[1990-91 338(b)]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1990-91
Statistical Inference II
Semestral-II Backpaper Examination

Date : 1.7.1991 Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Note : Attempt all questions. Marks are indicated in the margin.

Group A

1. Let $\alpha = \beta = .1$. Let $X \sim f_\theta(x) = \begin{cases} \exp[-(x-\theta)] & \text{if } x \geq \theta \\ 0 & \text{o.w.} \end{cases}$

   Find the SPRT for $H_0: \theta = 0$ vs. $H_1: \theta = 1$.

   Compute the ASN and OC under H_0. [13]

2. Let X_1, X_2 .. be i.i.d. Bernoulli trials with probability p of success, $0 < p < 1$. Consider testing $H_0: p = p_0$ vs $H_1 : p = 1-p_0$, $0<p_0<\frac{1}{2}$.

   Show that one can compute the EXACT OC.
   (You may consider only the case of $p \neq \frac{1}{2}$). [13]

3. Derive the Fundamental Identity of Sequential Analysis.

   OR

   Show that in Stein's two-stage procedure for a fixed-width confidence interval estimation of μ in $N(\mu,\sigma^2)$, σ^2 unknown, $\bar{X}_N$ is an unbiased estimator of μ. Obtain its variance. [14]

4. Assignments given in class. [10]

Group B NON-PARAMETRIC METHODS

Note : Use a separate answer booklet for this group. Answer as much as you can. The maximum you can score from this group is 50.

1.(a) Describe the 2-sample Wilcoxon rank-sum test. Find the null distribution of the Wilcoxon rank sum statistic W_s when $m = 3$, $n = 4$.

p.t.o.

1.(b) In a business administration course a set of lectures was given televised to one group and live to another. In each case an examination was given prior to the lectures and immediately following them. The difference between the two examination for the students in the two groups were as follows :

Live : 20.3, 23.5, 4.7, 21.9, 15.6, 20.2, 26.6, -9.4, 4.8, -1.6, 25.0

TV: 6.2, 15.7, 25.1, 4.9, 28.1, 17.2, 14.1, 31.2, 12.6, 9.4, 17.3, 23.4

Use the two-sided Wilcoxon test to test the hypothesis of no difference at significance level $\alpha=0.05$.

[2+4+14]

2.(a) Define the Kolmogorov-Smirnov Statistics $D^{+}_{m,n}$, $D_{m,n}$ for testing equality of 2 continuous distribution functions F and G on the basis of independent samples of sizes m and n respectively from F and G. Show that

$$P\left(D^{+}_{nn} \geq \frac{k}{n} \mid F = G\right) = \frac{\binom{2n}{n-k}}{\binom{2n}{n}}$$

$$k = 0, 1, \ldots n$$

[10+10]

3. Write short notes on

(a) Normal Scores test

(b) Siegel-Tukey test

(c) Wald - Wolfowitz Run test.

[7+7+7]

[1990-91 [318(b)]]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1990-91

Introduction to Stochastic Processes

Semestral-II Backpaper Examination

Date : 1.7.1991 Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Group A

1.(a) Consider a Markov chain having state space $\{0,1,2\}$ and transition matrix

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} .4 & .4 & .2 \\ .3 & .4 & .3 \\ .2 & .4 & .4 \end{bmatrix} \end{array}$$

Show that this chain has a unique stationary distribution π and find π.

(b) Consider a Markov chain having transition function P such that $P(x,y) = \alpha_y$, $x \in \mathcal{Y}$ and $y \in \mathcal{Y}$, where α_y are constants. Show that the chain has a unique stationary distribution π, given by $\pi(y) = \alpha_y$, $y \in \mathcal{Y}$.

[7+3]

2. Let X_n, $n \geq 0$ be a Markov chain state space $\mathcal{Y}$ in a subset of $\{0,1,2,\ldots\}$ and whose transition function P is such that

$\sum_y yP(x,y) = Ax+B$, $x \in \mathcal{Y}$, for some constants A,B.

(a) Show that $E(X_{n+1}) = A\,E(X_n) + B$

(b) Show that if $A \neq 1$, then

$$E(X_n) = \frac{B}{1-A} + A^n \left(E\,X_0 - \frac{B}{1-A}\right).$$

[10]

3. Consider the branching chain in which each particle is decomposed into some particles whose number follows the distribution of random variable ξ (≥ 0) after each unit time, independent of one another.

X_n = total number of particles at time n.

Let $X_0 = 1$.

Show that the extinction probability ρ is strictly less than 1 if $E(\xi) > 1$ and $\rho = 1$ if $E(\xi) \leq 1$.

[20]

p.t.o.

4. Let $S_n = X_1 + X_2 + X_3 + \ldots + X_n$

where X_i's are i.i.d. random variables

with $P(X_1 = 1) = P(X_1 = -1) = \frac{1}{2}$

Define $U_{2n} = \binom{2n}{n}/2^{2n}$,

then show that

(i) $P(S_1 \neq 0, S_2 \neq 0, \ldots, S_{2n-1} \neq 0, S_{2n} \neq 0) = u_{2n}$

(ii) $P(S_1 \geq 0, S_2 \geq 0, \ldots\ldots, S_{2n-1} \geq 0, S_{2n} \geq 0) = u_{2n}$ [10]

GROUP B

Attempt any two questions.

5.(a) Consider a birth and death process $(X(t): t \geq 0)$ with linear growth and immigration; specifically, the birth rates λ_n and death rates μ_n are of the form $\lambda_n = n\lambda + \alpha$ and $\mu_n = n\mu$, $n = 0,1,2,\ldots$, where $\alpha>0$, $\lambda>0$, $\mu>0$ and $\lambda \neq \mu$. Obtain the expression for $E(X(t)|X(0) = r)$ and discuss the asymptotic behaviour of the same as $t \to \infty$.

(b) Customers arrive at a store in groups. The arrivals of groups follow a poisson process with rate $\lambda>0$. The groups are independent and each group consists of either 1 or 2 persons with equal probabilities. (i) Show that arrivals of groups of size 1 also follow a Poisson process with a certain rate λ^* to be determined by you. (ii) If each group of size 1 makes purchases worth Rs R/- and each group of size 2 purchases goods worth Rs cR/-, obtain the average sale of the store by time $t>0$ (c is a positive constant).

[10+15]

6. For a renewal process $(X(t): t \geq 0)$ with associated distribution function $F(x)$, define $\mu_k(t) \underset{def}{=} E(X(t)^k)$, $k = 1,2,\ldots$, $t>0$.

(a) Comment on the finiteness of $\mu_k(t)$.

Contd.....

6.(b) Show that for $t>0$,

$$\mu_k(t) = z_k(t) + \int_0^t u_k(t - \tau)\, dF(\tau),$$

$k = 1,2,\ldots,$ where

$$z_k(t) = \int_0^t \sum_{j=0}^{k-1} C_j\, \mu_j(t - \tau)\, dF(\tau).$$

Hence, obtain $\mu_2(t)$ whenever $F(x)$ is distributed uniformly over $[0,1]$.

[12+13]

7.(a) Consider a Markov chain $(X(t): t \geq 0)$ where $X(t)$ denotes the number of accidents taking place in a certain city by time $t \geq 0$. The probability of an accident in $(t, t+h)$ is $\lambda_0 h + o(h)$ if no previous accidents have occurred, and is $\lambda_1 h + o(h)$ otherwise, independently of the actual number of accidents. Obtain $Pr(X(t) = k)$ for $k = 1,2,\ldots$.

(b) Describe how you would determine the optimal replacement period in an age replacement policy.

[13+12]

[1990-91] [328(b)]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1990-91

Design of Experiments

Semestral-II Backpaper Examination

Date : 26.6.1991 Maximum Marks : 100 Time : 3 Hours.

Note : Answer any THREE questions from Group A and any TWO questions from Group B. Marks allotted to a question are indicated in brackets [ ] at the end.

GROUP A

1.(a) State and explain the fundamental principles of experimental designs with examples.

(b) Identify the estimable functions of a randomised block design for v treatments in b blocks.

(c) Derive the expressions for the Bonferroni's and Tukey's methods for multiple comparisons.

[6+6+8 = 20]

2. Prove that the symbolic Kroneckler product of two pairs of MOLS (mutually orthogonal latin squares) of orders v_1 and v_2 is a pair of MOLS of order $v_1 v_2$. Describe a method of construction, with proof, of a complete set of MOLS of order s, where s is a prime or power of a prime. Explain with examples how the mutually orthogonal latin squares can be used as experimental designs.

[8+8+4 = 20]

3.(a) Develop the analysis of convariance under the usual fixed effects model :

$\Omega : \underset{\sim}{y} = X\beta + H\underset{\sim}{r} + \underset{\sim}{e}$, etc. stating clearly all the underlying assumptions.

(b) Apply the analysis to a randomised block design for v treatments and r blocks with one concomitant variable x, and give the ANOVACOV of this design.

[10+10 = 20]

4. Show how analysis of covariance can be used as an alternative to the missing plot technique. Use your results to obtain the 'estimates' of the missing values, and expressions for various variances of estimated elementary treatment contrasts for a randomised block design with two missing cells, affecting only one treatment, but two different blocks.

[8+12=20] p.t.o.

GROUP B

5.(a) Describe Yates Algorithm for 2^n experiment and prove that it requires exactly n steps to calculate the sum of squares of different factorial effects.

(b) For a 3^3 experiment write down the d.f. for different interaction components and the expressions for corresponding sum of squares. [10+10 = 20]

6.(a) Construct a confounded $(2^5, 2^3)$ design confounding the two independent factorial effects ABCD and BCE.

(b) Given the layout of a block in $(3^4, 3^2)$ experiment find out the complete set of confounded interaction components.

1111
1202
2012
0210
0001
2221
2100
0122
1020

[10+10 = 20]

7.(a) There are mk treatments arranged in m sets of k each and treatments of a set are assigned to a block and there are r replications. Show that the resulting design is orthogonal.

(b) Define the following BIBDs and write down the parameter of the designs.

(i) derived BIBD
(ii) residual BIBD
(iii) complementary BIBD.

[8+12 = 20]

[1990-91][328]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1990-91

Design of Experiments

Semestral-II Examination

Date : 6.5.1991 Maximum Marks : 100 Time: 3 Hours.

Note : Answer any THREE questions from Group A and any TWO questions from Group B. Marks allotted to a question are indicated in brackets [ ] at the end.

GROUP A

1.(a) Give an example of an experiment in which a graeco latin square of order 4 can be used. Give a non-randomised layout of your design, and point out one defect of your design. Write down the analysis of variance table of your design, indicating clearly how the various sums of squares are to be computed.

[2+3+2+3 = 10]

(b) Show that there exist at most (v-1) mutually orthogonal latin squares (MOLS) of order v. In an analogour manner show also that a set of (v-2) MOLS of order u can always be extended to a complete set of (v-1) MOLS of order v.

[5+5 = 10]

2.(a) Show that under a missing plot situation

(i) $SSE_{\tilde{\Omega}(\underset{\sim}{w}^*)} = SSE_{\Omega}$ where $\underset{\sim}{w}^*$ is any solution of $\underset{\sim}{w}^* = Z\,\hat{\beta}_{\tilde{\Omega}(\underset{\sim}{w}^*)}$, and

(ii) $\hat{\beta}_{\tilde{\Omega}(\underset{\sim}{w}^*)} = \hat{\beta}_{\Omega}$, where the symbols have their usual significances.

(b) Hence obtain the " estimates" for the missing values in cells (i,j) and (i', j'), $i \neq i' \in \{1,2,\ldots,v\}$, $j \neq j' \in \{1,2,\ldots,r\}$ in a randomised block design for v treatments in r blocks. Derive also the expressions for $V(\hat{\tau}_i - \hat{\tau}_j)$, $i \neq j \in \{1,2,\ldots,v\}$ for the resulting incomplete block design.

[(5+3)+(8+4) = 20]

3. Show how the analysis of covariance can be carried out easily using the results of the corresponding analysis of variance (start with the appropriate linear statistical model in each case and state clearly the underlying assumptions. Illustrate your results with respect to a latin square design with one concomitant variable.

[12+8 = 20]

p.t.o.

4. What is a split-plot design ? Write an appropriate linear model for a split-plot design in r randomised blocks with α main treatments and β sub-treatments, and indicate its analysis of variance, explaining clearly how the various sums of squares are computed. Give also the table of comparisons of interest and their estimated standard errors.

[2+3+10+5 = 20]

GROUP B

5.(a) What do you mean by a resolvable block design ? Show that for a resolvable BIBD with parameters (b,v,r,k,λ)

$$b \geq v + r - 1$$

(b) Either construct or prove non existence of the followi BIBDs.

(i) $v = b = 29$, $r = k = 8$, $\lambda = 2$

(ii) $b = 12$, $v = 9$, $r = 8$, $k = 6$, $\lambda = 5$.

[2+6+3+9 = 20]

6.(a) Show that if any particular block is deleted from a BIBD with $\lambda = 1$, the resulting design is still a connected design but not balanced.

(b) From the given layout of a particular block in $(3^4,3^2)$ experiment find out the set of all confounded interaction components.

2201
1212
0002
2010
1021
1100
2122
0220
0111

[8+12 = 20]

7. Construct the key blocks for a balanced confounded $(2^5, 2^3)$ design saving main effects and two factor interactions. Also obtain the expressions for sum of squares due to different factorial effects in the above design.

[15+5 = 20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year:1990-91
STATISTICS COMPREHENSIVE
SEMESTRAL EXAMINATION

Date:10 May 1991 | Maximum Marks:100 | Time:4 Hours

Figures in brackets [ ] indicate marks allotted to the questions.

The paper carries 125 marks. You may attempt any part of any question. However, the maximum you can score is 100.

1. On the basis of a single observation from a normal distribution with mean θ and variance $\theta^2(>0)$, where θ is unknown, derive the most powerful test of size 0.05 for the null hypothesis $H_0: \theta = 1$ against the alternative $H_1: \theta = 2$. Also discuss the existence of a uniformly most powerful test of size 0.05 for testing $H_0: \theta = 1$ against the alternative $H: \theta > 1$. [15]

2. (a) A random sample of size n was drawn from an exponential population with an unknown mean θ. Accidentally, the detailed data were lost, but the statistician could remember that Y of the n observations were greater than 10. On the basis of Y, find the maximum likelihood estimator, say $\hat{\theta}$, of θ. Find an expression for the sample variance of $\hat{\theta}$ and compare this with the variance of the maximum likelihood estimator of θ that one would have obtained had the complete data been available. Comment on your findings. [17]

   (b) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables each with p.d.f.

   $$e^{-(x-\theta)}(x-\theta), \quad \theta < x < \infty,$$

   where θ is an unknown parameter. Discuss analytically the derivation of the maximum likelihood estimator of θ. [10]

3. *(For this question, you need not give detailed proofs. Short answers can be given if you approach the problem in the right direction.)*

   (a) Suppose (X, Y) is uniformly distributed over the circle $x^2 + y^2 = 1$. Show that X and Y are not independent. [3]

PLEASE TURN OVER

(b) Suppose X_1, X_2 are *i.i.d.* as $\mathcal{N}(\mu, \sigma^2)$. Show that

$$E[\max(X_1, X_2)] = \mu + \frac{\sigma}{\sqrt{\pi}}.$$

[6]

(c) Suppose X is distributed either as $U(0,1)$ or as $U(\frac{1}{2}, \frac{3}{2})$. Find a sufficient statistic (other than X) for this two-member probability family based on one observation on X. [6]

4. A 2^3 factorial design is laid out in three blocks as shown below:

Block I	(1)	a		
Block II	c	ab		
Block III	b	ac	bc	abc

Identify the estimable (i.e., unconfounded) main effects and interactions, if any. [8]

5. Explain the phenomenon of 'regression to the mean' that led to the term *regression*. Give a theoretical support to this phenomenon. [12]

6. In a genetic study of *albinism*, families were contacted through an albino child. The albino children in the population were sampled independently, each with the same small known probability π of being selected. The frequency distribution of the 219 families selected in the sample by the number of children and the number of albino children is given below.

No. of Albino	*No. of Children*			
Children	4	5	6	7
1	22	25	18	16
2	21	23	13	10
3	7	10	18	14
4	0	1	3	5
5		1	0	1
6			1	0
7				0

CONTINUED

Let θ represent the probabiltiy that an offspring of a couple capable of producing an albino, is an albino.

(a) Formulate a model for the number of albino children in a family with k children, suitable for this situation. [10]

(b) Find the maximum likelihood estimate of θ, explaining your computations. [15]

7. In order to understand the concept of survival of the fittest, a sample of 1158 snails of the species *cepaea nemoralis* was collected in Manchester, England and their maximum shell diameters were measured. The snails were then left in an unheated room from the end of September 1968 until the middle of June 1969. Over this period about 80% of the snails died. The maximum shell diameters of the surviving snails were measured. The following table gives the data collected. The problem is to estimate a "fitness" or a "survival" function.

Maximum Shell Diameter (mm) x	Initial Frequency	Frequency of Survivors
21	21	1
22	93	6
23	255	44
24	343	70
25	289	62
26	128	36
27	29	7

Suppose that originally $X \sim \mathcal{N}(\mu_0, \sigma_0^2)$ and the probability of survival given $X = x$, is $p(x) = K.\exp[-c(x-\theta)^2]$. Estimate K, c, θ from the given data. [23]

1990-91 358

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) III Year : 1990-91

Physics II

Semestral-II Examination

Date : 8.5.1991 Maximum Marks : 100 Time : 3 Hours.

Group A

Note : Answer question no. 5 and any two of the rest.

1. Derive by Bose's statistics the Planck's law of radiation. Hence establish (i) Stefan-Boltzmann law and (ii) Wien's law of displacement.

8+3+4 = [15]

2.(a) Apply Maxwell-Boltzmann statistics to deduce either the monotomic ideal gas equation, or the barometric equation.

(b) Write down (do not deduce) the distribution function in FD-statistics. Explain the symbols used. Show that at absolute zero, the momentum space of an assembly of fermions is uniformly populated within a sphere of radius p_{mo} where

$$p_{mo} = \left(\frac{3Nh^3}{8\pi V}\right)^{\frac{1}{3}}$$

where N is the total number of fermions, V the total volume of the system and h, the Planck's constant.

7+(2+6) =[15]

3.(a) Prove that $\frac{1}{2}mv^2$, where $m = m_0/\sqrt{1-v^2/c^2}$, does not equal the kinetic energy of a particle moving at relativistic speeds.

Derive the relativistic expression of kinetic energy and show that it reduces to the usual expression $\frac{1}{2}mv^2$, when $v/c << 1$.

(b) An experimenter intends to study a beam of π-mesons of velocity 0.9c, where c is the velocity of light in free space. How far can he place his apparatus from the target where π-mesons are produced and still expect to get sufficient number of π-mesons ?

(5+4+2)+4 = [15]

p.t.o.

4.(a) What is Minkowski's space? Show that in such a space the Lorentz transformation amounts to a simple rotation about the origin.

(b) Establish the relations

(i) $E^2 = p^2c^2 + m_0^2c^4$

(ii) $p = m_0c\sqrt{\frac{1}{1-v^2/c^2} - 1}$

3+5+4+3 = [15]

5.(a) Prove the following

(i) $T = \left(\frac{\partial U}{\partial S}\right)_V$, (ii) $p = -(\partial F/\partial V)_T$

(b) Show that $\ln(x!) = x \ln x - x$ when x is very large.

(c) Suppose there are three cells in phase space :

1, 2 and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$; and let $W_1 = 2$ joules, $W_2 = 4$ joules, $W_3 = 6$ joules. If $\delta N_3 = -2$, find δN_1 and δN_2 such that $\delta N = 0$, and $\delta U = 0$.

(d) Apply MB-statistics to obtain the distribution of Brownian particles along the vertical axis z and hence indicate in outlines how Avogadro number can be obtained.

2+2+4+3+6+3 = [20]

Group B

Note : Answer Q.1 and any four of the rest.

Given : mass of an electron : 9.1×10^{-31} kg

charge of an electron : 1.6×10^{-19} C

Planck's constant : 6.62×10^{-34} JS

velocity of light in vacuum : 3×10^{8} m/S^2.

Symbols have their usual meanings.

1. Choose the correct answer(s).

(a) For a cubic lattice of side a, the distance between (110) planes is $a/\sqrt{2}$. The distance between (101) planes will be

(i) $a/\sqrt{3}$ (ii) $a/\sqrt{2}$ (iii) a (iv) $a/2$

Contd.....

1.(b) $[H,x]$ is equal to

(i) $i\hbar\ p/m$ (ii) $-i\hbar\ p/m$ (iii) $i\hbar^2\ p/m$

(iv) $i\hbar\ \frac{p^2}{m}$

(c) In the Einstein model of specific heat, the vibrational energy E of a solid element containing N atoms is

(i) $\frac{N}{2}\ \frac{h\nu}{e^{h\nu/KT}-1}$ (ii) $N\ \frac{h\nu}{e^{h\nu/KT}-1}$

(iii) $\frac{3N}{2}\ \frac{h\nu}{e^{h\nu/KT}-1}$ (iv) $3N\ \frac{h\nu}{e^{h\nu/KT}-1}$

(d) Suppose a particle is moving in a potential V(x). If V makes a sudden jump of infinite magnitude at $x=x_1$

(i) the wave function ψ has a finite discontinuity

(ii) $\frac{\partial\psi}{\partial x}\Big|_{x=x_1}$ has a finite discontinuity

(iii) ψ is continuous

(iv) $\frac{\partial\psi}{\partial x}\Big|_{x=x_1}$ is continuous.

(e) If a system is in one of the eigen states of $\hat{A}$, the uncertainty in the value of $\hat{A}$ is

(i) $\hbar/2$ (ii) zero (iii) $\hbar$ (iv) none of these.

(f) The value of Compton wavelength is

(i) 2.426×10^{-10} cm (ii) 2.426×10^{-10} m

(iii) 2.426×10^{-8} cm (iv) 2.426×10^{-8} m.

(g) With rise of temperature, conductivity of semiconductors in general

(i) increases, (ii) decreases

(iii) remains same (iv) varies in an unpredictable manner

(h) The effective number of electrons in an energy band in case of a one dimensional lattice of length L is given by $N_{eff} = \left(\frac{2Lm}{\pi\hbar^2}\right)\left(\frac{dE}{dK}\right)_{k=k_1}$. The effective number of electrons in a completely filled band

(i) reaches a maximum (ii) is zero

(iii) is half the total number of electrons in the band

(iv) cannot be said from this.

Contd.....

1.(i) In describing the change in time of a physical system in the Schrödinger picture
(i) the dynamical variables are constant in time
(ii) the state vectors are constant in time
(iii) state vectors vary
(iv) dynamical variables vary.

2.(a) What are Miller indices ?
(b) If electrons are incident on the lattice planes of a metal crystal lattice spacing 2.15Å) at a glancing angle of 16° in order to give a strong Bragg reflection in the first order, calculate the de Broglie wave-length associated with the electrons.

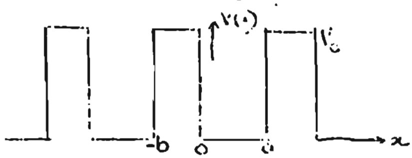
(c) The vibrational energy E of a crystal is given by

$$E = 9N\left(\frac{KT}{h\nu_D}\right)^3 KT \int_0^{x_m} \frac{x^3 dx}{e^x - 1}$$

where $x = \frac{h\nu}{KT}$ and $x_m = \frac{h\nu_D}{KT} = \frac{\Theta_D}{T}$, Θ_D being the Debye temperature. Given $6\zeta(4) = \frac{\pi^4}{15}$, calculate C_V for $T \ll \Theta_D$.

(2+2+4) = 8

3. The potential energy of an electron is periodic, and has the form as shown in the figure.



(a) Write down the Schrödinger equation for the system.
(b) What will be the nature of the solutions ?
(c) What will be the nature of the energy spectrum of the electron ?
(d) If $P = \frac{m V_0 b a}{\hbar^2}$, what happens to the energy spectrum as $P \to \infty$? Discuss briefly the physical meaning of the result.

(3+1+1+3) = 8

4.(a) H is the quantum mechanical Hamiltonian of a particle of momentum p. Calculate [H,p]. Hence show that $\frac{d}{dt}\langle p \rangle = -\langle \frac{\partial V}{\partial x} \rangle$.

Contd.....

4.(b) Given any time dependent ket of the form

$|\alpha_s(t)\rangle = e^{-iHt/\hbar}|\alpha_s(0)\rangle$ [subscript s refers to ket as viewed in the Schrödinger picture].

Discuss the time rate of change of a dynamical variable Ω_S and arrive at the Heisenberg equation of motion. (3+5)= 8

5.(a) Describe an idealised experiment to illustrate that as long as all agents (matter and light) have quantum properties, no measurement can, even in principle, lead to absolute precise determinations of positions and momenta.

(b) State the fundamental postulates of quantum mechanics.

(5+3) = 8

6.(a) A particle of energy E is incident on a potential barrier of height V, such that E < V. Is it possible for the particle to be found at x>a ?

Can such phenomenon be observed ? Give examples.

(b) Show that the de Broglie wavelength of an electron accelerated from rest through a potential difference of V volts is given by,

$$\lambda = \frac{h}{\sqrt{2m_0 eV}}\left(1+\frac{eV}{2m_0c^2}\right)^{-1/2}$$ relativistically.

(c) Find the eigen values and normalized eigen vectors of $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(3+3+2) = 8

7.(a) What do you understand by the term 'effective mass' of an electron ? How does effective mass vary with the wave vector K ?

(b) Show that $[x, \frac{\partial}{\partial x}] = -1$

(c) Starting from $H = (a^+a + 1/2)$, show that there is a lower bound of energy of the harmonic oscillator.

(3+2+3) = 8

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[1990-91] [368]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1990-91

Economics IV

Semestral-II Examination

Date : 8.5.1991 Maximum Marks : 100 Time: 3 Hours.

Note : Answer any FIVE questions. Marks allotted to each question are given within brackets.

1. Consider the regression equation

$$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + u_t, \quad t = 1,2,\ldots,T$$

where the error terms are assumed to be normally distributed with zero mean, constant variance and are uncorrelated over the observations. Discuss how you would test the following hypotheses.

(i) $\beta_1 = 0$;

(ii) $\beta_1 = 0,\ \beta_2 = 0$;

(iii) $\beta_1 = 0,\ \beta_2 = 0,\ \beta_3 = 0$;

(iv) $\beta_1 + \beta_2 + \beta_3 = 1$.

Justify your procedure mentioning the necessary theoretical results. [20]

2. Consider the following linear regression model

$$y_i = \beta x_i + u_i, \quad i = 1,2,\ldots, n;$$

$$E(u_i) = 0 \text{ for all } i$$

$$E(u_i^2) = \sigma^2 x_i^2$$

$$E(u_i u_j) = 0 \text{ for all } i \neq j$$

and $\sum_{i=1}^{n} x_i^2 = n$.

Prove that the ordinary least squares (OLS) estimator of β is unbiased but inefficient and that the formula for its estimated variance yields a downward biased estimate of the true variance. [20]

p.t.o.

3. Suppose we have the following linear regression model

$$y = \beta_1 x_1 + \beta_2 x_2 + u$$

where all the variables are measured from their respective means.
Now suppose for a given set of observations the least squares estimates of the regression coefficients are found to be $\hat{\beta}_1 = 1$ and $\hat{\beta}_2 = 1$. Further, the least squares estimates change drastically to $\hat{\beta}_1 = -1/2$ and $\hat{\beta}_2 = 3$ when a few observations are dropped from the given set.
What do you think of this result ? Why does such a result arise ? Suggest alternative ways of tackling the problem in case the results are unacceptable to you. [20]

4.(a) Briefly describe the adaptive expectations model. Also sketch out a suitable procedure of estimation of this model. [11]

(b) The model generating the $\{y_t\}$ series is presumed to be

$$y_t = \alpha y_{t-1} + u_t, \quad |\alpha| < 1$$

with

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad |\rho| < 1$$

and the ε_t's are independently and identically distributed with zero mean and constant variance σ^2.
Show that

$$\hat{\rho} = \sum_{t=2}^{n} e_t e_{t-1} \Big/ \sum_{t=2}^{n} e_{t-1}^2$$

where $e_t = y_t - \hat{\alpha} y_{t-1}$ and $\hat{\alpha}$ is the OLS estimator of α, is <u>not</u> a consistent estimator of ρ.

[9]

5.(a) Discuss with the help of a simple model why the OLS method of estimation is inappropriate for a linear model in which all the variables are measured with errors. [7]

Contd.....

5.(b) Write a short note on BLUS residuals. Also indicate how these residuals can be used for testing the presence of autocorrelation among the disturbances of a linear regression model.

[13]

6.(a) A model of meat market is specified as follows :

$$y_{1t} = \alpha_0 + \alpha_1 y_{2t} + \alpha_2 x_{1t} + u_{1t} \quad \text{(demand equation)}$$

$$y_{1t} = \beta_0 + \beta_1 y_{2t} + \beta_2 x_{2t} + \beta_3 x_{3t} + u_{2t} \quad \text{(supply equation)},$$

where the symbols have their usual meanings. Examine the identification status of each

equation. How does the position change if it is known a priori that $\beta_2/\beta_3 = k$, where k is a known constant ?

[12]

(b) State (no proof or justification needed) the salient features that distinguish each of the following estimation methods for estimating simultaneous system of equations.

(i) Indirect least squares;
(ii) Two-stage least squares;
and (iii) Three-stage least squares.

[8]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) III Year : 1990-91
Statistical Inference II
Semestral-II Examination

Date : 2.5.1991 Maximum Marks :100 Time: $3\frac{1}{2}$ Hours.

Group A

Note : Answer all questions. Marks are indicated in the margin.

1. Let $X_i = 1$ if a head shows up on the ith toss
   $= 0$, otherwise.

   Stop sampling after observing X_{n+1} if $\sum_{i=1}^{n} X_i = 0$ and $X_{n+1} = 1$. We want to estimate $1/\theta$, $0 < \theta \leq 1$, θ being the probability of obtaining a head in a single throw.

   (i) Show that the MLE is an unbiased estimator (of $1/\theta$)

   (ii) Obtain the variance of the MLE

   (iii) Verify whether the MLE attains the lower bound given by Wolfowitz's sequential version of Cramer-Rao's result. [13]

2. Let the common distribution of $X_1, X_2, \ldots$ have a density of the form
$$f(x|\theta) = \frac{1-\theta^2}{2} \exp\left\{-|x| + \theta x\right\}, \ |\theta| < 1.$$
Consider the SPRT of the hypothesis $H_0: \theta = -\frac{1}{2}$ vs

   $H_1: \theta = +\frac{1}{2}$. Show that EN can be expressed by an EXACT formula. (You may consider only the case $\theta \neq 0$). [13]

3. Write short notes on any two of the following :

   (i) Bahadur's derivation of $E(S_N - N\mu)^2 = \sigma^2 EN$ when $\mu = EZ$ could possibly be zero.

   (ii) Two-stage fixed width confidence interval for $\mu_1 - \mu_2$ where $\binom{X}{Y} \sim N_2\left(\binom{\mu_1}{\mu_2}, \Sigma\right)$, Σ unknown but positive definite.

   (iii) Generalization of Frechet-Cramer-Rao inequality for the sequential estimation problem when
$$\frac{\partial \log f_\theta(x)}{\partial \theta} = 0 \text{ at } \theta = \theta_0.$$
[14]

4. Assignments given in class. [10]

p.t.o.

Group B
(Non-parametric Methods)

Note : Use a separate answer-script for this group. Answer as many questions as you can. The maximum you can score from this group is 50.

1.(a) Suppose that X ~ F, where F is a continuous distribution function. Show that the random variable $Y = F(X)$ has uniform distribution on (0,1).

(b) For any 2 order statistics $X_{(r)}$ and $X_{(s)}$ (r<s) based on an independent random sample of size n from a population with continuous c.d.f. F, show that the event $X_{(r)} < \xi_p$ occurs if and only if either $X_{(r)}<\xi_p<X_{(s)}$ or $\xi_p>X_{(s)}$ (where ξ_p is the pth quantile 0<p<1 of F). Hence show that

$$P(X_{(r)}<\xi_p<X_{(s)}) = \int_0^p [B(r,n-r+1)]^{-1} u^{r-1}(1-n)^{n-r}\,du$$

$$- \int_0^p [B(s,n-s+1]^{-1} u^{s-1}(1-u)^{n-s+1}\,du$$

where B(m,n) denote the Beta function. [10+10]

2.(a) For arbitrary m,n find the probability

$$P_H(W_s = \tfrac{1}{2}\, n(n+1)P|)$$

under the hypothesis H of no treatment effect when W_s denote the two-sample Wilcoxon rank-sum statistic.

(b) In a study involving 3m objects 2m are assigned to treatment and m to control. Suppose the ordered set of observations turns out to have the pattern : XYYXYYX ..... XYYXYY. If large values of W_s are significant use the normal approximation to find the approximate significance probability of this observation when m is large. [5+10]

3. In a comparison of 2 drugs used for the relief of post-operative pain the following number of hours were observed for 16 patients of which 8 had been assigned to drug-A and the other 8 to drug-B.

A :	6.8,	3.1,	5.8,	4.5,	3.3,	4.7,	4.2,	4.9
B :	4.4,	2.5,	2.8,	2.1,	6.6,	0.0,	4.8,	2.3

Find the significance probability of the Kolmogorov-Smirnov test statistic for testing whether there are any difference between the two drugs. [10]

Contd.....

4.(a) Describe the Wilcoxon signed rank-sum test for testing that a continuous distribution F is symmetric about zero on the basis of N independent observations $Z_1, Z_2, \ldots, Z_N$ from F. Write down the null-distribution of the statistic V^+ for N = 5.

(b) Explain how the $M = \frac{N(N+1)}{2}$ ordered Walsh averages

$$\left(\frac{Z_i+Z_j}{2}, \ 1 \leq i \leq j \leq N \right), \ w_{(1)} \leq w_{(2)} \leq \cdots \leq w_{(M)}$$

can be used to obtain a confidence interval for the unknown point of symmetry θ of F.

[3+4+8]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) III Year : 1990-91

Introduction to Stochastic Processes

Semestral-II Examination

Date : 29.4.1991 Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Group A

1. Let $\{X_n\}_{n\geq 0}$ be a stochastic process with state space $\{0,1\}$.

$$P(X_{n+1} = 1 \mid X_n = 0) = p$$
$$P(X_{n+1} = 0 \mid X_n = 1) = q$$
$$P(X_0 = 0) = \pi_0(0)$$

Derive the limiting distribution of the process and show that it is identical with the stationary distribution of the same. [15]

2.(a) Consider a birth and death chain with state space $\{0,1,..,d\}$ and transition function of the form

$$P(x,y) = \begin{matrix} q_x & y = x-1 \\ r_x & y = x \\ p_x & y = x+1 \end{matrix}$$

where $p_x + q_x + r_x = 1$, for all x q_0, $p_d = 0$. Also p_x, $q_x > 0$ for all other cases. Calculate $P_x(T_a < T_b)$, where T_y is the first hitting time of y, with x as the initial state $a < x < b$.

(b) A gambler has respective probabilities of 9/19 and 10/19 of winning and losing each bet of one rupee. The gambler decides to quit playing as soon as his net winnings reach 25 rupees or his net losses reach 10 rupees. Find the probability that when he quits playing he will have 25 rupees. Find his expected loss. [12+8]

3. Consider a Markov Chain on $\{0,1,2\}$ having transition matrix

$$\begin{matrix} & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 1/2 & 1/2 & 0 \end{matrix}$$

(a) Show that the chain is irreducible.

(b) Find the period.

(c) Find the stationary distribution. [9]

p.t.o.

4. The transition function of a Markov Chain with state space $\mathcal{Y}$ is called doubly stochastic if $\sum_{x \in \mathcal{Y}} P(x,y) = 1,\ y \in \mathcal{Y}$. What is the stationary distribution of an irreducible chain having $d(<\infty)$ states and a doubly stochastic matrix as its transition matrix ? [6]

Group B

Note : Attempt any two questions.

5.(a) Let S_k denote the time to the k-th occurrence of the event under a Poisson process $(X(t): t > 0)$, $k = 1,2,\ldots$. Obtain the conditional joint distribution of $S_1, S_2, \ldots, S_n$ given $(X(t) = n)$.

(b) Let $(X(t) : t \geq 0)$ and $(Y(t): t \geq 0)$ be two independent Poisson processes with rates λ and v respectively. Obtain the probability that the number of occurrences of events under $X(t)$ within the time interval from the first to the $(r+1)$-th occurrences of events under $Y(t)$ equals n.

(c) In (b) above, let $Z(t) =_{def} X(t) + Y(t)$ and let U denote the time to the first occurrence of an event under $Z(t)$. What is the distribution of U ?

[9+8+8]

6.(a) Consider a renewal process $N(t) : t \geq 0)$ with expected interarrival time μ. Prove that

$$EN(t)/t = 1/\mu + o(1),$$

as $t \to \infty$.

(b) For a fixed $x > 0$, let $P(t)$ denote the probability that the residual life time of the unit in use at time t i.e. $Y(t) =_{def} (S_{N(t)+1} - t)$ exceeds x, S_n being the time to the n-th renewal, $n = 1,2,\ldots$. Using a renewal theoretic argument, show that

$$1 - P(t) = F(t+x) - \int_0^t [1-F(t+x-y)]\, dH(t)$$

where $H(t) = EX(t)$ and F is the distribution function of S_1. Hence obtain $P(t)$ when $F(t) = 1 - e^{-\lambda t}, t>0, \lambda>0$.

[13+12]

Contd.....

7. (a) For a Yule process $\{X(t) : t \geq 0\}$, with $X(0) = 1$, obtain $Pr\{X(t) = n\}$. Hence, or otherwise, obtain the probability density function of T_n, the time at which the population first reaches the value $n(>1)$.

(b) Consider a Poisson process $\{X(t) : t \geq 0\}$ with rate $\lambda > 0$. Obtain the correlation coefficient $\rho(t, t+s)$ between $X(t)$ and $X(t+s)$, $t > 0$, $s > 0$. Hence, write down the expression for $\rho(t, t')$.

[12+13]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I BACKPAPER EXAMINATION
DIFFERENTIAL EQUATIONS

Date: 2.1.91 Maximum Marks: 100 Time: 3 Hours.

Note: This paper carries 110 marks. You may answer all the questions. But the maximum you can score is 100.

1.(a) Find the general solution of the first order equation

$$(x+3x^3y^4)\,dy + y\,dx = 0$$

given that it has an integrating factor which is a function of $x.y$.

(b) A curve rises from the origin in the xy-plane into the first quadrant. The area under the curve from $(0,0)$ to (x,y) is one-third of the area of the rectangle with these points as opposite vertices. Find the equation of the curve. [10+10=20]

2.(a) An integral curve $y=u(x)$ of the differential equation $y''-4y'+29y=0$ intersects an integral curve $y=v(x)$ of the differential equation $y''+4y'+13y=0$ at the origin. The two curves have equal slopes at the origin. Also $u'(\pi/2)=1$. Determine $u(x)$ and $v(x)$.

(b) Find the general solution of

$$(x^2-1)y'' - 2xy' + 2y = (x^2-1)^2.$$ [10+10=20]

3. Show that the equation $4x^2y''-8x^2y'+(4x^2+1)y=0$ has only one Frobenius series solution. Find this solution. Also find the general solution. [15]

4. Show that $x=\infty$ is a regular singular point of the differential equation

$$(1-x^2)y''-2xy'+2y=0.$$

Find its exponents at $x=\infty$. Find two linearly independent solution of the form $x^{-\gamma}\sum_{k=0}^{\infty} C_k x^{-k}$, for the above equation, valid for $|x|>1$. [15]

5.(a) Find the two linearly independent solutions of the homogeneous system

$$\frac{dx}{dt} = 5x+4y$$

$$\frac{dy}{dt} = -x+y.$$

contd. 2/-

5.(b) If $I_n = \int_0^1 (\log x)^n x^\alpha \, dx$, $\alpha > 0$, $n=1,2,\ldots$

show that $(\alpha+1) I_n + n I_{n-1} = 0$. Hence, evaluate I_n. [10+10=20]

6.(a) Find the shape of a homogeneous rope of length l suspended at the points A = (0,1) and B = (1,1), assuming that the rope hangs in such away as to minimize its potential energy.

(b) Find the extremals of the functional

$$v[y(x),z(x)] = \int_{x_0}^{x_1} [2yz - 2y^2 + y'^2 - z'^2] \, dx.$$ [10+10=20

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INDIAN STATISTICAL INSTITUTE
B.STAT. (HONS.) III YEAR: 1990-91
SEMESTRAL-I BACKPAPER EXAMINATION
LINEAR STATISTICAL MODELS

Date: 2.1.91 | Maximum Marks: 100 | Time: 3 Hours.

Note: Attempt all questions.

1. Consider the model $(Y, X\beta, \sigma^2\Sigma)$ where Σ is a singular metrix. Show that you can transform the above model to an equivalent model with restrictions on β but with a positive definite dispersion metrix. [10]

2. Consider a two way classification model

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}, \quad \epsilon_{ij}\text{'s iid } N(0,\sigma^2).$$

$i = 1,2,\ldots,p$ and $j = 1,2,\ldots,q$.

Show that $\Sigma c_i\alpha_i$ is estimable iff $\Sigma c_i = 0$. [10]

3.(a) Explain clearly the step-wise procedure multiple linear regression.

(b) Comment on the uses and limitations of the stepwise multiple linear regression. [10+5=15]

4. Consider the following linear model:

$$y_1 = \alpha_1 + \alpha_2 + \epsilon_1$$
$$y_2 = \alpha_2 - \alpha_3 + \epsilon_2$$
$$y_3 = \alpha_1 - \alpha_3 + \epsilon_3$$
$$y_4 = \alpha_1 + 6\alpha_2 + 5\alpha_3 + \epsilon_4$$
$$y_5 = 2\alpha_1 + 5\alpha_2 + 3\alpha_3 + \epsilon_5$$

where $\epsilon_1, \ldots, \epsilon_5$ are i.i.d $N(0,\sigma^2)$.

(a) Is α_2 estimable? Give reasons for your answer.

(b) Write down the normal equations for α_1, α_2 and α_3.

(c) Obtain a least squares estimator of $(\alpha_1, \alpha_2, \alpha_3)^T$.

(d) Obtain the conventional unbiased estimator of σ^2.

(e) Let $y_1 = 2.8$, $y_2 = 5.1$, $y_3 = 2.1$, $y_4 = 27.5$ and $y_5 = 20.8$ be a realization of $y_1, \ldots, y_5$. Test the hypothesis $H: 3\alpha_1 + 5\alpha_2 + 2\alpha_3 = 19$ against $K: 3\alpha_1 + 5\alpha_2 + 2\alpha_3 > 19$ at 5% level of significance. [3+7+7+7+11=35]

contd. 2/-

5. Consider the following data on Y, X_1 and X_2

Y	42	33	75	28	91	55
X_1	7	4	16	3	21	8
X_2	33	41	7	49	5	31

(a) Fit multiple linear regression of Y on X_1 and X_2.

(b) Compute estimates of the standard errors of the estimated regression coefficients.

(c) Examine whether X_1 and X_2 are useful in predicting linearly. [15+5+10]

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INDIAN STATISTICAL INSTITUTE
B.STAT (HONS.) III YEAR.1990-91
SEMESTRAL-I EXAMINATION
LINEAR MODELS

Date: 30.11.90 | Maximum Marks:100 | Time: 4 Hours

Note: Attempt all the questions. The marks for each question are given in the brackets at the end of the question.

1. Consider the model $Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$, $E(\varepsilon)=0$, $D(\varepsilon)=\sigma^2 I$ where X_1 and X_2 are known matrices of orders $n \times r$ and $n \times s$ respectively and β_1, β_2 are unknown (but fixed) parameters belonging to $\mathbb{R}^r$ and $\mathbb{R}^s$ respectively. (Let P_B denote the orthogonal projector projecting vectors into $\mathcal{C}(B)$.)

(a) Show that $p'\beta_2$ is estimable iff $p \in \mathcal{C}(X_2'(I-P_{X_1}))$.

(b) Show that $p'\hat{\beta}_2$ is BLUE of $p'\beta_2$ (whenever it is estimable) iff $\hat{\beta}_2$ is a solution of $C\beta_2 = q$ where

$$C = X_2'(I-P_{X_1})X_2 \text{ and } q = X_2'(I-P_{X_1})Y.$$

[For the 'only if' part it is understood that $\hat{\beta}_2$ does not depend on p].

(c) Consider H: every estimable linear function of β_2 is zero. Write

$$R_0^2 = \min_{\beta_1,\beta_2} (Y-X_1\beta_1 - X_2\beta_2)'(Y-X_1\beta_1-X_2\beta_2)$$

and

$$R_1^2 = \min_{\beta_1,\beta_2 \text{ subject to } H} (Y-X_1\beta_1-X_2\beta_2)'(Y-X_1\beta_1-X_2\beta_2)$$

Show that $R_1^2 - R_0^2 = q'C^-q$ where C^- is any g-inverse of C.

(d) Explain the use of (c) in the analysis of two-way classified data with unequal number of observations per cell. (Assume a model without interaction.) [4+6+7+6=23]

2. Consider two alternative linear models $(Y, X\beta, \sigma^2 I)$ and $(Y, X\beta, \sigma^2\Sigma)$ $[\Sigma \neq I]$.

(a) Show that BLUES of $p'\beta$ (whenever it is estimable) under both the models are equal with probability 1 iff $\mathcal{C}(\Sigma X) \subseteq \mathcal{C}(X)$.

(b) Let Σ be an intra-class correlation matrix $(1-\rho)I+\rho 11'$. Show that the BLUEs of every estimable parametric function under both the models concide with probability 1 (irrespective of the value of ρ) iff 'either $1 \in \mathcal{C}(X)$ or $1'X=0$'. [7+5=12]

3.(a) Consider the linear regression model $(Y, X\beta, \sigma^2 I)$ where X is of full column rank. Denote BLUE of β by $\hat{\beta}$. Consider the model after dropping the i^{th} observation from the above model. Assume that the resulting X metrix is also of full column rank Denote the BLUE of β under this model by $\hat{\beta}_{(-i)}$. Show that

$$\hat{\beta} - \hat{\beta}_{(-i)} = \frac{(X'X)^{-1} x_i e_i}{1 - h_i}$$

where x_i' is the i^{th} row of X, $e_i = y_i - x_i' \hat{\beta}$ and h_i is the i^{th} diagonal element of $\frac{1}{\sigma^2} D(X\hat{\beta})$.

(b) Explain the use of the above result and suitable modifications of it in detecting influential observations in a linear regression model. [7+8=15]

4. The following table gives the yearly yield (Y) of wheat (in quintals) per acre at five agricultural stations for three consecutive years, together with two concomitant variables shoot height (Z) at ear emergence and number (W) of plants per foot at tillering.

Year	Variable	Agricultural Station 1	2	3	4	5	Total (rows)
1	Y	19.0	22.2	35.3	32.8	25.3	134.6
	Z	25.6	25.4	30.8	33.0	28.5	143.3
	W	14.9	13.3	4.6	14.7	12.8	60.3
2	Y	32.4	32.2	43.7	35.7	28.3	172.3
	Z	25.4	28.3	35.3	32.4	25.9	147.3
	W	7.2	9.5	6.8	9.7	9.2	42.4
3	Y	26.2	34.7	40.0	29.6	20.6	151.1
	Z	27.9	34.4	32.5	27.5	23.7	146.0
	W	18.6	22.2	10.0	17.6	14.4	82.8
Total (columns)	Y	77.6	89.1	119.0	98.1	74.2	458.0
	Z	78.9	88.1	98.6	92.9	78.1	436.6
	W	40.7	45.0	21.4	42.0	36.4	185.5

contd. 3/-

The following computations on the sums of squares and products of all the observations are also available.

$\Sigma Y^2 = 14678.58$, $\Sigma Z^2 = 12899.08$, $\Sigma W^2 = 2622.97$

$\Sigma YZ = 13634.33$, $\Sigma YW = 5464.99$ and

$\Sigma ZW = 5388.07$.

Assume a suitable model without interaction. Write down your model clearly. Analyse the data and give your comments on the appropriateness of the concomitant variables chosen.

5. In an investigation carried out for studying the effect of smoking on physical activity, 21 individuals were classified into one of three groups by smoking history and randomly assigned to one of three stress tests: bicycle engometer, treadmill or step test. The time until maximum oxygen uptake was recorded in minutes. The data are given below.

Time until maximum oxygen uptake

Stress test →

Smoking history	Bicycle	Treadmill	Step
None	12.8	16.2	22.6
	13.5	17.8	19.3
	11.2		18.9
Moderate		15.5	21.0
	10.9	13.8	15.9
		16.2	15.9
Heavy	9.2	13.2	16.2
	7.5	8.1	16.1
			17.8

The following computations are available. The sum of all the observations = 313.7 minutes. The sum of squares of all the observations = 9702.33.

Assume a two way classification model without interaction. Analyse the data and report your findings. [25]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I EXAMINATION
SAMPLE SURVEYS

Date: 28.11.90 Maximum Marks: 100 Time: 3 Hours

Note: Answer as many questions as you can; but the maximum marks alloted would be 100. In fact the paper carries 140 marks.

1.(a) Let there be u distinct units in a wr simple random sample of size n. Let K_r be the frequency with which the r^{th} distinct unit occurs in the sample.
Then show that

$$E(\bar{y}_n) = E(\bar{y}_u) = \bar{Y} \text{ and } V(\bar{y}_u) \leq V(\bar{y}_n)$$

where $\bar{y}_u = \sum_{r=1}^{u} y_r/u$ and $\bar{y}_n = \sum_{r=1}^{n} k_r y_r/n$

(b) In a population with N=5, the values of y_i are 8,3,1,11 and 4. If simple random samples of size 2 are drawn with replacement from this population, show by finding all possible samples that $V(\bar{y})$ satisfies the equation

$$V(\bar{y}) = \sigma^2/n, \text{ where } N\sigma^2 = \sum_{i=1}^{N} (y_i - \bar{Y})^2$$ [5+10+10=20]

2.(a) If the Cost function is of the form $C = c_0 + \Sigma t_n \sqrt{n_h}$ where C_0 and the t_h are known numbers, show that inorder to minimise $V(\bar{y}_t)$ for fixed total cost, n_h must be proportional to

$$(W_n^2 S_n^2/t_h)^{2/3} ; \quad W_n = \frac{N_h}{N}, \quad S_h^2 = \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_n)^2/(N_n - 1)$$

(b) Apply the rule in 2(a) to find the n_h's for a total sample of size 1000 under the following conditions:

Stratum	W_n	S_n	t_h
1	0.4	4	1
2	0.3	5	2
3	0.2	6	4

2.(c) With two strata, a sampler would like to have $n_1=n_2$ for administrative convenience, instead of using the values given by Neyman Allocation. If $V(\bar{y}_{st})$, $V_{opt}(\bar{y}_{st})$ denote the variances given by $n_1=n_2$ and Neyman Allocations respectively, show that

$$[V(\bar{y}_{st}) - V_{opt}(\bar{y}_{st})]/V_{opt}(\bar{y}_{st}) = [(r-1)/(r+1)]^2$$

where $r = \frac{n_1}{n_2}$ as given by Neyman Allocation. [5+10+10=25]

3.(a) Define a ratio estimator for a finite population total of a variable y when a sample of (x,y) values is taken from the population but the population total of x is known. Show that based on a simple random sample without replacement it may not be unbiased in general.

(b) The values of y,x in a population with N=6 are as follows:

y :	3	5	7	6	8	13
x :	1	2	2	3	3	3

check if the regression of y on x is a straight line through the origin.

Computing $\hat{R} = \frac{\bar{y}}{\bar{x}}$ for all 15 simple random samples with n=2, check if $\hat{R}$ is unbiased for $\frac{\bar{Y}}{\bar{X}}$. [10+15=25]

4. Suppose a sample of size n is selected with probabilities porportional to p_i without replacement from a population of size N; ($p_i > 0$ for all $i, \sum_1^N p_i = 1$).

Define $$t_1 = y_1/p_1$$

$$t_\lambda = y_1+y_2+\dots+y_{\lambda-1}+y_\lambda \frac{1-\sum_{j=1}^{\lambda-1} p_j}{p_\lambda}, \quad (\lambda=2 \dots n)$$

and $$t = \frac{1}{n}\sum_{\lambda=1}^{n} t_\lambda$$

Show that

(i) $E(t_\lambda) = Y = \sum_1^N y_i$ (ii) $E(t_\lambda t_\mu)=Y^2$ for $\mu \neq \lambda$ and

$V(t_\lambda) \leq V(t_{\lambda-1})$

If instead the units are selected with probabilities p_i with replacement and an estimator E based on a sample so drawn is taken as the mean of all the $\frac{y_i}{p_i}$ values in the sample then show that $V(t) \leq V(\bar{t})$

[5+5+10+5=25]

5.(a) Define first and second order inclusion probabilities, namely π_i and π_{ij}. Show that for any fixed sample size (n) design

$$\sum_{i=1}^{N} \pi_i = n \quad \text{and} \quad \sum_{i \neq j}\sum \pi_{ij} = n(n-1)$$

(b) There are 5 units in a population having the sizes 10,20,30,40, 50. A sample of 2 units is to be drawn with ppswor. Calculate the probabilities of inclusion in the sample of units and pairs of units required to verify that

$$\sum_{i=2}^{5} \pi_{1j} = \pi_1$$

[5+5+15=25]

6. Prove the following results:

(i) $\frac{\sigma^2}{n}\; 1+\overline{n-1} \quad {}_c = \sigma^2-\sigma_w^2$, when $\sigma^2 = \frac{1}{N} \sum_{r=1}^{k}\sum_{i=1} (y_{ri}-\bar{Y})^2$,

$\sigma_w^2 = \frac{1}{nk} \sum_{r=1}^{k} \sum_{i=1}^{n} (y_{ri}-\bar{y}_r)^2$ and ρ_e is the intra-class correlation coefficient.

(ii) For an r-stage design,

$$E(\hat{Y}) = E_1\, E_2 \ldots E_r(\hat{Y})$$

and

$$V(\hat{Y}) = V_1\, E_2 \ldots E_{r-1}\, E_r(\hat{Y}) + E_1 V_2 \ldots E_r(\hat{Y}) + \ldots + E_1\, E_2 \ldots E_{r-1}\, V_r(\hat{Y})$$

(iii) $b = B + \Sigma\, e_i(x_i-\bar{x}) \,/\, \Sigma(x_i-\bar{x})^2$

if $b = \Sigma(y_i-\bar{y})(x_i-\bar{x}) \,/\, \Sigma(x_i-\bar{x})^2$

$B = S_{xy} / S_x^2$

and $e_i = y_i-\bar{Y} - B(x_i-\bar{x})$

where the notations have the usual meaning. [5+5+5=15]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I EXAMINATION
STATISTICAL INFERENCE-I

Date: 23.11.90 Maximum Marks: 100 Time: 3 Hours

Note: Answer as many questions as you can. The maximum you can score is 100.

1.(a) $X_1, X_2, \ldots, X_n$ are independent observations from the normal distribution $N(\sigma, \sigma^2)$, $\sigma > 0$. Show that statistic $T(X_1, X_2, \ldots, X_n) = (\Sigma X_i, \Sigma X_i^2)$ is minimal sufficient for σ, but is not complete.

(b) $X_1, X_2, \ldots, X_n$ are independent observations from the normal distribution $N(\mu, \sigma^2)$ $-\infty < \mu < \infty$, $\sigma^2 > 0$. Obtain the UMVUE of $\frac{\mu}{\sigma}$. [(10+5)+10]

2. Find the Cramer-Rao lower bound for the variance of any unbiased estimator of $e^{-\theta}$ on the basis of n independent observations $X_1, X_2, \ldots, X_n$ from a Poisson distribution with mean θ, $\theta > 0$. Find the UMVUE of $e^{-\theta}$, compute the variance of this estimator and check whether the Crame'r-Rao lower bound is attained by this estimator. [6+9]

3.(a) Define a monotone likelihood ratio family (MLR) family of densities. Show that the family of densities

$$f_\theta(x) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < \theta < \infty$$

form an MLR family in the statistic $T(x) = x$.

(b) Define a uniformly most accurate upper confidence bound of level $(1-\alpha)$ for a real parameter θ. To determine the upper bound for the degree of radio-activity λ of a radio-active substance, the substance is observed till a count m has been obtained on a Geiger counter. The joint density of the time intervals T_i $(i = 1, 2, \ldots, m)$ between the $(i-1)$th and the i th counts is

$$p(t_1, t_2, \ldots t_m) = \lambda^m e^{-\lambda \sum_1^m t_i} \quad \text{if } t_i \geq 0 \text{ for all } i$$

$$= 0 \quad \text{otherwise.}$$

Show that the uniformly most accurate upper confidence bound of level $(1-\alpha)$ for λ is $\bar{\lambda} = \frac{c(\alpha)}{2\Sigma T_i}$ where $c(\alpha)$ is upper $100\alpha\%$ percentage point of χ^2_{2m} distribution. [(3+5)+12]

contd. 2/-

4.(a) Does there exist a UMP level α test for testing $H:\theta=0$ against $K:\theta \neq 0$ on the basis of iid observations $X_1, X_2, \ldots, X_n$ from $N(\theta,1)$ distribution ? Justify your answer.

(b) Define an unbiased test. Obtain the uniformly most powerful unbiased test of level α for the testing problem described in Q. 4(a). Obtain the expression for the power function of this test and check that this is an increasing function of $|\theta|$.

[6+(3+16+5)]

5.(a) State carefully the Neyman-Pearson lemma. Prove that the power β of the MP level α, $0 < \alpha < 1$, test for testing a simple hypothesis against a simple alternative is greater than or equal to α. What can you say about a strict inequality?

(b) Obtain the Bayes estimator of θ on the basis of one observatio X from the rectangular $R(0,\theta)$, $\theta > 0$ distribution where the prior density of θ is given by

$$h(\theta) \begin{cases} = \theta e^{-\theta} & \text{for } \theta > 0 \\ = 0 & \text{otherwise} \end{cases}$$

(assume squarred error loss) [(10+5)+10]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I EXAMINATION
DIFFERENTIAL EQUATIONS

Date: 21.11.90 Maximum Marks: 100 Time: 3 Hours

Note: This paper carries 115 marks. You may answer all the question. But the maximum you can score is 100.

1.(a) Find the general solution of the first order equation

$$2y(y^2-3x)\frac{dy}{dx}+(3y^2-x) = 0$$

given that it has an integrating factor $\mu(x,y)$ of the form $f(x+y^2)$.

(b) Find the shape of a mirror that reflects parallel to the x - axis, all the rays emanating from the origin (0,0,0).

[10+10=20]

2.(a) Find the general solution of the differential equation

$$xy''-2(x+1)y'+(x+2)y = x^3e^{2x}$$

for $x > 0$, given that the homogeneous equation has a solution of the form $y = e^{mx}$.

(b) By the method of Green's functions, solve the boundary value problem

$$y''+2y'+2y = 3e^{-x}\sin x, \quad y(0) = y(1) = 0.$$ [10+10=20]

3. Show that, for a non-negative integer n, the Bessel's equation

$$x^2y''+xy'+(x^2-n^2)y = 0$$

has only one, except for constant factors, Frobenius series solution $J_n(x)$. Show that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\,d\theta.$$ [20]

4. Show that $x = \infty$ is a regular singular point of the differential equation

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

Find its exponents at $x = \infty$. Find two linearly independent solutio[ns]

of the form $x^{-\gamma}\sum_{k=0}^{\infty} C_k x^{-k}$, for the above equation valid for $|x| > 1$.

[20]

contd. 2/-

5. Find two linearly independent solutions of the homogeneous system

$$\frac{dx_1}{dt} = x_1 + 3x_2$$

$$\frac{dx_2}{dt} = x_1 - x_2.$$

Also find the general solution of the non-homogeneous system

$$\frac{dx_1}{dt} = x_1 + 3x_2 + 4\sin 2t$$

$$\frac{dx_2}{dt} = x_1 - x_2.$$ [8+7=15]

6.(a) Find the stationary curve $y = y(x)$ of

$$\int_0^4 [xy' - y'^2]\, dx$$

which is determined by the boundary conditions

$y(0) = 0$ and $y(4) = 3$.

(b) A curve in the first quadrant joins (0,0) and (1,0) and has a given area A between the curve and x-axis. Show that the shortest such curve is an arc of a circle. [10+10=20]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I EXAMINATION
PHYSICS-I

Date: 19.11.90 Maximum Marks: 100 Time: 3 Hours

Note . 1) Question 1 is compulsory and select any four from the rest.
2) Fiques within square brackets after each question indicate marks.

1. Write Yes/No if the following statements are correct/wrong:

i) For a real gas the assumptions adopted in kinetic theory of gases may be supposed to apply perfectly.

ii) Phenomenon of heat conduction is reversible in thermodynamic sense.

iii) Diffusion represents transport of mass.

iv) In a viscous fluid there is no frictional effects between the layers.

v) Diffusion coefficient cannot be related to the translational Brownian motion of particles.

vi) The change in internal energy of a thermodynamical system is dependent on the process in which the transformation is effected.

vii) In a complete reversible cycle, the work done due to expansion and contraction at constant temperature are equal.

viii) Hamiltonian is always a constant of motion and equal to the total energy of the system.

ix) In Hamilton's principle the line integral $I = \int_{t_1}^{t_2} L\, dt$ has a stationary value for the correct path of motion out of all possible paths in phase space.

x) $2T = I_{xx}W_x^2 + I_{yy}W_y^2 + I_{zz}W_z^2 + 2\, I_{xy}W_xW_y + 2\, I_{yz}W_yW_z + 2\, I_{zx}W_zW_x$

where all I's $\neq 0$ is an expression for rotational kinetic energy of a rigid body where W_x, W_y, W_z are the components of angular velocity along principal axes.

xi) Bohr's theory of hydrogen atom is not in agreement with the Balmer formula for the frequency of radiation given

$$\frac{\nu}{c} = \frac{m\, e^4}{4\pi c\, \hbar^3}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad n, m = 1, 2 \ldots\ldots m > n.$$

and it agrees after Sommerfeld's modification.

xii) The hydrogen atom is stable only as electrostatic attractive force is balanced by the centrifugal force. [12 x $1\frac{1}{2}$=18]

contd. 2/-

2. Stating clearly the vander - Waals' corrections to represent the bahaviour of real gas, deduce the van der Waals equation in the form:

$$(p + \frac{a}{v^2})(v-b) = nRT$$

where the notations have their usual meanings. Also show that the co-volume 'b' is equal to four times the total molecular volume. [20]

3. Taking diffusion into consideration of Brownian particles in their translatory motion show that the mean square displacement depend upon the time τ, the temperature T, the viscosity η and the radius 'r' of the Brownian particle. [20]

Write short notes on reversible and irreversible processes and show that in an isothermal reversible cyclic process the net work would be zero. Show from the first law of thermodynamics that in an adiabatic change of an ideal gas PV^{γ} = const. where P and V are the pressure and volume and $\gamma = \frac{C_p}{C_v}$ = ratio of heat capaciti[es] at constant pressure and at constant volume. [4+4+4+8] = [

(i) What is Hamilton - Jacobi equation? Show how can you obtain a solution to the mechanical problem by the help of Hamilton's principal function as the generator of a canonical transformation.

(ii) Find out the solution of a one dimensional harmonic oscillator by Hamilton-Jacobi technique.

6.(i) Define Poisson Bracket of two functions with respect to the canonical variables. Show that the Poisson Bracket of the conjugate variable $[q_j, p_k]_{q,p} = \delta_{jk}$ = 0 if $j \neq k$, 1 if $j = k$

Prove the identity

$$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$$

where square brackets are Poisson brackets. From it show that if X and Y are both constant of motion the Poisson brack[et] of two constants of motion is itself a constant of motion.

[3+5·

7. Stating Bohr postulates in his theory of hydrogen atom deduce the expression for the quantized energy levels of the hydrogen atom. Justify Sommerfeld's modification and explain how he extended the Bohr's idea. [10+4+6]

Neatness [2]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) III YEAR: 1990-91
SEMESTRAL-I EXAMINATION
ECONOMICS-III

Date: 19.11.90 Maximum Marks: 100 Time: 3 Hours

Note: Q.No.1 is Home Assignment (compulsory). Answer ANY FOUR questions out of Question's 2 to 7. Marks allotted to each question are given in brackets.

1. Home Assignment (to be submitted by 28.11.90). [Topic: a critical evaluation of various methods of credit control used by the RBI] [12]

2.(a) What do you mean by the following terms: absolute level of living, incidence of poverty and relative level of living.

(b) Make a review of studies made in India on absolute level of living in rural India. [6+16=22]

3. Write a note on alternative explanations which have been advanced in the literature for the so-called industrial stagnation in India. [22]

4.(a) Discuss briefly the Minhas vaidyanathan decomposition scheme to determine the components of growth of crop output in India for the period 1951-54 to 1958-61.

(b) Comment on some of the important results of the study in respect of the regional variation in the contribution of different components in the growth of crop output of India. [16+6=22]

5. Elaborate the logic of Mathur-Ezekeil hypothesis to explain the inverse relationship between prices and marketable surplus of food grains (in India) Can you refer to any supplementary study on the above and asses the role of the same? [16+6=22]

6. Discuss any suitable framework to characterise the pattern of inter year changes in the balance of trade of India. [10+12=22]

Examine and interpret the table on Inter year change in Trade balances for India given below and suggest possible steps towards overcoming the persistent balance of payment deficits of India.

contd. 2/-

Inter-year changes in Balance of Trade components
in Million dollars

Year	Trade Balance	Quantity component	Price component	Terms of Trade effect	General Price level effect
1963-64	-103.0	-00.2	-102.8	-32.6	-20.2
1964-65	-103.0	-313.9	46.9	81.0	-34.1
1965-66	267.0	548.9	-543.9	-428.2	-120.5
1966-67	00.0	145.3	-115.3	-142.7	27.4
1967-68	446.0	391.6	54.4	17.8	36.6
1968-69	371.0	331.3	39.7	63.1	-53.4
1969-70	278.0	577.1	0.9	-0.1	1.0

7. Write short notes on any two of the following:
   (a) Block angularity in the structure of Indian economy.
   (b) Economic backwardness as a quasistable equilibrium system.
   (c) Self-sustaining characteristics of the village economy in India in the pre-British era. [11+11=22]

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