7^A when the temperaturesof the conductor are 0^oC and 100^oC respectively. Find the temperature co-efficient of the resistance of the conductor. What would be the magnitude of the current at 1200^oC temperature ? [7]

How do you account for the different temperature co-efficient of the resistance (of a conductor) in explaining the linear/non-linear characteristics ? With suitable graph explain the phenomena. [6]

2. Two resistances of 4Ω and 12Ω are connected in parallel with each other. A third resistance of 10Ω is connected in series with the combination. Find in each case what dc voltage should be applied across the whole circuit to pass 6^A through (a) 10Ω, (b) 12Ω resistor. Draw the neat circuit diagram and also calculate the pd across 4Ω in each cases. [9]

You have heard the 'shunting' of a galvanometer. What would be the value of shunt resistance wrt galvanometer resistance and why ? Hence clearly explain its utility. [4]

A bulb rated 110^V, 60^W is connected with another bulb rated 110^V, 100^W across a 220^V mains. Calculate the resistance to be joined in parallel with one of the bulb so that both of them may take their rated power. [7]

p.t.o.

3. There are m number rows of battery each of which is containing n number of cells having emf E and internal resistance r. An external resistance R is connected across the combination. Determine the condition for maximum obtainable current derived from this arrangement. How much efficiency you would be expecting from the system ? [6]

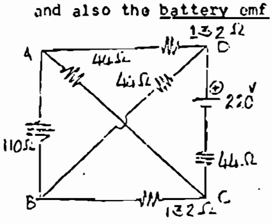
What combination would you prefer to derive maximum current using all the cells (as mentioned above) and considering their ideal nature (i.e. $r \simeq 0$) ? [2]

One hundred cells, each of which has emf of 2^V and internal resistance of 0.8Ω, are to be connected for sending maximum current through an external resistance of 5Ω. What combination would you prefer and why ? Find the value of current in each of the combination. [7]

Two resistors 1KΩ and 4KΩ were connected across 220^V supply. What would be the difference of voltage across 4KΩ(i) when measured by a voltmeter of 12KΩ resistance and (ii) calculated without using galvanometer ? [4]

4. State and explain, with suitable diagrams, Kirchhoff's current and voltage laws for network analysis. [6]

Briefly describe the relation of voltage drop across a resistor and also the battery emf wrt flowing current through them.



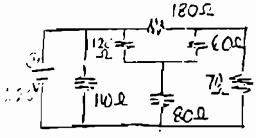
Along the rectangle ABCD different resistors are arranged as follows: 110Ω(AB), 132Ω(BC), 44Ω(AC), 132Ω(AD), 44Ω(BD) and 220^V in series with 44Ω(CD). Find with Kirchhoff's laws or otherwise the direction of current flowing through and potential difference across 110Ω. [10]

5. Three resistors, of different value, are connected across the side of an equilateral triangle in delta fashion. Mathematically deduce the relation of conversion it to an electrically equivalent star fashion of resistors along the medians of the same triangle. [7]

Contd....

Q.5. contd....

If possible, convert a star fashion of connection to an electrically equivalent delta configuration. [4]



Determine the current flowing through the 220^V supply and output voltage across 70Ω. [9]

6. Clearly state and explain with suitable diagrams, Thevenin's theorem for network analysis. Mention all the steps you may require. [8]

The four arms of a wheatstone bridge have the following fixed resistances in ohms :

AB = 900, BC=100, AD=450 and CD=50+40 (variable)

[The variable resistance has 2 extreme connections only]

A galvanometer of 15Ω internal resistance is connected across BD and a potential difference of 90^V is applied across AC. For two extreme connections of variable resistance in arm CD, find the value of current flowing through the galvanometer. [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1987-88

DESIGN OF EXPERIMENTS
Periodical Examination

Date : 24.2.88. Maximum Marks : 100 Time : 3 Hours

Note : Answer all questions. Marks allotted to each question are given within parentheses.

1. Obtain the layouts of the following designs : -
   (a) A CRD with 3 treatments A,B and C, the replication numbers being 6,5 and 10, respectively.
   (b) An RBD with 5 treatments in four blocks.
   (c) A 6x6 LSD. (6+6+8) = [20]

2. Writing R,C and E for the row M.S., column M.S. and error M.S., respectively of an mxm Latin square design prove that (i) an estimate of the error M.S. which would have been obtained if the row classification had not been made is

$$\frac{R + (m-1)E}{m}$$

and (ii) if the design had been completely randomised the same is

$$\frac{R + C + (m-1)E}{m+1}.$$

Hence estimate the efficiency of the Latin square relative to (i) randomised blocks made up by columns and (ii) unrestricted randomisation. (8+8+2+2) = [20]

3. The following table gives the layout and yields of 6 wheat varieties in an experiment in four randomised blocks.

Block - 1	V_2	V_3	V_6	V_1	V_4	V_5
Yield	30.6	27.7	24.9	27.8	16.2	16.2
Block - 2	V_1	V_4	V_6	V_2	V_5	V_3
Yield	27.3	15.0	22.5	28.8	17.0	22.7
Block - 3	V_6	V_2	V_4	V_3	V_1	V_5
Yield	22.7	31.0	14.1	34.9	28.5	17.7
Block - 4	V_4	V_6	V_5	V_2	V_3	V_1
Yield	14.1	22.7	17.7	39.5	36.8	38.5

p.t.o.

Q.3 contd....

(a) Analyse the data and determine the best variety.

(b) If the yield of V_2 on block 4 is known to be suspicious are you going to modify your conclusions ? (10+10)=[20]

4. In the experiment described below four materials were tested in each of four runs on a machine with four different positions. The letters A,B,C and D refer to four materials. The layout of the experiment is given below where the figures denote the loss in weight in a run of standard length.

Run \ Position	4	2	1	3
2	A(251)	B(241)	D(227)	C(229
3	D(234)	C(273)	A(274)	B(226)
1	C(235)	D(236)	B(218)	A(268)
4	B(195)	A(270)	C(230)	D(225)

(a) Analyse the data and give your comments.

(b) If the variation due to different positions of the machine is ignored, will you modify your conclusion ? (10+10)=[20]

5. Practical Assignments. [20]

[1987-88] [361]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1987-88

MULTIVARIATE DISTRIBUTIONS AND TESTS
Periodical Examination

Date : 22.2.88.   Maximum Marks : 100   Time : 3 Hours

Note : Answer any four questions.

1.(a) Define Multivariate normal density. Interprete its parameters in terms of first few moments.

(b) Let $X_{i\alpha} \sim N_2 \left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_{22}^2 \end{pmatrix} \right)$, $\alpha = 1,2$

Then find the distribution of $\left(\frac{x_{11}+x_{12}}{2}, \frac{x_{21}+x_{22}}{2} \right)$.

2.(a) Let $X_1, \ldots, X_N$ be independent random vectors such that $X_i \sim N_p\ (\mu_i, \Sigma)$. Then for orthogonal $C = (c_{ij})$, prove that

(i) $Y_i = \Sigma\, C_{ij} X_j \sim N_p\ (\nu_i, \Sigma)$, where $\nu_i = \Sigma\, C_{ij}\, \mu_j$

(ii) $Y_i, \ldots, Y_n$ are independent

(iii) $\Sigma Y_i Y_i' = \Sigma X_i X_i'$

(b) Using the above or otherwise prove that

(i) $\bar{X} \sim N_p\ (\mu, \frac{1}{n}\Sigma)$

(ii) $\bar{X}$ and S are independent, symbols have their usual meanings.

3.(a) Write the Hotelling's T^2 statistic for testing $H_o : \mu = \mu_o$ in a multivariate normal set up. Find its distribution under H_o.

(b) Let we have a random sample $X_1, \ldots, X_n$ from $N_p(\mu, \Sigma)$, Σ being unknown. Derive the Likelihood ratio test for the hypothesis $H_o : \mu = \mu_o$. Show that it is a function of T^2 defined earlier.

p.t.o.

4.(a) Let $(Z_{1\alpha}, Z_{2\alpha})$, $\alpha = 1,\ldots, n$ be independent, each pair with distribution $N_2\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right]$.

Prove that conditional distribution of $b = \Sigma Z_{1\alpha} Z_{2\alpha} / \Sigma Z_{1\alpha}^2$ and $V/\sigma^2 = \Sigma (Z_{2\alpha} - bZ_{1\alpha})^2 / \sigma^2$ given $Z_{1\alpha}$ ($\alpha = 1,\ldots,n$) is $N(\beta, \sigma^2/c^2)$, ($c^2 = \Sigma Z_{1\alpha}^2$) and $\chi^2 (n-1)$ respectively and that b and V are independent.

(b) On the basis of 15 trivariate observations the following correlation matrix is obtained.

$$\begin{pmatrix} 1 & .1 & .2 \\ .1 & 1 & .1 \\ .2 & .1 & 1 \end{pmatrix}$$

Test (i) H_0 ($\rho_{12} = 0$), (ii) H_0 ($\rho_{12.3} = 0$).

5.(a) Give an expression for multiple correlation coefficient of x_1 on $x_2,\ldots,x_p$. Find its distribution in the null case.

(b) In the example 4(b) test for H_0 ($\rho_{1.23} = 0$).

6. Write notes on any four of the following :

(i) Hotelling's Generalized T^2 statistics
(ii) Wishart distribution
(iii) Multiple Correlation Coefficient
(iv) Multivariate Normal density
(v) Total & partial correlation coefficients.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I BACKPAPER EXAMINATION
Statistical Inference

Date: 1.1.88 Maximum Marks: 100 Time: 3 hours

1. Let X a r.v. with density $f(., \theta)$. Consider the problem of testing $H_0: \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

(a) Define a level α test for H_0.

(b) For a randomized test, define the probabilities of type I error and types II error, denoted by α_1 and α_2, respectively.

(c) Obtain a test which minimizes

$$P_1 \alpha_1 + P_2 \alpha_2$$

where P_1 and P_2 are known positive constants.

(d) Show that the above test is MP of its size. [20]

2. Based on independent random samples from $N(\mu_1, \ {}^2)$ and $N(\mu_2, \ {}^2)$ obtain the LRT of size α for testing $\mu_1 = \mu_2$ against $\mu_1 \neq \mu_2$ when ^{2} is unknown. Derive a confidence interval of level $1-\alpha$ for $\mu_1-\mu_2$. [20]

3. Let X_1, X_2 be i.i.d according to the c.d.f

$$F(x,\theta) = 1-e^{-\theta x}, \quad x > 0$$
$$= 0, \quad x < 0$$

for a fixed $x > 0$, find the UMVUE of $1-e^{-\theta x}$. [15]

4. Let $X_1, \ldots X_n$ be the times (in months) until failure of on similar pieces of equipment. Suppose that X_i's are distributed with the common density

$$f(x,\theta) = \alpha e^{-\alpha x}, \quad x > 0$$
$$= 0, \quad x < 0$$

(a) Obtain the UMP test for $H_0: \alpha \geq \alpha_0$ against $H_1: \alpha < \alpha_0$. How would you obtain the cut-off point of the above test given the size α ?

contd. 2/-

4.(b) Suppose $\alpha_0 = 1/12$. Find the minimum sample size needed for the above test to achieve power at least 0.95 at $\alpha_1 = 1/15$ when $\alpha = 101$. [15

5. How would you test the equality of two Poisson distributions on the basis of independent samples of fixed sizes ? [10

6.(a) Show that the largest observation is minimal sufficient for $U(0,\theta)$.

(b) Obtain minimal sufficient statistics based on a random sample of size n from $N(\mu, \ ^2)$ when

(i) $-\infty < \mu < \infty, \ ^2 > 0$

(ii) $\mu = 0, \ ^2 > 0$

(iii) $-\infty < \mu < \infty, \ ^2 = 1$

(iv) $\mu = 0, \ = 1.$

(c) State Cramer Rao Inequality and discuss its role in statistical inference. [18

For clarity [2].

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I BACKPAPER EXAMINATION
Difference and Differential Equations

Date: 31.12.87 Maximum Marks: 100 Time: 3 hours

Note: Answer all the questions

1.(a) Solve the differential equations

(i) $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$

(ii) $y\,dy + by^2 dx = a\cos x\,dx$

(b) Use the fact that $y = x$ is an obvious solution of the following equation to find its general solution

$$y'' - \frac{x}{x-1}y' + \frac{1}{x-1}y = 0$$

[5+5+8=18]

2.(a) Establish completely Kepler's second law : the radius vector from the sun to a planet sweeps out equal areas in equal intervals of time.

(b) Two masses are M and m connected by a string which passes through a hole in a smooth horizontal plane, the mas m is hanging vertically. Show that M describes on the plane a curve whose differential equation is

$$\left(1+\frac{m}{M}\right)\frac{d^2u}{d\theta^2} + u = \frac{mg}{M}\,\frac{1}{h^2u^2}$$

(Here (r,θ) are polar coordinates, $u = 1/r$, h is a constant so that $d\theta/dt = hu^2$, and g is the acceleration due to gravity.)

[10+8=18]

3. Consider the equation $y'' + xy' + y = 0$.

(a) Find its general solution $y = \Sigma\, a_n x^n$ in the form

$$y = a_0y_1(x) + a_1y_2(x),$$

where $y_1(x)$ and $y_2(x)$ are power series.

(b) Use ratio test to verify that the two series $y_1(x)$ and $y_2(x)$ converge for all x.

(c) Show that $y_1(x)$ is the series expansion of $e^{-x^2/2}$. Use the fact to find a second independent solution, and convince yourself that the second solution is the function $y_2(x)$ found in (a).

[8+6+8=22]

Contd. 2/-

4.(a) Show that

$$\exp\left(\frac{x}{2}\left(t-\frac{1}{t}\right)\right) = \sum_{n=-\infty}^{\infty} J_n(x)\, t^n ,$$

where $J_n(x)$ is the Bessel function of order n.

Hence deduce

(i) $\cos(x \sin\theta) = \sum_{n=-\infty}^{\infty} J_n(x) \cos n\theta$

(ii) $\sin(x \sin\theta) = \sum_{n=-\infty}^{\infty} J_n(x) \sin n\theta$

(b) Prove the following recursion formula for Chebyshev polynomials

$$T_n(x) + T_{n-2}(x) = 2x\, T_{n-1}(x).$$

[(8+4+4)+10=26]

5.(a) Show that if f and g have the same Laplace transform, and g is continuous on $(0,\infty)$, then $f(t) = g(t)$ for every $t > 0$.

(b) Find the inverse Laplace transform of

$$\frac{p+3}{p^2+2p+5}$$

(c) Solve the following differential equation by the method of Laplace transforms

$$y'' + 4y = 4x,\ y(0) = 1,\ y'(0) = 5.$$

[8+8+8=24]

:53:

1987-88/342(b)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I BACKPAPER EXAMINATION
Sample Surveys

Date: 29.12.87 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours.

Note: Answer all questions. Marks allotted to each question are given in brackets( ).

1.(a) Define a 'sampling design'. What do you understand by the terms 'inclusion probability of a unit' and 'joint inclusion probability of a pair of units' for a sampling design.

(b) For a probability proportional to size with replacement sampling design of n draws, write down π_i, the probability of inclusion of a unit U_i and π_{ij}, the joint inclusion probability of a pair of units (U_i, U_j). (7+6=13)

2.(a) When stratified simple random sampling without replacement is used to estimate the population mean, derive an allocation of the fixed total sample size n to the strata which minimises the variance of the estimated mean. How does one implement this allocation in practice ?

(b) Write down the 'combined and separate ratio estimators' in stratified sampling. Which of these do you recommend ? Give reasons. (8+10=18)

3.(a) Define the term 'Intra Cluster Correlation Coefficient'. For a population of 14 clusters each of size 6, find the lower and upper bounds for the intra class correlation coefficient among the elements of the cluster.

(b) A population consists of N clusters of varying sizes M_i, $i=1,2,\ldots,N$. Suppose that n clusters are selected with probabilities proportional to the cluster size and with replacement. Write down an unbiased estimator of the population mean $\bar{Y}$ and an unbiased estimator of its variance. If it is decided to use the estimator $t = \sum_{i=1}^{n} M_i \bar{Y}_i / \sum_{i=1}^{n} M_i$ for $\bar{Y}$, where $\bar{Y}_i$ is the ith cluster mean, $i = 1,2,\ldots, N$, suggest a sampling scheme that makes t unbiased for $\bar{Y}$. (3+3)+(8+5)=(19)

A sample survey was conducted to estimate the total household expenditure in an urban area. The design adopted was a stratified two-stage one with census enumeration blocks as first stage units and households within them as second stage units. From each stratum, 4 blocks were selected with probability proportional

contd. 2/-

to population and with replacement and 4 households were selected from each selected block with equal probability and without replacement. The data on household expenditure for the sample households together with information on selection probabilities are given below :

Stratum	Sampled block	Inverse of probability of selection	Totals of households	Weekly household expenditure of sample households 1	2	3	4
I	1	67.63	189	110	201	120	114
	2	338.12	40	80	60	122	125
	3	101.50	135	122	210	171	105
	4	69.03	160	244	115	312	128
II	1	113.34	73	345	359	160	117
	2	441.00	26	97	179	144	85
	3	31.50	240	100	115	50	172
	4	661.57	14	102	40	126	148
III	1	15.80	287	122	176	108	140
	2	21.00	257	125	110	134	215
	3	48.89	68	300	115	67	110
	4	26.73	218	263	75	142	54

(a) Obtain an unbiased estimate of the total weekly household expenditure.

(b) Obtain an unbiased estimate of the sampling variance of the above estimate.

(c) What are the sources of non-sampling errors in the above survey and how do you assess them? (16+14+10=40)

5. In a recent survey of 64 students selected by random sampling with replacement who took a 'Backpaper Examination' it was found that 47 of them could not improve on their previous result. Estimate the proportion of students that took Backpaper examinations and could not improve, in the population. Also obtain an unbiased estimate of the sampling error of your estimate. Comment on the sampling design used in this survey and suggest any modification. (3+4+3=10)

: ss :

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I EXAMINATION
Stochastic Processes-2

Date: 23.11.67 | Maximum Marks: 100 | Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1. A telephone exchange has m channels. Calls arrive in the manner of a Poisson process with parameter λ ; they are accepted if there is an empty channel, otherwise they are lost (no waiting line is formed). The duration of each call is exponential with parameter μ. Let $X(t)$ denote the number of busy channels at time t. Find the birth and death rates for the process $X(t)$. Find the stationary distribution. [15]

2.(a) Write down the forward differential equation for a linear birth and death process X_t, $t \geq 0$ with birth rates $\lambda_n = n\lambda$ and death rates $\mu_n = n\mu$ $(\lambda > 0, \mu > 0, \mu > 0)$. Find $E(X_t \mid X_0 = 1)$.

(b) In case $\lambda = \mu$ prove that

$$P_{10}(t) = P(X_t = 0 \mid X_0 = 1) = \frac{\lambda t}{1+\lambda t}$$

$$P_{1n}(t) = P[X_t = n \mid X_0 = 1] = \frac{(\lambda t)^{n-1}}{(1+\lambda t)^{n+1}} \qquad n \geq 1$$

[5+5+15]

3. Let $\tau_1, \tau_2, \ldots$ be independent non-negative random variables with common distribution function $F(x)$ and density function $f(x)$.
Assume that

$$0 < \mu = E(\tau_1) < \infty.$$

(a) Define a stopping time N with respect to the sequence $\tau_1, \tau_2 \ldots$. State and prove Wald's equation. [2+8]

(b) Show that

$$\lim_{t\to\infty} \frac{E(N(t))}{t} = \frac{1}{\mu}$$

where $N(t)$, $t \geq 0$ is a renewal process with interarrival times $\tau_1, \tau_2 \ldots\ldots$ [10]

(c) Let $g(t) = E(t - S_{N(t)})$ where

$$S_n = \tau_1 + \cdots + \tau_n \qquad n \geq 1.$$

Show that $g(t)$ satisfies a renewal type equation

$$g(t) = h(t) + \int_0^t g(t-x)\, f(x)\, dx$$

p.t.o.

Show that $\lim_{t \to \infty} g(t) = \frac{E(\tau_1^2)}{2E(\tau_1)}$.

State carefully the renewal theorem that you use in finding the limit of g(t). [5+15]

4. Let X(t), $t \geq 0$ be the standard Brownian motion process.

(a) Let $Z_t = e^{-\alpha t} X(e^{2\alpha t})$ $t \geq 0$

($\alpha > 0$ fixed). Find the covariance function cov (Z_s, Z_t) and the mean function $E(Z_t)$ of the Z_t process. [10]

(b) Let $Y_t = \sup_{0 \leq u \leq t} X(u)$. Find the distribution function and the density function of the random variable Y_t, $t > 0$. [15]

:ss:

[illegible]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I BACKPAPER EXAMINATION
Stochastic Processes-2

Date: 28.12.87 | Maximum Marks: 100 | Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Suppose that customers arrive according to a Poisson Processes with rate λ and that each customer starts being served immediately upon arrival (infinitely many servers). Suppose that the service times are independent and exponentially distributed with parameter μ. Let X_t $t \geq 0$ be the number of customers in the process of being served at time t.

(a) Let $Y(t)$ denote the number of customers who arrive in the interval $[0,t]$. What is the conditional probability distribution of the actual times of arrival $S_1, S_2, \ldots S_n$ of the n customers given that $Y(t) = n$? (No proof is required)

(b) Using a) compute

$$P_{ij}(t)=P[X_t=j \quad X_0=i]$$ [5+15]

2. Consider a pure death process on $0,1,2,\ldots$ with death rates $\mu_0=0$, $\mu_n, n \geq 1$.

(a) Write down the forward equations

(b) Solve $P_{ij}(t)$ in terms of $P_{i,j+1}(t)$

(c) Find $P_{ii}(t)$, $P_{i,i+1}(t)$.

(d) If $\mu_n=n\mu$ $n \geq 0$, show that

$$P_{ij}(t) = \binom{i}{j} (e^{-\mu t})^j (1-e^{-\mu})^{i-j} \quad 0 \leq j \leq i$$ [5+5+5+10]

3. Let $N(t)$, $t \geq 0$ be a renewal process with interarrival times $_1, _2 \ldots$ where $_1$ has the distribution function F and density function f. Let $S_n = {}_1+\ldots+{}_n$ for ≥ 1.

(a) Show that $E(N(t)) = \sum_{n=1}^{\infty} P[S_n \leq t]$

(b) Show that $\frac{N(t)}{t} \longrightarrow \frac{1}{\mu}$ almost surely as t $t \longrightarrow \infty$ where $\mu = E(\quad)$.

contd. 2/-

3.(c) Let $m(t) = E(N(t))$

Show that $m(t) = F(t) + \int_0^t m(t-x)f(x)dx$

(d) Find the probability distribution of $N(t)$. [5+5+5+10]

4. Let $X(t)$, $t \geq 0$ be the standard Brownian motion process.

(a) Let $Z(t) = X(t) - t\,X(1)$ for $0 \leq t \leq 1$.

For $0 < t_1 < t_2 < \ldots < t_n$ find the probability distribution of $(Z(t_1), Z(t_2) \ldots Z(t_n))$.

(b) For $\alpha > 0$, let T_α be the first time $X(t)$ equals α. Find the density of the random variable T_α.

(c) Let $0 < t_1 < t_2$ and let

$A = X(t) = 0$ for some t in $[t_1, t_2]$

Compute $P(A)$ using the fact that $P[A \mid X(t_1) = \alpha] = P[T_\alpha < t_2 - t_1$

[5+15+15]

[1987-88.532]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I EXAMINATION
Statistical Inference

Date: 26.11.87n Maximum Marks: 100 Time: 3 hours

Group A
Answer any two questions [25+25]

1. Suppose that $X_1, \ldots, X_n$ are independently and identically distributed according to $U(0,\theta)$.

(a) Obtain the class of all MP size α tests for testing $\theta = 1/2$ against $\theta > 1/2$.

(b) Consider the following test for the above problem:

$$\phi_c(X) = 1, \text{ if } X(n) \geq c$$
$$= 0, \text{ otherwise.}$$

Is this test MP of its size?

(c) How large should n be so that ϕ_c has power 0.98 for $\theta = 3/4$ given that $\alpha = 0.05$?

2.(a) In testing $H: \mu \leq 0$ versus $K: \mu > 0$ based on a random sample size n from $N(\mu, \sigma^2)$ with σ^2 unknown, show that the one-sided t-test is LRT (for $\alpha < 1/2$)

(b) Sketch a proof to show that the above one-sided t-test is UMPU.

3.(a) Let X be distributed according to the Cauchy distribution with median θ. Define $U = 2x/(1+x^2)$. Show that $EU = 0$, $\text{Var } U = 1/2$ when $\theta = 0$.

(b) Obtain the initial region of the LMP size α test for $\theta = 0$ vs. $\theta > 0$ based on n independent observations on X. Use (a) to obtain an approximate cut-off point of this test.

(c) Show that the power of the above test tends to zero as $\theta \to \infty$ when $\alpha < 1/2$.

Group B
Answer any two questions [15+15]

1. Let X and Y be independent random variables with geometric distributions

$$f(x,y \mid \theta_1, \theta_2) = (1-\theta_1)(1-\theta_2)\, \theta_1^x\, \theta_2^y,$$

$$x = 0,1,\ldots;\ y = 0,1,\ldots$$

find a UMPU test of size $\alpha = .20$ for testing $\theta_1 \leq \theta_2$.

p.t.o.

2. Let $X_1, \ldots, X_n$ be the times to failure of n pieces of equipment. Assume that X_i's are independent exponentially distributed r.v.'s with the common mean $1/\lambda$. Obtain a uniformly most accurate level $(1-\alpha)$, upper confidence bound $\bar{q}$ for $q(\lambda) = 1 - e^{-\lambda t_0}$. (Hint: Consider testing $\lambda \geq \lambda_0$ against $\lambda < \lambda_0$).

3. Based on a random sample of size n from $N(\mu, 1)$, obtain the UMVUE of the c.d.f at a given point. Check whether the variance of this estimate attains the corresponding Cramer-Rao lower bound.

Group C [1...

1.(a) Show that the order statistics are minimal sufficient for the Cauchy distribution with median θ.

(b) Show that the c.d.f. of a r.v having densities with MLR property in a real parameter θ is decreasing in θ.

(c) Give examples to demonstrate

(i) MLE is biased,

(ii) Unbiased estimate is not consistent,

and (iii) Any statistic is sufficient.

for clarity [2].

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[1987-88/5..]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I EXAMINATION

Elective-4: Physical and Earth Sciences

Date: 20.11.87 | Maximum Marks: 100 | Time: 3 hours

Note: Attempt question no. 9 and any five from the rest.
Any attempt for extra question(s) will be penalised.

1. Describe the various processes involved when the hard parts of an organism get altered while it undergoes fossilization.
'Fossils are useful as economic tools' _______ how? [10+5]

2. What is the essential characteristic of silicate structure? How can pyroxene, amphibole, biotite, muscovite and quartz be differentiated on the basis of silicate structure?
Describe the role of Al in the structure of feldspars. [4+8+4]

3. Describe the principal differences between an acid lava and a basic lava.
What is a 'Primary magma'? To which view of the nature of primary magmas do you agree ________ single or two? Why? [5+4+5]

4. In what way Bowen's name is mainly associated with igneous petrology?
Describe in short the crystallization process of a basaltic magma. [6+10]

5. Describe the major factors and their effects in metamorphism.
Is there any difference between schistose structure and foliation?
Name one typical metamorphic rock from the Eastern Ghats and describe its mineralogical composition. [8+4+1+3]

6. How did Wegener depend his theory of continental drift?
What were the main criticisms against it?
Describe the importance of rock magnetism in the understanding of the drifting of the continents. [4+4+8]

7. Describe the role of the Mid-Oceamic Ridge System in the understanding of the Plate Technics model.
Explain in brief how magma/lava is generated in the subduction (convergant) zone.
What is the role of asthenosphere in plate tectonics? [8+4+4]

8. Write short notes on (any four):
Andesite; peridotite; progressive metamorphism; C^{14} method in radiometric age determination; woolly mammoths of Siberia and Alaska; hot spots.

p.t.o.

9. Fill up the blanks (any <u>ten</u>). Write down only one of the four choices for each blank.

(i) A pure sandstone (orthoquartzite) contains mineral constituents which are almost all __________ (feldspar/quartz/silica/chert).

(ii) The dark colour of an igneous rock is due to the presence of ________ (F_e-M_g/S_i-O/K-C_l/C_a-N_a).

(iii) K-feldspar is an important ________ (precious/economic/rock-fo-ng/crystallic) mineral.

(iv) Mountain-building activity is deeply associated with ________. (synchine/anticline/geosyncline/mantle).

(v) The term texture includes _________ (grain-size/structure/ cross-bedding/viscosity).

(vi) Al is an important element in _________ (magnetite/chalcopysite/ dolomite/andalusite).

(vii) A mineral assemblage which defines a particular P-T environment is called _______ (index/facies/retrogression/ profession).

(viii) A palaeontologist analyses remains of ancient organisms to trace their _________ (morphology/stratigraphy/evolution/rock-types).

(ix) The __________ (eyes/teeth/blood-cells/muscles) of a dinosaur were most likely to be preserved.

(x) Petroleum is a /an __________ (crystalline/mineral/organic/ inorganic) substance.

(xi) A plutonic rock is made up of __________ (fine-grained/ both coarse and fine grained/coarse-grained/clastic) minerals.

(xii) Pyrite is a/an _________ (precious/rock-forming/economic/ ordinary) mineral.

:SS:

[illegible]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I EXAMINATION
Elective-4: Economics

Date: 20.11.87 Maximum Marks: 100 Time: 3 hours 15 minutes

Note: Answer two questions from Group A and three questions from group B.

Group A

1.(a) Solve the following linear programming problem by the simplex method :

minimise $15x_1 + 10x_2 + x_3$

subject to

$$x_1 + 3x_2 + x_3 = 4$$
$$2x_1 + x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

(b) Also find an optimal solution to the dual problem. [18+7=25]

2.(a) Describe the structure of an input-output model (with an input matrix A). When would you call A 'productive'? (You may use Gale's definition.)

(b) Show that A is 'productive', if an only if $(I - A)^{-1}$ exists and is non-negative.

(c) Show that if A is 'productive', then $\lim_{t \to \infty} A^t = 0$, where $A^t = A.A.A\ldots$(t times). [4+14+7=25]

3. Consider a competitive economy in which each firm operates a set of linear activities requiring two types of inputs - plant capacities (which are given for each firm) and resources (which different firms compete for and whose supplies are given for the economy as a whole).
Show that such an economy can obtain its maximum possible total income, only if it is possible to assign prices to resources in such a way that not only will the available supplies of resources be sufficient to meet the total demand for them by all firms, but each firm will also be able to maximise its own profit. [25]

Group B

Obtain the commodity X commodity Input output table given the 'make matrix' and the 'commodity X industry table' below. (use commodity technology assumption).

p.t.o.

Table 1

Make Matrix

	Industries			
	1	2	3	Total
Commodities				
1	100	0	0	100
2	10	100	0	110
3	0	0	50	50
Total	110	100	50	260

Table 2

Commodity X Industry I-O table

	Industry				
	1	2	3	Final Demand	Total
Commodity					
1	20	30	0	50	100
2	30	20	20	40	110
3	10	20	10	10	50
Value added	50	30	20		100
Total	110	100	50	100	

(10)

2.(a) The following pay-off matrix represents the returns expected by a firm for five alternative investments and four different levels of sales. Which alternatives would the firm select if their decisions are based on the (a) maximin rule (b) maximax rule (c) Hurwitcz rule for $\lambda = .7$ and $\lambda = .3$, where λ = degree of optimism

	Levels of Returns			
	1	2	3	4
Alternative				
A	15	11	12	9
B	7	9	12	20
C	8	8	14	17
D	17	5	5	5
E	6	14	8	19

2.(b) A large water treatment facility is located in the flood plain of a river. The construction of a levee protect the facility during periods of flooding is under consideration. Data concerning the costs of construction and expected flood damage are shown below. The frequencies of river level reaching maximum height above normal in the last hundred years are also shown. Assuming the life of the levee to be 50 years, obtain the optimal height of the levels. (Rate of interest is 12 p.a.

p.t.o.

Feet (x)	No. of years river maximum level was x feet above normal	Damage if the river is x ft above level (in Rs.)	Construction cost of building x ft high (in Rs.)
0	48	0	0
5	24	5000	7000
10	16	10000	15000
15	6	20000	23000
20	4	40000	48000
25	2	60000	73000.

[8.0]

3. Consider an economy consisting of three order hierarchy of regions: national state and local. There are three industries only 1,2 and 3 which specialise in the production of national, state and local goods respectively. Goods are classified according to the size of their markets. Assume that the structure of production is same for all regions.
The final demand of each good in each region is given. As there are more than one state and local regions, distribution coefficients of national goods for different states and different local regions and those of state goods for different local regions are given. Indicate the steps for determining what goods and how much of them each region has to produce so that the production and demand of local goods are balanced within a local region, and that of state goods are balanced within a state region. However, the impact of changes in one local or state region is felt by all regions in the economy through changes in the output requirements of national goods for the production of state or local goods which may require input of national goods. [16]

4. A small company makes three different products 1,2,3. Each product requires work on two different machine types A,B. The shop has two varieties of type A machines which we shall designate by A_1, A_2 and three varieties of type B machines B_1, B_2 and B_3. Product 1 can be made on any variety of A machines, but must be processed on machines B_1 of type B. Finally product 3 can be made only on machine A_2 of type A and machine B_2 of type B. Product 1 can be produced in 6 different ways represented by the following combinations of machines (A_1, B_1), $(A_1\ B_2)$, (A_1, B_3), $(A_2\ B_1)$, $(A_2\ B_2)$, $(A_2\ B_3)$. Product 2 can be made in two ways represented by the combinations of machines (A_1, B_1), (A_2, B_1). Product 3 can be product in only one way i.e. (A_2, B_2) Given the informations below, obtain a

p.t.o.

Linear programming formulation for determining the optimum level of production of each of the products to be produced by each of the processes.

Time in minutes required by one unit of each product

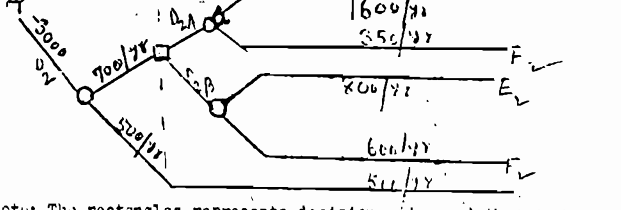
	Product			Total available time per week (minutes)	Cost at full capacity
Machine type	1	2	3		
A_1	10	15	-	8000	300
A_2	10	20	15	10000	321
B_1	12	15	-	4000	250
B_2	4	-	+1	70000	600
B_3	7	-	-	4000	200
Mat. cost	0.25	0.35	0.50		
Selling price	1.25	2.00	2.80		

[16]

5. Consider the structure of decision tree shown below reflecting possible alternative sequence of decision and the chance events covering two (decision point) periods. The forecasts of net revenues and costs are placed on the relevant branches of the tree. The length of each of the periods is 5 years.
When the possible events are $E_1 E_2$, $E_1 F_2$ and $F_1 F_2$, the estimated probabilities are

$$P(E_1E_2) = \frac{2}{5}, \quad P(E_1F_2) = \frac{1}{5} \quad \text{and} \quad P(FF_2) = \frac{2}{5}.$$

Obtain the optimal sequence of decision using the rolling back procedure.



Note: The rectangles represents decision nodes and the circles stand for chance nodes.

[16]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year
SEMESTRAL-I EXAMINATION
Sample Surveys

Date: 18.11.87 Maximum Marks : 100 Time: $3\frac{1}{2}$ hours

Note: Answer all questions. Marks allotted to each question are given in brackets( ).

1.(a) Define a 'sampling design' and a 'sampling scheme'.

(b) From a population of size N, one unit is drawn with probability of selection proportional to its size measure x. The rest of (n-1) units in the sample are selected from the remaining (N-1) units of the population by Simple Random Sampling With Out Replacement (SRSWOR). Show that, for this scheme, $\hat{R}=\bar{y}/\bar{x}$ is unbiased for $R=\bar{Y}/\bar{X}$, the ratio of population means of the study variable and the size variable.

(c) Calculate π_i, the probability of inclusion of the i th unit U_i for the design described in (b) above. (4+5+4)=(13)

2.(a) In stratified sampling, let C_0 be the overhead cost and C_i be the average cost per unit in the i th stratum, i=1,2,...,k. Derive an allocation of sample size to strata such that with a linear cost function of the form $C=C_0+\sum_{i=1}^{k} C_i n_i$, the variance of the estimated mean $\hat{\bar{Y}}_{st.}$ is minimum for a specified cost when SRSWOR scheme is used in all strata. What does this allocation reduce to when C_i's are equal ?

(b) Write down the 'combined and separate regression estimators' for the population total in stratified sampling. Which of these do you prefer? Give reasons. (7+11)=(18)

3.(a) Define the term 'Intra Cluster Correlation Coefficient'.

(b) A population consists of N clusters of varying sizes M_i, i=1,2,..,N. Suppose that n clusters are selected with probabilities proportional to the cluster size and with replacement (ppswr). Write down an unbiased estimator for the population mean $\bar{Y}$ and an unbiased estimator of its variance.

(c) In the above example, if simple random sampling with replacement design is used instead of ppswr, show how to obtain an approximate unbiased estimator for $\bar{Y}$ using the estimators $\sum_{i=1}^{n} M_i \bar{Y}_i / \sum_{i=1}^{n} M_i$ and $\frac{1}{n}\sum_{i=1}^{n} \bar{Y}_i$ (3+7+6)=(16)

p.t.o.

4. A sample survey was conducted to estimate the total household expenditure in an area. The design adopted was stratified two-stage one with census enumeration blocks as first stage units and households within them as second stage units. From the first stratum 4 blocks were selected from 40 blocks with probability poportional to population and with replacement and 4 households were selected from each selected block with equal probability and without replacement. From the second stratum, 6 blocks were selected by simple random sampling with replacement from the 72 blocks and 4 households are chosen from each selected block with equal probability and without replacement as before. The data on household expenditure for the sample households together with information on selection probabilities for the first stratum are given below:

Stratum	Sampled block	Total no. of households	Weekly household expenditure of sample households 1	2	3	4
I	1	189	110	281	120	114
	2	40	80	60	122	125
	3	135	122	210	171	105
	4	160	244	115	312	128

Inverses of probability of selection for sampled block i are 67.60, 338.12, 101.50, 69.03, i=1,2,3,4.

Stratum	Sampled block	Total no. of households	1	2	3	4
II	1	73	345	359	160	117
	2	26	97	179	144	85
	3	240	100	115	50	172
	4	14	102	40	126	148
	5	287	122	176	108	140
	6	257	125	110	134	215

(a) Obtain an unbiased estimate of the total weekly household expenditure.

(b) Obtain an unbiased estimate of the sampling variance of the above estimate. (22+18)=(40)

5.(a) In a recent survey of 205 households in an area selected by <u>SRS without replacement</u> it was found that in 124 households cricket matches were watched on the T.V. sets for more than 20 hours a week. Estimate the proportion of households that watched the matches for this duration in the population and obtain an unbiased estimate of its sampling error.

(b) Write down the sources of non-sampling error in the above survey. (8+5)=(13)

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[illegible]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.). III Year
SEMESTRAL-I EXAMINATION
Difference and Differential Equations

Date: 16.11.87 | Maximum Marks: 100 | Time: 3 hour

Note: Answer all the questions.

1. Answer (a), and any one of (b) and (c).

(a) Solve the following differential equations

(i) $(x + y)\frac{dy}{dx} + (x - y) = 0$

(ii) $3x(1 - x^2)\, y^2 \frac{dy}{dx} + (2x^2-1)y^3 = ax^3$

(b) Find the general solution of

$$y'' - 2y' + 5y = 25x^2 + 12$$

(c) Verify that one solution of

$$xy'' - (2x + 1)y' + (x + 1)y = 0$$

is given by $y = e^x$, and find the general solution. [5+5+8=18]

2.(a) Prove Kepler's first law of planetary motion : the orbit of each planet is an ellipse with the sun at one focus. What is the physical meaning of the eccentricity of the orbit?

2.(b) A boat is rowed with constant velocity u starting from a point A on the bank of a river, which flows with a constant velocity n u. If the boat points always towards a point B on the other bank exactly opposite to A, then show that the equation of the path of the boat is given by

$$\log r = - \log \cos \theta - \frac{1}{n}\log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + C,$$

where C is a constant. [9+9=18]

3.(a) Find two independent Frobenius series solutions of the differential equation

$$2x^2y'' + x(2x - 1)y' + y = 0$$

(b) Show that the general solution of the differential equation

$$(1 - x^2)y'' - xy' + p^2y = 0, \text{ p = nonnegative constant,}$$

near $x = 1$ is

$$y = C_1F\left(p, -p, \frac{1}{2}, \frac{1-x}{2}\right) + C_2\left(\frac{1-x}{2}\right)^{1/2} F\left(p+\frac{1}{2}, -p+\frac{1}{2}, \frac{3}{2}, \frac{1-x}{2}\right),$$

where F(a, b, c, x) denotes hypergeometric function. [10+8=18]

p.t.o.

4.(a) Establish the following formulas for Bessel functions

(i) $$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\,\sin x$$

(ii) $$J_0^2(x) + 2J_1^2(x) + 2J_2^2(x) + \dots = 1.$$

(b) Prove the Rodrigues' formula for Hermite polynomials

$$H_n(x) = (-1)^n\, e^{x^2} \frac{d^n}{dx^n} e^{-x^2}.$$ [8+7+8=23]

5.(a) Use Laplace transforms to show

$$\int_0^\infty \frac{\cos xt}{1+t^2}\,dt = \frac{\pi}{2}\cdot e^{-x}, \quad x > 0$$

(b) Find the inverse Laplace transform

$$L^{-1}\left[\frac{1}{(p^2+1)^2}\right]$$

(c) Solve the following integral equation

$$\phi(t) + \int_0^t (t-\xi)\,\phi(\xi)\,d\xi = \sin 2t.$$ [8+7+8=23]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1987-88

PERIODICAL EXAMINATION

Statistical Inference

Date: 11.9.1987 Maximum Marks: 75 Time: 3 hrs.

1. Either

(a) Suppose that the distribution of X is either U(0,1) or U(1,2). Find a non-trivial sufficient statistic (other than X). [8]

Or

(b) Let $X_1, \ldots, X_n$ be i.i.d. according to $N(\theta,1)$. Show directly (from the definition) that $\bar{X} = \frac{1}{n}\sum_1^n X_i$ is sufficient for θ $(-\infty < \theta < \infty)$. [8]

2. Let $X_1, \ldots, X_m$ be i.i.d. according to $N(\xi,\sigma^2)$ and $Y_1, \ldots, Y_n$ be i.i.d. according to $N(\eta,\tau^2)$; X_i's are independent of Y_j's. Find a minimal set of sufficient statistics in each of the following cases:

(i) ξ, η, $\sigma > 0$, $\tau > 0$ are arbitrary

(ii) $\sigma = \tau > 0$; ξ, η, σ are arbitrary

(iii) $\xi = \eta$; ξ, $\sigma > 0, \tau > 0$ are arbitrary.

For (ii), obtain the UMVUE of $(\xi - \eta)/\sigma$. [10]

3. Either

(a) Let $X_1, \ldots, X_n$ be a random sample from $U(0,\theta)$, $\theta > 0$. Obtain the MLE and UMVUE of θ. Consider estimates of the form $CX_{(n)}$ and find the one which has the smallest mean-squared error.

Or

(b) If $\hat{T}_1$ and $\hat{T}_2$ are UMVUE of $\psi(\theta)$ with finite variance, show that $\hat{T}_1 = \hat{T}_2$. [10]

4. Let X_1, X_2 be i.i.d. according to the c.d.f.

$$F(x,\theta) = 1 - e^{-\theta x}, \quad x > 0$$
$$= 0, \quad x < 0$$

$(\theta > 0)$. For a fixed $x > 0$, find the UMVUE of $1 - e^{-\theta x}$.

(Hint: Show that $X_1/(X_1+X_2)$ is independent of $X_1 + X_2$) [12]

p.t.o.

5. Either

(a) Let $X_1, X_2, \ldots, X_n$ be i.i.d. according to the p.d.f.

$$f(x,\theta) = \theta x^{\theta - 1}, \quad 0 < x < 1, \quad \theta > 0.$$

Using Cramer-Rao inequality show that

$$T(x) = -\sum_{1}^{n} \log X_i / n \quad \text{is a UMVUE of } 1/\theta.$$

(Hint: Find the moment generating function of X_1.)

[12]

Or

(b) A model that is often used for the time X to failure of an item is

$$P_\theta[X = k) = \theta^{k-1}(1-\theta), \; k = 1, 2, \ldots,$$

where $0 < \theta < 1$. Suppose that we only record the time of failure (in discrete periods), if failure occurs on or before time r and otherwise just note that the item has lived at least (r+1) periods. Thus we observe $Y_1, Y_2, \ldots, Y_n$ which are i.i.d.

(a) Find the probability function of Y_1.

(b) Show that the MLE of θ based on $Y_1, \ldots, Y_n$ is

$$\hat{\theta} = \left(\sum_{1}^{n} Y_i - n\right) / \left(\sum_{1}^{n} Y_i - m\right),$$

where m = number of indices i such that $Y_i = r + 1$.

[12]

6. Let $X_1, \ldots, X_n$ be i.i.d. according to Poisson (θ), $\theta > 0$.

(a) Find the UMVUE of $1 - e^{-\theta}$.

(b) Consider the following two estimates of $1 - e^{-\theta}$:

$$T_{1n} = \frac{\text{Number of } X_i\text{'s in } (X_1, \ldots, X_n) \text{ which are} \neq 0}{n}$$

$$T_{2n} = 1 - e^{-\bar{X}_n}, \quad \text{where } \bar{X}_n = \frac{1}{n}\sum_{1}^{n} X_i.$$

Obtain the ARE $e(\theta, T_1, T_2)$. [13]

7. Practical work and class performance. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1987-88

PERIODICAL EXAMINATION
Economic

Date: 9.9.87 Maximum Marks: 100 Time: 3 hours

Note: Answer ANY FOUR questions. All questions carry equal marks.

1.(a) Solve by the simplex method :

maximise $x_1 + 3x_2 + x_3$

subject to

$$2x_1 + x_2 + 2x_3 = 2$$
$$5x_1 + 3x_2 \leq 3$$
$$x_1 + 4x_2 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

[18]

(b) From the simplex tableau find also an optimal solution of the dual probelm.
[Hint: note that a unit vector (i.e., the first unit vector) is present in the system.] [7]

2. Consider a diet problem in which it is required to satisfy each of the nutrient requirements exactly (as equations rather than inequalities) and suppose a certain optimal diet uses (in positive quantities) only the foods $F_1, F_2, \ldots, F_k$. Now suppose the requirements are changed but can still be satisfied by some diet using the same set of foods. Prove that this new diet is automatically optimal. [25]
[Note: If you use the canonical equilibrium theorem you have to state and prove it. All other results can, however, be used without giving proofs.]

3.(a) Consider a standard minimisation problem, as follows: find an $x \geq 0$ such that cx is a minimum subject to $Ax \geq b$; (where c is a row rector, x and b are column vectors and A is a matrix). Consider now another row vector c^*, obtained from c by altering only the first component by $\lambda > 0$:

$c^* = c + \lambda e_1$, where e_1 is the first unit vector (row)

If $\bar{x} = (\bar{x}_j)$ is optimal for the original problem and $x^* = (x_j^*)$ is optimal for the altered problem, show that $x_1^* \leq \bar{x}_1$. Can you interpret this result in terms of an economic problem, say the diet problem? [10+7=17]

p.t.o.

(b) A function f of vectors c is called sub-additive, if $f(c_1 + c_2) \leq f(c_1) + f(c_2)$ for all (row)vectors c_1, c_2.

Let x^* be optimal for the following problem :
maximise cx, subject to $Ax \leq b$, $x \geq 0$.
The optimal value, cx^*, can be taken as a function of c.
Show that the optimal value of a standard maximum problem is a sub-additive function of the vector c. [8]

4.(a) Consider a system of simultaneous linear equations. Define a basic solution, a basic degenerate solution and a basic non-negative solution for this system. [4+3+1=8]

(b) Prove that whenever the system has a non-negative solution, it has also a basic non-negative solution. [17]

5.(a) Consider a canonical minimisation problem :
min $c'x$ subject to $Ax = b$, $x \geq 0$. Suppose in the process of solving this problem by the simplex method, one gets a tableau in which $z_k - c_k > 0$ for some a_k not in the current basis and $t_{ik} \leq 0$ for all i, i being an index for the ith basis vector; (all other symbols have their usual meanings).
Show that the given problem does not have any optimal solution. [15]

(b) Construct an example of a LP problem whose dual has a feasible solution, but which itself does not have any feasible solution. [10]

6. A scientist in ISI is conducting a sample survey on monthly household expenditures on services. A sample has been taken for each of the three types of households : h1, h2, h3 and per day at least 2,3 and 5 schedules are to be filled in for these three types of households, respectively.
Households can be investigated only in the morning or the afternoon of a day and accordingly, the scientist has engaged two separate teams of investigators to interview the households in these two sessions per day. The numbers of 'morning' and 'evening' investigators engaged are 4 and 7, respectively and an investigator of either team can interview only one household (i.e., fill in only one schedule) per day.
Let c_{ij} be the remuneration (in Rs.) to be paid to the ith team of investigators for filling in one schedule for the jth type of households and let the c_{ij} matrix be given as follows:

team of investigators	type of households		
	h1	h2	h3
morning	1	2	3
evening	2	4	6

p.t.o.

(a) Formulate a LP problem so as to minimise the total cost of filling in the schedules, subject to the constraints mentioned above. [12]

(b) Formulate the dual to the problem in (a). Try to interpret this dual problem. [8+5=13]

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PERIODICAL EXAMINATION
Physical and Earth Sciences

9.9.87 Maximum Marks: 100 Time: 3 hours

Note: Attempt Question No.1 and any five from the rest.

1. Fill up the blanks (any 10). Only write down one of the four choices for each blank.

(i) The overall density of the earth is _____ (4.5/5.0/5.5/6.0) gm/cc.

(ii) When a sea invades the land, it is called _______ (regression/transgression/onlap/overlap).

(iii) The dinosaur bones in the Indian Statistical Institute's geology museum have been obtained from continental rocks of __________ (Jurassic/Cambrian/Cenozoic /Palaeozoic) age.

(iv) Water may be considered to be a ______ (rock/mineral/Organic substance/crystal).

(v) The oldest evidence of life is obtained in southern Zimbabwe from a limestone which contains structures of ______ (tectonic/geological/organic/inorganic) origin.

(vi) A horizontal sequence of rock layers lying over a tilted sequence of rock layers has a/an ________ (metamorphic/sedimentary/unconformable/hypabyssal) contact.

(vii) Ca is an important element in ______ (magnetite/dolomite/tourmaline/agate).

(viii) Copper is extracted from a mineral known as _______ (goethite/pyrite/chalcopyrite/beryl).

(ix) Conglomerate is a sedimentary rock which consists of _________ (pebbles only/pebbles and matrix/matrix only/cement only)

(x) A crystal form which has all its faces well - developed is called ________ (anhedral/interlocking/subhedral/euhedral).

(xi) Common slate is a _________ (metamorphic rock/igneous rock/mineral/fossil).

(xii) Mudstone is a sedimentary rock in which there are more than __________ (15% / 20% / 30% / 40% / 50%) by volume of clasts of clay-silt size. [2x10=20]

2. Who is said to be the pioneer in suggesting that the solar system had a cold beginning? Who modified his hypothesis and how did the modifier explain the origin of the solar system? [2+2+12=16]

3. Can a mineral be a crystal and a crystalline or an amorphous substance?
How does a mineral form? Name three common rock-forming minerals.
What is a Gypsum? What is its usefulness? [6+4+3+1+2=16]

p.t.o.

4. What is understood by the Moh's scale of hardness? Describe the various ways you would use to determine the hardness of Magnetite (Fe_2O_3) which has a hardness ranging from 5.5. to 6.5. [6+10=16]

5. Describe the uses of the seismic wave study in an earthquake. What is the Low Velocity Zone? Does it occur above the Mohorovicic discontinuity (Moho)?
Describe in short the physical characteristics of the earth's crust. [6+2+2+6=16]

6. In a deep mine, it may be observed that with every 30 meter descent, there is an increase in temperature through 1°C. What is then the actual temperature at the earth's core?
What is the reason for the higher heat flow in the sialic crust?
Which hypothesis, hot earth or cold earth origin, supports the heat flow of the earth's interior? Why? [5+5+1+5=16]

7. What is the Geological Time-Scale? What is meant by the Palaeozoic time? What is the geological age of India's Gondwana coal and that of India's petroleum?
When did the first flowering plants appear in the earth?
Name the first bird to appear in the sky of the earth. [5+4+3+2+2=16]

8. What is a plutonic rock? What are its characteristics? Give an example of a plutonic rock and describe its major mineral constituents. [4+6+2+4=16]

9. Describe the ideas basing on which radiometric age determination of rocks is made.
Describe the K - Ar method employed in the determination of the age of a sedimentary rock. [8+8=16]

10. Write short notes (choose any three):
Earth as a Dynamo ; Primitive Hydrosphere ; Coal; Metamorphism of Mudstone ; Clastic Sedimentary Rock. [3x5+1=16]

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INDIAN STATISTICAL INSTITUTE
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PERIODICAL EXAMINATION

Stochastic Processes - 2

Date: 7.9.1987 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can.

1. Let $X(t)$, $t \geq 0$ be a homogeneous Poisson process with rate λ.

   (a) Calculate $P[X(s) = k \mid X(t) = n]$ for $0 < s < t$ and $k = 0, 1, 2, \ldots, n$.

   (b) Calculate $E[X(t).X(t+s)]$. (5+5) = [10]

2. Bus loads of passengers arrive at a bus terminus. Buses arrive in the pattern of a Poisson process at the rate λ. A bus contains j passengers with probability α_j $j = 1, 2, \ldots$. Let $X(t)$ denote the number of passengers that have arrived by time t.

   (a) Find $E[X(t)]$.

   (b) Find the generating function of $X(t)$.

   (7+8) = [15]

3. Define a non-homogeneous Poisson process $X(t)$, $t \geq 0$ with intensity function $\lambda(t)$, $t \geq 0$. Find the probability distribution of $X(t)$.

   (5+15) = [20]

4. Let $X(t)$, $t \geq 0$ be a pure birth process with birth rates α_i, where

$$\alpha_{2k+1} = \lambda_1 \quad \text{for } k = 0, 1, 2, \ldots$$
$$\alpha_{2k} = \lambda_2 \quad \text{for } k = 0, 1, 2, \ldots$$

Take $X(0) = 1$. Let $P_1(t) = P[X(t) = \text{odd}]$
and $P_2(t) = P[X(t) = \text{even}]$

Contd..... 2/-

Contd..... Q.No.4

(a) Derive the differential equation

$$P_1'(t) = -\lambda_1 P_1(t) + \lambda_2 P_2(t)$$
$$P_2'(t) = \lambda_1 P_1(t) - \lambda_2 P_2(t).$$

(b) Find $P_1(t)$, $P_2(t)$.

(8+12) = [20]

5. A telephone exchange has m channels. Calls arrive in the pattern of a Poisson process with parameter λ ; they are accepted if there is an empty channel otherwise they are lost. The duration of each call is a random variable whose distribution is exponential with parameter μ. The life times of separate calls are independent random variables. Find the stationary probabilities for the number of busy channels.

[15]

6. We have 2N balls labelled 1, 2, ..., 2N distributed in 2 boxes. A ball in box number i remains in that box for a random length of time that is exponentially distributed with parameters λ_i before going to the other box, $i = 0, 1$. The balls act independently of each other. Let $X(t)$ denote the number of balls in box-1 at time t, $t \geq 0$.

(a) Find the infinitisimal parameters for the process $X(t)$.

(b) Find $P_{i,o}(t) = P[X(t) = 0 \mid X(0) = i]$ for $i = 0, 1, \ldots, 2N$.

(c) Find $E[X(t) \mid X(0) = i]$.

(5+15+10) = [30]

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III Year
PERIODICAL EXAMINATION

Sample Surveys

2.9.87 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions

1.(a) For a probability proportional to size (p.p.s.) sampling design of n draws with replacement, (w.r.) obtain expressions for the probability of inclusion of a unit and joint probability of inclusion of a pair of units. [2+3=5]

(b) Explain under what circumstances p.p.s. sampling design (w.r.) will be better than a simple random sampling design (w.r.) [6]

(c) Show that for Lahiri's method of selection, the probability of selection of a unit is proportional to the size measure of that unit. [9]

2.(a) What are the advantages of stratified sampling? [4]

(b) If the cost function for a survey is of the form

$$C = C_0 + \sum_{i=1}^{k} t_i (n_i)^{\frac{1}{2}},$$

where C is the total cost and C_0 and t_i are known, obtain an allocation n_i to the strata which minimizes $V(\hat{\bar{Y}}_{st})$ for a fixed total cost where $\hat{\bar{Y}}_{st}$ is the unbiased estimator of the population mean on the basis of a stratified srs without replacement design. [10]

(c) A population of size 78 is divided into two strata of sizes 24 and 54 respectively. From the first stratum a probability proportional to size sample of size 4 is selected with replacement and the data on the study variable y and the auxiliary variable x is found to be as follows:

x :	124	476	98	216
y :	612	2131	499	1210.

p.t.o.

From the second stratum two independent circular systemetic samples of size 6 are selected and the data on y is as follows:

y - values

sample 1 :	212	144	200	189	196	187
sample 2 :	201	164	192	194	179	192.

It is also known that the total of x - values in stratum 1 is 6124.

(i) Estimate the population mean unbiasedly. [14]

(ii) Obtain an unbiased estimate of the variance of your estimate in (i) above. [22]

3.(a) Explain what you understand by 'intra-class correlation coefficient' ρ. [4]

(b) Obtain the variance of the estimate of the population mean in linear systematic sampling in term of ρ [assume n divides N]. [7]

(c) Explain why the variance of an estimator of the population mean based on a single systematic sample is not estimable. (give a brief answer) [3]

(d) Suppose that n divides N and the values on the study variate for the sample are $y_1, y_2 \cdots, y_n$ selected using linear systematic sampling method. Show that a biased alternate estimator for the $V(\hat{\bar{Y}})$ is given by

$$\frac{N-n}{Nn}\left[\sum_{i=1}^{n-1}(y_{i+1}-y_i)^2 / 2(n-1)\right] \qquad [6]$$

4. A simple random sample of size 3 is drawn from a population of size 4 with replacement. Show that the probabilities that the sample contains d distinct units d = 1,2,3 are $P_1 = 1/16$, $P_2 = 9/16$ and $P_3 = 3/8$ respectively. Show that the variance of $\bar{y}'$ is equal to $\frac{7}{32} S^2$ where $\bar{y}'$ is the sample mean over distinct units and $S^2 = \frac{1}{3}\sum_{i=1}^{4}(Y_i-\bar{Y})^2$ and compare this with $\bar{y}$, the conventional estimator of $\bar{Y}$, the population mean. [2+5+3=10]

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INDIAN STATISTICAL INSTITUTE
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PERIODICAL EXAMINATION
Difference and Differential Equations

31.8.87 | Maximum Marks: 100 | Time: 3 hours

Note: Answer all the questions

1. Solve any FOUR of the following first order equations.

(a) $x \sin\frac{y}{x} \cdot \frac{dy}{dx} = y \sin\frac{y}{x} + x$

(b) $(3y + 2x + 4)\,dx - (4x + 6y + 5)\,dy = 0$

(c) $(x^2y^3 + xy)\,dy = dx$

(d) $(\sin x \sin y - x e^y)\,dy = (e^y + \cos x \cos y)\,dx$

(e) $y(x^2+y^2+a^2)\frac{dy}{dx} + x(x^2+y^2-a^2) = 0$ ... [4x6=24]

2.(a) Show that there is exactly one function f, continuous on the positive axis, such that

$$f(x) = 1 + \frac{1}{x}\int_1^x f(t)\,dt \text{ for all } x > 0,$$

and find this function.

(b) Solve the following as linear equation

$$(1+x^2)\,dy + 2xy\,dx = \cot x\,dx$$

(c) Solve the initial-value problem

$$x\frac{dy}{dx} - 2y = 4x^3y^{1/2} \quad \text{on } (-\infty, +\infty)$$

with $y = 0$ when $x = 1$. [8+6+6=20]

3.(a) Find general and singular solutions of

$$y = xp + \sqrt{b^2+a^2p^2}, \quad p = dy/dx.$$

(b) Solve by the method of reduction of order

$$x^2y'' = 2xy' + (y')^2.$$

contd. 2/-

(c) Find the equation of the curve which cuts at a constant angle $\tan^{-1}\frac{m}{n}$ all the circles touching the y - axis at the origin.

Hence show that the orthogonal trajectories of the family of circles are given by

$$x^2 + y^2 = cy. \qquad [6+6+10=22]$$

4.(a) Solve the linear equations

(i) $(D^2 - 5D + 4)y = x^2 - 2x + 1$

(ii) $\frac{d^4y}{dx^4} + 2n^2 \frac{d^2y}{dx^2} + n^4 y = \cos mx$

(iii) $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2 \cos x$

(b) Find the general solution of any ONE of the following equations

(i) $(x^2 + x)y'' + (2-x^2)y' - (2 + x)y = x(x + 1)^2$

(ii) $(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2.$

[3x8+10=34]

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