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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

BACKPAPER EXAMINATION

Multivariate Distributions and Tests

Date: 6.7.1990 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

GROUP - A

Note: Answer any THREE questions.

1.(a) Let $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. Σ positive definite

(i) Obtain the characteristic function of the distribution.

(ii) Find the conditional distribution $\underset{\sim}{X}^{(1)'} = (X_1, \ldots, X_{p_1})$ given $\underset{\sim}{X}^{(2)'} = (X_{p_1+1}, \ldots, X_p)$.

(b) Let $\underset{\sim}{X} \sim N_3(0, \Sigma)$, where

$$\Sigma = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}.$$

Is there a value of ρ for which $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ are independent ?

[(7+7) + 8] = [22]

2. Let $\underset{\sim}{X}_\alpha \sim N_p(\underset{\sim}{\mu}, \Sigma)$, Σ, positive definite, $\alpha = 1 \ldots N(>p)$.

Define $\bar{\underset{\sim}{X}} = \frac{1}{N}\sum_{\alpha=1}^{N} X_\alpha$, $A = \sum_{\alpha=1}^{N}(\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})(\underset{\sim}{X}_\alpha - \bar{\underset{\sim}{X}})'$.

Prove that

(a) $\bar{\underset{\sim}{X}}$ and A are independently distributed.

(b) $\mathrm{Tr}\,\Sigma^{-1} A \sim \chi^2(p(N-1))$.

(c) $K|A|$ (where K is some constant to be determined) is distributed as product of p independent chi-square variables.

(8+6+8) = [22]

3. Work out the p.d.f. of the central wishart distribution.

[22]

p.t.o.

4. Let $\underset{\sim}{X}_{i\alpha} \sim N_p(\underset{\sim}{\mu}_i, \Sigma)$, $i = 1, 2, 3$, $\alpha = 1, 2, \ldots, N_i$. Suggest a suitable test for testing the hypothesis $H_0[2\underset{\sim}{\mu}_1 = \underset{\sim}{\mu}_2 + \underset{\sim}{\mu}_3]$ and hence derive the distribution of the test statistic under H_0.

(8+14) = [22]

5. Let $\underset{\sim}{X}_\alpha \sim N_p(\underset{\sim}{\mu}, \Sigma)$, $\alpha = 1, \ldots, N(>p)$. Derive the likelihood ratio test for testing the population multiple correlation of X_1 on $\underset{\sim}{X}^{(2)'} = (X_2, \ldots, X_p)$ to be zero. Hence derive the distribution of the sample multiple correlation under H_0.

(10+12) = [22]

GROUP - B

Note: Answer ALL the questions.

6. In a certain examination each student has to answer three essay-type questions which were valued independently by the two examiners A and B. The difference between the scores given by the two examiners (A minus B) for the i-th question was denoted by X_i, $i = 1, 2, 3$. The mean vector and dispersion matrix of X_1, X_2, X_3 based on 50 observations are given below.

	Mean	Dispersion matrix		
		X_1	X_2	X_3
X_1	2.54	16.90	22.01	12.35
X_2	-1.72		51.72	19.46
X_3	0.94			28.73

Examine simultaneously whether the average difference between the two examiners for each question is zero at 5% level.

[8]

7. In an investigation of the relation of the Wechsler Adult Intelligence Scale to age, following matrix of correlations was obtained from the measurements : digit span (X_1) and

Contd..... 3/-

vocabulary (X_2) subtests, chronological age (X_3) and years of formal education (X_4):

$$R = \begin{pmatrix} 1 & .45 & -.19 & .43 \\ & 1 & -.02 & .62 \\ & & 1 & -.29 \\ & & & 1 \end{pmatrix}$$

(a) Test the hypothesis of independence of the subtests (X_1, X_2) and age and education (X_3, X_4).

(b) Compute the partial correlation between X_1 and X_2 holding X_3 and X_4 constant and test the hypothesis of zero population partial correlation.

(8+8) = [16]

8. Practical Note Book. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

BACKPAPER EXAMINATION

Non-parametric and Sequential Methods

Date: 3.7.1990 Maximum Marks: 100 Time: 4 hours

Note: Use separate answerscripts for Groups A and B.

GROUP - A

Max. Marks: 80

Note: Answer as much as you can.

1.(a) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ be random samples from F(x) and G(x) respectively, which are assumed to be continuous. Derive a suitable test using empirical distributions for $H_0 : F \equiv G$ against H_1 : F is stochastically larger than G. Show that the sampling distribution of the test statistic is distribution free under H_0.

(b) Samples of 12 children each are drawn from two social groups and their IQ was recorded. Examine whether the distributions of IQ can be regarded as the same for both social groups.

Group 1 : 89, 95, 82, 101, 91, 85, 96, 93, 86, 99, 90, 84.

Group 2 : 103, 97, 100, 88, 86, 97, 105, 81, 86, 110, 99, 87.

(8+8) = [16]

2.(a) A random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$ is drawn from a bivariate population where the measurement is at least ordinal. Describe how you would get Spearman's rank correlation coefficient R for this data. Show that the coefficient satisfies all the desirable properties of a measure of association. Show further that if X and Y are independent, the distribution of R is symmetric about 0.

(b) The following data give the scores on two psychological characteristics of 12 patients. Examine whether the association between the two characteristics (as measured the grade by Spearman's correlation coefficient) can be regarded

Contd..... 2/-

as 0.5. (You can use the large sample approximation of the sampling distributions you need).

X	25	14	36	21	28	12	19	29	31	22	16	40
Y	14	12	13	12	15	12	11	11	10	13	12	12.

(8+8) = [16]

3.(a) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ be random samples from two independent populations with distributions $F(x)$ and $G(x)$ respectively. Describe the use of general linear rank order statistics for testing $H_0 : F(x) = G(x)$ against H_1 : F and G differ in their location parameter. (Discuss the case of finite sample sizes and large samples).

(b) Examine in detail the exact and asymptotic distribution of the test statistic under H_0 if expectations of order statistics from $N(0,1)$ are taken as scores.

(10+6) = [16]

4.(a) Three treatments are studied each in 6 blocks and the results are as given below. If the observations cannot be assumed to follow normal distribution, examine whether there is any difference in the treatments (Assume that there is no block-treatment interaction).

Block	Treatment 1	2	3
A	28	34	51
B	33	31	72
C	14	20	21
D	6	18	13
E	9	15	53
F	14	22	40

(b) The SAT scores of 10 students in region 1 and 8 students in region 2 are given below. Assuming the average scores are the same in the two regions examine whether there is more variability in region 1 than in region 2.

Region 1 : 510, 513, 618, 679, 710, 581, 635, 575, 700, 688

Region 2 : 710, 518, 634, 656, 547, 593, 632, 678.

(9+7) = [16]

5.(a) A random sample of n pairs of observations $(X_1, Y_1), \ldots, (X_n, Y_n)$ is observed from a bivariate population. Under suitable assumptions to be stated by you, devise a test for testing whether the means of the two random variables are the same against $E(X) > E(Y)$.

(b) An operator at a machine gave the following sequence of the results of the items produced at the end of the shift. Do you consider that the defective items are produced at random in the production line.

N N N N D D N N N N N D N D N N N D D D N N N N D
N N N N N D N D N N N N N D.

Where N stands for a nondefective item and D for a defective item produced by the machine.

(10+6) = [16]

GROUP - B

Sequencial Analysis

Max.Marks: 20

Note: Answer ALL the questions.

1.(a) Brefly describe Stien's two stage procedure for determining an interval estimate of a given length for the mean of a normal distribution when variance is unknown.

(b) Let Z be a random variable such that

(i) $P(Z > 0) > 0$ and $P(Z < 0) > 0$,

(ii) $\phi(t) = E(e^{tZ})$ exists for any real value t, and

(iii) $E(Z) \neq 0$.

Then show that there exists a $\gamma \neq 0$ such that $\phi(\gamma) = 1$.

(4+8) = [12]

2. If for the boundary points (A,B) with

$$A = \frac{1-\beta}{\alpha}, \qquad B = \frac{\beta}{1-\alpha}$$

the SPRT terminates with probability one and is of strength (α', β') then show that

$$\alpha' \leq \frac{\alpha}{1-\beta}, \quad \beta' \leq \frac{\beta}{1-\alpha} \quad \text{and} \quad (\alpha' + \beta') \leq (\alpha + \beta).$$

Explain its implications. [8]

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B.Stat. (Hons.) III Year : 1989-90

BACKPAPER EXAMINATION

Optimization Techniques

Date: 28.6.1990 Maximum Marks: 100 Time: 3 hours

1. Find $\xi_1, \xi_2, \xi_3 \geqq 0$ such that $8\xi_1 + 19\xi_2 + 7\xi_3$ is maximum subject to

$$3\xi_1 + 4\xi_2 + \xi_3 \leqq 25$$
$$\xi_1 + 3\xi_2 + 3\xi_3 \leqq 50.$$ [15]

2. Does the following problem have an optimal solution ? Give reasons for your answer.

Find $\xi_1, \xi_2 \geqq 0$ such that $2\xi_1 + 3\xi_2$ is maximum subject to:

$$-3\xi_1 + 2\xi_2 \leqq 0$$
$$\xi_1 - \xi_2 \leqq 2$$ [15]

3. Let $\bar{x} = (\bar{\xi}_1, \ldots, \bar{\xi}_m)$ be an optimal solution to the general linear programming problem of maximizing cx subject to arbitrary linear constraints. Let $c' = (\gamma_1 + \delta, \gamma_2, \ldots, \gamma_m)$ where $c = (\gamma_1, \ldots, \gamma_m)$ and $\delta > 0$ and let $x' = (\xi_1', \ldots, \xi_m')$ be an optimal solution to the problem of maximizing $c'x$ subject to the same constraints. Show that $\xi_1' \geqq \bar{\xi}_1$.

[15]

4. Let (A, b, c) stand for the problem:

Find $x \geqq 0$ such that cx is maximum subject to $xA = b$.

Suppose $\{c_k\}$ is a sequence of vectors with limit c and w_k is the value of (A, b, c_k) for $k = 1, 2, \ldots$. If w is the value of (A, b, c) then show that $w = \lim_{k \to \infty} w_k$.

[20]

Let (N, k) be a capacitated network. Let f be a flow from s to s' and (S, S') be a cut with respect to s and s'.

(i) Prove that $f(x,y) = k(x,y)$ for all $x \in S$ and $y \in S'$ if and only if f is a maximal flow and (S, S') a minimal cut.

Contd..... 2/-

(ii) Prove that if (S,S') and (R,R') are minimal cuts with respect to s and s', then so is $(S \cap R,\ S' \cup R')$.

[20]

6. Find the strategies and the pay off function for the following infinite game:

P_1 draws one of two cards marked 'H' and 'L' at random. He then bets an amount x, $1 \leq x \leq 2$.

P_2 hears P_1's bet and then either 'folds' or 'calls'. If he folds, he pays P_1 an amount 1. If he calls, P_1 shows his card. P_1 then wins or loses an amount x according as his card is marked H or L.

[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989 - 90

SUPPLEMENTARY EXAMINATION

Statistical Inference

Date: 18.6.1990 Maximum Marks: 65 Time: 3 hours

1. Give a formal statement of the following model identifying the probability law of the data and the parameter space. Write down the likelihood and obtain a set of (non-trivial) sufficient statistics.

   The number of eggs laid by an insect follows a Poisson distribution with unknown mean λ. Once laid, each egg has an unknown chance p of hatching and the hatching of one egg is independent of the hatching of the others. An entomologist studies the set of n such insects observing both the number of eggs laid and the number of eggs hatching for each nest.

   [10]

2. Let X have a Poisson distribution with mean θ.

   $$T(x) = 1, \text{ if } x = 0$$
   $$= 0, \text{ otherwise.}$$

   Calculate the Cramer-Rao bound for T and show that it is strictly less than the variance of T. Is T an UMVU estimate of its expectation ? Give reasons.

   (8+4) = [12]

3. State and prove the Rao-Blackwell Theorem. [10]

4. Suppose that $X_1, \ldots, X_n$ are independently and identically distributed according to the uniform distribution $U(0,\theta)$. Let $M_n = \max(x_1, \ldots, x_n)$, and let

   $$\delta_c(\underset{\sim}{X}) = \begin{cases} 1, & \text{if } M_n \geq c \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Compute the power function of δ_c and show that it is monotone increasing in θ.

   (b) In testing $H : \theta \leq \frac{1}{2}$ versess $K : \theta > \frac{1}{2}$, what choice of c would make δ_c have size exactly 0.05 ?

   (c) How large should n be so that δ_c as determined in (b) has power 0.98 for $\theta = \frac{3}{4}$?

   (4+4+4) = [12]

p.t.o.

5. Let $X_1, \ldots, X_n$ denote the times in days to failure of n similar pieces of equipment. Assume that the failure time X has the density

$$f(x,\theta) = \theta e^{-\theta x}, \quad x > 0$$
$$= 0, \quad \text{otherwise.}$$

Derive explicitly the UMP level α test for testing $\theta \geq \theta_0$ against $\theta < \theta_0$ based on $X_1, \ldots, X_n$.

[12]

6. A gambler observing a game in which a single die is tossed repeatedly gets the impression that 6 comes up about 18% of the time, 5 about 14% of time, while the other four numbers are equally likely to occur. Upon being asked to play, the gambler asks that he first be allowed to test his hypothesis by tossing the die N times.

(a) What test statistic should he use if the only alternative he considers is that the die is fair ?

(b) Show that if N = 2 the MP level 0.0324 test rejects his hypothesis if, and only if, two 6's are obtained.

(4+5) = [9]

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B.Stat. (Hons.) III Year : 1989-90

SEMESTRAL-II EXAMINATION

Elective-5 : Economics

Date: 10.5.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.No.1 and any TWO from the rest. The questions (including Practical Exercise) carries 110 marks but the maximum you can score is 100.

1. The table below gives the family-budget data for a few sample households belonging to four low-income classes in a country during a particular year.

	Yearly income per consumer unit in Rs.			
	below 600	600-	750-	1050-
No. of sample households	136	179	111	22
Average no. of consumer units per households	2.60	2.57	2.50	2.48
Average income per consumer unit	543.1	681.3	861.9	1232.0
Average expenditure on food per consumer unit	291.8	331.6	374.4	407.1

Calculate the income elasticity of demand for food, assuming the demand function to be of the constant elasticity form.

[28]

2.(a) Define the 'Specific Concentration Curve' (SCC) for an item of consumer expenditure.

(b) How is the SCC of (i) an inferior, (ii) a necessary and (iii) a luxury item related to the Lorenz curve and the egalitarean line ? Derive these relationships.

(c) Assuming that income $x \sim \Lambda(\mu, \sigma^2)$ and the Engel curve for an item is of the constant elasticity form, describe the alternative methods of estimation of Engel elasticity for the item using the SCC.

(5+21+7) = [33]

p.t.o.

3.(a) Discuss the problems of 'Identification' and 'Least Squares bias' likely to arise in the estimation of demand function from time series data. How does one overcome these problem

(b) Show that in a regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k,$$

with near-exact multicollinearity, variance of the least squares estimator $\hat{\beta}_j$ tends to ∞ as $R^2_{j.12,\dots,j-1,j+1,\dots,k}$ approaches 1, where $R^2_{j.12,\dots,j-1,\ j+1,\dots,k}$ is the coefficient of determination of the linear regression of x_j on the other regressors.

(18+15) = [33]

4.(a) Define constant, increasing and decreasing returns to scale for a homogeneous production function.

(b) Define the elasticity of substitution for a production function with two inputs-capital (K) and labour (L). Also, draw the isoquants corresponding to the cases where the elasticity of substitution between the inputs is (i) zero, (ii) infinity and (iii) positive and finite.

(c) Using a Cobb-Douglas production function show that constant and increasing returns to scale are incompatible with a determinate solution of the problem of profit maximisation under perfect competition.

(d) Obtain the elasticity of substitution for the CES production function. In what sense is it a generalisation of the Cobb-Douglas production function ?

(6+10+10+7) = [33]

Practical Exercise. [16]

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SEMESTRAL-II EXAMINATION

Elective-5 : Physical and Earth Sciences

Date: 10.5.1990 Maximum Marks: 100 Time: 3 hours

Note: You can answer ALL questions. Maximum score that you can obtain is 100. Draw neat diagram whenever necessary.

Assignments (Sessional). [15]

1.(a) Fill up the blanks: [10]

(i) In si, barrier potential is nearly _____ volt.

(ii) When a pentavalent material is added to a pure Ge crystal, we get _____ type _____ semiconductor.

(iii) A zener diode operatersin the _____ region.

(iv) In a transisto., the collector-base junction is always _____ biased.

(v) A transistor is said to be in saturation when V_{CE} equals nearly _____ volts.

(vi) _____ feed back is used in oscillator circuits whereas _____ feed back is employed in amplifier circuits.

(vii) A transformer works on the principle of _____ .

(viii) Hysterisis loss in a magnetic material is due to its _____ and _____.

(ix) The main purpose of lainmating the armature core of a d.c. generator is to reduce _____ loss.

(x) The Hysteresis loss in a magnetic material is converted into _____.

(b) Tick the answer which you think most appropriate: [10]

(i) Feed back in an amplifier helps to

(a) control its output
(b) increase its gain
(c) decrease its input impedance
(d) stabilize its gain.

p.t.o.

(ii) Improper biasing of a transistor circuit leads to:

(a) excessive heat production in collector.
(b) distortion in output signal.
(c) faultly location of load line.
(d) heavy loading of emitter terminal.

(iii) In case of a bipolar junction transistor α is

(a) positive and greater than 1.
(b) positive and less than 1.
(c) negative and greater than 1.
(d) negative and less than 1.

(iv) Leakage current I_{CO} of a transistor is due to

(a) reverse biasing of collectors base junction.
(b) minority carriers.
(c) majority carriers.
(d) increase in temperature.

(v) A P-N junction diode is mainly applied as

(a) an amplifier.
(b) an oscillator.
(c) a rectifier.
(d) a frequency generator.

(vi) The d.c. output voltage of a fullwave rectifier having a total secondary peak voltage of 100 v is _____ volt.

(a) 63.6 (b) 31.8 (c) 90 (d) 70.7

(vii) The depletion region around a p-N junction _____

(a) is quite wide
(b) contains mobile ions
(c) has no free charge carriers
(d) has a constant width.

(viii) You have to replace a 1500 Ω resistor by several 1000 Ω resistors. You would connect

(a) three in parallel
(b) three in series
(c) two in parallel and one in series
(d) two in parallel.

(ix) A 3 μF capacitor is series connected with a parallel combination of 2 μF and 4 μF capacitors, total capacitance is

(a) 3 μF (b) 13/3 μF (c) 2 μF (d) 9/11 μF.

(x) Current changing at the rate of .5 Amp./sec. induces an e.m.f of 2 v in 9 coil. The self inductance of the coil is

(a) 4 H (b) 2 H (c) 8 H (d) 1 H.

2.(a)

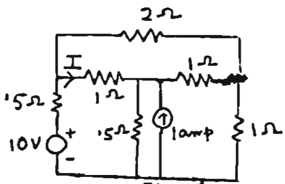

Fig. 1

Find I in fig.1. [8]

(b)

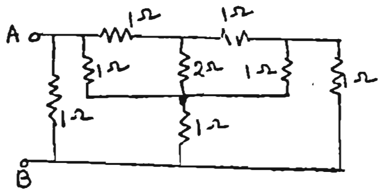

Fig. 2

Find the equivalent resistance between A and B. (Simplification in different steps should be clear) [5]

(c)

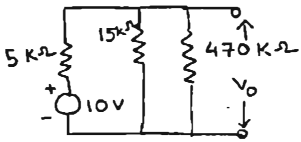

Fig. 3

Calculate V_o in fig. 3. [3]

3.

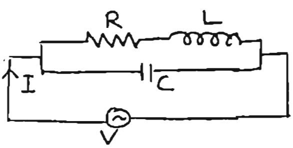

Fig. 4.

State the condition at which the circuit of fig.4 is in resonance. Hence deduce the current at resonance and resonant frequency in terms of the circuit components.

Why such a circuit is some times called rejecter circuit ?

(2+3+3+1) = [9]

In fig. 4, let $R = 30\,\Omega$, $L = 20$ mH and V is 25 V with frequency $1000/\pi$ Hz. If the capacitor C is varied until the current taken from the supply is minimum, find

(a) the capacitance of the circuit at that condition.
(b) the value of the current. [7]

4. Deduce the e.m.f. equation of a d.c. generator. Name the classes of generators classified according to the way in which their fields are excited.

(3+3) = [6]

A 4 pole, lapwound d.c. shunt generator has a useful flux/pole of 0.07 wb. The armature winding consists of 220 turns each of 0.004 Ω resistance. Calculate the terminal voltage when running at 900 r.p.m if the armature current is 50 A.

[6]

The no-load ratio of a 50 Hz, single phase transformer is 6000/250 V. Estimate the number of turns in each winding if the maximum flux is 0.06 wb in the core.

[4]

5. Describe the operation of a full wave rectifier using junction diode with neat diagram and deduce the output d.c. voltage, output r.m.s. voltage, ripple factor and rectification efficiency.

Name the different applications of junction diode. How zener diode is used as a voltage regulator ? What is LED ?

(6+1+1+1+1+2+2+2) = [16]

6. Give the name of the various methods used for biasing transistor circuits. Draw the diagram of any two biasing circuits. Explain the operation of a CE amplifier and mention its characteristics. What is a feed back amplifier? Deduce the gain of the amplifier with feed back in terms of feed back factor and the gain without feed back. State the condition for oscillation.

(2+4+3+3+1+2+1) = [16]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

SEMESTRAL-II EXAMINATION

Elective-5 : Biological Sciences

Date: 10.5.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions.

1.(a) Applying linearization theorem analyse the Volterra system and discuss its stability properties. Is the Volterra system structurally stable ? [10]

(b) Define a limit cycle. Give an example of a dynamical system with this limit cycle behaviour and discuss the nature of the trajectories in this case. [10]

2.(a) State the Bendixon's negative criterion for a closed trajectory of the dynamical system

$$\frac{dx}{dt} = \phi(x,y)$$
$$\frac{dy}{dt} = \psi(x,y)$$ [5]

(b) Verify that the following systems satisfy Bendixon's criterion:

(i) The undamped harmonic oscillator,

(ii) The Volterra system in the first quadrant. [5]

(c) State and discuss the Rashevsky - Turing theory of morphogenesis and explain by a simple mathematical model the development of polarity of a type typical in developing Biological systems. [10]

3.(a) Define discontinuous Markov Process. Write down the forward and backward system of Kolmogorov differential equations. [5]

(b) Derive the differential-difference equation for a Pure Birth Process. Find out the Yule-Furry distribution using the method of generating function. [15]

p.t.o.

4.(a) Give a simple formulation of a deterministic epidemic model which explains the characteristic maximum in the epidemic curve. [10]

(b) Derive the differential equation for the stochastic model of simple epidemics. Does this model yield the solution as that of the deterministic model ? [10]

5.(a) Prove that if $x_0 < \rho$, y decreases monotonically to zero and no epidemic occurs. If $x_0 > \rho$, y increases first and then decreases monotonically to zero.

(Hints: $\lim_{t \to \infty} x(t) = x_\infty$ exists and is the unique root of the threshold equation

$$x_0 \exp\left(\frac{m-n}{\rho}\right) - m = 0$$

where

x_0 = number of susceptibles at $t=0$
y = number of infectives
n = total population
ρ = removal rate
m = the unique root of the threshold equation). [10]

(b) Examine the stability of the equilibrium state in the basic deterministic model of recurrent epidemics. [10]

6.(a) State Rene Thom's theorem on generic properties on a class of potential functions. [5]

(b) Define gradient system in $\mathbb{R}^n$. Determine elementary catastrophe set for the following system in $\alpha - \beta$ plane

$$\dot{x} = -U'(x, \alpha, \beta)$$

where $U(x,\alpha,\beta) = \frac{x^4}{4} - \frac{\alpha x^2}{2} - \beta x$. [15]

7.(a) Explain entropy of a finite probability distribution. Show that under suitable assumptions the entropy function $H(p_1, p_2, \ldots, p_n)$ can only be represented by $-C \sum_{i=1}^{n} p_i \log_2$ C being arbitrary positive constant. [10]

(b) Show that the joint entropy H(X,Y) can never be greater than H(X) + H(Y). [10]

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INDIAN STATISTICAL INSTITUTE
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SEMESTRAL-II EXAMINATION

Multivariate Distributions and Tests

Date: 7.5.1990 Maximum Marks: 100 Time: 4 hours

GROUP - A

Note: Answer any THREE questions.

1.(a) Let $\underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_N$ be a random sample from $N_p(\underset{\sim}{\mu}, \Sigma)$, $\Sigma > 0$, $N > p$. Find the maximum likelihood estimate of $\underset{\sim}{\mu}$ and Σ. Hence show that the sample multiple correlation coefficient $R_{1.2,\ldots,p}$ is the m.l.e. of the corresponding population multiple correlation.

(b) Without deriving the distribution explicitly, show that the distribution of $R_{1.2,\ldots,p}$ involves $\underset{\sim}{\mu}$ and Σ only through the population multiple correlation coefficient.

(c) What are canonical correlations and variates between two sets of variates $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$? Suppose $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are jointly distributed as the normal distribution. Obtain the likelihood-ratio test for the independence between $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ based on a random sample of observations on $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$. Show that the likelihood-ratio statistic is a function of the sample canonical correlations.

(6+6+10) = [22]

2.(a) Let $A \sim W_p(n, \Sigma)$, $\Sigma > 0$. Consider the partition of

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{matrix} r \\ s \end{matrix}$$
$$\quad\quad\quad r \quad\quad s$$

and a similar partitioning of Σ. Prove that $A_{22} - A_{21} A_{11}^{-1} A_{12}$ is distributed as the wishart distribution

Contd..... 2/-

$W_s(n-r, \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12})$, and is independent of A_{11} and A_{12}.

(b) Let $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, where $\underset{\sim}{X}' = (x_1, x_2, \ldots, x_p)$. Derive the distribution of the sample partial correlation coefficient $r_{12.34, \ldots, p}$ based on a random sample of size N from the above distribution.

(12+10) = [22]

3. Let $\underset{\sim}{\bar{X}}$ and A be the mean vector and S.S. and S.P. matrix respectively in a random sample of size $N(>p)$ from $N_p(\underset{\sim}{\mu}, \Sigma)$, $\Sigma > 0$. Show that for a non-null vector $\underset{\sim}{a}$ $(p \times 1)$,

$$\underset{\underset{\sim}{a}}{\text{Max}} \frac{N(\underset{\sim}{a}'\underset{\sim}{\bar{X}})^2}{\underset{\sim}{a}' A \underset{\sim}{a}} = \frac{T^2}{N-1}$$

where T^2 is the Hotelling's T^2-statistic for testing $H_0[\mu = 0]$ against $H[\mu \neq 0]$.

Derive the distribution of T^2 under H_0.

Hence work out the distribution of the statistic

$$\frac{T_1^2}{N-1} = N\underset{\sim}{\bar{X}}'(A + N\underset{\sim}{\bar{X}}\underset{\sim}{\bar{X}}')^{-1}\underset{\sim}{\bar{X}}.$$

[6+10+6) = [22]

4. Let Y and $X_1, \ldots, X_p$ follow jointly a(p+1) - variate normal distribution and $b_1, \ldots, b_p$ be the sample partial regression coefficients of Y on $X_1, \ldots, X_p$ based on a random sample from this population. Derive the joint distribution of $b_1, \ldots, b_p$.

If $\beta_1, \ldots, \beta_p$ be the corresponding population partial regression coefficients, suggest a suitable test for testing $H_0[\beta_1 = \ldots + \beta_p = 0]$. Also give an outline of the derivation of the distribution of the test statistic under H_0.

(12+5+5) = [22]

GROUP - B

Note: Answer ALL questions.

5. A researcher considered three indices (X_1, X_2, X_3) measuring the severity of heart attacks. The values of these indices for N = 30 heart attack patients arriving at a hospital emergency room produced summary statistics:

$$\bar{X} = \begin{pmatrix} 46.1 \\ 57.3 \\ 50.4 \end{pmatrix} \qquad S = \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ & 80.2 & 55.6 \\ & & 97.4 \end{bmatrix}$$

Assume that all three indices are evaluated for each patient and they jointly follow a trivariate normal distribution.

(a) Test the hypothesis of equality of means of the three indices.

(b) Judge the differences in pairs of mean indices using 95% simultaneous confidence intervals.

(c) Test the hypothesis of equality of the variances of X_2 and X_3, when the population correlation $\rho_{23} = 0.75$.

(6+6+6) = [18]

6. In a reaction - time study 32 male and 32 female young normal subjects reacted to visual stimuli pre-ceded by warning intervals of different lengths. The sample covariance matrices of reaction times with preparatory intervals of 0.5 and 15 secs were

$$S_M = \begin{pmatrix} 4.32 & 1.88 \\ 1.88 & 9.18 \end{pmatrix} \qquad S_F = \begin{pmatrix} 2.52 & 1.90 \\ 1.90 & 10.06 \end{pmatrix}$$

where the elements are in units of 10^{-4} sec^2.
Test the hypothesis of a common covariance matrix in both the sexes. [6]

7. Practical Note Book. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90
SEMESTRAL-II EXAMINATION
Non-parametric and Sequential Methods

Date: 4.5.1990 Maximum Marks: 100 Time: 4 hours

Note: Use separate answerscript for each group. Answer as many questions as you can.

GROUP - A

Non-parametric Max.Marks: 80

1.(a) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be random samples from $F(x)$ and $G(x)$ which are assumed to be independent and continuous. Derive a suitable test using empirical distributions of the samples for $H_0 : F(x) = G(x)$ against $H_1 : F \neq G$. Show that the sampling distribution of the test statistic is distribution free under H_0 and indicate how you would obtain the sampling distribution of the test statistic.

(b) The heart beats of samples of 8 males and 8 females are recorded as follows. Can you conclude that the distributions of heart beats are the same for males and females ?

Males :	58,	76,	82,	74,	79,	65,	74,	86
Females :	66,	74,	69,	76,	72,	73,	75,	67.

(10+6) = [16]

2.(a) A batch of n students were given a written test and an oral interview. The scores in the test are marked in an interval scale from 0 to 100 and the rank in the interview is marked as an integer from 1 to n. Show how you can obtain an estimate of Kenall's measure of association in this situation. Show that the measure satisfies the desirable properties of a measure of association. Obtain expressions for the mean and variance of the estimate when the two characteristics are independent.

(b) Indicate how the estimate is altered if there are ties in either set of observations.

(12+4) = [16]

p.t.o.

3.(a) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be random samples from two independent populations with continuous distribution functions F and G respectively. Describe the use of general linear rank order statistics for testing $H_0 : F = G$ against suitable alternatives by showing them how you would use them in the case of finite samples and in the case when the samples are large.

(b) Examine in detail the exact and asymptotic distributions of the test statistic if Normal Scores are taken.

(10+6) = [16]

4.(a) Let $X_{i1}, \ldots, X_{in_i}$, $i = 1, \ldots, k$ random samples from k independent populations, the ith population having distribution $F_i(x)$, $i = 1, \ldots, k$, assumed to be continuous. Describe a test based on suitable scores for testing $H_0 : F_1 = \ldots = F_k$ against the alternative that the distributions may differ in their location parameter. Describe briefly how you would obtain the large sample distribution of the test statistic under H_0.

(b) Indicate briefly how you would adopt the above test when the data are on an ordinal scale, all the k populations having the same number of classes and the same type of ordering. (Members of different populations belonging to the same class are treated as equivalent).

(11+5) = [16]

5.(a) A random sample of n independent bivariate random variables (X_i, Y_i), $i = 1, \ldots, n$ where the measurement scales for X's and Y are nominal with two categories 0 and 1 is drawn. Derive a test for $H_0 : P(X = 0, Y = 1) = P(X = 1, Y = 0)$ against $H_1 : P(X = 0, Y = 1) \neq P(X = 1, Y = 0)$.

(b) Two packaging machines are calibrated to pack 50 gms. of a product. A random sample of n units from the first machine gave weights $X_1, \ldots, X_n$ and a random sample of m units from the second machine gave weights $Y_1, \ldots, Y_m$. Describe a test to test whether both machines have the same variability (No derivation of any sampling distribution is required).

(8+8) = [16]

6. Records. [10]

GROUP - B

Sequential Analysis

Max. Marks: 20

Note: Answer **ALL** questions.

1.(a) Explain the following terms:

(i) SPRT (ii) ASN (iii) O.C. function

(b) Consider a sequence of i.i.d. observations $x_1, x_2, \ldots$ on a random variable x and a sequencial decision procedure with a given stopping rule. Let n be the number of observations needed to come to a decision, z(x) be a function of x and H be some hypothesis specifying the probability distribution of x. Then show that

$$E(|z(x)||H) < \infty \text{ and } E(n|H) < \infty$$

$$\Rightarrow E(S_n|H) = E(z|H)\, E(n|H).$$

(Here $S_n = \sum_{i=1}^{n} z(x_i)$) (3x2+4) = [10]

2. State the SPRT for testing $H_0 : p = p_0$ vs. $H_1 : p = p_1$, $p_1 > p_0$ for point binomial variables with probability of success p. Apply this test for testing $H_0 : p = .3$ vs. $H_1 : p = .5$ with $\alpha = \beta = .05$ to the following sequence of observations:

0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1.

[10]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1989-90

SEMESTRAL-II EXAMINATION

Optimization Techniques

Date: 2.5.1990 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: The paper carries 110 marks. The maximum you can score is 100.

1. Consider a diet problem in which the nutrient requirements are to be satisfied exactly (i.e., as equations). Suppose there is an optimal diet using only the foods $F_1, \ldots F_k$. Let the requirements be now changed so that in the new situation there is a feasible diet using the same foods $F_1, \ldots F_k$. Show that this diet is optimal. [15]

2. In a certain plant there are n jobs $J_1, \ldots, J_n$ and m workers $I_1, \ldots, I_m$. To each i, j is associated a number $\alpha_{ij} \geqq 0$ which is the 'rating' of I_i for the job J_j. Write down a program for finding out what fraction of the total time I_i should work at J_j so that the total 'rating' is a maximum. (Assume that at any given time a person can work at only one job and only one person can work at a job.)

   Show that this problem always has an optimal solution.

   (5+10) = [15]

3. (a) Find $\xi_1, \xi_2 \geqq 0$ such that $\xi_1 - \xi_2$ is maximum subject to

$$-2\xi_1 + \xi_2 \leqq 5$$
$$\xi_1 - 2\xi_2 \leqq 2$$
$$\xi_1 + \xi_2 \leqq 5 .$$

[15]

   (b) Using the simplex method, show that there does not exist $\xi_1, \xi_2, \xi_3 \geqq 0$ such that

$$\xi_1 + 3\xi_2 - 5\xi_3 = 2$$
$$\xi_1 - 4\xi_2 - 7\xi_3 = 3 .$$

[15]

p.t.o.

4. Solve the 2 by 3 transportation problem whose cost matrix is $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$, the supplies are $\sigma_1 = 4$, $\sigma_2 = 7$ and the demands are $\delta_1 = 2$, $\delta_2 = 3$, $\delta_3 = 5$.

[15]

5. Let Γ be an $n \times n$ matrix game whose pay off matrix contains n distinct numbers $\alpha_1, \ldots, \alpha_n$, each α_i occuring in every row and in every column. Find a solution for the game.

[15]

6. Solve the following game:

There are three cards A, B, C in the deck. Each of two players P_1 and P_2 is dealt a card at random. P_1 looks at his own card and makes a guess as to which card is left in the deck. P_2 looks at his own card, hears P_1's guess and then makes a guess as to which card is left. If only one player guesses correctly, he wins an amount 1 from his opponent.

[20]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90
SEMESTRAL-II EXAMINATION
Design of Experiments

Date: 30.4.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any THREE questions. Each question carries 30 marks, the break-up being given in brackets [ ] at the end. SUBMIT YOUR PRACTICAL RECORDS To-day. THEY CARRY 20 marks. NOTE that the maximum that you can score is 100.

1.(a) Define a balanced incomplete block design (BIBD) with parameters v, b, r, k, λ, and show that any two blocks of such a design intersect in λ common treatments if and only if b = v.

(b) Describe a method of construction of the following series of BIBD, proving that the method works in general:

$$v = b = s^2 + s + 1, \quad r = k = s^2, \quad \lambda = s(s-1),$$

where s is a prime or power of a prime.

(c) Let N be the incidence matrix of D, a BIBD (v,b,r,k,λ). Consider the following block design D^* with the incidence matrix :

$$N^* = [N \; \vdots \; J_{vb} - N]_{v \times 2b}$$

where J_{vb} is a v x b matrix of all unities. Solve the reduced normal equations for $\{\hat{\tau}_i\}$ in terms of Q_i, i = 1, 2, ..., v, for the design D^*. Also give the analysis of variance for D^*.

(10+10+10) = [30]

2.(a) Show that under a missing plot situation

(i) $SSE_{\Omega^*(\underline{w}^*)} = SSE_{\Omega}$, where $\underline{w}^*$ is any solution of $\underline{w}^* = Z\hat{\beta}_{\Omega^*(\underline{w}^*)}$, and that (ii) $\hat{\beta}_{\Omega^*(w^*)} = \hat{\beta}_{\Omega}$,

where the symbols have their usual significances.

Contd..... 2/-

(b) Hence obtain the "estimates" for the missing values in cells (i,j) and (i',j'), $i \neq i' \in \{1, 2, \ldots, v\}$; $j \neq j' \in \{1, 2, \ldots, r\}$ in a randomised block design for v treatments in r blocks. Give also the expressions for $V(\hat{\tau}_i - \hat{\tau}_j)$, $i \neq j \in \{1, 2, \ldots, v\}$ for the resulting incomplete block design.

(c) Describe your concomitant variables and study variable to obtain the above "estimates" by using covariance with dumm; variables.

(10+15+5) = [30

3.(a) Develop the analysis of covariance under the usual fixed effects model

$\tilde{\Omega}: \underline{y} = X\underline{\beta} + H\underline{r} + \underline{e}$ etc, stating all the assump tions clearly.

(b) Apply the analysis to a latin square design for v treatmen with one concomitant variable x, and give the ANOVA/ACOV of this design.

(c) Also obtain $V(\sum_{i=1}^{v} \ell_i \quad i \quad \sum_{i=1}^{v} \ell_i = 0)$ under $\tilde{\Omega}$ for a lati square design.

(15+10+5) = [3c

4.(a) Give a balanced confounding scheme for a 2^5 factorial expe ment in blocks of 2^3 plots. Construct the key blocks for each of the replications of the suggested design.

(b) Give the analysis of variance of the confounded design at (a), indicating clearly how the various sum of squares of the effects can be computed.

(c) Identify all the confounded effects in one replication of a 2^8 expt. in blocks of size 2^4, of which the following constitute a block:

[efg, bgh, abefh, cfh, bdf, bce, a, bcdefgh, acegh, deh, cdg, adfgh, abcfg, abcdh, abdeg, acdef].

(10+12+8) = [3

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

PERIODICAL EXAMINATION

Elective - 5 : Physical and Earth Sciences

Date: 28.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any FIVE questions. Draw the circuit diagram or any figure whenever necessary.

1. What are the basic components used in electronic circuits ? Name different practical forms of passive components and the range of values in which they are normally available. What is colour coding ?

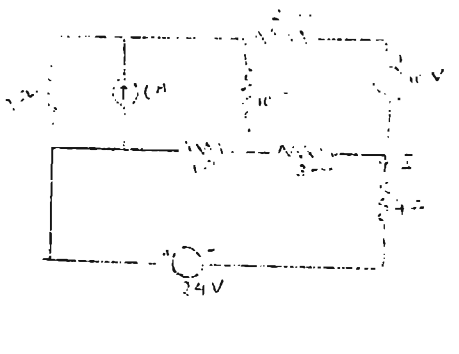
(2+8+3) = [13]

A capacitor is charged from a d.c. source through a resistor of .5 meg ohm. If the p.d. across it reaches 75% of its initial value in half a second, find its capacitance.

[6]

2. State and explain Kirchoff's laws applicable to an electrical net work.

(4+4) = [8]



Determine I.

[8]

Fig. 1

Contd..... 2/-

Contd..... Q.No.2

How a voltage source can be converted into a current source in solving network problems and vice versa. Indicate its application in the above network (Fig.1). [4]

3. State and explain Thevenin's theorem as applicable to d.c. circuits. (3+3) = [6]

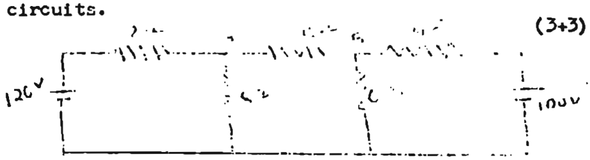


Fig. 2

Calculate the magnitude and direction of flow of current through 12 Ω resistance of Fig. 2. (Using Thevenin's theorem) [7]

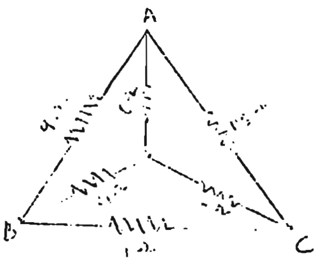


Fig. 3

Find equivalent resistance between the points AB of Fig. 3. [7]

State and explain Faraday's laws of electromagnetic induction. (3+3) = [6]

Contd..... 3/-

Contd..... Q.No.4

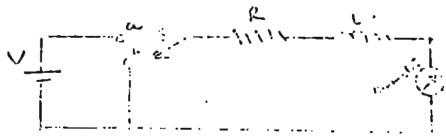


Fig. 4

Discuss the phenomenon that takes place when the switch 'S' of fig. '4' is suddenly thrown to the position 'a' and hence deduce the expression for current in the circuit. What happens when the switch 'S' is put to the position 'b'? What is meant by time constant of a circuit ?

(7+2) = [9]

A relay has a resistance of 300 Ω and is switched on to a 100 V d.c. supply. If the current reaches 63.2% of its final steady state value in 0.002 second, determine

(a) the time constant of the circuit.

(b) the inductance of the circuit. [5]

5. Obtain the expression of current in the following electrical circuits when an alternating voltage $v = V_m \text{Sin } wt$. is applied in each of them:

(a) circuit containing the voltage source and a resistance (pure).

(b) circuit containing the voltage source and a pure inductance.

(c) circuit containing the voltage source and a pure capacitance.

(3+3+3) = [9]

An alternating current is represented by $j = 70.7 \text{ Sin } 520\,t$. Calculate its (a) frequency, (b) r.m.s value, (c) average value. [3]

Contd..... Q.No.5

A voltage $v = 100 \sin 314t$ is applied to a circuit consisting of a 25 Ω resistor and a 80 μF capaciter in series. Determine (i) an expression for the current flowing at any instant, (ii) power factor, (iii) the power consumed and (iv) the p.d. across the capacitor at the instant when the current is 1/2 of its maximum value.

[8]

6. Discuss the phenomenon of resonance in a series L-C-R circuit. Find the expression for the resonant frequency. Why such a circuit is called an acceptor circuit ? What are Q-factor and bandwidth of a series circuit ? Find the expressions for each of them.

(5+2+1+2+2) = [12]

A series circuit consists of R = 10 Ω , L = 100/π mH ; C = 500/π μF . Find

(a) the current flowing when the applied voltage is 100 V at 50 Hz .

(b) the power factor of the circuit.

(c) what value of supply frequency would produce series resonance ?

[8]

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1989-90

PERIODICAL EXAMINATION

Elective - 5 - Economics

Date: 28.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any THREE questions.

1.(a) State Pareto's law and obtain the density function of the Pareto distribution. Also obtain the mean of the distribution stating the necessary parametric restriction. [8]

(b) Let $X \sim \Lambda(\mu, \sigma^2)$. Show that the median of the distribution is given by $\exp(\mu)$. [8]

(c) Define Lorenz curve, area of concentration and Lorenz ratio. What does a typical point (F, F_1) on a Lorenz curve indicate ? [8]

(d) Obtain the Lorenz curve in the form $\eta = h(\pi)$, where h is some function, $\eta = \frac{1}{\sqrt{2}}(F - F_1)$, and $\pi = \frac{1}{\sqrt{2}}(F + F_1)$. [6]

2.(a) Derive the Lorenz curve and Lorenz ratio for the Lognormal distribution. [12]

(b) For the following data on the distribution of assets per household in India for 1971-72, test whether Pareto's law holds. Also, obtain a suitably truncated distribution which is Paretean and estimate the Pareto coefficient and hence the Gini coefficient of inequality.

Asset group (Rs.)	Percentage of households
100 - 500	11.38
500 - 1000	8.36
1000 - 2500	15.49
2500 - 5000	16.09
5000 - 10000	18.31
10000 - 20000	15.40
20000 - 30000	6.24
30000 - 50000	4.83
50000 -	3.90

[18]

p.t.o.

3.(a) What are the desirable properties that a measure of inequality should possess ? [10]

(b) Show that the Gini coefficient satisfies the Pigou-Dalton principle of transfers, but does not satisfy the principle of diminishing transfers. [10]

(c) Show that Sen's poverty measure can be alternatively written as $HI_g(1 + G_g)$, where H : head count ratio, I_g : income gap ratio, and G_g : Gini coefficient of the distribution of poverty gaps. [10]

4.(a) State Engel's law. Define 'Engel elasticity' and discuss the types of commodity-classification that can be made on the basis of Engel elasticity. [10]

(b) Show that the average elasticity, η_M, of a commodity for a heterogeneous group of population can be obtained as

$$\eta_M = \frac{\sum_h E_h \eta_M^h}{\sum_h E_h}, \qquad \text{where}$$

M : total expenditure,

E_h : aggregate consumption of the commodity in the h-th homogeneous group,

and η_M^h : Engel elasticity of the commodity for the h-th group. [6]

(c) Describe the statistical criteria for the choice of an algebraic form of Engel curve. [6]

(d) Given the following data, plot the Engel curve for cereals assuming a double logarithmic form. Estimate the Engel elasticity from the graph.

Per capita expenditure per 30 days (Rs.)	
All items	Cereals
5.61	3.36
9.09	4.93
11.44	6.34
13.41	6.42
16.10	8.24
18.70	9.33
21.57	9.96
25.57	10.41
29.77	11.85
36.87	13.95
45.86	15.87
77.89	17.41

[8]

Practical Records. [10]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

PERIODICAL EXAMINATION

Elective - 5 : Biological Sciences

Date: 28.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Each question carries 16 (Sixteen) marks, 4 (four) marks for neatness.
Answer any SIX questions.

1. Define a dynamical system. Draw geometrically, in the XY-plane, the system trajectory of a two-dimensional dynamical system given by

$$\frac{dx}{dt} = f_1(x,y)$$

$$\frac{dy}{dt} = f_2(x,y).$$

2. Define open and closed systems. Determine the Hamiltonian of the closed system

$$\frac{dx}{dt} = f(x,p)$$

$$\frac{dp}{dt} = g(x,p).$$

Prove explicitly that the damped harmonic oscillator is not a conservative system and the undamped harmonic oscillation is not a limit cycle oscillation.

3. State the Lotka-Volterra predator-prey model. Study the stability properties of the system by constructing a suitable Lyapunov function. Is the system structurally stable ?

4. State the theorem regarding stability of the linear homogeneous system given by

$$\frac{dx_i}{dt} = \sum_{i=1}^{n} a_{ij} x_j, \quad i = 1, 2, \ldots, n$$

Contd..... 2/-

Contd..... Q.No.4

Where λ_i are the eigenvalues of the matrix (a_{ij}). Hence discuss the stability properties of the system:

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy .$$

5. Define the following:

   (a) Stability of a dynamical system.

   (b) Nodes, Centre, Vortex point, Spiral point, Saddle point, Structural stability and Limit cycle.

   (c) Bendixon's negative criterion.

   (d) Enzymo-cubstrate Kinetic model.

6. State and discuss the Rashevsky - Turing theory of morphogenesis. Establish by this theory the development of polarity typical of developing biological systems.

7. Construct a predator-prey model with increasing and diminishing returns.

   Investigate the stability of this system by constructing a suitable Lyapunov function.

8. Construct the generalized Gause model with the predator response function p(x).

   Discuss the conditions for persistence of both the species and investigate the stability properties of the equilibrium.

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INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1989-90

PERIODICAL EXAMINATION

Optimization Techniques

Date: 26.2.1990 Maximum Marks: 50 Time: 2 hours

Note: The paper carries 65 marks. The maximum you can score is 50.

1. Let P_0, P_1, ..., P_n be a set of geographical points. A good is produced at P_0 and desired at P_n. For each pair of points P_i and P_j, there is a nonnegative number γ_{ij} which is the maximum amount that can be shipped from P_i to P_j in a year. Formulate a linear program for maximizing the amount that can be received at P_n in a year. [10]

2. Let the following problem have a feasible solution:

   Find x such that xc is a maximum subject to xA = b.

   Show it has an optimal solution if and only if c is a linear combination of the columns of A. [10]

3. Let (I) be the standard problem:

   Maximize cx subject to $x \geqq 0$ and $xA \leqq b$.

   Let (I^*) be the dual of (I).

   Let $\phi(x,y) = xc + yb - xAy$, $x, y \geqq 0$. Show that $\phi(x, \bar{y}) \leqq \phi(\bar{x}, \bar{y}) \leqq \phi(\bar{x}, y)$ if and only $\bar{x}, \bar{y}$ are optimal solutions of (I) and (I^*) respectively. [15]

Find a nonnegative solution to the equations:

$$5\xi_1 + \xi_2 + 6\xi_3 - 5\xi_5 = 2$$

$$-7\xi_1 - \xi_2 - 2\xi_3 + \xi_4 + 2\xi_5 = -5$$

[15]

p.t.o.

5. Find a basis for the row space of the following matrix:

$$\begin{pmatrix} 1 & 0 & 3 & -2 \\ 2 & 1 & 2 & 0 \\ 0 & 1 & -4 & 4 \\ 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{pmatrix}$$

[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90
PERIODICAL EXAMINATION
Non-parametric and Sequential Methods

Date: 23.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer any FOUR questions.

1.(a) A random sample $X_1, \ldots, X_n$ is drawn from a continuous distribution. To test whether $F_0(x)$ is the c.d.f. of the population define the Kolmogorov-Smirnov statistics D_n^+, D_n^- and D_n and indicate how you would use them. Show that the sampling distributions of these statistics are distribution free under the null hypothesis, D_n^+ and D_n^- have the same distribution and obtain an expression for the c.d f.

(b) The following measurements (in cms.) are the head circumference (X) of 12 boys at the age of 16 weeks. Do an appropriate nonparametric test to see whether the distribution of X can be regarded as N(41, 4).
41.7, 41.8, 40.2, 41.1, 41.8, 42.5, 42.2, 41.6, 41.0, 42.8, 42.3, 40.6.

(15+10) = [25]

2.(a) A random sample $X_1, \ldots, X_n$ is drawn from a population with a continuous distribution. Derive a suitable test for testing H_0 : X_0 is the p_0^{th} quantile of the distribution against H_1 : the p_0^{th} quantile is larger than X_0. Describe how you would obtain a confidence interval of a given confidence coefficient for the p_0th quantile. (You are expected to obtain the sampling distns. of the test statistic).

(b) To determine the smoothness of paper manufactured by a process, the smoothness of a sample of 15 measurements are recorded as follows:

173, 135, 165, 185, 108, 120, 125, 135, 98, 150, 136, 104, 127, 120, 125.

Can you conclude that 75% of the production has the smoothness at least 110 ?

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(c) If the above measurements are part of a 100 observations, 43 of which are smaller than 96 and the rest larger than 165, can you conclude that the smoothness is at least 110?

(13+7+5) = [25]

3.(a) A random sample $X_1, \ldots, X_n$ of observations is drawn from a symmetric distribution. Derive a suitable test for H_0 : The median of the distribution is a specified value m_0 against H_1 : it is $> m_0$. (Obtain the exact and asymptotic null distributions of the test statistic).

(b) The I Q of 30 children are recorded as follows:

118, 116, 98, 100, 103, 109, 112, 141, 114, 93, 121, 124, 132, 118, 116, 110, 118, 98, 110, 118, 98, 110, 95, 110, 101, 116, 128, 151, 135, 134.

Can you conclude that the median I Q score is 110 ? (IQ is measured on a continuous scale).

(13+12) = [25]

4.(a) Random samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are drawn from two independent populations with distributions F and G respectively. Derive a test, stating your assumptions clearly, for H_0 : $F(x) = G(x)$ against H_1 : $F(x) = G(x + c)$ for some constant c . Derive the exact distribution of the test statistic and give its large sample approximation, under the null hypothesis.

(b) To see whether televised lectures are more helpful as a teaching method, a batch of 12 students were given coaching by television and another batch of 10 students were taught in a class room. The scores of the two batches of students in the final assessment are as follows:

T / Method :	58, 54.5, 53.5, 52.5, 73, 74, 71, 67, 28.5, 59.5, 24, 49
Class room Method :	77, 84, 70.5, 45.5, 48.5, 34.5, 82, 76, 61, 60

What can you conclude about the efficiency of teaching by television ?

(15+10) = [25]

5.(a) Show that, if Kolmogorov - Smirnov test for goodness of fit is used when the data are from a discrete distribution, the test would be more conservative than the test, had the sample come from a continuous distribution.

(b) Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ be random samples from two independent distributions F and G respectively. For testing $H_0 : F(x) = G(x)$, show that the Wilcoxon form of test statistic and Mann-Whitney form of test statistic give equivalent tests. Which form would you prefer ? Justify.

(c) Obtain the correction for the variance of the Wilcoxon-Mann-Whitney test when there are ties in the observations.

(8+9+8) = [25]

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B.Stat. (Hons.) III Year: 1989-90

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PERIODICAL EXAMINATION

Design of Experiments

Date: 21.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer as many as you can. Each question carries 20 marks. The maximum that you can score is 100.

1.(a) State and explain the three fundamental principles of experimental designs, explaining with examples all the objectives behind each of these principles.

(b) What is a randomised block design ? Under the usual linear model for such a design, indentify the class of all estimable functions, and develop the analysis of variance appropriate for such a design.

(6+14) = [20]

2.(a) Define mutually orthogonal latin squares (MOLS) of order v, and show that the maximum number of MOLS of order v is less than or equ l to (v-1).

(b) Describe, with an example, a method of construction of a complete set of MOLS of order s, where s is a prime or power of a prime, proving that the method works in general. What is Euler's conjecture about MOLS and what is its current status ?

(5+12+3) = [20]

3.(a) Give the two definitions of connectedness of a block design and show that they are equivalent.

(b) Suppose k observations of a block of a balanced incomplete block design (BIBD) got lost. Is the resulting design still connected ? Prove your answer.

(15+5) = [20]

4. In connection with the analysis of a general block design, prove the following results:

Contd..... 2/-

Contd..... Q.No.4

(a) Reduced normal equations for $\hat{\underline{\tau}}$ are given by $C\hat{\underline{\tau}} = Q$ where $C = r^{\delta} - NK^{-\delta}N'$ and $\underline{Q} = \underline{T} - NK^{-\delta}\underline{B}$.

(b) $\underline{l}'\underline{\tau}$ is estimable if and only if $\underline{l} \in$ Col. Space of C.

(c) Sum of squares due to the hypothesis $H_0: C\underline{\tau} = \underline{0}$ is given by $\sum_{i=1}^{v} \hat{\tau}_i Q_i$ and its degrees of freedom equals rank C.

(d) S.S. due to Blocks (unadj.) + S.S. due to Treatments (Adj.) = S.S. due to Blocks (Adj.) + S.S. due to Treatments (unadj.).

(4 x 5) = [20]

5.(a) Define a connected variance balanced block design, and prove that a connected block design is variance balanced if and only if all the off-diagonal elements of its C matrix are equal.

(b) Prove that a binary block design is orthogonal if and only if

$$N = \underline{r}\,\underline{k}'/n.$$

(c) Consider the following block design for 3 treatments in 5 blocks:

$$N = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Is this design connected, orthogonal, variance balanced ? Why ?

(10+5+5) = [20]

6. Submit your PRACTICAL RECORDS to the course Instructor on or before the last day of your Periodical Exams.

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

PERIODICAL EXAMINATION

Multivariate Distributions and Tests

Date: 19.2.1990 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. Total marks in the margin is 108 but maximum marks that you can score is 100.

1.(a) Prove that the dispersion matrix of a p-dimensional random vector is at least non-negative definite.

(b) Define a non-singular multinormal distribution. Let X be distributed as a p-dimensional non-singular normal distribution and $Z = PX$, where P is a $K \times p$ $(K \leq p)$ matrix.

Show that Z is distributed as a K-dimensional non-singular normal distribution.

(c) Let Σ be the positive definite disperson matrix of a p-dimensional random vector. Show that

$$|\Sigma| \leq \prod_{i=1}^{p} \sigma_{ii}$$

where $\sigma_{11}, \ldots, \sigma_{pp}$ are the diagonal elements of Σ.

(5+6+8) = [19]

2. Let $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$. Express the multiple correlation of X_1 on $\underset{\sim}{X}^{(2)} = (X_2, \ldots, X_p)'$ in terms of the elements of Σ. Hence show that this is the maximum correlation between X_1 and a linear combination $\underset{\sim}{\alpha}' \underset{\sim}{X}^{(2)}$, where $\underset{\sim}{\alpha}$ is a $(p-i) \times 1$ real vector.

(4+12) = [16]

3.(a) Let $X_1, \ldots, X_{N_1}$ and $Y_1, \ldots, Y_{N_2}$ be two independent samples from $N_p(\mu_1, \Sigma)$ and $N_p(\mu_2, \Sigma)$, $\Sigma > 0$ respectively. Obtain the maximum likelihood estimates of μ_1, μ_2 and Σ.

Contd..... 2/-

Contd..... Q.No.3

(b) Let $X_1, \ldots, X_N$ be a random sample from $N_p(\mu, \Sigma)$, $\Sigma > 0$.

(i) Prove that $A = \sum_{\alpha=1}^{N} (X_\alpha - \bar{X})(X_\alpha - \bar{X})'$ is positive definite with probability one if and only if $N > p$.

(ii) Let $A \sim W_p(n, \Sigma)$, $n \geq p$. Derive the c.f. of $\text{Tr}\, A\Sigma^{-1}$. Hence show that $\text{Tr}\, \Sigma^{-1} A$ is distributed as a chi-square with np d.f.

[12 + (7+9)] = [28]

4. Let $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$, where $\mu = (\mu_1, \ldots, \mu_p)'$. On the basis of a random sample of size $N (> p)$ from this population suggest an appropriate test for testing $H_0[\mu_1 = \ldots = \mu_p]$ against H[not all μ_i's are equal]. Hence derive the distribution of the test statistic under H_0 and H.

[20]

5.(a) The scores of students in three subjects A, B, C are supposed to be jointly normally distributed with means

$$\mu_1 = 52.2, \quad \mu_2 = 57.6, \quad \mu_3 = 43.5$$

variances $\sigma_1^2 = 77$, $\sigma_2^2 = 8.3$, $\sigma_3^2 = 6.2$, and total correlations $\rho_{12} = .36$, $\rho_{13} = .57$, $\rho_{23} = .48$.

What percentage of students in a large group is expected to have a total score between 100 and 200 ?

(b) Suppose that 11 and 12 observations are made on two random variables X_1 and X_2, where X_1 and X_2 are assumed to have $N_2(\mu_1, \Sigma)$ and $N_2(\mu_2, \Sigma)$ respectively. Sample mean vectors and pooled sample covariance matrix are

$$\bar{X}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \bar{X}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$S_{pooled} = \begin{bmatrix} 7.3 & -1.1 \\ -1.1 & 4.8 \end{bmatrix}.$$

Test the hypothesis $H_0[\mu_1 = \mu_2]$ against $H[\mu_1 \neq \mu_2]$.

(10+15) = [25]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.)III Year:1989-90

STATISTICAL INFERENCE

SEMESTRAL-I BACKPAPER EXAMINATION

Date : 2.1.1989 Maximum Marks : 100 Time : 3 Hours.

1.(a) Let $X_1,\ldots,X_n$ be a random sample from $U(0,\theta)$, $\theta>0$. Show that $X_{(n)} = \max(X_1,\ldots,X_n)$ is sufficient (directly from the definition).

(b) Show that the order statistics are minimal sufficient for the Cauchy distribution with median θ, $-\infty<\theta<\infty$.

(c) Suppose that the distribution of a random variable X can be any one of the following :

$$P(X=i)=1/3\ ;\ i = 0,1,2 \qquad P[X=i] = 1/3\ ;\ i=1,2,3.$$

Is there any sufficient statistic other than X ? [6+7+7]

2. Suppose X assumes values $-1,0,1,\ldots$ with probabilities

$$P(X=-1) = \theta,\quad P(X=k) = (1-\theta)^2\,\theta^k,\ k = 0,1,2,\ldots,$$

where $0<\theta<1$.

(a) Show that U is an unbiased estimator of zero if and only if $U(k) = ak$, $k = -1,0,1,\ldots$ for some a.

(b) Determine the class of all unbiased estimators of θ, and find the estimator in this class which has the smallest variance at $\theta = \theta_0$.

(c) Does there exist a UMVU estimator of θ ? [6+12+2]

3. Consider the problem of estimating $\Phi(\theta)$ based on a random sample of size n from $N(\theta,\sigma^2)$ where Φ is the cdf of $N(0,1)$ distribution. Determine the UMVU estimator, and compare its variance with the corresponding Cramer-Rao lower bound when σ^2 is known. Would you get the same UMVUE when σ^2 is unknown ? [10+10]

4. Let X be a random variable having density f_1 and f_0 under the hypotheses H_0 and H_1. For a test function φ, the probability of rejecting H_0 given X=x is given by $\varphi(X)$. Let φ^* be given by

$$\varphi^*(X) = \begin{cases} 1, & \text{if } f_0(X) > kf_0(X) \\ \gamma, & \text{if } f_1(X) = kf_0(X) \\ 0, & \text{if } f_1(X) < kf_0(X) \end{cases}$$

where $E[\varphi^*(X)\,|H_0] = \alpha$, and $0<k<\infty$.

(a) Show that φ^* minimizes $E[\varphi(X)|H_0]$ among all tests φ satisfying $E[\varphi(X)|H_1] \geq E[\varphi^*(X)|H_1]$.

(b) Show that the above test φ^* minimizes $p_0E[\varphi(X)|H_0] + p_1E[1-\varphi(X)|H_1]$, $(P_0>0,\ P_1>0,\ P_0+P_1=1)$ when $k = P_0/P_1$. [10+10]

p.t.o.

5.(a) Define uniformly most accurate lower confidence limit for a parameter θ. How would you obtain such a confidence limit based on a random sample of size n from $N(\theta,1)$?

(b) Let X be distributed as $N(\theta,1)$. It is desired to test $H_0:\theta=0$ against $H_1:\theta=1$ with the requirement that the probability of type I error $\leq .05$, and the probability of type II error $\leq .05$. Show that the above requirement can be met by taking n random observations on X with appropriate n. [13+7]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

DIFFERENCE AND DIFFERENTIAL EQUATIONS
SEMESTRAL-I BACKPAPER EXAMINATION

Date : 1.1.1990 Maximum Marks : 100 Time : 3 Hours.

Note : Answer all the questions.

1.(a) Find the general solution of the difference equation

$$y_{n+2} + y_{n+1} - 12y_n = 3^n + 10.$$

(b) If $I_n = \int_0^1 (\log x)^n x^\alpha \, dx$, $\alpha > 0$, n a non-negative integer, show that

$$(\alpha+1)I_n + n\, I_{n-1} = 0, \; n = 1,2,\ldots.$$

Hence evaluate I_n. [8+8 = 16]

2.(a) Find the general solution of the Bernoulli's equation

$$x \frac{dy}{dx} + y = x^4 y^3$$

(b) Solve the differential equation

$$xy - 1 + (x^2 - xy) \frac{dy}{dx} = 0$$

by finding an integrating factor. [8+8 = 16]

3.(a) Given that $y = e^x$ is a solution of the differential equation

$$(x^2 + x)\, y'' + (2 - x^2)\, y' - (2 + x)\, y = 0$$

find the general solution of

$$(x^2 + x)\, y'' + (2 - x^2)\, y' - (2 + x)\, y = x(x + 1)^2.$$

(b) Find the general solution of

$$y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x.$$ [12+8 = 20]

4. Let f(x,y) be continuous on the strip $a \le x \le b$, $-\infty < y < \infty$, and let f(x,y) satisfy the following Lipschitz condition :

$\exists K > 0$ such that

$$|f(x,y_1) - f(x,y_2)| \le K\,|y_1 - y_2| \;\; \forall x \in [a,b]$$

and $\forall\, y_1, y_2 \in (-\infty,\infty)$.

Show that, given $x_0 \in (a,b)$ and $y_0 \in (-\infty,\infty)$, there is a unique function $y = y(x)$ defined on $[a,b]$ which is a solution of the differential equation $\frac{dy}{dx} = f(x,y)$ on (a,b) and which satisfies the initial condition $y(x_0) = y_0$. [20]

p.t.o.

5. Show that $x = 1$ and $x = -1$ are regular singular points of the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \alpha(\alpha+1)\,y = 0$$

Find its indicial equation at $x = 1$.
Show that infinity is a regular singular point of the above equation. Find its indicial equation at $x = \infty$. [10]

6. Show that the equation

$$4x^2\frac{d^2y}{dx^2} - 8x^2\frac{dy}{dx} + (4x^2+1)y = 0$$

has only one Frobenius series solution valid on $(0,\infty)$. Find the general solution of the above equation valid on $(0,\infty)$. [10]

7. Solve the system of differential equations

$$\frac{dy_1}{dt} = -5y_1 - y_2 + e$$

$$\frac{dy_2}{dt} = y_1 - 3y_2 + e^{2t}.$$

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.)III Year : 1989-90
SAMPLE SURVEYS
SEMESTRAL-I BACKPAPER EXAMINATION

Date : 1.1.1990 Maximum Marks : 100 Time : 3 Hours.

Note : Answer all the questions. Figures in [ ] indicate marks allotted.

1.(a) Consider an SRSWR(N,n) sample s having $\nu(s)$ distinct units. Show that $E(\nu(s)) = N[1-(\frac{N-1}{N})^n]$. [3]

(b) Compare the performance of the sample mean $\bar{y}$ based on distinct units with that of the sample mean $\bar{y}$ based on all units (including repetitions). [7]

(c) Compare the better of the two in (b) with the sample mean based on an SRSWOR(N,n) sample data. [5]

2. For ppswor sample data,

(i) Write down the expression of the Des Raj estimator of the population mean, and derive its variance and an unbiased variance estimator. [2+4+4=10]

(ii) Obtain Murthy's estimator of the same in an explicit form for the case when the sample size is n=2 and derive an expression for its variance. [5+10=15]

3. Suppose for estimating the population mean stratified srswor sampling has been adopted with a sample size ≥ 2 from every stratum. Obtain an estimate of the gain in precision due to adoption of stratification as against the use of unstratified srswor procedure. [15]

4.(a) Suggest a method of sampling which would make the usual ratio estimator unbiased. [5]

(b) Under an SRSWOR(N,n) sampling, compare the performances of the ratio and the regression estimators of the population mean with that of the ordinary sample mean (in large samples). [15]

5.(a) Formulate the "matching" and "unmatching" problem in sampling a population on two successive occasions and find the optimum "matching" proportion. [10]

(b) Show that in repeated sampling of a population on successive occasions, the optimum "matching" proportion assumes the limitting value of $\frac{1}{2}$. [15]

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B.Stat.(Hons.) III Year :1989-90

STATISTICAL INFERENCE

SEMESTRAL-I EXAMINATION

Date : 27.11.1989 Maximum Marks : 90 Time : $3\frac{1}{2}$ Hours.

1. Let $X_1,\ldots,X_n$ be a sample from a population with density $p(x,\theta)$ given by

$$p(x,\theta) = \frac{1}{\sigma} \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)\right\}, \text{ if } x \geq \mu$$
$$= 0, \text{ otherwise.}$$

Where $\theta = (\mu,\sigma)$, $-\infty < \mu < \infty$, $0 < \sigma$.

(a) Show that min $(X_1,\ldots,X_n)$ is sufficient for μ when σ is fixed by applying the definition of sufficiency directly, as well as by using the factorization criterion.

(Hint : Use the joint density of the order statistics $X_{(1)} < \ldots < X_{(n)}$ and the density of $X_{(1)}$). [2+7]

(b) Find a one-dimensional sufficient statistic for σ when μ is fixed. [2]

(c) Exhibit a two-dimensional sufficient statistic for θ. [2]

(d) Is $X_{(1)}$ minimal sufficient when σ is fixed ? [4]

(e) Derive the maximum likelihood estimates of μ and σ. [5]

(f) Obtain the UMVU estimator of μ when σ is known. Will this estimator be UMVU when σ is unknown ? [5]

(g) Compare the mean-square errors of the m.l.e. and UMVUE of μ. [6]

(h) Obtain the UMVU estimator of σ when μ is known. What will be the UMVUE estimator of σ when μ is unknown ? [5]

(i) Suppose now $n = 2$, $\sigma=1$. Show that a test ϕ is UMP level α for testing $H_0: \mu = \mu_0$ against $H_1 : \mu < \mu_0$ if

$$E[\phi(\underset{\sim}{x})|H_0] = \alpha,$$
and $\phi(\underset{\sim}{x}) = 1$, if $x \notin S_0$,

where $S_0 = \{(x_1,x_2) : \mu_0 < x_i < \infty, i = 1,2\}$. [8]

p.t.o.

1.(j) Obtain a non-trivial upper confidence limit for μ at a confidence level $1-\alpha$.

[4]

2. Let $X_1,\dots,X_n$ be a random sample from $N(\theta,1)$. Obtain the Cramer-Rao lower bound for estimating $\Phi(\theta) = P_\theta(x>0)$, where Φ is the c.d.f of $N(0,1)$. Is this bound attained ? Give reasons. (Hint : Obtain the UMVUE of $\Phi(\theta)$).

[12]

EITHER

3. Let $X_1,\dots,X_n$ be a random sample from $N(0,\theta)$. Show that the following test is UMP unbiased size α for testing $\theta = 1$ against $\theta \neq 1$.

$$\phi(\underset{\sim}{x}) = \begin{cases} 1, & \text{if} \quad T < c_1 \text{ or } T > c_2 \\ 0, & \text{otherwise} \end{cases}$$

where $T = \sum_1^n X_i^2$,

$$\int_{c_1}^{c_2} g_n(t)dt = 1-\alpha,$$

$$g_{n+2}(c_2) = g_{n+2}(c_1),$$

g_n being the p.d.f of the chi-square distribution with n d.f.

[13]

OR

3.(a) Give a family of distributions of a random variable X for which $T(X) \equiv 0$ is sufficient.

[3]

(b) Let X be distributed either as uniform $U(0,1)$ or as uniform $U(\frac{1}{2}, \frac{3}{2})$. Derive a minimal sufficient statistic.

[5]

(c) Prove or disprove : If T_1 and T_2 are UMVU estimators of $\psi_1(\theta)$ and $\psi_2(\theta)$, respectively, then $T_1 + T_2$ is UMVU estimator of $\psi_1(\theta) + \psi_2(\theta)$.

[5]

Contd.....

4. Let X be a random variable having density f_0 and f_1 under the hypotheses H_0 and H_1, respectively. For a test function ϕ, the probability of rejecting H_0 given $X = x$ is given by $\phi[x]$. Consider the following test :

$$\phi^*(x) = \begin{cases} 1, & \text{if } f_1(x) > k\, f_0(x) \\ \gamma, & \text{if } f_1(x) = k\, f_0(x) \\ 0, & \text{if } f_1(x) < k\, f_0(x) \end{cases}$$

where $0 < k < \infty$.

(a) Show that

$$E[\phi(X)|H_1] \geq E[\phi^*(x)|H_1] \Rightarrow E[\phi(x)|H_0] = E[\phi^*(x)|H_0].$$

(b) Show that ϕ^* minimizes

$$p_0\, E[\phi(x)\ |H_0] + p_1\, E[1-\phi(x)|H_1],$$

where $0 < p_0, p_1 < 1$, $p_0 + p_1 = 1$, for $k = p_0/p_1$.

(c) Suppose $\beta(\alpha)$ be the power of a MP level-α test for testing H_0 against H_1. Show that $\beta(\alpha)$ is a nondecreasing function of α.

[5+5+5]

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INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1989-90

STOCHASTIC PROCESSES-2

SEMESTRAL-I BACKPAPER EXAMINATION

Date : 27.12.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Answer all the questions.

1. Consider a 2 state Markov chain with states 0 and 1, waiting times being Exp(λ), Exp(μ) respectively
   (i) Write down Kolmogorov's Equations for $p_{ij}(t)$.
   (ii) Solve them explicitly and write down the matrix $P(t)$.
   (iii) Calculate $\underset{t\to\infty}{\text{Lt}}\ P(t)$.
   (iv) Show how we could have thought of the process as an On-off system and derived the limit in (iii). [20]

2. (a) Define Brownian Bridge.
   (b) Let Z_t $0\leq t\leq 1$ be a Brownian Bridge. Define

$$X_t = (t+1)\, Z_{\frac{t}{t+1}} \qquad 0\leq t<\infty.$$

   Show that X_t is Brownian Motion.
   (c) If $(B_t)_{0\leq t<\infty}$ is a Brownian motion and $Y_t = \int_0^t B_s\, ds$ show that Y_t os a Gaussian process. Calculate $E(Y_t)$ and $Cov(Y_t, Y_u)$. [5+10+10]

3. (a) Define an On-off system.
   (b) Calculate $P(t)$ the probability that the system is on at time t.
   (c) State the Key Renewal theorem, explaining the notation involved.
   (d) Calculate $\underset{t\to\infty}{\text{Lt}}\ P(t)$ assuming that the cycle distribution is non lattice.
   (e) Explaining clearly how you bring in On-off system, evaluate $\underset{t\to\infty}{\text{Lt}}\ P(A_t \leq x)$ where A_t is the age at time t in a renewal process. [5+10+5+10+10]

4. Consider a Yule process (X_t) starting with one individual, that is, $X_0=1$. Given that $X_8=3$ show that the times of Births of the two individuals is like that of an order statistic from an appropriate distribution. [15]

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SAMPLE SURVEYS
SEMESTRAL-I EXAMINATION

Date : 1.12.1989 Maximum Marks : 100 Time : 4 Hours.

Note : Answer all the questions. Figures in [ ] indicate marks allotted.

1. It is desired to estimate a population proportion π using the sample proportion p based on an SRSWOR(N,n) sample data in such a way that the margin of error ($|\pi-p|$) does not exceed 0.05 with confidence coefficient 0.95. Using normal approximation, show that the sample size needed to ensure this is at the most 285 for large N. Show further that when N=500, the above can be achieved for a sample of size 182. [7+3=10]

2.(a) Describe ppswr and ppswor methods of sampling. [3]

(b) Consider a ppswor sample of n units from a population of size N. Denote by π_i the inclusion probability and by p_i the normed size measure of unit i, $1 \le i \le N$.

(i) Prove that for n=2, $p_i \gtrless p_j \iff \pi_i \gtrless \pi_j$. What happens for n>2 ? [3+1=4]

(ii) Write down the expression of the Symmetrized Des Raj estimator of the population mean. Show that it reduces to the sample mean when $X_1 = X_2 = \ldots = X_N$. [3+2=5]

(iii) Show that for an ordered sample s = (i,j), each of $t'_1 = \frac{Y_i}{p_i}$ and $t'_2 = \frac{Y_j(1-p_i)}{p_i p_j (N-1)}$ is an unbiased estimator of the population total. Are t'_1 and t'_2 uncorrelated ? [5+3=8]

(iv) Taking $t' = \frac{t'_1+t'_2}{2}$, derive the symmetrized form of t'. [5]

3.(a) Explain the ratio method of estimation of a finite population mean. [3]

(b) Write down the expression of the Hartley-Ross estimator of the population mean and show that it is unbiased. [2+5=7]

(c) Describe the Jack-knife technique for reducing bias of the ratio estimator. [5]

p.t.o.

4.(a) Describe briefly the method of stratified sampling. [5]

(b) Based on stratified srswor sample data, suggest an unbiased estimator of the overall population mean and derive an expression for its variance. For large stratum sizes, compare the performance of the suggested estimator with that of the mean based on unstratified srswor sample data. [2+4+4=10]

(c) What is Neyman's optimum allocation ? Under what conditions would optimum allocation suggest the sample sizes being proportional to the respective stratum totals ? [5+5=10]

5.(a) Explain double sampling technique in the context of regression method of estimation. [5]

(b) Suggest an estimator of the population mean of the study variable using double sampling data.
Derive large sample expression for the variance of the estimator proposed by you. [4+6=10]

6. Practical Assignments. .[10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

DIFFERENCE AND DIFFERENTIAL EQUATIONS
SEMESTRAL-I EXAMINATION

Date : 29.11.1989 Maximum Marks : 100 Time : 3 Hours.

Note : This paper carries 110 marks. You may answer all the questions. But the maximum you can score is 100.

1.(a) Find the general solution of the difference equation

$$y_{n+2} - \sqrt{2}\, y_{n+1} + y_n = 2^{3n/2}.$$

(b) If $u_n = \int_0^{\pi} \frac{\cos nx}{5-3\cos x}\, dx$,

show that $u_{n+2} + u_n = \frac{10}{3} u_{n+1}$, $n = 0,1,2,\ldots$.

Hence evaluate u_n.

[10+10=20]

2. Let $f(x,y)$ be continuous on the strip $a \le x \le b$, $-\infty < y < \infty$, and let $f(x,y)$ satisfy the following condition :

$\exists$ $K > 0$ such that

$$|f(x,y_1) - f(x,y_2)| \le K\,|y_1 - y_2| \quad \forall x \in [a,b] \text{ and } \forall y_1, y_2 \in (-\infty,\infty)$$

Show that, given $x_0 \in (a,b)$ and $y_0 \in (-\infty,\infty)$, there is a unique function $y = y(x)$ defined on $[a,b]$ which is a solution of the differential equation $\frac{dy}{dx} = f(x,y)$ on (a,b) and which satisfies the initial condition $y(x_0) = y_0$.

[20]

3.(a) Find the general solution of the Bernoulli's equation

$$xy^2 \frac{dy}{dx} + y^3 = x \cos x.$$

(b) Find the orthogonal trajectories of the family of curves

$$\frac{x^2}{2} + \frac{y^2}{3} = c, \; (c > 0).$$

[10+10=20]

4.(a) Given that $y(x) = x$ is a solution of

$$(x^2-1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

find the general solution of

$$(x^2-1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = (x^2-1)^2.$$

(b) Find the general solution of the differential equation

$$y''' + 3y'' + 3y' + y = xe^{-x}.$$

[12+8=20]

p.t.o..

5.(a) Consider the Bessel's equation

$$x^2y'' + xy' + (x^2-n^2)y = 0,$$ where n is a non-negative integer. Show that the above equation has only one (except for constant multiples) Frobenius series solution $J_n(x)$. Get a series expression for $J_n(x)$.

(b) Show that

$$J_n(x) = c.\frac{1}{\pi}\int_0^{\pi}\cos(n\theta - x\sin\theta)\,d\theta,$$ for some constant c.

[12+8=20]

6. Solve the system of differential equations

$$\frac{dy_1}{dt} = 4y_1 + y_2$$

$$\frac{dy_2}{dt} = -2y_1 + y_2 - 2e^t.$$

[10]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

STOCHASTIC PROCESSES-2
SEMESTRAL-I EXAMINATION

Date : 23.11.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Answer all the questions.

1.(a) Let N_t be a Nonhomogeneous Poisson process with intensity function $\lambda(t)$. Given that $N_5=3$ show that the times of occurrence of the three events are distributed like order statistics from an appropriate distribution.

(b) Show that the output process of an $M|G|1$ Queue is a nonhomogeneous Poisson process. What is its intensity function ?

[10+10]

2.(a) Let N_t be a renewal process with lifetime having mean μ and variance σ^2. Show that N_t is asymptotically normal with mean $\frac{t}{\mu}$ and variance $\sigma\sqrt{t/\mu^3}$.

(b) State and prove the inspection paradox.

[10+10]

3.(a) What is Equilibrium Renewal process ?

(b) Given t>0 calculate distribution of Y_t the residual lifetime at t for an Equilibrium Renewal process.

[5+10]

4.(a) For a continuous time Markov chain, assuming that $\underset{t\to 0}{\text{Lt}} \frac{P(t)}{t} = Q$, derive the Kolmogorov's Backward Equations.

(b) Write down the Backward Equations for a Birth and Death chain.

[10+10]

5.(a) Let $(B_t)_{t\geq 0}$ be Brownian Motion starting from 0. Calculate the conditional distribution of B_2 given $B_1=5$ and $B_3=9$.

(b) Let a>0 and $T(w) = \inf\{s: B_s=a\}$. Show that T is finite almost surely but $E(T) = \infty$.

[10=10]

6. Explain the following briefly :
   (a) Brownian Bridge.
   (b) Age at t in a Renewal process.
   (c) Continuous time Markov chain.

[15]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

ELECTIVE-4: BIOLOGICAL SCIENCES
SEMESTRAL-I EXAMINATIONS

Date : 20.11.1989   Maximum Marks : 100   Time : 3 Hours.

Note : Theory : Answer any three questions. Time $1\frac{1}{2}$ hours.
Theory : Full marks 50, each question has equal value.
Practical : Answer the given question. Time $1\frac{1}{2}$ hours.
Practical : Full marks 50.

Theory

1. Describe the criteria of inheritance due to a single, completely recessive, rare autosomal allele. Illustrate your answer with diagrams. Cite an example.

2. "A population in which two alleles $\underline{A}$ and $\underline{A}'$ occur in frequencies $\underline{p}$ and $\underline{q}$ respectively will consist, after one generation of random mating of the three genotypes $\underline{AA}$, $\underline{AA}'$ and $\underline{A'A'}$ in the equilibrium proportions $\underline{p}^2$, $\underline{2pq}$ and $\underline{q}^2$". What is this Law called ? Under what condition(s) do(es) this Law ideally operate? Applying this Law, estimate the allele proportions in a population having the following MN blood group phenotype frequencies:

$\overline{M} = 29.16\%$; $\overline{MN} = 49.58\%$; $\overline{N} = 21.26\%$.

3.1. What is the principle of blood grouping ? Write names of any five blood group systems.

3.2. In case of colour blindness if a carrier woman marries a normal man, then what types of offspring and in what proportions would you expect ?

3.3. Can blood group $\underline{O}$ children come from parents phenotypically $\underline{A} \times \underline{A}$ and $\underline{AB} \times \underline{AB}$?

3.4. What are the genotypes against phenotypes of the A_1A_2BO blood group system ?

4.1. Give a brief idea about the distribution of haptoglobin alleles in India in the world perspective.

4.2. State briefly the genetic control of Lactate Dehydrogenase (LDH) and describethe specific Indian variants of LDH.

Practical

1. Determine the ABO subtypes of the blood specimens provided against known anti-sera.

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B.Stat.(Hons.) III Year : 1989-90

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ELECTIVE-4 : ECONOMICS
SEMESTRAL-I EXAMINATION

Date : 20.11.1989 Maximum Marks : 100 Time : 3 Hours.

G.UP A

Note : Answer any two questions.

1. Consider a static Leontief input-output model where two goods are produced with goods and a single primary factor labour. Technology is given by the following matrix

$$A = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0 \end{pmatrix} \text{ and } a_{01} = 1, \quad a_{02} = 2$$

where a_{ij} and a_{0i} have their usual meaning. Assume further tha 12 units of labour are available and consumers spend half of their income on good 1 and the remaining half on good 2.

(a) Determine the relative price, the levels of consumption and the levels of output.

(b) If another technique A′ becomes available where

$$A' = \begin{pmatrix} 0.4 & 0.5 \\ 0.6 & 0 \end{pmatrix} \text{ with } a'_{01} = 1, \ a'_{02} = 2$$

determine which of the two techniques will be actually used.

(18+7)

2. In a von-Neumann growth model show that the interest factor cannot exceed the technological growth factor. Also, show that the price of a particular good is zero if the good grows by more than the technological growth rate and if a particular activity earns negative profits, its level of intensity is zero.

(13+12)

3. Show that in a two-sector, single primary factor Neo-Austrian model of production, the relative price is uniquely determined by the rate of interest provided there is no 'factor intensity reversal'. How will the instantaneous production possibility frontier change in such a model if there is an increase in the rate of growth ?

(15+10)

p.t.o.

GROUP B

Note : This part contains questions carrying 75 marks. You may answer any part and any question (and as many questions) as you like. But the maximum you may score is 50.

1. A motor car company manufactures for the home market and for export. Company policy is to export at least 60 percent of its cars. Not more than 150 cars can be produced each week. The delivery cost payable by the manufacturer are 5 and 20 respectively, for cars for the home market and for export, but the delivery firm specify that the total delivery costs must not be less than 1600 per week. Each car for the home market requires 120 man-hours to be spent on body work, 35 man-hours on the engine, and 10 man-hours on checking and testing. The corresponding figures for cars for export are 150, 40 and 15 man-hours respectively. The total number of man-hours available per week is 25000 for body work, 5500 for engine work, and 2000 for checking and testing. The net profit or car for export is K times that on a car for the home market, where K may lie between 2 and 0. Give a linear programming formulation of the problem if it is required to determine the number of cars of each kind if the company wishes to (1) maximise its net profit (2) minimise the excess of cars for export over those for the home market. (10)

2. (a) Find all the basic solution of the simultaneous equations

$$4x_1 + 5x_2 + 8x_3 + 7x_4 = 10$$

$$3x_1 + 2x_2 + 6x_3 + 9x_4 = 11$$

(b) Let $AX = b$ be a set of m simultaneous equation in N variables, with $m<N$ and $\text{rank}(A)=m$. Then prove, if the equations have a feasible solution ($X \geq 0$), they have a basic feasible solution. (3+9)

3. Explain in detail a single iteration of the simplex method. (10)

4.(a) For the maximising L.P. problem in the standard form, suppose that in the current basic feasible solution, $Z_j - c_j > 0$ for every column vector a_j of A which does not belong to B. Then prove that the current basic feasible solution is optimal.

(b) In the general linear programming problem the set of point representing all the feasible solutions is a closed convex set. (8+3)

Contd.....

5. In the Mathur-Hashim location model on defining the 'transport coefficients' what are the special observations mentioned ? Why has it been considered necessary to formulate a revised model for treating the 'Transport' sector as endogenous ? Why in the revised formulation do we notice absence of the production balance constraints ?

(4+4+4)

6. Find the optimal solution for the transportation problem for which the initial basic feasible solution is given as follows :

$x_{11}=6$ * 5	$x_{12}=8$ * 4	1	5
8	$x_{22}=2$ * 9	$x_{23}=4$ * 2	7
4	3	$x_{33}=1$ * 6	$x_{34}=4$ 2

The basic cells have been indicated by *

(5)

7. State whether the following statements are 'True' or 'False' (with brief reasons)

(a) The existence of secondary products in many economic activities results in the so called 'Aggregation problem' in the construction of Input Output tables.

(b) The informations contained in the 'Commodity x Industry input-Output table are derived from the 'Make Matrix'.

(c) Correct informations in respect of market prices solves the valuation problem in any exercise of Cost Benefit analysis for any situation.

(d) Mahalanobis Foursector Planning model is essentially an optimising model whilethe two sector model is a consistent typo planning model.

(e) Fully centrally controlled economies are perfectly compatible with the process of allocation of resources through existence of markets.

(3x5)

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90
ELECTIVE-4 : PHYSICAL AND EARTH SCIENCES
SEMESTRAL-I EXAMINATION

Date : 20.11.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Attempt question no.10 and any five from the rest.

1. The earth had either a hot or a cold beginning. Describe the one which you believe to be the best hypothesis. [16]

2. What is 'fold axis' ? What is meant by the 'dip' and the 'strike' of a bed ?

   Suppose you are visiting an area where previous workers have described a 'syncline' to occur. You find that it is actually an 'anticline'. Describe the reasons for your finding. [4+6+6]

3. S_i-O tetrahedron plays a significant role in the formation of silicate minerals. Describe its role in the formation of different kinds of igneous rocks. [16]

4. What is the importance of free Oxygen in the atmosphere ? When did it first form ? What could be the reason for its origin ? [2+4+10]

5. In 'plate tectonics', what is understood by 'plate' and 'tectonics' ? Describe in short the usefulness of the plate tectonics to the earth scientists. [6+10]

   OR

   Describe the role of rock magnetism in the understanding of the Plate Tectonics model.
   Explain in brief how magma/lava is generated in the subduction (convergent) zone.
   What is the role of asthenosphere in plate tectonics ? [3+4+4]

6. What do you understand by the terms 'bedding', 'bed' and 'lamination' ? In what ways cross-bedding and graded bedding are useful ? [10+6]

7. Describe the major factors and their effects in metamorphism. What is the difference between schistose structure and foliation ? Name one typical metamorphic rock from the Eastern Ghats and describe its mineralogical composition. [8+4+1+3]

p.t.o.

8. Describe the principal differences between an acid lava and a basic lava.

What is a 'primary' magma ? To which view about the nature of primary magmas do you agree - single or two ? Why ? [6+4+6]

9. Define a sedimentary rock. Describe how sediments are transported from one place to another place.

Suppose you are visiting an area where a deposit of glacial origin occurs. What are the criteria you would look for to prove that the deposit is indeed of glacial origin ? [4+6+6]

OR

What is a 'basin' and what kind of sedimentary processes go on in a basin ?

What is meant by 'sorting' of sediments ? In what way the fabric of a sedimentary deposit help in understanding the sedimentary processes ? [4+4+4+4]

10. Fill up the blanks (any ten). Write down only one of the four choices for each blank. (Please be careful not to attempt more than ten blanks; with each extra attempt one mark will be deducted.) [20]

(i) Sedimentary structures include ______ (grain-size/facies/sorting/stromatolite).

(ii) The term 'texture' includes ____ (grain-size/structure/cross-bedding/viscosity).

(iii) Presence of plant-eating four-legged animal remains in a sedimentary rock indicates that the environment of deposition of the rock was most probably ____ (mixed/continental marine/lagoon).

(iv) Asteroids are the product of ______(satellite/planet/meteor/comet).

(v) A marine sedimentary rock is best differentiated from a nonmarine one with the help of ______ (SiO_2/cross-bedding/iron-oxide content/fossils).

(vi) In metamorphism, a partial or wholesale change in mineral types takes place in ____ (gaseous/liquid/solid/biologic).

(vii) Shale is a ______ (sedimentary/metasomatic/metamorphic/igneous) rock.

(viii) Igneous rocks have a/an ____ (underlocking/crystalline/framework/matrix) texture.

(ix) The Tidal Hypothesis was proposed by _____(Moulton-Chamberlin/Kant-Laplace/Jeans-Jeffrays/Ringwood-Cameron).

(x) Lavas erupted by volcanoes are ____(basaltic/peridotitic/varied/granitic) in composition.

(xi) Magmas in areas of mountain-building activity are mostly _____(basaltic/granitic/andesitic/porphyritic) in composition.

(xii) Destructions in earthquakes are mainly caused by _____ (P waves/S waves/L waves).

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

Elective-4 : Economics
Periodical Examination

Date : 5.9.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Use separate answerscripts for each group.

GROUP A

Note : Answer any Two questions.

1.(a) Consider a simple Leontief system producing two goods with a single primary factor of production. Assuming that there is no joint production write down the Hawkins-Simon condition and interpret it economically. If instead, there were joint production, i.e. there were two activities where each activity produces both the goods (with $b_{ij} > 0$ as the amount of the ith good produced by the jth activity if run at the unit level), find out a condition which will guarantee that the system is viable.

(b) Prove that a Leontief system is viable if each row sum of the input-output matrix is less than unity. Interpret the result. (15+10)

2. Consider an economy producing two goods with two factors of production. Technology is given by $a_{11} = 1$, $a_{12} = 1$, $a_{21} = 1$, $a_{22} = 2$ where a_{ij} is the amount of the ith factor required to produce one unit of the jth good. Assume further that the consumers spend $2/5$th of their income on good 1 and $3/5$ th on good 2. Finally, let the endowments of the first and the second factors be 10 and 15 respectively.

(a) Determine the levels of output, the commodity prices and the factor prices.

(b) If the endowment of the first factor increases to 15, determine the commodity prices, the factor prices and the levels of output. (12+13)

3. State and prove the Non-Substitution Theorem for a Leontief economy. What are the economic implications of the theorem ? (25)

p.t.o.

GROUP B

Note : Answer any two questions.

1.(a) Define and illustrate 'Make Matrix'. Distinguish between 'Commodity Technology' and 'Industry Technology' assumption in the context of the construction of input output tables when there are secondary products along with principal products in many economic activities in the economy.

(b) Obtain the 'Commodity x Commodity' Input-output Table given in the 'Make Matrix' and the 'Commodity x Industry' table below.

Make Matrix				
	Industry			
Commodity	1	2	3	Total
1	100	0	0	100
2	10	100	0	110
3	0	0	50	50
Total	110	100	50	260

Commodity x Industry I-O Table					
	Industry				
Commodity	1	2	3	Final Demand	Total
1	20	30	0	50	100
2	30	20	20	40	110
3	10	20	10	10	50
Value added	50	30	20		100
Total	110	100	50		

(10+15)

2.(a) Derive the growth equations for the 'consumer goods' sector and 'capital goods' sector in the Mahalanobis two sector model of planning.

(b) How does the original four sector model of Mahalanobis look if we incorporate the following in the above ?
(Discuss also the consequences)

(1) Demand equations linking the increase in income and employment with demand for various consumer good sectors.

(2) The possibility of variation in the techniques of production.

(3) Controlling consumption through rationing or fiscal policies. (10+15)

3. Write short notes on any one of the following :

(a) (1) Shadow prices and the evaluation of public projects
(2) Private Profitability and Social Welfare.

(b) Decision tree analysis.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

Elective - 4 : Physical and Earth Sciences
Periodical Examination

Date : 8.9.1989   Maximum Marks : 100   Time : 3 Hours.

Note : Please attempt Question No.1 and any five from the rest.

1. Fill up the blanks (any 10). Only write down one of the four choices for each blank. (10x2)=20

   (i) ____________ (Silicates/Oxides/Carbonates/Sulphides) are the most common minerals.

   (ii) Emerald is the gemstone variety of ____________ (topaz/quartz/corundum/garnet).

   (iii) The ____________ (blood-cells/muscles/teeth/eyes) of a vertebrate animal are most likely to be preserved.

   (iv) The ____________ (thickness/chemical composition/ripple mark/colour) helps to determine the top surface of a sedimentary layer.

   (v) A rock composed of 40% by volume of angular pebbles is called ____________ (breccia/mudstone/eclogite/conglomerate).

   (vi) A horizontal sequence of sedimentary layers lying over a tilted sequence of sedimentary layers has a/an ____________ (tectonic/unconformable/sedimentary/metamorphic) contact.

   (vii) ____________ (Feldspar/Basalt/Peridotite/Granite) is a coarse grained light coloured plutonic rock.

   (viii) Pyrite is a/an ____________ (Precious/rock-forming/economic/ordinary) mineral.

   (ix) Marble is a ____________ (bed/mineral/rock/clast).

   (x) The Gondwana rocks of the Godavari Valley that are being mapped by the ISI geologists are ____________ (continental/marine/mixed/igueous) rocks.

   (xi) A palaeontologist analyzes fossils to trace their ____________ (morphology/stratigraphy/evolution/importance).

   (xii) A crystal form which does not have its faces developed is called ____________ (Subhedral/anhedral/hypidimorphic/enhedral).

p.t.o.

2. Define a mineral. What are its different modes of origin ? What is a crystalline substance ? Describe the method you would adopt to identify quartz in hand specimen.
(4+4+4+4)=16

3. What is a rock ? 'A rock has got metamorphosed' – What does this mean ? What kinds of physical changes are noted in a metamorphosed rock ?

   Write down the sequence of rocks that are formed when a mudstone is subjected to unidirectional stress over a long period of time.
(3+3+3+7)=16

4. Describe the various processes involved when the hard parts of an organism get altered while it undergoes fossilization.

   'Fossils are useful as economic tools' – elucidate.
(10+6)=16

5. A violent earthquake takes place at a certain time in a remote place. Describe the process you would follow to determine the time and location of the earthquake site. In what way the earthquakes are useful to the scientists ?
(8+8)=16

6. What is meant by 'rocks of Proterozoic age' ? Do you expect to find coal in such rocks ?

   Why is the study of the Cretaceous – Tertiary boundary so important to the geoscientists ?
   'The Archaean rocks contain graphite and thick layers of limestone : – What does this statement signify ?
(6+2+4+4)=16

7. Which one is said to be the most versatile among the various radiometric methods of age determination of rocks ? Describe it. Which rocks are said to give the best results in this method ?
   'There are some problems associated with this method' – What are those ?
(2+8+2+4)=16

8. Write short notes on (any four) :

   Texture of plutonic rocks; Texture of clastic sedimentary rocks; Magma; Low velocity zone; Shocked quartz; Appearance of first birds in geological time-scale.
(4x4)=16

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

Difference and Differential Equations
Periodical Examination

Date : 6.9.1989 Maximum Marks : 100 Time : 3 Hours

Note : This paper carries 110 marks. You may answer all the questions. But the maximum you can score is 100.

1. Find all the solutions of the differential equation

$$x\frac{dy}{dx} = y + x\,e^{-2y/x}.$$ [10]

2. Find the equation of the family of curves which are orthogonal to the family of curves

$$\frac{x^2}{2} + \frac{y^2}{3} = c, \quad c > 0.$$ [15]

3. Given that the differential equation

$$(e^y + xe^y) + xe^y\frac{dy}{dx} = 0$$

has an integrating factor which is a function of x only, find its general solution. [15]

4. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = e^x \operatorname{Cos} 2x$$

by the annihilator method. [15]

5. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\log x.$$ [15]

6. Show that $x = 1$ and $x = -1$ are regular singular points of the Legendre equation

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \alpha(\alpha+1)y = 0.$$

Find its indicial equation at $x = 1$.
Show that infinity is a regular singular point of the above equation. Find its indicial equation at $x = \infty$. [20]

7. Show that the equation

$$4x^2\frac{d^2y}{dx^2} - 8x^2\frac{dy}{dx} + (4x^2+1)y = 0$$

has only one Frobenius series solution valid in $(0,\infty)$. Find the general solution of the above equation valid in $(0,\infty)$. [20]

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INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1989-90

Statistical Inference
Periodical Examination

Date : 4.9.1989 Maximum Marks : 100 Time : 3 Hours.

1. Which of the following family of distributions are exponential families ? (Prove or disprove).

   (a) The uniform $\mathcal{U}(0, \theta)$ family

   (b) $P(x,\theta) = \frac{1}{9}$, $x \in \{0.1 + \theta, \ldots, 0.9 + \theta\}$

   (c) $N(\theta, \theta^2)$ family

   (d) $P(x,\theta) = \frac{2(x+\theta)}{1+2\theta}$, $0 < x < 1$, $\theta > 0$. [16]

2. Let $X_1, \ldots, X_n$ denote a sample from a population with one of the following densities. Find a non-trivial sufficient statistics and the MLE of θ in each case.

   (a) $P(x,\theta) = C\,\theta^c\, x^{-(c+1)}$, $x \geq \theta$, C constant > 0, $\theta > 0$.

   (b) $P(x,\theta) = \sqrt{\theta}\; x^{\sqrt{\theta}-1}$, $0 \leq x \leq 1$, $\theta > 0$. [12]

3. Let $(X_1, \ldots, X_m)$, $(Y_1, \ldots, Y_n)$ be independently distributed according to $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$, respectively. Find minimal sufficient statistics for the following three cases :

   (i) ξ, η, σ, τ are arbitrary, $-\infty < \xi, \eta < \infty$, $0 < \sigma, \tau$

   (ii) $\sigma = \tau$, and ξ, η, σ are arbitrary

   (iii) $\xi = \eta$ and ξ, σ, τ are arbitrary

   (iv) $\xi = \eta = 0$, σ and τ arbitrary > 0

   (v) $\sigma = \tau = 1$, $-\infty < \xi, \eta < \infty$. [20]

4. Let $(X_1, \ldots, X_n)$ be n i.i.d. Bernoulli random variables with probability of success being equal to θ $(0 < \theta < 1)$.

   (a) Find an unbiased estimator of $\theta(1-\theta)$ based on the sufficient statistic $T = \sum_1^n X_i$.

   (b) Show that there does not exist any unbiased estimator of $\theta/(1-\theta)$ whatever may be n. [5+8]

p.t.o.

5.(a) Let X have the distribution P_θ, $\theta \in \Theta$. Let $\mathcal{U}$ denote the class of all unbiased estimators of 0 which have finite variance for all θ. Let T be an unbiased estimator of $\psi(\theta)$ with $Var_\theta(T) < \infty$ for all θ. Show that T is the UMVU estimator of $\psi(\theta)$ if, and only if, $Cov_\theta(T, U) = 0$ for all $u \in \mathcal{U}$ and all $\theta \in \Theta$. [20]

(b) Suppose T_1 and T_2 are two UMVU estimators of θ with finite variance. Show that $T_1 = T_2$. [6]

6. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta,1)$. Prove directly that $\sum_1^n X_i$ is sufficient for $\theta \in R$. [10]

For clarity [3]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) III Year : 1989-90

Stochastic Processes
Periodical Examinations

Date : 31.8.1989 Maximum Marks : 100 Time : 3 Hours.

1.(a) Keeping part (b) below in mind define what is meant by a Poisson process with parameter λ.

(b) Let $(N_t)_{t \geq 0}$ be a PP(λ). Using your definition of Poisson process above, evaluate :

(i) $P(N_5 = 2, \quad N_8 = 3, \quad N_9 = 4)$.

(ii) $P(N_5 = 2, \quad N_8 = 1)$.

(iii) The conditional distribution of N_5 given that $N_8 = 7$.

[5+10]

2. The number of accidents upto time t in the four cities Bombay, Calcutta, Delhi, Madras be denoted by N_t^1 , N_t^2 , N_t^3 , N_t^4 respectively. Assume that they are independent and N_t^i is PP(λ_i) for i = 1,2,3,4. Denote by N_t the total number of accidents upto t in these four cities.

(i) Show that N_t is a Poisson process. What is its parameter ?

(ii) Given that $N_t = 10$ show that the conditional distribution of the 4-tuple $(N_t^1, N_t^2, N_t^3, N_t^4)$ is multinomial. What are the parameters of this multinomial ?

[7+8]

3. Let N(t) be a nonhomogeneous Poisson process with mean function $m(t) = t^2$.

(i) What is the intensity function $\lambda(t)$?

(ii) What is the distribution of $N_5 - N_2$?

(iii) Given that $N_7 = 1$, what is the conditional probability that the event occurred between t = 1 and t = 6 ?

(iv) Define $N^*(t) = N(\sqrt{t})$. Show that N^* is PP(1).

[5+5+5+5]

p.t.o.

4.(i) Explaining clearly the notation and hypotheses, state the Key Renewal Theorem.

(ii) What is an On-off system ? What is the renewal process associated with an On-off system ?

(iii) Suppose we have a usual renewal process $N(t)$ with a nonlattice renewal distribution F having mean μ. Show that

$$\underset{t \to \infty}{Lt} \quad P[X_{N(t)+1} \leq x] = \frac{1}{\mu} \int_0^x y dF(y)$$

for any $x > 0$. [10+10+10]

5.(i) For a renewal process, prove with usual notation,

$$m(t) = F(t) + \int_0^t m(t-x)\, dF(x).$$

(ii) Define the terms 'age at t' 'Residual life time at t' for a renewal process. Denote them by $A(t)$ and $Y(t)$ respectively.

(iii) Fill in the missing terms :

(a) $A(t) > x$ iff 0 events in the interval ____________

(b) $Y(t) > x$ iff 0 events in the interval ____________

(c) $P(Y(t) > x) = P(A(\_) > -)$. [15+5+10]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1989-90

Periodical Examinations

Sample Surveys

Date : 28.8.1989 Maximum Marks : 100 Time : 3 Hours.

Note : Attempt all questions. Marks allotted to each question are given in brackets ( ).

1.(a) Define the terms 'Sampling Design' and 'Sampling Scheme'. What do you understand by the 'inclusion probability of a unit, π_i' and 'joint inclusion probability of a pair of units π_{ij}'. Calculate π_i and π_{ij} for a Simple Random Sampling (SRS) with Replacement design of n draws.

(3+3+2+2+2+3) = (15)

(b) Let the population size be 3 and the sample size be 2 and let $s_1 = \{U_1, U_2\}$, $s_2 = \{U_1, U_3\}$ and $s_3 = \{U_2, U_3\}$. Under the SRS design let $p(s_i) = 1/3$ for i = 1,2,3. Define the estimator t by

$$t = \begin{cases} t_1 = (y_1+y_2)/2 \text{ if } s_1 \text{ occurs} \\ t_2 = (y_{1/2}) + (2y_{3/3}) \text{ if } s_2 \text{ occurs} \\ t_3 = (y_{2/2}) + (y_{3/3}) \text{ if } s_3 \text{ occurs.} \end{cases}$$

Show that t is unbiased for $\bar{Y}$ and that there exist populations (Y_1, Y_2, Y_3) for which $V(t) \nleq V(\bar{y})$, where $\bar{y}$ is the conventional sample mean. What does this example show ?

(3+5+2) = (10)

(c) Out of 105 students sampled using a SRSWOR design from a school of 1241 science students, 72 expressed that they would like to study computer science. Estimate the proportion of students preferring computer science and also obtain an approxomate 95% confidence interval for the proportion.

(3+7) =(10)

2.(a) How do you select a 'linear systematic sample of size n' from a population of size N ?

(b) Suggest an unbiased estimator for the population mean $\bar{Y}$ of a characteristic y based on the above design. Describe a modification of the above design which makes the sample mean an unbiased estimator of $\bar{Y}$ when N is not divisible by n.

(c) When the values of the y-characteristic are known to be of the form $Y_i = \alpha + \beta i$ and when the population size is a multiple size, would you prefer systematic sampling to simple random sampling ? Give reasons. (2+8+10) = (20)

p.t.o.

3.(a) When do you use a probability proportional to size sampling technique ? (4)

(b) Show that the selection of units by 'Lahiri's Method' does indeed give the probability of selection for a unit U_i equal to X_i / X where X_i is the size measure of the unit U_i and $X = \sum_{i=1}^{N} X_i$. (7)

(c) A sample of 6 hospitals is drawn from a population of 74 hospitals with probability of selection proportional to the size (x, the no. of beds) with replacement and the number of discharges is observed :

sampled hospital	x no. of beds	y no. of discharges
21	21	105
32	101	524
43	14	73
54	6	31
25	41	200
36	12	64

It is also known that the total number of beds in the 74 hospitals is 2949.

(i) Estimate the average number of discharges in the population. (11)

(ii) Calculate an estimate of the coefficient of variation of the above estimate. (14)

(iii) If, by mistake, a sampler treats the sample as a without replacement sample obtained in the above order, what would be his estimate of the average number of discharges ? Comment on that. (7+2) = (9)