INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Backpaper Examination
Design of Experiments

Date: 5.7.89 Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours.

Note: Answer any four of questions 1-5. Marks allotted to each question are given within parentheses.

1.(a) What is a treatment contrast? When are two such contrasts said to be orthogonal? Show that in a 2^4-experiment the main effects and interaction effects are mutually orthogonal. How would you obtain the SS due to a main effect or interaction effect in a 2^4-experiment?

(b) Below is given an incomplete Key-block of a 2^4 factorial experiment conducted in two 8-plot blocks:-

Incomplete Key-block : (1) ac bc acd ... .

Search out the other 4 treatment combinations for the Key-block and also the confounded interaction. (12+8)=[20]

2. Writing R, C and E for the row M.S., column M.S. and error M.S., respectively of an m x m Latin square design prove that (i) an estimate of the error variance which would have been obtained if the row classification had not been made is

$$\frac{R + (m-1) E}{m}$$

and (ii) if the design had been completely randomised the same is

$$\frac{R + C + (m-1) E}{m + 1}.$$

Hence estimate the efficiency of the Latin square relative to (i) randomised blocks made up by columns and (ii) unrestricted randomisation. (8+8+2+2) = [20]

3.(a) What is a split-plot design? Why is it said that this design confounds main effects? Give the analysis of this design when the whole-plot treatments are arranged into a p x p Latin square.

(b) Obtain the estimates of standard errors for different types of treatment comparison.

(2+2+10+6) = [20]

p.t.o.

4. The following table gives the plan and the yields of a 2^4 field experiment on beans. The yields are given in lbs and the manurial factors were as follows:-

Dung (D) : 0, 1 fon/acre, Nitrochalk (N) : 0, 0.4 cwt N/acre

Super-phosphate (P) : 0, 0.6 cwt P_2O_5 per acre

Muriate of Potash (K) : 0, 1.0 cwt K_2O per acre

	Block I				Block II			
	p	k	d	npk	dp	nk	dk	pk
	46	56	54	37	50	44	43	51
Replication 1	dnk	dnp	dpk	n	dnpk	(1)	dn	np
	41	48	55	41	43	57	40	49
	Block III				**Block IV**			
	npk	d	p	dnk	nk	dp	(1)	np
	42	41	33	34	44	53	58	40
Replication 2	n	dnp	k	dpk	pk	dk	dnpk	dn
	48	53	51	45	56	52	54	42

Analyse the data and interpret your results. [20]

5. An experiment on sugar-cane conducted in four randomised blocks using plots of size 37'x12' gave the following values of number of plants per plot (x) and weight of cane in Kg. (y).

The three treatments used were:

Nitrogen - 350 lbs./acre as ammonium sulphate - N,

Phosphorous - 450 lbs./acre as superphosphate - P,

Potash - 150 lbs./acre as sulphate of potash - K.

Treatment

Block	N x	N y	P x	P y	K x	K y
1	41	122	41	81	42	80
2	40	120	50	80	38	82
3	38	138	46	79	54	65
4	41	121	42	75	40	58

Analyse the data to find out whether the treatments had any effect, after eliminating the dependence of y on x. In case of significance judge the treatments pairwise to obtain the best treatment.

Is the measurement of concomitant variable worth while? (10+6+4)=[2

6. Practical Assignment. [2

INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Backpaper Examination
Optimization Techniques

Date: 3.7.89 | Maximum Marks: 100 | Time: $2\frac{1}{2}$ Hours

Note: Answer all questions

1.(a) Write the dual of the LP: max $c^T x$ subject to $Ax=b$, $x \geq 0$. State the duality theorem for this LP and its dual. [12]

(b) Prove that the system "$Dx=d$, $x \geq 0$" has a solution iff

$$D^T y \geq 0 \Rightarrow d^T y \geq 0. \quad [12]$$

2. Assuming the max-flow min-cut theorem, prove that in any (0,1)-matrix the maximum number of independent 1's equals the minimum number of lines (rows and columns) covering all the 1's. [22]

3. Consider the linear program

$$\min \ \eta$$

subject to $Ay - \underset{\sim}{1}\eta \leq 0$, $\underset{\sim}{1}^T y = 1$, $y \geq 0$,

where

$$A = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 1 & -1 & 0 & 3 \\ 2 & 1 & -3 & 4 \end{bmatrix}$$

and $\underset{\sim}{1}$ denotes a vector of all 1's with appropriate order.

(a) Convert the above LP to the standard form and solve it by the 2-phase simplex method. [40]

(b) Write the dual of the above LP. If y_0 and η_0 form an optimal solution of the primal, show that y_0 is an optimal mixed strategy for the column player and η_0 is the value of the game for the matrix game with pay-off matrix A. [14]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1988-89
Semestral-II Back-paper Examination
Nonparametric and Sequential Methods

Date: 30.6.89 Maximum Marks: 100 Time: 3 Hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Define the Wilcoxon rank-sum statistic W_s for the 2-sample problem. Find $E(W_s)$ and Var (W_s) under the null hypothesis. Show that the null distribution of W_s is symmetric about its mean. By using a suitable normal approximation compute approximately

$$P(W_s \leq 240)$$

under the null hypothesis when $m = n = 20$. [3+(2+5)+6+4]

2. In a study of the comparative tensile strength of tape-closed and sutured wonnds, the following results were obtained on 10 rats, 40 days after incisions made on their backs had been closed by suture or by surgical tape.

Tensile strength

Rat	1	2	3	4	5	6	7	8	9	10
Tape	659	984	397	574	447	479	676	761	647	577
Suture	452	587	460	787	351	277	234	516	577	513

Use a Wilcoxon-signed-rank-sum test to test the hypothesis of no difference against the alternative that tape closed wonnds are stronger, at level of significance $\alpha=0.05$. Find significance probability or the P-value of the observed data. [20]

3.(a) Define the Kolmogorov-Smirnov statistic $D_{m,n}$ for the 2-sample problem. Calculate $D_{m,n}$ for the sample arrangement XXYYYX. Find $P[D_{3,3} \geq d]$ under the null hypothesis F=G where d is the observed value of $D_{3,3}$ for XXYYYX by actual enumeration.

(b) Define the Siegel-Tukey statistic for the 2-sample problem. Describe the situation when it is more appropriate to be used than the Wilcoxon statistic. [(5+5+10)+10]

4.(a) Describe Wald's sequential probability ratio test of a simple hypothesis against a simple alternative. [5]

(b) Derive the approximate equations between the constants A,B and theprobabilities of error α,β of SPRT(A,B) for $A < 1 < B$. [10]

contd. 2/-

4.(c) For testing $H : \mu = 0$ against $K : \mu = 10$ using SPRT(A,B) on the basis of observations from $N(\mu,\sigma^2)$, $\sigma^2=100$, find A,B from Wald's approximations when $\alpha=\beta=0.05$. Compute the power function and the ASN function of this SPRT as a function of μ, for $\mu=10, 15, 20$. [5+15]

5. Describe how one can use Stein's 2-stage sampling procedure for obtaining a confidence interval of length less than or equal to 2d (a given number) for the mean μ of a normal population with unknown mean μ and unknown variance σ^2. [10

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Examination
Elective-5: Economics

Date: 12.5.89 Maximum Marks: 100 Time: 3 Hours

Note: This question paper carries a total of 130 marks. You can answer any part of any question; but the maximum you can score is 100. Marks allotted to each question are given in brackets.

1.(a) Discuss the general properties of a Lorenz curve. [8]

(b) Explain clearly the significance of Lorenz curve comparison of different income profiles. [6]

(c) Obtain the expression for the Lorenz ratio of a two-parameter lognormal distribution. [6]

2.(a) Discuss briefly the various criteria used for choosing a suitable form for the Engel curve for a given set of cross-section data on consumer expenditure. [15]

(b) In the context of estimating a demand function for a commodity with an aggregate market transaction time-series data, discuss the following statement.
"The identification problem is concerned with the question of whether any specific equation in a model can in fact be estimated. It is not a question of the method of estimation nor of sample size; but of whether meaningful estimates of structural coefficients can be estimated." [10]

3. Write short notes on any two of the following.

(a) Use of dummy variables in estimating demand functions.

(b) Measurement of poverty.

(c) Economy of scale in household consumptions. (10x2)=[20]

4.(a) A single-product firm produces output with two inputs, viz., capital and labour following a production function which is homogeneous of degree one in the input quantities. Also, the firm faces competetive markets for its product and the inputs. Show that if the factor shares are constant irrespective of the levels of input and output prices, then the firm's production function is of Cobb-Douglas form.

(b) Write down the properties of Cobb-Douglas production function.

(c) Write briefly about the rationale and the major problems of estimating a Cobb-Douglas production function for an economy as a whole. (10+10+10)=[30]

p.t.o.

5. The following table is based on a budget enquiry. At the time of analysis, the sample households were ranked in ascending order of per capita income and then grouped in such a manner that each group included 20 per cent of the estimated population.

Quantile group from bottom	Average per person per month	
	Income (Rs.)	Expenditure on education(Rs.)
1st	137	5
2nd	216	16
3rd	372	37
4th	528	68
5th	923	152

Fit a semi-logarithmic Engel curve $y = \alpha + \beta \log x$ by appropriate least squares method, taking x as the per capita income and y the per capita expenditure on education. Then compute the Engel elasticity at x = Rs.500 using the fitted curve. [25]

6. Practical Records. [10]

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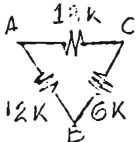
INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Examination
Elective-5: Physical and Earth Sciences

Date: 12.5.89 | Maximum Marks: 100 | Time: 3 Hours

Note: All questions carry equal marks. You can attempt all questions but maximum score will be 100.
Draw the necessary figures, circuit diagrams, graphs and clearly workout the calculations.

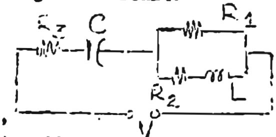
1. Write down the <u>methods</u> used in interconnecting 3-phase circuits. Derive the transformation of one method to other and vice-versa. Sketch the necessary diagrams neatly. [1+5+4+2=12]



A network connection ABC with resistances of 18KΩ, 12KΩ and 6KΩ across CA, AB and BC are given. Now across A the + ve pole of 24^V battery, across B and C resistances of 10KΩ and 9KΩ are joined. The free ends of these resistances and the negative pole of the battery are earthened. Find out the input current and output voltage across 9K. Sketch all necessary diagrams. [1+3+2.5+1.5=8]

2. Derive expressions for average and r.m.s values of the instanteneous alternating current i = Im Sin θ. Find out the amplitude factor Ka and state its importance. [3+2+2=7]

Write down the step by step procedure of getting overall impedance and the input current flowing through the given circuit. [6]



If R_2 and R_3 are withdrawn and R_1=50Ω, L=0.2H and C is made to vary with given supply of 200^V, $50H_z$, find out the value of the capacitor C at unity power factor and also the capacitive voltage at that condition. [7]

3. A realistic coil (having some industance L and resistance R and a variable pure capacitance C connected to a supply voltage of V∠0^o. Consider the following combinations-(a) all in series, (b) all are parallel, compute the following: (i) condition for resonance(series/ parallel) (ii) current at resource, (iii) equivalent impedance, (iv) resonant frequency, (v) nature of the resonance and (vi) Q-factor. Also state the reasons for the special name of the circuit with respect to resonant frequency. Draw the necessary circuit diagrams. [1+2+7x1]x2=20

p.t.o.

4. Clearly explain the purposes of inserting an additional third electr in the space-charge region of a triode. Define the different triode tube co-efficient along with their inter-relationships. [4+4=8]
Explain the principle used in operating a triode as an amplifier. Draw the circuit diagrams with all the signal wave forms you have come-across in implementing this operation. [1+5+2=8]

Distinguish between class A and class B amplifiers in respect of grid biasing, operating point, plate current flow and efficiency.

5. How does tho depletion layer form in a P-N junction and what electrical behaviours it possesses in normal condition? Clearly explain with suitable diagrams and graphs the nature of current flows with the majority and minority carriers when this depletion layer is (i) forward biased and (ii) reverse biased. How does the reverse-saturation current originate? Draw the combined V/I characteristic of a P-N junction. [3+4+4+2+1=14]

How does an N-P-N transistor form? Name the different types of circuit configurations to be used. Draw these configurations using a P-N-P transistor. [2+1+3=6]

6. Using resistance and capacitance, form (a) the differentiating and (b) the integrating circuit. In each case draw the circuit diagram. Mentioning the necessary conditions, derive the output voltage and its form when a square wave type voltage source in applied at the input. [(1+1+3+1)x2=12]

How does a zener diode operate? Using a zener diode draw the circuit diagram of a voltage regulator for fluctuating input voltage (dc). Explain its operation. Find the value of series resistance R_s. [2+1+4+1=8]

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year-1988-89
Semestral-II Examination
Optimization Techniques

Date: 8.5.89 Maximum Marks: 100 Time: 3 Hours

Note: Answer all questions. The paper carries 115 marks but the maximum you can score is 100.

1. Consider a transportation problem with the cost matrix

$$\begin{bmatrix} 40 & 25 & 20 & 20 & 50 \\ 80 & 30 & 60 & 70 & 60 \\ 60 & 10 & 35 & 50 & 60 \end{bmatrix} \begin{matrix} 5 \\ 5 \\ 7 \end{matrix}$$
$$\begin{matrix} 3 & 6 & 2 & 4 & 2 \end{matrix}$$

where supplies and demands are given in the margins.

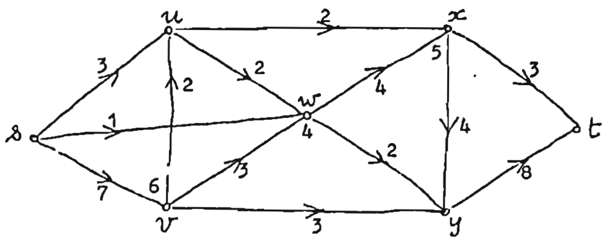
(a) Find a basic feasible solution using Vogel's method. [8]

(b) Starting with the basic feasible solution obtained in (a), obtain an optimal transportation schedule. [6]

2. (a) Assuming the existence of a maximum flow, prove the max-flow min-cut theorem. [15]

(b) If $(S_1, \bar{S}_1)$ and $(S_2, \bar{S}_2)$ are minimum cuts in a network, prove that $(S_1 \cap S_2, \bar{S}_1 \cup \bar{S}_2)$ is also a minimum cut. [6]

(c) Find a maximum flow in the following network. Here the capacities of the arcs and the capacities of the vertices v, w, x are given alongside. [18]



3. (a) Assuming the max-flow min-cut theorem, prove P.Hall's theorem on SDR's. [12]

(b) Show that if $S_1, S_2, \ldots, S_8$ are 3-element subsets of $X = \{1, 2, \ldots, 10\}$ and each element of X belongs to at most three S_i's, then $S_1, \ldots, S_8$ have an SDR. [6]

p.t.o.

4. (a) Find optimum pure strategies separately for the two players in the game with pay-off matrix

$$\begin{pmatrix} 2 & 1 & 3 & -2 \\ 1 & -1 & 0 & 3 \\ 2 & 1 & -3 & 4 \end{pmatrix}$$

Does the game have a solution in pure strategies? Why? [8]

(b) Find a solution in mixed strategies for the game in (a). [20]

(c) Prove that if the row player of a matrix game has 4 pure strategies and the column player has 7 pure strategies, then the column player has an optimal mixed strategy in which at least 3 of the pure strategies receive probability zero. [6]

5. Assignment. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Examination
Nonparametric and Sequential Methods

Date: 5.5.89 Maximum Marks: 100 Time: 3 Hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Consider a linear rank sum statistic of the form $T = \sum_{1}^{m+n} a_i U_i$ for the 2-sample problem where U_i is a random variable taking the values 0 and 1 according as the ith order statistic in the combined sample is an X or a Y. Compute the Expectation and the variance of T under the null hypothesis. Show that the null distribution of T is symmetric about its mean if

either $m = n$

or $a_i + a_{(m+n)-i+1}$ = constant for $i=1,2,\ldots, m+n$.

[(3+5)+12]

2. In a business administration course a set of lectures was given televised to one group and live to another. In each case an examination was given prior to the lectures and immediately following them. The difference between the 2 examination scores for the 2 groups were as follows.

Live : 20.3, 23.5, 4.7, 21.9, 15.6, 20.3, 26.6, 21.9, -9.4, 4.7, -1.6, 25.0.

TV : -6.2, 15.6, 25.0, 4.7, 28.1, 17.2, 14.1, 31.2, 12.6, 9.4, 17.2, 13.4.

(a) Use the one-sided Wilcoxon test to test the hypothesis of no difference against the alternative that the Televised lectures give worse performance than live ones, at significance level $\alpha = 0.05$. (use normal approximations)

(b) Assuming that the difference of examination scores is Normal with $\sigma^2=64$ for $m=n=12$ find the approximate power of the above Wilcoxon test for the shift model if televised lectures decreases the examination scores by 5 i.e. $\Delta=-5$. [20+10]

3.(a) Define the Wilcoxon signed-rank-sum statistic V_s for the one sample problem. Find $E(V_s)$ and Va (V_s) under the null-hypothesis

(b) Find the exact null distribution of V_s when N=4, by complete enumeration. [(3+2+5)+10]

4.(a) Define the normal scores test for the 2-sample problem.

(b) Define the Kolmogorov-Smirnor statistic $D_{m,n}$ for the 2-sample problem. Calculate $D_{m,n}$ for the sample arrangement XXYXXYY.

[5+(3+7)]

p.t.o.

5.(a) Let $X_1, X_2, \ldots$ be a sequence of Bernoulli random variables with constant probability p of success. Consider the problem of test $H : p = 1/3$ against $K : p = 2/3$ by using SPRT(A, B) where $\log A = -a \log 2$ and $\log B = b \log 2$, a, b being positive integers.

(i) Show that the Wald approximations $\alpha = \frac{1 - A}{B - A}$ and $\beta = \frac{A(B-1)}{B-A}$ become exact equalities in this case

(ii) Compute the power function $\pi(p)$ and ASN function $E(N \quad p)$ for this SPRT. [6+12]

6. Describe how one can use Stein's 2-stage sampling procedure for obtaining an unbiased estimate of the mean μ of a normal populati[on] $N(\mu, \sigma^2)$ such that the variance of the estimate is less than a pre-assigned number > 0. [1(

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year (1988-89)
Semestral-II Examination
Design of Experiments

Date: 2.5.89 | Maximum Marks: 100 | Time: $3\frac{1}{2}$ Hours

Note: Answer any four of questions 1-5. Marks allotted to each question are given within parentheses.

1.(a) Calculate the amount of information contained in various types of 2^n-experiment in $4r$ replications.

(b) Estimate the efficiency of split-plot design relative to randomised blocks. Give an interpretation of the split-plot design which brings out its similarity with a confounded experiment.

(c) Suggest an unbiased estimator of the whole-plot error variance in case of a split-plot design. (8+7+2+3)=[20]

2.(a) Show that in the analysis of covariance for an RBD the sum of squares due to the hypothesis that there are no treatment effects can be written as

$$\left(T_{yy} - \frac{T_{xy}^2}{T_{xx}}\right) + \left(\frac{T_{xy}}{T_{xx}} - \frac{E_{xy}}{E_{xx}}\right)^2 \frac{T_{xx}E_{xx}}{E_{xx}}$$

where the symbols carry their usual meanings.
Interpret the significance of the two components.

(b) Estimate the average value of the variance of the difference between two estimated treatment effects.

(c) How would you examine whether the use of the concomitant variable is worthwhile or not? (10+5+5) = [20]

3.(a) Construct a 2^4 factorial experiment in 4 replicates (including randomisation) each of 2 randomised blocks, confounding all the second order interactions.

(b) Consider a 3^3 factorial experiment involving 3 factors A, B and C each at the levels 0, 1 and 2. Construct two replicates of this experiment in blocks of 9 plots each confounding the interactions AB^2C and AB^2C^2 respectively. (10+10) = [20]

4. For a factorial experiment with three factors N, P and K, each at two levels, the design and yield per plot (in a suitable unit) are given below.

Square-I				Square-II			
(1)	n	pk	npk	(1)	p	npk	nk
25	30	24	36	24	26	30	28
np	p	nk	k	np	n	k	pk
30	27	32	32	36	28	32	20
nk	k	np	p	npk	nk	(1)	p
34	39	30	32	45	41	34	29
pk	npk	(1)	n	k	pk	np	n
36	42	44	46	35	39	32	41

Identify the confounded interactions and analyse the experiment.
(8+12)=[20]

p.t.o.

5. A variety - manurial experiment was conducted with 4 varieties V_1, V_2, V_3 and V_4 as whole-plot treatments and 3 manures M_1, M_2 and M_3 as sub-plot treatments. The plan and yield (in a suitable unit) are shown below where whole plots are demarcated by double lines.

(a) Analyse the data to find out if there are any effects due to manure, variety or interaction between variety and manure after identifying the design according to which the whole-plot treatments are arranged.

(b) Estimate the standard errors for different types of treatment comparison.

V_1M_1	V_1M_2	V_1M_3	V_3M_2	V_3M_1	V_3M_3	V_2M_2	V_2M_1	V_2M_3	V_4M_2	V_4M_1	V_4M_3
94	147	112	248	220	218	250	147	297	275	110	440
V_3M_1	V_3M_2	V_3M_3	V_4M_1	V_4M_2	V_4M_3	V_1M_2	V_1M_1	V_1M_3	V_2M_2	V_2M_1	V_2M_3
124	265	340	95	290	370	135	71	124	180	160	340
V_2M_1	V_2M_2	V_2M_3	V_1M_1	V_1M_2	V_1M_3	V_4M_1	V_4M_2	V_4M_3	V_3M_1	V_3M_2	V_3M_3
145	220	235	78	155	115	130	262	483	135	196	260
V_4M_1	V_4M_3	V_4M_2	V_2M_1	V_2M_3	V_2M_2	V_3M_2	V_3M_3	V_3M_1	V_1M_2	V_1M_3	V_1M_1
175	323	450	175	246	250	296	191	114	122	145	81

(12+8)=[2

6. Practical Assignments. [2

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Semestral-II Examination
Multivariate Distributions and Tests

Date: 28.4.89 Maximum Marks: 100 Time: 3 Hours.

Note: Answer any four questions. Questions carry equal marks.

1. Let: $\overset{p\times 1}{Y} \longrightarrow N_p(\mu, \Sigma)$.

(a) If $Y = \begin{pmatrix} Y_1^{p_1 \times 1} \\ Y_2^{p_2 \times 1} \end{pmatrix}$. Find the conditional distribution of Y_2 given Y_1.

(b) Show that $(Y - \mu)^1 A(Y-\mu)$ is χ^2 iff

$$\Sigma A \Sigma A \Sigma = \Sigma A \Sigma.$$

2.(a) If $(Z_{1\alpha}, Z_{2\alpha})$, $\alpha = 1, \ldots, n$ are independent each pain with distribution $N\left(\binom{0}{0}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ & \sigma_2^2 \end{pmatrix}\right)$ then prove that the conditional

distributions of $b = \Sigma Z_{2\alpha} Z_{1\alpha} / \Sigma Z_{1\alpha}$ and $V/\sigma^2 = \Sigma(Z_{2\alpha} - bZ_{1\alpha})^2/\sigma^2$ given

$Z_{1\alpha} = Z_{1\alpha}(\alpha=1,\ldots,n)$ are $N(\beta_1\ \sigma^2/c^2)$ $(\sigma^2 = \sigma_2^2(1-\rho^2),\ c^2 = \Sigma\ Z_{1\alpha}{}^2)$

and ^{2} with n-1 d-f. respectively and that b and V are independent. Hence or otherwise prove that if $\rho = 0$

then $\sqrt{n-1}\ \frac{r}{\sqrt{1-r^2}}$ is t_{n-1}.

(b) On the basis of an experiment on 40 jute plants, the correlation coeffecient between weights of dry jute fibre extracted from the plant and the weights of the jute plant is found to be .8895. Test the hypothesis (i) $H_0: \rho = 0$ against $H_1: \rho \neq 0$ (ii) $H_0: \rho = .8$ against $H_1: \rho \neq .8$.

.(a) Define feofellig's T^2 statistic for Testing $H_0: \mu : \mu$ on the basis of a sample of size n from a distribution $N(\mu, \Sigma)$. Show that it is also the likelihood ratio test for the above problem. Find the distribution of the statistic under H_0.

p.t.o.

3(b) The heart weights in grams of 12 female and 15 male cats are given below. Does the heart of a male cat on an average weight more than that of a female cat?

Weight of hearts (in grams)

Male Cat	12.7	15.6	9.1	12.8	8.3	11.2	9.4	8.0	14.9	10.7	13.6	9.6	11.7	9.[illegible]
Female Cat	7.4	7.3	7.1	9.0	7.6	9.5	10.1	10.2	10.1	9.5	8.7	7.2		

4.(a) Let θ_1 denote random variables which are quadratic forms in items of random sample of size n from a distribution which is $N(0,\sigma^2)$. Let A and B denote respectively the real symmetric matrices of θ_1 and θ_2. The prove that random variables θ_1 and θ_2 are stochastically indepen dent iff AB = 0.

(b) Let $X_1, \ldots\ldots, X_n$ denote a random sample of size n from $N(0,\sigma^2)$. Prov that ΣX_1^{*2} and every quadratic form which is non identically zero in $x_1, \ldots, x_n$ are stochastically dependent.

5.(a) Let q_1 and q_2 be the prior probability of drawing an observation f: a population π_1 with density $P_1(x)$ and a population π_2 with density $P_2(n)$ respectively, and let the cost of misclassifying an observati from π_1 as from π_2 be c(2/1) and an observation from π_2 as from π_1 c(1/2). Define the region of classification so that average cost of classification of an observation is minimised.

(b) Apply the above procedure when the two populations are $N(\mu_i, \Sigma)$, i = 1,2. Calculate the two probabilities of misclassifications.

:SS:

INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Periodical Examination
Elective-5: Economics

Date: 1.3.89 Maximum Marks: 100 Time: 3 Hours

Note: This paper carries a total of 125 marks. You can answer any part of any question. The maximum you can score is 100. Marks allotted to each question are given in brackets.

1.(a) Obtain the expressions for the mean, median and mode of a lognormal distribution.

(b) State and prove the theorem concerning the jth moment distribution of a lognormal distribution and hence find the Lorenz ratio for this distribution. (10+12)=[22]

2.(a) Let $x \geq 0$ be a continuous random (size) variable with finite mean μ and distribution function $F(.)$. Show that the Lorenz ratio L for the distribution can be expressed as

$$L = 1 - \left[\int_0^\infty (1-F(u))^2 \, du \right] / \mu.$$

(b) Write a short note on the social - welfare approach to measuring income inequality. (13+11)=[24]

3.(a) Discuss about the universality of Pareto law of income distribution.

(b) Find the equation of the Lorenz curve for an exactly Paretean income distribution over the income range (c,∞) where $c(>0)$ is the subsistence income.

(c) Suppose you are given some income data and you plot a graph showing $\log T_x$ against $\log N_x$, where N_x is the number of earners earning x or more and T_x is the total income of these N_x persons. What would be the equation of the graph if the income distribution is Paretean? (8+9+5)=[22]

4.(a) Describe briefly how a demand function for an item of consumption is specified stating clearly the assumptions made and the restrictions to be satisfied (by the demand function).

(b) Justify the use of 'per capita' formulation of an Engel relation. Do you think that such a formulation is always adequate? Give reasons for your answer.

(12+10)=[22]

p.t.o.

5. Consider the following distribution of population by per capita household expenditure on all items per 30 days (PCE).

PCE classes (Rs.)	estimated p.c. of population	average PCE (Rs.)
0-11	15.80	8.40
11-15	16.40	14.24
15-21	30.80	19.36
21-28	17.40	25.28
28-35	11.10	30.12
35-43	5.30	40.19
43 and above	3.20	55.30

(a) For the data given above, fit a Paretean distribution over the appropriate range.

(b) Obtain the value for the Sen's measure of poverty from the above data by considering the poverty line to be defined at Rs.40.

(16+19)=[35]

1SS1

INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Periodical Examination
Elective-5: Physical and Earth Sciences

Date: 1.3.89 Maximum Marks: 100 Time: 3 hours

Note: Draw all the circuit diagrams, graphs and figures with suitable labeling and neatly. Rough calculation should be properly shown. Each questions carry equal marks 20. You may attempt all questions but maximum score will be 100.

1. Define specific resistance of a material. Write down mathematical expression of resistance in terms of it. Briefly explain the change of temperature on the resistance of different materials. [6]

A lead wire and an alluminium wire are connected together in parallel. The current flowing in the respective wires is in the ratio of 39:40. The lead wire is 65% longer than the alluminium wire and the ratio of their specific resistance is 98:13. Find the ratio of their cross-sectional areas. [6]

What do you mean by non-linearity in resistance? Draw the voltage-current curve of linear and non-linear resistances with brief explaination. [8]

2. A number of cells ((N = m x n) have been arranged in a mixed grouping to deliver current through an external load R. If the p.d. and internal resistance of each cell are E and r respectively find the magnitude of current. [6]

Find, through calculation, which arrangement will give optimum current in either of the conditions: external resistance $\gg$ / $\ll$ internal resistances of the battery. [8]

What arrangement of cells you prefer to get maximum current through an external resistance of 600 Ohm from the given 36 cells each of which has a p.d. of 2 volt and internal resistance of 150 Ohm? Find current. [6]

3. State and explain clearly Thevenin's theorem applicable to a network containing resistors and cell (E, r). Deduce an expression of current flowing through the resistance concerned. [9]

Four arms of a Wheat stone bridge contain resistences P, Q, R and S respectively. Across the diagonal of it a cell (E, r) is connected and over the other diagonal a galvanometer (internal resistance G) is attatched to indicate the flow of current I_G through that. Find the value of I_G in terms of given parameters. Deduce null condition. [11]

p.t.o.

4. Considering all the necessary parameters describe a magnetic circuit. What is its reluctance? [5]

Write down the important features of similarities and dissimilarities between the magnetic circuit and electric circuit. [7]

With suitable experiment, explain the phenomena of mutual inductance. Find out co-efficient of mutual inductance and also mutually induced e.m.f (e_M). [8]

5. Across a series combination of capacitance (C) and resistance (R) a supply voltage is applied as follows:

(i) D.C. voltage E; the capacitance is allowed to charge fully. Define time-constant (τ) of the given circuit. What voltage would the capacitor attained after a time of 5τ? Clearly draw the curves showing the voltage building over and current flowing through the capacitor wrt time. [8]

(ii) Fully charged capacitor is allowed to discharge through the close-circuit resistance after the removal of the battery. Deduce the magnitude of discharging current. Wrt time draw the graph of charging and discharging current before and after the removal of battery. [5]

(iii) A.C. voltage $E = E_m \sin wt$; Deduce the current I flowing through the circuit. Why the current is not in phase to the applied voltage? Draw the curves of I and E wrt time. [7]

6. Find out the effective resistance along the solid diagonal of a cube made of electrically equivalent 12 wires made from same material. [4]

The four arms of a Wheatstone bridge have the following resistances in Ohms : AB = 100, BC = 10, CD = 5 and DA = 60. A galvanometer of 15 Ohm resistance is connected across BD. Calculate the current through the galvanometer when a p.d. of 10 volt is maintained across AC. [9]

A steady voltage of 5kilovolt is applied across two circular parallel metal plates each having an area of one square metre and 18 cm apart. Between the plates are two layers of dielectric of thickness t_1=6cm, t_2=12 cm and relative permittivities $\epsilon_{r1} = 3$, $\epsilon_{r2} = 4$ respectively. Find the voltage gradient E_1 and E_2 and the capacitance C between the plates (given $\epsilon_0 = 8.854 \times 10^{-12}$). [7]

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Periodical Examination
Non-parametric and Sequential Methods

Date: 24.2.89 — Maximum Marks: 100 — Time: 3 hours

Note: Answer all questions.

1.(a) Find the exact probability distribution of Wilcoxon Rank sum statistic W_s for the 2-sample problem under the null hypothesis when m=3 and n=3. [10]

(b) Find the expression for the expectation and the variance of Mann-Whitney statistic $W_{xy} = W_s - \frac{n(n+1)}{2}$ when the X's and the Y's are distributed according to F and G respectively. [10]

(c) Suppose a new post-surgical treatment is being compared with a standard treatment by observing the recovery times of 9 treatment subjects and 9 controls. Suppose that the recovery times (no. of days) are as follows:

Controls : 20, 21, 24, 30, 32, 36, 40, 43, 54

Treatment: 19, 22, 25, 26, 28, 29, 34, 37, 38

Compute the value of the Wilcoxon statistic W_s and find the significance probability or the P-value of this data. [15]

(d) Show that the power function $\pi_F(\Delta)$ of the Wilcoxon-rank-sum test for the 2-sample problem under the shift alternative $G(x) = F(x - \Delta)$ is a non-decreasing function of Δ.

2.(a) Define the Kolmogorov-Smirnov two-sample statistic $D_{m,n}$. [5]

(b) Find the value d of $D_{m,n}$ for the sample arrangement xyyxyy. Find the P-value of $P_H[D_{m,n} \geq d]$ for this observed d. [15]

(c) Consider the following alternative form of Siegel-Tukey statistic (known as Ansari-Bradley statistic) which is obtained by assigning score 1 to both the smallest and the largest observations, score 2 to both the 2nd-smallest and the 2nd-largest observations and so on. Let T be the sum of scores corresponding to the treatment observations y. Find the null distribution T for m=n=3. Find E(T) under the null hypothesis when m=n. [10+5]

3.(a) In a taste test 7 subjects were asked to express a preference between a domestic and a more expensive imported brand of chocolate. If 5 out of 7 prefer the imported brand find the significance probability of this result when you are using a sign test. [10]

(b) Find the exact null distribution of the Wilcoxon signed-rank-sum statistic V_s for the one-sample problem when N=5 by actual enumeration. [10]

p.t.o.

Cumulative probability for the null distribution of Wilcoxon Rank sum statistic W_s for m=n=9

a	$P(W_s \geq a)$	a	$P(W_s \geq a)$
45	0.000	66	0.047
46	0.000	67	0.057
47	0.000	68	0.068
48	0.000	69	0.081
49	0.000	70	0.095
50	0.000	71	0.111
51	0.001	72	0.129
52	0.001	73	0.149
53	0.001	74	0.170
54	0.002	75	0.193
55	0.003	76	0.218
56	0.004	77	0.245
57	0.005	78	0.273
58	0.007	79	0.302
59	0.009	80	0.333
60	0.012	81	0.365
61	0.016	82	0.398
62	0.020	83	0.432
63	0.025	84	0.466
64	0.031	85	0.500
65	0.039		

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INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Periodical Examination
Optimization Techniques

Date: 27.2.89 | Maximum Marks: 100 | Time: 3 hours

Note: Answer all questions. The maximum you can score is 100 though the paper carries 110 marks.

1. Consider the linear program P : maximize $c^T x$ subject to $Ax = b,\ x \geq 0$ where A is an m x n matrix with full row rank. Let J be a feasible basis for P.

   (a) Define the tableau w.r.t. J in terms of A, b, c, J. [5]

   (b) Prove that if $z_j - c_j \geq 0$ for all j (with usual notation), then J is optimal for P, the dual D of P has an optimal solution and that the optimal values of P and D are equal. [14]

   (c) If $z_k - c_k \leq 0$ and $y_{ik} \leq 0$ for all i and some k, then prove that the objective function in P has no upper bound. [14]

2. Describe clearly and completely (but without proofs) the simplex method of solving a (general) linear programme using the two-phase technique and the anticycling rule. [22]

3. Let the tableau of the linear programme P considered in Question 1 w.r.t. an optimal feasible basis J be known and let $k \notin J$ be fixed. If c_k is altered without changing any other c_j, what is the range of values for c_k in which J remains optimal? Why? [10]

4. Solve the following linear programme by the two-phase method (find an optimal solution and the optimal value).

$$\begin{aligned} \text{minimize} \quad & 2x_1 - 3x_2 + 6x_3 \\ \text{subject to} \quad & 3x_1 - 4x_2 - 6x_3 \leq 2, \\ & 2x_1 + x_2 + 2x_3 \geq 11, \\ & x_1 + 3x_2 - 2x_3 \leq 5, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$ [35]

Assignment. [10]

:S:

INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Periodical Examination
Design of Experiments

Date: 22.2.89 . Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours

Note: Answer all questions. Marks allotted to each question are given within parentheses.

1.(a) How would you measure the relative efficiency of two designs?

(b) Show that the "amount of information" which the observed difference between two treatment means supplies about the true difference is estimated by

$$\frac{n+1}{n+3} \cdot \frac{r}{2E}$$

where r is the common replication number of the two treatments, n is the error d.f. and E is the observed error M.S..

(c) Estimate the efficiency of the Latin Square design relative to completely randomised design. (2+8+10)=[20]

2.(a) Write down the expression for the "treatment sum of squares" in Latin Square design. Show that this can be expressed as half the sum of squares of all ordered pairwise differences of estimates of treatment effects.

(b) Explain the 'missing plot technique' in case of a Latin Square design and suggest an accurate test of significance for the varietal differences showing that an upward bias is introduced if the analysis is carried out as usual incorporating an estimate of the missing observation.

(1+4+15)=[20]

3. The following table relates to data from an agricultural experiment conducted in 4 randomised blocks. The first figure in each cell represents the treatment and the second figure represents its yield.

(a) Analyse the data and draw suitable conclusions.

(b) In the table, the control treatment denoted by (0) has been replicated 4 times in each block. To utilise so much resources towards control may be considered as wastage. As such considering the first yield for the control treatment in each block as the only yield for the control treatment and neglecting the remaining yields for the control treatment, examine if the conclusions drawn in (a) are to be modified.

p.t.o.

O	A	B	C	D	O	O	E	F	O	G	H
269	283	252	212	138	100	197	263	282	216	230	145
E	F	A	O	G	B	C	O	O	O	H	D
95	127	80	134	107	89	41	74	88	25	42	62
G	O	D	A	O	H	F	B	E	O	C	D
124	211	194	222	102	193	128	42	162	191	107	67
A	O	G	C	O	H	F	O	D	B	O	E
193	209	109	153	29	9	17	19	23	19	44	18

(12+8)=[20]

4. Five treatments are tested in a Latin Square. The details of the layout and the data are given below denoting the treatments by A, B, C, D and E. The yields are in grams per plot of size 20'x18'.

13 (A)	17 (C)	12 (B)	13 (E)	12 (D)
13 (C)	14 (E)	13 (D)	15 (B)	12 (A)
11 (D)	11 (A)	16 (E)	12 (C)	14 (B)
12 (E)	11 (B)	12 (A)	10 (D)	17 (C)
15 (B)	16 (D)	14 (C)	18 (A)	18 (F)

(a) Analyse the data for comparing the treatment effects.

(b) Obtain the standard error of the difference between the estimates of the effects A and D.

(c) Is it more efficient than a randomised block design with rows as blocks? (12+4+4)=[20]

5. Practical Assignments [20]

:SS:

INDIAN STATISTICAL INSTITUTE
B.Stat. III Year: 1988-89
Multivariate Distributions and Tests
Periodical Examination

Date: 20.2.89 | Time: 3 hours

Note: Answer all five questions.

1.(a) Let $x_1, \ldots, x_n$ be a random sample from a normal population $N_p(\mu, \Sigma)$ Find the m.l.e. of μ and Σ How does Σ changes when it is known that $\mu = 0$?

(b) Show that $N \hat{\Sigma} = \sum_{\alpha=1}^{N-1} Z_\alpha Z_\alpha'$ where $Z_\alpha \longrightarrow N_p(0, \Sigma)$

$\alpha = 1, \ldots, N-1$ and are independent.

2.(a) Let $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{matrix} q \\ p-q \end{matrix} \longrightarrow N_p\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \right)$

Obtain the conditional distribution of x_1 given x_2.

(b) Hence or otherwise find the distribution of

$(x-\mu)' \Sigma^{-1} (x-\mu) - (x_2-\mu_2)' \Sigma_{22}^{-1} (x_2-\mu_2)$

3.(a) Let $x_1, \ldots, x_N$ be an independent sample form $N(\mu, (\sigma_{ij}\delta_{ij}))$.

Then find the density of sample correlation matrix.

(b) Hence or otherwise show that in a random sample of size N from a bivariate normal population with population correlation coefficient $\rho = 0$

$$\frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$$ has student's

t distribution with N-2 d.f.

4.(a) Define multiple correlation coefficient of x_1 on $x_2, \ldots, x_p$.

Show that multiple correlation coefficient is the maximum correlation coefficient between x_1 and all linear combinations of $x_2, \ldots, x_p$. Hence or otherwise show that multiple correlation coefficient can't be negative.

(b) From a data on yield (x_1) in hundred weights per acre, spring rain fall in inches (x_2) and accumulated temperature above 42°F in spring (x_3) from on English Country over 20 years, the following correlation matrix is obtained.

1.00	0.80	-0.40
	1.00	-0.56
		1.00

Test whether $\rho_{1.23}$ is significantly different from zero. What does the test signifies ?

p.t.o.

5.(a) Following data gives the values of the correlation coefficient between stature and sitting height of Muslims in eight districts of Bengal as obtained in the Bengal Anthropometric survey in 1945.

Examine of the correlation coefficients can be regarded as equal except for fluctuations of sampling.

correlation between statrum and sitting heights of Muslims in Eight districts of Bengal

District	Sample size	correlation coefficient
Barisal	131	.719
Burdwan	59	.696
Dacca	337	.824
Faridpur	77	.685
Murshidabad	124	.701
Mayman Singh	299	.547
Nadia	170	.793
Rangpur	139	.687

(b) The correlation coefficient between the weight of a green plant and weight of dry fibre in an experiment on jute in a village of West Bengal in 1953 based on 40 plants as a sampl. is .8895.

Can the population correlation between weight of green plant and weight of dry fibre be (i) significantly different from zero ? (ii) Significantly different from 0.8 ?

:SS:

1988-89	322(b)

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Stochastic Processes - 2

Date: 2.1.1989 Maximum Marks: 100 Time: 3 hours

Note: Answer ALL questions.

1. Let $\{X_t, t \geq 0\}$ be a continuous time-parameter Markov Chain with countable state space S and stationary transition probabilities $((p_{ij}(t)))$. Assume that $\lim_{t \to 0+} p_{ij}(t) = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$; $i, j \in S$. Prove that (a) if for $i, j \in S$ and $t_0 > 0$ $p_{ij}(t_0) > 0$, then $p_{ij}(t) > 0$ for all $t > t_0$.

   (b) $\lim_{t \to \infty} p_{ij}(t)$ exists for every $i, j \in S$.

   (8+12) = [20]

2. Let $\{X_t, t \in \mathbb{R}\}$ be a stochastic process. When is the process called a Gaussian process ?

   Let $Z_{11}, Z_{12}, Z_{21}, Z_{22}, \ldots, Z_{n1}, Z_{n2}$ be 2n independent random variables such that Z_{ki} are $N(0, \sigma_k^2)$ random variables $i = 1,2$, $k = 1, 2, \ldots, n$. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be reals. Define

   $X_t = \sum_{k=1}^{n} (Z_{k1} \cos \lambda_k t + Z_{k2} \sin \lambda_k t)$, $t \in \mathbb{R}$. Show that $\{X_t, t \in \mathbb{R}\}$ is a Gaussian process.

   Prove : Two Gaussian processes $\{X_t, t \in \mathbb{R}\}$ and $\{Y_t, t \in \mathbb{R}\}$ are identically distributed if $E(X_t) = E(Y_t)$ and $\text{cov}(X_s, X_t) = \text{cov}(Y_s, Y_t)$ for all $s, t \in \mathbb{R}$.

   Illustrate with the aid of an example that the condition "Gaussian" cannot be dropped in the above assertion.

   (b) Let $\{X_t, t \in T\}$, where $T = \mathbb{R}$ or $\mathbb{Z}$, be a stochastic process. When is the process $\{X_t\}$ said to be stationary ? Prove that a Gaussian process is stationary if it is second order stationary.

   (3+7+7+5+3+5) = [30]

p.t.o.

3. Describe carefully the two-state Birth and death process. Write down the Kolmogorov Forward differential equations and solve for $p_{ij}(t)$, $i, j \in \{0, 1\}$.

[25]

4. Let $\{X_t, t \in T\}$ be as in Q.No.1. Define the Infinitesimal generator $((q_{ij}))$ of the process. Establish that

$$\sum_{j \in S} q_{ij} \leq 0, \quad \text{for } i \in S.$$

Define (a) stable (b) instantaneous states of the Markov Chain $\{X_t, t \in T\}$. When is the chain called <u>conservative</u> ?

Prove: Every finite state continuous time-parameter Markov Chain is conservative.

(3+7+3+4+8) = [25]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Elective - 4 : Economics

Date: 30.12.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions.

1.(a) A factory is engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and for one unit of B are 3, 1 and 3 hours respectively. The profits on each unit of A and B are Rs.2/- and Rs.3/- respectively.

Assuming that 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembly time are available, pose a linear programming problem in terms of maximising the profit on the items manufactured.

(b) Solve the following L.P. problem by simplex method:

$$\text{Min } Z = 2x_1 + x_2 - x_3$$

Subject to

$$x_1 + x_2 + x_3 \leq 3$$
$$x_2 + x_3 \geq 2$$
$$x_1 + x_3 = 1$$

$(12\frac{1}{2} + 12\frac{1}{2}) = [25]$

2. If you are given the 'Make Matrix' and 'Commodity x Industry Table' for an economy, how would you obtain the Commodity x Commodity Table and the Industry x Industry Table on the basis of (a) Commodity Technology assumption (b) Industry Technology assumption. Discuss the logic of your procedure.

[25]

3.(a) An objective function $Z = 4x_1 - x_2 - 3x_3 + 2x_4$ has finite maximum subject to certain linear constraints. The set of Basic Feasible solutions of the given constraints are

Contd..... 2/-

Contd..... Q.No.3.(a)

$x_1 = (2,4,0,0)$, $x_2 = (0,5,0,0)$, $x_3 = (0,0,2,1)$
$x_4 = (1,0,0,3)$.
Find out the maximum value of the objective function.

(b) What is the maximum number of non-zero components in a basic solution ? What is the minimum number of zeros in a basic solutions ?

(c) What is the criterion for the existence of an unbounded solution of a L.P. problem ?

(10+10+5) = [25]

4. Write short notes on the following:

(1) Shadow Price.
(2) Artificial variable (related to the Simplex Algorithm).
(3) Valuation problem in the construction of I-O tables.

$(8\frac{1}{3} + 8\frac{1}{3} + 8\frac{1}{3})$ = [25]

5. How you can use the 'Decision tree analysis' technique to obtain the optimal sequence of decisions when the probabilities of future events are known.

[25]

6.(a) The following pay-off matrix represent the returns expected by a firm for five alternative investments and four differen levels of sales. Which alternatives would the firm select if their decisions are based on (a) Maxmin rule, (b) Maximax rule, (c) Hurwitcz rule, for $\lambda = .7$, and for $\lambda = .3$, where λ = degou of optimism.

Alternative	Levels of Returns			
	1	2	3	4
A	15	11	12	9
B	7	9	12	20
C	8	8	14	17
D	17	5	5	5
E	6	4	8	19 .

Contd...... Q.No.6

(b) Find the initial Basic Feasible solution of the following transportation problem using Vogel's Approximation method:

				a_i
	6	7	8	25
	9	3	5	12
b_j	13	17	10	

$(12\frac{1}{2} + 12\frac{1}{2}) = [25]$

:bcc:

1988-89	342(b)

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Sample Surveys

Date: 29.12.1988 Maximum Marks: 100 Time: 3 hrs.

Note: Answer any FOUR questions. Each question carries equal marks.

1.(a) With two strata, a sampler would like to have $n_1 = n_2$ for administrative convenience, instead of using the values given by the Neyman's optimum allocation. If $V(\bar{y}_{st})$ and $V_{opt}(\bar{y}_{st})$ denote the variances given by the $n_1 = n_2$ and the Neyman's allocations, respectively, show that the fractional increase in variance is

$$\frac{V(\bar{y}_{st}) - V_{opt}(\bar{y}_{st})}{V_{opt}(\bar{y}_{st})} = \left(\frac{r-1}{r+1}\right)^2,$$

where $r = \frac{n_1}{n_2}$ as given by Neyman's allocation.

(b) The N_i's, σ_i's (in kg) and c_i's (in Rs.) are given for 5 strata into which a population is divided in a certain crop survey. Obtain the optimum values of n_i's and the corresponding variance of the estimator if the population mean is to be estimated and if the total approved cost of the survey is Rs.5000/- and the overhead cost is Rs.550/-.

i	N_i	σ_i(in Rs.)	c_i(in Rs.)
1	3780	28.5	3.50
2	5260	18.6	2.75
3	8200	27.6	2.25
4	4160	27.2	3.00
5	2980	16.8	2.50

(12+13) = [25]

2.(a) Obtain an approximate expression for the mean square error of the ratio estimator of a population total. When is the ratio estimator optimum ?

Contd..... 2/-

Contd..... Q.No.2

(b) The following table shows the data for household sizes (x) and total monthly income (y) for two villages. Draw a simple random sample of size 4 for each village and estimate the total monthly income for the villages using x as an auxiliary variable. Also obtain an estimate of mean square error.

	sl.no.	household size (x)	monthly income ('000 Rs.)
Village 1	1	5	1.2
	2	11	2.7
	3	7	1.4
	4	6	1.3
	5	7	3.2
	6	12	4.1
	7	8	3.2
	8	6	1.8
	9	4	0.7
Village 2	1	3	0.8
	2	5	0.9
	3	4	1.1
	4	9	2.1
	5	6	1.1
	6	7	1.2
	7	6	1.4

(5+8 + 12) = [25]

3.(a) Explain a cluster sampling design. When it should be used ? Obtain the variance of an unbiased estimator of population mean under this design in terms of intracluster correlation coefficient when the clusters are selected by simple random sampling. Hence obtain the efficiences of a cluster sampling procedure with respect to a comparable simple random sampling procedure.

(b) In a study of the possible use of sampling to cut down the work in taking inventory in a stock room, a count is made of the value of the articles on each of 36 shelves in the room. The values to the nearest rupees are as follows:
29, 38, 42, 44, 45, 47, 51, 53, 53, 54, 56, 56, 56, 58, 58, 59, 60, 60, 60, 60, 61, 61, 61, 62, 64, 65, 65, 67, 67, 68, 69, 71, 74, 77, 82, 85.

Contd..... 3/-

Contd..... Q.No.3.(b)

The estimate of the total value made from a sample is to be correct within Rs.200, apart from a 1 in 20 chance. An advisor suggests that a simple random sample of 12 shelves will meet the requirements. Do you agree ?

[($\overline{3+2}$ + 6+3) + 11] = [25]

4.(a) Show that for any sampling design with positive first-order inclusion probabilities π_i , $\hat{Y} = \sum_{i \in s} y_i/\pi_i$ is an unbiased estimator for Y.

(b) Derive the variance of $\hat{Y}$.

(c) In sampling with unequal probability for samples of size 2, when the first unit is selected with PPS and the second unit with PPS from the remaining units, prove that yetes and Grundy's variance estimator would be non-negative.

5. A population contains N fsu's, M_i , being the number of subunits in the ith fsu. A sample of n fsu's is selected with replacement and with selection probabilities p_i, $\sum_i p_i = 1$. If the ith fsu occurs λ_i times, λ_i independent subsamples of m_i subunits are selected from it by SRSWOR. Suggest an unbiased estimator for population total. Obtain the variance of your estimator and an unbiased estimator of the variance.

6. The following table gives the number of indoor patients treated (y) and number of beds (x) in 10 hospitals of a certain city. ⓐ select a sample of size 2 by PPSWOR and compute Des Raj, Muthy, and Horvitz Thompson estimators for total number of indoor patients treated in the hospitals. Also estimate the variances of the estimators used.

serial no. of hospitals	1	2	3	4	5	6	7	8	9	10
indoor patients treated (y)	100	125	66	98	120	340	98	112	48	60
number of beds (x)	125	168	98	100	140	380	100	228	98	100

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1988-89	332(b)

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

BACKPAPER (SEMESTRAL-I) EXAMINATION

Statistical Inference

Date: 28.12.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Suppose that $X_1, X_2, \ldots, X_n$ are n independent observation from a probability distribution with density

$$f_\theta(x) \begin{cases} = 3x^2 \theta e^{-\theta x^3} & \text{for } x > 0 \\ = 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$ is an unknown parameter.

(a) Compute $h(\theta) = E_\theta(X_1^3)$.

(b) What is the UMVUE of $h(\theta)$? Calculate the Cramer - Rao lower bound for the variance of an unbiased estimator of $h(\theta)$. Does the UMVUE of $h(\theta)$ attain this bound ?

(5+5+5+5) = [20]

2 (a) Obtain the Bayes estimator of θ on the basis of one observation X from the uniform distribution on the interval $(0, \theta)$, $\theta > 0$, where the prior density of θ is given by

$$g(\theta) = \theta e^{-\theta} \quad \text{for } \theta > 0$$
$$= 0 \quad \text{otherwise}$$

(assume squarred error loss).

(b) Define an admissible estimator. Show that if T is an unbiased estimator of $h(\theta)$ and $b \neq 0$ is a known constant, then $T+b$ is not admissible for estimating $h(\theta)$. [Assume squared error loss].

(c) Give an example to show that the UMVUE need not be admissible.

(7+5+5+8) = [25]

p.t.o.

3.(a) State the Neyman-Pearson Lemma carefully.

(b) Prove the part of the lemma giving a sufficient condition for a test to be MP level α for testing a simple Hypothesis against a simple alternative.

(c) Let $X_1, X_2, \ldots, X_n$ be IID, discrete random variables with probability mass function $f_\theta(x)$, $x = 1, 2, 3$ and $0 < \theta < 1$, where

$$f_\theta(1) = P_\theta[X_1 = 1] = \theta^2$$

$$f_\theta(2) = 2\theta(1-\theta)$$

$$f_\theta(3) = (1-\theta)^2.$$

Let N_1, N_2, N_3 be the number of X_i's which are equal to 1, 2 and 3 respectively. Show that the test which rejects H if and only if $2N_1 + N_2 \geq C$ is UMP level α for testing $H : \theta \leq \theta_0$ against $K : \theta > \theta_0$ where C and α satisfy

$$P_{\theta_0}[2N_1 + N_2 \geq C] = \alpha .$$

(6+9+10) = [25]

4. Let $X_1, X_2, \ldots, X_n$ be independent observations from a normal distribution with unknown mean θ and variance $\sigma^2 = 9$.

(a) Show that the UMP level α $(0 < \alpha < 1)$ test for testing $H : \theta = 0$ against $K : \theta \neq 0$ does not exist.

(b) Define an unbiased test. Show that an unbiased test $\varphi(\underline{x})$ of level α for the testing problem in (a) satisfies

$$E_{\theta=0}[\varphi(\underline{X})] = \alpha$$

and

$$E_{\theta=0}[\bar{X}\varphi(\underline{X})] = 0 .$$

Write down the UMPU test of level α for the testing problem in (a) - (No derivation). Find the smallest value of n such that the probability of 2nd type of error of this test (with $\alpha = 0.10$) against the alternative $\theta = 5$ does not exceed 0.05.

[You can use R.M.M. tables]

(7+4+5+4+10) = [30]

Contd..... 3/-

5. Find the Bayes estimator of θ on the basis of an observation X from a Poisson distribution with mean $\theta > 0$ where the prior density of θ is

$$g(\theta) = \frac{1}{2} e^{-\theta/2} \quad \text{for } \theta > 0$$
$$= 0 \quad \text{otherwise}$$

Assume squarred error loss. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year: 1988-89
BACKPAPER (SEMESTRAL-I) EXAMINATION
Difference and Differential Equations

Date: 26.12.1988 Maximum Marks: 100 Time: 3 hours

Note. Answer ALL questions.

1. (i) Solve the difference equation

$$y_{n+2} + y_{n+1} - 56y_n = 2^n (n^2 - 3)$$ [6]

(ii) Solve the simultaneous difference equations

$$2U_{n+1} - 3U_n + 5V_n = 2$$
$$2U_n + V_{n+1} - 2V_n = 7$$ [9]

(iii) By the method of difference equations, evaluate

$$\int_0^{\pi} \frac{\cos n\theta}{\cos\theta - \cos\alpha}\, d\theta,$$ n being an integer and $0 < \alpha < \pi$. [10]

2.(a) State and prove the convolution theorem for Laplace transforms. [6]

(b) Use (a) to find the inverse Laplace transform of

$$\frac{s}{(s-1)(s^2+1)}.$$ [4]

(c) Find the function f(t) given that

$$f(t) = t^2 - \int_0^t e^u f(t-u)\,du.$$ [5]

3. Solve the simultaneous differential equations

$$\frac{dx}{dt} + 2\frac{dy}{dt} - x - 5y = \sin 2t + \cos 2t$$

$$\frac{dx}{dt} + 3\frac{dy}{dt} - 2x - 6y = 9e^t$$

given that x = y = 0 when t = 0. [20]

p.t.o.

4.(a) Carefully state, without proof, Picard's theorem on the <u>local</u> existence and uniqueness of solution of the initial-value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval $|x-x_0| \leq h$. Under what condition does this become a global solution ? (Prove your statement). Use the latter to sketch a proof of the existence and uniqueness of the solution of the initial-value problem

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x),$$

$y(x_0) = y_0$, $y'(x_0) = y'_0$, where P, Q, R are assumed to be continuous functions on $[a,b]$ and $x_0 \varepsilon [a,b]$.

(5+10+5) = [20]

(b) Solve the following initial-value problem by Picard's method and compare the result with the exact solution:

$$\begin{cases} \dfrac{dy}{dx} = -z \ , & y(0) = 1 \\ \dfrac{dz}{dx} = -y \ , & z(0) = 0 \end{cases}$$

[10]

5. Prove that the Bessel functions $J_n(x)$ of integral order are linked together by the identity

$$e^{x/2(t-1/t)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)[t^n + (-1)^n t^{-n}]$$

and use it to prove Bessel's integral formula

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\,d\theta .$$

(5+5) = [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Statistical Inference

Date: 26.11.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1.(a) Let $X_1, X_2, \ldots, X_n$ be independent observations from $N(\theta, \theta^2)$ distribution where $\theta > 0$ is an unknown parameter. Show that $(\sum_1^n X_i, \sum_1^n X_i^2)$ is a sufficient statistic but it is not complete.

(b) Let $X_1, X_2, \ldots, X_n$, $n \geq 2$ be independent observations from $N(\theta, 1)$ distribution. Obtain the UMVUE of $e^{-\theta^2/2}$.

(10+10) = [20]

2. Let X be a random variable having the Binomial $B(n, \theta)$ distribution where $0 < \theta < 1$ is the unknown parameter.

(a) Using the Beta distribution $Be(a,b)$ as prior and squarred error loss, show that the Bayes estimator of θ is $\delta(X) = \frac{X+a}{n+a+b}$.

(b) Compute the risk function $R(\theta, \delta)$ of the estimator (X).

(c) Show that by taking $a = b = \sqrt{n}/2$ we get the Bayes estimator $\frac{X+\sqrt{n}/2}{n+\sqrt{n}}$ whose risk function is a constant. Hence prove that $\frac{X+\sqrt{n}/2}{n+\sqrt{n}}$ is a minimax estimator θ. Is the usual estimator X/n of θ minimax ?

(6+4+15) = [25]

Obtain the Cramer - Rao lower bound and the 2nd Bhattacharya lower bound for the variance of an unbiased estimator $2\theta^2 - \theta$ on the basis n independent observations from $N(\theta, 1)$ distribution. What is the UMVUE of $2\theta^2 - \theta$? Check that the UMVUE of $2\theta^2 - \theta$ attains the 2nd Bhattacharya lower bound.

(4+6+4+6) = [20]

p.t.o.

4.(a) Define a MLR family of densities.

(b) $X_1, X_2, \ldots, X_n$ are the times until failures of n similar pieces of instruments. It is assumed that $X_1, X_2, \ldots, X_n$ are IID observations from Weibull distribution with density

$$f_\theta(x) \begin{cases} = \theta c x^{c-1} e^{-\theta x^c} & \text{for } x > 0 \\ = 0 & \text{otherwise} \end{cases}.$$

Here c is a known constant and $\theta > 0$ is the unknown parameter of interest. Show that $Y = X^c$ has the density $\theta e^{-\theta y}$ for $y > 0$. Hence show that $2\theta \sum_1^n X_i^c$ has χ^2_{2n} -distribution. Show that the UMP level α-test for testing $H : \theta \leq \theta_0$ against $K : \theta > \theta_0$ is a non-randomised test which rejects H if and only if $\sum_1^n X_i^c \leq \frac{x_{2n}(\alpha)}{2\theta_0}$ where $x_{2n}(\alpha)$ is the lower $100\alpha\%$ point of χ^2_{2n} -distribution.

(5+5+5+10) = [25]

5.(a) Does there exist UMP level α-test for testing $H : \theta = 0$ against $K : \theta \neq 0$ on the basis n independent observations from $N(\theta, 1)$ distribution ? Justify your answer.

(b) Define an unbiased test. Write down the UMP unbiased test of size α for the testing problem in (a). Obtain the expression of the power function of this test in terms of the distribution function $\Phi(x)$ of standard normal distribution. Show that the power function is an increasing function of $|\theta|$.

(10+5+5+5) = [25]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) Part III

SEMESTRAL-I EXAMINATION 1988-89

Elective-4: Physical & Earth Sciences

Date: 24.11.88 Max. marks 100 Time: 3 hours

Note: Attempt any five questions. Maximum score for each question is 20. Answers should be brief and to the point. Draw sketches wherever necessary.

1.(a) Briefly discuss the principle of radiometric dating of rocks. (16)

(b) Why is C^{14} method of radiometric dating inapplicable to rocks older than 70,000 years ? (4)

2.(a) What is a mineral ? Is sugar cube a mineral ? Name four important groups of rock-forming silicate minerals. (2 + 1 + 2) = 5

(b) What is polymorphism ? How are calcite and aragonite related to each other in this respect ? (3 + 2) = 5

(c) What is a rock ? What are the major groups of rocks ? Name a monomineralic rock-type from each of the groups. (3 + 1½ +1½)=6

(d) What differences would you expect between an arkose and a granite ? (4)

3. Draw neat sketches and label them to illustrate the following :

(a) True dip and apparent dip of an inclined plane.

(b) Axial surface of a fold.

(c) Similar fold.

(d) Parallel fold.

(e) Oblique-slip fault. (4 x 5) = 20

4.(a) Briefly describe the broad geologic setting under which the following varieties of metamorphic rocks are generally formed :

(i) Blue-schist (ii) Amphibole-hornfels (iii) Migmatite

(4 x 3) = 12

(b) What are the metamorphic equivalents of (i) Sandstone (ii) Limestone (iii) Shale (iv) Basalt (1 x 4) = 4

(c) Arrange the following index minerals in order of increasing grade of metamorphism:

garnet, sillimanite, chlorite, biotite

(4)

5.(a) Indicate the parameters for classification of igneous rocks. (3)

(b) What sort of textural differences would you expect between a volcanic rock and its plutonic equivalent ? Give reasons for such differences. (3 + 2) = 5

p.t.o.

5.(c)(i) Separate the following rock types into two columns, one for the volcanic varieties and the other for their plutonic equivalents. (ii) Arrange the volcanic varieties in order of decreasing SiO_2 content and (iii) Match each of them against their plutonic equivalents:

- Granite, Basalt, Dacite, Rhyolite, Gabbro, Andesite, Granodiorite, Diorite (4 + 4 + 4)=12

6.(a) What is a fossil ? Mention the modes of their preservation. Briefly discuss the utilities of studying fossil. (2+4+8) = 14

(b) What major changes took place in the organic community on the earth (as evident from fossil record) during
(i) early Cambrian
(ii) Cretaceous-Tertiary transition (3 + 3) = 6

7. What is Gondwanaland ? Briefly discuss the geological evidences in favour of continental drift. ((4 + 16)=20

8.(a) Identify the characteristic features of mid-oceanic ridges. Discuss the palaeomagnetic evidences in favour of sea-floor spreading. (3 + 6)= 9

(b) What is a subduction zone ? Describe the geological features associated with such zones. (3 + 6) = 9

(c) Indicate the reason for the origin of the Himalayas in terms of plate-tectonics concept. (2)

9. Write short notes on any four of the following:

(a) Tillite, (b) Lithostratigraphic unit (c) Craton (d) Delta
(e) Unconformity (f) Pelagic sediments (g) Oceanic crust
(h) Transform fault (5x4) = 20

10. Attempt any eight of the following:

(a) During the pre-Devonian period the earth's landscape was markedly different from that of today. Why ?

(b) A large granite pluton has been later metamorphed into granite-gneiss. Radiometric age determined from a whole rock sample of the same by Rb-Sr method is found to be greater than that determined from a mineral grain of the same sample by the same method. Suggest a plausible reason for this discrepancy.

(c) Both alluvial fans and deltas have similar drainage patterns in having a channel that develops a number of distributaries. Is there a common cause for this similarity of behaviour ? Justify your answer.

(d) Mean palaeocurrent direction measured from aeolian sediments in any area invariably coincides with that determined from closely associated fluvial sediments. Is this statement correct ? Justify.

(e) A plumb-line (a weight suspended on a strong) originally hangs in a vertical position. At the foothills of a mountain (system) one would expect the plumb-bob to be deflected towards the mountain because of the gravitational attraction of its mass. The observed deflection is, however, considerably less than expected. How do you explain this phenomenon ?

contd...3

(f) While a vast majority of modern rivers flow along linear courses, deposits formed by rivers in the rock record are found to be often laterally extensive. Why this is so ?

(g) On the scarp face along a E-W trending road, lower boundary of a sedimentary layer occurs at a fixed elevation all along the road. What would you conclude about strike and dip direction of that layer ?

(h) Extensive solid solution is noted between albite ($NaAl\ Si_3O_8$) and anorthrite ($Ca\ Al_2\ Si_2O_8$) but not between albite and potash feldspar ($K\ Al\ Si_3O_8$). How do you account for this ?

(i) Why do aeolian sediments show a relatively narrower scatter in their grain-size distribution as compared to that of fluvial sediments ?

(j) How do you account for the paucity of pre-Jurassic pelagic sediments on the ocean floor ?

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Elective-4 : Economics

Date: 24.11.1988 Maximum Marks: 130 Time: 3 hrs. and 45 minutes

Note: Answer as many questions (and any part of any question) as you like.

1.(a) What are the advantages of using a unit basis as an initial basis in solving a Linear Programming problem by simplex method ?

Or,

In a Transportation problem with 4 origins and 4 destinations, may the variables x_{11}, x_{22}, x_{23}, x_{24}, x_{31}, x_{34}, x_{42}, x_{44} be considered as basic variables ? Give reasons for your answer.

(b) The initial basic feasible solution for a transportation problem is obtained as follows. Obtain the optimum basic feasible solution to the T.P. and find out the corresponding value of the objective function.

	1	2	3	4
1	$c_{11}=5$	18, $c_{12}=3$	1, $c_{13}=6$	$c_{14}=2$
2	$c_{21}=4$	$c_{22}=7$	12, $c_{23}=9$	25, $c_{24}=1$
3	16, $c_{31}=3$	$c_{32}=4$	18, $c_{33}=9$	$c_{34}=5$

(4+16) = [20]

2.(a) In the revised model of optimum location and flows formulated by Mathur and Hashim the sector 'Transport' has been treated as an endogenous sector. Do you think that this is an improvement over the original formulation ? Give reasons for your answer.

(b) Sketch the revised formulation of Mathur-Hashim model.

(5+15) = [20]

p.t.o.

3.(a) Derive the growth equations for capital goods and consumption goods sectors in Mohalanobis two-sector model. Using these equations explain the choice problem implicit in this model.

(b) Obtain the 'Commodity x Commodity' Input output table given the 'Make Matrix' and the commodity x Industry table below (use commodity technology assumption).

Make Matrix

Commodity	Industry 1	Industry 2	Industry 3	Total
1	100	0	0	100
2	10	100	0	110
3	0	0	50	50
Total	110	100	50	260

Commodity x Industry I-O table

Commodity	Industry 1	Industry 2	Industry 3	Final demand	Total
1	20	30	0	50	100
2	30	20	20	40	110
3	10	20	10	10	50
Value added	50	30	20		100
Total	110	100	50		

(10+10) = [20]

4.(a) An incomplete initial simplex tablean of a maximisation problem is given below. Slack vectors $a_5(e_1)$ and $a_6(e_2)$ constitute the initial unit basis. Complete the initial tablean. Write down the original problem. Is the solution optimal ? If not, find the vector which will enter in the next basis and which vector will leave the current basis ?

C_B	Vectors in the basis	b	a_1	a_2	a_3	a_4	a_5	a_6
		6	2	-4	1	2		
..	...	15	3	1	0	3		
$Z_j - C_j$		0	-3	2	3	-1		

(b) A large water treatment facility is located in the flood plain of a river. The construction of a leve. to protect the facility during periods of flooding is under consideration. Data concerning the costs of construction and expected flood damages are shown below. The frequencies of river level reaching maximum height above normal in the last hundred years are also shown. Assuming the life of the levee to be 50 years, obtain the optimal leight of the levee (Rate of interest is 12 per unit).

Contd..... 3/-

Contd..... Q.No.4.(b)

Feet(x)	No. of years river level was x feet above normal	Damage if the river is x ft above levee (in Rs.)	Construction cost of building x ft high (in Rs.)
0	48	0	0
5	24	5000	7000
10	16	10000	15000
15	6	20000	23000
20	4	40000	48000
25	2	60000	73000

(7+8) = [15]

5. State 'True' or 'False' (with brief reasons):

(a) The informations contained in the 'Commodity x Industry Input-output table' are derived from the 'Make Matrix'.

(b) It is a practice to treat all noncompetitive imported inputs going to different sectors to constitute an exogenous row while presenting an Input-output table.

(c) The existence of secondary products in many economic activities results in the socalled 'Aggregation Problem' in the construction of Input-output tables.

(d) For the maximisation L.P. problem in the standard form, suppose that in the current basic feasible solution there exists at least one value of K for which $Z_K - C_K < 0$ and $\beta_{Ki} \leq 0$ for all i, $i = 1, \ldots, m$. Then the problem has an unbounded solution with an unbounded value of the objective function (Reasons not required) (usual notations).

(e) The condition that a set of m equations with n variables $AX = b$, $(n > m)$ are linearly independent is that Rank $(A) = n$.

(3x5) = [15]

6. (a) Consider the linear programming problem

$$\text{Minimise } Z = x_1 - 3x_2 + 2x_3$$

$$\text{Subject to } \begin{aligned} 3x_1 - x_2 + 2x_3 &\leq 7 \\ -2x_1 + 4x_2 &\leq 12 \\ -4x_1 + 3x_2 + 8x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

Find the dual.

Contd..... 4/-

Contd..... Q.No.6

(b) In a Linear Programming problem

$$\text{Max} \quad Z = 2x_1 - 3x_2 + x_3 + 2x_4$$

$$\text{Subject to} \quad x_1 + 2x_2 + x_4 \geq 5$$

$$3x_2 + x_3 - 2x_4 \geq 3$$

$$x_j \geq 0$$

$$j = 1, \ldots, 4.$$

Do we need to introduce artificial variables to get an initial unit basis ? What are the initial unit vectors ? What will be the initial Basic Feasible Solution ? Construct the initial table.

(c) Discuss briefly the logic of 'Decision Tree Analysis'.

(6+9) = [15]

7.(a) For the maximisation linear programming problem in the standard form suppose that in the current basic feasible solution $Z_j - C_j \geq 0$ for every column vector a_j of A which does not belong to B. Then prove that the current basic feasible solution is optimal.

(b) Derive the expression for the optimal economic order quantity for the inventory control problem when there is finite delivery rate and no back ordering.

(12+8) = [20]

8.(a) Formulate Leontief Input-output model as a linear programming problem. Obtain the dual of the above.

(b) Discuss Manne's model for optimal time phasing of Investment when demand for the product is growing at an arithmetic rate and a part of the demand for the product is met by Import.

(6+14) = [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Sample Surveys

Date: 21.11.1988 — Maximum Marks: 100 — Time: 3 hours

Note: 1. Answer any FOUR questions
2. Each question carries equal marks.
3. Practical note book carries 10 marks.

1. Suppose from a population of size N a sample s is selected with positive inclusion probabilities π_i, π_{ij} and let ν_s be the effective sample size of s. Then show that

(a) $\sum\limits_{i\neq j=1}^{N}\sum^{N} \pi_{ij} = E(\nu_s)(E(\nu_s) - 1) + Var(\nu_s)$.

(b) $Var(\sum\limits_{i\varepsilon s} y_i/\pi_i) = \sum\limits_{i<j=1}^{N}\sum^{N} (\pi_i\pi_j - \pi_{ij}) \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$

when ν_s is same for all s.

(c) $\sum\limits_{i\varepsilon s} \frac{y_i^2}{p_\beta}\frac{a_i}{\alpha_i} + \sum\limits_{i\neq j\varepsilon s}\sum \frac{y_i y_j}{p_\beta}\frac{a_{ij}}{\alpha_{ij}}$ is an unbiased estimator

for $Var(\sum\limits_{i\varepsilon s}\frac{y_i}{\pi_i})$ where $a_i = (\frac{1}{\pi_i} - 1)$, $a_{ij} = (\frac{\pi_{ij}}{\pi_i\pi_j} - 1)$,

$\alpha_i(\alpha_{ij})$ = number of samples containing ith(i and jth) unit with positive probabilities. P_s = selection probability for sample s.

2.(a) Define ordered and unordered samples.

(b) If a sample s of size n is selected by PPSWR method with $P_i(i = 1, 2, ..., N)$ as the normed size measure for the ith unit, show that the estimators

t_1 = Hansen-Hurwitz estimator, t_2 = Symmetrized form of t_1

and t_3 = Horvitz-Thompson estimator are unbiased for population total Y, and $Var(t_2) \leq Var(t_1)$.

p.t.o.

3.(a) Suppose two units i and j are selected with $\{P_i\}$ $i = 1,\ldots,N$ as the selection probability of ith unit in the first draw and $P_{j/i} = P_j/(1-P_i)$ as the conditional selection probability for jth unit at the second draw given that the ith unit is selected in the first draw. Show that

(i) $t_1 = y_i/P_i$, $t_2 = Y_i + Y_j/P_{j/i}$ and $t = \frac{t_1 + t_2}{2}$ are unbiased estimators for the population total Y.

(ii) $Var(t) \leq \frac{1}{2} Var(t_1)$.

(b) Find approximate expression for bias and mean square error for the regression estimator of population total under SRSWOR method of sampling.

4. A population contains N fsu's M_i being the number of ssu's in the ith fsu. A sample of n fsu's is selected by PPSWOR method with $\pi_i(\pi_{ij})$ as the inclusion probabilities for the ith (ith and jth) fsu respectively. If ith fsu appears in the sample a subsample of m_i subunits is selected by SRSWOR method of sampling. Suggest an unbiased estimator for population mean. Obtain the expression for the variance of the proposed estimator and an unbiased estimator of the variance.

5. A list of 20 villages in a tehsil arranged in ascending order of geographical area (x) is given together with village-wise area (y) under winter paddy.

(a) Draw 3 systematic samples each of 6 villages. Making use of 18 sample observations estimate the relative efficiency of systematic sample to that of SRSWOR for estimating total area under paddy (Y) based on a sample of 6 villages.

(b) Select 5 villages with probability proportional to size (x) and with replacement using Lahiri's method and give an unbiased estimate for the variance of the village wise area under winter paddy

$$\sigma^2 = \frac{1}{20} \sum_{i=1}^{\infty} (y_i - \bar{Y})^2 \quad (\text{where } \bar{Y} = Y/20).$$

Contd..... 3/-

Contd..... Q.No.5.(b)

serial no. of villages (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
geographical area (x)	65	70	72	78	79	79	80	82	82	85	86	88	88	88	90	91	91	95	99	98
area under winter paddy (y)	30	35	41	40	49	48	52	55	60	66	62	68	69	75	68	72	77	78	78	80

6. For estimating the total yield of paddy Y in a district, a statified two-stage sampling design was adopted, where 2 villages were selected from each stratum with PPSWOR, size being the geographical area, and 4 plots were drawn from each sample village by SRSWOR method for ascertaining the yield of paddy. Using the information given below estimate unbiasedly Y and obtain its rse.

Stratum	Sample village	Inverse probability	Total no. of plots	Yield of paddy (in Kg.)			
				1	2	3	4
1	1	440.21	28	104	182	148	87
	2	660.43	14	64	132	156	200
2	1	21.00	256	124	111	135	216
	2	16.80	258	123	177	106	138
3	1	67.68	189	281	120	114	110
	2	60.14	42	80	61	118	124

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.)-III Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Difference and Differential Equations

Date: 18.11.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions, or parts thereof, as you can. The maximum that you can score is 100 marks.

1. (i) Prove that the difference equation $(E-a)^2 y_n = 0$ has solution $y_n = (A + Bn) a^n$. Use it to solve the difference equation $y_{n+3} - 5y_{n+2} + 8y_{n+1} - 4y_n = 0$ given that $y_0 = 3$, $y_1 = 2$, $y_4 = 22$. (2+4) = [6]

(ii) Find the general solution of the difference equation

$$y_{n+2} - \sqrt{2}\, y_{n+1} + y_n = 2^{\frac{3n}{2}}. \qquad [6]$$

(iii) If $I_n = \int_0^1 (\log x)^n x^\alpha \, dx$ where $\alpha > 0$ and n is a positive integer, show that $(\alpha + 1) I_n + n\, I_{n-1} = 0$ and hence find I_n. [8]

2.(a) Define the Laplace transform $\tilde{f}(p) = L[f(x)]$ of a function f(x) and discuss some conditions under which it exists. Prove that

(i) $L[f'(x)] = p\tilde{f}(p) - f(0)$, $L[f''(x)] = p^2 \tilde{f}(p) - pf(0) - f'(0)$ (2+2) = [4]

(ii) $L[\frac{f(x)}{x}] = \int_p^\infty \tilde{f}(p)\, dp$ [3]

(iii) $L[\int_0^x f(x-y)\, g(y)\, dy] = \tilde{f}(p)\, \tilde{g}(p)$. [6]

(These results may be used in answering (b) and (c) below).

(b) Evaluate the integral $\int_0^\infty \frac{e^{-ax} \sin bx}{x} dx$. [5]

Contd..... 2/-

(c) Show that the solution of

$$\frac{d^2y}{dt^2} + 2a\frac{dy}{dt} + (a^2 + b^2)y = f(t)$$

is $\frac{1}{b}\int_0^t e^{-a\lambda} \sin(b\lambda)\, f(t-\lambda)\, d\lambda$ given that $\frac{dy}{dt} = y(t) = 0$ at $t = 0$ and $b > 0$. [7]

3.(a) Use the methods described in class to find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = 5x + 4y, \\ \frac{dy}{dt} = -x + y \end{cases}$$ [6]

(b) Find by the method of variation of parameters the particular solution of the simultaneous differential equations

$$\frac{dx}{dt} + 3\frac{dy}{dt} + x = 0.$$

$$2\frac{dx}{dt} - \frac{dy}{dt} + 4y = 11e^t$$

which makes $x = 0$, $y = 1$ when $t = 0$. [9]

4.(a) State precisely, <u>without proof</u>, Picard's theorem on the local existence and uniqueness of solution of the initial-value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval $|x - x_0| \le h$. [5]

(b) Let $f(x,y)$ be defined on the rectangle $R = [-1,1] \times [-1,1]$ as follows:

$$f(x,y) = \begin{cases} 0 & \text{if } x = 0 \\ 2\frac{y}{x} & \text{if } x \neq 0 \text{ and } |y| \le x^2 \\ 2x & \text{if } x \neq 0,\ y > x^2 \\ -2x & \text{if } x \neq 0,\ y < -x^2 \end{cases}$$

(i) Prove that $|f(x,y)| \le 2$ for all $(x,y) \in R$. (ii) For each constant C, $|C| \le 1$, show that $y = Cx^2$ is a solution of the initial-value problem $y' = f(x,y)$ with $y = 0$ when $x = 0$. Show also that the graphs of each of these solutions over $(-1,1)$ lies in R. (iii) Apply the method of successive

Contd..... 3/-

Contd..... Q.No.4.(b)

approximation to this initial - value problem, starting with initial guess $y_0(x) = 0$. Determine $y_n(x)$ and show that the approximations converge to a solution of the problem on (-1,1) (iv) Repeat (iii) with initial guess $y_0(x) = x$. Determine $y_n(x)$ and show that the approximations converge to a solution different from any in part (ii). (v) Explain why the solution of the equation in (ii) is non-unique.

[20]

5.(a) Find the general solution of the differential equation

$$(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$$

near the singular point $x = -1$. Examine the nature of the point $x = \infty$ for this equation.

(4+6) = [10]

(b) Defining the Legendre polynomials $P_n(x)$ as

$$P_n(x) = \frac{1}{n!\,2^n} \frac{d^n}{dx^n} [(x^2 - 1)^n],$$

prove that $y = P_n(x)$ satisfies the differential equation

$$(1-x^2) \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

Use the (well-known) orthogonality properties of $P_n(x)$ to show that if, for $-1 < x < 1$, the function $f(x)$ can be expanded in the form $f(x) = \sum_{n=0}^{\infty} a_n P_n(x)$, then

$$a_n = \frac{2n + 1}{2^{n+1}\, n!} \int_{-1}^{1} f^{(n)}(x)\,(1-x^2)^n\,dx.$$

Find the values of a_n for $f(x) = x^3$.

(5+4+3) = [12]

(c) Compute in closed form the Laplace transform of $J_0(x)$, the Bessel function of order 0.

[8]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

FIRST SEMESTRAL EXAMINATION

Stochastic Processes - 2

Date: 16.11.1988 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: There are 5 questions carrying 120 marks. Maximum one can score is 100.

1. Let $\{X_t, t \geq 0\}$ be a continuous time-parameter Markov Chain with countable state space S and stationary transition probabilities $((p_{ij}(t)))$. Assume that $\lim_{t\to 0+} p_{ij}(t) = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = i \end{cases}$; $i, j \in S$. Prove that (a) if for $i, j \in S$ and $t_o > o$ $p_{ij}(t_o) > 0$, then $p_{ij}(t) > 0$ for all $t > t_o$.

   (b) $\lim_{t\to\infty} p_{ij}(t)$ exists for every $i, j \in S$.

   (8+12) = [20]

2. Suppose S is a countable set and suppose for every $i, j \in S$ there exists a function $p_{ij} : (0, \infty) \to \mathbb{R}$ satisfying the following three conditions: for every i, j and s, t,

$$p_{ij}(t) > 0;$$

$$S_i(t) = \sum_{k \in S} p_{ik}(t) \leq 1;$$

$$\sum_{k \in S} p_{ik}(s)\, p_{kj}(t) = p_{ij}(s+t).$$

   (a) For a fixed $i \in S$, show that either $S_i(t) = 1$ for all $t > 0$ or $S_i(t) < 1$ for all $t > 0$.

   (b) Let $\lim_{t\to 0+} p_{ij}(t)$ exist and be denoted by u_{ij}; $i, j \in S$. Show that for every i, j in S

$$u_{ij} \geq 0; \quad \sum_{k \in S} u_{ik} \leq 1 \quad \text{and} \quad u_{ij} \geq \sum_{k \in S} u_{ik} u_{kj}.$$

   Also show that $u_{ij} = \sum_{k \in S} u_{ik} u_{kj}$ if $\sum_{k \in S} u_{ik} = 1$

   (14+11) = [25]

p.t.o.

Let $\{X_t, t \in T\}$ be a stochastic process, where $T = \mathbb{Z}$ or $\mathbb{R}$. When do you say that $\{X_t\}$ is a second order process ? When is a second order process $\{X_t\}$ called second order stationary ? Define the cross-covariance function of two second order processes $\{X_t, t \in T\}$, $\{Y_t, t \in T\}$.

Let $Z_{11}, Z_{12}, Z_{21}, Z_{22}, \ldots, Z_{n1}, Z_{n2}$ be 2n independent random variables such that Z_{ki} are $N(0, \sigma_k^2)$ random variables $i = 1, 2$; $k = 1, 2, \ldots, n$. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be reals. Define

$$X_t = \sum_{k=1}^{n} (Z_{k1} \cos \lambda_k t + Z_{k2} \sin \lambda_k t), \; t \in \mathbb{R}.$$

Show that $\{X_t, t \in \mathbb{R}\}$ is a second order stationary stochastic process. (State clearly the result you may make use of).

((3+4+4 + (11+3)) = [25]

4. Let $\{X_t, t \in T\}$ be as in Q.1. Define the Infinitesimal generator $((q_{ij}))$ of the process. Establish that

$$\sum_{j \in S} q_{ij} \leq 0, \text{ for } i \in S.$$

Define (a) stable (b) instantaneous states of the Markov Chain $\{X_t, t \in T\}$.

(3+7+2) = [12]

5.(a) Define carefully a one-dimensional Wiener process $\{W_t, t \in \mathbb{R}\}$. Show that Wiener process is a second order process which is not second order stationary. Prove that it is also a Gaussian process.

(b) Let $\{W_t, -\infty < t < \infty\}$ be the Standard Wiener process. Find the distribution of $W_1 + W_2 + \ldots + W_n$ where $n \geq 1$.

Put $X_t = e^{-\alpha t} W_{2\alpha t}, -\infty < t < \infty$, $\alpha > 0$ fixed.

Show that $\{X_t\}$ is a stationary Gaussian process. (State clearly the result you would use in this context.)

(6+10+8+7+7) = [38]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

PERIODICAL EXAMINATION

Elective-4: Economics

Date: 2.9.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions. Answer questions from Group A and Group B in separate answer books.

GROUP-A

1. Consider a simple Leontief system where two goods are produced. The technology matrix A is given by

$$\begin{bmatrix} 1/2 & 1 \\ 1/8 & 1/2 \end{bmatrix}$$

where the elements a_{ij} of the A matrix represent the amount of the ith good required to produce 1 unit of the jth good. Moreover, to produce 1 unit of each good, 1 unit of labour is required. The economy is endowed with 240 units of labour.

(a) Derive the equation for the consumption possibility locus.

(b) If the wage rate is equal to unity, find out the price of each good.

(c) Suppose the demand function is given by $C_1/C_2 = P_2/P_1$ where C_1, C_2 are consumption of the two goods and P_1, P_2 are prices. Find out the values of C_1 and C_2.

(15+5+5) = [25]

2. What is meant by the viability of a static Leontief input-output system ? State and interpret the Hawkins-Simsns condition of viability in a 2 x 2 static Leontief system.

(5+20) = [25]

3. Explain the von Neuman method and the Hicksian Neo-Austrian method of incorporating fixed capital in a production system. Show that in the Neo-Austrian structure, a negative relationship exists between the wage rate and the rate of interest.

(10+15) = [25]

p.t.o.

GROUP-B

1. Discuss the structure of Mahalanobis Four Sector Planning model. If we want to introduce some degree of freedom in the Mahalanobis model by allowing the possiblity of technological choice or some control over consumption how they affect the structure of the Mahalanobis Model ? Do you think that given targets of income and employment can meet the requirement of planning for the development process of the economy ?

2. The following pay-off matrix represents the returns expected by a firm for five alternative investments and four different levels of business possibilities. Which alternative would the firm select if their decisions are based on (a) maximin rule, (b) maxi max rule and (c) Hurwitcz rule for $\lambda = .7$ and $\lambda = .4$ where λ = degree of optimism ?

Alternatives	Business Possibilities 1	2	3	4
A	15	11	12	9
B	7	9	12	20
C	8	8	14	17
D	17	5	5	5
E	6	14	8	19

3. Assume a single firm producing a product and meeting the entire domestic requirement for the product. Demand for the product grows over time at a constant rate. Plant life is infinite. Investment cost is subject to economies of scale. Assume also that constant growth of plant size is optimal. Determine the optimal time phasing of investment. (Assume constant time discount factor).

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

PERIODICAL EXAMINATION

Elective-4: Physical and Earth Sciences

Date: 2.9.1988 Maximum Marks: 100 Time: 3 hours

Note: Attempt question no.4 and any FOUR from the rest. Draw sketches wherever necessary.

1. Discuss critically any one of the hypotheses regarding the origin of the earth. [20]

2. What is Bowen's reaction series ? Discuss briefly the origin of ultrabasic and granitic rocks from a parent basaltic magma. A granitic rock is found to grade imperceptably into a metamorphic rock. How do you explain this ?

(8+7+5) = [20]

3. Draw a neat cross-section (to the scale) of the earth to show the crust, lithosphere, low-velocity zone, core-mantle boundary and inner core. Describe the behaviour of seismic waves in these zones. What do you infer about the physical characters of the zones from the behaviour of seismic waves ?

(10+5+5) = [20]

4. Indicate the correct answer (Attempt any ten).

(a) Age of the oldest fossil algae is (i) >1.0 B.Y., (ii) >1.5 B.Y., (iii) >2.8 B.Y., (iv) >4.5 B.Y.

(b) Internal structure of quartz consists of (i) 3-D network of silicate tetrahedra, (ii) single chain of silicate tetrahedra, (iii) double chain of silicate tetrahedra, (iv) none of the above.

(c) Overall density (in gm/c.c.) of the earth is (i) 2.5 - 3.0, (ii) 3.0 - 4.0, (iii) 5.0 - 5.5, (iv) 5.5 - 6.5.

(d) During the last 2 billion years of the earth's history the main reason for oxygen enrichment in the atmosphere is (i) photosynthesis, (ii) volcanic eruptions, (iii) chemical disintegration of oxides, (iv) disintegration of radioactive minerals.

Contd..... 2/-

Contd..... Q.No.4

(e) The fold that closes upward with the youngest rock at its core is a/an (i) antiformal anticline, (ii) synformal syncline, (iii) synformal anticline, (iv) antiformal syncline.

(f) S-waves are cut-off at the mantle-core boundary because of (i) high density of the core material, (ii) weak nature of the waves, (iii) iron-rich composition of the core, (iv) liquid state of the outer core.

(g) Interference colour of a mineral is related to (i) its specific gravity, (ii) double refraction of light rays, (iii) colour of the mineral, (iv) refractive indices of minerals adjacent to it.

(h) Mechanically deposited sedimentary rocks are best identified by their (i) feldspar-rich mineralogical composition, (ii) coarse grain size, (iii) well developed bedding structure, (iv) sedimentary structures like ripple-marks and cross-beds.

(i) A mineral shows white colour, white streak, vitreous lustre, moderate hardness (approx. 3.0), 3 sets of cleavage and effervescing strongly in dilute Hcl is (i) quartz, (ii) olivine (iii) zeolite (iv) calcite.

(j) In the eastern part of a terrain the average orientation of beds is $30^{o}/45^{o}$ -> NW and that in the western part is $35^{o}/60^{o}$ -> SE. The structure is a/an (i) synform, (ii) antiform, (iii) normal fault, (iv) reverse fault.

(k) As compared to present day, a much higher temperature of the earth's interior during first 1 billion years of its history was due to (i) higher amount of solar radiation at that time, (ii) higher amount of heat generated from radioactive minerals during that period, (iii) higher rate of meteoric impact at that time, (iv) higher temperature of the parent star from which the earth was born.

(2x10) = [20]

5. Write short notes on <u>any four</u>:

(a) Interference colour; (b) Metamorphic textures; (c) Classification of fold; (d) Solid solution; (e) Fault; (f) Plate-tectoines.

(4x5) = [20]

p.t.o.

6. Answer briefly:

(a) What are the evidences which suggest that the earth's magnetic field originates from its molten core ?

(b) What are the diagnostic characters of igneous, metamorphic and sedimentary rocks on the basis of which they can be identified in the field ?

(c) In spite of the fact that the basic component of the silicate minerals is the silicate tetrahedra, they show most diverse properties among different qroups. Can you provide an explanation for this diversity ?

(d) Why would you not expect generation of basaltic magma in the upper ten km. of the continental crust ? What kind of intrusive rocks would you expect in a hypothetical planet in which the magmas had the same composition as the earth's crust and in which all magmas had cooled very slowly and all crystals had maintained equilibrium with the melt ?

(4 x 5) = [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1988-89

PERIODICAL EXAMINATION

Difference and Differential Equations

Date: 26.8.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer questions 1 and 2, and any three of the remaining four questions. Marks assigned to questions are given in parenthesis.

1. Let P_1 and P_2 be analytic on an open interval $(x_0 - r, x_0 + r)$, say $P_1(x) = \sum_{n \geq 0} b_n(x - x_0)^n$, $P_2(x) = \sum_{n \geq 0} c_n(x - x_0)^n$. Prove that the differential equation

$$y'' + P_1(x)\, y' + P_2(x)\, y = 0$$

has two independent solutions u_1 and u_2 which are analytic in the same interval. [15]

Find two independent power-series solutions of the equation

$$y'' - 2xy' + 2\alpha y = 0 \quad \text{(where } \alpha \text{ is a constant)}$$

on an interval of the form $(-r, r)$. Show that one of these solutions is a polynomial when α is a non-negative integer. [10]

2. Define the notions of a regular singular point and indicial equation associated with the second order differential equation

$$y'' + P_1(x)\, y' + P_2(x)\, y = 0 \quad \ldots\ldots \quad (1)$$

Assuming that the roots of the indicial equation are real, describe completely, without proof, two independent solutions of (1). [8]

For the equation $x^2 y'' + xy' + x^2 y = 0$, show that

$$y_1 = \sum_{n \geq 0} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$$ is one solution and that the second independent solution is

$$y_2 = y_1 \log x + \sum_{n \geq 1} \frac{(-1)^{n+1}}{2^{2n}(n!)^2} \left(1 + \frac{1}{2} + \ldots + \frac{1}{n}\right) x^{2n}.$$ [12]

p.t.o.

3. Find the general solution of each of the following equations:

(i) $y'' + 4y = 4\cos 2x + 6\cos x + 8x^2 - 4x$ [8]

(ii) $(x^2-1)y'' - 2xy' + 2y = (x^2-1)^2$ [6]

(iii) $(1-x^2)y' - xy = axy^2$, a being a constant. [6]

4. (i) If $M(x,y)\,dx + N(x,y)\,dy = 0$ is not an exact equation but if

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx}$$

is a function $g(z)$, where $z = xy$, show that $\mu = e^{\int g(z)\,dz}$ is an integrating factor of this equation. Use this, or otherwise, to solve $xdy + ydx + 3x^3y^4dy = 0$.

[10]

(ii) Solve : $(y+x)dy = (y-x)dx$. [5]

5. A point Q moves on a straight line L and a point P pursues Q in such a way that the distance from P to Q has a constant value $k > 0$. If P is initially not on L, find the curve of pursuit, i.e. the curve traced out by P. Prove that such a curve is orthogonal to the family of circles with radius k and centres on the y-axis. [20]

6. Find the shape of a curved mirror such that light from a source at the origin will be reflected in a beam of rays parallel to the x-axis.

[20]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988 - 89

PERIODICAL EXAMINATION

Statistical Inference

Date: 31.8.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer as much as you can. The maximum you can score is 100.

1. Let $X_1, X_2, \ldots, X_n$ be independent observations from a discrete distribution which assigns probabilities $\frac{1}{3}$ to each of $\theta-1$, θ and $\theta+1$. Let θ range over all integers.

   (a) Determine the class of all unbiased estimator of zero.

   (b) Show that no non-constant function of θ has UMVUE.

   (5+5) = [10]

2. Let $X_1, X_2, \ldots, X_n$ be independent observations from a probability distribution with density

$$f_\theta(x) = \theta x^{\theta-1} \quad \text{for } 0 < x < 1$$
$$= 0 \quad \text{otherwise}$$

   where $\theta > 0$ is an unknown parameter

   (a) Check that $\sum_1^n \log X_i$ is a sufficient statistic.

   (b) Obtain the Cramer - Rao lower bound for the variance of an unbiased estimator of $\frac{1}{\theta}$ on the basis of $X_1, X_2, \ldots, X_n$

   (c) Using (b) show that $-\frac{1}{n}\sum_1^n \log X_i$ is the UMVUE of $\frac{1}{\theta}$.

   (4+6+10) = [20]

3. Consider the family of probability distributions $\mathcal{P} = \{P_\theta : \theta = 1, 2, \ldots\}$ for a random variable X where each P_θ is defined by

$$P_\theta[X=i] = \frac{1}{\theta} \quad \text{for } i = 1, 2, \ldots, \theta$$
$$= 0 \quad \text{otherwise}$$

   (a) Show that the family of probability distributions $\mathcal{P}$ is complete.

   (b) Show that $2X-1$ is the UMVUE of θ.

Contd..... 2/-

Contd..... Q.No.3

(c) Show that even if you omit one single P_K from $\mathcal{P}$ i.e., if for example you consider $\mathcal{P}_0 = \mathcal{P} - \{P_K\}$, then $\mathcal{P}_0$ is not complete. Determine the class of all unbiased estimators of zero for $\mathcal{P}_0$. Obtain the UMVUE of θ on the basis of X whose unknown distribution is known to belong to $\mathcal{P}_0$.

(6+4+6+6+8) = [30]

4. Let $X_1, X_2, \ldots, X_n$ be independent observations from a probability distribution with density

$$f_\theta(x) = e^{-(x-\theta)} \quad \text{for} \quad x > \theta$$
$$= 0 \quad \text{otherwise}$$

where $-\infty < \theta < \infty$ is an unknown parameter

(a) Compute $g(\theta) = P_\theta[X_1 \geq 0]$.

(b) Show that $T = \underset{i}{\text{Min}}\, X_i$ is a sufficient.

(c) Show that $X_1 - T$ and T are independent.

(d) Find that the UMVUE of $g(\theta) = P_\theta[X_1 \geq 0]$.

(5+5+5+15) = [30]

5. (a) Let $X_1, X_2, \ldots, X_n$ be iid according to Poisson distribution $P(\lambda)$. Find the UMVUE of λ^3.

(b) Show that the correlation coefficient between the UMVUE T of an unbiasedly estimable parametric function $h(\theta)$ and any other unbiased estimator T_1 of $h(\theta)$ is

$$\sqrt{\frac{Var_\theta(T)}{Var_\theta(T_1)}}.$$

(10+10) = [20]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

PERIODICAL EXAMINATION

Sample Surveys

Date: 29.8.1988 Maximum Marks: 100 Time: 3 hours

Note: Answer question ONE and any FOUR from the remaining. Maximum marks for each subdivision of a question has been indicated in the margin.

1.(a) Define the following terms in the context of survey sampling:

(i) Finite Population
(ii) Sample
(iii) Sample Space
(iv) Sampling design
(v) Estimator.

(b) Define first and second order inclusion probabilities of units in a design. Let $t = \sum_{i \in S} a_i y_i$ be a linear estimator of a population parameter θ, where $a_1, \ldots, a_N$ are known constants attached to units 1, ..., N in the population. Find

(i) $E(t)$ (ii) $E(t^2)$.

[(3 x 5) + (2+2+4+5)] = [28]

2.(a) Define a simple random sampling without replacement SRSWOR procedure. Show that under SRSWOR, an unbiased estimator of variance of sample mean exists. Compare a SRSWOR with a simple random sampling with replacement (SRSWR) procedure.

(b) A SRSWR of 2070 farms is taken from a population of 20760 farms and information is collected on the number of cattles on each farm. The following data were obtained:

$$\sum_{i=1}^{n} y_i = 25,881 \; ; \quad \sum_{1}^{n} y_i^2 = 599,486.$$

Estimate the total number of cattle in the population along with rse. Also find 95% confidence interval for the total number of cattle.

[(3+5+3) + (1+4+2)] = [18]

p.t.o.

3.(a) Assuming a linear cost function find optimum sample size n_h for different strata in a stratified random sampling when total cost of the survey is assumed to be fixed.

(b) For a socio-economic survey, all the villages in a region including the uninhabitated ones were grouped into 4 strata on the basis of their attitude above sea-level and population density and from each strata some villages were selected.

Stratum No.	Total no. of villages	Total no. of households in sampled villages				
1	140	43	84	98		
2	470	50	147	62	87	
3	256	228	262	110		
4	1500	17	34	25	36	0

(i) Estimate the total number of households and its rse.

(ii) Examine whether there has been any gain due to use of stratification as compared to unstratified SRSWR.

(8+10) = [18]

4.(a) A survey was conducted in a village consisting of 625 households by covering a sample of 50 households drawn with SRSWOR to estimate the average monthly household expenditure on toilet goods. The estimate turned out to be Rs.4.20 with a standard error of Rs.0.17. Using this information determine the sample size needed to estimate the same characteristic in a neighbouring village on the basis of a sample selected with SRSWR such that the length of the confidence interval at 95% of the true value. State the assumption involved in finding the sample size.

(b) Prove that under certain assumptions (to be stated by you)

$$V_{ran} \geq V_{prop} \geq V_{opt}.$$

Can it hold that $V_{prop} > V_{ran}$? Explain, the symbols have their usual meanings.

(8+7+3) = [18]

5.(a) Obtain the large sample formulae for the variance of ratio estimator of population total. Also obtain two estimators for the variance. Find an expression for the bias of the estimator and compare it with its standard error.

Contd..... Q.No.5

(b) In stratified random sampling obtain two ratio estimators of population total and make an analytic study of their precisions.

[(5+2+4+2)+5] = [18]

6.(a) Considering clusters of equal sizes, compare the variance of the estimator based on cluster sampling with that of the mean per unit estimator based on a comparable simple random sampling procedure.

(b) In compare the precision of ratio estimator with that of mean per unit estimator in SRSWOR.

(12+6) = [18]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1988-89

PERIODICAL EXAMINATION

Stochastic Processes-2

Date: 23.8.1988 Maximum Marks: 50 Time 2 hours

Note: In the sequel T stands for set of non-negative real numbers, T_o for the set of positive real numbers and S a countable set. Answer all the questions.

1. When is the continuous time-parameter stochastic process $\{X_t, t\in T\}$ which assumes values in the state space S, called a continuous time-parameter Markov Chair ? When do you say that the Chain has stationary transition probabilities ? If the Chain has stationary transition probabilities, state and prove the Chapman-Kolmogorov equation.

(3+2+5) = [10]

2. A transition matrix is an array of functions $((p_{ij}))$, $i, j \in S$, defined on T_o and satisfying the following three conditions : for every i, j and s , t,

$$p_{ij}(t) \geq 0 ;$$

$$\sum_{j\in S} p_{ij}(t) = 1 ;$$

$$\sum_{k\in S} p_{ik}(s)\, p_{kj}(t) = p_{ij}(s+t).$$

(a) Show that for any transition matrix $((p_{ij}))$ and for any fixed $h>0$ the expression $\sum_j |p_{ij}(t+h) - p_{ij}(t)|$ is non-increasing as t increases.

(b) Suppose a transition matrix $((p_{ij}))$ moreover satisfies

$$\lim_{t\to 0} p_{ij}(t) = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Also define $p_{ij}(0) = \delta_{ij}$.

Contd..... 2/-

Contd..... Q.No.2 (b)

Show that such a transition matrix satisfies the following:

(i) For every i, j, $t \geq 0$ and $h > 0$

$$| p_{ij}(t+h) - p_{ij}(t) | \leq 1 - p_{ii}(h).$$

(ii) For every i, $t \geq 0$, $p_{ii}(t) > 0$.

(6+5+9) = [20]

3. Let $((p_{ij}))$, $i, j \in S$, be an array of functions defined on T_o and satisfy the following three conditions : for every i, j and s, t,

$$p_{ij}(t) > 0 ;$$

$$S_i(t) = \sum_j p_{ij}(t) \leq 1 ;$$

$$\sum_k p_{ik}(s)\, p_{kj}(t) = p_{ij}(s+t).$$

Fix $i \in S$. Show that either $S_i(t) = 1$ for all t or $S_i(t) < 1$ for all t. (Hint: Show that if there exist $j \in S$ and $t_o \in T_o$ such that $S_j(t_o) < 1$ then $S_i(t) < 1$ for $t > t_o$. Now consider the set $\{ t \in T_o : S_i(t) < 1 \}$. Show that this set is either empty or the infimum of the set is zero).

[14]

4. Consider a Birth and Death chain with state space $S = \{ 0, 1, 2, 3, \ldots\ldots \}$ and one-step transition matrix $((p_{ij}))$ where

$$p_{i,i+1} = r_i, \quad p_{ii} = s_i, \quad p_{i,i-1} = t_i \quad \text{for } i \geq 1$$

$$p_{oo} = s_o, \quad p_{o1} = r_o \qquad \text{and}$$

$$0 \leq r_i,\ s_i,\ t_i\ ; \quad r_i + s_i + t_i = 1 \quad \text{for } i \geq 1$$

$$r_o + s_o = 1\ ; \quad r_i > 0 \text{ for } i \geq 0,\ t_i > 0 \text{ for } i \geq 1.$$

Find a necessary and sufficient condition for transience. (State clearly the theorem you may use in this context).

[12]

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